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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-5: Analog Circuits + Materials Science
Electronic Devices & Circuits-1 + Advanced Electronics Topics-1
Analog and Digital Communication Systems-2

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Roll No : E C 1 9 M B D L A 3 1 1

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

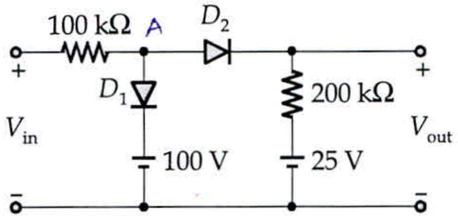
Question No.	Marks Obtained
Section-A	
Q.1	46
Q.2	46
Q.3	—
Q.4	49
Section-B	
Q.5	30
Q.6	—
Q.7	45
Q.8	—
Total Marks Obtained	216

Signature of Evaluator

Cross Checked by

Section A : Analog Circuits + Materials Science

1 (a) Consider the circuit shown in the figure below:



By assuming that the diodes are ideal, develop the transfer characteristic curve of the above circuit.

[12 marks]

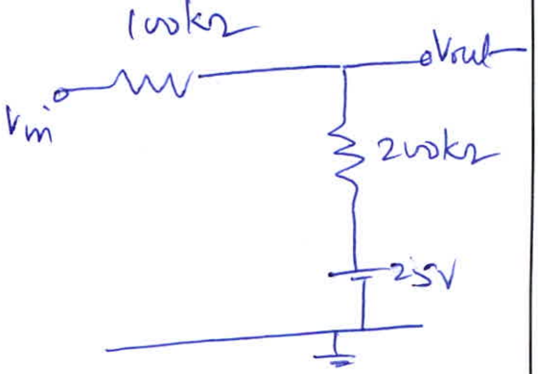
For $V_{in} < 25\,V$

Both the diodes are off & $V_{out} = 25\,V$

For $25 < V_{in} < 100\,V$

1

D_2 ON, D_1 OFF



$$V_{out} = 25 + 200 \times \frac{V_{in} - 25}{300}$$

$$V_{out} = \frac{25}{3} + \frac{2}{3} V_{in} \rightarrow ①$$

For $V_A \geq 100\,V \Rightarrow V_{in} \geq 137.5$

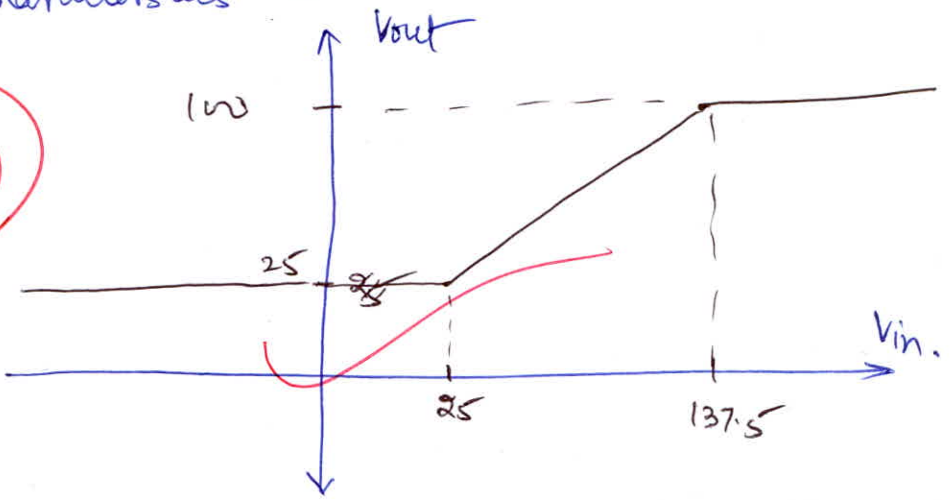
Both the diodes are ON.

$V_A = 100 \Rightarrow V_{in} = \frac{300 - 25}{2} = 137.5\,V$

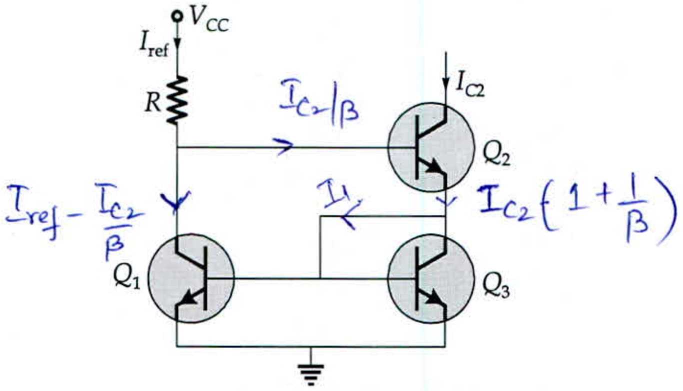
$V_{out} = 100\,V. \rightarrow ②$

From the above equations, we can draw the transfer characteristics as shown

10



Q.1 (b) Consider the Wilson current mirror circuit as shown in the figure below:



Assume that the three transistors to be matched with $V_{BE1} = V_{BE3}$ and $\beta_1 = \beta_2 = \beta_3 = \beta$. Derive an expression for I_{C2} in terms of I_{ref}.

[12 marks]

$$I_{B2} = \frac{I_{C2}}{\beta} \rightarrow ①$$

$$I_{C1} = I_{ref} - I_{B2} = I_{ref} - \frac{I_{C2}}{\beta} \rightarrow ②$$

By KCL,

$$I_{E2} = I_{C2}\left(1+\frac{1}{\beta}\right) \rightarrow ③$$

②

As Q_1 & Q_3 are matched,

$$I_{C3} = I_{C1} = I_{ref} - \frac{I_{C2}}{\beta} \rightarrow ④$$

By KCL,

$$I_1 = I_{C2}\left(1+\frac{1}{\beta}\right) - I_{ref} + \frac{I_{C2}}{\beta} \rightarrow ⑤$$

Also,

$$I_1 = 2I_B$$

or $I_B = \dfrac{I_1}{2}$. $\rightarrow$ ⑥

①

Also,

$$I_{C1} = \beta I_{B1} \rightarrow ⑦$$

substitute eqⁿ ④ and ⑥ in eqⁿ ⑦

$$\Rightarrow I_{ref} - \frac{I_{C2}}{\beta} = \frac{\beta}{2}\left(I_{C2}\left(1+\frac{2}{\beta}\right) - I_{ref}\right)$$

⑧

$$\Rightarrow I_{ref} - \frac{I_{C2}}{\beta} = \frac{I_{C2}\beta}{2} + I_{C2} - \frac{I_{ref}\beta}{2}$$

$$\Rightarrow I_{ref}\left(1+\frac{\beta}{2}\right) = I_{C2}\left(1+\frac{\beta}{2}+\frac{1}{\beta}\right)$$

$$\Rightarrow \boxed{I_{C2} = I_{ref}\left[\frac{1+\frac{\beta}{2}}{1+\frac{\beta}{2}+\frac{1}{\beta}}\right]}$$ Answer.

Q.1 (c) A long narrow rod (having cubic structure) has an atomic density of 5×10^{28} atoms/m^3. Each atom has a polarizability of 10^{-40} F-m^2. Calculate the internal electric field in the rod when an external axial field of 1 V/m is applied.

[12 marks]

Given Information –

~~N = 5~~ $\alpha = 10^{-40}$ F-m^2.

$E_{ext} = 1$ V/m

cubic structure $\Rightarrow \gamma = \frac{1}{3}$

1

$N = 5 \times 10^{28}$ atoms/m^3.

Internal field E_i is given by the equation –

$$E_i = E + \frac{\gamma P}{\epsilon_0} \rightarrow ①$$

where P is the polarization

and,

$$P = N\alpha E_i \rightarrow ②$$

Substitute ② in ① to get.

$$E_i\left(1 - \frac{N\alpha}{3\epsilon_0}\right) = E.$$

$$E_i = \frac{E}{1 - \frac{N\alpha}{3\epsilon_0}}.$$

10

$$= \frac{1}{1 - \frac{5 \times 10^{28} \times 10^{-40}}{3 \times 8.85 \times 10^{-12}}}$$

$$E_i = 1.23 \text{ V/m}$$

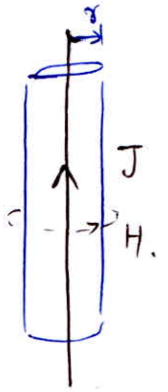
1 (d) Explain Silsbee's rule for superconductors. Also give some applications of superconductors.

[12 marks]

Silsbee's rule states that when a current I_c is passed thru a superconducting material, a field developes around it. If the superconductor loses its superconductivity, then such field is called the critical field and such current is called critical current.

consider a superconducting rod carrying a current density J Amps/m²

2



the field developed around it = H A/m

By amperes circutal law,

$$H = \frac{I}{2\pi r} \rightarrow ①$$

If $H = H_c$, then

$$H_c = \frac{I_c}{2\pi r} = \frac{J_c \times \cancel{\pi} r^2}{2\cancel{\pi} r}$$

4

$$\Rightarrow \boxed{H_c = \frac{J_c r}{2}}$$

Superconductors are widely used in various applications around the world. Some of them are listed below —

(i.) Magnetic leviation transportation

These are used in Maglev trains where the extremely high speed transportation is possible by avoiding physical contacts

(ii.) Magnetic Resonance Imaging.

super conductors have their uses in medical field also eg MRI.

2

.1 (e) Write short notes on the following nanomaterials:

(i) Quantum dots

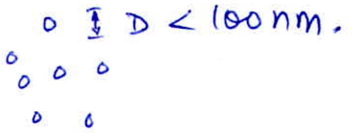
(ii) Carbon nanotubes

[6 + 6 marks]

(i.) Quantum Dots

These are 1D nanoparticles which have all the three dimensions in nano range i.e, less than 100nm. Quantum dots are used in ~~ma~~ textile industry for production of fire resistant etc suits.

2

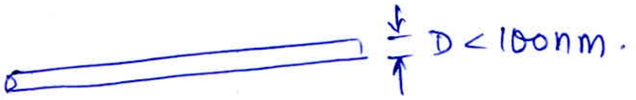


(ii.) Carbon Nanotubes

→ These are 2D Nanoparticles.

→ Two dimensions are in the nano range.

→ One dimension is larger than nano range



3

→ They are manufactured using top down approach

→ They find use in medical industry.

→ Carbon nanotubes are used to cure interartery diseases and various cardiac ailments.

Do nc
write i
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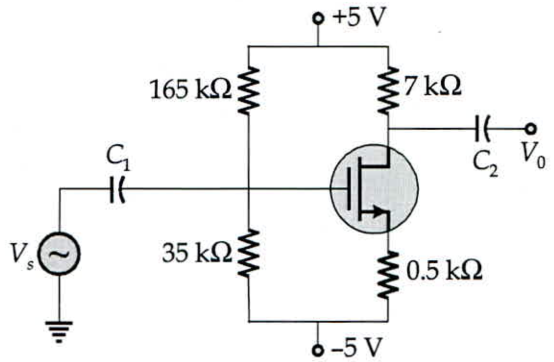
2 (a) (i) Consider the common source transistor circuit shown in the figure below:



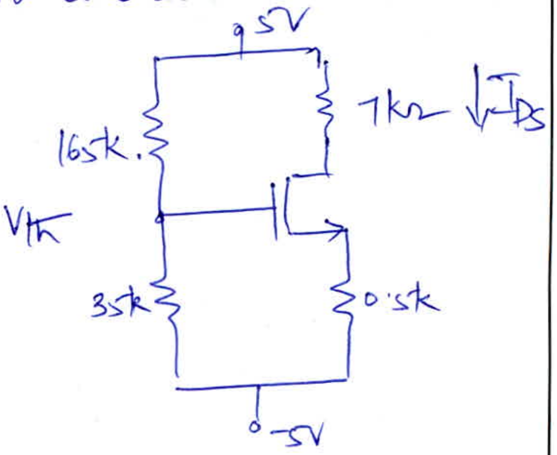
The transistor parameters are $V_{TN} = 0.8$ V, $K_n = \dfrac{\mu_n C_{ox} W}{2L} = 1\,\text{mA/V}^2$ and $\lambda = 0$.

Calculate the value of small signal voltage gain V_0/V_s of the circuit.

(ii) A differential amplifier has input voltages $V_1 = 1$ mV and $V_2 = 3$ mV. The amplifier has differential gain $A_d = 5 \times 10^3$ and CMRR = 1000. Calculate the output voltage of the amplifier.

[15 + 5 marks]

(i.) Consider the DC equivalent circuit :—



By KVL

~~$5 - (7.5) I_{DS}$~~

$$V_{th} = \frac{35}{165+35} \times 10 - 5$$

$V_{th} = -3.25$ V. → ①

By KVL

$V_{th} - V_{GS} - I_{DS}(0.5) = -5.$

$+3.25 + V_{GS} + \frac{I_{DS}}{2} = +5$

$V_{GS} + \frac{I_{DS}}{2} = 1.75$ → ②

Also,

$$I_{DS} = \frac{1}{2} \frac{\mu_n C_{ox} W}{L} \cdot (V_{GS} - 0.8)^2 = (V_{GS} - 0.8)^2 \rightarrow ③$$

From ② and ③

$2V_{GS} + V_{GS}^2 + 0.64 - 1.6V_{GS} = 3.5$

or $V_{GS}^2 + 0.4 V_{GS} = 2.86$

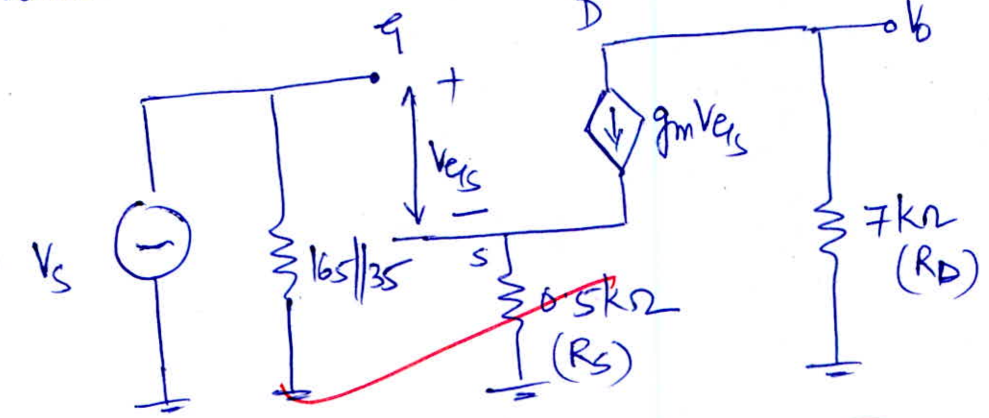
$$\boxed{V_{GS} = 1.5V}$$

$$\boxed{I_{DS} = 0.49\ mA}$$

$$g_m = \frac{dI_{DS}}{dV_{GS}} = 2k_n \cdot (V_{GS} - V_T) = 2 \times 1 \times 0.7$$

$$= 1.4\ mA/V \rightarrow ③$$

draw the small signal equivalent circuit



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By KVL,

$$V_S = V_{GS} + g_m V_{GS} R_S$$

$$V_S = V_{GS}(1 + g_m R_S) \rightarrow ④$$

By KVL

$$V_0 = -g_m V_{GS} R_D \rightarrow ⑤$$

divide eqⁿ ⑤ by ④

$$A_V = \frac{V_0}{V_S} = \frac{-g_m R_D}{1 + g_m R_S}$$

$$A_V = \frac{-1.4 \times 7}{1 + 1.4 \times 0.5} = \underline{-5.76}$$

(ii.) $V_1 = 1 mV$

$V_2 = 3 mV$

differential input $= V_d = V_2 - V_1 = 2 mV.$ → ①

Common mode input $= V_c = \frac{V_1 + V_2}{2} = 2 mV.$ → ②

$A_d = 5 \times 10^3$

$CMRR = \frac{A_D}{A_c} = 1000$

2

$\Rightarrow A_c = 5.$ → ③

Output voltage is given by

$|V_0| = A_d V_d + A_c V_c$

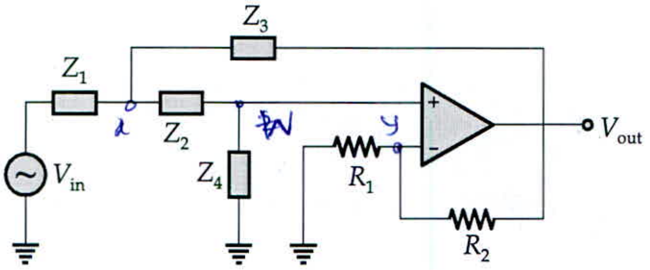
$= 5 \times 10^3 \times 2 + 5 \times 2$

$= 10 \times 10^3 + 10$

≈ 10010 mvolts.

$=$ 10.01 volts.

Q.2 (b) Consider the circuit shown in the figure below:



The figure represents a second order active filter system.

(i) Derive an expression for V_{out}/V_{in}.

(ii) If each of the impedance elements Z_1 through Z_4 are replaced by a resistor of value R, then find the value of V_{out}/V_{in}.

[20 marks]

Apply Nodal at x.

$$\frac{x - V_{in}}{Z_1} + \frac{x - V_{out}}{Z_3} + \frac{x - W}{Z_2} = 0 \quad \rightarrow ①$$

As current through op amp is zero,

$$W = x \frac{Z_4}{Z_2 + Z_4} \quad \text{(By voltage division)} \quad \rightarrow ②$$

3

Also,

$$y = \frac{V_{out} R_1}{R_1 + R_2} \quad \rightarrow ③$$

As the op amp is having a -ve FB, we can apply virtual short.

$$W = y \Rightarrow x \frac{Z_4}{Z_2 + Z_4} = \frac{V_{out} R_1}{R_1 + R_2}$$

$$\text{Or } x = V_{out} \left(\frac{R_1}{R_1 + R_2}\right) \underbrace{\left(\frac{Z_2 + Z_4}{Z_4}\right)}_{\rightarrow z'} \quad \rightarrow ④$$

$$\Rightarrow W = V_{out} \underbrace{\left(\frac{R_1}{R_1 + R_2}\right)}_{\rightarrow R'} \quad \rightarrow ⑤$$

put ④ & ⑤ in ①

$$\frac{V_{out}\, z'}{z_1} - \frac{V_{in}}{z_1} + \frac{V_{out}(z'-1)}{z_3} + \frac{V_{out}(z'-R')}{z_2} = 0$$

$$V_{out}\left[\frac{z'}{z_1} + \frac{z'-1}{z_3} + \frac{z'-R'}{z_2}\right] = \frac{V_{in}}{z_1}$$

Or $$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{z_1\left(\frac{z'}{z_1} + \frac{z'-1}{z_3} + \frac{z'-R'}{z_2}\right)}}$$

3

where $z' = \frac{R_1}{R_1+R_2} \cdot \frac{z_2+z_4}{z_2}$ and $R' = \frac{R_1}{R_1+R_2}$

3

(ii) $z_1 = z_2 = z_3 = z_4 = R$.

$\Rightarrow z' = \frac{2R_1}{R_1+R_2}$

$$\frac{V_{out}}{V_{in}} = \frac{1}{R\left(\frac{2R_1}{R_1+R_2} + \frac{2R_1}{R_1+R_2} - 1 + \frac{2R_1}{R_1+R_2} - \frac{R_1}{R_1+R_2}\right)}$$

5

$$= \frac{1}{\frac{5R_1}{R_1+R_2} - 1}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{4R_1-R_2}}$$

Q.2 (c) **(i)** For a dielectric, establish an expression for the relationship between the polarizability and permittivity. How does this relation lead to Clausius-Mossotti equation?

(ii) When an NaCl crystal is subjected to an electric field of 1000 V/m, the resulting polarization is 4.3×10^{-8} C/m^2. Calculate the relative permittivity of NaCl.

[15 + 5 marks]

For a dielectric,

$$P = N\alpha E_i \quad \rightarrow ①$$

where E_i = Internal field.

$$E_i = E + \frac{\gamma P}{\epsilon_0} \quad \rightarrow ②$$

Also,

$$P = \epsilon_0(\epsilon_r - 1)E \quad \rightarrow ③$$

From ① & ②

$$P = N\alpha\left(E + \frac{\gamma P}{\epsilon_0}\right)$$

$$P = N\alpha E + \frac{\gamma N\alpha P}{\epsilon_0}$$

$$\Rightarrow P\left[1-\frac{\gamma N\alpha}{\epsilon_0}\right] = N\alpha E$$

$$\text{or } P = \frac{N\alpha E}{1-\frac{\gamma N\alpha}{\epsilon_0}} \rightarrow ④$$

put ④ in ③

$$\frac{N\alpha \cancel{E}}{1-\frac{\gamma N\alpha}{\epsilon_0}} = \epsilon_0(\epsilon_r - 1)\cancel{E}$$

$$\boxed{\epsilon_r - 1 = \frac{\frac{N\alpha}{\epsilon_0}}{1-\frac{N\alpha}{3\epsilon_0}}} \rightarrow ⑤$$

$\left[\text{For cubic crystals, } \gamma = \frac{1}{3}\right]$

13

Add '3' on both LHs & RHs

$$\epsilon_r + 2 = \frac{\frac{N\alpha}{\epsilon_0}}{1-\frac{N\alpha}{3\epsilon_0}} + 3 = \frac{3}{1-\frac{N\alpha}{3\epsilon_0}} \rightarrow ⑥$$

divide eq^n ⑤ & eq^n ⑥

$$\boxed{\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}}$$

Claussius Massotti Eq^n

Numerical

$E = 1000$ V/m

$P = 4.3 \times 10^{-8}$

From ③ , $P = \epsilon_0(\epsilon_r - 1)E$

1

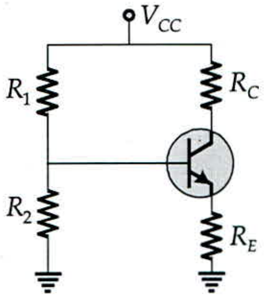
$$4.3\times10^{-8} = \epsilon_0(\epsilon_r - 1)\times 1000$$

$$\epsilon_r - 1 = \frac{4.3\times10^{-8}}{1000\times 8.85\times10^{-12}} = 4.858$$

$\Rightarrow \epsilon_r = 4.858 - 1$

$\Rightarrow \boxed{\epsilon_r = 3.858}$ (Answer)

.3 (a) Consider the voltage divider biasing circuit shown in the figure below:



For this circuit,

(i) Derive an expression for stability factor S [i.e., the variation of I_C w.r.t. I_{CO}].

(ii) Derive an expression for stability factor S' [i.e., the variation of I_C w.r.t. V_{BE}].

(iii) Derive a relation between S and S'.

[20 marks]

Q.3 (b) What are the types of cubic crystal structure? Derive the atomic packing factor of all the cubic crystal structures.

[20 marks]

Q.3 (c) Electron drift mobility in indium (In) has been measured to be 6 $\text{cm}^2\,\text{V}^{-1}\text{s}^{-1}$. At room temperature (27°C), the resistivity of In is $8.37 \times 10^{-8}\ \Omega\,\text{m}$ and its atomic mass and density are $114.82\ \text{g mol}^{-1}$ and $7.31\ \text{g cm}^{-3}$ respectively.

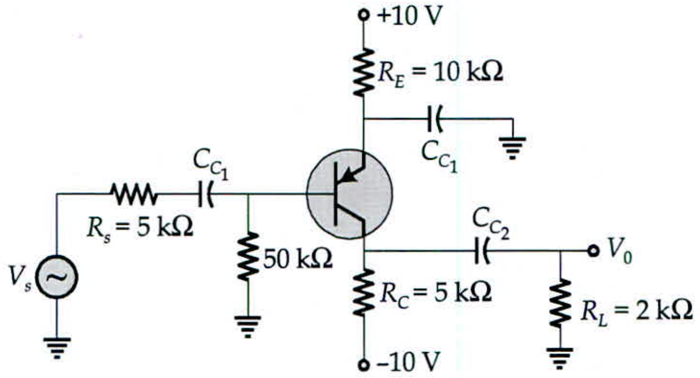
(i) Based on the resistivity value, determine the effective number of free electrons donated by each In atom in the crystal.

(ii) If the mean speed of conduction electrons in In is $1.74 \times 10^{8}\ \text{cm s}^{-1}$, what is the mean free path?

(iii) Calculate the thermal conductivity of In at room temperature.

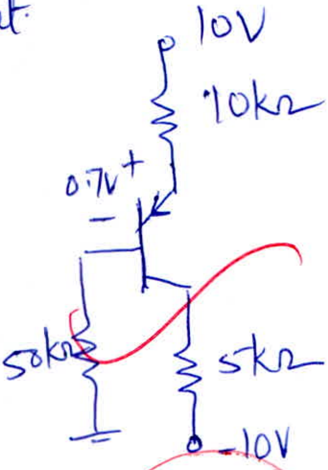
[20 marks]

Q.4 (a) Consider a *p-n-p* transistor shown in the figure below. The transistor has $V_{EB(\text{on})} = 0.7$ V, $\beta = 150$ and $V_A = \infty$. Draw a neat and labelled graph for DC and AC load line. Mark the *Q*-point on the graph.



[20 marks]

Consider DC Equivalent circuit



By KVL

$10 - 10 I_E - 0.7 - 50 I_B = 0$

$I_E = (\beta+1) I_B = 151 I_B$

$\Rightarrow 10 - 0.7 = (1510 + 50) I_B$

or $I_B = 5.96\ \mu A$.

$I_C = \beta I_B = 0.894$ mA.

~~V_{EE}~~ By KVL

$10 - I_E \times 10 - V_{EC} - 5 I_C = -10$

$V_{EC} \approx 20$ ~~A~~ $- 15 I_C = 6.59$ volts

$V_{CC} - I_C(R_C + R_E) - V_{EC} = -V_{EE}$

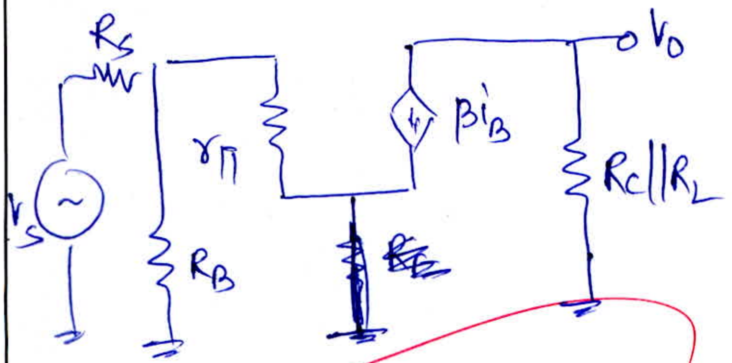
or $I_C = \dfrac{V_{CC} + V_{EE}}{R_C + R_E} + \left[\dfrac{-\cancel{V_{EE}}\,1}{R_C + R_E}\right] V_{EC}$

$\uparrow$ y intercept. $\Downarrow$ 1.33 mA

$\uparrow$ slope $= -0.06 \times 10^{-3}$

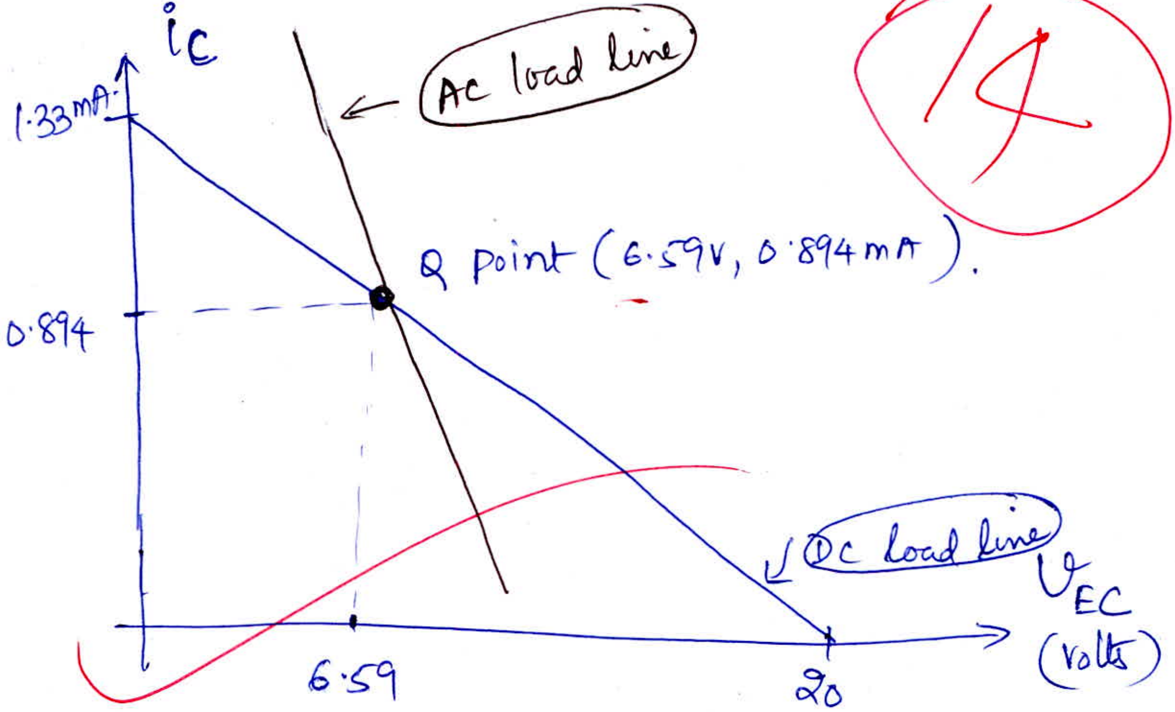
2

Draw AC Equivalent.



$$-i_c \; R_C \| R_L = V_c$$

or $i_c = -\dfrac{V_0}{R_C \| R_L} = \dfrac{-V_c}{1.428}$ → ② slope $= -0.7$ ✓



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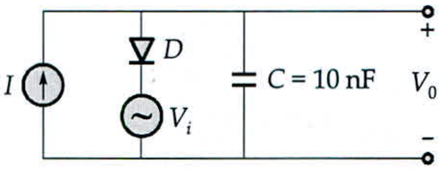
Q.4 (b) Consider the circuit shown in the figure below:



I is DC current and V_i is a sinusoidal signal with small amplitude and frequency of 100 kHz. Thus for small signal input and output voltages V_i and V_0, calculate:

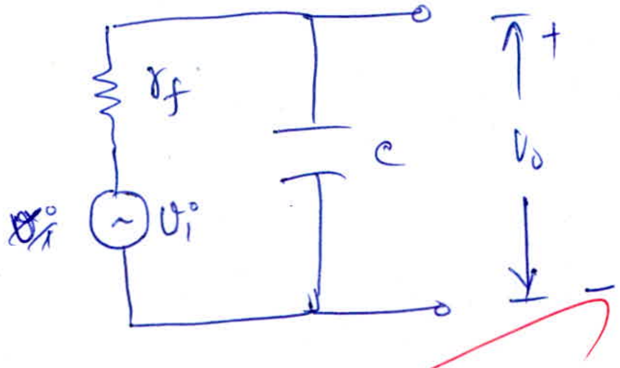
(i) Phase angle difference between V_i and V_0.

(ii) The value of DC current I for which the phase shift between V_i and V_0 is –45°. (Assume V_T = 25 mV)

(iii) The range of phase shift that is achieved as I is varied over the range of 0.1 to 10 times of the value obtained in part (ii).

[20 marks]

(i.) drawing Small signal ac Equivalent



Where $r_f = \cancel{\frac{I}{V_T}}\ \frac{V_T}{I}$

$$\frac{V_0}{v_i} = \frac{-jX_C}{r_f - jX_C} = \cancel{\frac{v_i}{1+j\omega r_f C}}\ \frac{\frac{1}{j\omega C}}{r_f + \frac{1}{j\omega C}}$$

$$V_0 = \frac{V_i^\circ}{1+j\omega r_f C}$$

$$\phi_{V_0} = \phi_{V_i^\circ} - \tan^{-1}(\omega r_f C)$$

$$\Delta\phi = -\tan^{-1}(\omega r_f C) = -\tan^{-1}(2\pi \times 100 \times 10^{3} \times 10^{-8} \times r_f)$$

$$\Delta\phi = -\tan^{-1}(6.283 \times 10^{-3} \times r_f)$$

(ii.) For $\Delta\phi = -45$.

$$\tan^{-1}(6.283 \times 10^{-3} \times r_f) = 45^\circ$$

$$6.283 \times 10^{-3} \times r_f = 1.$$

$$r_f = 159.15\,\Omega.$$

$$\Rightarrow I_d = \frac{V_T}{r_f} = \frac{0.025}{159.15} = 0.157 \text{ mA}.$$

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(iii.) When $I = 0.0157$ mA.

$$r_f = \frac{V_T}{I} = 1592\,\Omega$$

$$\Delta\phi = -\tan^{-1}(6.283 \times 10^{-3} \times 1592) = -84.29^\circ$$

When $I = 1.57$ mA

$$r_f = 1.59\,\Omega$$

$$\Delta\phi = -\tan^{-1}(6.283 \times 10^{-3} \times 1.592) = -5.71^\circ$$

When I is varied from 0.0157 mA to 1.57 mA

$\Delta\phi$ varies from -84.29° to -5.71°.

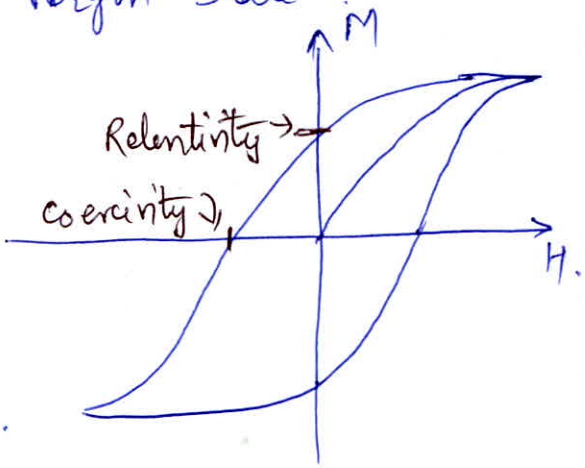
Q.4 (c) (i) What do you understand by magnetic hysteresis? Differentiate between hard and soft magnetic materials?

(ii) In a magnetic material, the field strength is found to be 10^6 A/m. If the magnetic susceptibility of the material is 0.5×10^{-5}, calculate the intensity of the magnetization and the magnetic flux density in the material.

[12 + 8 marks]

Magnetic Hystersis

It refers to the property of a ferromagnetic material to repeat its ~~hys~~ history of magnetisation when the excitation is varied. However, the material never goes back into its virgin state



The curve representing the M v/s H characterstic is called. Hysterysis loop / curve.

Hard Magnetic Material	Soft Magnetic Material
① Hard to magnetize & demagnetize	① Easy to magnetize and demagnetize
② ~~Used to~~ Have large value of Retentivity & Coercivity	② Small values of retentivity & coercivity.
③ Wide / Broad M-H curve	③ Narrow M-H curve.
④ ~~loss~~ Large Hystersis loss	④ Less Hystersis loss
⑤ Small permeability	⑤ Large permeability
⑥ Used to manufacture permanent magnets	⑥ Used to manufacture Electromagnets
Eg - AlNiCo, CuNiFe	Eg - Fe-Si, ~~Cs~~ soft steel

9

(ii.) Given,

$X_m = 0.5 \times 10^{-5}$

$H = 10^6$ A/m.

$M = X_m H$

$= 0.5 \times 10^{-5} \times 10^{6}$

$= 0.5 \times 10 = 5$ A/m.

Magnetic flux density

$B = \mu_0 (H + M)$

$= 4\pi \times 10^{-7} (5 + 10^6)$

$\Rightarrow$ $B = 1.256$ Wb/m^2

7

Section B : Electronic Devices & Circuits-1 + Advanced Electronics Topics-1 + Analog and Digital Communication Systems-2

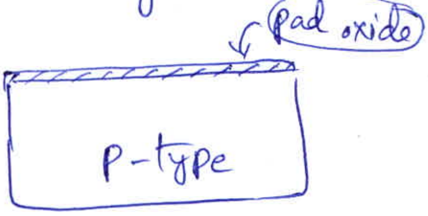
5 (a) With neat diagrams, explain the Local Oxidation of Silicon (LOCOS) isolation technique used in IC fabrication.

[12 marks]

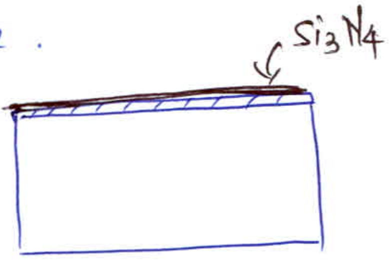
Local oxidation of silicon is used to develope SiO_2 for oxide isolation technique. The steps involved are —

① Take a p type substrate formed using cz method.

② develop pad oxide.



③ Apply Si_3N_4 liquid over pad oxide.



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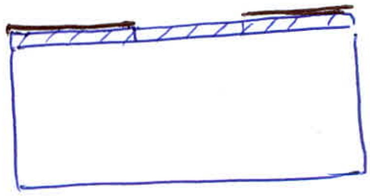
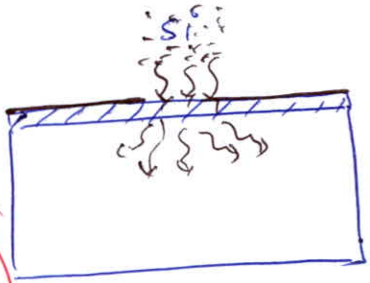
④ open a window in Si_3N_4 using photolithography



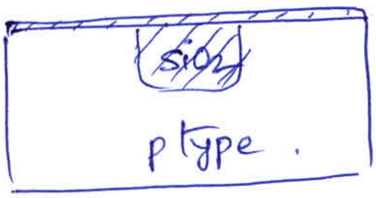
⑤ Thermal oxidation at 1000°C.

$$Si + 2H_2O \xrightarrow{1000°C} SiO_2 + 2H_2$$



5

⑥ Si_3N_4 stripping



Q.5 (b) **(i)** The oxide removal rate and the removal rate of a layer underneath the oxide (called a stop layer) are *r* and 0.1*r* respectively. To remove 1 μm of oxide and a 0.01 μm stop layer, the total removal time is 5.5 minutes. Find the oxide removal rate (*r*).

(ii) Calculate the Al average etch rate and etch rate uniformity on a 200 mm diameter silicon wafer, assuming the etch rates at the center, left, right, top and bottom of the wafer are 750, 812, 765, 743 and 798 nm/min respectively.

[6 + 6 marks]

(i.) $$\frac{1}{r} + \frac{0.01}{0.1r} = 5.5$$

Where r is in μm/min }

$$\Rightarrow \frac{1}{r} + \frac{0.1}{r} = 5.5$$

$$\frac{1.1}{r} = 5.5$$

5

or $$\boxed{r = \frac{1}{5} = 0.2\ \mu m/\text{min}.}$$

(ii) Average etch rate $= \dfrac{750+812+765+743+798}{5}$

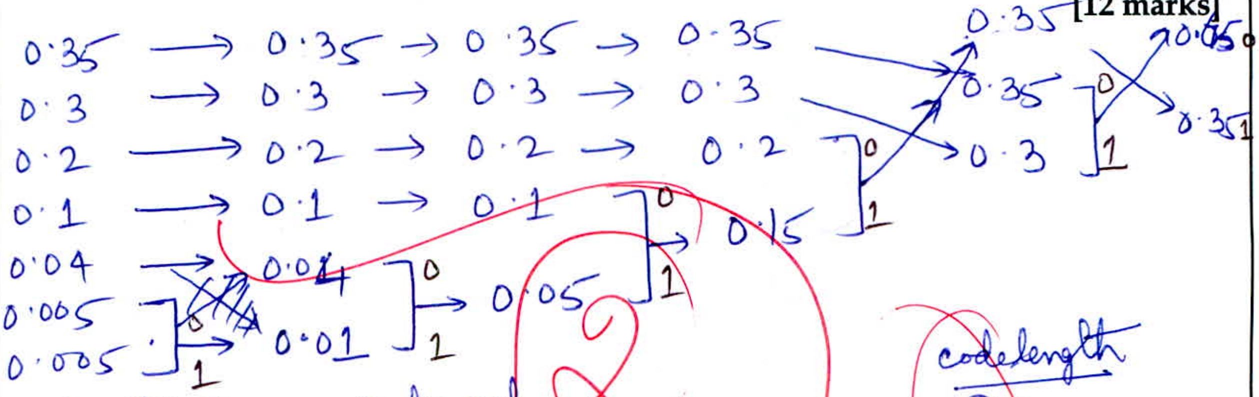
$= 773.6$ nm/min

②

Also find Etch rate uniformly

Q.5 (c) A source emits seven symbols with probabilities 0.35, 0.3, 0.2, 0.1, 0.04, 0.005, 0.005. Give Huffman coding for these symbols and calculate average bits of information and average binary digits of information per symbol.

[12 marks]



2

probability		codeword	codelength
0·35	→	1	①
0·3	→	0 1	②
0·2	→	1 0	②
0·1	→	1 1 0	③
0·04	→	1 1 1 0	④
0·005	→	1 1 1 1 0	⑤
0·005	→	1 1 1 1 1.	⑤

1

Average bits of Information

= Average code length = ~~1×0·35~~ $\sum l_i p_i$

= $1 \times 0.35 + 0.3 \times 2 + 0.2 \times 2 + 0.1 \times 3 + 0.04 \times 4 + 0.005 \times 5 \times 2$

= 1·86 bits

Average binary digits of Information per symbol

= Entropy

= $\sum p_i \log \frac{1}{p_i}$

= 2·109 bits/symbol.

2

.5 (d) The distribution (with respect to energy) of electron concentration in the conduction band is given by density of allowed quantum states times the probability that state being occupied by an electron. i.e., $n(E) = g_C(E)\,f(E)$

where, $g_C(E)$ = Density of allowed states, $f(E)$ = probability of state being occupied. Assuming that Boltzmann approximation in a semiconductor is valid, calculate the ratio of $n(E)$ at $E = E_C + 4kT$ to that at $E = E_C + (kT/2)$. Here, k = Boltzmann constant, E_C = edge of the conduction band and T = temperature in °K.

[12 marks]

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$f(E_c+4kT) = \frac{1}{1+e^{(E_c-E_F)/kT}\cdot e^{4}} \rightarrow ①$$

$$f(E_c+kT/2) = \frac{1}{1+e^{(E_c-E_F)/kT}\,e^{1/2}} \rightarrow ②$$

from ①

$$f(E_c+4kT) = \frac{1}{1+\frac{n_1}{N_c}e^{4}} \qquad \left[\text{As } n = N_c\, e^{(E_c-E_F)/kT}\right]$$

$$\approx \frac{N_c}{n_1 e^{4}}$$

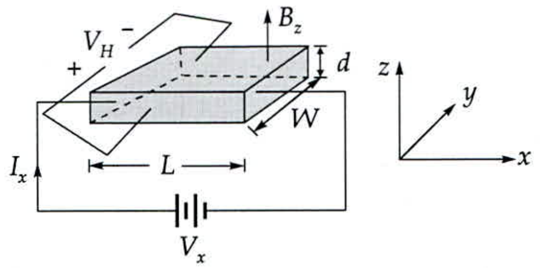
From ②

$$f\left[E_c + \frac{kT}{2}\right] = \frac{1}{1 + \frac{n_2}{N_c} e^{1/2}} \approx \frac{N_c}{n_2 e^{1/2}}$$

$$\frac{n(E_c+4kT)}{n\left(E_c+\frac{kT}{2}\right)} \times \frac{f[E_c+4kT]}{f\left[E_c+\frac{kT}{2}\right]} = \frac{e^{1/2}}{e^4} \times \frac{\cancel{N_c}}{\cancel{N_c}}$$

$$= e^{-7/4}.$$

(e) Consider a silicon Hall effect device which is used for the experiment as shown below:



The device has dimensions $d = 5 \times 10^{-3}$ cm, $W = 5 \times 10^{-2}$ cm and $L = 0.5$ cm. The electrical parameters measured as the result of the experiment are $I_x = 0.5$ mA, $V_x = 1.25$ V and $B_z = 6.5 \times 10^{-2}$ T. If the induced Hall electric field is $E_{Hy} = -16.5$ mV/cm, then determine:

(i) Hall voltage (V_H)

(ii) The type of semiconductor

(iii) The majority carrier concentration

[12 marks]

(i.) Hall voltage $= V_H$.

$$V_H = -\int E \cdot dl$$

$$\Rightarrow V_H = -\int(-16.5\ \text{mV/cm})(-dy)$$

$$V_H = -16.5\ \text{mV/cm} \times \int dy = -16.5\ \text{mV/cm} \times W$$

$$V_H = -16.5\ \text{mV/cm} \times 5 \times 10^{-2}\ \text{cm} = -0.825\ \text{mV}$$

(ii.) As the voltage has −ve polarity, the semiconductor is N type.

⑨

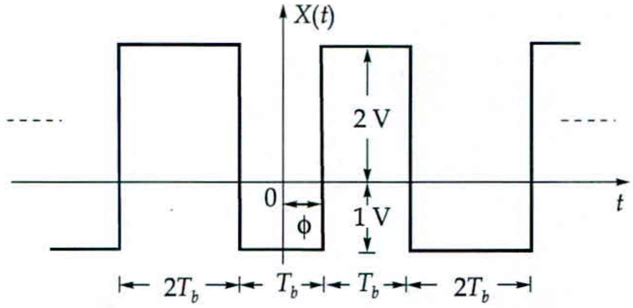
(iii.)
$$V_H = \frac{BI}{\rho W}$$

$$\rho = \frac{BI}{V_H W} = \frac{6.5 \times \cancel{10}^{-2} \times 0.5 \times \cancel{10}^{-3}}{0.825\ \cancel{\text{mV}} \times \cancel{10}^{-3} \times 5 \times \cancel{10}^{-2} \times 10^{-2}}$$

$$\rho = 78.78\ \text{C/m}^3$$

$$\Rightarrow N_D = \frac{\rho}{e} = 4.924 \times 10^{20}\ /\text{m}^3.$$

Q.6 (a) Consider the random binary wave shown below:



In this binary wave, logic-1 is represented with positive rectangular pulse and logic-0 is represented with negative rectangular pulse, both with different amplitudes. ϕ is an independent random variable uniformly distributed in the range $[0, T_b]$, where T_b is the bit duration. Determine and sketch the auto-correlation function of $X(t)$. Assume that logic-1 and logic-0 are occurring with equal probability.

[20 marks]

Q.6 (b) A 1 cm long bar of n-type Ge has a cross section of 1 mm × 1 mm. The resistivity of material is 20 Ω-cm and the lifetime of the carriers is 100 microseconds.

(Assume μ_n = 3800 cm^2/V-s, μ_p = 1800 cm^2/V-s and intrinsic carrier concentration $n_i = 2.5 \times 10^{13}/\text{cm}^3$).

(i) Calculate the resistance of the bar.

(ii) Calculate the donor concentration.

(iii) Calculate the resistance of the bar when it is illuminated such that excess electron-hole pairs are generated at a rate of 10^{15} cm^{-3} s^{-1}, uniformly all over the bar.

[20 marks]

(c) **(i)** Binary data (equiprobable bits) with a rate of 1 Mbps is transmitted through an AWGN channel using different modulation schemes. The two sided power spectral density of the channel noise is 0.5×10^{-11} W/Hz and the carrier signal used in the transmitters is $5\cos(2\pi f_c t)$ mV. In each case of different modulation schemes, the signals are received by their respective correlator receivers with exact phase synchronisation and with optimum threshold detection. Find the average symbol error probability for modulation schemes BASK, BFSK and BPSK.

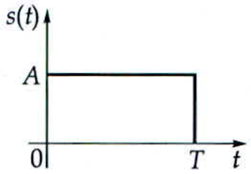
(ii) Suppose that two signals $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean white noise process is correlated with $s_1(t)$ and $s_2(t)$ separately, to yield the following variables:

$$n_1 = \int_0^T s_1(t)n(t)\,dt \text{ and } n_2 = \int_0^T s_2(t)n(t)\,dt$$

Prove that n_1 and n_2 are orthogonal. **[15 + 5 marks]**

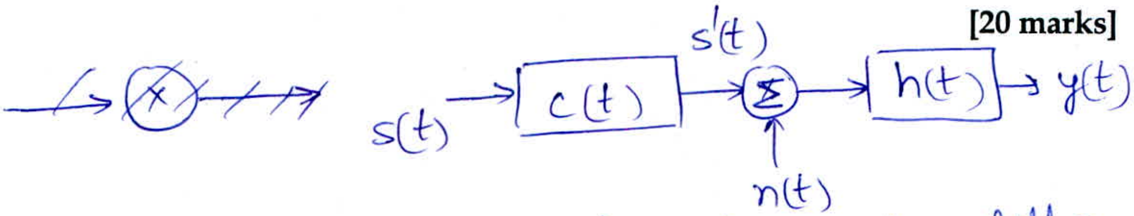
Do
wri
this

Q.7 (a) Consider the signal shown in the figure below:



This signal is passed through a channel and applied to a filter matched to the signal $s(t)$ at the receiving end. If the channel is not ideal, but has an impulse response $c(t) = \delta(t) + \frac{1}{2}\delta\left(t - \frac{T}{2}\right)$, then determine and sketch the output of the matched filter.

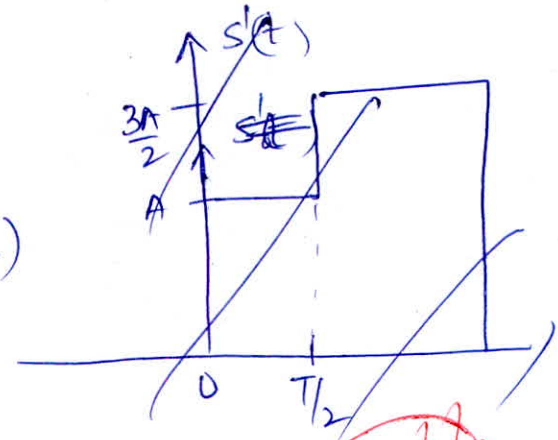
[20 marks]

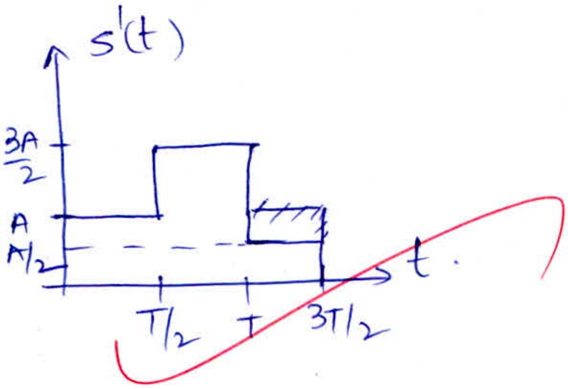


Let the output of channel just before the filter be s'(t)

$s'(t) = s(t) * c(t)$

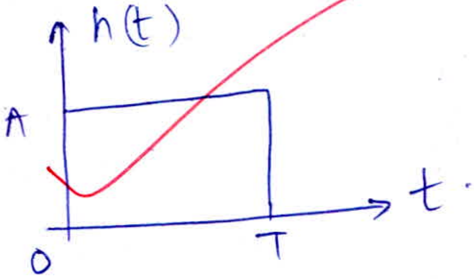
$= s(t) + \frac{s(t-T/2)}{2}$





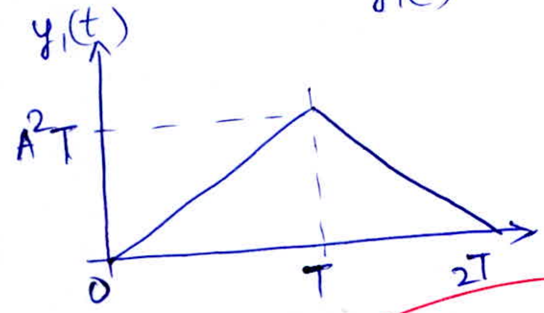
But the filter is matched to s(t)

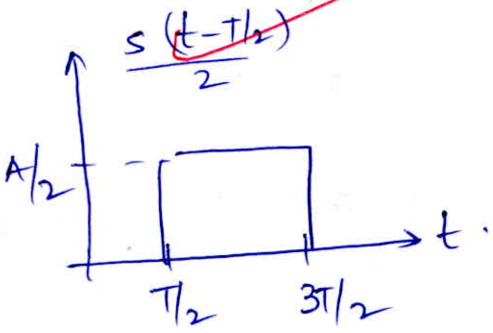
$h(t) = s(T-t)$



$$\Rightarrow y(t) = s'(t) * h(t).$$

$$= \underbrace{s(t) * h(t)}_{y_1(t)} + \underbrace{\frac{s(t-T/2)}{2} * h(t)}_{y_2(t)}$$





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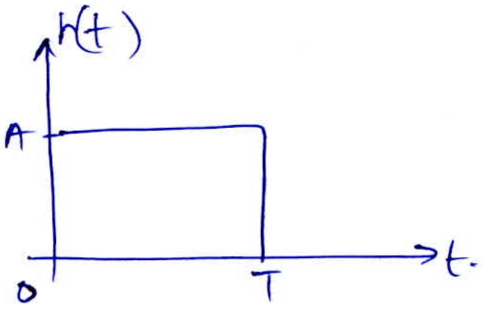

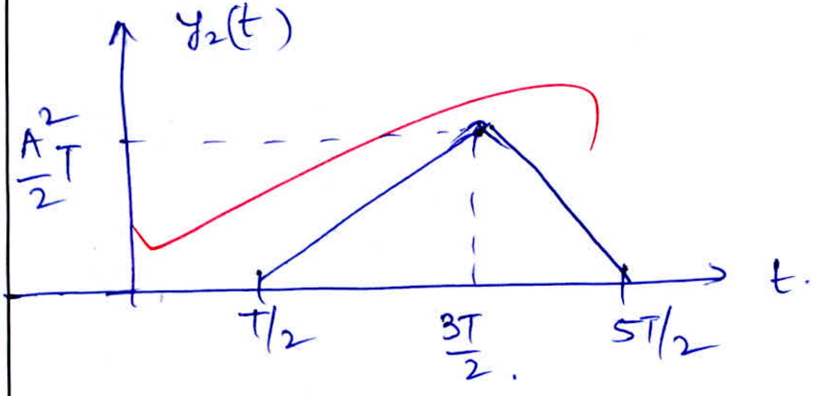

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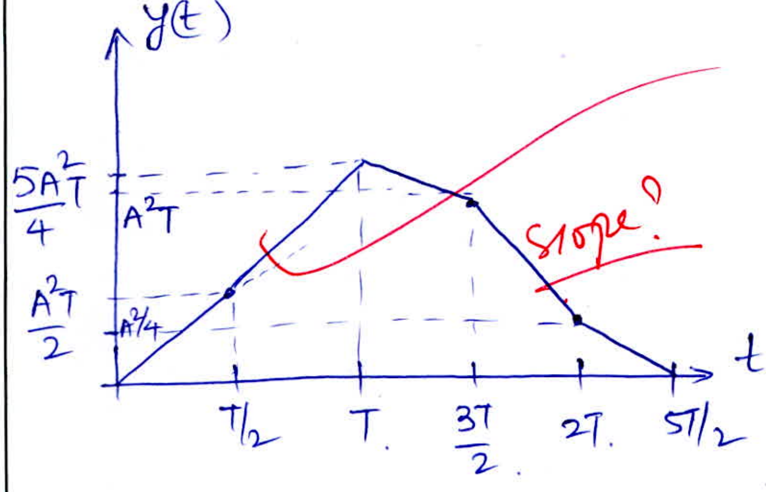
$$y(t) = y_1(t) + y_2(t)$$



Q.7 (b) Explain the basic steps involved in the fabrication of a CMOS transistor using silicon on sapphire (SOS) process.

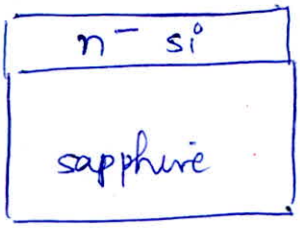
[20 marks]

Silicon on Sapphire (SOS) process is used to manufacture twin tub CMOS and avoid CMOS latch up condition using a silicon dioxide (SiO_2) insulator.

Steps –

Step① Take a sapphire substrate.

Step② Form a ~~n~~ intrensic (or n^-) type si layer by epitaxy



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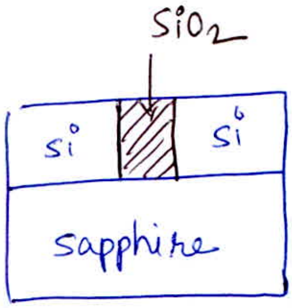
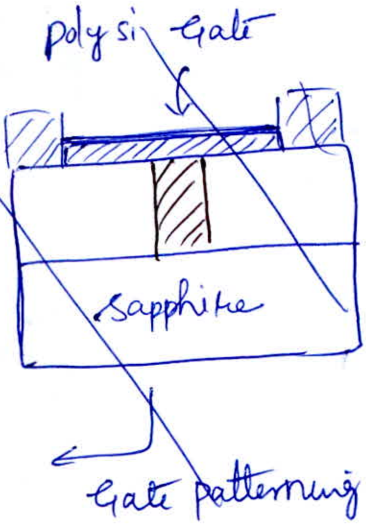
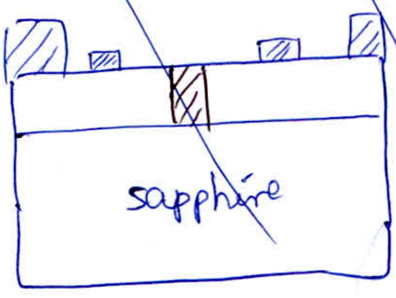
Step③ Form SiO_2 insulation by LOCOS

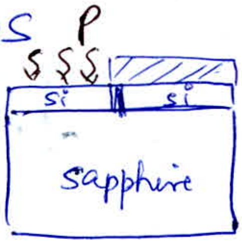


~~Step④ substrate doping for PMOS Gate formation~~

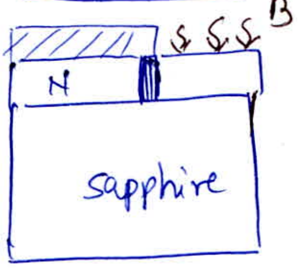




step⑤ ~~source~~ Substrate doping for NMOS

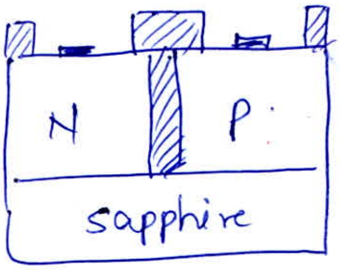


step⑥ substrate doping for PMOS

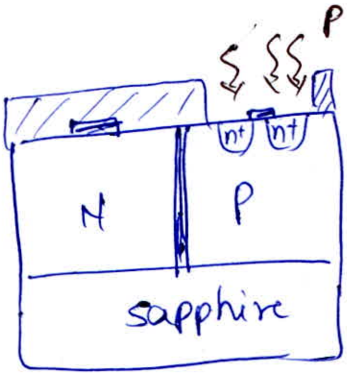


step⑦ Gate formation

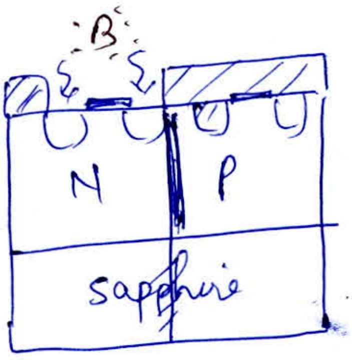
2



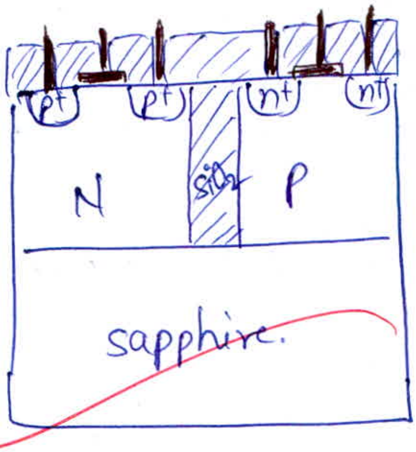
step⑧ source & drain diffusion for NMOS



step⑨ source & drain diffusion for PMOS



step⑩ metallization.



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Q.7 (c) A *p*-type lightly doped semiconductor has electron mobility μ_n, hole mobility μ_p, intrinsic carrier concentration n_i and the acceptor impurity concentration N_A.

- **(i)** Derive an expression for the hole concentration '*p*' in terms of n_i, μ_n and μ_p, such that the conductivity of the semiconductor is minimum.
- **(ii)** Derive an expression for the minimum conductivity of the semiconductor.
- **(iii)** If $n_i = 1.5 \times 10^{10}$ cm^{-3}, $\mu_n = 1300$ cm^2/V-sec and $\mu_p = 500$ cm^2/V-sec, then calculate the value of minimum conductivity.
- **(iv)** If there is 100% ionization of doping atoms, then calculate the value of acceptor impurity concentration (N_A).

[20 marks]

(i)-)

Conductivity $\sigma = q n \mu_n + q p \mu_p$. → ①

By law of mass action

$np = n_i^2$. → ②

From ① & ②

$\sigma = q \frac{n_i^2}{p} \mu_n + q p \mu_p$. → ③

differentiate WRT p & equate to zero

$$\frac{d\sigma}{dp} = -\frac{q n_i^2 \mu_n}{p^2} + q\mu_p = 0$$

$\Rightarrow p^2 = n_i^2 \frac{\mu_n}{\mu_p}$ or $\boxed{P_{min} = n_i \sqrt{\frac{\mu_n}{\mu_p}}}$ → ④

(ii.) $\sigma_{min} = ?$

From ③ & ④ ~~& ②~~

$$\sigma_{min} = q\frac{n_i^2}{p_{min}}\mu_n + q\,p_{min}\,\mu_p$$

$$= q\frac{n_i^2\mu_n}{n_i}\cdot\sqrt{\frac{\mu_p}{\mu_n}} + q\,n_i\sqrt{\frac{\mu_n}{\mu_p}}\cdot\mu_p$$

$$\boxed{\sigma_{min} = 2q\,n_i\sqrt{\mu_p\mu_n}}$$

(iii.) Given,

$n_i = 1.5\times10^{10}$

$\mu_n = 1300$

$\mu_p = 500$

$$\Rightarrow \sigma_{min} = 2q\,n_i\sqrt{\mu_p\mu_n}$$

$$= 2\times1.6\times10^{-19}\times1.5\times10^{10}\times\sqrt{1300\times500}$$

$$= 3.86\times10^{-6}\ ℧cm^{-1}\ (℧ cm)^{-1}$$

(iv.) By law of charge Neutrality,

total +ve charge = total −ve charge.

$$\cancel{N_D}^{0} + p = N_A + n$$

$$\Rightarrow N_A = p - n = p - \frac{n_i^2}{p} \rightarrow ⑤$$

From ④ and ⑤

$$N_A = n_i\sqrt{\frac{\mu_n}{\mu_p}} - n_i\sqrt{\frac{\mu_p}{\mu_n}}.$$

$$\Rightarrow N_A = n_i\left[\sqrt{\frac{\mu_n}{\mu_p}} - \sqrt{\frac{\mu_p}{\mu_n}}\right]$$

$$= 1.5\times10^{10}\left[\sqrt{\frac{1300}{500}} - \sqrt{\frac{500}{1300}}\right]$$

$$\boxed{N_A = 1.488\times10^{10}\ cm^{-3}}$$

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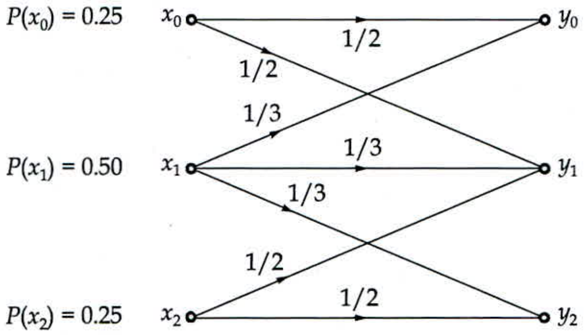
a) Consider the discrete memoryless channel shown below:



Determine the mutual information $I(X; Y)$.

[20 marks]

Q.8 (b) For a boron diffusion in silicon at 1000°C, the surface concentration is maintained at 10^{19} cm^{-3} and the diffusion time is 1 hour. Assume that the diffusivity (D) of Boron in Silicon at 1000°C is 2×10^{-14} cm^{2}/s. Determine:

(i) The total number of dopant atoms per unit area of semiconductor.

(ii) The distance of the location from the surface where the dopant concentration reaches 10^{15} cm^{-3}. Assume that $\text{erfc}^{-1}(10^{-4}) = 2.75$.

(iii) The gradient of the diffusion profile at the surface.

(iv) The gradient of the diffusion profile at the distance from the surface obtained in part (ii).

[20 marks]

Q.8 (c) **(i)** Find the expression for reverse saturation current I_0 in a p-n junction diode in terms of intrinsic carrier concentration n_i.

(ii) Find an expression for the reverse saturation current in terms of the conductivity of the device and prove that, $I_0 = AV_T \dfrac{b\sigma_i^2}{(1+b)^2}\left[\dfrac{1}{L_p\sigma_n} + \dfrac{1}{L_n\sigma_p}\right]$ where, $b = \dfrac{\mu_n}{\mu_p}$

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work