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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-5 : Basic Electronics Engineering + Analog Electronics

+ Electrical Materials

+ Electrical Machines - 1 + Power Systems - 2

Name : Ankit Jayal

Roll No : EE19MTDLA010

Test Centres

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	51
Q.2	35
Q.3	
Q.4	55
Section-B	
Q.5	38
Q.6	
Q.7	30
Q.8	
Total Marks Obtained	209

Signature of Evaluator

Cross Checked by

[Signature]

K. Sudharshan.

Section A : Basic Electronics Engg. + Analog Electronics + Electrical Materials

a) A conducting bar of 20 μm length, 2 μm wide and 1 μm thick is taken. Find the resistance of the bar if it is

(i) n-doped Silicon with $N_D = 10^8/cm^3$.

(ii) p-doped Silicon with $N_A = 10^{10}/cm^3$.

take $\mu_n = 2.5 \mu_p = 1200 cm^2/Vs$ and n_i for Silicon is $1.5 \times 10^{10}/cm^3$.

[12 marks]

$\sigma = ne\mu_n + p q \mu_p$ (Intrinsic semiconductor)

i) n-doped Silicon

$N_D = 10^8/cm^3$

~~$n + N_D = n + N_A$~~ $\therefore p = \frac{n_i^2}{n}$

now $p + N_D = n + N_A$ ($\because N_A = 0$)

$\therefore n = p + N_D$ now $np = n_i^2 \Rightarrow n^2 - nN_D - n_i^2 = 0$

$n = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2} \approx 1.5 \times 10^{10}/cm^3$

now $p = \frac{n_i^2}{n} = 1.49 \times 10^{10}/cm^3$

now $\sigma = ne\mu_n + p q \mu_p = 6.02 \times 10^{-9} + 1.44 \times 10^{-26}$

$\sigma = 2.88 \times 10^{-6} + 1.44 \times 10^{-6} \approx 4 \times 10^{-6}$

now $\rho = \frac{1}{\sigma} = 25 \times 10^4 \Omega cm$

now $R = \frac{\rho l}{A} = \frac{25 \times 10^4 \times 20 \times 10^{-4}}{1 \times 10^{-4} \times 2 \times 10^{-4}} = 2.5 \times 10^{10} \Omega$

$R = 2.5 \times 10^{10} \Omega$

ii) p-doped Silicon

$N_A = 10^{10}/cm^3$

now $p + N_D = n + N_A$ ($\because N_D = 0$)

$p = n + N_A \Rightarrow n = p - N_A$

using $np = n_i^2 \Rightarrow (p - N_A)p = n_i^2 \Rightarrow p^2 - pN_A - n_i^2 = 0$

$\therefore p = \frac{N_A + \sqrt{N_A^2 + 4n_i^2}}{2} = 2.08 \times 10^{10}/cm^3$

Now $n = n_i^2/p = 1.08 \times 10^{10} / \text{cm}^3$

Now $\sigma = ne\mu_n + pe\mu_p = 2.07 \times 10^{-6} + 1.59 \times 10^{-6}$
 $\sigma = 3.667 \times 10^{-6}$

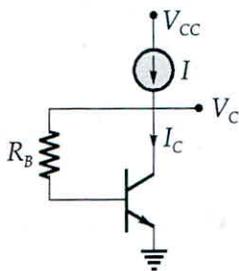
$\therefore \rho = 272702.48 \Omega \text{cm}$

Now $R = \frac{\rho l}{A} = \frac{272702.48 \times 20 \times 10^{-4}}{20 \times 10^{-4} \times 1 \times 10^{-4}} = 2.72 \times 10^{10}$

$R = 2.72 \times 10^{10} \Omega$ On



Q.1 (b) A circuit that can provide a very large voltage gain for a high resistance load is shown in figure below. Find the value of current I and R_B to bias the BJT at $I_C = 3 \text{ mA}$ and $V_C = 1.5 \text{ V}$ for $\beta = 90$.



[12 marks]

Solution

$I = I_B + I_C$

KVL in inner loop

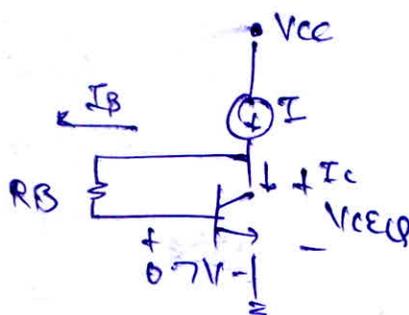
$-V_C + I_B R_B + V_{BE} = 0$

$\Rightarrow I_B = \frac{1.5 - 0.7}{R_B}$

$\therefore I_C = 3 \text{ mA} \Rightarrow I_B = \frac{3}{\beta} \text{ mA} = 33.33 \mu\text{A}$

$\therefore R_B = \frac{1.5 - 0.7}{33.33 \times 10^{-6}} \approx 24 \text{ k}\Omega$

$R_B = 24 \text{ k}\Omega$



$$\text{Now } I = I_C + I_B$$

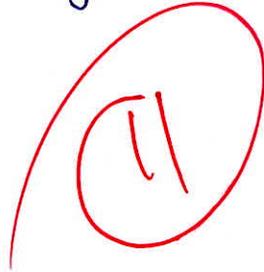
$$= (\beta + 1) I_B$$

$$I = 3.03 \text{ mA} \quad \text{Ans}$$

check $V_{CE} = 1.5 \text{ V} \quad (V_E = 0)$

\therefore transistor is in active region

$$\begin{aligned} \therefore I &= 3.03 \text{ mA} \\ R_B &= 24 \text{ k}\Omega \end{aligned} \quad \left. \vphantom{\begin{aligned} \therefore I &= 3.03 \text{ mA} \\ R_B &= 24 \text{ k}\Omega \end{aligned}} \right\} \text{Ans}$$



- Q.1 (c) A material with magnetic property such that when it was placed in a magnetic field, $B = 4 \text{ Wb/m}^2$, magnetic field intensity was found to be 4800 A/m . If \vec{H} is reduced to 640 A/m and $B = 1.8 \text{ Wb/m}^2$, then calculate the percentage change in magnetization M of the material.

[12 marks]

Solution case (i) $B = 4 \text{ Wb/m}^2 \therefore H = 4800 \text{ A/m}$

$$\text{Now } B = \mu_0 (H + M)$$

$$\Rightarrow M = \frac{B}{\mu_0} - H = 3.178 \times 10^6 \text{ A/m}$$

case (ii)

$$B = 1.8 \text{ Wb/m}^2 \therefore H = 640 \text{ A/m}$$

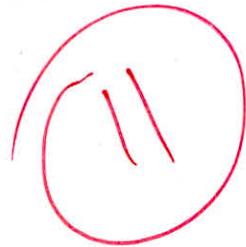
$$\text{Now } M = \frac{B}{\mu_0} - H = 1.43 \times 10^6 \text{ A/m}$$

% change in magnetization

$$\% \text{ change } M = \frac{M_2 - M_1}{M_1} \times 100$$

$$\boxed{\% \text{ M change} = -55\%}$$

\therefore magnetization reduces by about 55% in 2nd case



- d) What is the significance of 'Magnetic dipole' and 'Magnetization' phenomena in magnetic materials? Explain clearly with the help of definition and mathematical derivation. How are above two phenomena related to each other?

[12 marks]

Magnetic Dipole

Magnetic dipole is defined as current flowing in a closed loop and mathematically can be given as:

$$\vec{p}_m = I \times A \hat{n}$$



where I : current flowing in loop

A : Area of loop

\hat{n} : direction according to Right Hand thumb Rule wrt plane of loop

Magnetization

magnetisation can be defined as the magnetic dipoles per unit volume

Now magnetisation can also be defined as formation & alignment of magnetic dipoles when external magnetic field is applied to a magnetic material

$$\text{ie } \vec{M} = \frac{\sum_{i=1}^{NAV} \vec{p}_m}{\Delta V} \quad N: \text{No. of magnetic dipoles/m}^3$$

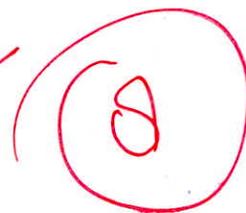
$$\vec{M} = \frac{NAV \vec{p}_m}{\Delta V} \quad \therefore \boxed{\vec{M} = N \vec{p}_m}$$

where \vec{p}_m : magnetic dipole moment

\vec{M} : magnetization

magnetic dipole arise in a material due to three basic phenomenon:

- (i) nucleus spin angular momentum
- (ii) electron orbital angular momentum
- (iii) electron spin angular momentum



now the first 2 factors are often neglected & electron spin angular momentum is considered.

Number of unpaired electrons in '3d' orbital of material decide the magnetic dipole moment of material



$\therefore \vec{p}_m = 4 \vec{p}_B$

\vec{p}_B : Bohr magneton

- e) What are type-I and type-II superconductors? Draw the magnetization versus magnetic field characteristic for type-I and type-II superconductors. Why superconductivity is observed for signals upto radio frequencies?

[4 + 4 + 4 marks]

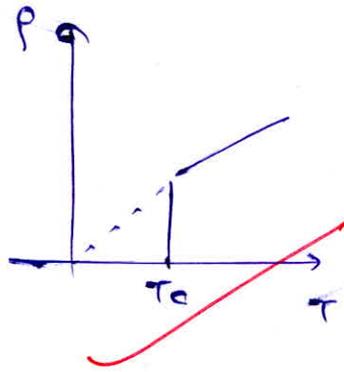
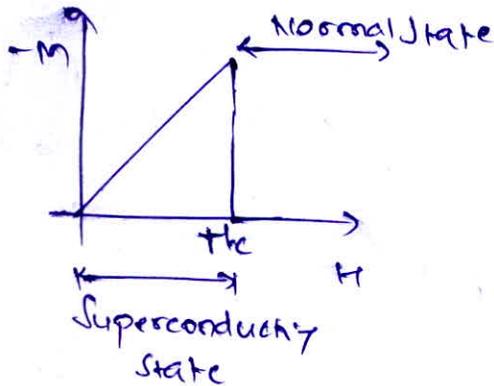
Type I Superconductors

These are also called as Soft or ideal Superconductors as these superconductors follow complete Meissner's effect and Silsbee Rule

- i) They have low values of critical magnetic field and transition temperatures

- ii) They change from Superconducting to Normal state and vice versa is abrupt and no intermediate state is there

Curves

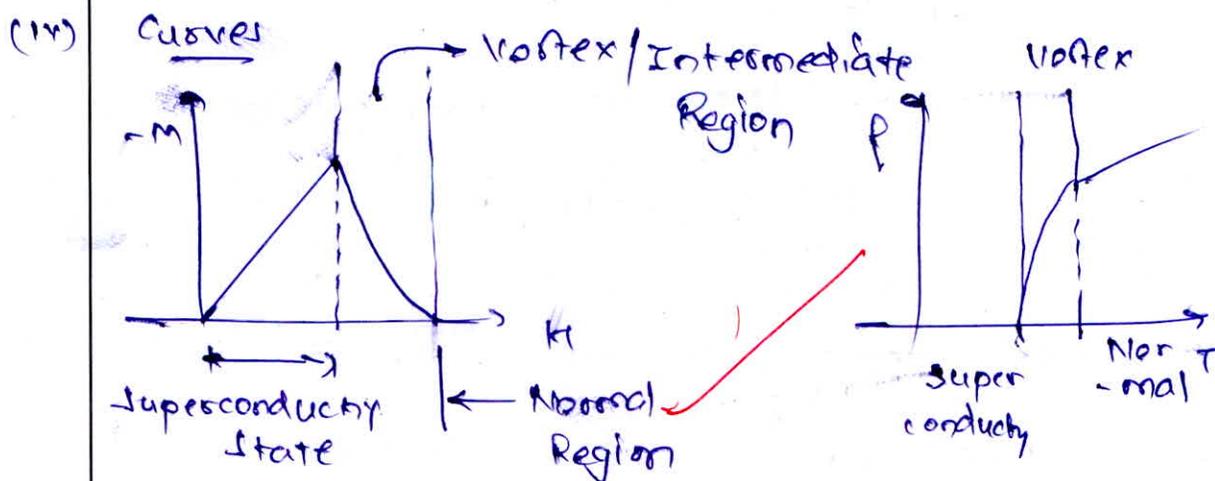


Type II Superconductors

These are also called as non ideal / Hard Superconductors as they follow Silsbee Rule & Meissner's effect partially only

- i) They have '2' values of critical magnetic field and have high values of H_c & transition temperature

(iii) They change from Superconducting to Normal state & vice versa gradually showing an intermediate/Vortex State in between the transition.



Superconductivity is associated with perfect conductivity of material i.e. in superconducting state material exhibits '0' Resistivity

Now As the frequency range is increased, then At higher frequencies, Resistance & hence Resistivity of the material increases due to skin effect and after radio frequencies the resistance of the material becomes almost equal to resistance in the normal state of material and hence Superconductivity is destroyed after radio frequencies

(10)

- a) The copper crystal has FCC unit cell configuration. If radius of Cu atom is 0.148 nm and atomic mass of Cu is 63.5 gmol⁻¹ then calculate atomic packing fraction (APF), the atomic concentration in a unit cell and density of Cu atom in gcm⁻³.
(Take Avogadro number : 6.023 × 10²³ mol⁻¹)

[20 marks]

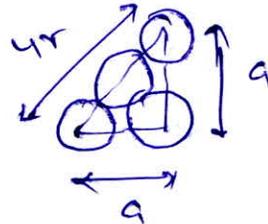
For Cu Crystal : FCC Structure

$$\therefore \text{effective number of atoms/unit cell} = \frac{1}{8} \times 8 + 6 \times \frac{1}{2} = 4$$

Now given radius = 0.148 nm

For FCC

$$r = \frac{\sqrt{2}a}{4}$$



where a : lattice parameter of unit cell

$$\therefore a = \frac{4r}{\sqrt{2}} = 0.4186 \text{ nm}$$

Now For FCC

$$\text{Atomic packing Fraction} = \frac{\text{effective no. of atoms/unit cell} \times \text{volume of atom}}{\text{volume of unit cell}}$$

$$\text{APF} = \frac{4 \times \left(\frac{4}{3} \pi r^3\right)}{a^3} = \frac{16}{3} \pi \left(\frac{r}{a}\right)^3 = 0.74$$

$$\therefore \boxed{\text{Atomic packing Fraction} = 0.74} \quad \text{Ans.}$$

$$\text{Now Atomic mass 'M'} = 63.5 \text{ g mol}^{-1}$$

$$\text{Now } \rho = \frac{\text{No. of effective atoms/unit cell} \times \text{mass of one atom}}{\text{volume of unit cell}}$$

$$\rho = \frac{4 \times M / N_A}{a^3} \quad \therefore N_A = 6.022 \times 10^{23} \text{ (Avogadro No.)}$$

$$\rho = \frac{4 \times 63.5 / 6.022 \times 10^{23}}{(0.4186 \times 10^{-9})^3} \text{ gm/m}^3$$

$$\rho = 5.75 \times 10^6 \text{ gm/m}^3$$

$$\rho = 5.75 \times 10^3 \text{ kg/m}^3$$

$$\Rightarrow \boxed{\rho = 5750 \text{ kg/m}^3}$$

↓
Density of Cu atom

Now As We know

Atomic concentration

$$(N) = \frac{N_A \rho}{M}$$

$$N = \frac{6.022 \times 10^{23} \times 5750 \times 10^3 \text{ g/m}^3}{63.5 \text{ gm mol}^{-1}}$$

$$\boxed{N = 5.45 \times 10^{28} / \text{m}^3}$$

Ans.

∴ conclusion

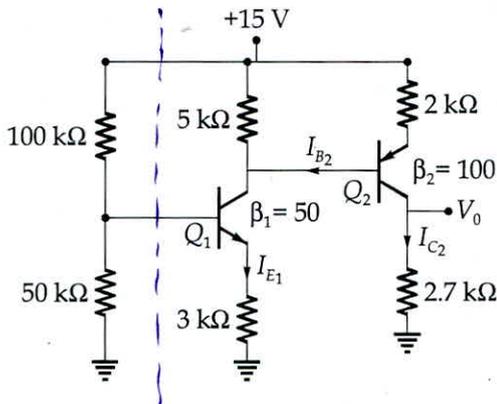
$$\text{APF} = 0.74$$

$$\text{Atomic concentration } - N' : 5.45 \times 10^{28} / \text{m}^3$$

$$\text{Density of Cu atom } - \rho' : 5750 \text{ kg/m}^3$$

Ans.

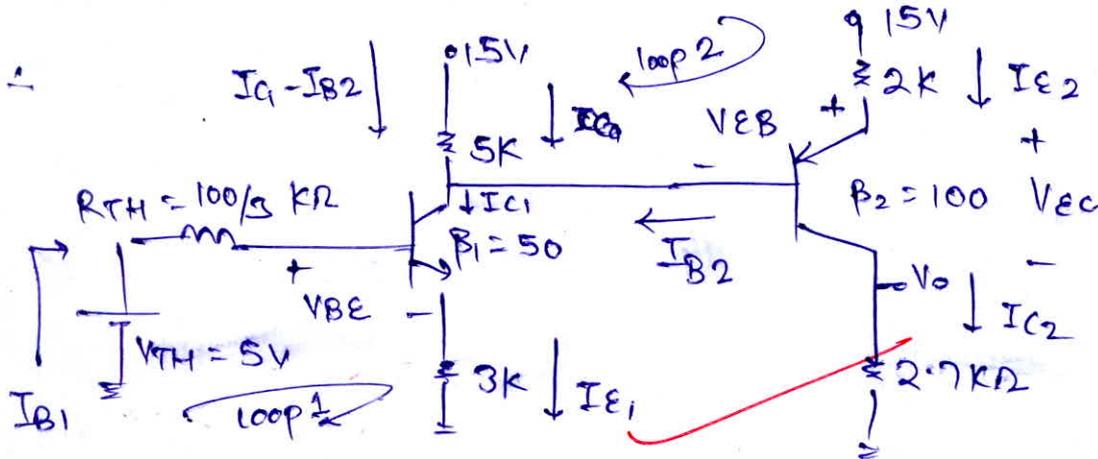
(b) In the below configuration, calculate the values of I_{B2} , I_{C2} , I_{E1} and V_0 .



[20 marks]

for Redrawing circuit by applying thevenin's in input side

$$V_{TH} = \frac{50}{100+50} \times 15 = 5V \quad ; \quad R_{TH} = 50K \parallel 100K = \frac{100}{3} K\Omega$$



Now KVL in input loop 1

$$-V_{TH} + R_{TH}(I_{B1}) + V_{BE} + (I_{E1})3K = 0$$

$$\Rightarrow I_{B1} (R_{TH} + (1+\beta_1)3K) = V_{TH} - V_{BE}$$

$$\Rightarrow I_{B1} = \frac{5 - 0.7}{\frac{100}{3} + 51 \times 3} \text{ mA} = 23.07 \mu\text{A}$$

$$I_{B1} = 23.07 \mu\text{A} \quad ; \quad I_{C1} = 1.154 \text{ mA} \quad ; \quad I_{E1} = 1.177 \text{ mA}$$

Now

KVL in loop 2

$$-15 + (2K) I_{E2} + V_{EB2} - 5K(I_{C1} - I_{B2}) - 15 = 0$$

$$\Rightarrow I_{B2} (1 + \beta_2) 2K - 5K(I_{C1}) + (5K) I_{B2} = 30 - 0.7$$

$$\Rightarrow I_{B2} = \frac{30 - 0.7 + 5 \times 1.154}{(1 + \beta_2) 2K + 5K}$$

$$I_{B2} = \frac{43.67 \mu A}{169.42} \quad ; \quad I_{C2} = \frac{4.36 \text{ mA}}{16.94} \quad ; \quad I_{E2} = \frac{11.48 \text{ mA}}{17.11}$$

Now check for Q₁

$$-15 + 5K(I_{C1} - I_{B2}) + V_{CE1} + 3K(I_{E1}) = 0$$

$$\Rightarrow V_{CE1} = \frac{15 - 3 \times 1.177 - 5 \times 10^3 (1.154 \text{ m} - 113.67 \mu)}{1}$$

$$\boxed{V_{CE1} = 6.26 \text{ V}} \quad (Q_1 \text{ in Active Region})$$

Q₂

$$-15 + 2K(I_{E2}) + V_{CE2} + (2.7K) I_{C2} = 0$$

$$\Rightarrow \boxed{V_{CE2} = -38.632} \quad (\text{can't be in Active Region})$$

Assume in Saturation (Q₂)

$$\Delta \quad \boxed{V_{CE2} = 0.2 \text{ V}}$$

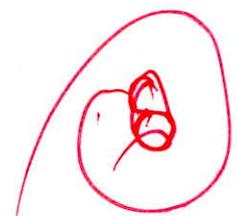
$$\text{Now } -15 + (2K) I_{E2} + V_{CE2} + (2.7K) I_{C2} = 0$$

$$\Rightarrow \text{Assu: } I_{B2} = \frac{15 - 0.2}{2K(1 + \beta_2) + 2.7K \beta_2} = 31.35 \mu A$$

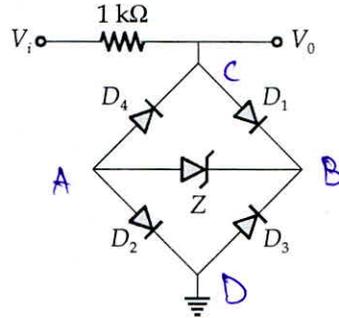
$$\therefore I_{C2} = 3.13 \text{ mA} \quad ; \quad I_{E2} = 3.166 \text{ mA}$$

$$\boxed{V_o = I_{C2} \times 2.7K = 8.451 \text{ V}} \quad \text{Ans}$$

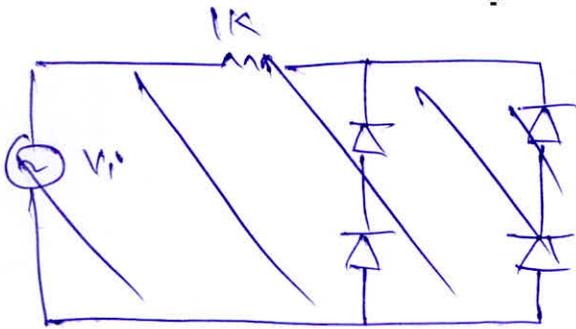
$$\therefore \begin{cases} I_{B2} = 31.35 \mu A \\ I_{C2} = 3.13 \text{ mA} \\ I_{E1} = 1.177 \text{ mA} \end{cases} \quad \text{Ans}$$



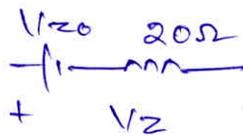
(c) Sketch the transfer characteristics of the circuit given below for $-20\text{ V} \leq V_i \leq 20\text{ V}$. Assume that diodes can be represented by a piece-wise linear model with $V_{D0} = 0.65\text{ V}$ and $r_D = 20\ \Omega$. Assuming that the specified zener voltage at a current of 10 mA is 8.2 V and $r_Z = 20\ \Omega$. Represent the Zener by a piece-wise linear model.



[20 marks]



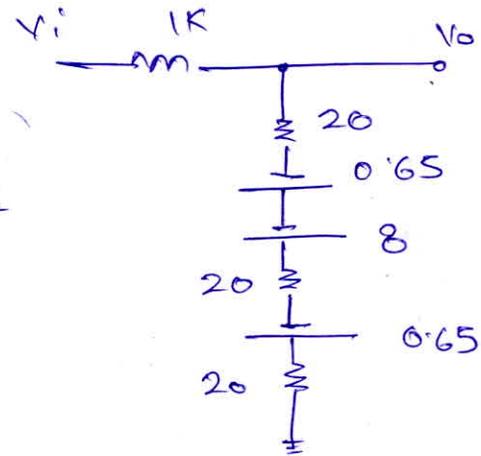
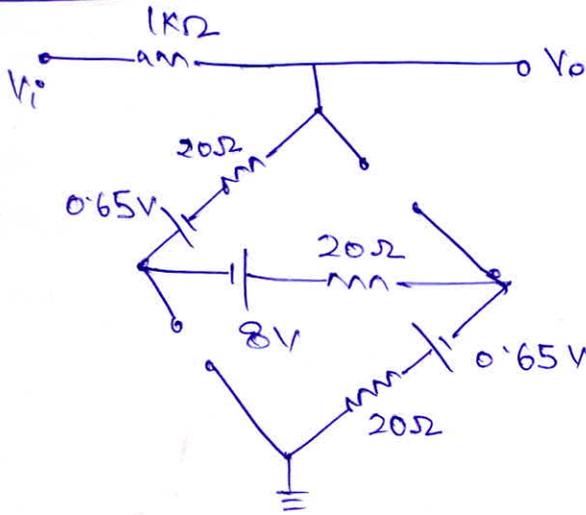
zener diode



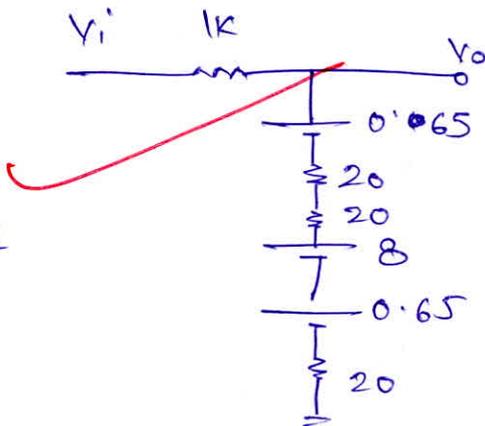
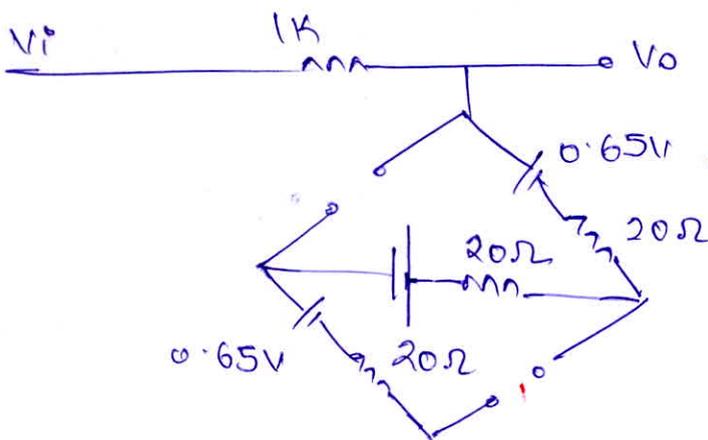
$$8.2 = V_{Z0} + 20 \times 10^{-2}$$

$$V_{Z0} = 8\text{V}$$

equivalent model ($V_i < 0$)



equivalent model ($V_i > 0$)



Now during $V_i < 0$

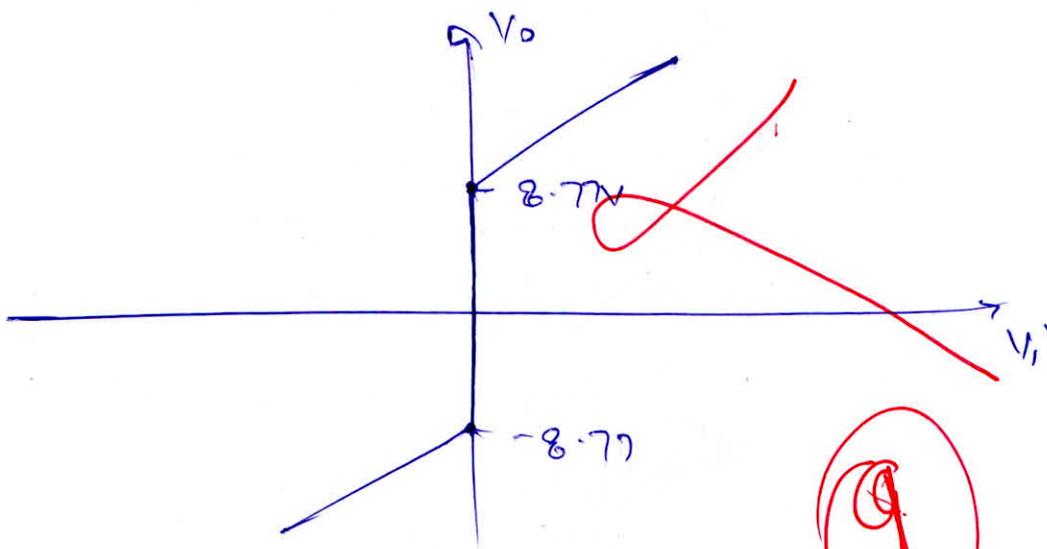
$$V_o = \frac{60V_i + (-9.3) \times 1K}{1K + 60}$$

$$V_o = \left(\frac{60}{1060} V_i - 8.77 \right) V$$

During $V_i > 0$

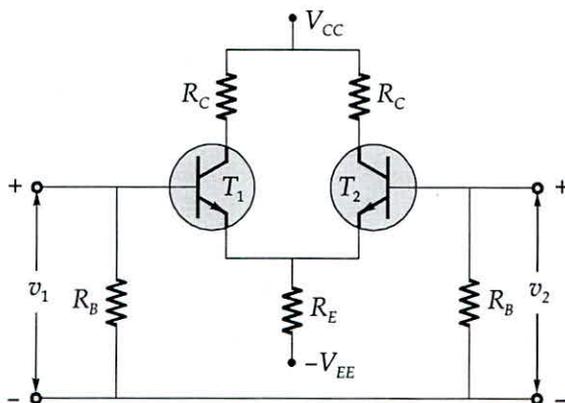
$$V_o = \frac{60V_i + 9.3 \times 1K}{1K + 60} = \frac{60V_i}{1060} + 8.77V$$

$$V_o = \left(\frac{60V_i}{1060} + 8.77 \right) V$$



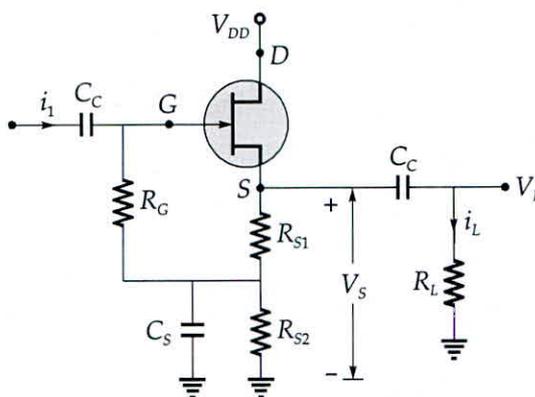
Transfer characteristics

- (a) (i) The BJT in the differential amplifier circuit shown below have negligible leakage current and $\beta_1 = \beta_2 = 60$. Also $R_C = 6.8 \text{ k}\Omega$, $R_B = 10 \text{ k}\Omega$ and $V_{CC} = V_{EE} = 15 \text{ V}$. Find the value of R_E needed to bias the amplifier such that $V_{CEQ1} = V_{CEQ2} = 8 \text{ V}$.



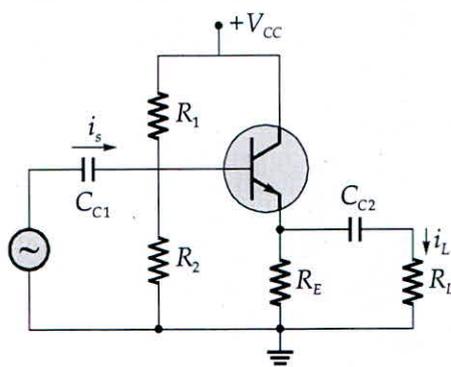
[10 marks]

- Q.3 (a) (ii) In the circuit shown below $R_G \gg R_{S1}, R_{S2}$. The JFET is described by $I_{DSS} = 10 \text{ mA}$, $V_P = 4 \text{ V}$, $V_{DD} = 15 \text{ V}$, $V_{DSQ} = 10 \text{ V}$ and $V_{GSQ} = -2 \text{ V}$. Find the value of R_{S1} and R_{S2} to set amplifier at above Q-point and also find the value of V_S .



[10 marks]

Q.3 (b) Consider the amplifier circuit shown below:

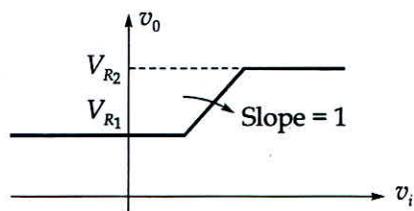


The parameters of BJT and the circuit are, $\beta = 80$, $V_{CC} = 10\text{ V}$, $V_{CEQ} = 5\text{ V}$, $V_{BE(\text{on})} = 0.7\text{ V}$ and $R_E = R_L = 500\ \Omega$. Design the values of R_1 and R_2 such that the mid-band current

gain $A_i = \frac{i_L}{i_S} = 8$. Assume that $V_T = 26\text{ mV}$.

[20 marks]

- (c) (i) Design a clipper circuit for the characteristics curve given below:



[10 marks]

- Q.3 (c) (ii) Define the 'mobility' of electrons in conductive materials and derive relation showing components of random velocity and average drift velocity to deduce expression for electron mobility.
- (iii) If a conductor material has following data as shown below:
- Density : 9.40 gram/cc
Resistivity : 1.72×10^{-8} ohm-m
Atomic weight : 63.5
- Compute the mobility and the average time of collision of electrons in the conductors if valance electron for each conductor material atom is 1.

[10 marks]

- Q.4 (a) (i) Consider a diode with mean lifetime of holes to be 10 nsec and $\eta = 1$. If a forward current of 0.1 mA is flowing in diode then determine the diffusion capacitance. (Assume room temperature to be 300 K).

[5 marks]

For a diode

Solution

As we know, Diffusion Capacitance

$$C_D = \frac{I_{\text{forward}} \times \tau}{\eta V_T}$$

where $V_T = \text{Volt equivalent temperature} = \frac{T(K)}{11,600}$

 $I_{\text{forward}} = 0.1 \text{ mA}$ given $\tau = \text{mean lifetime of holes} = 10 \text{ nsec}$ $\eta = 1$

$$C_D = \frac{0.1 \times 10^{-3} \times 10 \times 10^{-9}}{300/11,600} \text{ F}$$

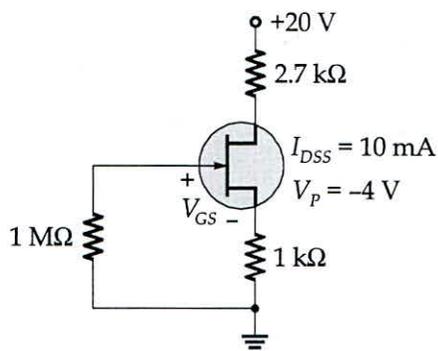
$$= 3.866 \times 10^{-11} \text{ F}$$

$$C_D = 38.66 \text{ PF}$$

An.

5

- (a) (ii) Determine V_{GSQ} , I_{DQ} and V_{DS} for the self bias circuit shown in figure below.



[15 marks]

Assuming, Saturation Region operation
Now KVL in input loop

$$+V_{GS} + I_D(1K) = 0$$

$$\Rightarrow \boxed{I_D = -(V_{GS}) \text{ mA}}$$

Now by Shockley's equation

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\Rightarrow (-V_{GS}) \text{ mA} = 10 \text{ mA} \left(1 - \frac{V_{GS}}{-4}\right)^2$$

$$\Rightarrow -V_{GS} = 10 \left(1 + \frac{V_{GS}}{4}\right)^2$$

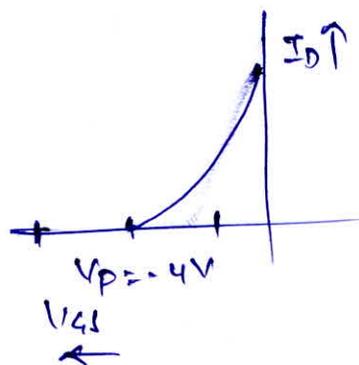
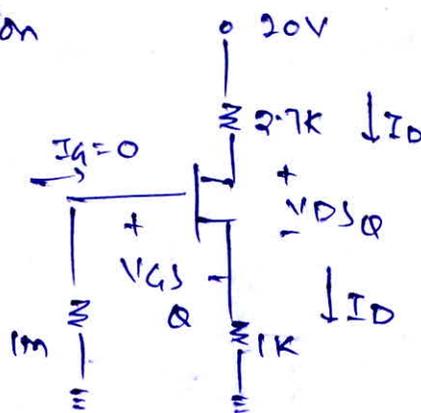
$$\Rightarrow \frac{10}{16} V_{GS}^2 + 10 + 5V_{GS} = -V_{GS}$$

$$\Rightarrow 0.625V_{GS}^2 + 6V_{GS} + 10 = 0$$

Solving $V_{GS} = -2.146 \text{ V}$, -7.45 V
(Selected) (Neglected)

$$\text{Now } \boxed{V_{GSQ} = -2.146 \text{ V}}$$

$$I_D = -V_{GS} \text{ mA} = 2.146 \text{ mA}$$



KVL in o/p loop

$$-20 + (2.7K)I_D + V_{DSQ} + (I_D)1K = 0$$

$$\Rightarrow (V_{DS})_Q = 20 - 2.7 \times 2.146 - 2.146$$

$$(V_{DS})_Q = 12.06V$$

check

$$(V_{DS})_Q = 12.06V ; (V_{GS})_Q - V_T = -2.146 + 4 \\ = 1.854V$$

$$\therefore (V_{DS})_Q > (V_{GS})_Q - V_T$$

\(\therefore\) Assumption correct

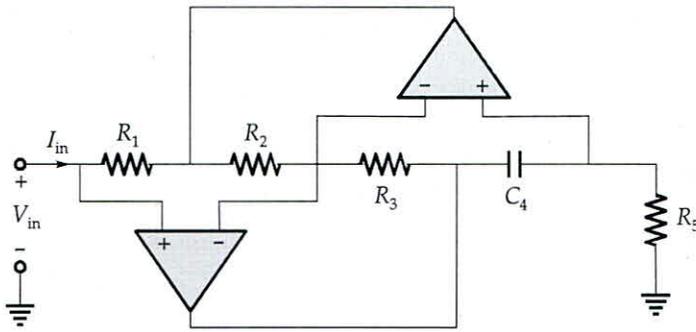
$$\therefore (V_{GS})_Q = -2.146V$$

$$(I_D)_Q = 2.146mA$$

$$(V_{DS})_Q = 12.06V$$

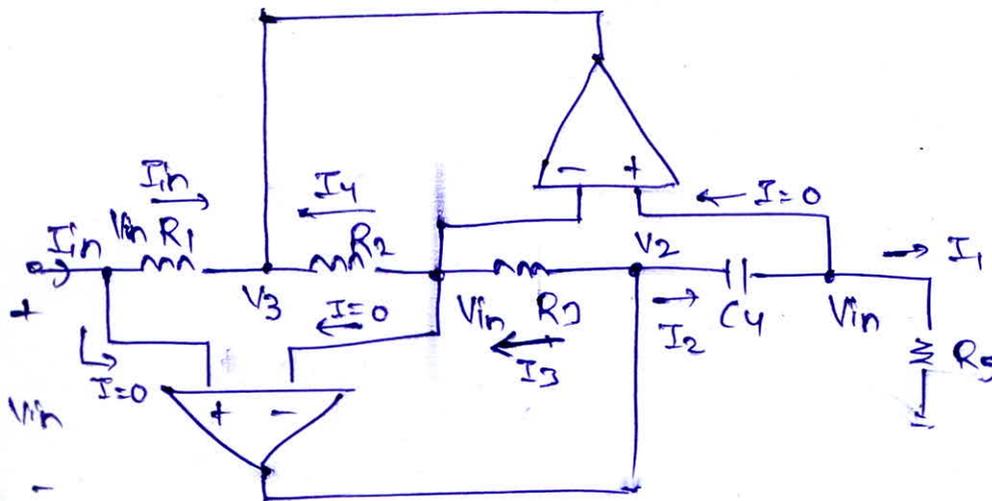
(14)

(b) Consider the circuit with ideal op-amps, shown in the figure below:



Calculate the input impedance $Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)}$ and comment on the result obtained.

[20 marks]



$$\text{clearly } I_1 = \frac{V_{in}}{R_5}$$

$$\text{ALSO } I_2 = I_1 = V_{in}/R_5$$

$$\text{Now } (V_2 - V_{in})sC_4 = I_2 = V_{in}/R_5$$

$$\Rightarrow \boxed{V_2 = V_{in} + \frac{V_{in}}{sR_5C_4}}$$

$$\text{Now } I_3 = \frac{-V_{in} + V_2}{R_3} = \frac{V_{in}}{sR_5C_4R_3}$$

$$\text{clearly } I_4 = I_3 = \frac{V_{in}}{sR_5C_4R_3}$$

$$\text{Now } \frac{V_{in} - V_3}{R_2} = I_4 = \frac{V_{in}}{sR_5C_4R_3}$$

$$\therefore V_3 = V_m - \frac{V_{in} R_2}{s R_5 (C_4 R_3)}$$

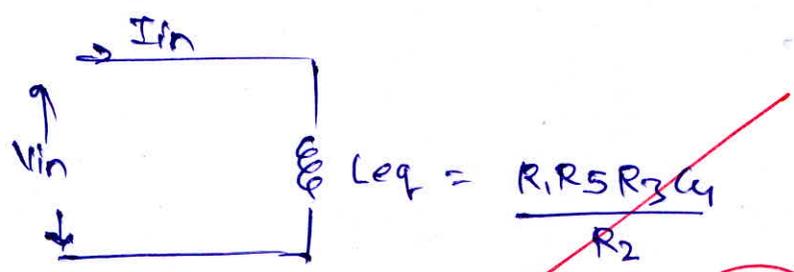
$$\text{Now } I_{in} = \frac{V_{in} - V_3}{R_1} = \frac{V_{in} R_2}{s R_1 R_5 R_3 C_4}$$

$$\therefore \boxed{\frac{V_{in}}{I_{in}} = \frac{s R_1 R_5 R_3 C_4}{R_2}} \text{ Ans}$$

$$\therefore Z_{in}(s) = s \left(\frac{R_1 R_5 R_3 C_4}{R_2} \right) = s L_{eq}$$

where $L_{eq} = \frac{R_1 R_5 R_3 C_4}{R_2}$

Thus the above circuit behaves as an inductor and can be redrawn as:



Ans



(c) Explain briefly the polarization occurring in dielectric materials. What are different types of polarization occurring in dielectric material?

If a dielectric material contains 3.2×10^{19} polar molecules/m³ and the relative permittivity of material is $\epsilon_r = 2.4$ with applied external electric field $\vec{E} = 10^4 \vec{a}_x$ V/m, then calculate the value of polarization and dipole moment in each molecule. (Consider all molecules have same dipole moment).

[20 marks]

Polarization:

The formation and orientation of electric dipoles in a dielectric under the influence of an applied electric field is known as polarization.

On a macroscopic scale, polarization is defined as electric dipole moment per unit volume.

$$\vec{P} = \frac{\sum_{i=1}^{N\Delta V} \vec{p}_i}{\Delta V} \quad \text{where } \vec{p}_i = \text{electric dipole moment of } i\text{th dipole}$$

$$\vec{P} = \frac{(N\Delta V)\vec{p}}{\Delta V}$$

$$\Rightarrow \boxed{\vec{P} = N\vec{p}}$$

\vec{P} : polarization

N : no. of dipoles / volume

Different types of polarization are as follows:

Electronic polarization: In a dielectric, bound e^- are predominant. When an external electric field is applied, the e^- cloud & nucleus gets shifted and separated from each other by some distance thus creating an electric dipole. This phenomenon is known as electronic polarization.

$$\vec{P} = N\alpha_e \vec{E} \quad \text{where } \alpha_e = (\text{Cm}^2/\text{V})$$

(ii) Ionic polarization : occurs in materials having net +ve & -ve charge. This is elastic displacement of bonds of the material due to application of electric field.



(iii) orientational polarization : This occurs in materials having partial ionic bond/polar covalent bond and contains permanent dipoles " \vec{p}_p ". The external electric field applies a Torque on " \vec{p}_p " to align them towards its own direction. This process of orientation of permanent dipoles is known as orientational polarization.

$$\vec{P}_0 = \left(\frac{N p_p^2}{3KT} \right) E \quad ; \quad \alpha_0 = \frac{p_p^2}{3KT}$$

Now given $P = (N \alpha_e E + N \alpha_i E + \frac{N p_p^2}{3KT} E)$

$$N = 3.2 \times 10^{19} / \text{m}^3 \quad ; \quad \epsilon_r = 2.4, \quad \vec{E} = 10^4 \hat{a}_x \text{ V/m}$$

$$\therefore P = \epsilon_0 (\epsilon_r - 1) \vec{E} \Rightarrow \boxed{P = 1.24 \times 10^{-7} \text{ C/m}^2} \quad \text{Ans}$$

↓
polarization

$$\text{Also } P = N \alpha E = N \vec{p}$$

$$\Rightarrow \vec{p} = P/N = \frac{1.24 \times 10^{-7}}{3.2 \times 10^{19}} = 3.875 \times 10^{-27} \text{ C-m}$$

$$\boxed{\vec{p} = 3.875 \times 10^{-27} \text{ cm}} \quad \text{Ans}$$

dipole moment on each molecule

(18)

Section B : Electrical Machines-1 + Power Systems-2

(a) Draw the reactance diagram of the system whose bus admittance matrix is given below. First, second, third and fourth rows refer to buses 1, 2, 3 and 4 respectively.

$$Y_{bus} = j \begin{bmatrix} -3.78 & 1.25 & 2.5 & 0 \\ 1.25 & -3.42 & 1.11 & 1.0 \\ 2.5 & 1.11 & -4.89 & 1.25 \\ 0 & 1.0 & 1.25 & -2.31 \end{bmatrix}$$

[12 marks]

action Sum of Row 1 : ~~-3.78j~~ -0.03j

∴ Bus 1 is connected to Ref. bus

Sum of Row 2 : -0.06j

∴ Bus 2 also connected to Ref bus

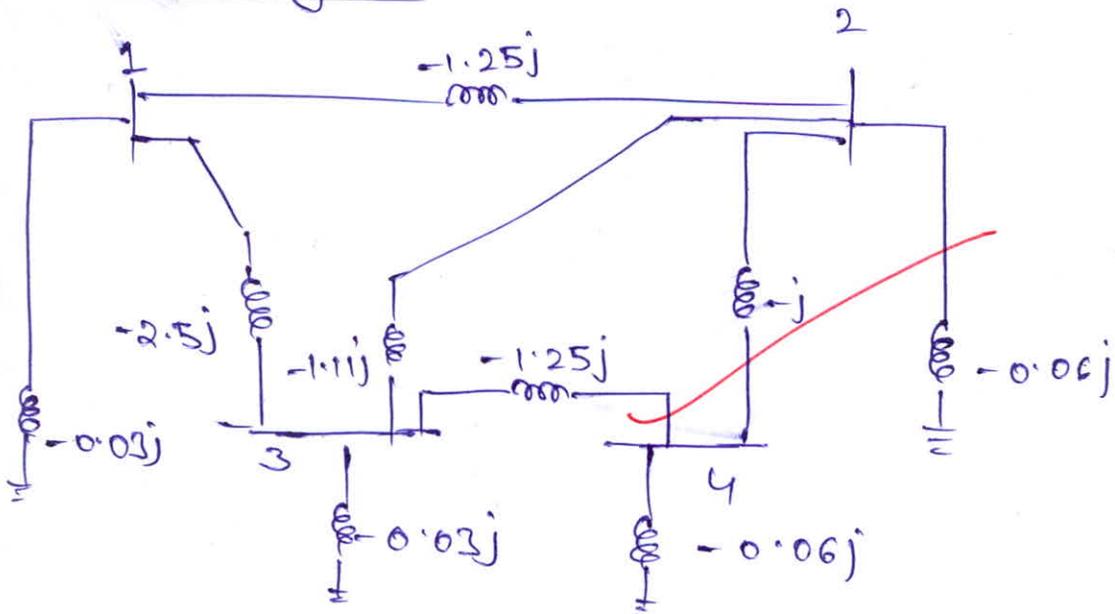
Sum of Row 3 : -0.03j

∴ Bus 3 also connected to Ref bus

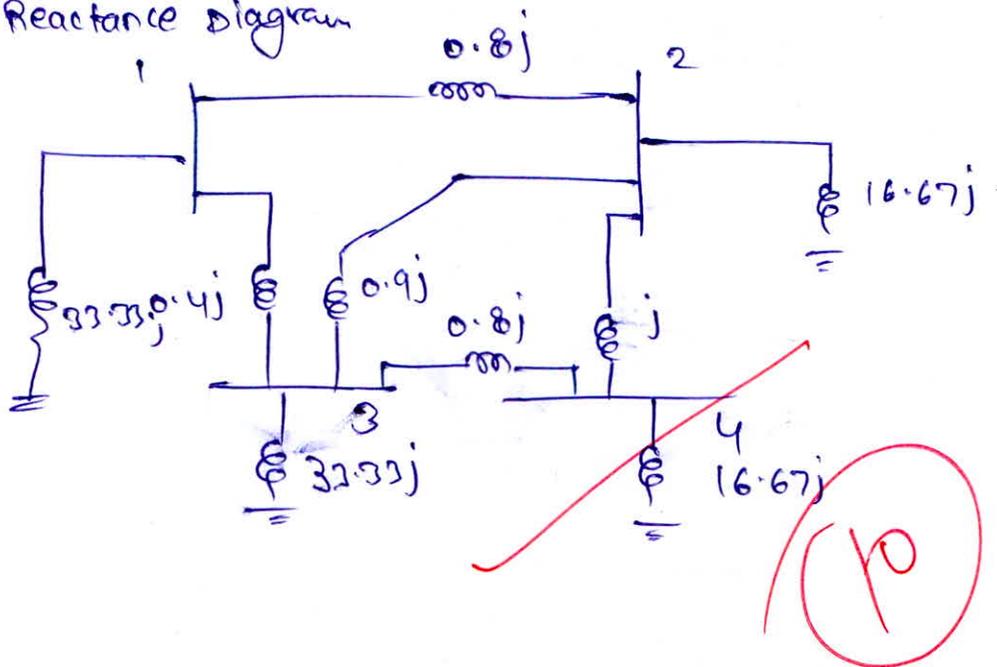
Sum of bus 4 : -0.06j

∴ Bus 4 always connected to Ref bus

Now Reactance Diagram (shown values are Admittances)



∴ Reactance diagram



Q.5 (b) A voltage of $(200 \sin \omega t - 50 \sin 3\omega t)$, 50 Hz is applied to a 250 turn transformer winding having negligible resistance and leakage reactance. Deduce an expression for flux and find its maximum value. By what percentage will eddy current loss in the iron core be reduced if the applied voltage is altered to $200 \sin \omega t$?

[12 marks]

Solution

$$v = 200 \sin \omega t - 50 \sin 3\omega t$$

$$\therefore \text{induced voltage (E)} = -\frac{d\lambda}{dt} = -N \frac{d\phi}{dt}$$

$$\text{now } \phi = \frac{1}{N} \int v dt = \frac{1}{N} \int (200 \sin \omega t - 50 \sin 3\omega t) d\omega t$$

$$\phi = \frac{1}{N} (200 \cos \omega t - \frac{50}{3} \cos(3\omega t)) \text{ wb}$$

now For max. value of Flux

$$\phi = \phi_m / \omega t = 0 \Rightarrow \phi_m = (200 - 50/3) \frac{1}{N}$$

$$\phi_m = \frac{183.33}{N} \text{ wb}$$

$$\therefore \Phi_{\max} = \frac{183.33}{250} \text{ Wb}$$

$$\therefore \Phi_{\max} = 0.733 \text{ Wb} \quad \text{Ans.}$$

Now For eddy Current loss

$$P_{\text{eddy}} \propto f^2 B_m^2 \propto v^2 \quad \because (B_m \propto v/f)$$

Now when

$$v = 200 \sin \omega t - 50 \sin 3\omega t$$

$$V_{\text{rms}} = \sqrt{\frac{200^2}{2} + \frac{50^2}{2}} = 145.77 \text{ V}$$

when $v = 200 \sin \omega t$

$$V_{\text{rms}} = \frac{200}{\sqrt{2}} = 141.42 \text{ V}$$

$$\text{Now } \frac{P_{e2}}{P_{e1}} = \left(\frac{141.42}{145.77} \right)^2 = 0.9412$$

$$\text{Now \% Reduction} = \frac{P_{e1} - P_{e2}}{P_{e1}} \times 100$$

$$= \left(1 - \frac{P_{e2}}{P_{e1}} \right) \times 100$$

$$\% \text{ Reduction in eddy Current loss} = 5.88\%$$

Ans

- Q.5 (c) An industry load of 200 kW at 0.75 p.f. lagging is fed from the 3- ϕ , 11 kV distribution feeders. It is required to maintain the 0.9 p.f. lag at the drawl point. Find the rating of capacitor installed at industrial drawl point.

[12 marks]

Solution

load: 200 kW at 0.75 lag pf

At drawl point : 200 kW at 0.9 pf lag

Let \vec{S}_c be the Rating of capacitor installed at the industrial drawl point

$$\therefore \vec{S}_{\text{input}} = \vec{S}_c + \vec{S}_{\text{load}}$$

$$\vec{S}_{\text{load}} = \frac{200}{0.75} \left[\cos^{-1}(0.75) \right] = \frac{800}{3} \left[41.4^\circ \text{ KVA} \right]$$

$$\vec{S}_{\text{input}} = \frac{200}{0.9} \left[\cos^{-1}(0.9) \right] = \frac{2000}{9} \left[25.84^\circ \text{ KVA} \right]$$

$$\text{now } \vec{S}_c = \vec{S}_{\text{input}} - \vec{S}_{\text{load}}$$

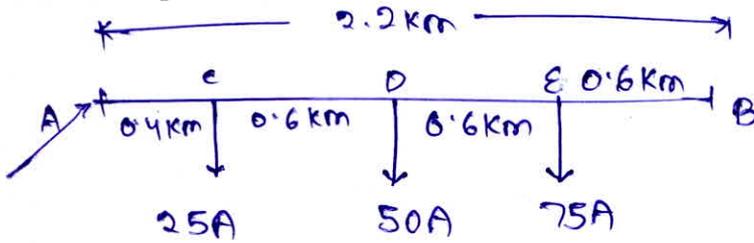
$$\boxed{\vec{S}_c = -79.492j \text{ KVAR}}$$

indicates capacitor

$$\therefore \text{Rating of capacitor} = 79.492 \text{ KVAR} \quad \text{Ans}$$



- (d) A 2-wire DC distributor cable AB is 2.2 km long and supplies loads of 25 A, 50 A, 75 A at 0.4 km, 1 km and 1.6 km from the point A. Each conductor has a resistance of $0.05 \Omega/\text{km}$. Calculate the potential difference at each point if potential difference of 400 volts is maintained at point A. [12 marks]



$$V_A = 400 \text{ V}$$

$$\text{conductor Resistance} = 0.05 \Omega/\text{km} = r$$

$$\therefore \text{Distributor Resistance} = 2r = 0.1 \Omega/\text{km}$$

$$\text{NOW Total Current Supplied From A} = 25 + 50 + 75 = 150 \text{ A}$$

$$\text{NOW } V_C = V_A - (r_{AC} \times I)$$

$$V_C = 400 - 0.4 \times 0.1 \times 150 = 394 \text{ V Ans} \Rightarrow V_C = 394 \text{ V}$$

$$\text{NOW } V_D = V_A - (r_{AC} \times I) - (r_{CD} \times (I - 25))$$

$$= 394 - (0.6 \times 0.1 \times 125)$$

$$V_D = 386.5 \text{ V} \text{ Ans}$$



$$V_E = V_D - (r_{DE}) \times (I - 75)$$

$$= 386.5 - 0.6 \times 0.1 \times 75$$

$$V_E = 382 \text{ V} \text{ Ans.}$$

$$\text{NOW } V_B = V_E = 382 \text{ V} \text{ Ans}$$

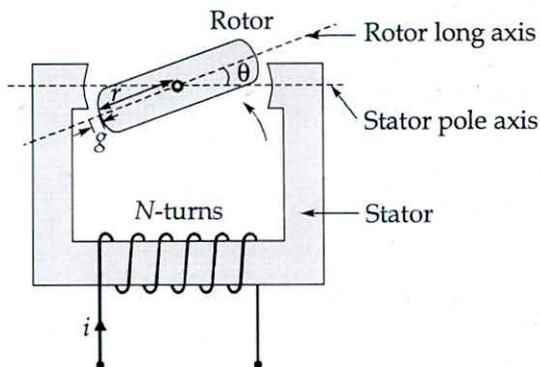
At all points

$$\begin{aligned}
 V_A &= 400\text{V} \\
 V_B &= V_E = 382\text{V} \\
 V_C &= 394\text{V} \\
 V_D &= 386.5\text{V} \\
 V_E &= 382\text{V}
 \end{aligned}$$

} An.

Q.5 (e) For the electro-mechanical configuration shown in figure, assume all the field energy is present in the overlapping regions. Radius is r and the airgap length is g . Calculate the magnitude of torque, when the maximum flux density in the airgap is limited to 2.2 T. The other data are as follows:

- Radius, $r = 50\text{ mm}$,
- Gap length, $g = 2\text{ mm}$,
- Length normal to radius is $l = 10\text{ mm}$.



[12 marks]

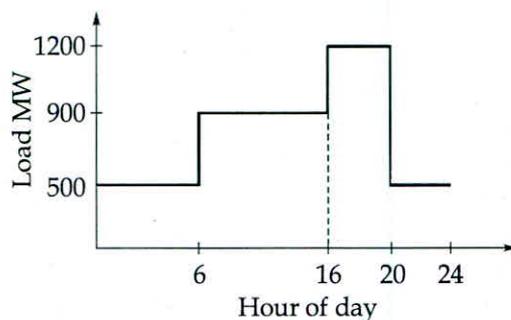
Solution

Q.6 (a) The fuel cost characteristics of two thermal plants are as under,

$$C_1 = 7700 + 52.8P_1 + 5.5 \times 10^{-3} P_1^2 \text{ Rs/hour}$$

$$C_2 = 2500 + 15P_2 + 0.05 P_2^2 \text{ Rs/hour}$$

The limit of generation for the two units are $200 \leq P \leq 800$ MW. The load curve is shown in figure below. Find the daily operating schedule to minimize the operating costs. The cost of taking a unit off and then putting it on is Rs 1000.00.



[20 marks]

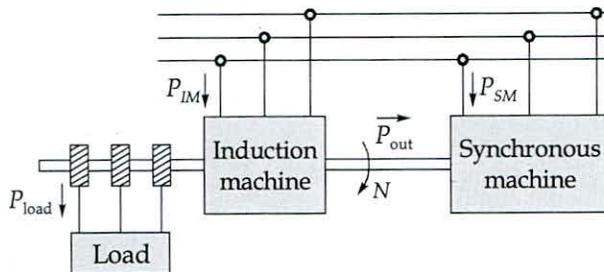
- Q.6 (b) (i) The incremental fuel costs for two units of a plant are $\lambda_1 = \frac{df_1}{dP_{g1}} = 0.012P_{g1} + 8.0$;
 $\lambda_2 = \frac{df_2}{dP_{g2}} = 0.008P_{g2} + 9.6$ where f is in (Rs/hour) and P_g is in megawatts (MW). If both units operate at all times and maximum and minimum loads on each unit are 550 and 100 MW respectively then find λ of the plant in Rs/MWh versus plant output in MW for economic dispatch as total load varies from 200 to 1100 MW.
- (ii) Find the saving in Rs/hour for economic dispatch of load between the units of part (i) compared with their sharing the output equally when the total plant output is 600 MW.

[20 marks]



c) A 3- ϕ wound-rotor induction machine is mechanically coupled to a 3- ϕ synchronous machine as shown in figure. The synchronous machine has 4-poles and the induction machine has 6-poles. The stator of the two machines are connected to a 3- ϕ , 50 Hz supply. The rotor of the induction machine is connected to a 3- ϕ resistive load. Neglect rotational losses and stator resistance losses. The load power is 1 p.u. The synchronous machine rotates at the synchronous speed.

- (i) The rotor rotates in the direction of the stator rotating field of the induction machine. Determine the speed, frequency of the current in the resistive load, and power taken by the synchronous machine and by the induction machine from the source.
- (ii) Repeat part (i) if the phase sequence of the stator of the induction machine is reversed.



[20 marks]



a) The primary, secondary and tertiary winding of a three-winding transformer are rated as 11 kV, 6 MVA, star/3.3 kV, 3 MVA, star/400 V, 3 MVA, delta respectively. The short circuit tests on this transformer gave the following results:

Secondary shorted ; primary excited : 500 V, 100 A

Tertiary shorted ; primary excited : 600 V, 100 A and

Tertiary shorted ; secondary excited : 100 V, 200 A

- (i) Find the per unit leakage reactances of the star equivalent circuit. Neglect resistance.
 (ii) The primary is energized at rated voltage and the secondary is open circuited. For a three-phase balanced short circuit at the tertiary terminals, calculate the short circuit current and the secondary terminal voltage.

[20 marks]

for primary wdg : 11kV, 6MVA

$$I_{base} = \frac{6 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 314.92 \text{ A}$$

Secondary wdg : 3.3kV, 3MVA

$$I_{base} = \frac{3 \times 10^6}{\sqrt{3} \times 3.3 \times 10^3} = 524.86 \text{ A}$$

Tertiary wdg : 400V, 3MVA

$$I_{base} = \frac{10^6 \times 3}{\sqrt{3} \times 400} = 4330.12 \text{ A}$$

Now Secondary Shorted, primary excited

$$V = \frac{500}{11 \text{ kV}} \text{ PU} \quad ; \quad I = \frac{100}{314.92} \text{ PU}$$

$$= 0.04545 \text{ PU} \quad ; \quad I = 0.3175 \text{ PU}$$

$$\text{Now } X_{12} = \frac{0.04545}{0.3175} = 0.143 \text{ PU}$$

Similarly Tertiary Shorted, primary excited

$$V = \frac{600}{11 \text{ kV}} \text{ PU} \quad ; \quad I = \frac{100}{314.92} \text{ PU} = 0.3175 \text{ PU}$$

$$= 0.05454 \text{ PU}$$

$$\text{Now } X_{13} = \frac{0.05454}{0.3175} \text{ PU} = 0.1717 \text{ PU}$$

Now Tertiary Shorted, Secondary excited

$$V = \frac{100}{3.3kV} \text{ pu} ; I = \frac{200}{524.86} \text{ pu}$$

$$= 0.03 \text{ pu} \quad = 0.381 \text{ pu}$$

$\therefore X_{23} = 0.0787 \text{ pu}$

Now $X_{12} = X_1 + X_2 = 0.143 \text{ --- (i)}$

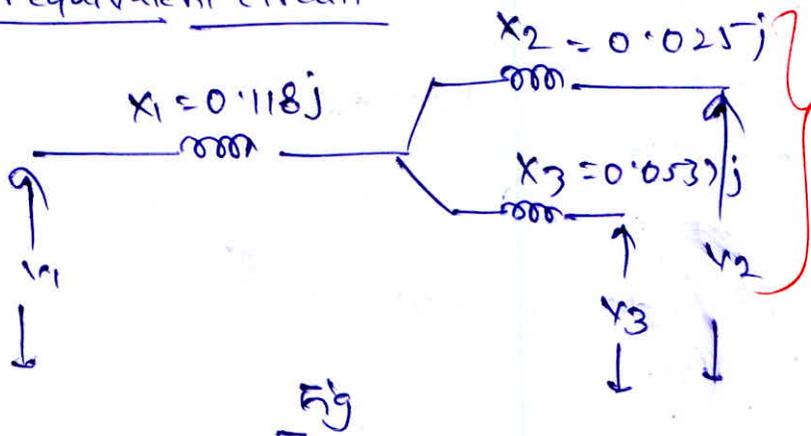
$X_{23} = X_2 + X_3 = 0.0787 \text{ --- (ii)}$

$X_{13} = X_1 + X_3 = 0.1717 \text{ --- (iii)}$

(i)

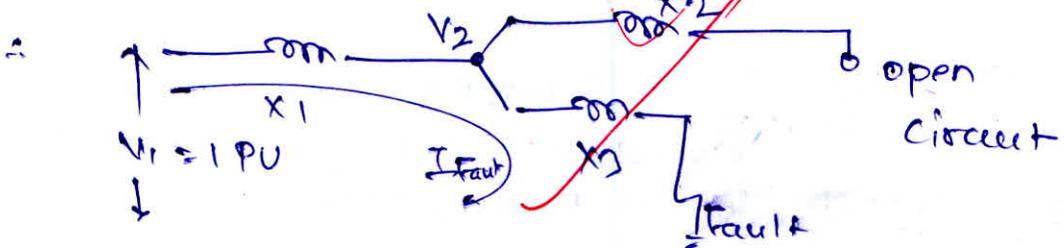
Solving $X_1 = 0.118 \text{ pu} ; X_2 = 0.025 \text{ pu} ; X_3 = 0.0537 \text{ pu}$

\therefore Star equivalent circuit



(ii) Primary excited, Secondary open

Short circuit at tertiary terminal.



$$\therefore I_{\text{Fault}} = \frac{1}{X_1 + X_3} = \frac{1}{0.118j + 0.0537j}$$

$$\therefore \boxed{I_{\text{Fault}} = 5.824 \text{ pu}} \text{ Ans}$$

$$\begin{aligned} \therefore \text{Short circuit current} &= 0.5824 \text{ PU} \\ &= 5.824 \times 314.92 \text{ A} \\ &= 1834.13 \text{ A} \end{aligned}$$

Now $V_2 = I_{\text{Fault}} \times X_2 = 5.824 \times 0.0537 \text{ PU}$

$$V_2 = 0.3127 \text{ PU}$$

Secondary terminal voltage

KV?

16

~~$V_2 = 0.3127 \text{ x}$~~

b) A 4-pole, 50-Hz turbo-alternator is rated at 45 MW, 0.8 pf lag and has an inertia of 25000 kg-m². It is connected via a transmission system to another set whose corresponding data is 2-pole, 50 Hz, 60 MW, 0.75 lag, 9000 kg-m². Calculate the inertia constant of each set on its own rating and that of the single equivalent set connected to an infinite bus-bar and on a base rating of 100 MVA.

[20 marks]

Alternator 1

$P = 4, f = 50 \text{ Hz}, S = \frac{45}{0.8} = 56.25 \text{ MVA}; J = 25 \times 10^3 \text{ kgm}^2$

Alternator 2

$P = 2, f = 50 \text{ Hz}, S = \frac{60}{0.75} = 80 \text{ MVA}; J = 9000 \text{ kgm}^2$

NOW AS WE KNOW

Inertia constant 'M'

$$H = \frac{KE}{S} = \frac{\frac{1}{2} J \omega_m^2}{S} \quad ; \quad \omega_m = \frac{4\pi f}{P} \text{ (mech rad/sec)}$$

$$\Rightarrow H = \frac{\frac{1}{2} MW}{S} \quad [\because m = J \omega]$$

Now Alternator 1

$$H_1 = \frac{1}{2} \times 25 \times 10^3 \times \left(\frac{4\pi \times 50}{4} \right)^2 \times 10^{-6} \quad \text{MJ/MVA}$$

56.25

$$H_1 = 5.48 \text{ MJ/MVA}$$

Alternator 2

$$H_2 = \frac{1}{2} \times 9000 \times \left(\frac{4\pi \times 50}{2} \right)^2 \times 10^{-6} \quad \text{MJ/MVA}$$

80

$$H_2 = 5.55 \text{ MJ/MVA}$$

∴ on their own Ratings

$$\left. \begin{array}{l} H_1 = 5.48 \text{ MJ/MVA} \\ H_2 = 5.55 \text{ MJ/MVA} \end{array} \right\} \text{Ans}$$

on a base Rating of 100 MVA

$$\therefore H_1 = K E = \text{constant}$$

$$\therefore H_1 S_1 = H_2 S_2$$

$$\therefore H_{\text{new}} = H_{\text{old}} \left(\frac{S_{\text{old}}}{S_{\text{new}}} \right)$$

$$\therefore H_{1 \text{ new}} = 5.48 \times \frac{56.25}{100} = 3.08 \text{ MJ/MVA}$$

$$H_{1 \text{ new}} = 3.08 \text{ MJ/MVA} \quad \text{Ans}$$

Alternator 2

$$H_{2\text{new}} = H_{2\text{old}} \times \frac{S_{\text{old}}}{S_{\text{new}}}$$

$$= 5.55 \times \frac{80}{100}$$

$$\therefore H_{2\text{new}} = 4.44 \text{ MVA/mVA}$$

For equivalent set connected to infinite bus

if coherently swinging

$$H_{\text{eq}} = H_{1\text{new}} + H_{2\text{new}} = 7.52 \text{ MVA/mVA Ans}$$

if non coherent swinging

$$H_{\text{eq}} = \frac{H_1(\text{new})H_2(\text{new})}{H_{1\text{new}} + H_{2\text{new}}} = 1.818 \text{ MVA/mVA Ans}$$

6

Q.7 (c) The following test results are obtained for a 3- ϕ , 280 V, 60 Hz, 6.5 A induction machine.

Block-rotor test : 44 V, 60 Hz, 25 A, 1250 W

No load test : 208 V, 60 Hz, 6.5 A, 500 W

The average resistance measured between two stator terminals is 0.27Ω .

Determine:

- (i) the no load rotational loss.
- (ii) the output power in horse power (hp) at $s = 0.1$.
- (iii) the efficiency.

(Take 1 hp = 746 W)

[20 marks]

Solution For induction machine (Assuming Δ connected)

(i) No Load Test ($s=0$)

$V_{NL} = 208V$
 $W_{NL} = 500W$
 $I_{NL} = 6.5A$

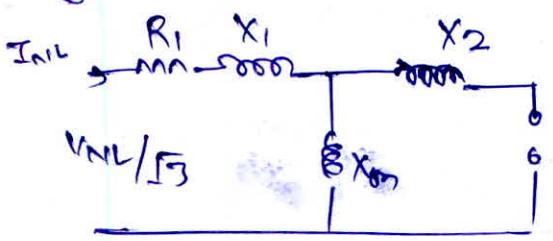


Fig. equivalent circuit under No Load

$$\therefore Z_{NL} = \frac{V_{NL}}{I_{NL}} = \frac{208/\sqrt{3}}{6.5} = 18.475 \Omega$$

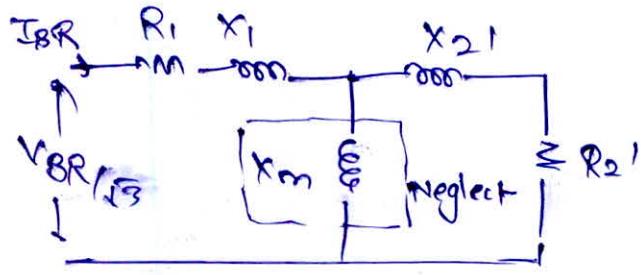
$$R_{NL} = \frac{W_{NL}/3}{I_{NL}^2} = \frac{500}{3 \times 6.5^2} = 3.944 \Omega$$

$$X_{NL} = (Z_{NL}^2 - R_{NL}^2)^{1/2} = 18.05 \Omega$$

From Fig. clearly $X_{NL} = X_1 + X_m = 18.05$ — (1)

Blocked rotor test ($s=1$)

$V_{BR} = 44V$
 $I_{BR} = 25A$
 $W_{BR} = 1250W$



Equivalent circuit under Blocked Rotor

From readings

$$Z_{BR} = \frac{V_{BR}}{I_{BR}} = \frac{44/\sqrt{3}}{25} = 1.016 \Omega$$

$$R_{BR} = \frac{W_{BR}/3}{I_{BR}^2} = \frac{1250}{3 \times 25^2} = 0.67 \Omega$$

$$\text{Now } X_{BR} = \sqrt{Z_{BR}^2 - R_{BR}^2} = 0.7637 \Omega$$

$$\text{Now } X_{BR} = X_1 + X_2' \quad \text{--- } \times$$

$$\text{Here } X_1 = X_2' \quad \therefore X_1 = X_2' = \frac{X_{BR}}{2} = 0.382 \Omega$$

Now From eqn (1)

$$X_m = X_{NL} - X_1 = 18.05 - 0.382$$

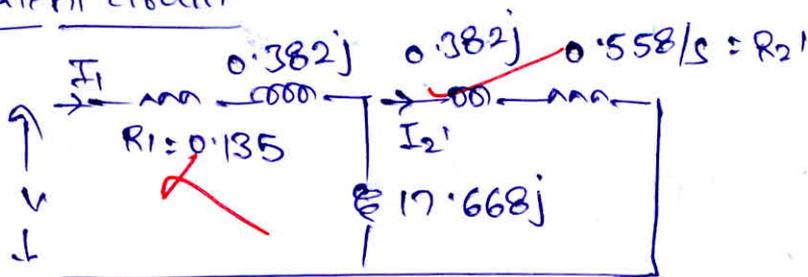
$$\therefore X_m = 17.668 \Omega$$

$$\text{Also } R_2' = \left(\frac{X_2' + X_m}{X_m} \right)^2 \times (R_{BR} - R_1)$$

$$R_2' = \left(\frac{0.382 + 17.668}{17.668} \right)^2 \times \left(0.67 - \frac{0.27}{2} \right)$$

$$R_2' = 0.558 \Omega$$

Now equivalent circuit



No load Rotational loss

$$\text{During No load test } P_{rot} = P_{NL} - 3 \times I_{NL}^2 R_1$$

$$P_{rot} = 500 - 3 \times 6.5^2 \times \frac{0.27}{2} = 500 - \frac{34.22}{2}$$

$$P_{rot} = 482.89 \text{ W}$$

now (ii) For output power at $s=0.1$

$$I_1 = \frac{V}{Z_{eq}} \quad ; \quad Z_{eq} = (0.135 + 0.382j) + 17.688j \parallel \left(\frac{0.382j}{0.1} + 5.58 \right)$$

$$\therefore Z_{eq} = 5.5 \angle 24.28^\circ \Omega$$

$$\text{now } I_1 = \frac{280/\sqrt{3}}{5.5 \angle 24.28^\circ} = 29.4 \angle -24.28^\circ \text{ A}$$

$$\therefore P_{input} = \sqrt{3} \times 280 \times 29.4 \times \cos(24.28)$$

$$\boxed{P_{input} = 12997.05 \text{ W}}$$

$$\text{Now } I_2' = I_1 \times \frac{17.688j}{17.688j + 0.382j + 5.58} = 27.497 \angle -7.12^\circ \text{ A}$$

$$\text{now } P_g = 3 I_2'^2 R_2' / s = 3 \times 27.497^2 \times 5.58$$

$$P_g = 12656.86 \text{ W}$$

$$P_m = (1-s) P_g = 0.9 P_g = 11391.17 \text{ W}$$

$$\text{now } P_{output} = P_m - P_{rot} = 10908.28 \text{ W}$$

$$\therefore \boxed{P_{output} (\text{hp}) = 14.622 \text{ hp}} \quad \text{Ans}$$

$$(iii) \quad \eta = \frac{P_o/p}{P_i/p} \times 100 = \frac{10908.28}{12997.05} \times 100$$

$$\boxed{\eta = 83.92\%} \quad \text{Ans}$$

⊖

Figure below shows the single line diagram of a sample 3-bus power system. Data for this system are given in table-1 and table-2.

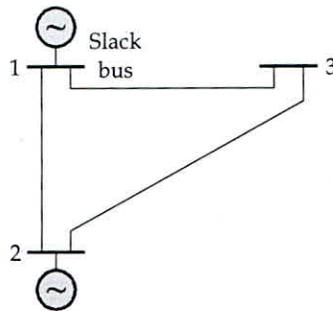


Table 1: Scheduled generation and loads and assumed bus voltage

Bus code <i>i</i>	Assumed Bus voltage	Generation		Load	
		MW	MVAr	MW	MVAr
1 (slack bus)	$1.05 + j0.0$	-	-	0	0
2	$1 + j0.0$	50	30	305.6	140.2
3	$1 + j0.0$	0	0	138.6	45.2

Base MVA = 100

Table 2: Line impedance

Bus code <i>i - k</i>	Impedance Z_{ik} (p.u.)
1 - 2	$0.02 + j0.04$
1 - 3	$0.01 + j0.03$
2 - 3	$0.0125 + j0.025$

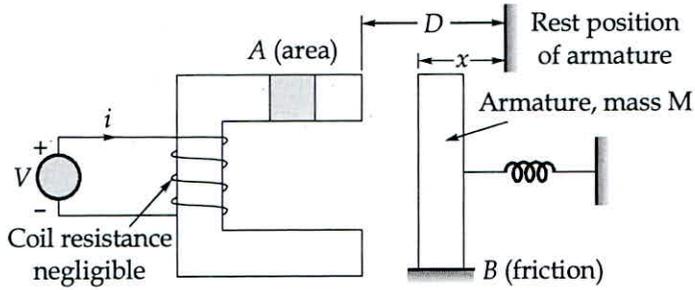
Using the Gauss-Seidel method, determine the phasor values of voltages at buses 2 and 3. Perform one iteration only.

[20 marks]

For the electromechanical system shown in figure, the area of cross-section of core is A and the air-gap flux density under steady operating condition is $B(t) = B_m \sin \omega t$.

Find:

- (i) the coil voltage.
- (ii) the force of field origin as a function of time.
- (iii) the displacement of armature as a function of time.



[20 marks]

Q.8 (c) A 1- ϕ 10 kVA, 2400/240 V, 50 Hz distribution transformer has the following characteristics.

Core loss at rated voltage = 100 W

Copper loss at half load = 60 W

(i) Determine the per unit rating at which the transformer efficiency is maximum. Also determine this efficiency if the load power factor is 0.9 (lag).

(ii) The transformer has the following load cycles

no load for 6 hours.

70% full load for 10 hours at 0.8 p.f.

90% full load for 8 hours at 0.9 p.f.

Determine the all-day efficiency of the transformer:

(iii) If the above transformer is connected as autotransformer then, determine the maximum kVA rating and for this rating determine the efficiency when delivering full load at 0.8 power factor lagging.

[20 marks]

Space for Rough Work

Space for Rough Work

$$HS = \frac{1}{2} \frac{mv^2}{r} \quad e = cv$$

$$\frac{2HS}{\omega} = M \quad r + \frac{e}{v} = c$$

$$\frac{KW}{KVA} = pf$$

$$P + MD = n + \frac{N}{A}$$

$$m = (P + MD)$$

$$\partial P = n i^2$$

$$n - MDn = n i^2$$

$$m^2 = n MD - n i^2 = 0$$

$$m = \frac{MD \pm \sqrt{MD^2 + 4n i^2}}{2}$$

$$\frac{2}{150} \frac{of}{P} \times \frac{2\pi}{60}$$

$$\frac{4\pi f}{P}$$

$$a \rightarrow aD$$

$$\frac{4}{aD}$$



