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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-5 : Basic Electronics Engineering + Analog Electronics

+ Electrical Materials

+ Electrical Machines - 1 + Power Systems - 2

Name : Kartikeya Singh

Roll No :

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Test Centres

Delhi Bhopal Noida Jaipur Indore
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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

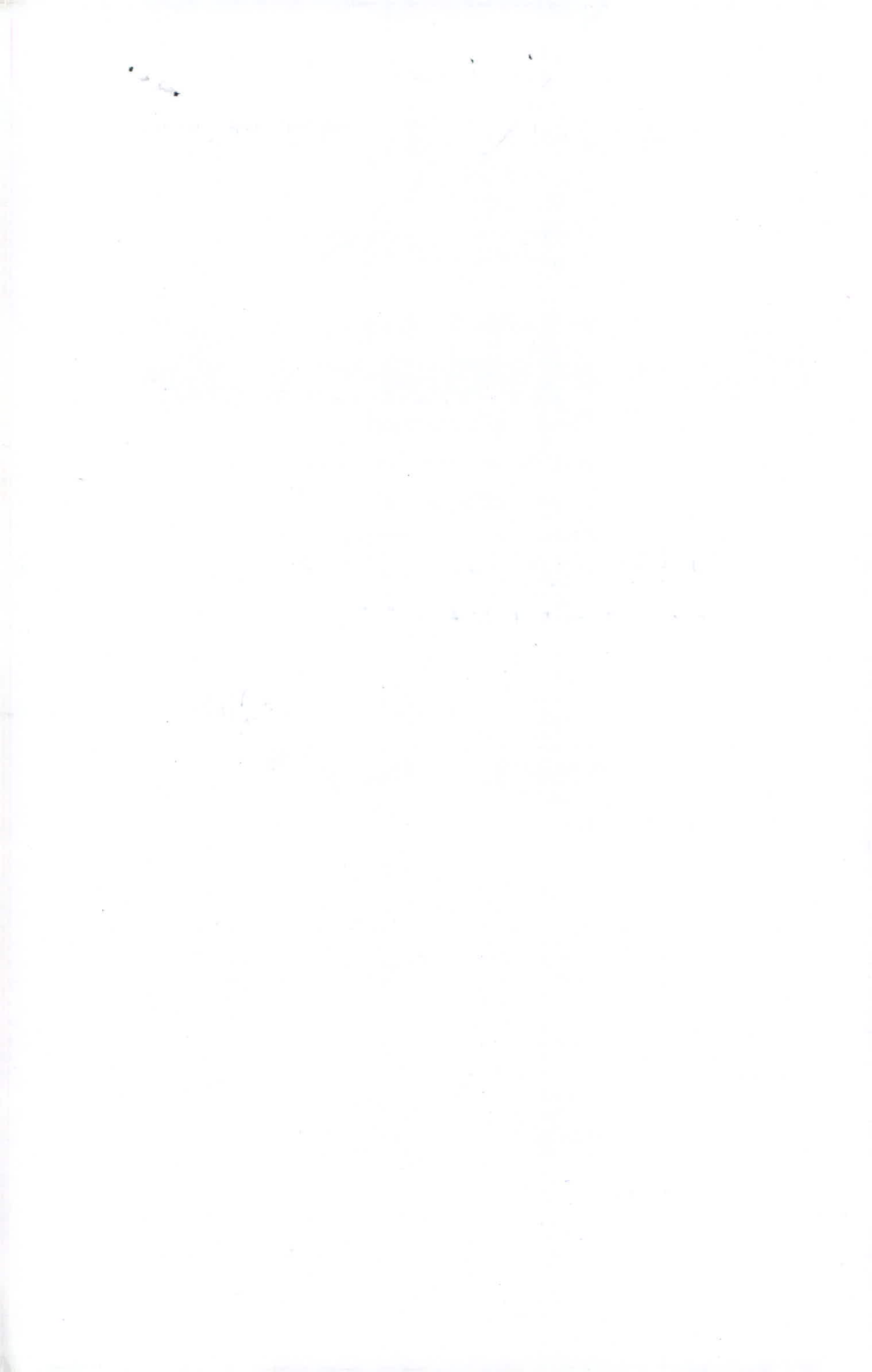
Question No.	Marks Obtained
Section-A	
Q.1	53
Q.2	
Q.3	55
Q.4	58
Section-B	
Q.5	34
Q.6	
Q.7	
Q.8	50
Total Marks Obtained	250

Good

Signature of Evaluator

Sourabh.kumar

Cross Checked by



Section A : Basic Electronics Engg. + Analog Electronics + Electrical Materials

(a) A conducting bar of 20 μm length, 2 μm wide and 1 μm thick is taken. Find the resistance of the bar if it is

(i) n-doped Silicon with $N_D = 10^8/\text{cm}^3$.

(ii) p-doped Silicon with $N_A = 10^{10}/\text{cm}^3$.

take $\mu_n = 2.5 \mu_p = 1200 \text{ cm}^2/\text{Vs}$ and n_i for Silicon is $1.5 \times 10^{10}/\text{cm}^3$.

[12 marks]

(i) n doped Silicon $N_D = 10^8/\text{cm}^3$, $N_A = 0$

Now using charge Neutrality

$$p + N_D = n$$

$$10^8 + p = n \quad \text{--- (1)}$$

Using law of mass action $n_i^2 = np$.

$$\text{hence } p = \frac{(1.5 \times 10^{10})^2}{n} \quad \text{--- (2)}$$

Substituting eq (2) in eq (1)

$$10^8 + \frac{(1.5 \times 10^{10})^2}{n} = n$$

$$n^2 - 10^8 n - (1.5 \times 10^{10})^2 = 0$$

Solving we get $n = 1.505 \times 10^{10} / \text{cm}^3$

$$\text{hence } p = \frac{(1.5 \times 10^{10})^2}{1.505 \times 10^{10}} = 1.495 \times 10^{10} / \text{cm}^3$$

Conductivity

$$\sigma = (p\mu_p + n\mu_n)q$$

$$= 1.6 \times 10^{-19} \left(1.495 \times 10^{10} \times \frac{1200}{2.5} + 1.505 \times 10^{10} \times 1200 \right)$$

$$\sigma = 4.0377 \times 10^{-6} \text{ } \cancel{\text{cm}^{-1} \text{ cm}^{-1}} \text{ cm}^{-1} = \cancel{4.0377} \times 10^{-6}$$

$$\text{hence Resistance} = \frac{l}{\sigma A} = \frac{1}{4.0377 \times 10^{-6}} \times \frac{20 \times 10^{-4}}{2 \times 1 \times 10^{-8}} \quad (\text{all are in cm})$$

$$= \underline{2.4766 \times 10^{10} \text{ ohms}}$$

Case (ii) $N_A = 10^{10} / \text{cm}^3$

$$N_D = 0$$

$$p = n + N_A$$

$$\frac{n_i^2}{n} = n + N_A$$

$$n^2 + n N_A - n_i^2 = 0$$

$$n^2 + 10^{10} n - (1.5 \times 10^{10})^2 = 0$$

$$n = 1.081 \times 10^{10} / \text{cm}^3$$

$$\text{and } p = \frac{(1.5 \times 10^{10})^2}{1.081 \times 10^{10}} = 2.081 \times 10^{10} / \text{cm}^3$$

hence $\sigma = (n \mu_n + p \mu_p) q$

$$\sigma = (1.081 \times 10^{10} \times 1200 + 1200 \times 2.081 \times 10^{10}) \times 1.6 \times 10^{-19}$$

$$\sigma = 3.674 \times 10^{-6} \text{ } \overset{2.5}{\Omega\text{-cm}^{-1}}$$

$$R = \frac{l}{\sigma A}$$

$$R = \frac{1}{3.674 \times 10^{-6}} \times \frac{20 \times 10^{-4}}{2 \times 1 \times 10^{-8}} \text{ } \overset{\text{cell}}{\text{cm}}$$

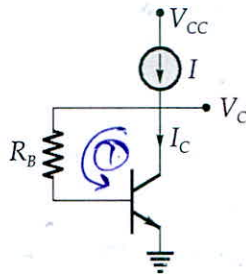
$$R = 2.7218 \times 10^{10} \Omega$$



Optimum utilization of space

Q.1 (b)

A circuit that can provide a very large voltage gain for a high resistance load is shown in figure below. Find the value of current I and R_B to bias the BJT at $I_C = 3 \text{ mA}$ and $V_C = 1.5 \text{ V}$ for $\beta = 90$.



[12 marks]

$$V_C = 1.5 \text{ V}, I_C = 3 \text{ mA}$$

applying KVL loop ①

$$-V_C + I_B R_B + V_{BE} = 0$$

$$I_B R_B = V_C - V_{BE}$$

$$I_B R_B = 1.5 - 0.7$$

$$I_B R_B = 0.8 \text{ V}$$

$$R_B = \frac{0.8}{I_B} = \frac{0.8}{(I_C / \beta)} = \frac{90 \times 0.8}{3 \times 10^{-3}}$$

$$R_B = 24000 \Omega \text{ or } 24 \text{ k}\Omega$$

Now applying KCL at V_C

$$I_e = I_c + I_B.$$

$$I = (1 + \beta) I_B.$$

$$= 91 \times \frac{I_c}{\beta}.$$

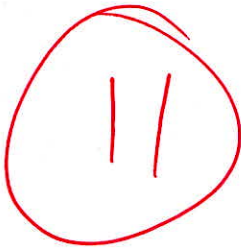
$$= \frac{91}{90} \times 3 \times 10^{-3}$$

$$= 3.03 \times 10^{-3} \text{ A}$$

or

$$I = \underline{3.03 \text{ mA}}$$

Good



- Q.1 (c) A material with magnetic property such that when it was placed in a magnetic field, $B = 4 \text{ Wb/m}^2$, magnetic field intensity was found to be 4800 A/m . If \vec{H} is reduced to 640 A/m and $B = 1.8 \text{ Wb/m}^2$, then calculate the percentage change in magnetization M of the material.

[12 marks]

Case ①

$$B = 4 \text{ Wb/m}^2$$

$$H = 4800 \text{ A/m}$$

We have $B_1 = \mu_0 (H_1 + M_1)$

$$H_1 + M_1 = \frac{B_1}{\mu_0}$$

$$M_1 = \frac{B_1}{\mu_0} - H_1$$

$$M_1 = \frac{4}{4\pi \times 10^{-7}} - 4800 = 3178298.86 \text{ A/m}$$

Case ②

$$B = 1.8 \text{ Wb/m}^2$$

$$H = 640 \text{ A/m}$$

hence $M_2 = \frac{B_2}{\mu_0} - H_2$

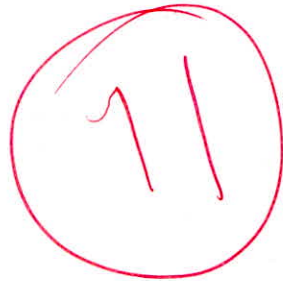
$$= \frac{1.8}{4\pi \times 10^{-7}} - 640 = 1431754.48 \text{ A/m}$$

% change in $\frac{M_2 - M_1}{M_1} \times 100$

$$\frac{1431754.48 - 3178298.86}{3178298.86} \times 100$$

$$= -54.95\%$$

Good



- (d) What is the significance of 'Magnetic dipole' and 'Magnetization' phenomena in magnetic materials? Explain clearly with the help of definition and mathematical derivation. How are above two phenomena related to each other? [12 marks]

Magnetic dipole:- A current loop of small size produces a magnetic field is equivalent to a magnetic dipole or a small magnetic bar which have North and south poles separated by small distance is known as magnetic dipole.

→ In the materials atoms, or molecules are also treated as magnetic dipole because of angular momentum of charge in electrons (mainly) and Nucleus. Dipole moment is given as

$$\vec{p} = I \vec{A}$$

$I = \text{Current}$
 $A = \text{area vector}$

Magnetisation:- Magnetization is process of creation of magnetic dipole and mathematically is defined as magnetic dipole moment per unit volume.

$$\vec{M} = \frac{1}{\Delta V} \sum_{n=0}^n \vec{p}_{m_n}$$

Where $n =$ no. of ^{dipoles} electrons per unit volume and μ_m is dipole moment of each dipole.

* Magnetization $\boxed{\vec{M} = n\vec{\mu}_m}$ A/m.

assuming all dipoles have dipole moment of μ_m value and $n \rightarrow$ no. of dipoles per unit volume.

Significance:-

Magnetic dipoles are nothing but generators of magnetic field and every material is made up of magnetic dipoles. Degree of magnetization is defined by Magnetization vector \vec{M} which is magnetic dipole moment per unit volume. Gives an idea about ~~how deeply a spe~~ magnetic characteristics of materials.

*Neat handwriting
and presentation*

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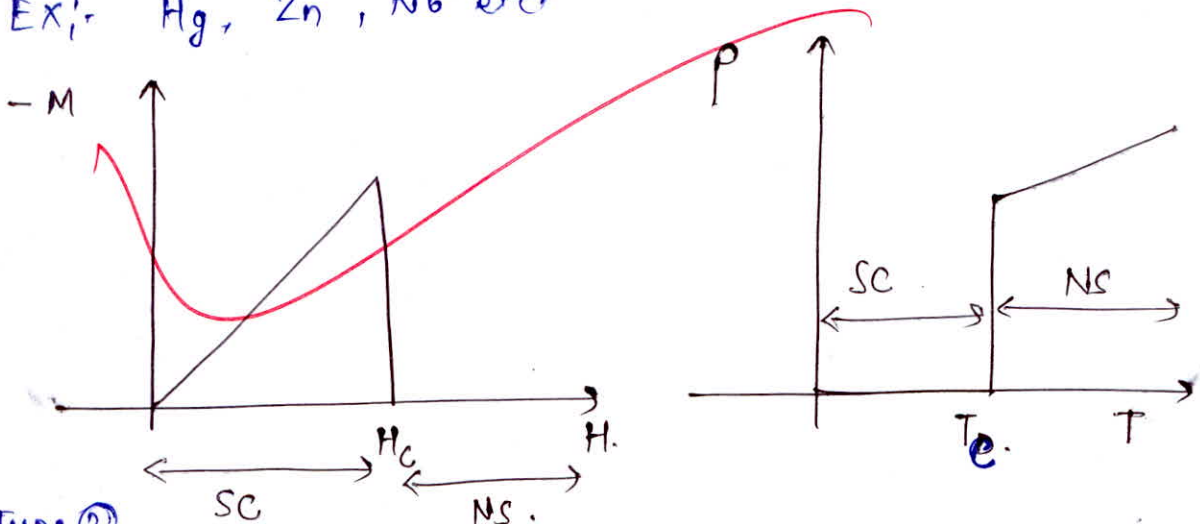
(e) What are type-I and type-II superconductors? Draw the magnetization versus magnetic field characteristic for type-I and type-II superconductors. Why superconductivity is observed for signals upto radio frequencies?

[4 + 4 + 4 marks]

Type I :-

Type I superconductors are called soft superconductors they have small values of H_c and T_c (Critical field and temperature) They follow Meissner effect and Sibley rule completely. They change from normal state to S.C. state and vice versa abruptly at temperature T_c and critical field H_c .

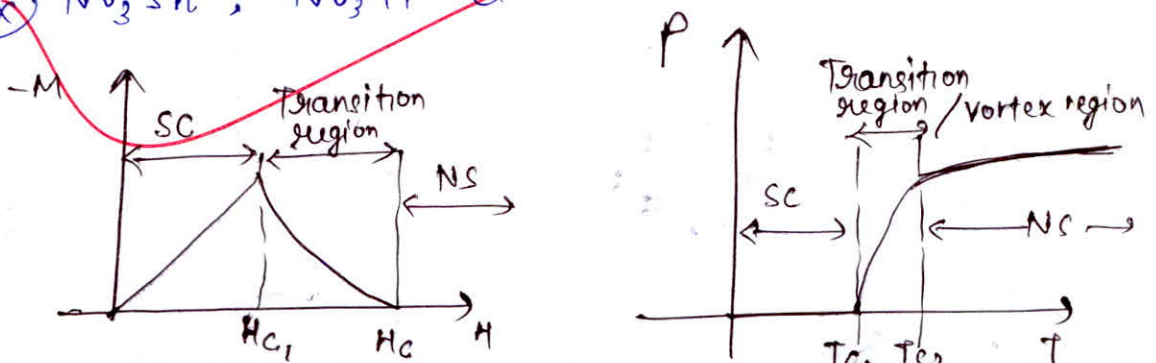
EX: Hg, Zn, Nb etc.



Type II

Type II superconductors are called hard superconductors they have higher values of H_c and T_c . They don't follow or show Meissner effect and Sibley rule completely. They change from Normal state to S.C. state and vice versa gradually.

EX: Nb_3Sn , Nb_3Ti etc.



Super conductivity is observed upon radio frequency beyond which super conductivity gets destroyed because beyond radio frequency skin effect dominates and increases resistance of material to high value destroying super conductivity.

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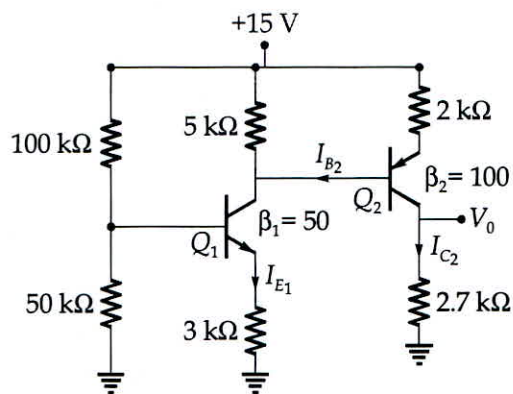


- (a) The copper crystal has FCC unit cell configuration. If radius of Cu atom is 0.148 nm and atomic mass of Cu is 63.5 gm mol⁻¹ then calculate atomic packing fraction (APF), the atomic concentration in a unit cell and density of Cu atom in gcm⁻³.
(Take Avogadro number : 6.023×10^{23} mol⁻¹)

[20 marks]

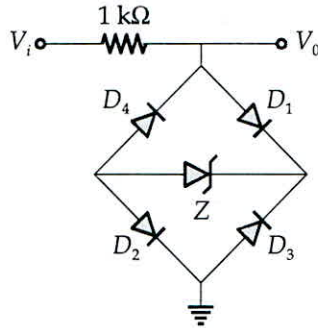


(b) In the below configuration, calculate the values of I_{B_2} , I_{C_2} , I_{E_1} and V_0 .



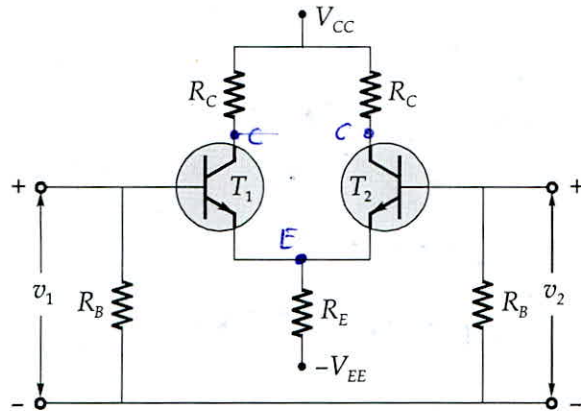
[20 marks]

- (c) Sketch the transfer characteristics of the circuit given below for $-20\text{ V} \leq V_i \leq 20\text{ V}$. Assume that diodes can be represented by a piece-wise linear model with $V_{D0} = 0.65\text{ V}$ and $r_D = 20\ \Omega$. Assuming that the specified zener voltage at a current of 10 mA is 8.2 V and $r_Z = 20\ \Omega$. Represent the Zener by a piece-wise linear model.



[20 marks]

- (a) (i) The BJT in the differential amplifier circuit shown below have negligible leakage current and $\beta_1 = \beta_2 = 60$. Also $R_C = 6.8 \text{ k}\Omega$, $R_B = 10 \text{ k}\Omega$ and $V_{CC} = V_{EE} = 15 \text{ V}$. Find the value of R_E needed to bias the amplifier such that $V_{CEQ1} = V_{CEQ2} = 8 \text{ V}$.



[10 marks]

DC equivalent or DC Analysis

KVL in loop ①

$$V_{BE1} + 2I_E R_E - V_{EE} = 0$$

$$R_E I_E = \frac{V_{EE} - V_{BE1}}{2}$$

$$= \frac{15 - 0.7}{2}$$

$$R_E I_E = 7.15 \text{ V} \quad \text{--- ①}$$

KVL in loop ②

$$-V_{CC} + I_C R_C + V_{CE1} + 2I_E R_E - V_{EE} = 0$$

$$I_C R_C + 8 + 2I_E R_E = V_{EE} + V_{CC} = 15 + 15$$

$$I_C R_C + 2I_E R_E = 30 - 8$$

$$I_C R_C = 22 - 7.15 \times 2$$

$$I_C R_C = 7.7 \text{ Volts}$$

$$I_C = \frac{7.7}{6.8 \text{ k}} = 1.1323 \text{ mA}$$

from eq ①

$$I_E R_E = 7.15 \text{ V}$$

$$R_E = \frac{7.15}{1.13235 \times \frac{(\beta+1)}{\beta}}$$

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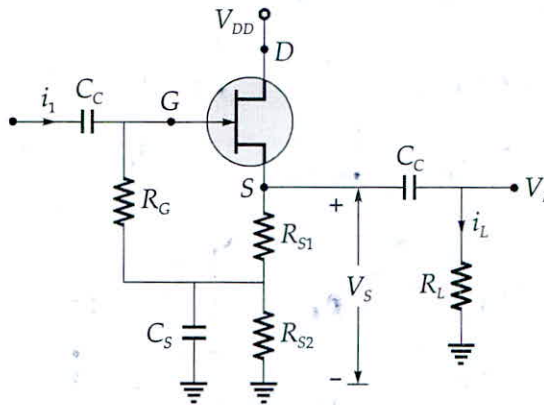
$$R_E = \frac{7.15 \times 60}{61 \times 1.3235 \text{ mA}}$$

Slight deviation

$$R_E = 6.21 \text{ k}\Omega$$

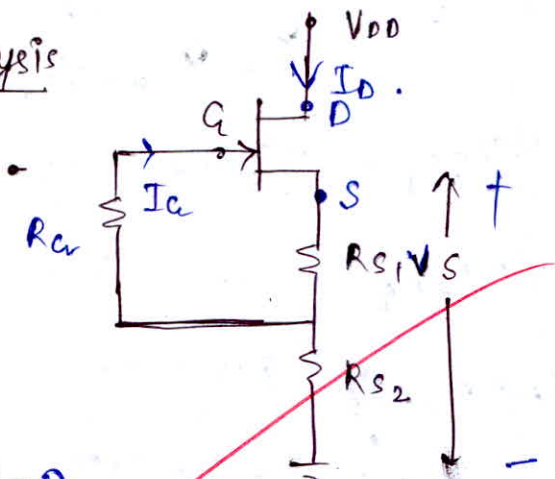
$$6.08 \text{ k}\Omega$$

- Q.3 (a) (ii) In the circuit shown below $R_G \gg R_{S1}, R_{S2}$. The JFET is described by $I_{DSS} = 10 \text{ mA}$, $V_P = 4 \text{ V}$, $V_{DD} = 15 \text{ V}$, $V_{DSQ} = 10 \text{ V}$ and $V_{GSQ} = -2 \text{ V}$. Find the value of R_{S1} and R_{S2} to set amplifier at above Q-point and also find the value of V_S .



[10 marks]

DC Analysis



here $I_G = 0$

$$\text{hence } V_{GS} = V_G - V_S$$

$$= \frac{I_D \times R_{S2}}{R_{S1} + R_{S2}} - I_D \times (R_{S1} + R_{S2})$$

$$V_{GS} = -I_D R_{S1}$$

$$-I_D R_{S1} = -2$$

$$I_D R_{S1} = 2V \quad \text{--- (1)}$$

Now

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 \quad (V_P = -4V \text{ for n channel})$$

$$I_D = 10 \times 10^{-3} \left(1 - \frac{-2}{-4} \right)^2 \text{ mA}$$

$$= 10 \left[1 - \frac{1}{2} \right]^2$$

$$= 10 \times \frac{1}{4} = 2.5 \text{ mA}$$

hence from eq (1)

$$R_{S1} = \frac{2V}{2.5 \text{ mA}} = 0.8 \text{ k}\Omega \text{ or } 800\Omega$$

$$R_{S1} = 800\Omega$$

and $V_{DSQ} = 10V$

$$V_{DD} - I_D (R_{S1} + R_{S2}) = 10$$

$$\frac{15-10}{2.5 \text{ mA}} = R_{S1} + R_{S2}$$

$$R_{S1} + R_{S2} = 2 \text{ k}\Omega$$

$$R_{S2} = 2 - 0.8$$

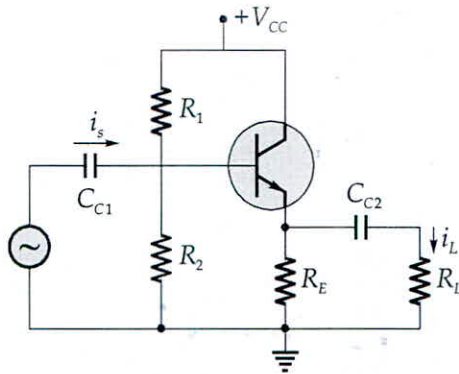
$$= 1.2 \text{ k}\Omega \text{ or } \underline{\underline{1200\Omega}}$$

Now $V_S = I_D (R_{S1} + R_{S2})$

$$= 2.5 \times 10^{-3} \times (1.2 + 0.8) \times 10^3$$

$$V_S = 2.5 \times 2 = \underline{\underline{5V}}$$

Q.3 (b) Consider the amplifier circuit shown below:



The parameters of BJT and the circuit are, $\beta = 80$, $V_{CC} = 10\text{ V}$, $V_{CEQ} = 5\text{ V}$, $V_{BE(\text{on})} = 0.7\text{ V}$ and $R_E = R_L = 500\ \Omega$. Design the values of R_1 and R_2 such that the mid-band current gain $A_i = \frac{i_L}{i_s} = 8$. Assume that $V_T = 26\text{ mV}$.

DC analysis

[20 marks]

Now KVL in
loop ①

$$V_{CC} = V_{CE} + I_E R_E$$

$$10 - 5 = I_E R_E$$

$$I_E R_E = 5$$

$$I_E = \frac{5}{500} = 0.01\text{ A or } 10\text{ mA}$$

hence

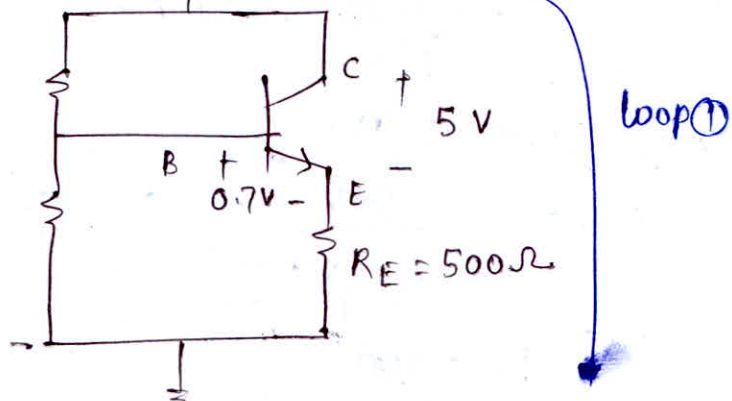
so

$$g_m = \frac{I_c}{V_T}$$

$$g_m = \frac{9.876\text{ mA}}{26} = 0.37986\text{ A/V}$$

and $g_m \mu_\pi = \beta$

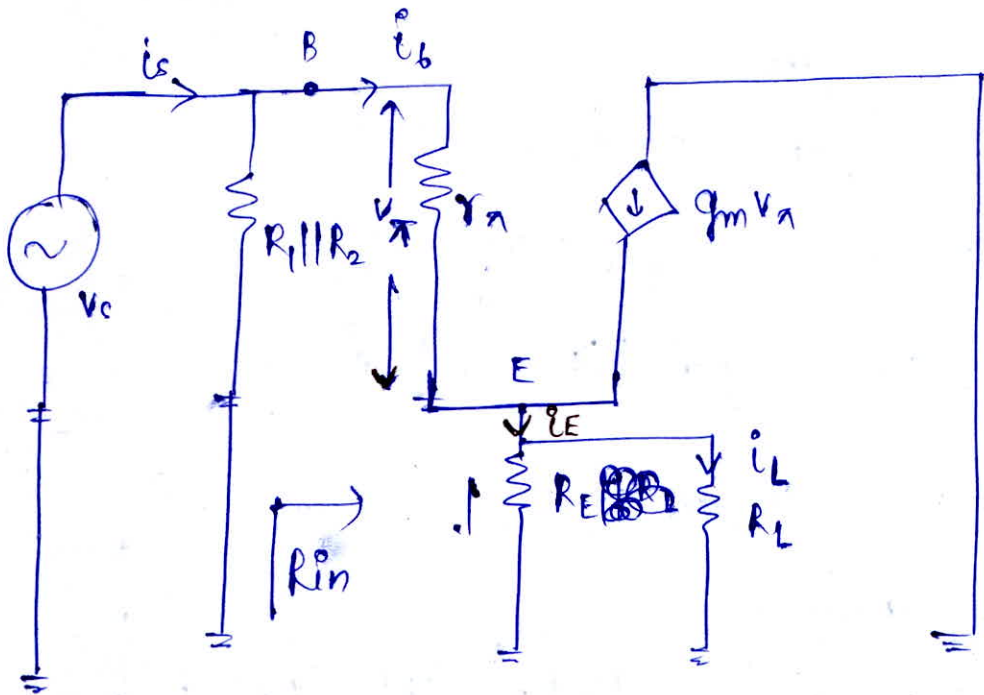
$$\mu_\pi = \frac{80}{0.37986} = 210.6\ \Omega$$



$$I_c = \frac{80 \times 10}{81}$$

$$I_c = 9.876\text{ mA}$$

AC model



here $i_L = i_E \times \frac{R_E}{R_E + R_L}$

$i_L = i_E \times \frac{500}{1000} = i_E / 2$ — (i)

and KCL at E

$i_b + g_m v_{\pi} = i_E$

$i_E = i_b + g_m \times i_b \times r_{\pi}$

$i_E = i_b (1 + \beta)$

$i_E = 81 i_b$ — (ii)

hence $\frac{i_L}{i_b} = \frac{i_E / 2}{i_E / 81} = \frac{81}{2} = \underline{\underline{40.5}}$

Now $\frac{i_L}{i_s} = \frac{i_L}{i_b} \times \frac{R_1 || R_2}{R_1 || R_2 + R_{in}}$

$$\beta = 40.5 \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_{in}}$$

$$\begin{aligned} R_{in} &= r_x + (1 + \beta)(r_e \parallel r_L) \\ &= 210.6 + 81 \times (250) \\ &= 20.46 \text{ k}\Omega \end{aligned}$$

$$\beta / 40.5 = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + 20.46 \text{ k}\Omega}$$

$$R_{Th} + 20.46 \text{ k}\Omega = R_{Th} \times \frac{40.5}{8}$$

$$\frac{20.46}{4.0625} = R_{Th} \quad (4.0625)$$

$$R_{Th} = 5.0363 \text{ k}\Omega$$

from DC analysis

$$V_{Th} = R_{Th} i_b + i_e R_E + V_{BE}$$

$$V_{Th} = \frac{5.0363 \times 10}{81} + 0.7$$

$$+ 10 \times 0.50$$

$$V_{Th} = 6.3217 \text{ V}$$

Good

$$V_{Th} = \frac{V_{cc} R_2}{R_1 + R_2} = \frac{V_{cc} R_{Th}}{R_1}$$

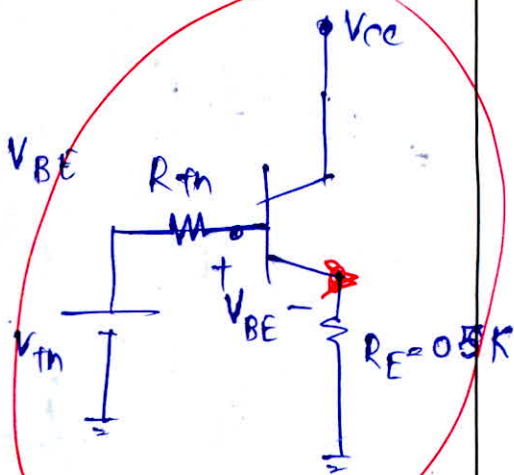
$$\left(R_{Th} = \frac{R_1 R_2}{R_1 + R_2} \right)$$

$$6.3217 = \frac{10 \times 5.0363 \text{ k}}{R_1}$$

$$R_1 = 7.9668 \text{ k}\Omega$$

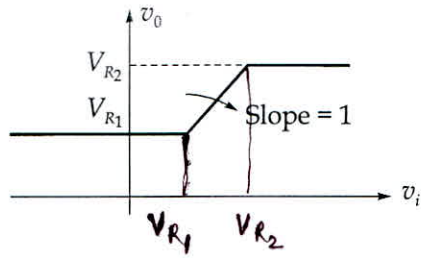
$$R_2 = \frac{R_{Th} R_1}{R_1 - R_{Th}} = \frac{5.0363 \times 7.9668}{7.9668 - 5.0363}$$

$$R_2 = 13.691 \text{ k}\Omega$$



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(c) (i) Design a clipper circuit for the characteristics curve given below:



[10 marks]

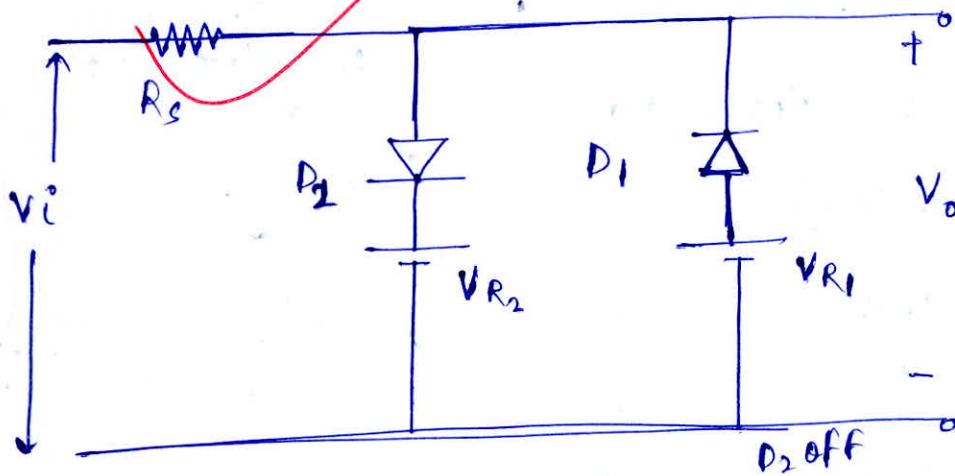
→ from the characteristic

$$V_o = V_{R1} \text{ for } V_i < V_{R1}$$

$$V_o = V_{R2} \text{ for } V_i > V_{R2}$$

$$V_o = V_i \text{ for } V_{R1} < V_i < V_{R2}$$

$$V_{R2} > V_{R1}$$



When $V_i < V_{R1}$ diode D_1 is ON, and $V_o = V_{R1}$

When $V_i > V_{R2}$ diode D_2 is ON and $V_o = V_{R2}$

When $V_{R1} < V_i < V_{R2}$ both diodes OFF $V_o = V_i$

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- Q.3 (c) (ii) Define the 'mobility' of electrons in conductive materials and derive relation showing components of random velocity and average drift velocity to deduce expression for electron mobility.
- (iii) If a conductor material has following data as shown below:
 Density : 9.40 gram/cc
 Resistivity : 1.72×10^{-8} ohm-m
 Atomic weight : 63.5
- Compute the mobility and the average time of collision of electrons in the conductors if valance electron for each conductor material atom is 1.

[10 marks]

Mobility :- Mobility of electrons is defined as drift velocity per unit electric field in the direction of field. It gives an idea of carrier free movement.

$$\mu = \frac{V_d}{E}$$

Now as we know that

$$\text{Current } I = \frac{Q}{t}$$

$$I = \frac{Ne}{t}$$

$$\text{time } t = \frac{l}{v_d}$$

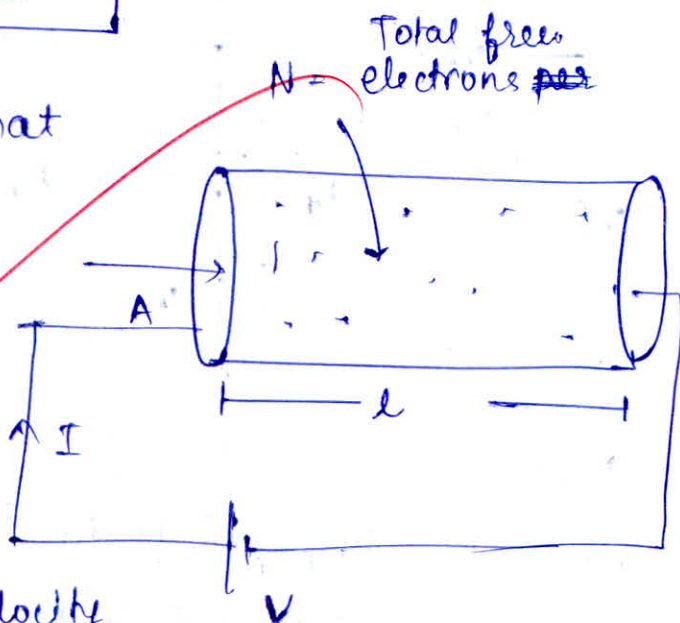
where v_d = drift velocity

$$I = \frac{Ne v_d}{l}$$

Now $J = \sigma E$ (as per ohme law),

$$\text{So } J = \frac{I}{A} = \frac{Ne v_d}{lA} = \sigma E$$

$$\text{hence } \frac{Ne v_d}{lA} = \sigma E$$



Where $\frac{N}{\Delta A} = \frac{\text{electrons}}{\text{Volume}} = \text{free electrons per m}^3 = n$

so

$$n e v_d = G E$$

$$\mu = \frac{v_d}{E} = \frac{G}{n e}$$

* Force on charged electron is electric field

$$F = e E$$

accelerating $a = F/m$

$$a = \frac{e E}{m}$$

and drift velocity

$$v_d = a \tau \quad (\tau = \text{avg collision time})$$

$$v_d = \frac{e E \tau}{m}$$

hence

$$\mu = \frac{v_d}{E} = \frac{e \tau}{m}$$

(iii) $\rho = 9.4 \text{ gram/cc}$

$$\rho = 1.72 \times 10^{-8} \text{ ohm-m}$$

$$M = 63.5$$

electrons per cm^3

$$n = \frac{N_A \rho}{M}$$

$$n = \frac{6.023 \times 10^{23} \times 9.4}{63.5}$$

$$n = 8.9159 \times 10^{28} / \text{m}^3$$

mobility

$$\mu = \frac{1}{\rho n e}$$

$$\mu = \frac{1}{1.72 \times 10^{-8} \times 8.9159 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$\mu = 4.075 \times 10^{-3} \text{ m}^2/\text{sec-V}$$

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Average time of collision = ?

- Q.4 (a) (i) Consider a diode with mean lifetime of holes to be 10 nsec and $\eta = 1$. If a forward current of 0.1 mA is flowing in diode then determine the diffusion capacitance. (Assume room temperature to be 300 K).

[5 marks]

Diffusion capacitance

$$C_D = \frac{\tau I_D}{n V_T}$$

When τ = mean life time of holes.

$$\eta = 1$$

I_D = forward current

$$V_T = 26 \text{ mV for } 300 \text{ K}$$

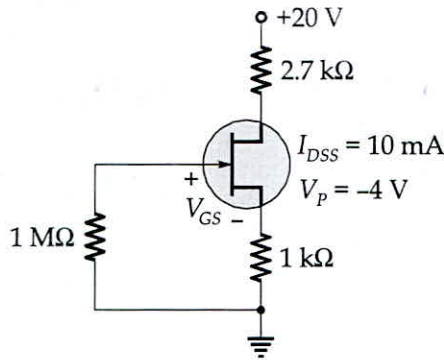
$$C_D = \frac{10 \times 10^{-9} \times 0.1 \times 10^{-3}}{1 \times 26 \times 10^{-3}}$$

$$C_D = \frac{10 \times 10^{-9} \times 0.1}{26} = 3.846 \times 10^{-11} \text{ F}$$

$$C_D = 38.46 \text{ pF}$$

5

4 (a) (ii) Determine V_{GSQ} , I_{DQ} and V_{DS} for the self bias circuit shown in figure below.



[15 marks]

hence $V_{GS} = V_G - V_S$

$$V_{GS} = 0 - I_D \times 1000 \quad (V_G = 0 \text{ as } I_G = 0)$$

$$V_{GS} = -I_D \times 1000$$

$$I_D = \frac{-V_{GS}}{1000} \quad \text{--- (1)}$$

and $I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$

$$\frac{-V_{GS}}{1000} = 10 \times 10^{-3} \left[1 + \frac{V_{GS}}{4}\right]^2$$

$$\frac{-V_{GS}}{10} = 1 + \frac{V_{GS}^2}{16} + \frac{2V_{GS}}{4}$$

$$\frac{V_{GS}^2}{16} + V_{GS} \left(\frac{1}{2} + \frac{1}{10}\right) + 1 = 0$$

Solving we get

$$V_{GS} = -2.1467 \text{ V (✓)}$$

$$= -7.4532 \text{ V (X)}$$

Take

$V_{GS} = -2.1467 \text{ V}$ because $|V_{GS}|$ can't be greater than $|V_P|$ hence second option is invalid

$$\boxed{V_{GS} = -2.1467 \text{ V}}$$

$$\begin{aligned} I_{DS} &= \frac{-V_{GS}}{1000} \\ &= \frac{+2.1467}{1000} = 2.1467 \times 10^{-3} \text{ A} \\ &= 2.1467 \text{ mA} \end{aligned}$$

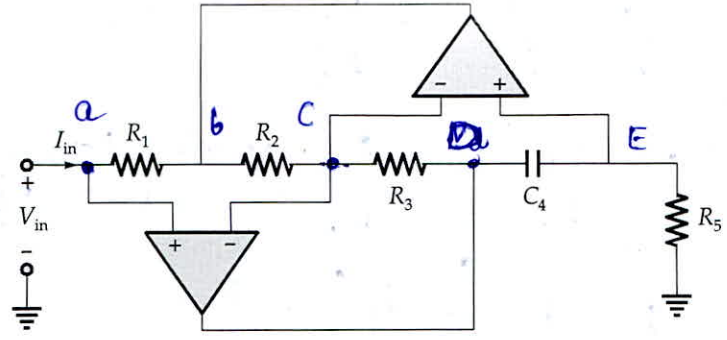
Now

$$\begin{aligned} V_{DS} &= V_{DD} - R_D I_D - R_S I_D \\ &= 20 - 2.7\text{K} \times 2.1467 - 2.1467 \times 1\text{K} \end{aligned}$$

$$V_{DS} = 12.087 \text{ Volts}$$

15

4 (b) Consider the circuit with ideal op-amps, shown in the figure below:



Calculate the input impedance $Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)}$ and comment on the result obtained.

By virtual short,

$$V_a = V_{in} = V_c \quad (\text{because } V_+ = V_-)$$

[20 marks]

$$V_c = V_e = V_{in}$$

Now KCL at a -

$$I_{in} = \frac{V_a - V_b}{R_1} = \frac{V_{in} - V_b}{R_1} \quad \text{--- (i)}$$

KCL at c

$$\frac{V_b - V_c}{R_2} = \frac{V_c - V_d}{R_3}$$

$$\frac{V_b - V_{in}}{R_2} = \frac{V_{in} - V_d}{R_3} \quad \text{--- (ii)}$$

KCL at e

$$\frac{V_d - V_e}{\frac{1}{C_4 s}} = \frac{V_e}{R_5}$$

$$C_4 s (V_d - V_e) = \frac{V_e}{R_5}$$

$$V_d C_4 s = V_e \left(\frac{1}{R_5} + C_4 s \right)$$

$$V_d = \frac{V_e}{C_4 s} \left(\frac{1}{R_5} + C_4 s \right) = \frac{V_{in}}{R_5 C_4 s} \left[1 + \frac{1}{R_5 C_4 s} \right]$$

from eq (ii)

$$\frac{V_b}{R_2} = V_{in} \left(\frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{V_D}{R_3}$$

$$V_b = V_{in} \left(1 + \frac{R_2}{R_3} \right) - \frac{R_2}{R_3} \times V_{in} \left(1 + \frac{1}{R_3 C_4 S} \right)$$

$$V_b = V_{in} \left(1 + \frac{R_2}{R_3} - \frac{R_2}{R_3} - \frac{R_2}{R_3 R_3 C_4 S} \right)$$

$$V_b = V_{in} \left(1 - \frac{R_2}{R_3 R_3 C_4 S} \right)$$

from eq (i)

$$I_{in} R_1 = V_{in} - V_b$$

$$= V_{in} - V_{in} \left[1 - \frac{R_2}{R_3 R_3 C_4 S} \right]$$

$$I_{in} R_1 = \frac{V_{in} R_2}{R_3 R_3 C_4 S}$$

$$\boxed{\frac{V_{in}}{I_{in}} = Z_{in} = \frac{R_1 R_3 R_3 C_4 S}{R_2}}$$

* Above circuit behaves as an inductor

$$\text{with } L_{eq} = \frac{R_1 R_2 R_3 C_4}{R_2}$$

(gyrator circuit)

converts capacitance to equivalent inductance.

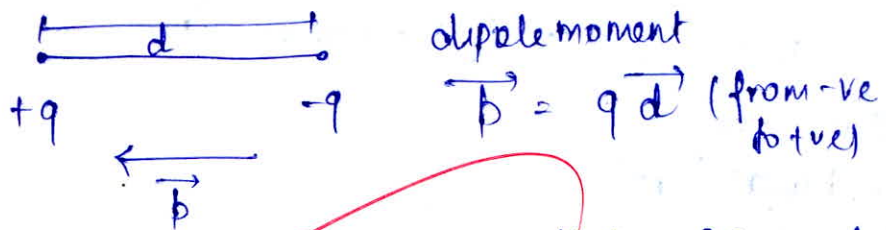
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4 (c) Explain briefly the polarization occurring in dielectric materials. What are different types of polarization occurring in dielectric material?

If a dielectric material contains 3.2×10^{19} polar molecules/m³ and the relative permittivity of material is $\epsilon_r = 2.4$ with applied external electric field $\vec{E} = 10^4 \vec{a}_x$ V/m, then calculate the value of polarization and dipole moment in each molecule. (Consider all molecules have same dipole moment).

[20 marks]

Every material is made of electrical dipole because of charges in atomic structure. A dipole is defined as small ^{opposite} charges separated by small distance



but when electric field is not applied, either material does not possess dipole moment of permanent nature because +ve and -ve charge centres coincide or it may contain permanent dipole moment but they are randomly oriented and hence net moment = 0

On application of electric field, material gets polarized by separation of +ve and -ve charge centres (induced polarisation) or by orientation of permanent dipoles in the field direction. This is the creation of induced dipole. This process is called polarisation.

Polarization is given by dipole moment per unit volume

$$\vec{P} = N \vec{p}$$

where $N =$ no. of dipoles per unit volume

$p =$ dipole moment of each dipole

There are following types

- i) Electronic polarisation (induced by charge separation)
- ii) Ionic ————— (stretching of ionic bond)
- iii) orientational ————— (orientation of permanent dipoles)
- iv) Space charge ————— (trapping of carriers)

(ii) Given

$$\epsilon_r = 2.4$$

$$N = 3.2 \times 10^{19} \text{ Molecules / m}^3$$

Polarisation

$$\vec{P} = \vec{E} \epsilon_0 \chi_e$$

$$\vec{P} = (\epsilon_r - 1) \epsilon_0 \vec{E}$$

$$= (2.4 - 1) \times 8.85 \times 10^{-12} \times 10^4 \hat{a}_x$$

$$= 1.239 \times 10^{-7} \hat{a}_x \text{ C/m}^2$$

Good

dipole moment of each molecule

$$\vec{p} = \frac{\vec{P}}{N} = \frac{1.239 \times 10^{-7} \hat{a}_x}{3.2 \times 10^{19}}$$

$$\vec{p} = 3.871 \times 10^{-27} \hat{a}_x \text{ C-m}$$

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Section B : Electrical Machines-1 + Power Systems-2

5 (a) Draw the reactance diagram of the system whose bus admittance matrix is given below. First, second, third and fourth rows refer to buses 1, 2, 3 and 4 respectively.

$$Y_{bus} = j \begin{bmatrix} -3.78 & 1.25 & 2.5 & 0 \\ 1.25 & -3.42 & 1.11 & 1.0 \\ 2.5 & 1.11 & -4.89 & 1.25 \\ 0 & 1.0 & 1.25 & -2.31 \end{bmatrix} \begin{matrix} \rightarrow \Sigma = -0.03j \\ \Sigma = -0.06j \\ \Sigma = -0.03j \\ \Sigma = -0.06j \end{matrix}$$

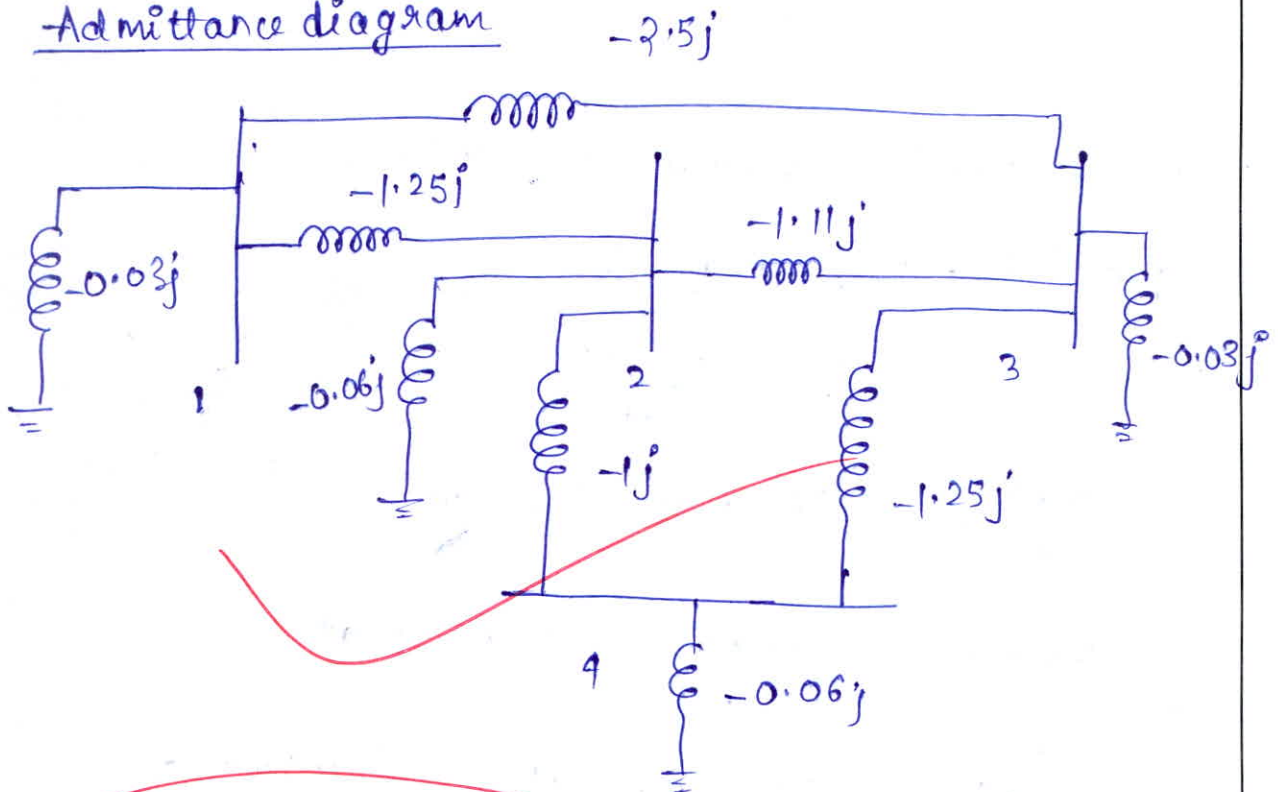
[12 marks]

We can get shunt element on each bus by summation of each row.

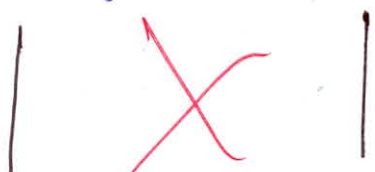
hence shunt element on each bus (admittance)

$$\begin{aligned} 1 &= -0.03j \\ 2 &= -0.06j \\ 3 &= -0.03j \\ 4 &= -0.06j \end{aligned}$$

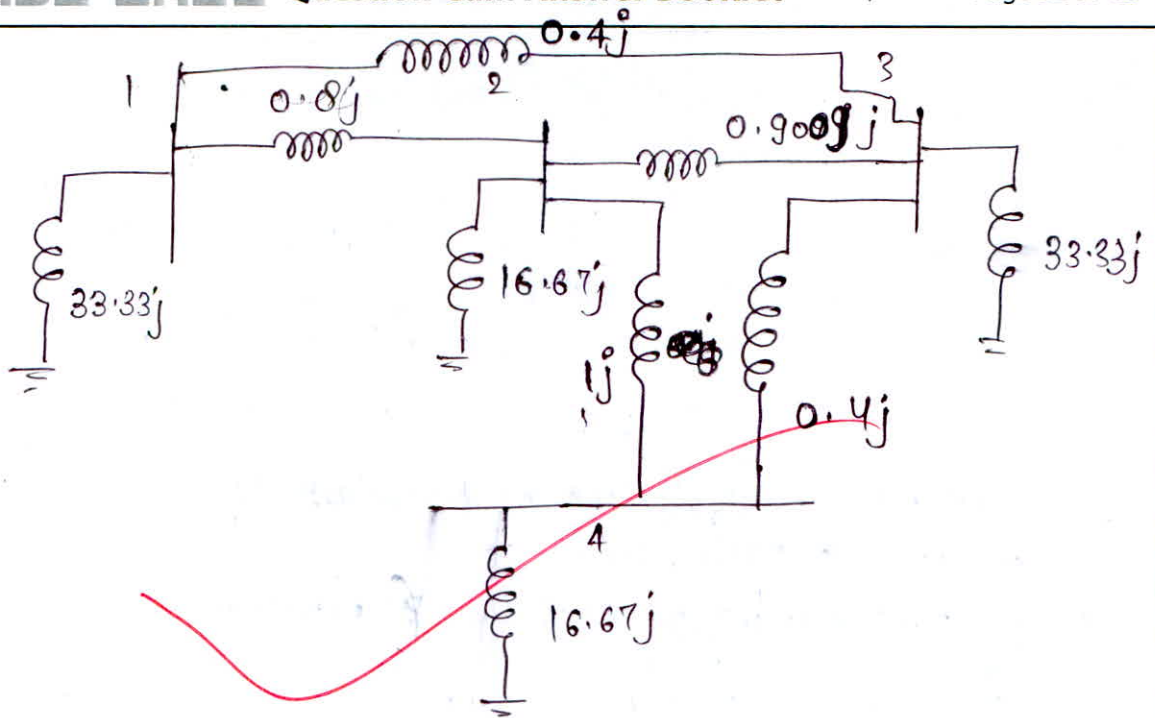
Admittance diagram



Reactance diagram



9



- Q.5 (b) A voltage of $(200 \sin \omega t - 50 \sin 3\omega t)$, 50 Hz is applied to a 250 turn transformer winding having negligible resistance and leakage reactance. Deduce an expression for flux and find its maximum value. By what percentage will eddy current loss in the iron core be reduced if the applied voltage is altered to $200 \sin \omega t$?

[12 marks]

$$\text{Voltage } V = 200 \sin \omega t - 50 \sin 3\omega t$$

~~flux~~ A/c to Faraday's law

$$E = V = -\frac{d\phi}{dt} N$$

$$-N \frac{d\phi}{dt} = 200 \sin \omega t - 50 \sin 3\omega t$$

$$-N d\phi = (200 \sin \omega t - 50 \sin 3\omega t) dt$$

$$-\int d\phi = \frac{\int 200 \sin \omega t dt - 50 \sin 3\omega t dt}{N}$$

$$\phi = -\frac{1}{250} \left[-\cos \omega t \times \frac{200}{\omega} + \frac{50}{3\omega} \cos 3\omega t \right]$$

$$\phi = +\frac{200}{250\omega} \cos \omega t - \frac{50}{250 \times 3\omega} \cos 3\omega t$$

2

$$\phi = \frac{200}{250 \times 100\pi} \cos \omega t - \frac{50}{250 \times 300\pi} \cos 3\omega t$$

$$\phi = (2.546 \cos \omega t - 0.212 \cos 3\omega t) \times 10^{-3} \text{ Wb}$$

$$\phi = (2.546 \cos \omega t - 0.212 \cos 3\omega t) \text{ mWb}$$

for max value of flux $\frac{d\phi}{dt} = 0$

hence when $E = V = 0$, ϕ will be maximum

$$E = 200 \sin \omega t - 50 \sin 3\omega t = 0$$

$$200 \sin \omega t = 50 \sin 3\omega t = 0$$

Solving above eq we get

$$\phi_{\max} =$$

$$\phi_{\max} = \sqrt{2.546^2 + 0.212^2}$$

$$= 2.5548 \text{ mWb}$$

- Q.5 (c) An industry load of 200 kW at 0.75 p.f. lagging is fed from the 3- ϕ , 11 kV distribution feeders. It is required to maintain the 0.9 p.f. lag at the drawl point. Find the rating of capacitor installed at industrial drawl point.

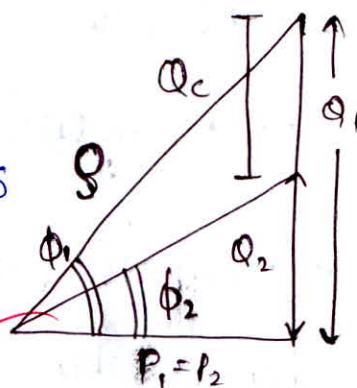
[12 marks]

Without Capacitor

$$P_1 = 200 \text{ kW}$$

$$Q_1 = 200 \tan \cos^{-1} 0.75$$

$$Q_1 = 176.38 \text{ KVAR}$$

With capacitor (

$$P_2 = 200 \text{ kW (same because capacitor consumes no real power)}$$

$$Q_2 = 200 \tan \cos^{-1} 0.9$$

$$= 96.86 \text{ KVAR}$$

It means that reactive power demand is reduced by using capacitor by an amount which is equal to its capacity

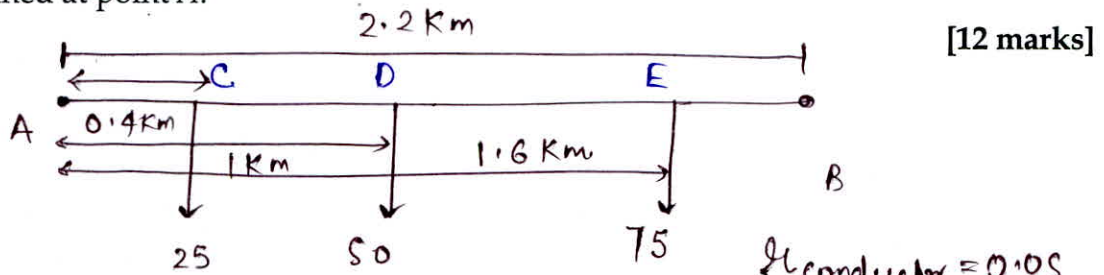
$$\begin{aligned} Q_c &= Q_1 - Q_2 \\ &= 176.38 - 96.86 \\ &= 79.5155 \text{ KVAR} \end{aligned}$$

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~~Assing~~ Assume bank is in star and $f = 50 \text{ Hz}$

$$\begin{aligned} C &= \frac{Q_c/3}{V_{ph}^2 \omega} \\ &= \frac{79.5155 \times 10^3}{3 \times \left(\frac{11000}{\sqrt{3}}\right)^2 \times 100\pi} \\ &= \underline{\underline{2.091 \text{ mF}}} \end{aligned}$$

- d) A 2-wire DC distributor cable AB is 2.2 km long and supplies loads of 25 A, 50 A, 75 A at 0.4 km, 1 km and 1.6 km from the point A. Each conductor has a resistance of $0.05 \Omega/\text{km}$. Calculate the potential difference at each point if potential difference of 400 volts is maintained at point A.



[12 marks]

$$V_A = 400 \text{ V given}$$

$$R_{\text{conductor}} = 0.05 \frac{\Omega}{\text{km}}$$

$$R_{\text{loop}} = 0.05 \times 2 \frac{\Omega}{\text{km}}$$

Voltage at C :-

$$V_C = V_A - (I_1 + I_2 + I_3) \times 0.4 \times 0.05 \times 2$$

$$V_C = 400 - (25 + 50 + 75) \times 0.4 \times 0.05 \times 2$$

$$V_C = 394 \text{ volts.}$$

Voltage at D :-

$$V_D = V_C - (I_2 + I_3) \times (1 - 0.4) \times 0.05 \times 2$$

$$V_D = 394 - (50 + 75) \times 0.6 \times 0.05 \times 2$$

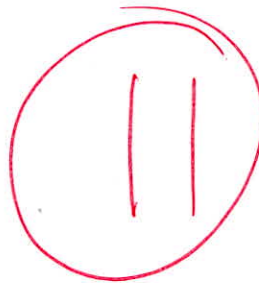
$$V_D = 386.5 \text{ volts.}$$

Voltage at E

$$V_E = V_D - (I_3) \times (1.6 - 1) \times 0.05 \times 2$$

$$= 386.5 - (75) \times 0.6 \times 0.05 \times 2$$

$$= 382 \text{ Volts.}$$

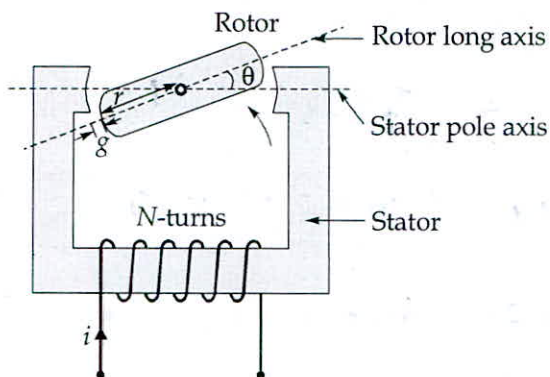


- Q.5 (e) For the electro-mechanical configuration shown in figure, assume all the field energy is present in the overlapping regions. Radius is r and the airgap length is g . Calculate the magnitude of torque, when the maximum flux density in the airgap is limited to 2.2 T. The other data are as follows:

$$\text{Radius, } r = 50 \text{ mm,}$$

$$\text{Gap length, } g = 2 \text{ mm,}$$

$$\text{Length normal to radius is } l = 10 \text{ mm.}$$



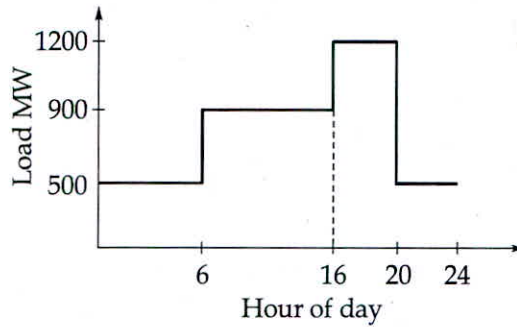
[12 marks]

Q.6 (a) The fuel cost characteristics of two thermal plants are as under,

$$C_1 = 7700 + 52.8P_1 + 5.5 \times 10^{-3} P_1^2 \text{ Rs/hour}$$

$$C_2 = 2500 + 15P_2 + 0.05 P_2^2 \text{ Rs/hour}$$

The limit of generation for the two units are $200 \leq P \leq 800$ MW. The load curve is shown in figure below. Find the daily operating schedule to minimize the operating costs. The cost of taking a unit off and then putting it on is Rs 1000.00.

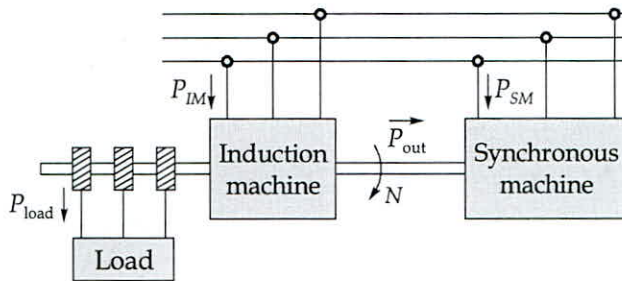


[20 marks]

- Q.6 (b) (i) The incremental fuel costs for two units of a plant are $\lambda_1 = \frac{df_1}{dP_{g1}} = 0.012P_{g1} + 8.0$;
 $\lambda_2 = \frac{df_2}{dP_{g2}} = 0.008P_{g2} + 9.6$ where f is in (Rs/hour) and P_g is in megawatts (MW). If both units operate at all times and maximum and minimum loads on each unit are 550 and 100 MW respectively then find λ of the plant in Rs/MWh versus plant output in MW for economic dispatch as total load varies from 200 to 1100 MW.
- (ii) Find the saving in Rs/hour for economic dispatch of load between the units of part (i) compared with their sharing the output equally when the total plant output is 600 MW.

[20 marks]

- 2) A 3- ϕ wound-rotor induction machine is mechanically coupled to a 3- ϕ synchronous machine as shown in figure. The synchronous machine has 4-poles and the induction machine has 6-poles. The stator of the two machines are connected to a 3- ϕ , 50 Hz supply. The rotor of the induction machine is connected to a 3- ϕ resistive load. Neglect rotational losses and stator resistance losses. The load power is 1 p.u. The synchronous machine rotates at the synchronous speed.
- (i) The rotor rotates in the direction of the stator rotating field of the induction machine. Determine the speed, frequency of the current in the resistive load, and power taken by the synchronous machine and by the induction machine from the source.
- (ii) Repeat part (i) if the phase sequence of the stator of the induction machine is reversed.



[20 marks]

4) The primary, secondary and tertiary winding of a three-winding transformer are rated as 11 kV, 6 MVA, star/3.3 kV, 3 MVA, star/400 V, 3 MVA, delta respectively. The short circuit tests on this transformer gave the following results:

Secondary shorted ; primary excited : 500 V, 100 A

Tertiary shorted ; primary excited : 600 V, 100 A and

Tertiary shorted ; secondary excited : 100 V, 200 A

- (i) Find the per unit leakage reactances of the star equivalent circuit. Neglect resistance.
- (ii) The primary is energized at rated voltage and the secondary is open circuited. For a three-phase balanced short circuit at the tertiary terminals, calculate the short circuit current and the secondary terminal voltage.

[20 marks]

- b) A 4-pole, 50-Hz turbo-alternator is rated at 45 MW, 0.8 pf lag and has an inertia of 25000 kg-m². It is connected via a transmission system to another set whose corresponding data is 2-pole, 50 Hz, 60 MW, 0.75 lag, 9000 kg-m². Calculate the inertia constant of each set on its own rating and that of the single equivalent set connected to an infinite bus-bar and on a base rating of 100 MVA.

[20 marks]

- Q.7 (c) The following test results are obtained for a 3- ϕ , 280 V, 60 Hz, 6.5 A induction machine.
Block-rotor test : 44 V, 60 Hz, 25 A, 1250 W
No load test : 208 V, 60 Hz, 6.5 A, 500 W

The average resistance measured between two stator terminals is 0.27 Ω .

Determine:

- (i) the no load rotational loss.
- (ii) the output power in horse power (hp) at $s = 0.1$.
- (iii) the efficiency.

(Take 1 hp = 746 W)

[20 marks]

Figure below shows the single line diagram of a sample 3-bus power system. Data for this system are given in table-1 and table-2.

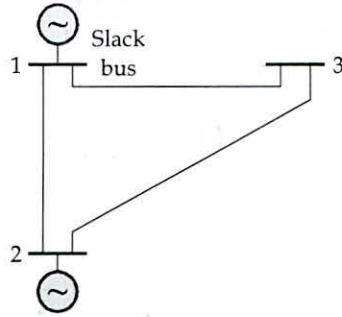


Table 1: Scheduled generation and loads and assumed bus voltage

Bus code <i>i</i>	Assumed Bus voltage	Generation		Load	
		MW	MVA _r	MW	MVA _r
1 (slack bus)	$1.05 + j0.0$	-	-	0	0
2	$1 + j0.0$	50	30	305.6	140.2
3	$1 + j0.0$	0	0	138.6	45.2

Base MVA = 100

Table 2: Line impedance

Bus code <i>i - k</i>	Impedance Z_{ik} (p.u.)
1 - 2	$0.02 + j0.04$
1 - 3	$0.01 + j0.03$
2 - 3	$0.0125 + j0.025$

Using the Gauss-Seidel method, determine the phasor values of voltages at buses 2 and 3. Perform one iteration only.

$$Y_{bus} = \begin{bmatrix} 20 - 50i & -10 + 20i & -10 + 30i \\ -10 + 20i & 26 - 52i & -16 + 32i \\ -10 + 30i & -16 + 32i & 26 - 62i \end{bmatrix} \quad [20 \text{ marks}]$$

$$\text{Now } S_2 = P_2 + jQ_2 = \frac{(50 - 305.6) + j(30 - 140.2)}{100} = -2.556 - 1.102j \text{ pu}$$

$$\text{hence } V_2^{(1)} = \frac{1}{Y_{22}} \left[\frac{S_2^*}{V_2^{(0)}} - Y_{21} V_1 - Y_{23} V_3^{(0)} \right]$$

$$V_2^{(1)} = \frac{1}{26 - 52j} \left[\frac{-2.556 + j1.102}{120^\circ} - (-10 + 20j) \times 1.05 - (-16 + 32j) \times 120 \right]$$

$$V_2^{(1)} = 0.983 \angle -1.798^\circ \text{ pu}$$

$$S_3 = P_3 + jQ_3 = \frac{(0 - 138.6) + j(0 - 45.2)}{100}$$

$$= -1.386 - j0.452 \text{ pu}$$

$$V_3^{(1)} = \frac{1}{Y_{33}} \left[\frac{S_3^*}{V_3^{(0)*}} - Y_{31} V_1 - Y_{32} V_2^{(1)} \right]$$

$$V_3^{(1)} = \frac{1}{26 - 62j} \left[\frac{-1.386 + j0.452}{120^\circ} - (-10 + 30j) \times 1.05 - (-16 + 32j) \times 0.983 \angle -1.798^\circ \right]$$

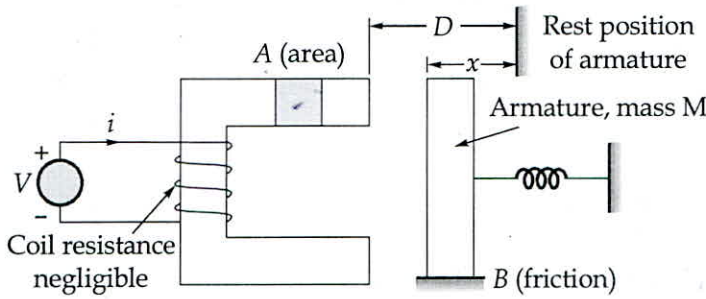
$$V_3^{(1)} = 1.00170 \angle -2.012 \text{ pu}$$

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For the electromechanical system shown in figure, the area of cross-section of core is A and the air-gap flux density under steady operating condition is $B(t) = B_m \sin \omega t$.

Find:

- (i) the coil voltage.
- (ii) the force of field origin as a function of time.
- (iii) the displacement of armature as a function of time.



[20 marks]

i here reluctance of air gap,

$$= \frac{2 \times (D-x)}{\mu_0 A}$$

hence inductance ~~of coil~~ = $\frac{N^2}{\text{Reluctance}}$

$$= \frac{N^2 \mu_0 A}{2(D-x)}$$

energy stored in air gap ~~W =~~ $\frac{1}{2} L i^2$

$$W = \frac{1}{2} \left(\frac{N^2 \mu_0 A}{2(D-x)} \right) i^2 \quad \text{--- (1)}$$

Now energy stored is also given as

$$W = \frac{1}{2} \frac{B^2}{\mu_0} \times \text{volume of air gap.}$$

$$= \frac{1}{2} \times \frac{B_m^2 \sin^2 \omega t}{\mu_0} \times 2(D-x) \times A$$

$$= \frac{1}{2} \times \frac{2(D-x) \times A B_m^2 \sin^2 \omega t}{\mu_0}$$

force

$$F = \frac{dW}{dx} = \frac{(-1) B_m^2 A \sin^2 \omega t}{\mu_0}$$

$$F = \frac{-B_m^2 A \sin^2 \omega t}{\mu_0}$$

Now coil voltage

using faraday's law

$$V = E = -N \frac{d\phi}{dt}$$

(as there is no

$$V = -N \times \frac{d}{dt} [B \cdot A]$$

$$V = -N \times A \frac{dB}{dt}$$

$$V = -NA \frac{d}{dt} [B_m \sin \omega t]$$

$$V = -NA B_m \times \omega \cos \omega t$$

$$V = -N \omega A B_m \cos(\omega t)$$

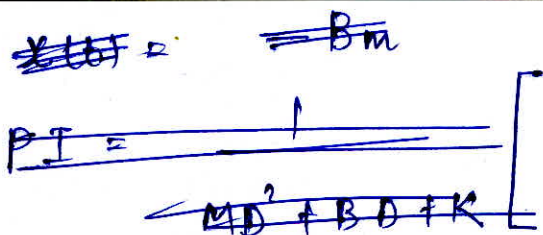
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ii Now apply force balance at armature

$$F = M \frac{d^2 x}{dt^2} + B \frac{dx}{dt} + Kx$$

$$M \frac{d^2 x}{dt^2} + B \frac{dx}{dt} + Kx = \frac{-B_m^2 A \sin^2 \omega t}{\mu_0}$$

$$(MD^2 + BD + K)x(t) = \frac{-B_m^2 A \sin^2 \omega t}{\mu_0}$$



Q.8 (c) A 1- ϕ 10 kVA, 2400/240 V, 50 Hz distribution transformer has the following characteristics.

Core loss at rated voltage = 100 W

Copper loss at half load = 60 W

(i) Determine the per unit rating at which the transformer efficiency is maximum. Also determine this efficiency if the load power factor is 0.9 (lag).

(ii) The transformer has the following load cycles

no load for 6 hours.

70% full load for 10 hours at 0.8 p.f.

90% full load for 8 hours at 0.9 p.f.

Determine the all-day efficiency of the transformer:

(iii) If the above transformer is connected as autotransformer then, determine the maximum kVA rating and for this rating determine the efficiency when delivering full load at 0.8 power factor lagging.

[20 marks]

~~Core losses $\propto V^2$~~

Core loss at rated voltage = ~~25~~ 100 W

Copper losses at rated load = $(2)^2 \times 60$
= 240 W

$$\begin{aligned} (i) \quad S_{\eta_{\max}} &= S_{FL} \left(\frac{P_i}{P_{cu}(FL)} \right)^{1/2} \\ &= 10 \left(\frac{100}{240} \right)^{1/2} \\ &= 6.4549 \text{ KVA} \end{aligned}$$

$$\text{Sat } \eta_{\max} = \frac{6.4549}{10} = 0.6455 \text{ pu}$$

$$\begin{aligned} (ii) \quad \eta_{\max} &= \frac{6.4549 \times 10^3 \times 0.9}{6.4549 \times 10^3 \times 0.9 + 2 \times 100} \\ &= 0.966719 \\ &\text{or} \\ &96.6719\% \end{aligned}$$

(ii) Load table

	Duration (hr)	Pf	O/p in kWhr
No load	6	—	0
70% full load	10	0.8	$0.7 \times 10 \times 0.8 \times 10 = 56$
90% of full load	8	0.9	$0.9 \times 10 \times 0.9 \times 8 = 64.8$

total kWhr output = $0 + 56 + 64.8$
 $= 120.8 \text{ kWhr}$

Copper loss in kWhr

$$= \frac{(0.7)^2 \times 10 \times 240 + (0.9)^2 \times 8 \times 240}{1000}$$

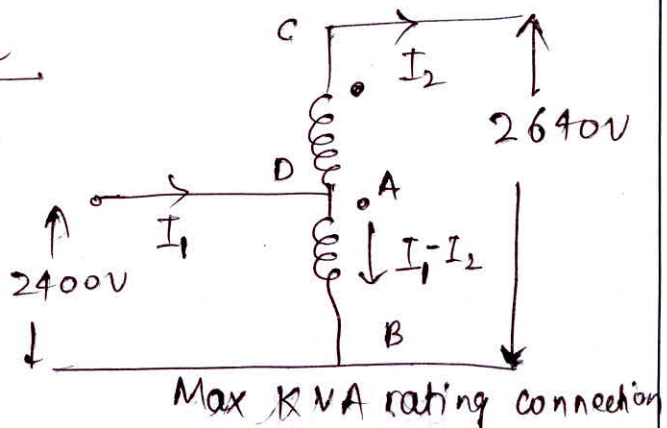
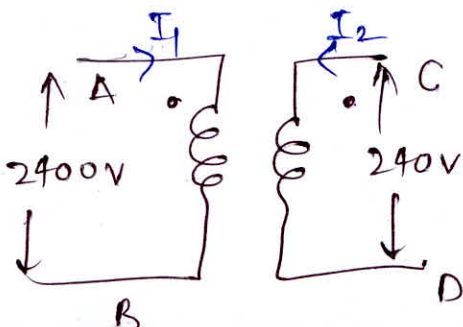
$= 2.7312 \text{ kWhr}$

Core loss in kWhr = $\frac{100 \times 24}{1000}$

$= 2.4 \text{ kWh}$

$\eta_{\text{all day}} = \frac{120.8}{120.8 + 2.7312 + 2.4} = 0.95925$
 or 95.925%

(iii) Auto transformer



$$I_{2(\text{rated})} = \frac{10000}{240} = 41.67 \text{ A}$$

and hence KVA rating = $\frac{41.67 \times 2640}{1000}$

$$\boxed{\text{KVA rating} = 110 \text{ KVA}}$$

• η at full load 0.8 Pf lagg. (

$$\eta = \frac{110 \times 10^3 \times 0.8}{110 \times 10^3 \times 0.8 + \text{Full load losses}}$$

Full losses \Rightarrow same as two winding Xmer

$$\Rightarrow 100 + 240$$

$$\Rightarrow 340 \text{ W}$$

$$\eta = \frac{110 \times 10^3 \times 0.8}{110 \times 10^3 \times 0.8 + 340}$$

Good

$$= 0.99615$$

$$\boxed{\eta_{\text{auto}} \text{ or } \underline{99.615\%}}$$

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Space for Rough Work

Space for Rough Work

$$\frac{P}{A} = 52 \text{ cm}$$
