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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-5: Flow of Fluids, Hydraulic Machines and Hydro Power
Design of Concrete and Masonry Structures-1
Strength of Materials-2

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	42
Q.2	
Q.3	58 -2 = 56
Q.4	
Section-B	
Q.5	43
Q.6	48
Q.7	60
Q.8	
Total Marks Obtained	251 - 2 = 249

Good attempt to question.

Signature of Evaluator

Cross Checked by

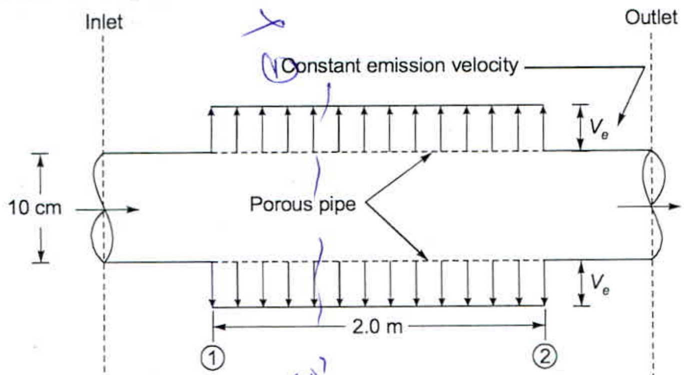
Can improve presentation little more.

Section A : Flow of Fluids, Hydraulic Machines and Hydro Power

.1 (a) A circular pipe 10 cm in diameter has a 2 m length which is porous. In this porous section the velocity of exit is known to be constant as shown in figure. If the velocities at inlet and outlet of the porous section are 2.0 m/s and 1.2 m/s respectively. Estimate

(i) the discharge emitted out through the walls of the porous pipe and

(ii) the average velocity of this emitted discharge.



[12 marks]

Ans

Given dia = 10cm = 0.1m , 2m = ℓ $V_{inlet} = 2$ m/s, $V_{outlet} = 1.2$ m/s

$Q_{inlet} = V_{inlet} \times A = 2 \times \frac{\pi}{4} \times 0.1^2 = 0.0157 \ m^3/sec$

$Q_{outlet} = V_{outlet} \times A = 1.2 \times \frac{\pi}{4} \times 0.1^2 = 9.425 \times 10^{-3} \ m^3/sec$

Let us consider a section x–x at a distance of x from ① of length dx.

(i) discharge emitted out through the walls of the porous pipe

$Q_{emitted} = Q_{inlet} - Q_{outlet}$

$= 0.0157 - 9.425 \times 10^{-3}$

$= 6.27522 \times 10^{-3} \ m^3/sec$

12

$\boxed{Q_{emitted} = 6.27522 \ lit/sec}$

(ii) velocity emitted $= \dfrac{Q_{emitted}}{Area} = \dfrac{6.27522 \times 10^{-3}}{\pi \times 0.1 \times 2}$

$\boxed{V_{emitted} = 9.9873 \times 10^{-3} \ m/s}$

Q.1 (b) **(i)** Explain forced vortex flow occurring in a centrifugal pump.

(ii) Water is flowing through a smooth pipe of 100 mm diameter at rate of 0.036 m^3/s. Determine

(a) Darcy's friction factor

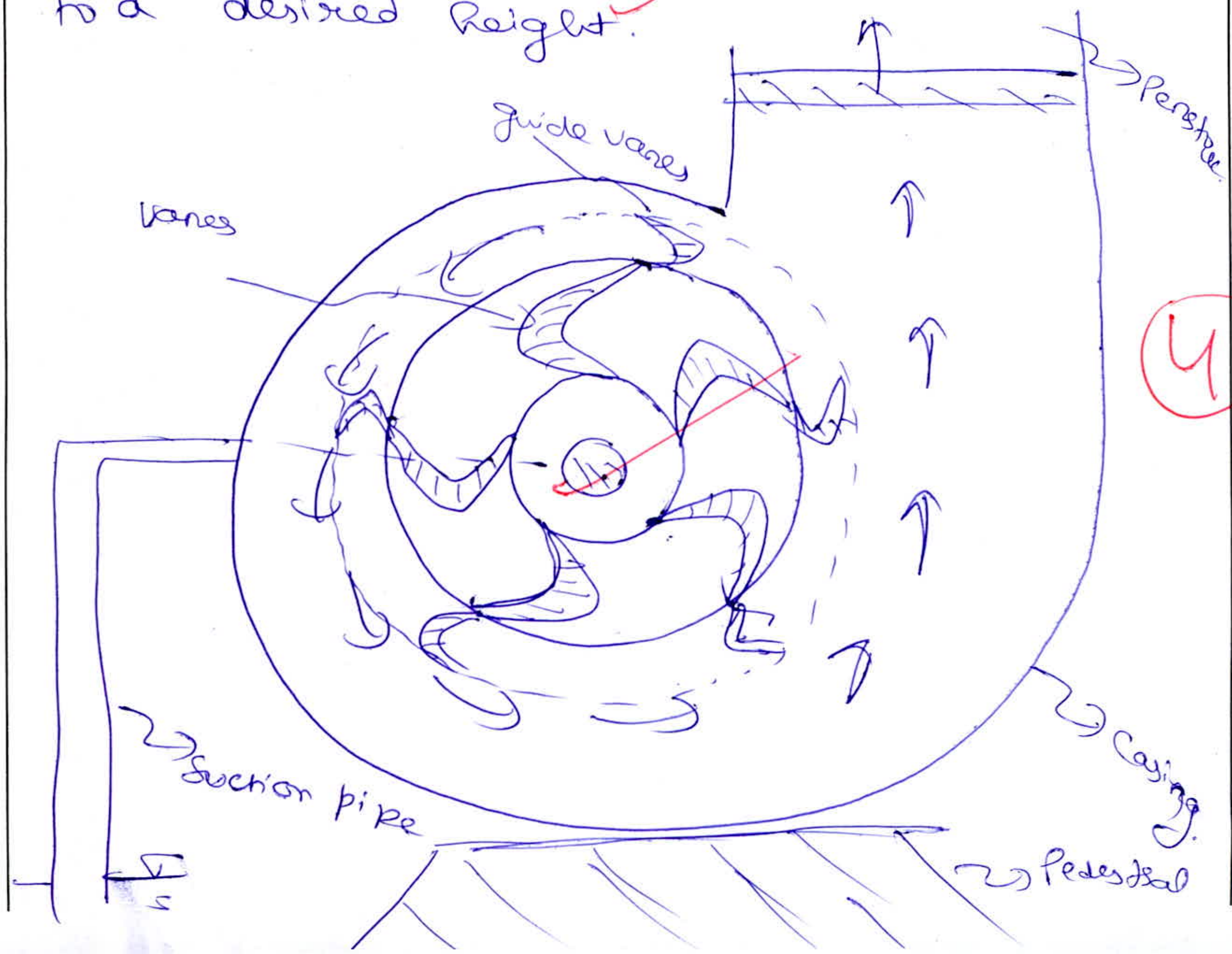
(b) Normal thickness of viscous sub layer

Take kinematic viscosity = 10^{-6} m^2/s and f (Darcy's friction factor) = $0.0032 + \frac{0.221}{R_e^{0.237}}$

Ans 1 b **[6 + 6 marks]**

(i) Forced vortex flow is a flow in which the liquid flows in circular direction under the action of external force.

In centrifugal pumps. As the energy is applied to the pumps, it exerts a torque on the water making it flow in a circular direction which will increase both kinetic & pressure energy of the water so that it can be pumped to a desired height.



4

(ii) Given $d = 100\,mm = 0.1\,m$. $\nu = 10^{-6}\,m^2/sec$

$Q = 0.036\,m^3/sec$

$$V = \frac{Q}{A} = \frac{0.036}{\frac{\pi}{4} \times 0.1^2} = 4.58366\,m/s$$

$$Re = \frac{VD}{\nu} = \frac{4.58366 \times 0.1}{10^{-6}} = 458366.2361$$

$$f = 0.0032 + \frac{0.221}{Re^{0.237}}$$

$$f = 0.0032 + \frac{0.221}{(458366.2361)^{0.237}}$$

$$\boxed{f = 0.013262}$$

Normal thickness of viscous sub layer.

$$y' = \frac{11.6\,\nu}{V^*}$$

$$V^* = \bar{U}\sqrt{f/8}$$

$$V^* = 4.58366\sqrt{\frac{0.013262}{8}}$$

$$V^* = 0.186625\,m/s$$

$$y' = \frac{11.6 \times 10^{-6}}{0.186625}$$

$$y' = 6.2156 \times 10^{-5}\,m$$

$$\boxed{y' = 0.06215\,mm}$$

6

Do not write in this margin

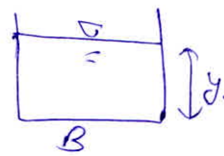
Q.1 (c) Show that at the critical state of flow, the specific energy in a rectangular channel is equal to 1.5 times the depth of flow. Also find at critical flow condition whether the depth of flow will be greater or less than $\frac{2}{3}$ times specific energy for a trapezoidal channel.

[12 marks]

Ans 1 (c)

we know

$$E = y + \frac{v^2}{2g}.$$



$v = \frac{Q}{A} = \frac{Q}{By}$

Let B = width, y = depth of rectangular channel.

$$E = y + \frac{Q^2}{2gB^2y^2}.$$

$$\frac{dE}{dy} = 1 - \frac{\cancel{2}Q^2}{\cancel{2}gB^2y^3} = 0.$$

$\left[\frac{dE}{dy} = 0\right]$ [As E is Minimum at critical depth]

Let $q = \frac{Q}{B}$ = discharge / width.

$$1 - \frac{q^2}{gy^3} = 0 \qquad \frac{q^2}{g} = y_c^3 \Rightarrow y_c = \left(\frac{q^2}{g}\right)^{1/3}.$$

$$E_c = y_c + \frac{q^2}{2gy_c^2}.$$

As

$$E_c = y_c + \frac{y_c^3}{2y_c^2} \qquad \left[\text{As } \frac{q^2}{g} = y_c^3\right]$$

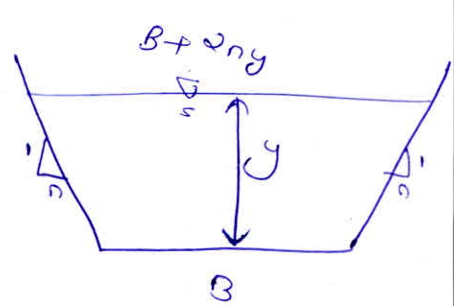
$$E_c = y_c + \frac{y_c}{2} = \frac{3}{2}y_c$$

$$\boxed{E_c = \frac{3}{2}y_c}$$

for a Trapezoidal channel

for critical condition

$$\frac{Q^2T}{gA^3} = 1$$



$$\frac{Q^2 \times (B+2ny)}{g\,(B+ny)^3y^3} = 1$$

$A = (B + B+2ny)\,y/2$

$A = (B+ny)y$

Q

7

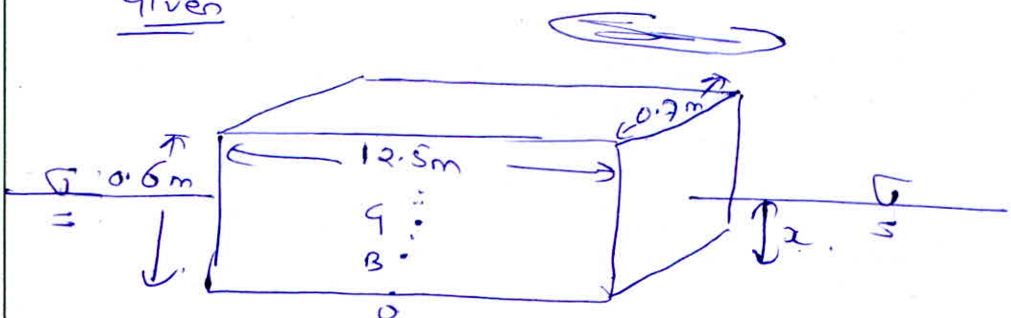
??

Φ2 (B P2ny) s

Q.1 (d) An empty tank with all sides closed is 12.5 m long, 0.7 m broad and 0.6 m high. The surface of sheet metal weighs 363 N/m^2 and the tank is allowed to float in fresh water with 0.6 m side vertical. Determine the state of equilibrium.

[12 marks]

Ans 1 (d) Given



weight of the tank = $\gamma \times$ Surface Area

$$= 363 \times [(12.5 \times 0.6 + 12.5 \times 0.7 + 0.7 \times 0.6) 2]$$

$$= 12102.42 \text{ N}$$

wt. of tank $= 12.10242$ kN

As the tank is floating, so displaced volume of water is equal to the weight of the tank

$\rho g V_{displaced} = \text{Weight},$

$10^3 \times 9.81 \times 12.5 \times 0.2 \times x = 12102.42$

$\Rightarrow x = 0.141\,m$

$OG = \frac{h}{2} = \frac{0.6}{2} = 0.3\,m$

$OB = \frac{x}{2} = \frac{0.141}{2} = 0.0705\,m$

$BG = OG - OB = 0.2295\,m.$

$BM = \frac{I_{min}}{V_{displaced}} - BG$

$\frac{I_{min}}{V_{displaced}} = \frac{\frac{12.5 \times 0.2^3}{12}}{12.5 \times 0.2 \times 0.141} - 0.2295$

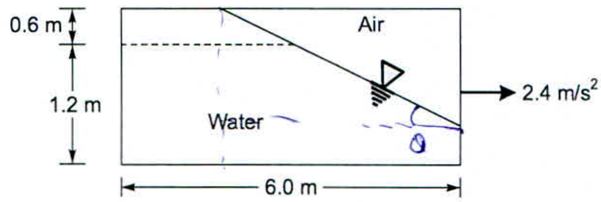
$= 0.289598 - 0.2295$

$BM = 0.06\,m > 0.$

Hence the given tank is stable.

12

Q.1 (e) A closed tank 6 m long, 2 m wide and 1.8 m deep initially contains water to a depth of 1.2 m. The top has an opening in the front part to have air space at atmospheric pressure. If the tank has given a horizontal acceleration at a constant value of 2.4 m/s^2 along its length, calculate the total pressure force on the top of the tank.



[12 marks]

Ans 1(e)

$$\tan\theta = \frac{a_x}{g} = \frac{2.4}{9.81}$$

$$\theta = \tan^{-1}\frac{2.4}{9.81} = 13.7473^\circ$$

?

1

Q.2 (a) A cylinder 0.25 m in radius and 2 m in length rotates coaxially inside a fixed cylinder of the same length and 0.30 m radius. Olive oil of viscosity 4.9×10^{-2} Ns/m^2 fills the annular space between the cylinders. A torque 4.9 N-m is applied to the inner cylinder. After constant velocity is attained, calculate the velocity gradient at the cylinder walls, the resulting rpm, and the power dissipated by fluid resistance ignoring end effect.

[20 marks]

Q.2 (b) A pump impeller is 37.5 cm in diameter and discharges water with velocity components of 2 m/s and 12 m/s in the radial and tangential directions respectively. The impeller is surrounded by a concentric cylindrical chamber with parallel sides, the outer diameter being 45 cm. If the flow in this chamber is a free-spiral vortex, find the components of velocity of water on leaving and the pressure rise in the shroud if there is no loss.

[20 marks]

Q.2 (c) **(i)** Many researchers believe that the problem of air-entertainment in free surface vortex formation at intakes is influenced by forces of viscosity and surface tension. Show that for dynamic similarity between model and prototype, the following relationship must be satisfied:

$$\left(\frac{\mu V}{\sigma}\right)_m = \left(\frac{\mu V}{\sigma}\right)_p$$

Also prove that by use of the same liquid results in the "equal-velocity" concept of model testing.

(ii) Water from a reservoir flowing through a rigid 150 mm diameter pipe, with a velocity 2.4 m/s is completely stopped by closure of a valve situated 1100 m from the reservoir, determine the maximum rise in pressure, when valve closure takes place

(1) In one second and

(2) In five seconds

Without damping of pressure wave. Consider the velocity of sound in water as 1432 m/s.

[10 + 10 marks]

Q.3 (a) An inward flow reaction turbine has inlet and outlet diameters of 1.2 m and 0.6 m respectively. The breadth at the inlet is 0.25 m and at the outlet it is 0.35 m. At a speed of rotation of 250 rpm, the relative velocity at entrance is 3.5 m/s and is radial. Calculate the (i) absolute velocity at entrance and the inclination to the tangent of the runner, (ii) discharge and (iii) the velocity of flow at the outlet.

[20 marks]

Ans 3 (a)

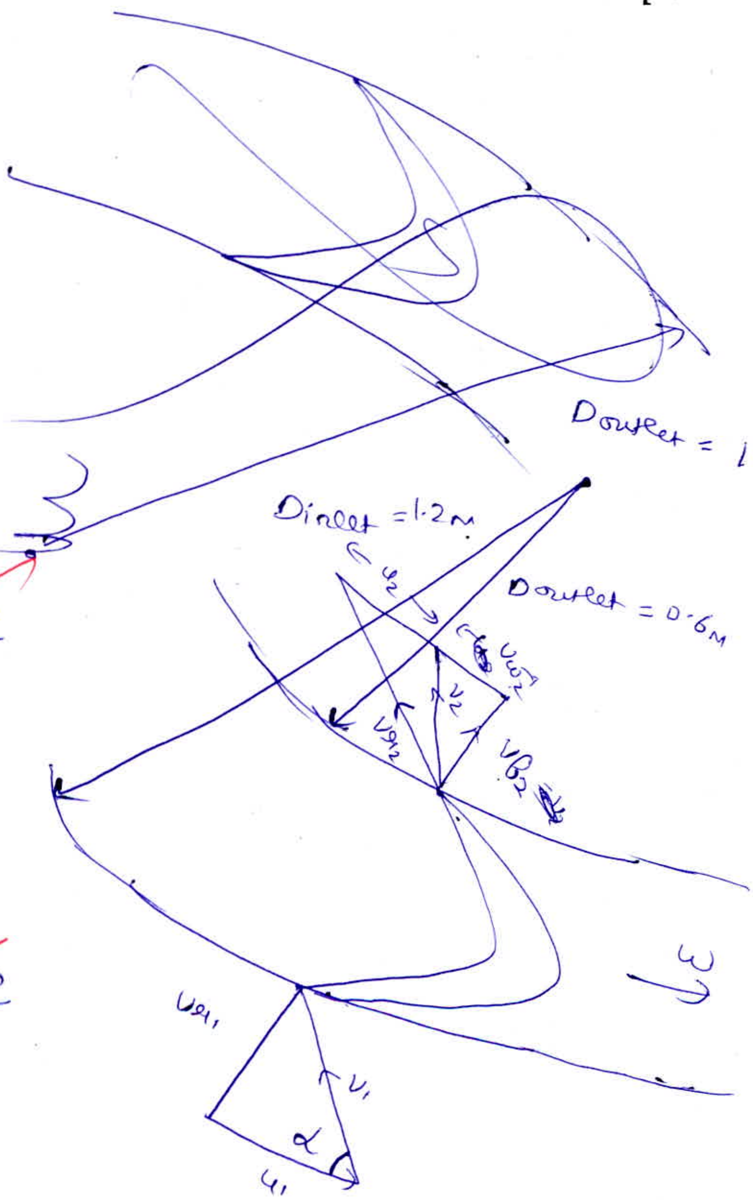
Given

$B_{inlet} = 0.25 m.$

$B_{outlet} = 0.35 m.$

$N = 250 rpm.$

$V_{r1} = 3.5 m/s$



$$u_1 = \frac{\pi D_1 N}{60}$$

$$u_1 = \frac{\pi \times 1.2 \times 250}{60}$$

$u_1 = 15.708 m/s$

$$u_2 = \frac{\pi D_2 N}{60}$$

$$u_2 = \frac{\pi \times 0.6 \times 250}{60}$$

$u_2 = 7.854 m/s$

$V_{r1} = V_{f1} = 3.5 m/s.$

(i) $\tan\alpha = \frac{V_{r1}}{u_1} = \frac{3.5}{15.708} \Rightarrow \boxed{\alpha = 12.56124^\circ}$

$V_1 = \sqrt{V_{r1}^2 + u_1^2} = \sqrt{3.5^2 + 15.708^2}$

$\boxed{V_1 = 16.093 m/s}$

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(ii) $Q = \pi D_1 B_1 V_{f_1}$

$Q = \pi \times 1.2 \times 0.25 \times 3.5$

$\boxed{Q = 3.29867 \, m^3/sec}$

(iii) $Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$

$= 1.2 \times 0.25 \times 3.5 = 0.6 \times 0.35 \times V_{f_2}$

$\Rightarrow \boxed{V_{f_2} = 0.5 \, m/s}$

20

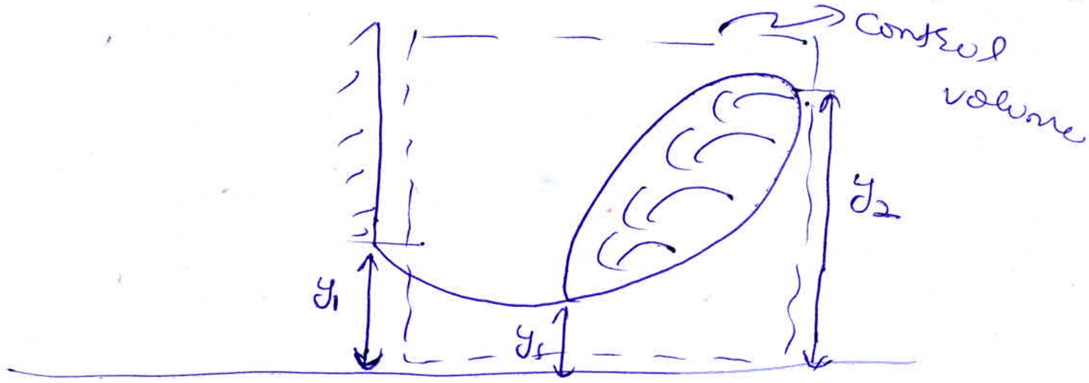
Q.3 (b) Show that for a submerged hydraulic jump just downstream of a sluice gate, in a horizontal rectangular channel,

$$\frac{y_s}{y_1} = \sqrt{2F_1^2\left(\frac{y_1}{y_2}-1\right)+\left(\frac{y_2}{y_1}\right)^2}$$

where y_1 is the depth of opening of the sluice gate, y_2 is the depth of flow downstream of the submerged hydraulic jump, y_s is the water depth on the downstream side of the sluice gate and F_1 is the Froude number of flow through the sluice opening.

[20 marks]

Ans 3 (b)



$P_1 + M_1 = P_2 + M_2$ (conserving specific force in control volume)

$P_1 - P_2 = M_2 - M_1$

$\gamma A_1 \frac{y_s}{2} - \gamma A_2 \frac{y_2}{2} = \rho Q [v_2 - v_1]$

$\gamma \frac{y_s^2}{2} - \gamma \frac{y_2^2}{2} = \rho q^2 \left[\frac{1}{y_2} - \frac{1}{y_1}\right]$

$\frac{\gamma}{2}\left[y_s^2 - y_2^2\right] = \rho q^2 \left[\frac{y_1 - y_2}{y_1 y_2}\right]$

$\frac{\cancel{\rho} g}{2}\left[y_s^2 - y_2^2\right] = \cancel{\rho} q^2 \frac{[y_1 - y_2)}{y_1 y_2}$

$y_s^2 - y_2^2 = \frac{2q^2}{g} \frac{[y_1 - y_2]}{y_1 y_2}$

divided the whole equation by y_1^2

$\left(\frac{y_s}{y_1}\right)^2 - \left(\frac{y_2}{y_1}\right)^2 = \frac{2q^2}{g y_1^3}\left[\frac{y_1}{y_2} - 1\right]$

$$\left(\frac{y_s}{y_1}\right)^2 - \left(\frac{y_2}{y_1}\right)^2 = 2F_1^2\left(\frac{y_1}{y_2} - 1\right)$$

As $F_1^2 = \frac{q^2}{g\,y_1^3}$.

$$\left(\frac{y_s}{y_1}\right)^2 = 2F_1^2\left(\frac{y_1}{y_2} - 1\right) + \left(\frac{y_2}{y_1}\right)^2$$

$$\boxed{\frac{y_s}{y_1} = \sqrt{2F_1^2\left(\frac{y_1}{y_2} - 1\right) + \left(\frac{y_2}{y_1}\right)^2}}$$

Hence proved

2

Q.3 (c) (i) What is meant by local and convective acceleration? For a one dimensional flow described by $V(x, t)$, derive the expression for convective acceleration in terms of velocity and its gradient.

(ii) A rectangular channel 5.2 m wide has a discharge of 10 m^3/sec at a velocity of 1.25 m/s. At a certain section the bed width is reduced to 3.0 m through a smooth transition. A smooth flat hump is to be built in this contracted section to cause critical flow for flow measurement purposes. Estimate the height of the hump necessary for this purpose. (Assume no loss of energy at the transition.)

[10 + 10 marks]

Ans 3(c) (i) the local acceleration. ~~which~~ ~~is~~ also known as temporal acceleration is the rate of change of velocity with time

$$a_x = \frac{\partial V_x}{\partial t}, \quad a_y = \frac{\partial V_y}{\partial t}, \quad a_z = \frac{\partial V_z}{\partial t}$$

the convective acceleration is rate of change of velocity with their space coordinates

$$a_x = V_x \frac{\partial V_x}{\partial x} + V_y \cdot \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z}$$

$$a_y = V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z}$$

$$a_z = V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z}$$

for one dimensional flow.

$a_x = \frac{\partial V}{\partial t}$ (local acceleration)

$a_x = V_x \frac{\partial V_x}{\partial x}$ (convective acceleration)

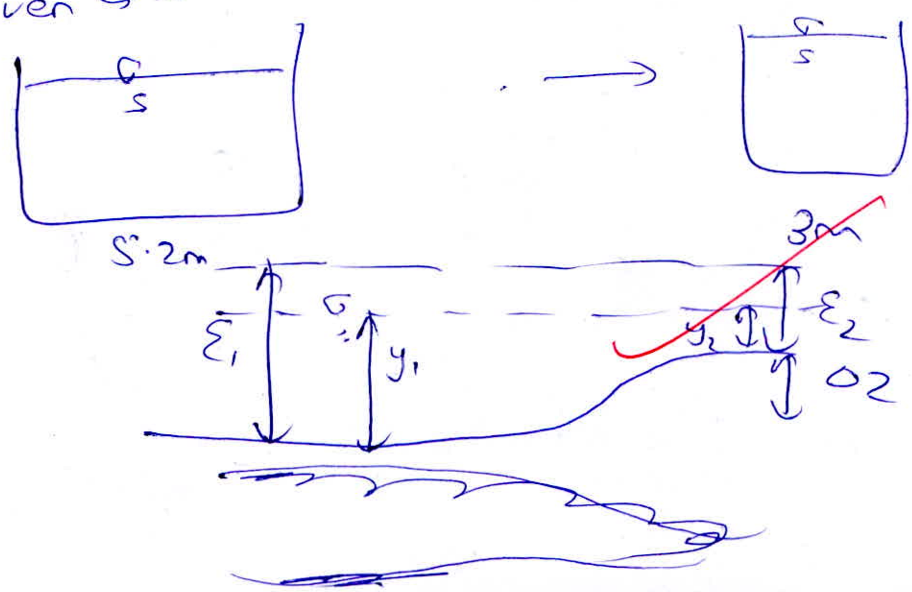
$a_{total} = a_{local} + a_{convective}$

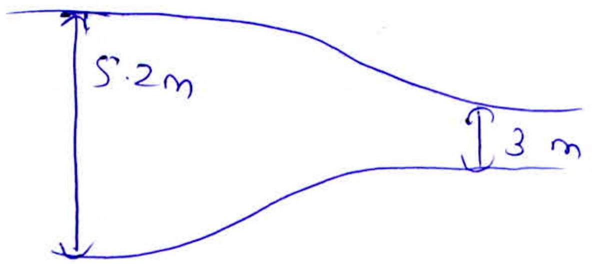
$$\boxed{a_{total} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}}$$

8 −2

= 6

(ii) Given $Q = 10\,m^3/s$ $V = 1.25\,m/s$





$Q = AV \Rightarrow Q = ByV$ $\qquad 10 = 5.2 \times y \times 1.25$

$\Rightarrow y = 1.53846\ m.$

$E_1 = y_1 + \frac{V^2}{2g} \Rightarrow E_1 = 1.53846 + \frac{1.25^2}{2\times 9.81}$

$\boxed{E_1 = 1.618\ m}$

$E_1 = E_2 + \Delta Z.$

Given that, E_2 is such that to cause a critical flow for flow measurement purposes.

$q = \frac{Q}{B_2} = \frac{10}{3} = 3.333\ m^3/s/m$

$y_c = \left(q^2/g\right)^{1/3} = \left(\frac{(10/3)^2}{9.81}\right)^{1/3} = 1.0424\ m.$

$E_c = \frac{3}{2} y_c = \frac{3}{2} \times 1.0424 = 1.56358\ m$

$E_c = E_2 = 1.56358\ m$

$E_1 = E_c + \Delta Z$

$1.618 = 1.56358 + \Delta Z$

10

$\Delta Z = 0.0544\ m.$

$\boxed{\Delta Z = 5.44\ cm}$

Q.4 (a) **(i)** For the velocity profile, $\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ on a flat plate, find out the average velocity and kinetic energy correction factor.

(ii) Calculate the friction drag on a flat plate 15 cm wide and 45 cm long placed longitudinally in a stream of oil of relative density 0.925 and kinematic viscosity 0.9 stoke, flowing with a free stream velocity of 6.0 m/s. Also, find the thickness of the boundary layer and shear stress at the trailing edge.

[10 + 10 marks]

Q.4 (b) A stream is spanned by a bridge which is a single masonry arch in the form of a parabolic arch, the crown being 2.5 metre above the springings which are 9 meters apart. The overall width of the bridge is 6 metres. During a flood the stream rises to a level 2 metres measured in the direction of the stream above the springings. Calculate the force tending to lift the bridge from its foundations if the arch remains water tight.

[20 marks]

Q.4 (c) **(i)** Define bulk modulus of elasticity of a fluid. What is the SI unit of bulk modulus of elasticity? Discuss the factors affecting bulk modulus of elasticity of a fluid. Why liquids are generally considered incompressible?

(ii) Show that the theoretical discharge in an open channel flow may be expressed as:

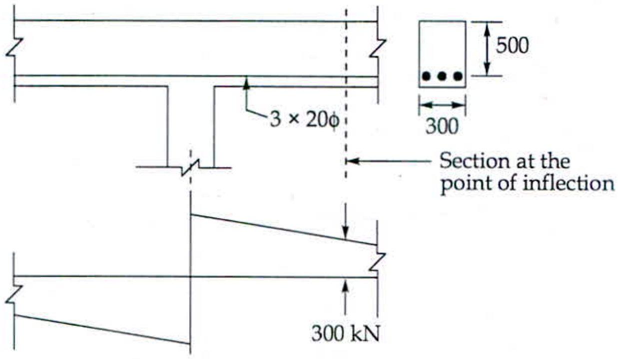
$$Q = A_2 \sqrt{\frac{2g(\Delta y - h_f)}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

where A_1 and A_2 are the cross-sectional areas of flow at sections (1) and (2) respectively, Δy is the drop in the water surface between the two sections and h_f is the energy head loss between the two sections.

[10 + 10 marks]

Section B : Design of Concrete and Masonry Structures-1 + Strength of Materials-2

Q.5 (a) Check for bond stress at the point of inflection of a continuous beam as shown in figure, if it is subjected to an ultimate shear force of 300 kN at the point of inflection. Consider concrete of grade M20 and steel of grade Fe415. [Take design bond stress for M20 concrete = 1.2 N/mm²]



Ultimate shear force diagram

Continuous beam

Given
M20/Fe415
$\tau_{bd} = 1.2$
$V_u = 300$ kN.

[12 marks]

Ans 5 (a) $L_d \leq 1.3\frac{M}{V} + L_0$

$0.36 f_{ck} x_u B = 0.87 f_y A_{st}$.

$0.36 \times 20 \times x_u \times 300 = 0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 20^2$

$x_u = 157.53778$ mm $< x_{u\,lim}$

$x_{u\,lim} = 0.48d = 0.48 \times 500 = 240$ mm

So, it is a under Reinforced section.

$M_u = 0.36 f_{ck} x_u B (d - 0.42 x_u)$

$= 0.36 \times 20 \times 157.53778 \times 300 \times (500 - 0.42 \times 157.53778) \times 10^{-6}$

$M_u = 147.62557$ kNm

Assume width of support = 300 mm

clear cover = 30 mm

$L_d = \frac{\phi(0.87 f_y)}{4 \tau_{bd}} \Rightarrow \frac{20 \times 0.87 \times 415}{4 \times 1.2 \times 1.6}$.

$L_d = 940.2343$ mm.

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$L_s = \frac{w}{2} - \bar{x} = \frac{300}{2} - 30 = 120 mm$

$1.3 \frac{M_u}{V_u} + L_s = \frac{1.3 \times 147.62557 \times 10^6}{300 \times 10^3} + 120$

Lo2 maxm

12 ϕ

d

$1.3 \frac{M_u}{V_u} + L_s = 759.7108 mm < L_d.$

Hence the given ~~bea~~ beam is not safe in bond stress.

8

Q.5 (b) State the assumptions made while analyzing the reinforced concrete beam using Limit State of Flexure as per **IS 456:2000 Code.**

[12 marks]

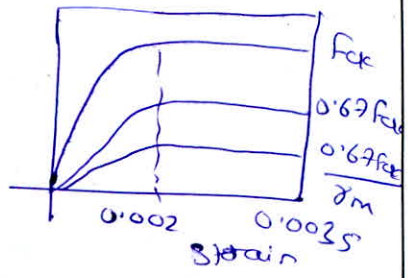
Ans (b) the Assumption made while analyzing Reinforced Concrete Beam using LSM as per IS 456: 2000 are

1) Plane section before bending remain plane even after bending. which signifies that the Strain profile is Linear

2) Maximum Strain ~~at the out~~ in the Concrete at outermost compression fibre is 0.0035 in bending

3) Concrete on the tension side is Ignored.

4) The Stress Strain relationship ~~behave~~ in the concrete can be ~~at~~ Trapezoidal, rectangular, Parabola or any other Shape which is in Substantional aggrement with the test results As per IS 456 : 2000 it is given as



0.67 fck strength is considerd stress in case of concrete to account the Shape factor and a material factor of Safety of 1.5 is applied in addition to this.

5) the Stress - Strain relationship ~~of~~ in case of Steel is also Considered from the test results.

0.87 fy strength of steel is considered and a material factor of safety of 1.15 is applied in addition to this.

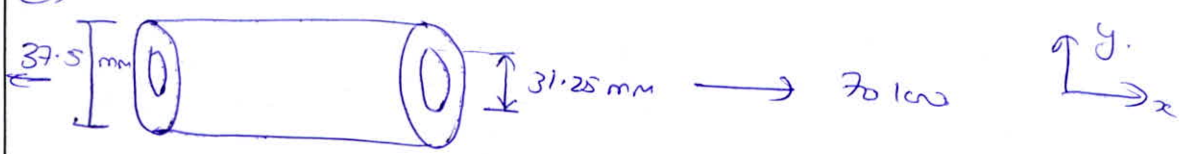
9

6) the strain in Steel in tension shall not be less than $0.002 + \frac{0.87 f_y}{E_s}$

Q.5 (c) Three exactly similar mild steel tube specimens have the external and internal diameters 37.5 mm and 31.25 mm respectively. One of these specimens was tested in pure tension and limit of proportionality was recorded to be 70 kN. The second specimen was tested in torsion whereas the third was tested in torsion with superimposed bending moment of 350 Nm. If the failure criterion is the maximum shear stress, determine the torque at which the two specimens would have failed?

[12 marks]

Ans 5 (c)

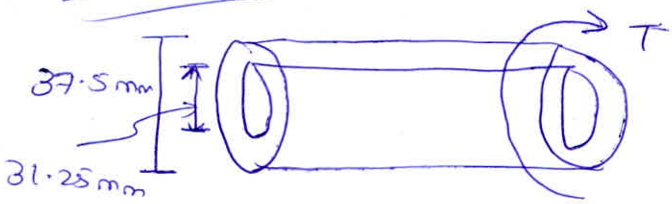


$$\sigma_x = \frac{70\times10^3}{\frac{\pi}{4}(37.5^2-31.25^2)} = 207.4223\ N/mm^2.$$

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} \Rightarrow \frac{207.4223-0}{2}$$

$$\boxed{\tau_{max} = 103.7111\ N/mm^2}$$

In 2nd Specimen



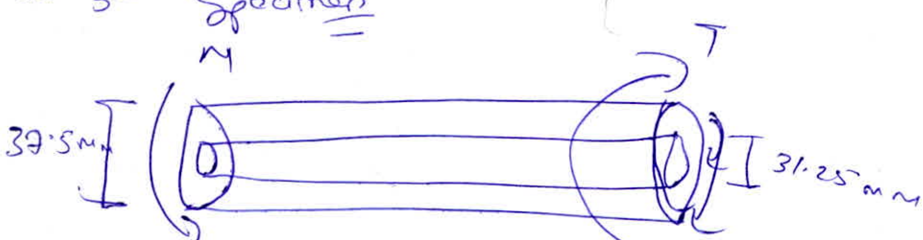
$$\tau_{max} = \frac{T}{J}r.$$

$$103.7111 = \frac{T}{\frac{\pi}{32}[37.5^4-31.25^4]}\times 37.5/2.$$

$$T = 555989.5833\ N\text{-}mm.$$

$$T = 0.5559\ kNm \Rightarrow \boxed{T = 555.989\ N\text{-}m}$$

In 3rd Specimen



$$\cancel{\tau_{max} = \frac{16}{\pi D^3}\sqrt{M^2+T^2}}$$

$$T_{max} = \sqrt{M^2+T^2}$$

$$\tau_{max} = \frac{T_{max}}{J} \times r$$

$$103.7111 = \frac{T_{max}}{\frac{\pi}{32}\left[37.5^4 - 31.25^4\right]} \times 37.5/2$$

$$T_{max} = 555.989 \text{ N-m}$$

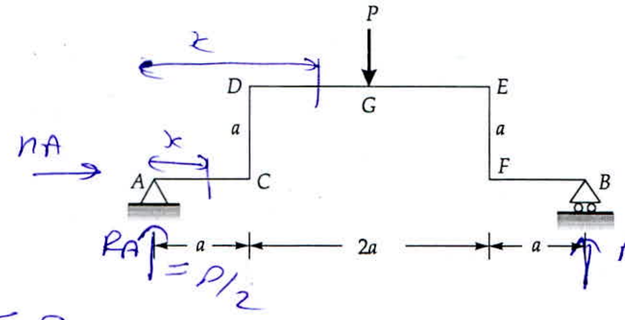
$$\sqrt{M^2+T^2} = 555.989$$

$$\sqrt{350^2+T^2} = 555.989$$

$$\boxed{T = 432 \text{ N-m}}$$

12

5 (d) Find the central deflection of the framed beam using strain energy method as shown in figure. [EI is constant]



[12 marks]

Ans 5(d)

$\Sigma f_y = 0 \qquad R_A + R_B = P$

$\Sigma f_x = 0 \qquad H_A = 0$

$\Sigma M_A = 0 \qquad R_B \times 4a = P \times 2a \Rightarrow \boxed{R_B = P/2}$

$\boxed{R_A = P/2}$

Members	M_x	$m_x = \frac{\partial M_x}{\partial P}$	$\delta_G = \int_0^l \frac{M_x m_x \, dx}{EI}$
AC	$\frac{P}{2}x$	$\frac{x}{2}$	$\delta_G = \int_0^a \frac{\frac{P}{2}x \cdot \frac{x}{2} \, dx}{EI}$
DC	$\cancel{0}$ $\frac{P}{2}a$	0	$\delta_G = 0.$
DG	$\frac{P}{2}x$ $\frac{P}{2}(a+x)$	$\frac{x}{2}$	$\delta_G = \int_a^{2a} \frac{\frac{P}{2}x \cdot \frac{x}{2} dx}{EI}$
FB	$\frac{P}{2}x$	$\frac{x}{2}$	$\delta_G = \int_0^a \frac{\frac{P}{2}x \frac{x}{2} dx}{EI}$
EF	$\cancel{0}$	0	0
EF	$\frac{P}{2}\cancel{x}$	$\frac{x}{2}$	$\delta_G = \int_a^{2a} \frac{\frac{P}{2}x \frac{x}{2} dx}{EI}$

$$\delta_{G_{total}} = 2\left[\int_0^a \frac{P x^2 dx}{4EI} + \int_a^{2a} \frac{P x^2 dx}{4EI}\right]$$

$$\delta_{G\,total} = \frac{2P}{4EI}\left[\int_0^a x^2 dx + \int_a^{2a} x^2 dx\right]$$

$$S_{a\,total} = \frac{P}{2EI}\left[\left[\frac{x^3}{3}\right]_0^a + \left[\frac{x^3}{3}\right]_a^{2a}\right]$$

$$S_{a\,total} = \frac{P}{2EI}\left[\frac{a^3}{3} + \frac{1}{3}\left[8a^3 - a^3\right]\right]$$

$$= \frac{P}{2EI}\left(\frac{a^3}{3} + \frac{7a^3}{3}\right)$$

$$S_{a\,total} = \frac{8Pa^3}{3 \times 2EI} = \frac{4Pa^3}{3EI}$$

$$\boxed{S_{a\,total} = \frac{4Pa^3}{3EI}}$$

4

2

Q.5 (e) A machine component is made of a material whose ultimate strength in tension, compression and shear are 40 N/mm^2, 110 N/mm^2 and 55 N/mm^2 respectively. At the critical point in the component, the state of stress is represented by

$$\sigma_x = 25 \text{ N/mm}^2 \text{ and } \sigma_y = -75 \text{ N/mm}^2$$

Find the maximum value of the shear stress τ_{xy} which will cause failure of the component?

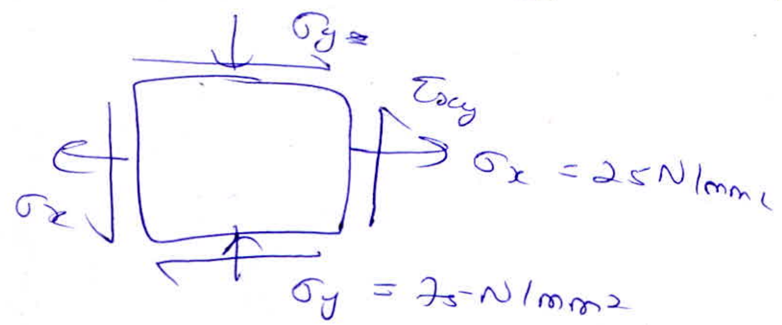
[12 marks]

Ans 5 (e)

Given $\sigma_x = 25$ N/mm^2

$\sigma_y = -75$ N/mm^2

$\sigma_{Tu} = 40$ N/mm^2 $\quad \sigma_{cu} = 110$ N/mm^2 $\quad \tau_u = 55$ N/mm^2



$$\sigma_{p_1}/\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{p_1}/\sigma_{p_2} = \frac{25-75}{2} \pm \sqrt{\left(\frac{25+75}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{p_1}/\sigma_{p_2} = -25 \pm \sqrt{50^2 + \tau_{xy}^2}$$

$$\sigma_{p_1} = -25 + \sqrt{50^2 + \tau_{xy}^2}$$

$$\sigma_{p_2} = -25 - \sqrt{50^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{\sigma_{p_1} - \sigma_{p_2}}{2} = \sqrt{50^2 + \tau_{xy}^2}$$

~~Op~~ for Safety

$$\sigma_{p_1} \leq \sigma_{Tu}$$

$$-25 + \sqrt{50^2 + \tau_{xy}^2} \leq 40. \Rightarrow \sqrt{50^2 + \tau_{xy}^2} \leq 65$$

$$\boxed{\tau_{xy} \leq 41.533 \text{ MPa}}$$

$\sigma_{P_1} \leq \sigma_{cu}$

$-25 - \sqrt{50^2 + \tau_{xy}^2} \leq -110.$

$\boxed{\tau_{xy} = 68.7386 \ N/mm^2.}$

$\tau_{max} \leq \tau_u$

$\sqrt{50^2 + \tau_{xy}^2} \leq 55$

$\boxed{\tau_{xy} = 22.913 \ N/mm^2}$

$\text{Max } \tau_{xy} = \text{Min}\left[\tau_{xy_1}, \tau_{xy_2}, \tau_{xy_3}\right]$

$= \text{Min}\left[41.533, 68.7386, 22.913\right]$

$\boxed{\text{Max } \tau_{xy} = 22.913 \ N/mm^2}$

12

Q.6 (a) Design a rectangular beam section of 300 mm width and 500 mm effective depth which is subjected to an ultimate bending moment of 50 kNm, ultimate shear force of 50 kN and ultimate torsional moment of 40 kNm. Consider concrete of grade M20 and steel of grade Fe415. [Assume effective cover = 35 mm]

$p_t(\%)$	≤ 0.15	0.25	0.5	0.75	1
$\tau_c\left(N/mm^2\right)$	0.28	0.36	0.48	0.56	0.62

[20 marks]

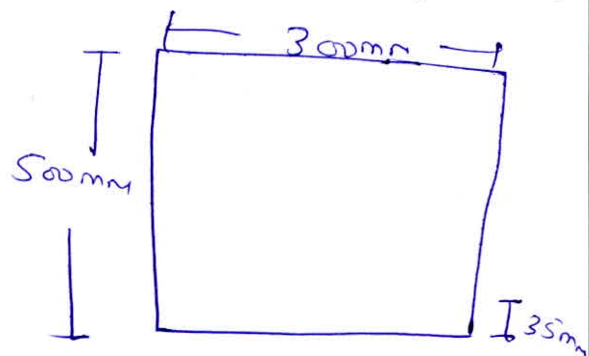
Ans 6 (a)

Given $M_u = 50$ kNm

$V_u = 50$ kN

$T_u = 40$ kNm

M20 / Fe415



$M_{eq} = M_u + \underbrace{\frac{T_u}{1.7}\left[1+\frac{D}{B}\right]}_{M_{ue}}$

$d = 465$ mm

$V_{eq} = V_u + \frac{1.6 T_u}{B}$

$M_{ue} = \frac{T_u}{1.7}\left[1+\frac{D}{B}\right] = \frac{40}{1.7}\left[1+\frac{500}{300}\right]$ 535

$= 62.745$ kNm 65.49 > 50 kNm

$M_{ue_2} = M_{ue} - M_u = 62.745 - 50 = 12.745$ kNm.

It is provided on the opposite side.

M_{ue_1} $M_{ue} + M_u = 62.745 + 50 = 112.745$ kNm

$M_{u\,lim} = 0.36 f_{ck}\, x_{u\,lim}\, B\,(d - 0.42\, x_{u\,lim})$

$x_{u\,lim} = 0.48 \times 465 = 223.2$ mm

$M_{u\,lim} = 0.36 \times 20 \times 223.2 \times 300 \times (465 - 0.42 \times 223.2) \times 10^{-6}$

$M_{u\,lim} = 178.9869$ kNm $> M_{ue_1}$

Under Reinforced Section.

$\therefore A_{st_1} = \frac{M_u}{0.87 f_y (d - 0.42 x_u)} = \frac{112.745 \times 10^6}{0.87 \times 415 \times (465 - 0.42 \times 223.2)}$

$A_{st1} = 841.117\ mm^2 = 4-18\phi$

$$A_{st2} = \frac{M_{ue2}}{0.87 f_y (d-d'_c)} = \frac{12.745\times10^6}{0.87\times415\times(465-35)}$$

$= 82.092\ mm^2$

$$V_{eq} = V_u + 1.6\frac{T_u}{B} = 50 + 1.6\times\frac{40}{0.3}$$

$V_{eq} = 263.333\ kN.$

$$\tau_v = \frac{V_{eq}}{Bd} = \frac{263.333\times10^3}{300\times465} = 1.8877\ N/mm^2 < \tau_{c\,max}$$

$$P_t\% = \frac{4\times\frac{\pi}{4}\times18^2}{300\times465}\times100 = 0.72966\%$$

Assume $\tau_c = 1.2\ N/mm^2$.

Provide 2L − 8ϕ Stirrups.

$$\frac{A_{sv}}{B S_v} = \frac{\tau_v - \tau_c}{0.87 f_y}$$

$$\frac{2\times\frac{\pi}{4}\times8^2}{300\times S_v} = \frac{1.8877-1.2}{0.87\times415}$$

$\Rightarrow S_v = 175.9328\ mm$

Minimum Shear Reinforcement

$$\frac{A_{sv}}{B S_v} = \frac{0.4}{0.87 f_y} \Rightarrow \frac{2\times\frac{\pi}{4}\times8^2}{300\times S_v} = \frac{0.4}{0.87\times415}$$

$\Rightarrow S_v = 302.472\ mm$

As per torsional formula.

$d_1 = 500 - 2\times35$
$= 430\,mm$
$b_1 = 300 - 2\times35$
$b_1 = 230\,mm$

$$S_v = \frac{0.87 f_y A_{sv} d_1}{\left[\frac{V_u}{2.5} + \frac{T_u}{b_1}\right]}$$

$$S_v = \frac{0.87\times415\times2\times\frac{\pi}{4}\times8^2\times430}{\left[\frac{50\times10^3}{2.5} + \frac{40\times10^6}{230}\right]}$$

$S_v = 80.4875\,mm.$

~~provide 2L 8ϕ @ 80 c/c~~

Spacing of lateral ties = Min $\begin{cases} x_1 \\ \frac{x_1+y_1}{4} \\ 300\,mm \end{cases}$

$x_1 = 300 - 2\times35 - 8\times2 = 214\,mm$

$y_1 = 500 - 2\times35 - 8\times2 = 414\,mm$

$\frac{x_1+y_1}{4} = \frac{214+414}{4} = 157\,mm$

$\begin{cases} 214\,mm \\ 157\,mm \\ 300\,mm \end{cases}$ ~~provide~~ $\Rightarrow 157\,mm$

side faced reinforcement ??

provide 2L 8ϕ @ 80 c/c

detailing diagram?

16

Q.6 (b) **(i)** A ring beam of water tank has a diameter of 12.5 m. It is subjected to outward radial force of 25 kN/m. Design the section of ring beam using M25 and Fe415. Assume $m = 11$ and allowable stress in tension as 1.2 N/mm^2.

(ii) Calculate the development length in tension and compression for a single mild steel bar of diameter ϕ in concrete of grade M20. Assume $\tau_{bd} = 1.2$ N/mm^2.

[14 + 6 marks]

Ans 6 (b) $T = 25$ kN/m. M20/Fe415.

$\sigma_{st} = 130$ N/mm2. $m = 11$, $\sigma_{bt} = 1.2$ N/mm2

$A_{st\,req} = \frac{T}{\sigma_{st}} = \frac{25 \times 10^3}{130} = 192.307$ mm2

provide 8ϕ

Spacing $= \frac{1000}{192.307} \times \frac{\pi}{4} \times 8^2$

$= 261.38$

8ϕ @ 260 c/c.

$\frac{T_h}{1000t + (m-1)A_{st}} \leq \sigma_{bt}$

A_{st} provided $= \frac{1000}{260} \times \frac{\pi}{4} \times 8^2 = 193.3287 \text{ mm}^2$

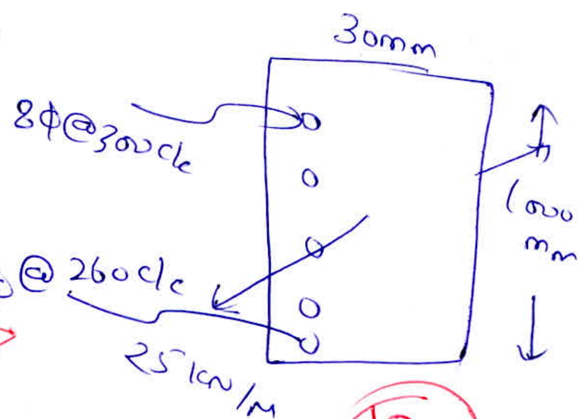
$$\frac{25 \times 10^3}{1000 \times t + (11-1) \times 193.3287} \leq 1.2$$

$$\frac{25 \times 10^3}{1.2} \leq 1000t + 10 \times 193.3287$$

$$t \geq 18.9 \text{ mm.}$$

Size of ring beam?

provide $t = 30 \text{ mm.}$



σ_{bt} developed

$$= \frac{T}{1000t + (m-1)A_{st}}$$

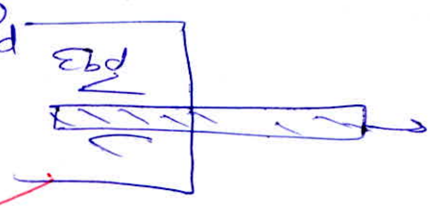
$$= \frac{25 \times 10^3}{1000 \times 30 + (10 \times 193.3287)}$$

$$= 0.78288 \text{ N/mm}^2 < 1.2 \text{ N/mm}^2 \quad \underline{\underline{\text{ok}}}$$

R

(ii) development length in tension.

$$\frac{\pi}{4}\phi^2(0.87 f_y) = \tau_{bd} \times \pi \phi L_d$$



$$L_d = \frac{\phi(0.87 f_y)}{4\tau_{bd}}$$

$$L_d = \frac{\phi(0.87 \times 250)}{4 \times 1.2}$$

$$\boxed{L_d = 45.3125\phi}$$

development length in compression.

$$l_d = \frac{\phi(0.87 f_y)}{4 \tau_{bd}}$$

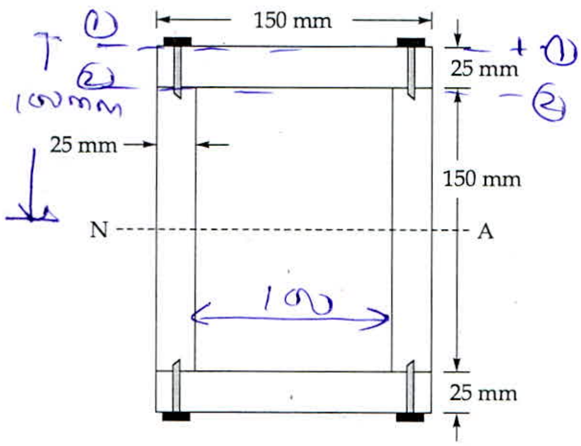
[In compression τ_{bd} is increased by 25%.]

$$l_d = \frac{\phi(0.87 \times 250)}{4 \times 1.2 \times 1.25}$$

$$\boxed{l_d = 36.25\phi}$$

6

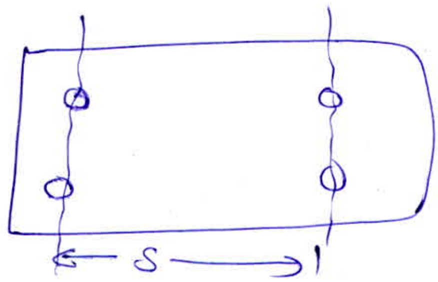
.6 (c) The box beam as shown in figure below is made up of four 150 mm × 25 mm wooden planks connected by screws. Each screw can safely transmit a shear force of 1250 N. Estimate the minimum necessary spacing of screws along the length of the beam if the maximum shear force transmitted by the cross-section is 5000 N. Also determine the shear stress distribution across the section.



Given
Planks = 150 mm × 25 mm
V_{max}/screw = 1250 N.
V_{max}/cross section = 5000 N

[20 marks]

Ans 6 (c)



Top view of beam
Let spacing = S.

$\varphi = A\bar{y}$

$\varphi = 150 \times 25 \times \left(75 + \frac{25}{2}\right)$
$= 328125 \text{ mm}^3$

$\tau_{screw} = \dfrac{V\varphi}{It}$

$I_{NA} = \dfrac{150 \times 200^3}{12} - \dfrac{100 \times 150^3}{12}$

$I_{NA} = 71.875 \times 10^6 \text{ mm}^4$

$\tau_{screw} = \dfrac{5000\left[150 \times 25 \times \left(75 + 25/2\right)\right]}{71.875 \times 10^6 \times 25 \times 2}$

$\tau_{screw} = \dfrac{21}{46} = 0.45652 \text{ N/mm}^2$

~~τ_{max} Area = V safe.~~

τ_{max} in one screw $= \dfrac{0.45652}{2} = 0.22826 \text{ N/mm}^2$

=

Assume diameter of screw = 10 mm

$\tau_{max} A = \phi V_{safe}$

$0.22826 \times 10 \times S = (1250 \times 2)$

$\boxed{S = 547.619\ mm = 54.762\ cm}$

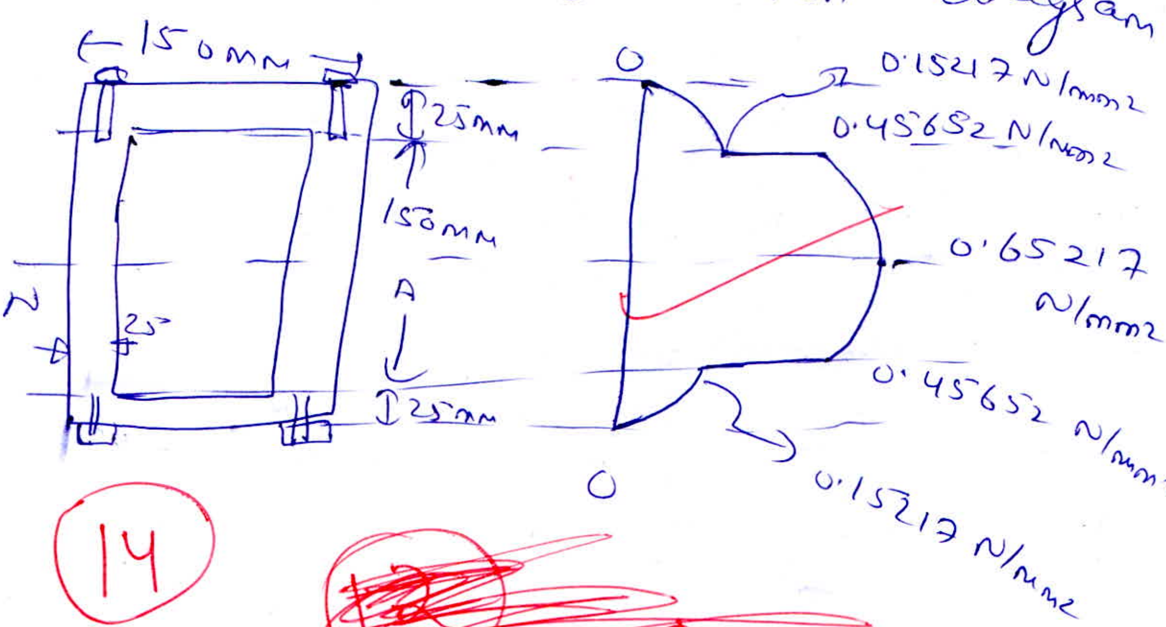
$\tau_{top} = \frac{VQ}{It} \Rightarrow \tau_{2-2\,top} = \frac{5000 \times \left[150 \times 25 \times \left(75 - \frac{25}{2}\right)\right]}{71.875 \times 10^6 \times 150}$

$\tau_{2-2\,top} = 0.15217\ N/mm^2$

$\tau_{NA} = \frac{VQ}{It} = \frac{5000 \times 6 \left[150 \times 25 \times \left(75 + \frac{25}{2}\right) + 50 \times 75 \times 75/2\right]}{71.875 \times 10^6 \times 50}$

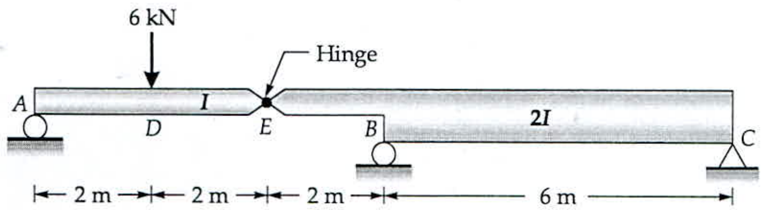
$\tau_{NA} = 0.65217\ N/mm^2$

Shear stress distribution diagram



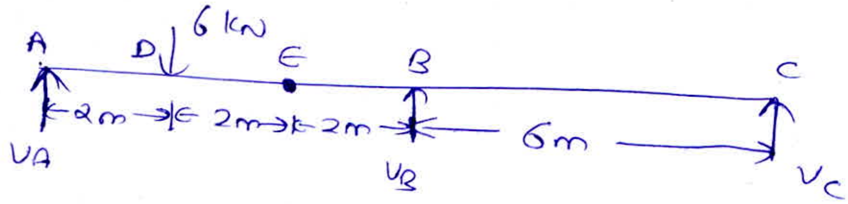
14

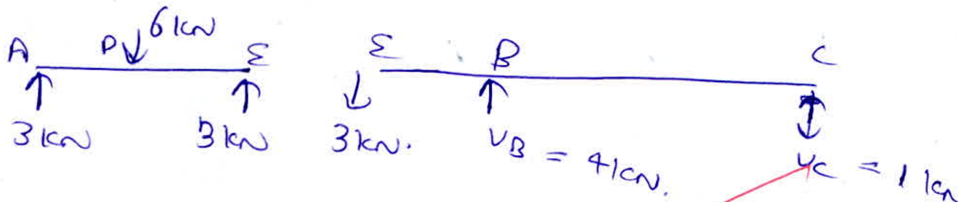
Q.7 (a) A hinged beam system is loaded as shown below. Determine the slope at point E and D. Also determine the deflection at D. Use Conjugate beam method.



[20 marks]

Ans 7 (a)





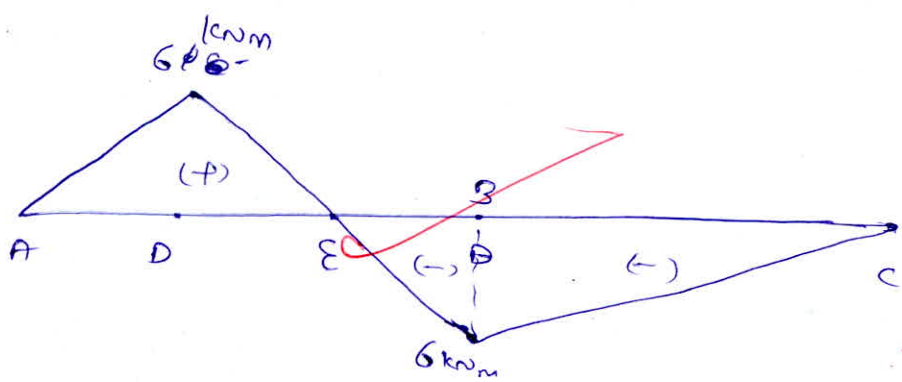
$\Sigma M_E = 0 \Rightarrow V_A \times 4 = 6 \times 2$

$\Rightarrow V_A = 3\,kN$, $V_E = 6 - 3 = 3\,kN$.

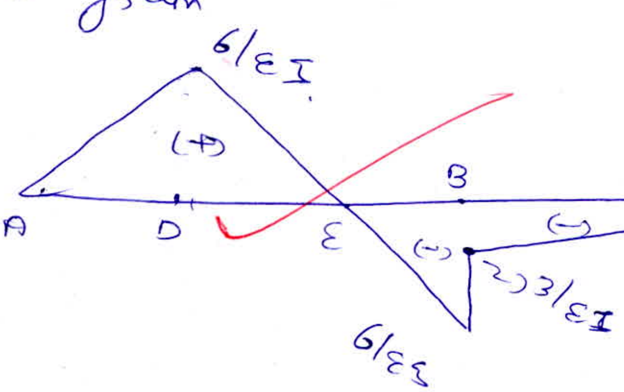
$\Sigma M_C = 0 \quad 3 \times 8 = V_B \times 6 \Rightarrow V_B = 4\,kN$,

$V_A + V_C = 3\,kN \Rightarrow V_C = 3 - 4 = -1\,kN$.

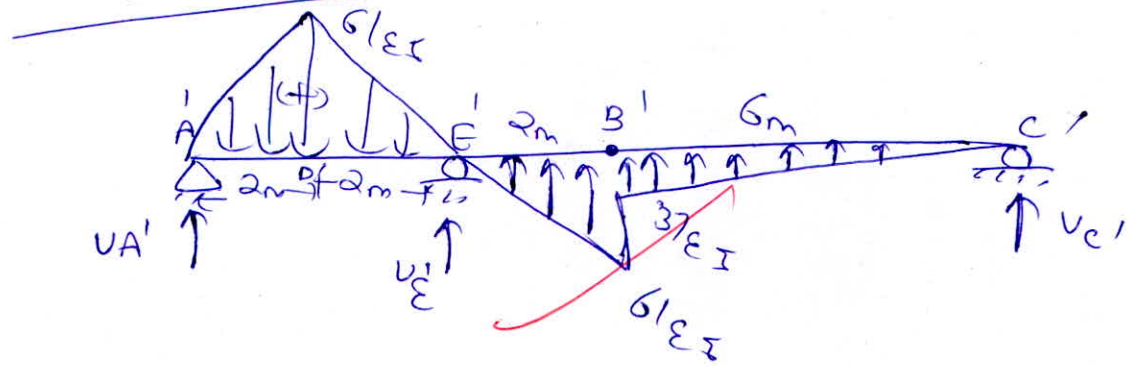
BMD



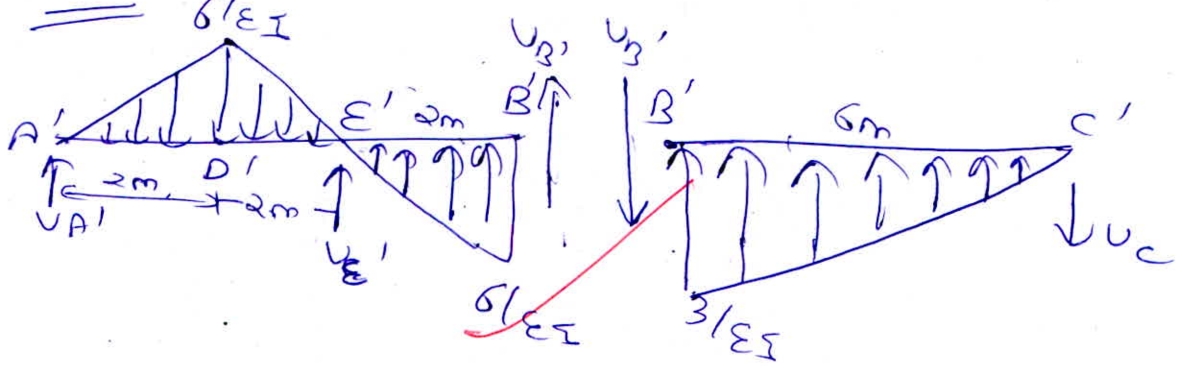
M/EI diagram



Conjugate Beam



FBD



$\Sigma M_B = 0 \Rightarrow V_C' \times 6 = \frac{1}{2} \times \frac{3}{EI} \times 6 \times \frac{1}{3} \times 6$

$V_C' = \frac{3}{EI}$

$V_B' + V_C' = \frac{1}{2} \times \frac{3}{EI} \times 6$

$V_B' + \frac{3}{EI} = \frac{9}{EI} \Rightarrow V_B' = 6/EI$

$V_A' + V_E' + \frac{6}{EI} + \frac{1}{2} \times \frac{6}{EI} \times 2 = \frac{1}{2} \times \frac{6}{EI} \times 4$

$V_A' + V_E' + \frac{12}{EI} = \frac{12}{EI}$

$V_A' + V_E' = 0$

$V_A' = -V_E'$

$\Sigma M_A = 0.$

$$V_B' \times 6 + \frac{1}{2} \times \frac{6}{EI} \times 2 \times \left[4 + \frac{2}{3} \times 2\right] + V_E' \times 4$$

$$= \frac{1}{2} \times \frac{6}{EI} \times 2 \times 4$$

$$\frac{6}{EI} \times 6 + \frac{32}{EI} + V_E' \times 4 = \frac{24}{EI}$$

$$V_E' = \cancel{\frac{-15.5}{EI}} \quad \frac{-11}{EI}$$

$$V_A' = \cancel{\frac{15.5}{EI}} \quad \frac{11}{EI}$$

Slope at D = Shear force at D'

$$= \frac{11}{EI} - \frac{1}{2} \times \frac{6}{EI} \times 2$$

$$\boxed{\text{Slope at D} = \cancel{\frac{9.5}{EI}} \quad \frac{5}{EI}}$$ (Clockwise)

deflection at d = ~~M~~ bending moment at D'

$$= V_A' \times 2 - \frac{1}{2} \times \frac{6}{EI} \times 2 \times \frac{1}{3} \times 2.$$

$$= \frac{11}{EI} \times 2 - \frac{4}{EI}$$

$$\boxed{\text{deflection at D} = \cancel{\frac{27}{EI}} \quad \frac{18}{EI}}$$ (downwards)

Slope at left of E = Shear force at left of E'

$$= \frac{11}{EI} - \frac{1}{2} \times \frac{6}{EI} \times 4$$

$$\boxed{\text{Slope at left of E} = \frac{-1}{EI}}$$ (Anticlockwise)

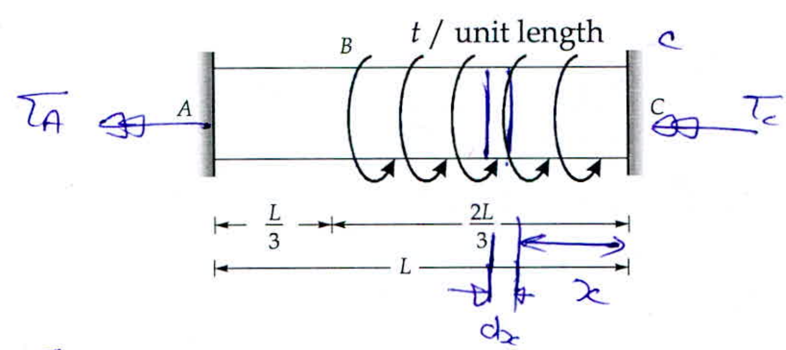
Slope at right of E = Shear force at right of E'

$= \frac{6}{EI} + \frac{1}{2} \times \frac{6}{EI} \times 2$

$\boxed{\theta_E \text{ Right} = \frac{12}{EI} \text{ (clockwise)}}$

20

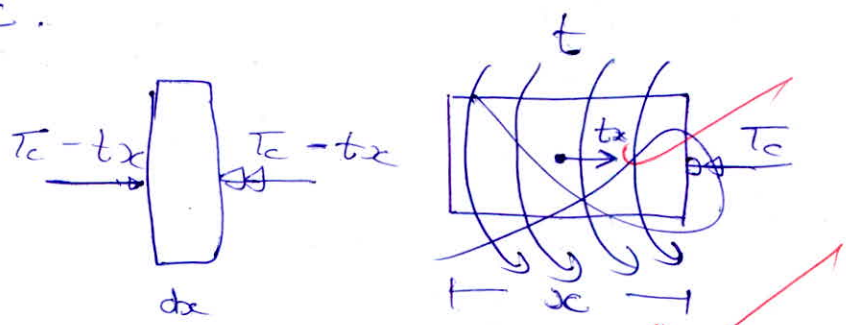
Q.7 (b) A solid circular cross-section shaft is clamped at both ends and loaded by a twisting moment t per unit length as shown in figure below. Determine the reactive twisting moment at each end of the bar.



Ans 7 (b) **[20 marks]**

$T_A + T_c = t \times \frac{2L}{3} = \frac{2tL}{3}$

Consider a section dx at distance of x from C.



$\theta_{CB} = \int_0^{2L/3} \frac{T\,dx}{GJ} = \int_0^{2L/3} \frac{(T_c - tx)\,dx}{GJ}$

$$\theta_{CB} = \frac{1}{GJ}\left[\left[T_c x\right]_0^{2l/3} - \left[t\frac{x^2}{2}\right]_0^{2l/3}\right]$$

$$\theta_{CB} = \frac{1}{GJ}\left[\left(T_c \times \frac{2l}{3}\right) - \frac{t}{2}\left[\frac{4l^2}{9}\right]\right]$$

$$\theta_{CB} = \frac{1}{GJ}\left[\frac{2lT_c}{3} - \frac{2tl^2}{9}\right]$$

$$\theta_{BA} = \frac{T_{BA}L}{GJ} = \frac{\left(T_c - t\frac{2l}{3}\right)\times\frac{L}{3}}{GJ.}$$

$$\theta_{A} = \theta_{CB} + \theta_{BA} = 0$$

$$\theta_{CA} = \frac{1}{GJ}\left[\frac{2l}{3}T_c - \frac{2tl^2}{9}\right] + \left[\frac{T_cL}{3} - \frac{t2l^2}{9}\right]\frac{1}{GJ}$$

$$\theta_{CA} = \frac{1}{GJ}\left[\frac{2l}{3}T_c + \frac{T_c l}{3} - \frac{2tl^2}{9} - \frac{2tl^2}{9}\right] = 0$$

$$\frac{1}{GJ}\left[T_c l - \frac{4tl^2}{9}\right] = 0$$

$$T_c l = \frac{4tl^2}{9}$$

$$\boxed{T_c = \frac{4tl}{9}}$$

$$T_A = \frac{2tl}{3} - \frac{4tl}{9}$$

$$\boxed{T_A = \frac{2}{9}tl}$$

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7 (c) Design a reinforced concrete rectangular section of size 250 × 500 mm for a factored moment of 225 kN. The grades of concrete and HYSD steel are M20 and Fe415, respectively. [Take effective cover = 50 mm, f_{sc} = 353 MPa]

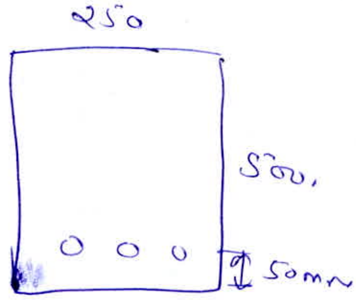
[20 marks]

Ans 7 (c) Given $f_{sc} = 353$ MPa

M20 / Fe415

$M_u = 225$ kNm

$d' = 50$ mm $b \times d = 250 \times 500$ mm



$M_{u\,lim} = 0.36 f_{ck}\, x_{u\,lim}\, B\,(d - 0.42\, x_{u\,lim})$

$x_{u\,lim} = 0.48 d = 0.48 \times 450 = 216$ mm.

$M_{u\,lim} = 0.36 \times 20 \times 216 \times 250 \times (450 - 0.42 \times 216) \times 10^{-6}$

$= 139.688$ kNm < 225 kNm

Hence doubly Reinforced Section is Required.

$A_{st1} = \dfrac{M_{u\,lim}}{0.87 f_y (d - 0.42 x_u)} = \dfrac{139.688 \times 10^6}{0.87 \times 415 \times (450 - 0.42 \times 216)}$

$A_{st1} = 1076.858$ mm2

$A_{st2} = \dfrac{M_u - M_{u\,lim}}{0.87 f_y (d - d_0')} = \dfrac{(225 - 139.688) \times 10^6}{0.87 \times 415 \times (450 - 50)}$

$A_{st2} = 590.7215$ mm2.

$A_{st} = A_{st1} + A_{st2} = 1076.858 + 590.7215$

$= 1667.58$ mm2

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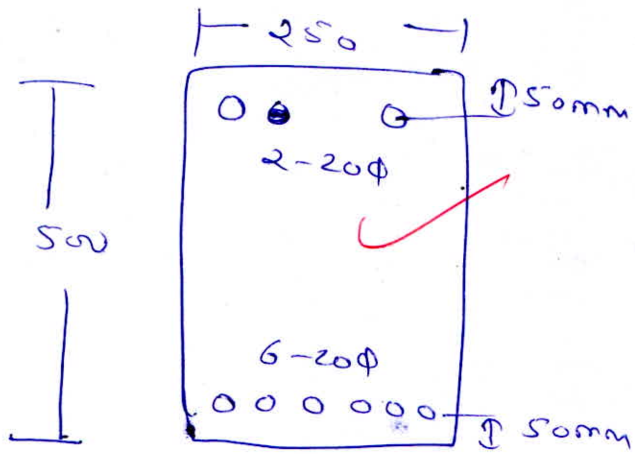
$$Asc = \frac{Mu - Mu_{lim}}{(f_{sc} - 0.45 f_{ck})(d - d_c)}$$

$$Asc = \frac{(225 - 139.688)\times 10^6}{(353 - 0.45\times 20)(450 - 50)}$$

$Asc = 620\ mm^2$

2-20φ



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(a) (i) A rectangular beam section of 300 mm width and 500 mm effective depth is reinforced with 5 bars of 20 mm ϕ, out of which 2 bars have been bent at 45°. Determine the shear resistance of the bent up bars and additional shear reinforcement required if it is subjected to an ultimate shear force of 300 kN. Consider concrete of grade M20 and steel of grade Fe415.

$p_t(\%)$	≤0.15	0.25	0.5	0.75	1
$\tau_c\left(N/mm^2\right)$	0.28	0.36	0.48	0.56	0.62

(ii) Determine the ultimate load capacity of a circular column of 400 mm diameter reinforced with 6 × 25 mm ϕ bars adequately tied with (i) lateral ties and (ii) spirals. Consider concrete of grade M25 and steel of grade Fe415.

[10 + 10 marks]

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8 (b) A staircase consists of 14 steps, each of 300 mm tread and 180 mm rise, plus two landings of each 1.25 m length. The width of staircase is 1.4 m. Design the staircase for a live load of 5 kN/m^2. Use M20 grade concrete and Fe415 reinforcement.

[20 marks]

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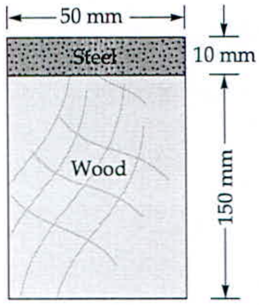
Q.8 (c) **(i)** A wooden beam 50 mm wide and 150 mm deep is reinforced by gluing a steel plate 10 mm thick and 50 mm wide on the top of section. The beam is simply supported over its ends which are 5 m away from each other. The beam carries a point load of 500 kN at mid of beam. Calculate maximum shear stress at the junction of wood and steel plate. Take $m = 20$.



(ii) Find the dimensions of a hollow steel shaft of internal diameter 0.6 times the external diameter, to transmit 150 kW at 250 rpm, if the shearing stress is not to exceed 70 N/mm^2. If a bending moment of 3000 Nm is now applied to the shaft, find the speed at which it must be driven to transmit the same power for the same value of maximum shearing stress.

[10 + 10 marks]

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Space for Rough Work

$$\frac{A_{sv}}{B\,S_v} = \frac{\tau_v - \tau_c}{0.87 f_y}$$

$$S_v = \frac{0.87 f_y A_{sv} d}{\tau_v - \tau_c}$$

$$\frac{0.87 f_y A_{sv} d}{\left(\frac{V_u}{2.5} + \frac{\tau_c}{bd}\right)}$$

$$\int_0^{2\theta/3} \frac{(\tau_c - t x)}{45}\, dx$$

$$\tau_c \theta - \frac{t\theta^2}{2} = 0$$

$$\tau_c = \frac{t\theta}{2}$$

$$\tau_c - t\frac{2x}{3}$$

$$\frac{\left(\tau_c - t\frac{2\theta}{3}\right)\frac{x}{3}}{45}$$

Space for Rough Work