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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-5: Flow of Fluids, Hydraulic Machines and Hydro Power
Design of Concrete and Masonry Structures-1
Strength of Materials-2

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	50
Q.2	
Q.3	60
Q.4	
Section-B	
Q.5	51 − 2 = 49
Q.6	
Q.7	46
Q.8	58
Total Marks Obtained	265 − 2 = 263

Good presentation & handwriting. Keep it up!

Signature of Evaluator .... Cross Checked by ....

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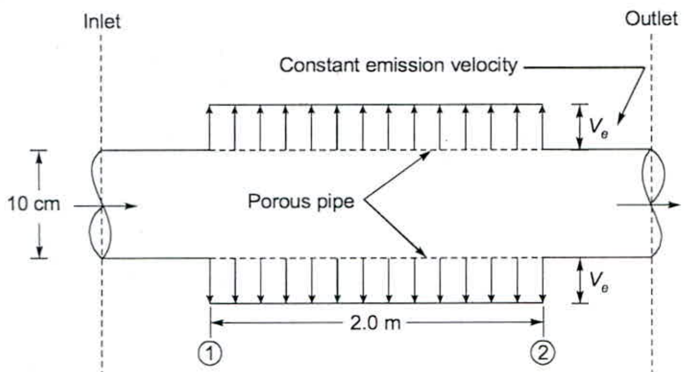
Section A : Flow of Fluids, Hydraulic Machines and Hydro Power

.) A circular pipe 10 cm in diameter has a 2 m length which is porous. In this porous section the velocity of exit is known to be constant as shown in figure. If the velocities at inlet and outlet of the porous section are 2.0 m/s and 1.2 m/s respectively. Estimate

(i) the discharge emitted out through the walls of the porous pipe and

(ii) the average velocity of this emitted discharge.



[12 marks]

Acc. to the law of conservation of mass :→

incoming Discharge = outgoing discharge.

$\frac{\pi}{4} \times ~~100~~ 0.1^2 \times 2 = Q_{porous} + \frac{\pi}{4} \times 0.1^2 \times 1.2$

$Q_{porous} = 6.283$ lit/sec

(ii) Area of the emitting section = πDL

$= \pi \times 0.1 \times 2.$

$= 0.2\pi \ m^2$

Discharge of emitting section = $6.283 \times 10^{-3} \ m^3/s.$

$$\text{Avg. velocity} = \frac{Q}{\text{Area}}$$

$$= \frac{6.283 \times 10^{-3}}{0.2 \times \pi}$$

$$\boxed{V = 1\ \text{cm/sec}}$$

12

) **(i)** Explain forced vortex flow occurring in a centrifugal pump.

(ii) Water is flowing through a smooth pipe of 100 mm diameter at rate of 0.036 m^3/s. Determine

(a) Darcy's friction factor

(b) Normal thickness of viscous sub layer

Take kinematic viscosity = 10^{-6} m^2/s and f (Darcy's friction factor) = $0.0032 + \frac{0.221}{R_e^{0.237}}$

[6 + 6 marks]

Given Diameter = 0.1 m.

$Q = 0.036 m^3/s$.

$\nu = 10^{-6} m^2/s$

$f = 0.0032 + \frac{0.221}{(Re)^{0.237}}$

(a) Reynold's number = $\frac{V.D}{\nu}$.

$= \frac{0.036}{\frac{\pi}{4} \times 0.1^2} \times 0.1 \times \frac{1}{10^{-6}}$

$Re = 458366.236$.

Friction factor $f = 0.0032 + \frac{0.221}{(458366.236)^{0.237}}$

$f = 0.01326$

(b) thickness of laminar sub layer = $\frac{11.6 \nu}{u_*}$

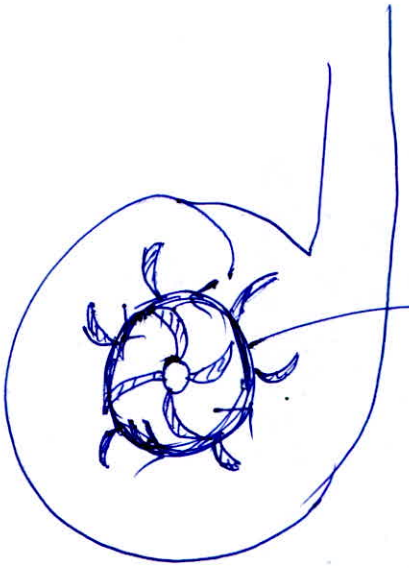
$u_* = V\sqrt{\frac{f}{8}} = \frac{0.036}{\frac{\pi}{4} \times 0.1^2}\sqrt{\frac{0.01326}{8}} = 0.1866 m/s$

$$\delta^* = \frac{11.6\nu}{u_*} = \frac{11.6\times10^{-6}}{0.1866}$$

$$\boxed{\delta^* = 0.062\ mm.}$$

6

(ii)



* Due to rapid movement of the vanes the water trapped b/w the guide vanes & moving vanes undergoes forced vortex motion.

1

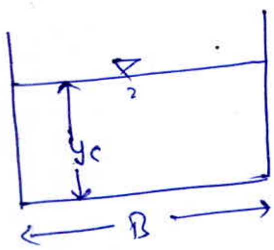
) Show that at the critical state of flow, the specific energy in a rectangular channel is equal to 1.5 times the depth of flow. Also find at critical flow condition whether the depth of flow will be greater or less than $\frac{2}{3}$ times specific energy for a trapezoidal channel.

[12 marks]

For Critical flow in a channel :→

$$\frac{Q^2 T}{A^3 g} = 1$$

for Rectangular channel :→ $T = B$. $A = B \cdot y_c$.



$$\frac{Q^2 T}{A^3 g} = \frac{Q^2 \cdot B}{B^3 \cdot y_c^3 \cdot g} = 1.$$

$$\therefore y_c = \left(\frac{q^2}{g}\right)^{1/3}.$$

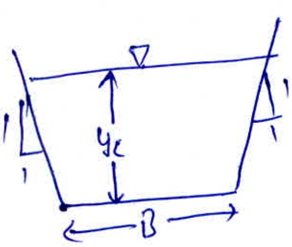
* Energy at this depth :→

$$E_c = y_c + \frac{V^2}{2g} = y_c + \frac{Q^2}{B^2 \cdot y_c^2} \times 2g.$$

$$E_c = y_c + \frac{q^2}{g} \cdot \frac{y_c^{-2}}{2} = y_c + \frac{y_c}{2} \Rightarrow \boxed{E_c = \frac{3y_c}{2}}$$ Hence Proved.

∴ For a trapezoidal channel :→

$$\frac{Q^2 \times (B + 2y_c)}{0.5(B \times 2 + 2y_c) y_c \times g} = 1.$$



As rectangular channel is also a trapezoidal channel with 90° slope.

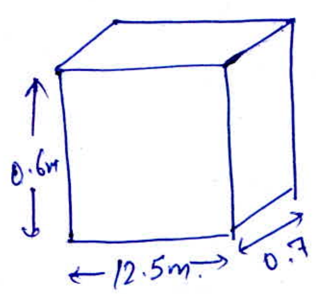
In in that case, $E_c = \frac{2}{3} y_c$??

(7)

Q.1 (d) An empty tank with all sides closed is 12.5 m long, 0.7 m broad and 0.6 m high. The surface of sheet metal weighs 363 N/m^2 and the tank is allowed to float in fresh water with 0.6 m side vertical. Determine the state of equilibrium.

[12 marks]

Ans



W sheet metal = 363 N/m^2

Surface area of cuboid.

$= 12.5 \times 0.6 \times 2 + 0.7 \times 0.6 \times 2 + 12.5 \times 0.7 \times 2$

$= 33.34 \, m^2$

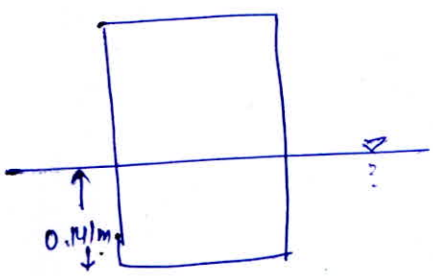
Weight of cuboid = $33.34 \times 363 = 12.102$ kN.

For state of equilibrium by floating, same weight of water is to be displaced.

Depth of Immersion is $12.5 \times 0.7 \times x = \frac{12.102 \times 10^3}{1000 \times 9.81}$

$x = 0.141m.$

Hence, At equilibrium, cuboid floats with 0.141m immersion in water.



Centre of Gravity is at $\frac{h}{2}$.

$= 0.3m$ from the bottom.

Centre of Buoyancy $= \frac{x}{2}$.

$= 0.0705m.$

from Bottom.

As Centre of Buoyancy is below Centre of gravity.

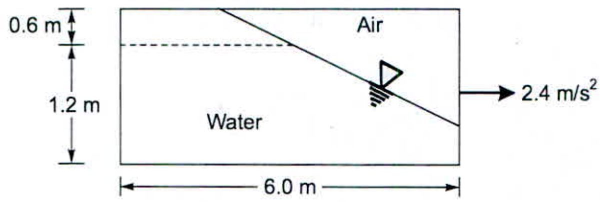
$$GM = \frac{I}{V} - BG = \frac{\frac{12.5 \times 0.7^3}{12}}{0.141 \times 12.5 \times 0.7} - (0.3 - 0.0705)$$

$GM = 0.06 > 0$, +ve.

12

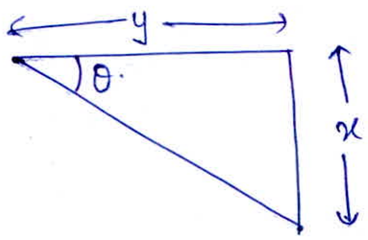
Hence tank is in stable equilibrium.

Q.1 (e) A closed tank 6 m long, 2 m wide and 1.8 m deep initially contains water to a depth of 1.2 m. The top has an opening in the front part to have air space at atmospheric pressure. If the tank has given a horizontal acceleration at a constant value of 2.4 m/s² along its length, calculate the total pressure force on the top of the tank.



[12 marks]

Ans



$\tan\theta = \frac{a_x}{g}$

$\tan\theta = \frac{2.4}{9.81}$

$\frac{x}{y} = \frac{2.4}{9.81}$ —— (1)

By conservation of Volume :→

$\frac{1}{2} \times y \times x = 0.6 \times 6$

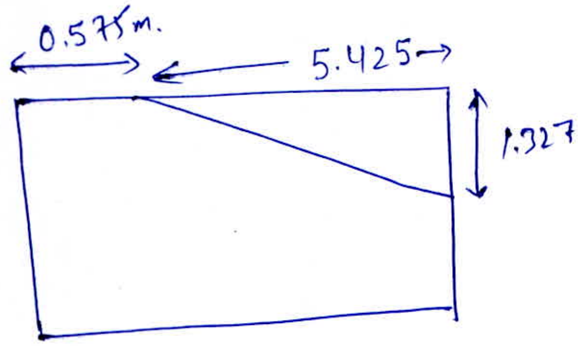
$xy = 7.2$ —— (2)

By solving (1) + (2) :→

$\frac{2.4}{9.81} y \cdot y = 7.2$

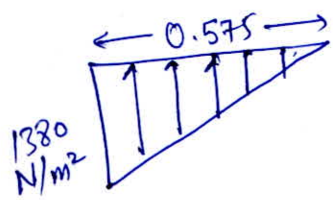
$y = 5.425 \text{ m.}$

$x = 1.327 \text{ m.}$



As No. component of acceleration due to gravity results in pressure on top.

Pressure on top of tank at corner = $\rho \times a_x \times 0.575m$.
$= 1000 \times 2.4 \times 0.575$
$= 1380$ Pa.



Total Pressure force = $0.5 \times 1380 \times 0.575 \times 2$.
$\Rightarrow F = 793.5$ N

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Q.2 (a) A cylinder 0.25 m in radius and 2 m in length rotates coaxially inside a fixed cylinder of the same length and 0.30 m radius. Olive oil of viscosity 4.9×10^{-2} Ns/m^2 fills the annular space between the cylinders. A torque 4.9 N-m is applied to the inner cylinder. After constant velocity is attained, calculate the velocity gradient at the cylinder walls, the resulting rpm, and the power dissipated by fluid resistance ignoring end effect.

[20 marks]

Q.2 (b) A pump impeller is 37.5 cm in diameter and discharges water with velocity components of 2 m/s and 12 m/s in the radial and tangential directions respectively. The impeller is surrounded by a concentric cylindrical chamber with parallel sides, the outer diameter being 45 cm. If the flow in this chamber is a free-spiral vortex, find the components of velocity of water on leaving and the pressure rise in the shroud if there is no loss.

[20 marks]

Q.2 (c) **(i)** Many researchers believe that the problem of air-entertainment in free surface vortex formation at intakes is influenced by forces of viscosity and surface tension. Show that for dynamic similarity between model and prototype, the following relationship must be satisfied:

$$\left(\frac{\mu V}{\sigma}\right)_m = \left(\frac{\mu V}{\sigma}\right)_p$$

Also prove that by use of the same liquid results in the "equal-velocity" concept of model testing.

(ii) Water from a reservoir flowing through a rigid 150 mm diameter pipe, with a velocity 2.4 m/s is completely stopped by closure of a valve situated 1100 m from the reservoir, determine the maximum rise in pressure, when valve closure takes place

(1) In one second and

(2) In five seconds

Without damping of pressure wave. Consider the velocity of sound in water as 1432 m/s.

[10 + 10 marks]

) An inward flow reaction turbine has inlet and outlet diameters of 1.2 m and 0.6 m respectively. The breadth at the inlet is 0.25 m and at the outlet it is 0.35 m. At a speed of rotation of 250 rpm, the relative velocity at entrance is 3.5 m/s and is radial. Calculate the (i) absolute velocity at entrance and the inclination to the tangent of the runner, (ii) discharge and (iii) the velocity of flow at the outlet.

[20 marks]

$D_1 = 1.2m.$

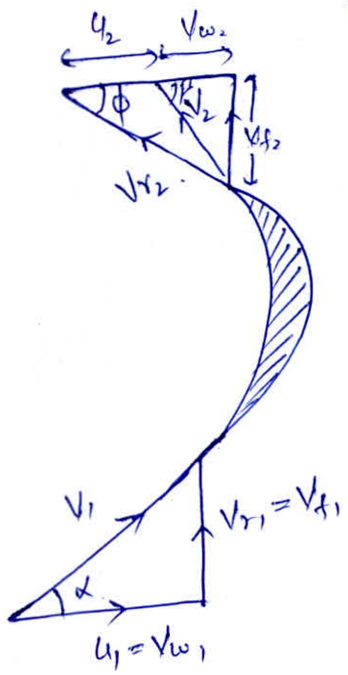
$D_2 = 0.6m.$

$B_1 = 0.25m.$

$B_2 = 0.35m.$

$N = 250 rpm.$

$V_{r_1} = 3.5 m/s.$



$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 250}{60}$

$= 15.71 m/s.$

$V_{f_1} = 3.5 m/s.$

$V_{w_1} = u_1 = 15.71 m/s.$

i) Absolute velocity :-

$V_1 = \sqrt{V_{f_1}^2 + V_{w_1}^2}$

$= \sqrt{15.71^2 + 3.5^2}$

$V_1 = 16.09 m/s.$

Inclination $\alpha = \tan^{-1} \frac{V_{f_1}}{V_{w_1}} = 12.56°$

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(ii) Discharge = $\pi D_1 B_1 \times V_{f_1}$

$= \pi \times 1.2 \times 0.25 \times 3.5$

$Q = 3.298\ m^3/s$

(iii) Velocity of flow at outlet :→

$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$

$\pi \times 1.2 \times 0.25 \times 3.5 = \pi \times 0.6 \times 0.35 \times V_{f_2}$

$V_{f_2} = 5\ m/s$

20

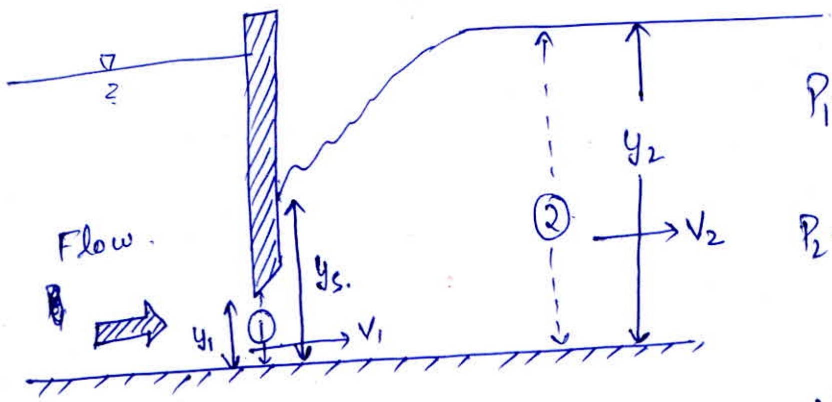
) Show that for a submerged hydraulic jump just downstream of a sluice gate, in a horizontal rectangular channel,

$$\frac{y_s}{y_1} = \sqrt{2F_1^2\left(\frac{y_1}{y_2}-1\right)+\left(\frac{y_2}{y_1}\right)^2}$$

where y_1 is the depth of opening of the sluice gate, y_2 is the depth of flow downstream of the submerged hydraulic jump, y_s is the water depth on the downstream side of the sluice gate and F_1 is the Froude number of flow through the sluice opening.

[20 marks]

Assuming horizontal, frictionless channel.



$P_1 \rightarrow$ Pressure thrust at section ①.

$P_2 \rightarrow$ Pressure thrust at section ②.

$V_1 \rightarrow$ velocity at section ①

$V_2 \rightarrow$ velocity at section ②

Acc. to the momentum eq^n :→

$$P_1 + (-P_2) = M_2 + (-M_1)$$

$$\gamma_w \cdot \bar{y_s} \cdot (y_s \cdot B) - \gamma_w \bar{y_2} \cdot (y_2 B)$$

$$= \rho Q A (V_2^2 - V_1^2) = \rho A_2 V_2^2 - \rho A_1 V_1^2$$

~~A - B.~~

$$\gamma_w \cdot B \cdot \frac{y_s^2}{2} - \gamma_w \cdot \frac{y_2^2}{2} \cdot B = \rho \frac{Q^2}{A_2^2} \cdot A_2 - \rho \frac{Q^2}{A_1^2} \cdot A.$$

$$= \rho \frac{Q^2}{A_2^2} \cdot A_2 - \rho \frac{Q^2}{A_1} \cdot A_1 = \rho Q^2 \left(\frac{1}{A_2} - \frac{A1}{A1}\right)$$

$$= \rho Q^2 \left(\frac{1}{B \cdot y_2} - \frac{1}{B y_1}\right).$$

$$= \frac{\rho Q^2}{B}\left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$

$$\frac{B\gamma_w}{2}\left[y_s^2 - y_2^2\right] = \frac{\rho Q^2}{B}\left[\frac{1}{y_2} - \frac{1}{y_1}\right]$$

$$\frac{\gamma_w}{2}\left[y_s^2 - y_2^2\right] = \rho q^2\left[\frac{1}{y_2} - \frac{1}{y_1}\right]$$

$$\frac{1}{2}\left[y_s^2 - y_2^2\right] = \rho\frac{q^2}{\rho g}\left[\frac{1}{y_2} - \frac{1}{y_1}\right]$$

$$\frac{1}{2}\left[y_s^2 - y_2^2\right] = \frac{q^2}{g y_1}\left[\frac{y_1}{y_2} - 1\right]$$

Dividing by y_1^2

$$\frac{1}{2}\left[\frac{y_s^2}{y_1^2} - \frac{y_2^2}{y_1^2}\right] = \frac{q^2}{g y_1^3}\left[\frac{y_1}{y_2} - 1\right].$$

$$\frac{q^2}{g y_1^3} = F_1^2$$

$$\frac{1}{2}\left[\left(\frac{y_s}{y_1}\right)^2 - \left(\frac{y_2}{y_1}\right)^2\right] = F_1^2\left[\frac{y_1}{y_2} - 1\right].$$

$$\boxed{\frac{y_s}{y_1} = \sqrt{2F_1^2\left(\frac{y_1}{y_2} - 1\right) + \left(\frac{y_2}{y_1}\right)^2}}$$

Hence Proved.

(20)

(c) **(i)** What is meant by local and convective acceleration? For a one dimensional flow described by $V(x, t)$, derive the expression for convective acceleration in terms of velocity and its gradient.

(ii) A rectangular channel 5.2 m wide has a discharge of 10 m^3/sec at a velocity of 1.25 m/s. At a certain section the bed width is reduced to 3.0 m through a smooth transition. A smooth flat hump is to be built in this contracted section to cause critical flow for flow measurement purposes. Estimate the height of the hump necessary for this purpose. (Assume no loss of energy at the transition.)

[10 + 10 marks]

Local acceleration → Rate of change of velocity vector with respect to time in a flow.
$\left(\frac{\partial v}{\partial t}\right)$

Convectione acceleration → Rate of change of velocity vector with respect to space in a flow.
$\left(v\frac{\partial v}{\partial s}\right)$

Velocity of flow: $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$.

Velocity varies with space + time.

$\vec{V} = f(x, t)$.

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k} \doteq u\hat{i}$

In x - direction. acceleration :-

$a_x = V \cdot \frac{\partial v}{\partial s} = v \cdot \frac{\partial v}{\partial s}$

$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

$\frac{\partial v}{\partial s} = \frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}$

$v \cdot \frac{dv}{ds}$ for x - directi

$a_x = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z}$

Similarly as the flow is one directionl.

$\boxed{a_x = u \cdot \frac{\partial u}{\partial x}}$ as $v = w = 0$.

Where $u \rightarrow$ velocity

$\frac{\partial u}{\partial x} \rightarrow$ gradient.

10

width = 5.2m.

Q = 10 m³/sec.

V = 1.25 m/s.

B_{min} = 3m.

Discharge Intensity at original section = $\frac{10}{5.2}$ m³/s/m.

" " " throat " = $\frac{10}{3}$ m³/s/m.

Energy at u/s of the section = $y + \frac{V^2}{2g}$.

$y = \frac{10}{1.25 \times 5.2} = 1.538m.$

E = 1.6181 m.

Let the height of hump be. Δz.

Critical flow at d/s section.

Critical Energy = $\frac{3}{2} y_c' = \frac{3}{2}\left[\frac{(10/3)^2}{9.81}\right]^{1/3}$

= 1.5635 m.

Energy Conservation :→

$E_1 = E_c + \Delta z$.

1.6181 = 1.5635 + Δz.

Δz = 0.0545 m.

10

Hump required = 0.0545m.

Q.4 (a) **(i)** For the velocity profile, $\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ on a flat plate, find out the average velocity and kinetic energy correction factor.

(ii) Calculate the friction drag on a flat plate 15 cm wide and 45 cm long placed longitudinally in a stream of oil of relative density 0.925 and kinematic viscosity 0.9 stoke, flowing with a free stream velocity of 6.0 m/s. Also, find the thickness of the boundary layer and shear stress at the trailing edge.

[10 + 10 marks]

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•) A stream is spanned by a bridge which is a single masonry arch in the form of a parabolic arch, the crown being 2.5 metre above the springings which are 9 meters apart. The overall width of the bridge is 6 metres. During a flood the stream rises to a level 2 metres measured in the direction of the stream above the springings. Calculate the force tending to lift the bridge from its foundations if the arch remains water tight.

[20 marks]

:)

(i) Define bulk modulus of elasticity of a fluid. What is the SI unit of bulk modulus of elasticity? Discuss the factors affecting bulk modulus of elasticity of a fluid. Why liquids are generally considered incompressible?

(ii) Show that the theoretical discharge in an open channel flow may be expressed as:

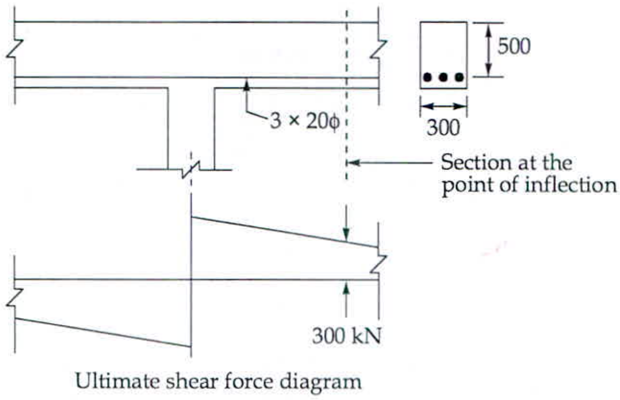
$$Q = A_2 \sqrt{\frac{2g\left(\Delta y - h_f\right)}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

where A_1 and A_2 are the cross-sectional areas of flow at sections (1) and (2) respectively, Δy is the drop in the water surface between the two sections and h_f is the energy head loss between the two sections.

[10 + 10 marks]

Section B : Design of Concrete and Masonry Structures-1 + Strength of Materials-2

Q.5 (a) Check for bond stress at the point of inflection of a continuous beam as shown in figure, if it is subjected to an ultimate shear force of 300 kN at the point of inflection. Consider concrete of grade M20 and steel of grade Fe415. [Take design bond stress for M20 concrete = 1.2 N/mm²]



Ultimate shear force diagram

Continuous beam

[12 marks]

Ans

Factored shear force = 300kN.

M20, Fe415.

Tension Reinf^{n} = 3 − 20ϕ

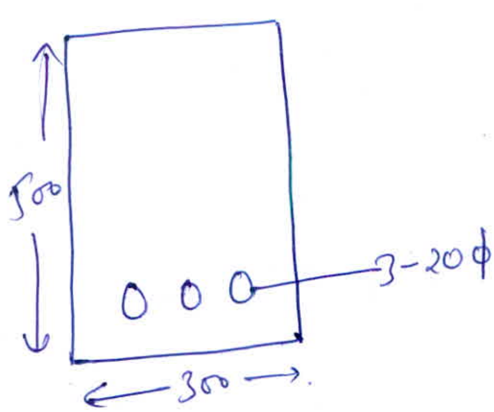
τ_{bd} = 1.2 MPa.

Development Length req = $\frac{0.87 f_y \cdot \phi}{4 \tau_{bd}}$

= $\frac{0.87 \times 415 \times 20}{4 \times 1.2 \times 1.6}$

= 940.23 mm.

Section :→



Neutral axis = $x_u = \frac{0.87 f_y . A_{st}}{0.36 \times f_{ck} . B}$

$= \frac{0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 20^2}{0.36 \times 20 \times 300}$

$x_u = 157.54 mm$.

x_u limiting = 0.48 d = 0.48 × 500.

= 240 mm

So, Under Reinf^n Section.

MOR = $0.36 f_{ck} . B . x_u (d - 0.42 x_u)$

$= 0.36 \times 20 \times 300 \times$ ~~1.57~~ $157.54 (500 - 0.42 \times 157.54)$

$M_{u1} = 147.627 kNm$.

$\frac{M_{u1}}{V_u} = \frac{147.627 \times 10^3}{300} =$ +Lo 492.09 mm. < L_d req.

Not ~~safe,~~

Anchorage required = 448.14 ≈ 450mm.

Check, Ld < $\frac{Mu}{Vu}$ + Lo.

(8) − 2 = (6)

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Q.5 (b) State the assumptions made while analyzing the reinforced concrete beam using Limit State of Flexure as per **IS 456:2000 Code.**

[12 marks]

Ans

Assumptions:

(i) Plane section before bending remains plane after bending

→ It serves as ~~ease~~ of advantage during analysis.

→ linear strain.

(ii) ~~Ka~~ Minimum stress in the tensile steel is

$$\frac{0.87 f_y}{E_s} + 0.002.$$

(iii) ~~Ea~~ Strain at the most compressed concrete fiber is 0.0035.

(iv) Stress - Strain curve of the concrete in compression with linear, parabolic or any shape which conforms to the test results.

(v). Concrete below the ~~ka~~ NA ~~or~~ (Concrete in Tension doesn't take any tension. whole of the tension is taken by steel.

7

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) Three exactly similar mild steel tube specimens have the external and internal diameters 37.5 mm and 31.25 mm respectively. One of these specimens was tested in pure tension and limit of proportionality was recorded to be 70 kN. The second specimen was tested in torsion whereas the third was tested in torsion with superimposed bending moment of 350 Nm. If the failure criterion is the maximum shear stress, determine the torque at which the two specimens would have failed?

[12 marks]

For first case. → Principle stress = $\frac{P}{A}$.

$$\sigma_{P_1} = \frac{70 \times 10^3}{\frac{\pi}{4}(37.5^2 - 31.25^2)} = 207.42 \text{ MPa}.$$

max. shear stress = $\frac{207.42}{2}$ = 103.71 MPa.

max shear strength = 103.71 MPa.

(i) Let Torsion applied is T.

max shear stress = $\frac{16T}{\pi D^3(1-k^4)}$

where $k = \frac{31.25}{37.5} = \frac{5}{6}$.

failure Torque = $\frac{16 \times T \times 10^6}{\pi \times 37.5^3 \left(1-\left(\frac{5}{6}\right)^4\right)} = 103.71$

T = 555.98 Nm.

(ii) Let failure Torque be T'

Bending ~~Torque~~ Moment = 350Nm.

Equivalent ~~shear~~ Torsion = $\sqrt{M^2+T^2}$

$T_{eq} = \sqrt{350^2+T^2}$

At failure.

$$\frac{16 T_{eq} \times 10^6}{\pi \times 37.5^3 \left(1-\left(\frac{5}{6}\right)^4\right)} = 103.71$$

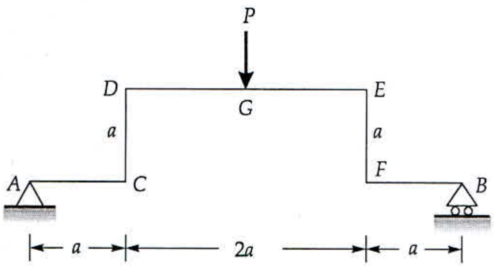
$T_{eq} = 555.98$

$T' = 431.99.$

$T' = 431.99 = 432$ Nm.

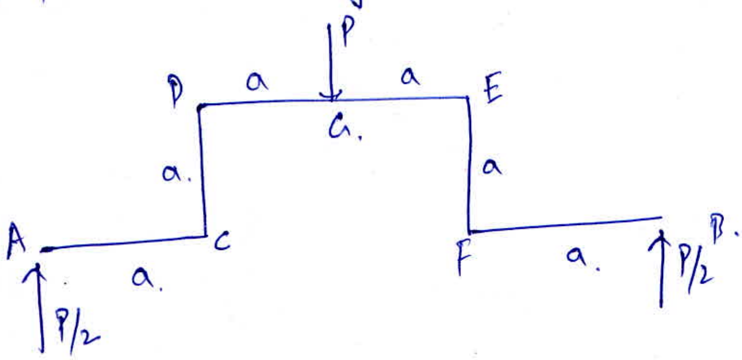
12

) Find the central deflection of the framed beam using strain energy method as shown in figure. [EI is constant]



[12 marks]

As it is a symmetrical section.



Strain Energy stored = $2 \times U_{AC} + 2 \times U_{DC} + 2\,U_{DG}$.

$U_{AC} = \int_0^a \frac{M_x^2\,dx}{2EI} = \int_0^a \frac{\left(\frac{P}{2}x\right)^2 . dx}{2EI} = \frac{P^2}{4 \times 2EI}\int_0^a x^2 . dx.$

$= \frac{P^2}{8EI} \times \frac{a^3}{3} = \frac{P^2 a^3}{24EI}$

$U_{CD} = \int_0^a \frac{M_x^2\,dx}{2EI} = \int_0^a \frac{\left(\frac{P}{2}a\right)^2 . dx.}{2EI} = \left(\frac{P}{2}a\right)^2 \frac{\times a.}{2EI}$

$= \frac{P^2 a^3}{8EI}$

$U_{DG} = \int_0^a \frac{M_x^2\,dx}{2EI} = \int_0^a \frac{\left(\frac{P}{2}a + \frac{P}{2}x\right)^2 . dx.}{2EI} = \frac{7}{24}\frac{P^2 a^3}{EI}$

$\Rightarrow \frac{P^2}{4}a^2 . a + \frac{P^2}{4} . \frac{a^3}{3} = \frac{7}{24}\frac{P^2 a^3}{EI}$

$$U_{Total} = \left(2\times\frac{1}{24} + 2\times\frac{1}{8} + 2\times\frac{7}{24}\right)\times\frac{P^2a^3}{EI}$$

$$U_{Total} = \frac{11}{12}\frac{P^2a^3}{EI}.$$

deflection, $\frac{\partial U}{\partial P} = \frac{11}{12}\frac{2Pa^3}{EI}$

$$\delta_a \Rightarrow \frac{11}{6}\frac{Pa^3}{EI}$$

12

A machine component is made of a material whose ultimate strength in tension, compression and shear are 40 N/mm^2, 110 N/mm^2 and 55 N/mm^2 respectively. At the critical point in the component, the state of stress is represented by

$$\sigma_x = 25 \text{ N/mm}^2 \text{ and } \sigma_y = -75 \text{ N/mm}^2$$

Find the maximum value of the shear stress τ_{xy} which will cause failure of the component?

[12 marks]

Check with $\sigma_{p_1} = 40$ MPa.

~~40 = 25 − 75~~

(i) $\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$(40)\ \sigma_{p_1} = \frac{25 + (-75)}{2} \pm \sqrt{\left(\frac{25+75}{2}\right)^2 + \tau_{xy}^2}$

$\tau_{xy} = 41.53$ MPa. —①.

(ii) check with $\sigma_{p_2} = -110$ MPa.

$-110 = \frac{25-75}{2} - \sqrt{\left(\frac{25+75}{2}\right)^2 + \tau_{xy}^2}$

$\tau_{xy} = 68.74$ MPa —②

(iii) check with τ_{max}.

$\tau_{max} = \sqrt{\left(\frac{25+75}{2}\right)^2 + \tau_{xy}^2}$

$\tau_{xy} = 22.91$ MPa.

Max shear stress that can be applied = 22.91 MPa.

(12)

Design a rectangular beam section of 300 mm width and 500 mm effective depth which is subjected to an ultimate bending moment of 50 kNm, ultimate shear force of 50 kN and ultimate torsional moment of 40 kNm. Consider concrete of grade M20 and steel of grade Fe415. [Assume effective cover = 35 mm]

p_t (%)	≤0.15	0.25	0.5	0.75	1
τ_c (N/mm^2)	0.28	0.36	0.48	0.56	0.62

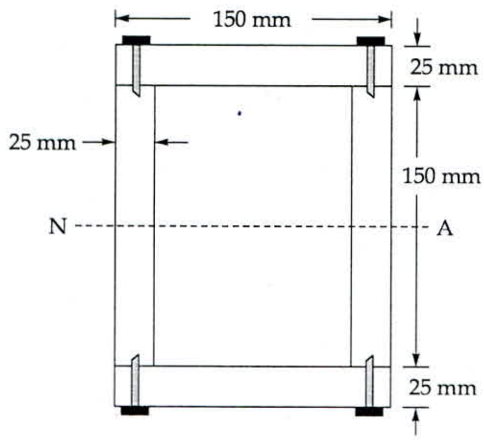
[20 marks]

Q.6 (b) **(i)** A ring beam of water tank has a diameter of 12.5 m. It is subjected to outward radial force of 25 kN/m. Design the section of ring beam using M25 and Fe415. Assume $m = 11$ and allowable stress in tension as 1.2 N/mm^2.

(ii) Calculate the development length in tension and compression for a single mild steel bar of diameter ϕ in concrete of grade M20. Assume $\tau_{bd} = 1.2$ N/mm^2.

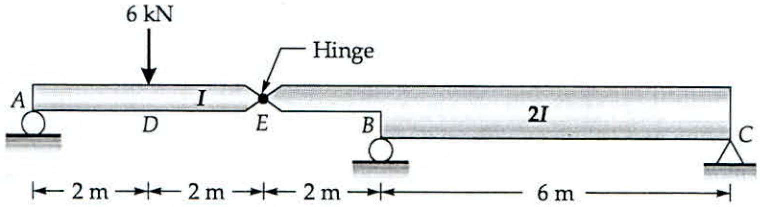
[14 + 6 marks]

The box beam as shown in figure below is made up of four 150 mm × 25 mm wooden planks connected by screws. Each screw can safely transmit a shear force of 1250 N. Estimate the minimum necessary spacing of screws along the length of the beam if the maximum shear force transmitted by the cross-section is 5000 N. Also determine the shear stress distribution across the section.

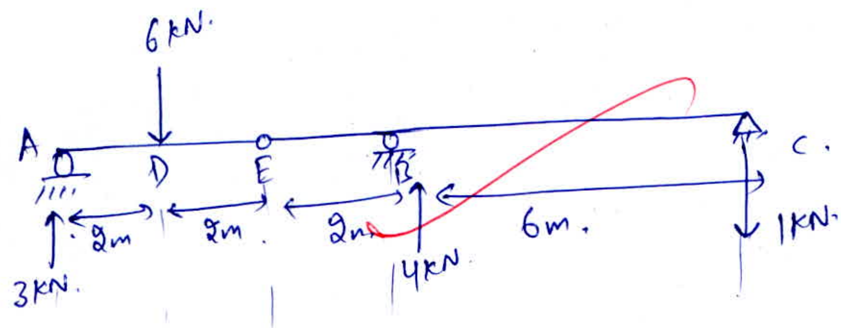


[20 marks]

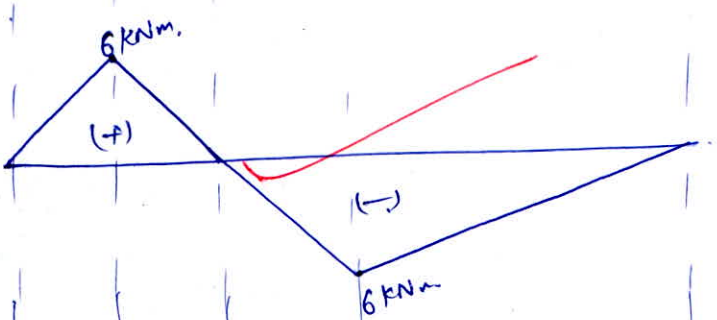
A hinged beam system is loaded as shown below. Determine the slope at point E and D. Also determine the deflection at D. Use Conjugate beam method.



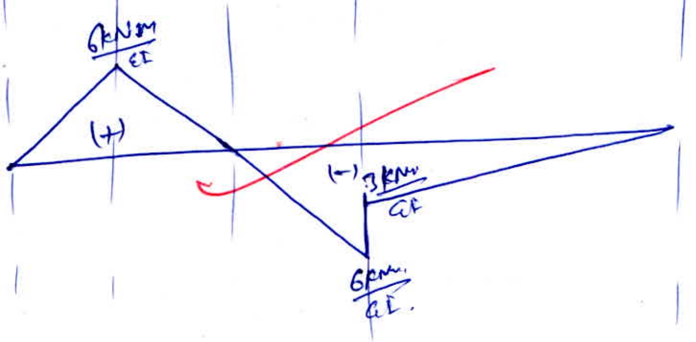
[20 marks]



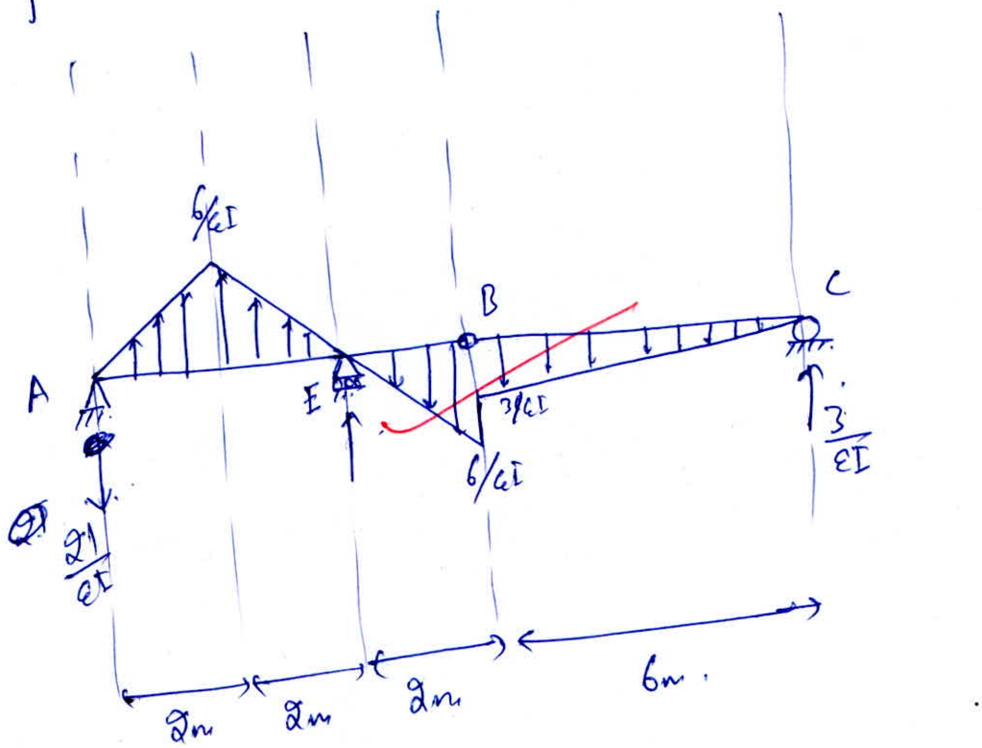
BMD →



$\frac{M}{EI}$ →



Conjugate beam :→



$$R_c = \frac{0.5 \times 6 \times \frac{3}{EI} \times \frac{6}{3}}{6} = \frac{3}{EI}$$

$$R_E = \frac{-0.5 \times 4 \times 6 \times \frac{2}{EI} + 0.5 \times 6 \times \frac{2}{EI} \times \left(4 + \frac{2}{3} \times 2\right) + 0.5 \times 6 \times \frac{3}{EI} \times \left(6 + \frac{6}{3}\right)}{4}$$

Wro… Calc…

$$R_E = \frac{20}{EI} \quad -\frac{11}{EI}$$

$$R_A = \frac{-0.5 \times 6 \times \frac{4}{EI} \times 4 + 0.5 \times 6 \times 2 \times \frac{2}{3} \times \frac{20}{EI} \times 2}{6}$$

$$= -\frac{21}{EI} \quad \frac{11}{EI}$$

Slope at E = ~~$\frac{20}{EI}$ (clockwise)~~

~~Slope at~~ $-\frac{21}{EI} + 0.5 \times 4 \times 6 = \frac{-9}{EI}$

LHS of E = $\frac{9}{EI}$ anticlockwise.

RHS of E = $-\frac{9}{EI} + \frac{20}{EI} = \frac{11}{EI}$ clockwise

Slope at D = $-\frac{21}{EI} + 0.5 \times 2 \times 6$.

$\theta_D = \frac{15}{EI}$ anticlockwise.

Deflection at D :→ Moment at D.

$-\frac{21}{EI} \times 2 + 0.5 \times 2 \times \frac{6}{EI} \times \frac{2}{3}$

$\delta_D = \frac{-38}{EI}$

Deflection = $\frac{38}{EI}$ downwards.

X

5

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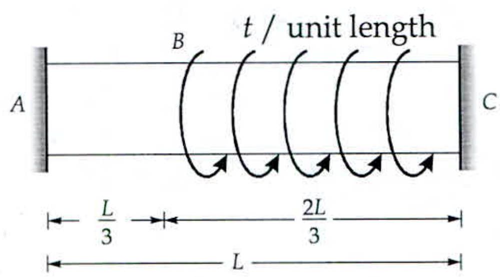
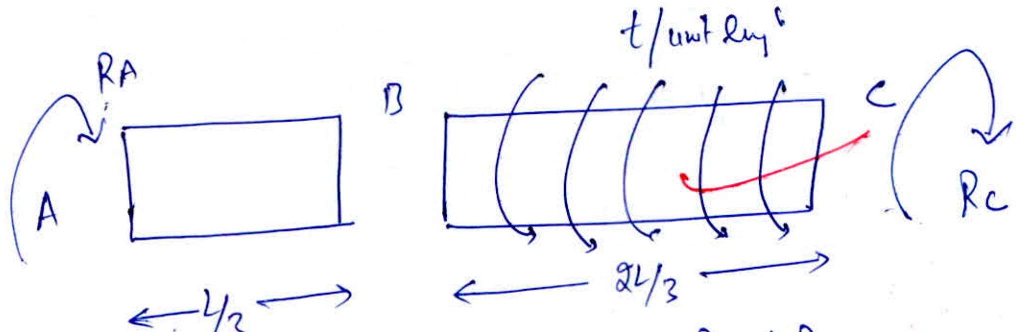
Q.7 (b) A solid circular cross-section shaft is clamped at both ends and loaded by a twisting moment t per unit length as shown in figure below. Determine the reactive twisting moment at each end of the bar.

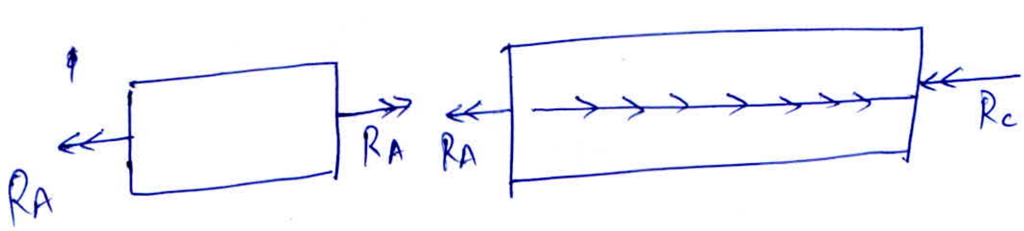


[20 marks]

Ans



Let the Reactions be R_A + R_c.



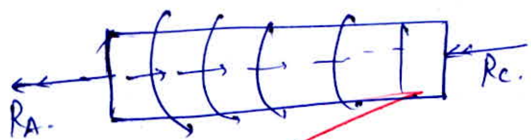
Total angle of twist between A & C = 0.

$$R_A + R_C = t \times \frac{2L}{3} = \frac{2tL}{3}$$

$$R_A + R_C = \frac{2tL}{3} \quad \text{--- (1)}$$

* Angle of twist is zero.

for span BC :-



$$\left(\frac{TL}{GJ}\right)_{AB} + \left(\frac{TL}{GJ}\right)_{BC} = 0.$$

$$\frac{R_A \cdot L/3}{GJ} + \int_0^{2L/3} \frac{(R_A - tx)\,dx}{GJ} = 0.$$

$$\frac{R_A \cdot L/3}{GJ} + \frac{1}{GJ}\left[R_A\left(\frac{2L}{3}\right) - t\left[\frac{\left(\frac{2L}{3}\right)^2}{2}\right]\right] = 0.$$

$$\frac{R_A L}{3GJ} + \frac{2R_A L}{3GJ} - \frac{t\,4L^2}{9 \times 2\,GJ} = 0.$$

$$R_A = \left(\frac{4tL^2}{9 \times 2}\right) \Big/ L.$$

$$R_A = \frac{4tL}{9 \times 2} = \frac{2tL}{9}.$$

$$\boxed{R_A = \frac{2tL}{9} \qquad R_C = \frac{4tL}{9}.}$$

20

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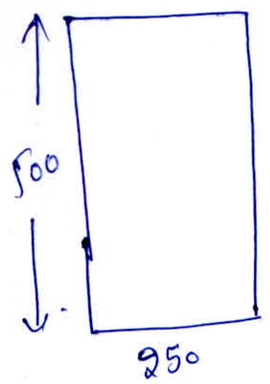
Design a reinforced concrete rectangular section of size 250 × 500 mm for a factored moment of 225 kN. The grades of concrete and HYSD steel are M20 and Fe415, respectively. [Take effective cover = 50 mm, f_{sc} = 353 MPa]

[20 marks]



Factored BM = 225 kNm.

M20, Fe415.

eff cover = 50mm.

f_{sc} = 353 MPa.

d = 450 mm.

Limiting Moment of Resistance = $0.138 f_{ck} bd^2$

$= 0.138 \times 20 \times 250 \times 450^2$

M_{u_1} = 139.725 kNm.

Unbalanced BM = 85.275 kNm. = M_{u_2}

Section will be designed as Doubly Reinforced section.

x_u limiting = $0.48 \times d$ = 0.48×415 = 199.2 mm.

Tension Reinf^n for M_{u_1} (A_{st_1}):

$$A_{st_1} = \frac{139.725 \times 10^6}{0.87 \times 415 \times (450 - 0.42 \times 199.2)}$$

= 1056.39 mm²

Ast_2 for Unbalanced. Mu_2.

$$Ast_2 = \frac{Mu_2}{0.87 \times 415 \times (d - d_c)}$$

Assuming $d_c = 50mm$.

$$Ast_2 = \frac{85.275 \times 10^6}{0.87 \times 415 \times (450 - 50)}$$

$$= 590.465 \, mm^2$$

Compression side Reinf^n.

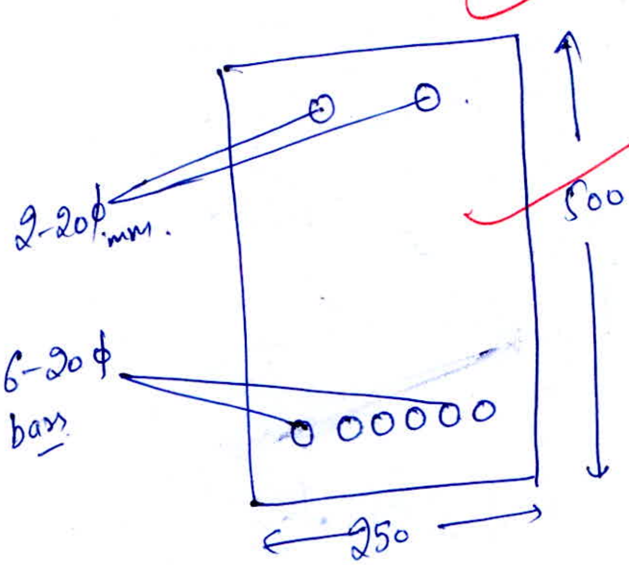
$$Asc = \frac{Mu_2}{(f_{sc} - 0.45 f_{ck}) \times (d - d_c)}$$

$$Asc = \frac{85.275 \times 10^6}{(353 - 0.45 \times 20) \times (400)}$$

$$= 619.73 \, mm^2$$

$Ast = 1646.85 \, mm^2$. Provide. 6 - 20ϕ mm bar.

$Asc = 619.73 \, mm$ Provide 2 - 20ϕ mm bar.



20

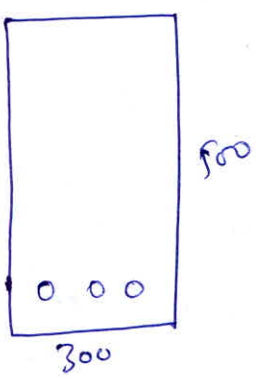
(i) A rectangular beam section of 300 mm width and 500 mm effective depth is reinforced with 5 bars of 20 mm ϕ, out of which 2 bars have been bent at 45°. Determine the shear resistance of the bent up bars and additional shear reinforcement required if it is subjected to an ultimate shear force of 300 kN. Consider concrete of grade M20 and steel of grade Fe415.

p_t (%)	≤0.15	0.25	0.5	0.75	1
τ_c (N/mm^2)	0.28	0.36	0.48	0.56	0.62

(ii) Determine the ultimate load capacity of a circular column of 400 mm diameter reinforced with 6 × 25 mm ϕ bars adequately tied with (i) lateral ties and (ii) spirals. Consider concrete of grade M25 and steel of grade Fe415.

[10 + 10 marks]

M20, Fe415



Factored shear force = 300 kN.

Shear Resisted by Concrete :→

% pt = $3 \times \frac{\pi}{4} \times 20^2 = \frac{942.47}{300 \times 500} \times 100$

P_t = ~~0.7854 %~~ 0.6283 %.

corresponding $\tau_c = 0.48 + \frac{0.56 - 0.48}{0.75 - 0.5} \times (0.6283 - 0.5)$

= 0.521 MPa.

Shear resisted = $0.521 \times 300 \times 500 \times 10^{-3}$

= 78.15 kN.

Unbalanced shear = 221.84 kN

Shear capacity of bent up bars = $A_{sv} \times 0.87 f_y \times \sin\alpha$.

$= 2 \times \frac{\pi}{4} \times 20^2 \times 0.87 \times 415 \times \sin 45^\circ$

Bars Capacity = 160.4 KN.

Addition shear reinf^n required for shear :-

$\max\left(\frac{V_{ue}}{2}, V_{ue} - V_b\right)$

$= \left(\frac{221.84}{2}, 221.84 - 160.4\right)$

Design Stirrup Capacity = 110.92 KN.

Providing 10 mm φ stirrups. 2-legged.

$S_v = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 10^2 \times 500}{110.92 \times 10^3}$

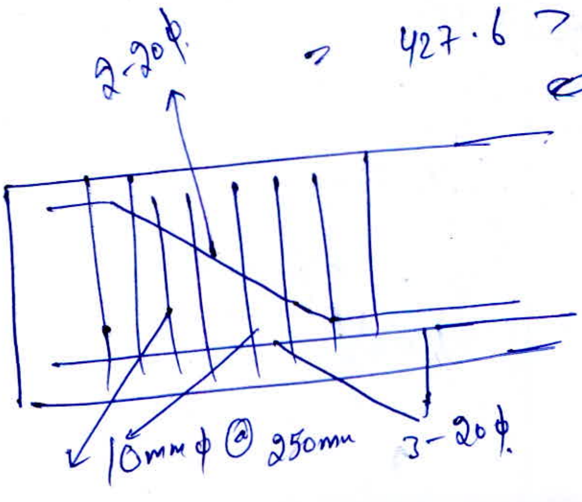
$S_v = 255.65$ mm.

Provide 10mm φ stirrup at 250 mm spacing $< 0.75 \times 500$ (375)

min spacing $= \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 10^2 \times 500}{0.4 \times 500 \times 300}$

$= 427.6 > 300$

> 250 mm. Ok



10

ii) (a) lateral ties. →

$P_u = 0.4 f_{ck} \cdot A_c + 0.67 f_y \cdot A_{st}$

$P_u = 0.4 \times 25 \times \frac{\pi}{4} \times 400^2 + (0.67 \times 415 - 0.4 \times 25) \times 6 \times \frac{\pi}{4} \times 25^2$

$P_u = 2046.11$ kN

10

(b) Spirals. → (Load carrying capacity ↑ by 5%)

$P_u = 1.05(2046.11)$

$P_u = 2148.41$ kN.

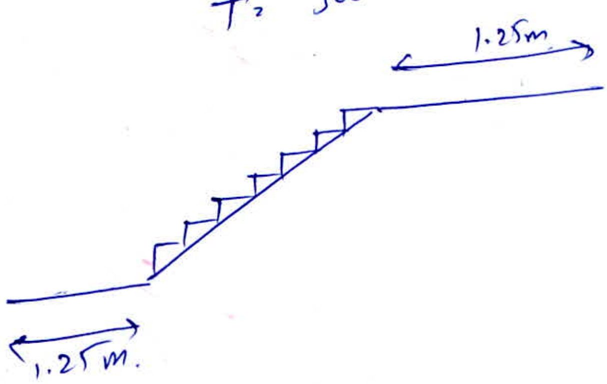
A staircase consists of 14 steps, each of 300 mm tread and 180 mm rise, plus two landings of each 1.25 m length. The width of staircase is 1.4 m. Design the staircase for a live load of 5 kN/m^2. Use M20 grade concrete and Fe415 reinforcement.

[20 marks]

$R = 180$ mm.

$T = 300$ m.

Let the depth of waist slab = $\frac{5.45 \times 10^3}{20 \times 1.2} = 250$ mm.



width of stair ca. = 1.4m.

L.L = 5 kN/m^2

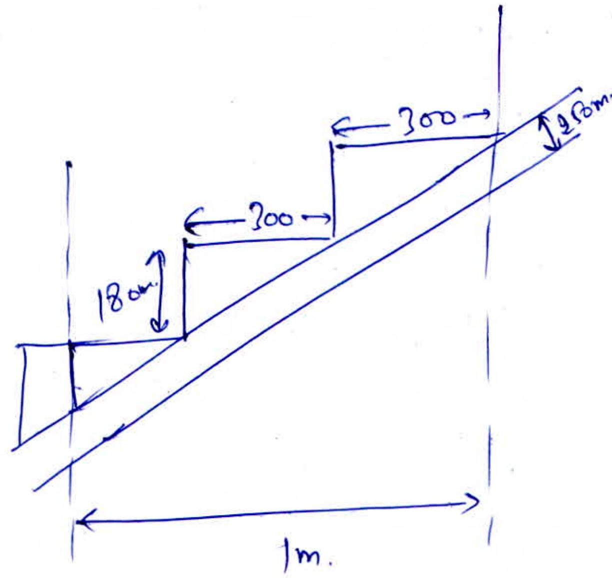
M20, Fe415

Going = $14 \times 300 = 4.2$ m.

effective length. $4.2\text{m} + \frac{1.25}{2} + \frac{1.25}{2}$

$= 5.45$ m.

Loads. For 1m. length horizontal & 1m width.



DL → Slab load = 1.1662 × 1 × 25 × 0.25

= ~~29.15 kN/m~~ 7.29 kN/m.

Steps = $\frac{1000}{300}$ × 0.5 × 0.18 × 0.3 × 25

= 2.25 kN/m.

LL → 5 kN/m² × 1m = 5 kN/m.

Total = 14.54 kN/m.

Factored Load = 14.54 × 1.5 = 21.81 kN/m

Factored Bending moment = $\frac{wl_{eff}^2}{8}$

= $\frac{21.81 \times 5.45^2}{8}$

= 80.9469 kNm.

Taking eff cover - 50 mm.

MOR = 110.4 KNm.

U/s section

So, $A_{st} = \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 80.4 \times 10^6}{20 \times 1000 \times 200^2}}\right] \times 1000 \times 200$

$= 1294.85 \text{ mm}^2$

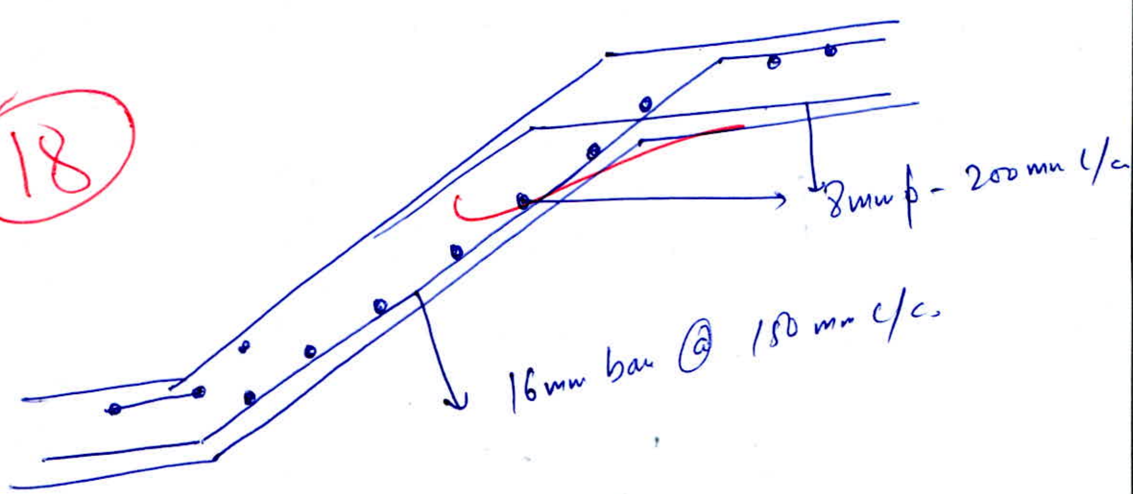
Provide 16 mm bars @ 150 mm c/c.

Distribution → 0.12% of 1000 × 200

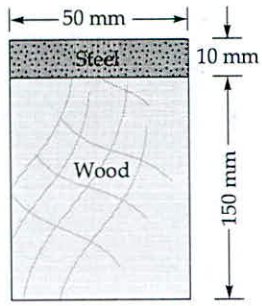
= 240 mm².

Provide 8mm φ @ 200 mm c/c.

18



Q.8 (c) **(i)** A wooden beam 50 mm wide and 150 mm deep is reinforced by gluing a steel plate 10 mm thick and 50 mm wide on the top of section. The beam is simply supported over its ends which are 5 m away from each other. The beam carries a point load of 500 kN at mid of beam. Calculate maximum shear stress at the junction of wood and steel plate. Take $m = 20$.



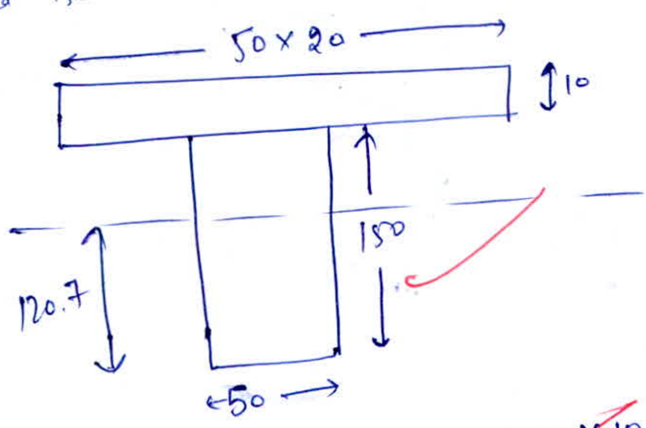
(ii) Find the dimensions of a hollow steel shaft of internal diameter 0.6 times the external diameter, to transmit 150 kW at 250 rpm, if the shearing stress is not to exceed 70 N/mm^2. If a bending moment of 3000 Nm is now applied to the shaft, find the speed at which it must be driven to transmit the same power for the same value of maximum shearing stress.

[10 + 10 marks]

Ans (i) Bending Moment = $\frac{PL}{4} = \frac{500 \times 5}{4} = 625$ kNm.

max. Shear force = 250 kN

Transformed section as



$$\bar{y}_b = \frac{50 \times 150 \times 75 + 1000 \times 10 \times 155}{50 \times 150 + 1000 \times 10}$$

$= 120.7$ mm.

$$I_{NA} = \frac{50 \times 150^3}{12} + (50 \times 150 \times (75 - 120.7)^2)$$
$$+ \frac{1000 \times 10^3}{12} + (155 - 120.7)^2 \times 1000 \times 10$$

$I_{NA} = 41574408\ mm^4$

Shear stress at junction $= \frac{VA\bar{y}}{Ib}$.

$V = 250\ kN$.

$$= \frac{250 \times 10^3 \times 50 \times 20 \times 10 \times (155 - 120.7)}{41574408 \times 50}$$

10

$\tau_{max} = 41.25\ MPa$.

(ii) $\tau_{max} = 70\ MPa$.

Torque $\Rightarrow \frac{2\pi NT}{60} = 150 \times 10^3$

$T = 5.73\ kNm$.

$$\frac{16 \times 5.73 \times 10^6}{\pi \times D^3 (1 - 0.6^4)} = 70$$

$D = 72.24\ mm$. 4 $d = 46.94\ mm$

*.

For same Equivalent Torque.

$T_{eq} = 5.73$ kNm.

$\sqrt{3^2 + T^2} = 5.73^2$

$T = 4.88$ kNm

As Power is same.

$150 = \frac{2\pi \times N \times 4.88}{60}$

$N = 293.5$

10

Space for Rough Work

$F = \frac{V}{\sqrt{g}}$

$F^2 = \frac{V^2}{gy} = \frac{q^2}{gy_1^3}$