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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-5: Flow of Fluids, Hydraulic Machines and Hydro Power
Design of Concrete and Masonry Structures-1
Strength of Materials-2

Name : ABUZAR GAFFARI

Roll No : CE19MTINAG07

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	55
Q.2	—
Q.3	38
Q.4	58
Section-B	
Q.5	56+1=57
Q.6	—
Q.7	58
Q.8	—
Total Marks Obtained	265+1 = 266

Excellent command on subjects
Improve presentation.

Signature of Evaluator

Cross Checked by

Good accuracy.

Excellent command on subjects.
Just little bit improve your way of presenting the answers.

Overall very good.

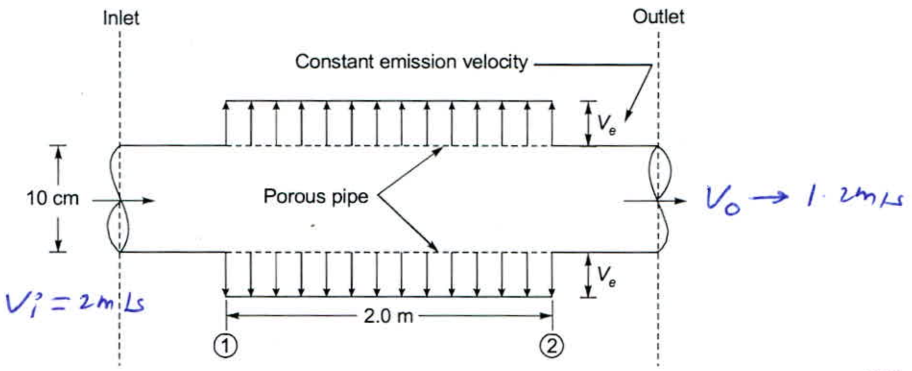
Do not write in this margin

Section A : Flow of Fluids, Hydraulic Machines and Hydro Power

(a) A circular pipe 10 cm in diameter has a 2 m length which is porous. In this porous section the velocity of exit is known to be constant as shown in figure. If the velocities at inlet and outlet of the porous section are 2.0 m/s and 1.2 m/s respectively. Estimate

(i) the discharge emitted out through the walls of the porous pipe and

(ii) the average velocity of this emitted discharge.



[12 marks]

Soln

(i) By continuity equation

$Q_{in} = Q_{out}$

$2 \times \frac{\pi}{4} \times 0.1^2 = 1.2 \times \frac{\pi}{4} \times 0.1^2 + Q_{sides}$

$Q_{walls} = \frac{\pi}{4} \times 0.1^2 \times (2 - 1.2)$

$\boxed{Q_{walls} = 6.283 \times 10^{-3}\ m^3/s = 6.283\ lps}$

(ii) $Q_{walls} = V_{avg} \times Area$

$\frac{6.283}{10^3} = V_{avg} \times \pi \times 0.1 \times 2$

From walls → $\boxed{V_{avg} = 0.01\ m/s}$

12

(b) **(i)** Explain forced vortex flow occurring in a centrifugal pump.

(ii) Water is flowing through a smooth pipe of 100 mm diameter at rate of 0.036 m^3/s. Determine

(a) Darcy's friction factor

(b) Normal thickness of viscous sub layer

Take kinematic viscosity = 10^{-6} m^2/s and f (Darcy's friction factor) = $0.0032 + \frac{0.221}{R_e^{0.237}}$

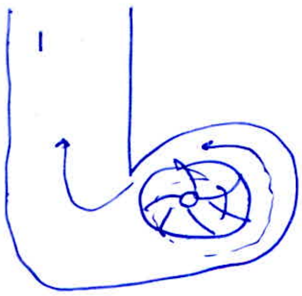
[6 + 6 marks]

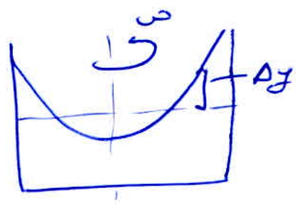
Soln

(i) In forced vortex flow, an external torque acts on fluid.

In pump this torque supplied by rotating runner vanes, by rotating fluid in pump, its energy is increased.

Water enters at impeller eye and flows out into gradually expanding spiral, velocity ↓es and pressure ↑es





3

At suction side pressure is low, at outlet high pressure maintained by pump.

(ii) $D = 0.1 m$ $\quad Q = 0.036 m^3/s$

$$\boxed{v_{avg} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = 4.584 m/s}$$

$$\boxed{Re = \frac{v_0 D}{\nu} = \frac{4.584 \times 0.1}{10^{-6}} = 4.584 \times 10^5}$$

$$f = 0.0032 + \frac{0.221}{Re^{0.237}}$$

$$f = 0.0032 + \frac{0.221}{(4.584\times10^{5})^{0.237}}$$

$$\boxed{f = 0.0133}$$

(b) $$\frac{u_{avg}}{u_*} = \sqrt{\frac{8}{f}}.$$

$$\boxed{u_* = u_{avg}\sqrt{\frac{f}{8}} = 0.187 \text{ m/s}}$$

$$\delta' = \frac{11.6\,\nu}{u_*} = \frac{11.6\times10^{-6}}{0.187} = 62.06\times10^{-6}\text{ m}$$

$$\boxed{\delta' = 0.062 \text{ mm}}$$ → viscous sub layer thickness.

6

(c) Show that at the critical state of flow, the specific energy in a rectangular channel is equal to 1.5 times the depth of flow. Also find at critical flow condition whether the depth of flow will be greater or less than $\frac{2}{3}$ times specific energy for a trapezoidal channel.

[12 marks]

Soln

$$E = y + \frac{Q^2}{2gA^2}$$

rect. channel $A = By$ $\quad Q/B = q$

$$E = y + \frac{q^2}{2gy^2}$$

$\frac{dE}{dy} = 0 \Rightarrow$ critical state

$$1 - \frac{q^2}{2g}(2y^{-3})$$

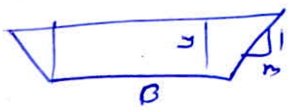
$$\boxed{1 = \frac{q^2}{gy^3}}$$

$$\boxed{E = y + \frac{y^3}{2y^2} = 1.5y} \quad \boxed{y = y_{cr}}$$

$$\boxed{E = 1.5y_c}$$

(ii) At critical flow for trapezoidal channel

$$\frac{Q^2T}{A^3g} = 1$$



$$Q^2(B+2my) = [\quad]^3 g$$

$$E = y + \frac{Q^2}{2gA^2} = y + \frac{Ag}{T2g}$$

$$E = y + \frac{A}{2T}$$

~~E = y +~~

$$E = y + \frac{A}{2T}$$

check $\frac{A}{2T} = \frac{By + my^2}{2(B + 2my)} < \frac{y}{2}$

hence $E = y + \frac{A}{2T} \leq y + \frac{y}{2}$

$$\boxed{y > \frac{2E}{3}}$$

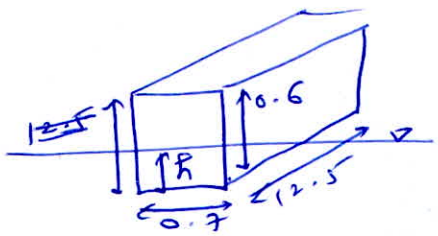
$$\boxed{\text{Depth} > \frac{2E}{3}}$$

12

Q.1 (d) An empty tank with all sides closed is 12.5 m long, 0.7 m broad and 0.6 m high. The surface of sheet metal weighs 363 N/m^2 and the tank is allowed to float in fresh water with 0.6 m side vertical. Determine the state of equilibrium.

[12 marks]

Soln



Now

Weight = Buoyancy

$363 \times 12.5 \times 0.7 \times 0.6 = \gamma_w \times h \times 0.7 \times 12.5$

$$\boxed{h = 0.0222 \text{ m}}$$

weight of tank = $363 \times [2][0.6 \times 0.7 + 0.6 \times 12.5 + 12.5 \times 0.6]$

$\boxed{W = 12.10242 kN}$

$W = F_B$

$12.10242 = \gamma_w \times 0.7 \times 12.5 \times h$

$\boxed{h = 0.141 m}$ → submerged depth.

• M
- G
• B

Now
for stability

$GM > 0$

$MB > BG$

$\frac{I}{V_{sub}} > \frac{0.6}{2} - \frac{h}{2}$

$\frac{\frac{12.5 \times 0.7^3}{12}}{0.7 \times 12.5 \times 0.141} > 0.3 - 0.0705$

$\boxed{0.2896 > 0.2295 m}$

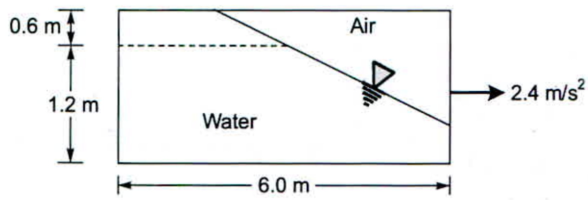
Hence $GM > 0$ stable vertically

$\boxed{GM = 0.06 m}$

12

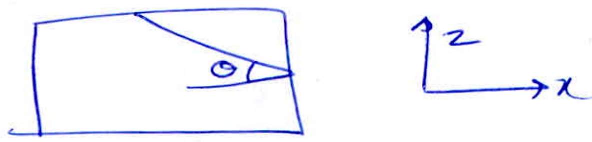
Do not write in this margin

Q.1 (e) A closed tank 6 m long, 2 m wide and 1.8 m deep initially contains water to a depth of 1.2 m. The top has an opening in the front part to have air space at atmospheric pressure. If the tank has given a horizontal acceleration at a constant value of 2.4 m/s^2 along its length, calculate the total pressure force on the top of the tank.

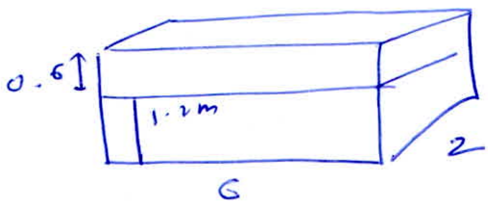


[12 marks]

Soln

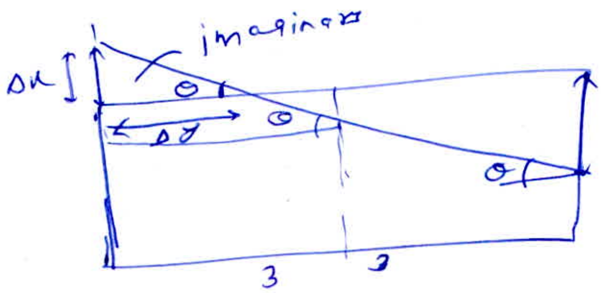


$$\boxed{\tan\theta = -\frac{a_x}{g_{eff}}}$$



$$\boxed{\tan\theta = \left|\frac{-2.4}{9.81}\right| = 0.2446}$$

Lets suppose tank was sufficiently highthen.



$$\frac{\Delta x + 0.6}{3} = 0.2446$$

$$\boxed{\Delta x = 0.1338\text{m}}$$

F_{top} = weight of imaginary water

$\frac{\Delta x}{\Delta y} = \tan\theta = 0.2446$

$\boxed{\Delta y = 0.547m}$

$F_{top} = \gamma_w \times \frac{1}{\cancel{2}} \times \Delta x \times \Delta y \times \cancel{2}$

$F_{top} = 9.81 \times 0.1338 \times 0.547$

$\boxed{F_{Top} = 0.7179 \text{ kN}}$

0.791 kN

10

Q.2 (a) A cylinder 0.25 m in radius and 2 m in length rotates coaxially inside a fixed cylinder of the same length and 0.30 m radius. Olive oil of viscosity 4.9×10^{-2} Ns/m^2 fills the annular space between the cylinders. A torque 4.9 N-m is applied to the inner cylinder. After constant velocity is attained, calculate the velocity gradient at the cylinder walls, the resulting rpm, and the power dissipated by fluid resistance ignoring end effect.

[20 marks]

Q.2 (b) A pump impeller is 37.5 cm in diameter and discharges water with velocity components of 2 m/s and 12 m/s in the radial and tangential directions respectively. The impeller is surrounded by a concentric cylindrical chamber with parallel sides, the outer diameter being 45 cm. If the flow in this chamber is a free-spiral vortex, find the components of velocity of water on leaving and the pressure rise in the shroud if there is no loss.

[20 marks]

Q.2 (c) **(i)** Many researchers believe that the problem of air-entertainment in free surface vortex formation at intakes is influenced by forces of viscosity and surface tension. Show that for dynamic similarity between model and prototype, the following relationship must be satisfied:

$$\left(\frac{\mu V}{\sigma}\right)_m = \left(\frac{\mu V}{\sigma}\right)_p$$

Also prove that by use of the same liquid results in the "equal-velocity" concept of model testing.

(ii) Water from a reservoir flowing through a rigid 150 mm diameter pipe, with a velocity 2.4 m/s is completely stopped by closure of a valve situated 1100 m from the reservoir, determine the maximum rise in pressure, when valve closure takes place

(1) In one second and

(2) In five seconds

Without damping of pressure wave. Consider the velocity of sound in water as 1432 m/s.

[10 + 10 marks]

3 (a) An inward flow reaction turbine has inlet and outlet diameters of 1.2 m and 0.6 m respectively. The breadth at the inlet is 0.25 m and at the outlet it is 0.35 m. At a speed of rotation of 250 rpm, the relative velocity at entrance is 3.5 m/s and is radial. Calculate the (i) absolute velocity at entrance and the inclination to the tangent of the runner, (ii) discharge and (iii) the velocity of flow at the outlet.

[20 marks]

Soln

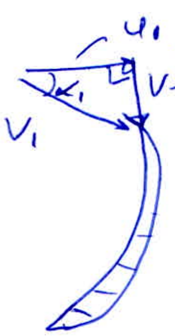
Given

$D_1 = 1.2 m$ $\quad B_1 = 0.25 m$

$D_2 = 0.6 m$ $\quad B_2 = 0.35 m$

$N = 250$ rpm

Inlet Δ



$$\boxed{V_{r_1} = V_{f_1} = 3.5 \, m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} = 15.71 \, m/s$$

$$u_2 = \frac{\pi D_2 N}{60} = 7.85 \, m/s$$

(i)

$$V_1 = \sqrt{u_1^2 + V_{r_1}^2} = \sqrt{15.71^2 + 3.5^2}$$

Absolute entrance velocity ⇒ $\boxed{V_1 = 16.095 \, m/s}$

$$\tan\alpha_1 = \frac{V_{r_1}}{u_1} = \frac{3.5}{15.71}$$

inclination of V_1 to tangent of runner $\boxed{\alpha_1 = 12.56°}$

(ii) $Q = \pi D_1 B_1 V_{f_1}$ [Neglect vane thickness]

$Q = \pi \times 1.2 \times 0.25 \times 3.5$

$\boxed{Q = 3.298 \ m^3/s}$ → Discharge

(iii) Now

$Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$

$1.2 \times 0.25 \times 3.5 = 0.6 \times 0.35 \times V_{f_2}$

velocity of flow at outlet → $\boxed{V_{f_2} = 5 \ m/s}$

20

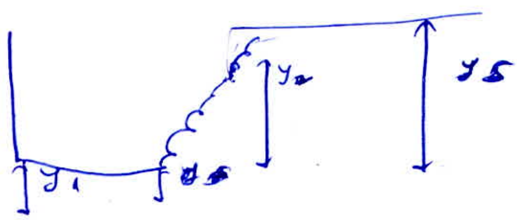
(b) Show that for a submerged hydraulic jump just downstream of a sluice gate, in a horizontal rectangular channel,

$$\frac{y_s}{y_1} = \sqrt{2F_1^2\left(\frac{y_1}{y_2}-1\right)+\left(\frac{y_2}{y_1}\right)^2}$$

where y_1 is the depth of opening of the sluice gate, y_2 is the depth of flow downstream of the submerged hydraulic jump, y_s is the water depth on the downstream side of the sluice gate and F_1 is the Froude number of flow through the sluice opening.

[20 marks]

Soln



Let discharge intensity be q m²/s

$F_1^2 = \dfrac{q^2}{g y_1^3}$

Prove using momentum equation

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8F_1^2} - 1\right] \qquad \frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)}$$

Now y_s

0

.3 (c) (i) What is meant by local and convective acceleration? For a one dimensional flow described by $V(x, t)$, derive the expression for convective acceleration in terms of velocity and its gradient.

(ii) A rectangular channel 5.2 m wide has a discharge of 10 m^3/sec at a velocity of 1.25 m/s. At a certain section the bed width is reduced to 3.0 m through a smooth transition. A smooth flat hump is to be built in this contracted section to cause critical flow for flow measurement purposes. Estimate the height of the hump necessary for this purpose. (Assume no loss of energy at the transition.)

[10 + 10 marks]

Soln

(i) Local acceleration → acceleration OR rate of change of velocity with respect to time (partial derivative)

$$\boxed{a_t = \frac{\partial v}{\partial t}}$$

convective acceleration → Rate of change of velocity along space

$$\boxed{a_s = v\frac{\partial v}{\partial s}}$$

$$\boxed{a_{total} = a_t + a_s}$$

$V(x, t)$

$$\text{acceleration} = \frac{dV}{dt}$$

$$\boxed{dV = \frac{\partial V}{\partial t} \times dt + \frac{\partial V}{\partial x} \times dx}$$

Divide by dt

$$a_{total} = \frac{dV}{dt} = \frac{\partial V}{\partial t} \times \cancel{\frac{dt}{dt}} + \frac{\partial V}{\partial x} \times \underbrace{\frac{dx}{dt}}_{V}$$

$$\boxed{a_{total} = \frac{\partial V}{\partial t} + \underbrace{V \frac{\partial V}{\partial x}}_{\text{convective acceleration}}}$$

8

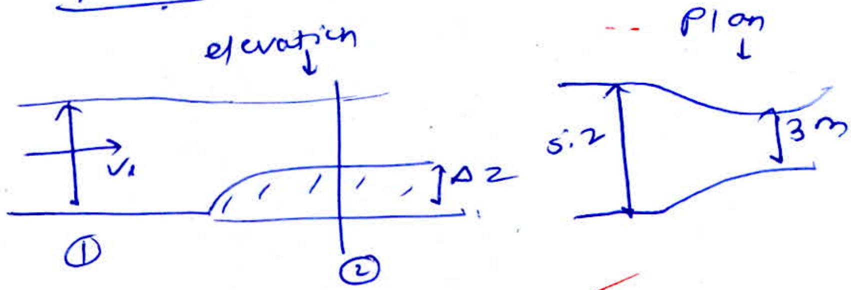
$$\boxed{a_{convective} = V(x,t) \frac{\partial V(x,t)}{\partial x}}$$

$$\boxed{a_c = \text{velocity} \times \text{gradient}}$$

(ii) $Q = 10 \, m^3/s$
$B_1 = 5.2 \, m$
$V_1 = 1.25 \, m/s$

$B_2 = 3 \, m$

Now



$$\boxed{E_1 = E_2 + \Delta z}$$ → energy equation.

Now

At section ② flow is critical.

$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$, $E_2 = E_c = 1.5 y_c$

$q = \frac{Q}{3} = \frac{10}{3}$

At ② → $\boxed{y_c = 1.042 m}$ $\boxed{E_2 = E_c = 1.5 y_c = 1.564 m}$

At ①

$E_1 = y_1 + \frac{V_1^2}{2g}$ $\left[\cancel{V_1}\right]$ Q

$y_1 = \frac{Q}{BV_1} = \frac{10}{5.2 \times 1.25} = 1.538 m$

$$\boxed{E_1 = 1.538 + \frac{1.25^2}{19.62} = 1.618 m}$$

$E_1 = E_2 + \Delta z$

$$\boxed{1.618 - 1.564 = \Delta z = 0.054 m}$$

hump height → $\boxed{\Delta z = 5.41 cm}$

10

Q.4 (a) **(i)** For the velocity profile, $\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ on a flat plate, find out the average velocity and kinetic energy correction factor.

(ii) Calculate the friction drag on a flat plate 15 cm wide and 45 cm long placed longitudinally in a stream of oil of relative density 0.925 and kinematic viscosity 0.9 stoke, flowing with a free stream velocity of 6.0 m/s. Also, find the thickness of the boundary layer and shear stress at the trailing edge.

[10 + 10 marks]

Soln

(i) $V_{avg} = \frac{\int V\,dA}{\int dA}$

$$V_{avg} = \frac{\int_0^\delta U_\infty \times \frac{1}{2}\left[3\left(\frac{y}{\delta}\right) - \frac{y^3}{2\delta^3}\right] \cancel{b}\,dy}{\cancel{b} \times \delta}$$

$$V_{avg} = \frac{\frac{U_\infty}{2}\left[\frac{3}{2}\delta - \frac{\delta}{4}\right]}{\delta}$$

$$\boxed{V_{avg} = \frac{11\,U_\infty}{2 \times 8}} = \frac{11\,U_\infty}{16} \quad \boxed{U_{avg} = \frac{5\,U_\infty}{8}}$$

(ii) Kinetic energy correction factor $= \alpha$

$$\alpha = \frac{\int v^3\,dA}{V_{avg}^3\,A}$$

$$(V_{avg})^3 A\,\alpha = \int_0^\delta \left[\frac{3}{2}t - \frac{t^3}{2}\right]^3 \times b\,dy\,U_\infty^3 \quad \left[t = \frac{y}{\delta}\right]$$

Let $t = \frac{y}{\delta}$

$\delta\,dt = dy$

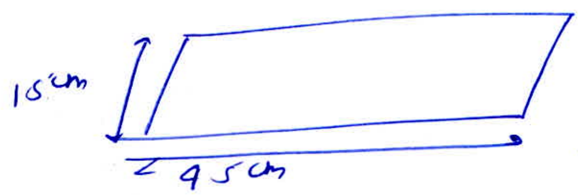
$$= \int_0^1 \left[1.5t - \frac{t^3}{2}\right]^3 b\,\delta\, dy$$

$$\alpha = \frac{\frac{131}{320}\, b \times \delta}{V_{avg}^3 \times b \times \delta} = \frac{131\, u_\infty^3}{320\left[\frac{5u_\infty}{8}\right]^3}$$

10

$$\boxed{\alpha = 1.6768}$$ → K.E corr. factor.

(ii)



Given

$\rho = 925\ kg/m^3$

$\nu = 0.9 \times 10^{-4}\ m^2/s$

$V_0 = 6\ m/s$

Let's check

$$Re_L = \frac{V_0 L}{\nu} = \frac{6 \times 0.45}{0.9 \times 10^{-4}} = 3 \times 10^4$$

$$Re_L = 3 \times 10^4 < 5 \times 10^5$$

B.L. throughout Laminar

As per Blasius.

$$C_{F\,avg} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{3 \times 10^4}}$$

$$\boxed{C_{F\,avg} = 7.667 \times 10^{-3}}$$

$$F_{\text{one side}} = \frac{1}{2} \rho C_F A V_0^2$$

$$F_{\text{one side}} = \frac{1}{2} \times 925 \times 7.667 \times 10^{-3} \times 0.15 \times 0.45 \times 6^2$$

$$\boxed{F_{\text{one side}} = 8.617\ N}$$

$$\boxed{F_D = 2F_0 = 17.233\ N}$$

At trailing edge

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$$

$x = L$

$$\delta = \frac{5L}{\sqrt{Re_L}} = \frac{5 \times 0.45}{\sqrt{3 \times 10^9}} =$$

$$\boxed{\delta = 1.299 \text{ cm}}$$

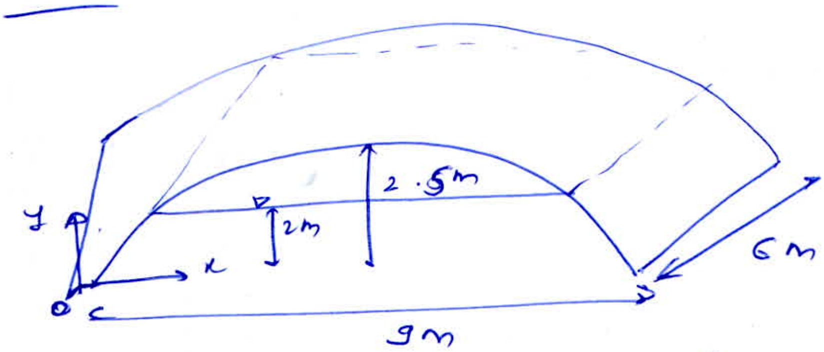
$$\boxed{\frac{\tau_w}{\frac{1}{2}\rho V_0^2} = \frac{0.664}{\sqrt{Re_L}}} \rightarrow \Rightarrow \boxed{\tau_{w_{x=L}} = 63.83 \text{ N/m}^2}$$

10

4 (b) A stream is spanned by a bridge which is a single masonry arch in the form of a parabolic arch, the crown being 2.5 metre above the springings which are 9 meters apart. The overall width of the bridge is 6 metres. During a flood the stream rises to a level 2 metres measured in the direction of the stream above the springings. Calculate the force tending to lift the bridge from its foundations if the arch remains water tight.

[20 marks]

Soln



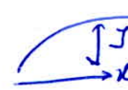
Lets write equation of parabola about O (left edge)

$$y = a x (9 - x)$$

at $x = 4.5m$, $y = 2.5m$

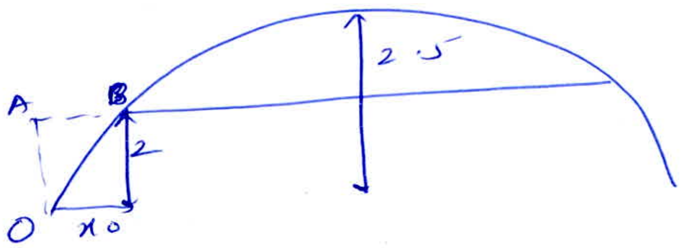
$$2.5 = a \; 4.5 \times 4.5$$

$$\boxed{a = \frac{10}{81}}$$



$$\boxed{y = \frac{10}{81}(9x - x^2)} \rightarrow \text{about } O$$

Now



find x_0

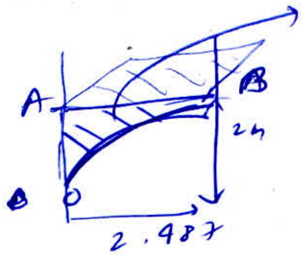
$$2 = \frac{10}{81}(9x_0 - x_0^2)$$

$$90x_0 - 10x_0^2 - 162 = 0$$

$$\boxed{x_0 = 2.487m}$$

Force on curved surface $OAB = \gamma_w \times V_{OAB} \times 2$
volume

weight of liquid in this column.



$$\text{Area } OAB = \int_0^{2.487} (2 - y)\, dx$$

$$A_{OAB} = \int_0^{2.487} \left[2 - \frac{10}{81}(9x - x^2)\right] dx$$

$$A_{OAB} = 2.171 m^2$$

volume of water in $OAB = A_{OAB} \times$ width

$= 2.171 \times 6$

$V = 13.026 m^3$

Each side $= 2V = 26.05 m^3$

$$\boxed{F_{up} = 2V \times \gamma_w = 255.57 kN} \uparrow$$

20

Do not write in this margin

4 (c) (i) Define bulk modulus of elasticity of a fluid. What is the SI unit of bulk modulus of elasticity? Discuss the factors affecting bulk modulus of elasticity of a fluid. Why liquids are generally considered incompressible?

(ii) Show that the theoretical discharge in an open channel flow may be expressed as:

$$Q = A_2 \sqrt{\frac{2g\left(\Delta y - h_f\right)}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

where A_1 and A_2 are the cross-sectional areas of flow at sections (1) and (2) respectively, Δy is the drop in the water surface between the two sections and h_f is the energy head loss between the two sections.

[10 + 10 marks]

Soln

(i) Bulk modulus of Elasticity of fluid is defined as $\boxed{K = -\frac{dP}{dV/V}}$.

Measures compressibility of fluid

It is pressure that must be applied for unit volumetric strain / dilation

Unit → MPa or N/mm² or N/m²

Factors are :- 1) density of fluid

2) Ambient pressure 3) poisson ratio

4 Modulus of elasticity of fluid.

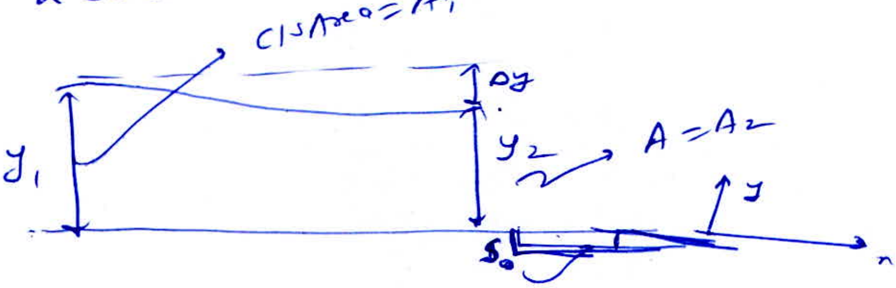
8

Liquids are generally incompressible because their densities are already high and molecules are comparatively very close as compared to gases.

Hence to compress it, large pressure is required thus 'K' for liquids high

(ii) In OCF

By energy equation between 2 sections



$E_1 = E_2 + \text{head loss}$

$E \rightarrow$ specific energy

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_f$$

$$y_1 - y_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_f \qquad [y_1 - y_2 = \Delta y]$$

$$\Downarrow$$

$$\Delta y - h_f = \frac{Q^2}{2g}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]$$

$$2g(\Delta y - h_f) = \frac{Q^2}{A_2^2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

$$\frac{2g(\Delta y - h_f)A_2^2}{1 - \left(\frac{A_2}{A_1}\right)^2} = Q^2$$

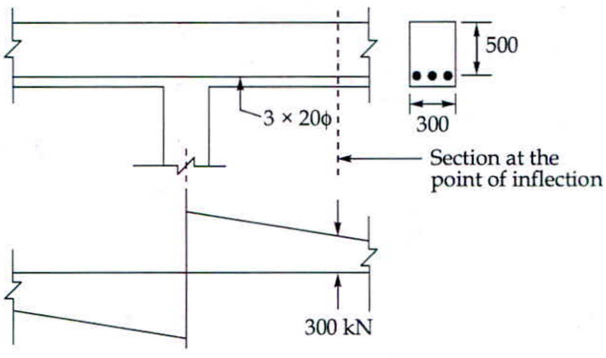
10

$$\boxed{\sqrt{\frac{2g(\Delta y - h_f)}{1 - \left(\frac{A_2}{A_1}\right)^2}} \times A_2 = Q}$$

Hence proved

Section B : Design of Concrete and Masonry Structures-1 + Strength of Materials-2

Q.5 (a) Check for bond stress at the point of inflection of a continuous beam as shown in figure, if it is subjected to an ultimate shear force of 300 kN at the point of inflection. Consider concrete of grade M20 and steel of grade Fe415. [Take design bond stress for M20 concrete = 1.2 N/mm²]



Ultimate shear force diagram

Continuous beam

[12 marks]

Soln

$V_u = 300\,kN$ M20 Fe415

$\tau_{bd} = 1.2\ N/mm^2$

$L_d \leq \frac{M_1}{V_u} + L_0$

$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 1.2 \times 1.6}$

$\boxed{L_d = 940.23\,mm}$

Now

$M_1 = 0.87 f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{ck} b} \right]$

$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.48\,mm^2$

$M_1 = 0.87 \times 415 \times 942.48 \left[500 - \frac{415 \times 942.48}{20 \times 300} \right]$

$\boxed{M_1 = 147.958\ kNm}$

$\frac{M_1}{V_u} = 493.17 mm.$

at pt. of inflection $L_o = d$ or 12ϕ

$= 500$; $240mm$

$\boxed{L_o = 500mm}$

$L_d = 940.23 < \frac{M_1}{V_u} + L_o = 993.195mm$

$\boxed{L_d \le \frac{M_1}{V_u} + L_o}$ → Hence safe in bond

12

Q.5 (b) State the assumptions made while analyzing the reinforced concrete beam using Limit State of Flexure as per **IS 456:2000 Code.**

[12 marks]

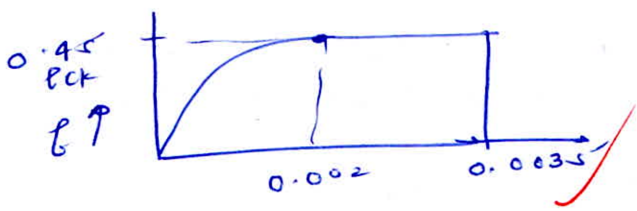
Soln

① Plane sections normal to axis remain plane even after bending thus strain is linear over depth of section

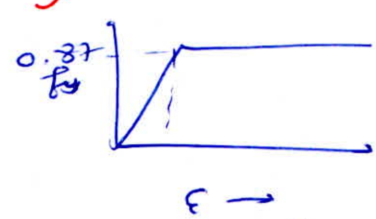
② Tensile strength of concrete ignored

③ Perfect bond between steel & concrete

④ Design stress strain diagram for concrete is parabolic rectangular



⑤ For steel



⑥ Failure will always occur by crushing of concrete in compression when it reaches strain of 0.0035

⑦ Partial FOS for steel → 1.15
concrete → 1.5

⑧ Maximum strain in tensile reinforcement at failure must not be less than

$$\varepsilon_{max\,t} \geq \frac{0.87 f_y}{E_s} + 0.002.$$

This ensures steel has yielded prior to concrete crushing

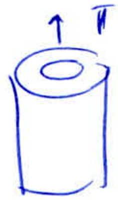
10+1=9

Q.5 (c) Three exactly similar mild steel tube specimens have the external and internal diameters 37.5 mm and 31.25 mm respectively. One of these specimens was tested in pure tension and limit of proportionality was recorded to be 70 kN. The second specimen was tested in torsion whereas the third was tested in torsion with superimposed bending moment of 350 Nm. If the failure criterion is the maximum shear stress, determine the torque at which the two specimens would have failed?

[12 marks]

Soln

Now sample – 1



$$\frac{T}{A} = \sigma = \frac{70\times10^3}{\frac{\pi}{4}\times[37.5^2-31.25^2]}$$

So $\boxed{\sigma_{max} = 207.42\ N/mm^2}$

$$\tau_{max} = \frac{\sigma_0}{2}$$

(ii) specimen – 2

$$\tau_{max} = \frac{16T}{\pi D^3} \quad \frac{T\times D/2}{\frac{\pi}{32}[D^4-d^4]}$$

$$= 16\times T$$

$$\frac{\sigma_0}{2} = \frac{T\times 37.5/2}{\frac{\pi}{32}[37.5^4-31.25^4]}$$

$$\boxed{T_{max} = 0.5559\ kNm}$$

(iii) $\sigma_{1,2} = \frac{1}{2}[M \pm \sqrt{M^2+T^2}]$

$$\left|\frac{\sigma_1-\sigma_2}{2}\right| = \frac{\sigma_0}{2}$$

$$\sigma = \frac{M\times D/2}{\frac{\pi}{64}[D^4-d^4]} = \frac{32MD}{\pi[D^4-d^4]}$$

Do not write in this marg

$$\tau = \frac{16TD}{\pi[D^4 - d^4]}$$

$$\sigma_1 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,2} = \frac{16D}{\pi[D^4 - d^4]}\left[M \pm \sqrt{M^2+T^2}\right]$$

$$\left|\frac{\sigma_1 - \sigma_2}{\cancel{2}}\right| \leq \frac{\sigma_y}{\cancel{2}}$$

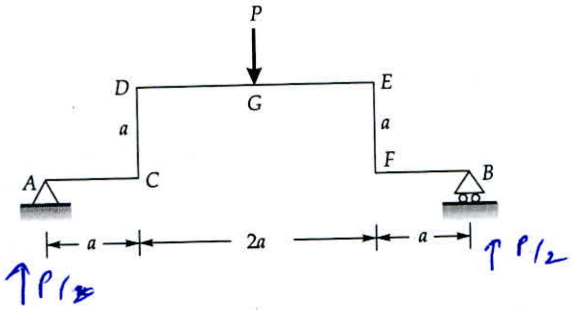
$$\frac{16D}{\pi[D^4-d^4]}\left[2\sqrt{M^2+T^2}\right] \leq 207.2$$

$$M^2 + 0.35^2 \leq 0.5559^2$$

$$\boxed{T = 0.4319 \text{ kNm}}$$

12

Q.5 (d) Find the central deflection of the framed beam using strain energy method as shown in figure. [EI is constant]

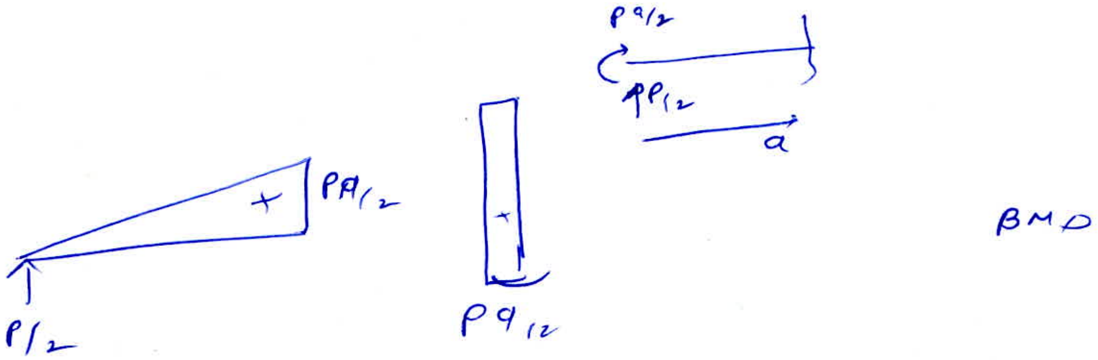


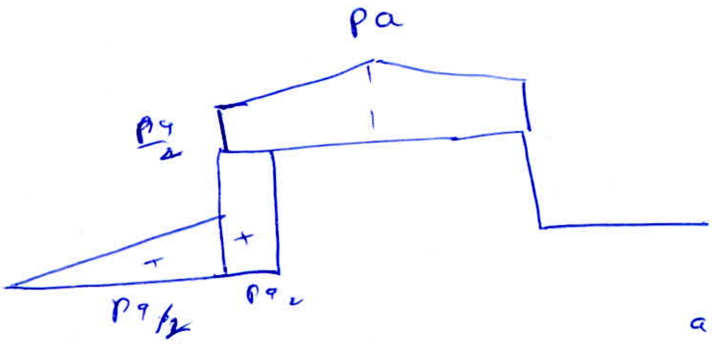
[12 marks]

Soln

$$U = \int \frac{M^2\,dx}{2EI}$$

$$\Delta = \frac{\partial U}{\partial P} = \int \frac{M \frac{\partial M}{\partial P}\,dx}{EI}$$





12

$$\Delta = 2 \times \left[\int_0^a \frac{\frac{P}{2}x \times \frac{x}{2}}{EI}\,dx + \int_0^a \frac{\frac{Pa}{2} \times \frac{a}{2}}{EI}\,dx + \int_0^a \frac{\left[\frac{Pa}{2} + \frac{P}{2}x\right]\left[\frac{a}{2} + \frac{x}{2}\right]}{EI}\,dx \right]$$

$$\Delta = 2 \times \left[\frac{Pa^3}{12EI} + \frac{Pa^3}{4EI} + \frac{7Pa^3}{12EI} \right]$$

$$\boxed{\Delta = \frac{11Pa^3}{6EI}}$$

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.5 (e) A machine component is made of a material whose ultimate strength in tension, compression and shear are 40 N/mm², 110 N/mm² and 55 N/mm² respectively. At the critical point in the component, the state of stress is represented by

$$\sigma_x = 25 \text{ N/mm}^2 \text{ and } \sigma_y = -75 \text{ N/mm}^2$$

Find the maximum value of the shear stress τ_{xy} which will cause failure of the component?

[12 marks]

Sol^n

principal stresses

$\sigma_{1,2}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = -25 \pm \sqrt{50^2 + \tau_{xy}^2}$$

For tension

$$-25 + \sqrt{50^2 + \tau^2} \leq 40$$

$$\boxed{\tau_{xy} \leq 41.53 \text{ N/mm}^2}$$

For compression

$$25 + \sqrt{50^2 + \tau^2} = 110$$

$$\boxed{\tau \leq 68.79 \text{ N/mm}^2}$$

10 + 2 = 12

$$\sqrt{50^2 + \tau^2} \leq 55$$

$$\boxed{\tau \leq 22.913 \text{ N/mm}^2}$$

$$\boxed{\tau_{xy\,min} = \tau = 22.913 \text{ N/mm}^2}$$

.6 (a) Design a rectangular beam section of 300 mm width and 500 mm effective depth which is subjected to an ultimate bending moment of 50 kNm, ultimate shear force of 50 kN and ultimate torsional moment of 40 kNm. Consider concrete of grade M20 and steel of grade Fe415. [Assume effective cover = 35 mm]

p_t (%)	≤0.15	0.25	0.5	0.75	1
τ_c (N/mm^2)	0.28	0.36	0.48	0.56	0.62

[20 marks]

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Q.6 (b) **(i)** A ring beam of water tank has a diameter of 12.5 m. It is subjected to outward radial force of 25 kN/m. Design the section of ring beam using M25 and Fe415. Assume $m = 11$ and allowable stress in tension as 1.2 N/mm^2.

(ii) Calculate the development length in tension and compression for a single mild steel bar of diameter ϕ in concrete of grade M20. Assume $\tau_{bd} = 1.2$ N/mm^2.

[14 + 6 marks]

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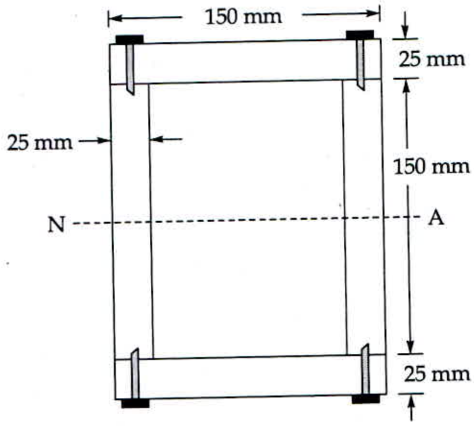
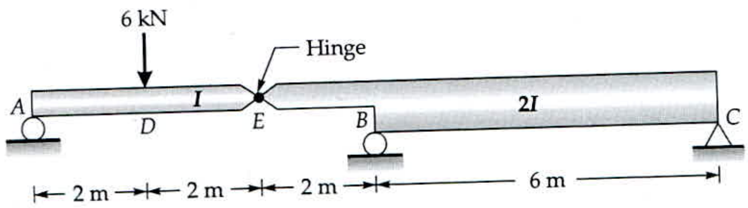
.6 (c) The box beam as shown in figure below is made up of four 150 mm × 25 mm wooden planks connected by screws. Each screw can safely transmit a shear force of 1250 N. Estimate the minimum necessary spacing of screws along the length of the beam if the maximum shear force transmitted by the cross-section is 5000 N. Also determine the shear stress distribution across the section.



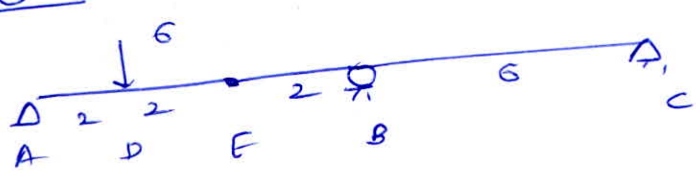
[20 marks]

.7 (a) A hinged beam system is loaded as shown below. Determine the slope at point E and D. Also determine the deflection at D. Use Conjugate beam method.



[20 marks]

Soln



$\Sigma M_E = 0$

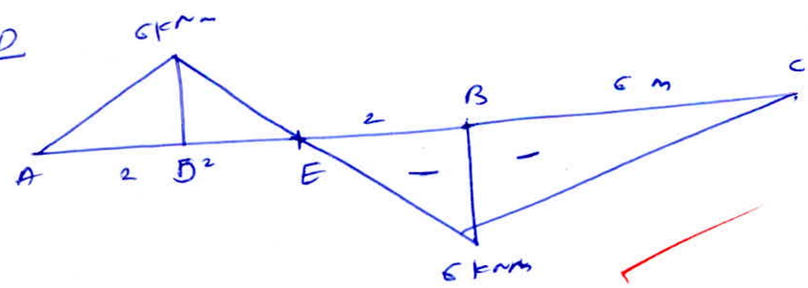
$R_A \times 4 = 6 \times 2$ $\boxed{R_A = 3\,kN}$

$\boxed{R_B + R_C = 6 - 3 = 3\,kN}$

$\Sigma M_E = 0$ $R_B \times 2 + 8 R_C = 0$

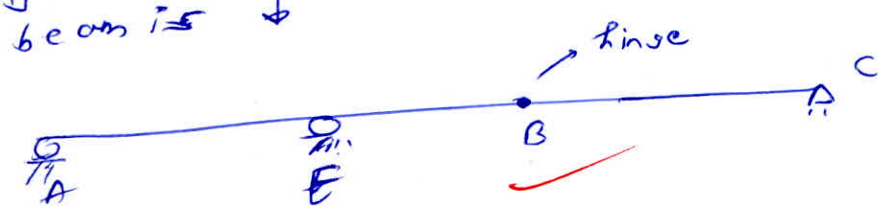
$\Rightarrow$ $\boxed{R_B = 4}$ $\boxed{R_C = -1\,kN}$

BMD

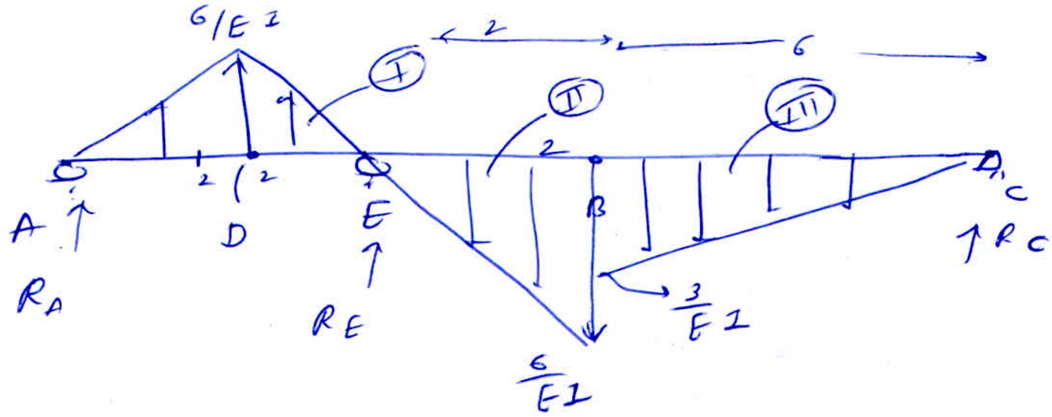


Now

Conjugate beam is ↓



$\frac{M}{EI}$ diagram



$$\text{Area-I} = \frac{1}{2} \times 4 \times \frac{6}{EI} = \frac{12}{EI}$$

$$A_{II} = \frac{1}{2} \times 2 \times \frac{6}{EI} = \frac{6}{EI}$$

$$A_{III} = \frac{1}{2} \times 6 \times \frac{3}{EI} = \frac{9}{EI}$$

$$R_A + R_E + R_C = \frac{6+9}{EI} - \frac{12}{EI} = \frac{3}{EI}$$

$\Sigma M_B = 0$

$$R_C \times 6 = \frac{9}{EI} \times \frac{6}{3} = \frac{3}{EI}$$

$$\boxed{R_C = \frac{3}{EI} \uparrow}$$

$\Sigma M_A = 0$

$$R_E \times 4 + R_C \times 12 + \frac{12}{EI} \times 2$$

$$= \frac{6}{EI} \times \left(4 + 2 \times \frac{2}{3}\right)$$

$$+ \frac{9}{EI} \times \left[6 + \frac{6}{3}\right]$$

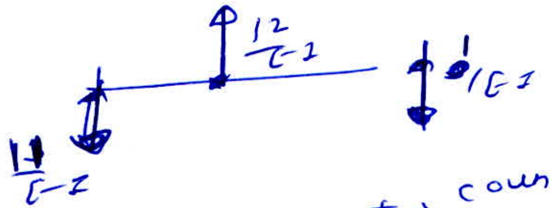
$$\boxed{R_E = \frac{\cancel{44}\,11}{EI} \uparrow} = \frac{11}{EI} \uparrow$$

$R_A = \frac{3 - \cancel{11} - 3}{EI} = \cancel{-11}\ \frac{-11}{EI}$

$\boxed{R_A = \frac{\cancel{44}\ 11}{EI} \downarrow}$

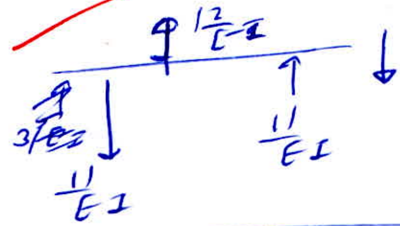
slope at pt. E to left of E

$\theta_{E-} =$



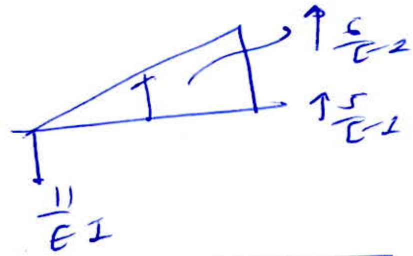
$\boxed{\theta_{E-} = \frac{+1}{EI}}$ i.e. +→ counter clockwise → $\frac{1}{EI}$

θ_{E+}



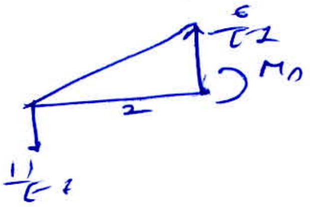
$\boxed{\theta_{E+} = +\frac{12}{EI}}$ ↺ c.c.w

$\theta_D =$



$\boxed{\theta_D = \frac{-5}{EI}}$ i.e clockwise

$\Delta_D = M_D =$

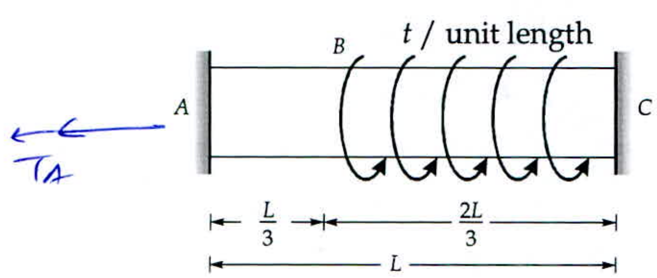


$M_D + \frac{22}{EI} = \frac{1}{2} \times 2 \times \frac{6}{EI} \times \frac{2}{3}$

$\boxed{M_D = \frac{-18}{EI}}$ $\boxed{\Delta_D = \frac{18}{EI} \downarrow}$

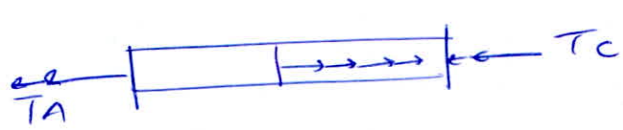
20

Q.7 (b) A solid circular cross-section shaft is clamped at both ends and loaded by a twisting moment t per unit length as shown in figure below. Determine the reactive twisting moment at each end of the bar.



[20 marks]

Soln



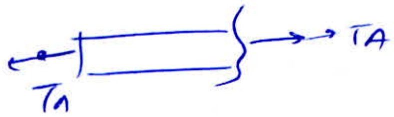
$\Sigma T = 0$

$$\boxed{T_A + T_C = t \times \frac{2L}{3}}$$

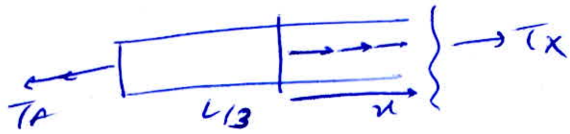
$$\boxed{\phi_{A/C} = 0}$$ Angle of twist b/w A & C

$\phi_{A/B} + \phi_{B/C} = 0$

In AB



In BC



20

$$\boxed{T_x = T_A - tx}$$

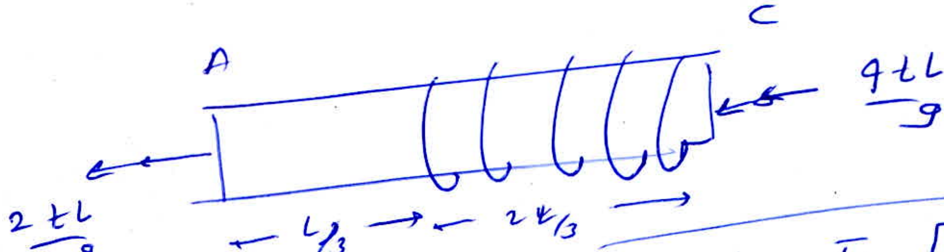
$$\phi = \int \frac{T\,dx}{GJ}$$

$$\frac{T_A \times L/3}{GJ} + \int_0^{2L/3} \frac{(T_A - tx)\,dx}{GJ} = 0$$

$$\frac{T_A \times L}{3} + \frac{T_A \times 2L}{3} - \frac{t}{2} \times \left(\frac{2L}{3}\right)^2 = 0$$

$$\boxed{T_A = \frac{2tL}{9}}$$

$$\boxed{\text{and } T_C = \frac{2tL}{3} - \frac{2tL}{9} = \frac{4tL}{9}}$$



As seen from C

$$\frac{4tL}{9} = T_C \text{ [C.W.] clockwise}$$

$$T_A = \frac{2tL}{9} \text{ [C.C.W.]}$$

7 (c) Design a reinforced concrete rectangular section of size 250 × 500 mm for a factored moment of 225 kN. The grades of concrete and HYSD steel are M20 and Fe415, respectively. [Take effective cover = 50 mm, f_{sc} = 353 MPa]

[20 marks]

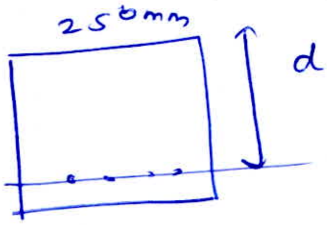
Soln

Given.

M20

Fe415

f_{sc} = 353 MPa



$d = 500 - 50$
$= 450$ mm

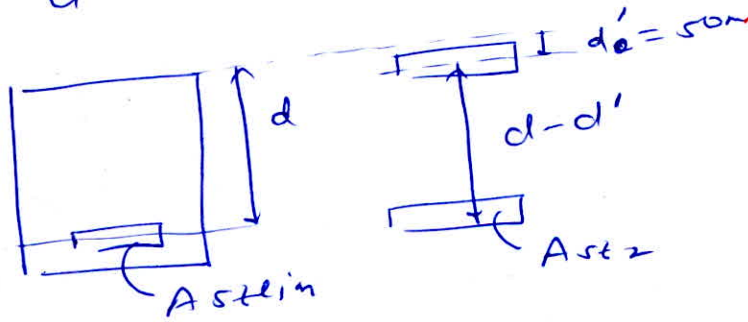
$b = 250$ mm

$M_{ulim} = 0.138 f_{ck} b d^2 = 0.138 \times 20 \times 250 \times 450^2$

$\boxed{M_{ulim} = \cancel{172.5}\ 139.725 \text{ kNm}}$

$M_u = 225$ kNm > M_{ulim}

Hence we have to design doubly reinforced section



Now

Total Area of steel in tension

$\boxed{A_{st} = (A_{st})_{lim} + A_{st2}}$

$0.36 f_{ck} b x_{ulim} = 0.87 f_y A_{st\,lim}$

$0.36 \times 20 \times 250 \times 0.48 \times 450 = 0.87 \times 415\ A_{st\,lim}$

$\boxed{A_{st\,lim} = 1076.86 \text{ mm}^2}$

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Now

$Mu - Mulim = 0.87 f_y Ast_2 [d - d']$

$[225 - 139.725] \times 10^6$

$\boxed{Ast_2 = 590.46 mm^2}$

Now $\frac{d'}{d} = \frac{50}{450} = 0.11 < 0.2$

$\boxed{f_{cc} = 0.45 f_{ck} = 9 \, M/mm^2}$

compression steel

$Mu - Mulim = [f_{sc} - f_{cc}] Asc [d - d']$

$[225 - 139.725] \times 10^6$

$= [353 - 9] Asc \times 400$

$\boxed{Asc = 619.73 mm^2}$

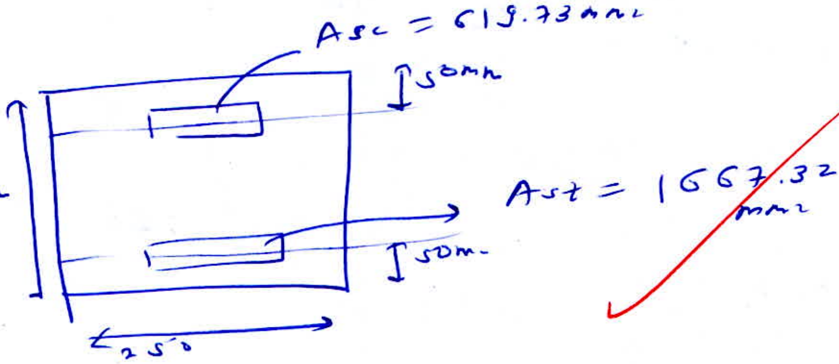
$\boxed{Ast = Astlim + Ast_2 = 1667.32 \, mm^2}$

$\boxed{Asc = 619.73 mm^2}$

18

Check minm. tension steel $= \frac{0.85}{f_y} \times bd = 231 mm^2$ ok

draw the detailing diagram properly.



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3 (a) (i) A rectangular beam section of 300 mm width and 500 mm effective depth is reinforced with 5 bars of 20 mm ϕ, out of which 2 bars have been bent at 45°. Determine the shear resistance of the bent up bars and additional shear reinforcement required if it is subjected to an ultimate shear force of 300 kN. Consider concrete of grade M20 and steel of grade Fe415.

$p_t (\%)$	≤ 0.15	0.25	0.5	0.75	1
$\tau_c \left(N/mm^2\right)$	0.28	0.36	0.48	0.56	0.62

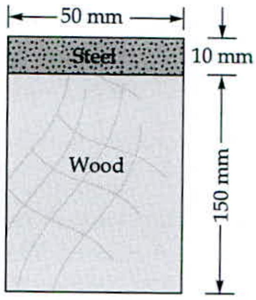
(ii) Determine the ultimate load capacity of a circular column of 400 mm diameter reinforced with 6 × 25 mm ϕ bars adequately tied with (i) lateral ties and (ii) spirals. Consider concrete of grade M25 and steel of grade Fe415.

[10 + 10 marks]

Q.8 (b) A staircase consists of 14 steps, each of 300 mm tread and 180 mm rise, plus two landings of each 1.25 m length. The width of staircase is 1.4 m. Design the staircase for a live load of 5 kN/m^2. Use M20 grade concrete and Fe415 reinforcement.

[20 marks]

Q.8 (c) (i) A wooden beam 50 mm wide and 150 mm deep is reinforced by gluing a steel plate 10 mm thick and 50 mm wide on the top of section. The beam is simply supported over its ends which are 5 m away from each other. The beam carries a point load of 500 kN at mid of beam. Calculate maximum shear stress at the junction of wood and steel plate. Take $m = 20$.



(ii) Find the dimensions of a hollow steel shaft of internal diameter 0.6 times the external diameter, to transmit 150 kW at 250 rpm, if the shearing stress is not to exceed 70 N/mm^2. If a bending moment of 3000 Nm is now applied to the shaft, find the speed at which it must be driven to transmit the same power for the same value of maximum shearing stress.

[10 + 10 marks]

OOOO

Space for Rough Work

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