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## ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

### Electronics & Telecommunication Engineering

**Test-3: Analog and Digital Communication Systems**

**Network Theory-1 + Microprocessors and Microcontroller-1**

**Digital Circuits-2 + Control Systems-2**

Name : \_\_\_\_\_

Roll No : **E C 1 9 M T N D A 6 0 6**

#### Test Centres

#### Student's Signature

Delhi <input type="checkbox"/>	Bhopal <input type="checkbox"/>	Noida <input checked="" type="checkbox"/>	Jaipur <input type="checkbox"/>	Indore <input type="checkbox"/>
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#### Instructions for Candidates

- Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
- Answer must be written in English only.
- Use only black/blue pen.
- The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
- Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
- Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

#### FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	<b>48</b>
Q.2	<b>—</b>
Q.3	<b>48</b>
Q.4	<b>—</b>
Section-B	
Q.5	<b>46</b>
Q.6	<b>22</b>
Q.7	<b>45</b>
Q.8	<b>—</b>
<b>Total Marks Obtained</b>	<b>209</b>

Signature of Evaluator

*Sumeet*

Cross Checked by

*JTA*

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Remarks : Very well presented and good accuracy!  
Keep it up!



## Section A : Analog and Digital Communication Systems

1 (a)

Let  $X(t)$  be a real WSS process and another process  $Y(t) = \hat{X}(t)$ . i.e.,  $Y(t)$  is the Hilbert transform of  $X(t)$ .  $R_X(\tau)$  and  $R_Y(\tau)$  denote the auto-correlation function of  $X(t)$  and  $Y(t)$  respectively, and  $R_{XY}(\tau)$  denotes the cross-correlation function of  $X(t)$  and  $Y(t)$ . Then prove that the following two relations are true.

$$R_1: R_Y(\tau) = R_X(\tau)$$

$$R_2: R_{XY}(-\tau) = -R_{XY}(\tau)$$

Given:  $y(t) = \hat{x}(t)$   $\quad \text{①}$ ;  $\hat{x}(t) \rightarrow \text{Hilbert Transform of } x(t)$  [12 marks]

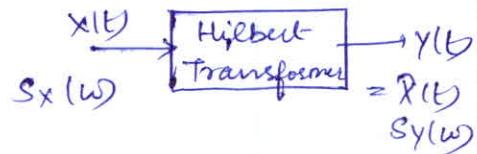
To prove is  $R_Y(\tau) = R_X(\tau)$

Taking Fourier Transform of eq^①: 
$$Y(\omega) = -j \operatorname{sgn}(\omega) X(\omega)$$

If PSD of  $x(t)$  is  ~~$S_x(f)$~~   $S_x(\omega)$

Then PSD of  $y(t)$  is  ~~$S_y(f)$~~

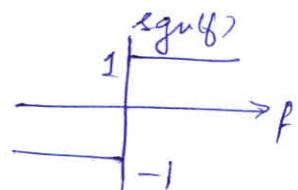
$$S_y(\omega) = S_x(\omega) |H(f)|^2$$



where  $H(f) = -j \operatorname{sgn}(f)$

$$\therefore S_y(f) = S_x(f) | -j \operatorname{sgn}(f) |^2$$

$$S_y(f) = S_x(f) \quad (\because \operatorname{sgn}^2(f) = 1)$$



Taking Inverse Fourier Transform

$$R_Y(\tau) = R_X(\tau)$$

Hence Proved.

To prove:  $R_{XY}(-\tau) = -R_{XY}(\tau)$

$y(t) = \hat{x}(t) \Rightarrow$  Taking Fourier Transform

$$Y(\omega) = -j \operatorname{sgn}(\omega) X(\omega)$$

$$|Y(\omega)| = |X(\omega)|$$

$$\angle Y(\omega) = \angle X(\omega) - \pi/2$$

$\therefore$   ~~$Y(\omega)$~~  signal  $y(t)$  is odd signal  $\Rightarrow$

i.e. provides  $-90^\circ$  phase shift to input signal.

$$\because \text{odd} \quad \text{so} \quad R_{XY}(-\tau) = R_{XY}(\tau)$$

~~$R_{XY}(-\tau) = R_{XY}(\tau)$~~

**Q.1 (b)** Consider a single-tone AM signal as follows:

$$s(t) = [1 + \mu \cos \omega_m t] \cos \omega_c t$$

If  $\mu = \frac{1}{2}$  and the upper sideband component is attenuated by a factor of 2, then determine the expression for the envelope of the resulting modulated signal.

Soln: Given:  $s(t) = [1 + \mu \cos \omega_m t] \cos \omega_c t$  [12 marks]

$$= \cos \omega_c t + \mu \cos \omega_m t \cos \omega_c t$$

$$= \cos \omega_c t + \frac{\mu}{2} [\cos((\omega_c + \omega_m)t) + \cos(\omega_c - \omega_m)t]$$

$$= \cos \omega_c t + \frac{\mu}{2} \cos(\omega_c + \omega_m)t + \frac{\mu}{2} \cos(\omega_c - \omega_m)t$$

{ using  $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ }

since:

upper sideband component is attenuated by a factor of 2 so

$$s(t) = \cos \omega_c t + \frac{1}{2} \cdot \frac{\mu}{2} \cos(\omega_c + \omega_m)t + \frac{\mu}{2} \cos(\omega_c - \omega_m)t$$

$$= \cos \omega_c t + \underbrace{\frac{\mu}{4} \cos(\omega_c + \omega_m)t}_{\text{USB}} + \underbrace{\frac{\mu}{2} \cos(\omega_c - \omega_m)t}_{\text{LSB}}$$

Now

$$\begin{aligned}
 s(t) &= \cos \omega_c t + \frac{\mu}{4} [\cos \omega_m t \cos \omega_c t - \sin \omega_m t \sin \omega_c t] \\
 &\quad + \frac{\mu}{2} [\cos \omega_c t \cos \omega_m t + \sin \omega_c t \sin \omega_m t] \\
 &= \left(1 + \frac{\mu}{4} \cos \omega_m t + \frac{\mu}{2} \cos \omega_m t\right) \cos \omega_c t \\
 &\quad + \left[\frac{\mu}{2} \sin \omega_m t - \frac{\mu}{4} \sin \omega_m t\right] \sin \omega_c t
 \end{aligned}$$

$$\boxed{s(t) = \left(1 + \frac{3\mu}{4} \cos \omega_m t\right) \cos \omega_c t + \left(\frac{\mu}{4} \sin \omega_m t\right) \sin \omega_c t}$$

$\therefore$  Envelope of resulting modulated signal is given as:

$$A \cos(f_{c}t) + B \sin(f_{c}t) \xrightarrow{\text{Envelope}} = \sqrt{A^2 + B^2}$$

Similarly, in given question:

envelope of modulated signal

$$= \sqrt{\left(1 + \frac{3\mu}{4} \cos \omega_m t\right)^2 + \left(\frac{\mu}{4} \sin \omega_m t\right)^2}$$

$$= \sqrt{1 + \frac{9\mu^2}{16} \cos^2 \omega_m t + \frac{\mu^2}{16} \sin^2 \omega_m t + \frac{3\mu}{2} \cos \omega_m t}$$

$$= \sqrt{1 + \frac{\mu^2}{2} \cos^2 \omega_m t + \frac{\mu^2}{16} + \frac{3}{2} \mu \cos \omega_m t}$$

$$\therefore \mu = Y_2$$

$$\text{so } \text{Envelope} = \sqrt{1 + \frac{\cos^2 \omega_m t}{8} + \frac{1}{64} + \frac{3}{4} \cos \omega_m t}$$

$$\text{Envelope} = \sqrt{\frac{65}{64} + \frac{3}{4} \cos \omega_m t + \frac{\cos^2 \omega_m t}{8}}$$

Any

- Q.1 (c)** Over the interval  $|t| \leq 1$ , an angle modulated signal is given by,  $s(t) = 10 \cos 13000t$ .  
 Carrier frequency  $\omega_c = 10000 \text{ rad/s}$ .
- If it is a PM signal with  $k_p = 1000 \text{ rad/V}$ , then determine  $m(t)$  over the interval  $|t| \leq 1$ .
  - If it is an FM signal with  $k_f = 1000 \text{ rad/s/V}$ , then determine  $m(t)$  over the interval  $|t| \leq 1$ .

Soln) Angle modulated signal,  $s(t) = 10 \cos (13000t)$  [6 + 6 marks]

i) If  $s(t) = PM$  signal;  $k_p = 1000 \frac{\text{rad}}{\text{V}}$ , To find  $m(t)$

General eqn of PM signal is:

$$s(t) = A_c \cos [2\pi f_c t + k_p m(t)] \quad \text{--- (1)}$$

Given eqn:  $s(t) = 10 \cos [(3000 + \underbrace{10000}_{w_c} t)]$  (given)

$$s(t) = 10 \cos [2\pi f_c t + 3000t] \quad \text{--- (2)}$$

Comparing eqn (1) & (2)

$$k_p m(t) = 3000 t$$

$$\because k_p = 1000 \Rightarrow 1000 m(t) = 3000 t$$

$$m(t) = 3t$$

Ans.;  $|t| \leq 1$

ii) If  $s(t) = FM$  signal;  $k_f = 1000 \text{ rad/s/V}$ , To find  $m(t)$

General eqn of FM signal is:-

$$s(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] \quad \text{--- (3)}$$

Given eqn:  $s(t) = 10 \cos [\underbrace{(10000 + 3000)}_{w_c} t]$

$$s(t) = 10 \cos [2\pi f_c t + 3000 t] \quad \text{--- (4)}$$

Comparing eqns (3) and (4), we get

$$2\pi k_f \int_0^t m(\tau) d\tau = 3000 t$$

$$\Rightarrow 2f \times \left(\frac{1000}{2\pi}\right) \int_0^t m(\tau) d\tau = \frac{3}{3000} t \quad k_f = \frac{1000}{2\pi} \text{ Hz/V}$$

$$\Rightarrow \int_0^t m(\tau) d\tau = 3t$$

→ differentiating both sides wrt 't'

$m(t) = 3$

;  $H \leq 1$

Ans

(12)

- 1 (d) Two continuous random variables X and Y are related as,  $Y = aX + b$ . If 'a' and 'b' are positive constants, then derive the relation between the differential entropies of the two random variables.

Given : 
$$Y = ax + b \quad ; \quad a > 0, b > 0$$
 [12 marks]

To find : Relation between Differential entropies of X and Y

steps : Differential Entropy of a continuous random variable is given by :

$$H(X) = \int_{-\infty}^{\infty} f_X(x) \log_2 \frac{1}{f_X(x)} dx$$

where  $X \rightarrow$  continuous random variable

Now : Similarly :

$$H(Y) = \int_{-\infty}^{\infty} f_Y(y) \log_2 \frac{1}{f_Y(y)} dy. \quad \text{--- (1)}$$

Given  $Y = ax + b$ .

we know:

$$f_Y(y) = f_X(x) \cdot \left| \frac{dx}{dy} \right| \quad \text{--- (2)}$$

using eq<sup>n</sup>-② in eq<sup>n</sup>-①

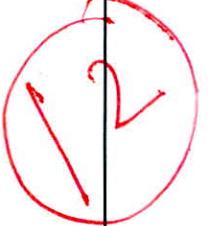
$$H[Y] = \int_{-\infty}^{\infty} f_X(x) \left| \frac{dx}{dy} \right| \log_2 \left( \frac{1}{f_X(x) \left| \frac{dx}{dy} \right|} \right) dy \quad \textcircled{3}$$

$$\because Y = ax + b \Rightarrow \frac{dy}{dx} = a \Rightarrow \left| \frac{dx}{dy} \right| = \frac{1}{a} \quad \textcircled{4}$$

Substituting ④ in ③

$$\begin{aligned} H[Y] &= \int_{-\infty}^{\infty} f_X(x) \left( \frac{1}{a} \right) \cdot \log_2 \left( \frac{a}{f_X(x)} \right) (a dx) \\ &= \int_{-\infty}^{\infty} f_X(x) \left[ \log_2 a - \log_2 f_X(x) \right] dx \\ &= \underbrace{\log_2 a \int_{-\infty}^{\infty} f_X(x) dx}_{\text{equal to } 1} + \underbrace{\int_{-\infty}^{\infty} f_X(x) \log_2 \frac{1}{f_X(x)} dx}_{\text{Differential entropy of } X} \\ &\quad \text{By property of probability density function.} \end{aligned}$$

$$H[Y] = \log_2 a + H[X]$$

 Relation between differential entropies of random variables  $x$  and  $y$  is

$$H[Y] = H[X] + \log_2 a$$

Hence proved.

1 (e)

What are the advantages and disadvantages of delta modulation compared to PCM? With the help of a sketch, mention various noises associated with delta modulation. How will you overcome these noises?

Advantages of Delta Modulation over PCM [12 marks]

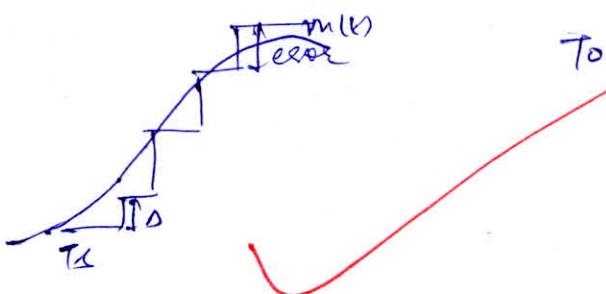
- ① No: of bits per symbol required is very less as in Delta Modulation  $n=1$  bit
- ② Bandwidth requirement is very less as compared to PCM
- ③ Quantization Error is very less in Delta Modulation

Disadvantage of Delta Modulation:

- ① Circuit becomes complex
- ② costlier circuit
- ③ Slope overload ~~error~~ & Granular Noises are the problem.

Various Noises Associated with Delta Modulation are:

- ① Slope overload error (when  $m(t)$  is fast varying signal)



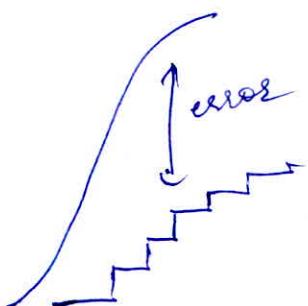
To minimize this

$$\left| \frac{dm(t)}{dt} \right|_{\max} \leq \frac{\Delta}{T_s}$$

$\Delta \rightarrow$  Step size

$T_s \rightarrow$  Sampling Interval

iv) Granular Noise: when  $m(t)$  is slow varying signal



To minimize this

$$\left| \frac{dm(t)}{dt} \right|_{\max} \gg \frac{\Delta}{T_s}$$

~~(\*)~~ → slope overload errors is  $\Rightarrow$  more harmful than granular noise  
 $\because$  it affects at low frequency.



to avoid

2 (a)

Two random variables  $X$  and  $Y$  are independent and identically distributed, each with a Gaussian density function with mean equal to zero and variance equal to  $\sigma^2$ . If these two random variables denote the coordinates of a point in the plane, find the probability density function of the magnitude and the phase of that point in polar coordinates.

[20 marks]





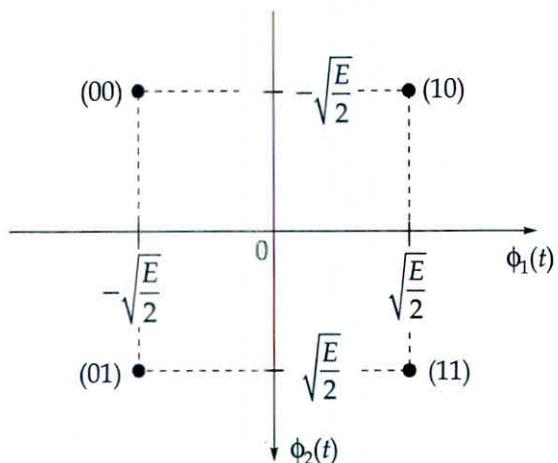
**Q.2 (b)**

A double conversion superheterodyne receiver is designed with  $f_{IF(1)} = 30 \text{ MHz}$  and  $f_{IF(2)} = 3 \text{ MHz}$ . Local oscillator frequency of each mixer stage is set at the lower of the two possible values. When the receiver is tuned to a carrier frequency of 300 MHz, insufficient filtering by the RF and first IF stages results in interference from three image frequencies. Determine those three image frequencies.

**[15 marks]**



**Q.2 (c)** Consider the signal-space diagram of a coherent QPSK system as shown in the figure below:



$\phi_1(t)$  and  $\phi_2(t)$  are two orthonormal basis functions, which are represented as,

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t); \quad 0 \leq t \leq T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t); \quad 0 \leq t \leq T$$

All the four message symbols are occurring with equal probability and they are transmitted through an AWGN channel with two-sided noise power spectral density of  $\frac{N_0}{2}$ . Suggest a receiver model to reproduce the symbols at channel output and derive an expression for the probability of symbol error.

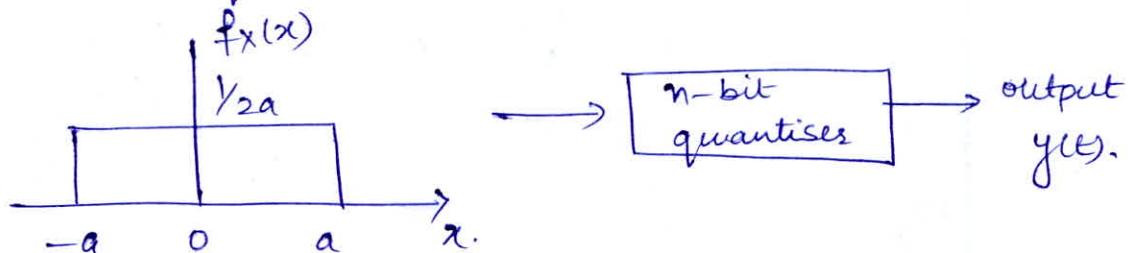
[25 marks]



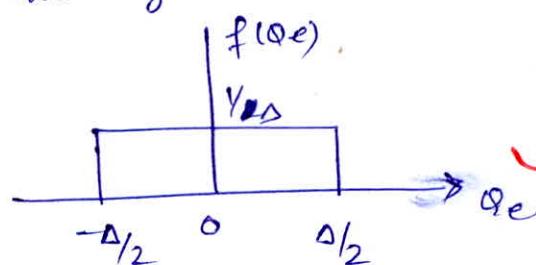


- .3 (a) The samples of a stationary random process  $X(t)$ , whose amplitude is uniformly distributed in the range  $[-a, a]$ , are applied to an  $n$ -bit uniform mid-riser quantizer. Derive an expression for the signal-to-quantization noise ratio at the output of the quantizer, with suitable assumptions. Using the expression obtained, find the signal-to-quantization noise ratio for an 8-bit quantizer.

Soln) Given:  $X(t)$  is uniformly distributed in range  $[-a, a]$  & applied to  $n$ -bit uniform mid-riser quantizer. [20 marks]



Let Quantizer characteristic is uniform: (given)



Signal to Noise Ratio at op:

$$\begin{aligned}
 \text{Signal Power} : &= E[X^2(t)] \\
 &= \int_{-\infty}^{\infty} x^2(t) \cdot f_X(x) dx \\
 &= \int_{-a}^{a} \left(\frac{1}{2a}\right) \cdot x^2(t) dx \\
 &= \left(\frac{1}{2a}\right) \left|\frac{x^3}{3}\right|_{-a}^a \\
 &= \frac{1}{2a} \times \frac{1}{3} \times (a^3 + a^3) \\
 P_S &\Rightarrow \boxed{\frac{a^2}{3}}
 \end{aligned}$$

Quantization Noise Power :  $P_{NQ} = E[Qe^2]$

$$P_{NQ} = \int_{-\Delta/2}^{\Delta/2} Qe^2 f(Qe) dQe = \int_{-\Delta/2}^{\Delta/2} Qe^2 \left(\frac{1}{2\Delta}\right) dQe$$

$$= \frac{1}{2\Delta} \cdot \left| \frac{Qe^3}{3} \right|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{2\Delta} \left( \frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right) = \frac{\Delta^3/4}{2\Delta}$$

$$\boxed{P_{NQ} = \frac{\Delta^2}{12}}$$

$$\Rightarrow \boxed{P_{NQ} = \frac{\Delta^2}{12}}$$

∴ Signal to Quantization Noise Ratio is

$$SNR = \frac{P_s}{P_{NQ}} = \frac{a^2/3}{\Delta^2/12} = 4 \left( \frac{a}{\Delta} \right)^2$$

where  $\Delta = \frac{V_{pp}}{2^n} = \frac{(2a)}{2^n}$

$$\therefore SNR = 4 \left( \frac{a \cdot 2^n}{2a} \right)^2$$

$$\boxed{SNR = 2^{2n}}$$

$$\begin{aligned} SNR \text{ in dB} &= 10 \log_{10} 2^{2n} \\ &= 20n \log_{10} 2 = 6.02n \text{ dB} \end{aligned}$$

Now : using this Relation,

$SNR$  of  $n=8$  bit quantizer is :

$$S N_q R = 2^{2 \times 8} = \underline{\underline{2^16}}$$

$$(S N_q R) \text{ in dB} = 10 \log_{10} (2^{16}) \\ = 160 \log_{10} 2 \\ = \underline{\underline{48.164}} \text{ dB}$$

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- .3 (b) A binary channel matrix is given by,

		Outputs	
		$y_1$	$y_2$
Inputs	$x_1$	$\frac{2}{3}$	$\frac{1}{3}$
	$x_2$	$\frac{1}{10}$	$\frac{9}{10}$

If  $P(x_1) = 1/3$  and  $P(x_2) = 2/3$ , then determine:  $H(x)$ ,  $H(x|y)$ ,  $H(y)$ ,  $H(y|x)$  and  $I(x;y)$

$P[x] = \left[ \begin{array}{cc} \frac{1}{3} & \frac{2}{3} \end{array} \right]$ ,  $P\left[\frac{y}{x}\right] = \begin{matrix} y_1 & y_2 \\ x_1 & \left[ \begin{array}{cc} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{array} \right] \\ x_2 & \left[ \begin{array}{cc} y_{11} & y_{12} \\ y_{21} & y_{22} \end{array} \right] \end{matrix}$  [20 marks]

$$H[X] = - \sum_{i=1}^2 P(x_i) \log_2 p(x_i)$$

$$= \frac{1}{3} \log_2 3 + \frac{2}{3} \log_2 1.5$$

$$\boxed{H[X] = \frac{1}{3} [1.584 + 1.1699] = \underline{\underline{0.917}} \text{ bits per symbol.}}$$

$$P[X, Y] = P[X] \cdot P\left[\frac{Y}{X}\right]$$

$$= \left[ \begin{array}{cc} \frac{1}{3} & 0 \\ 0 & \frac{2}{3} \end{array} \right] \cdot \left[ \begin{array}{cc} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{array} \right] = \left[ \begin{array}{cc} \frac{2}{9} & \frac{1}{9} \\ \frac{1}{15} & \frac{3}{5} \end{array} \right]$$

$$\therefore P\left[\frac{X}{Y}\right] = \frac{P(X, Y)}{P(Y)_d} = \begin{bmatrix} 0.769 & 0.022824 \\ 0.23 & 0.977176 \end{bmatrix}$$

$$P[Y] = P[X] \cdot P\left[\frac{Y}{X}\right] = \left[\frac{1}{3} \quad \frac{2}{3}\right] \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$$

$$P[Y] = \left[ \frac{\frac{2}{9} + \frac{1}{15}}{\frac{13}{45}} \quad \frac{\frac{1}{9} + \frac{3}{5}}{\frac{32}{45}} \right]$$

$$P[Y] = \left[ \frac{13}{45} \quad \frac{32}{45} \right]$$

$$\begin{aligned} H[Y] &= \sum_{j=1}^2 P[y_j] \log_2 P[y_j] \\ &= \frac{13}{45} \log_2 \left( \frac{45}{13} \right) + \frac{32}{45} \log_2 \left( \frac{45}{32} \right) \\ &= 0.5175 + 0.349 \end{aligned}$$

$$H[Y] = 0.8665 \text{ bits/symbol}$$

$$\begin{aligned} \therefore H\left[\frac{X}{Y}\right] &= - \sum_{j=1}^2 \sum_{k=1}^3 P(x_j, y_k) \log_2 P\left(\frac{x_j}{y_k}\right) \\ &= - \left[ \frac{2}{9} \log_2 (0.769) + \frac{1}{9} \log_2 (0.1562) \right. \\ &\quad \left. + \frac{1}{15} \log_2 (0.23) + \frac{2}{5} \log_2 (0.343) \right] \end{aligned}$$

$$= - [-0.0842 - 0.2976 - 0.1413 - 0.737]$$

$$H\left[\frac{X}{Y}\right] = 1.2601 \text{ bits/symbol}$$

$$\begin{aligned}
 H\left[\frac{Y}{X}\right] &= - \sum_{j=1}^2 \sum_{k=1}^2 p(x_j, y_k) \log_2 p\left(\frac{y_k}{x_j}\right) \\
 &= - \left[ \frac{2}{9} \log_2 (2/3) + \frac{1}{9} \log_2 (1/3) \right. \\
 &\quad \left. + \frac{1}{15} \log_2 (1/10) + \frac{3}{5} \log_2 (9/10) \right] \\
 &= - [-0.1299 - 0.1761 - 0.221 - 0.0912]
 \end{aligned}$$

$\boxed{H\left[\frac{Y}{X}\right] = 0.6182 \text{ bits/symbol}}$

$$I(X;Y) = H[Y] - H\left[\frac{Y}{X}\right]$$

$$= \cancel{0.917}$$

$$= 0.8665 - 0.6182$$

$I(X;Y) = \underline{\underline{0.2483}} \text{ bits/symbol}$

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- Q.3 (c)**
- In a DSBSC system, the message signal  $m(t)$  is multiplied with the carrier signal  $c(t) = 4\cos(2\pi f_c t)$  to form a modulated signal  $s(t)$ . If  $m(t) = 2\text{sinc}(2t) - \text{sinc}^2(t)$  and  $f_c = 100$  Hz, then determine and sketch the spectrum of the modulated signal  $s(t)$ . Assume that,  $\text{sinc}(t) = (\sin \pi t) / \pi t$ .
  - The spectrum of the message signal  $m(t)$  is shown below in Figure (a). This signal is processed by the system shown below in Figure (b).

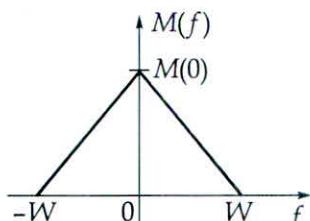


Figure (a)

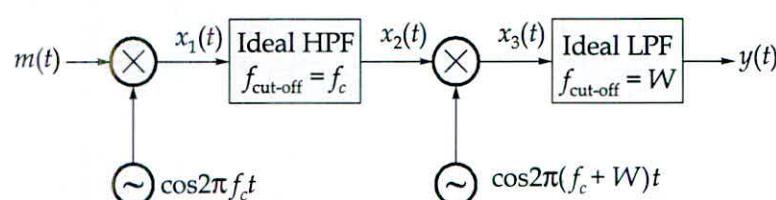


Figure (b)

If each filter has a passband gain of 1, then determine and sketch the spectrum of the output signal  $y(t)$ . Assume that  $f_c \gg W$ .

[8 + 12 marks]

Sol) Given DSBSC system:

$$m(t) = 2\text{sinc}(2t) - \text{sinc}^2(t)$$

$$c(t) = 4\cos(2\pi f_c t) \quad ; \quad f_c = 100 \text{ Hz}$$

DSBSC modulated signal is given by:

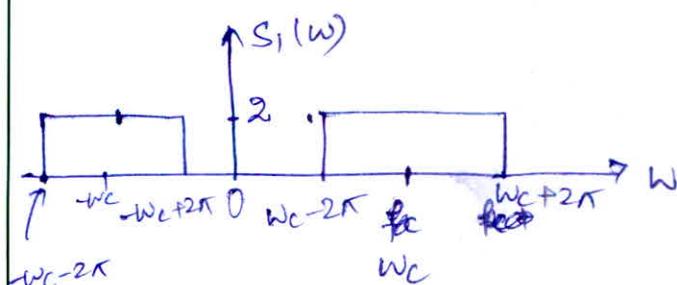
$$s(t) = m(t) \cdot c(t)$$

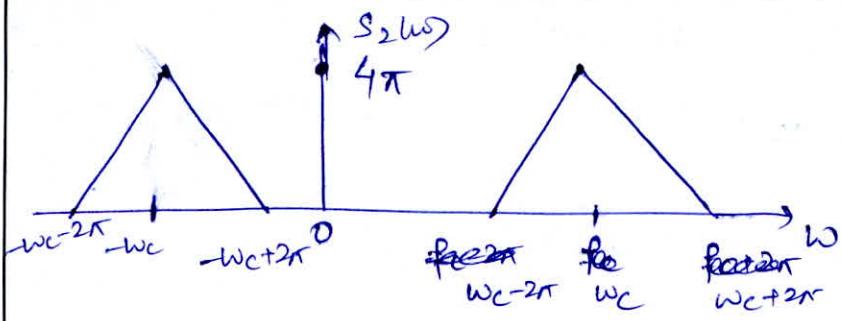
$$= [2\text{sinc}(2t) - \text{sinc}^2(t)] \cdot [4\cos(2\pi f_c t)]$$

$$= 8 \frac{\sin 2\pi t}{2\pi t} \cos(2\pi f_c t) - 4\text{sinc}^2(t) \cos 2\pi f_c t$$

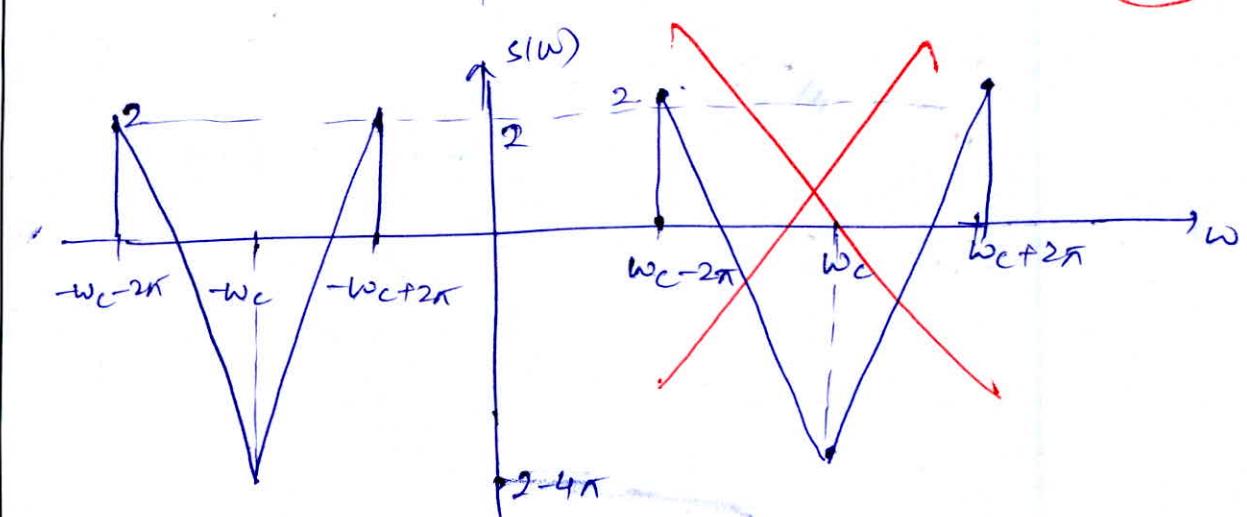
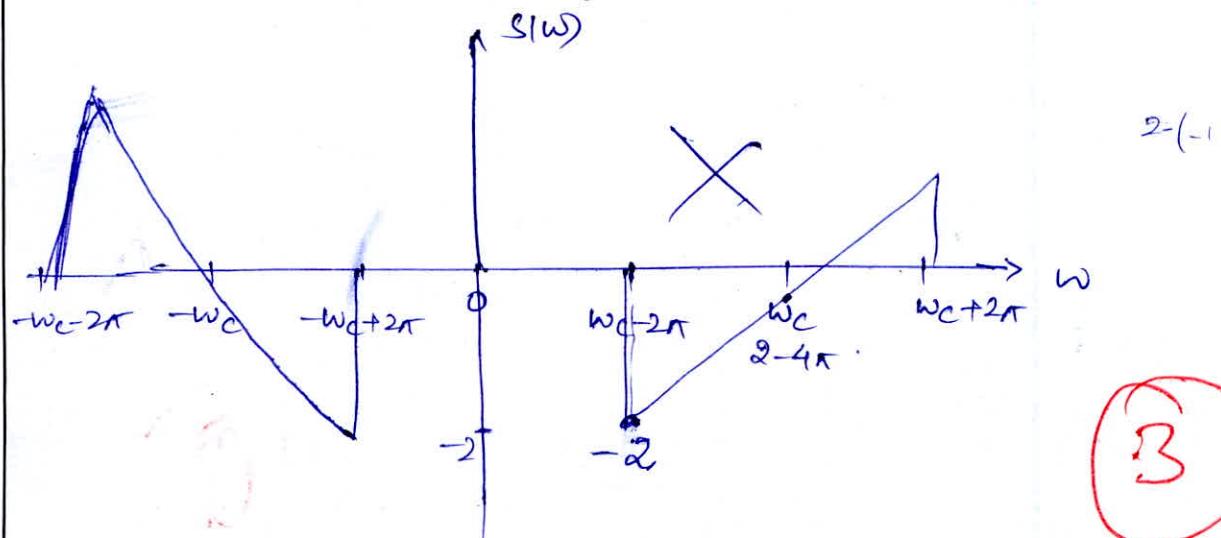
$$s(t) = \underbrace{4 \frac{\sin(2\pi t)}{\pi t} \cos 2\pi f_c t}_{s_1(t)} - \underbrace{4 \left(\frac{\sin^2 \pi t}{\pi t}\right)^2 \cos 2\pi f_c t}_{s_2(t)}$$

Spectrum of  $s_1(t)$  &  $s_2(t)$  are as:



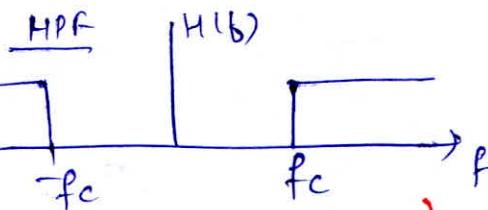
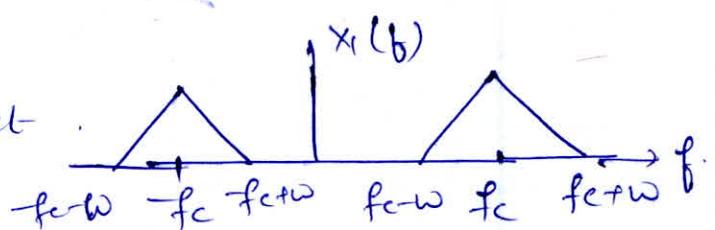


∴ overall spectrum of  $s(t)$  is  $S_1(\omega) - S_2(\omega)$

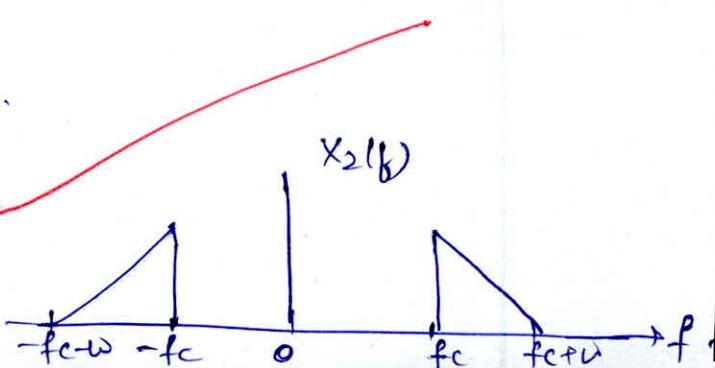


Q8

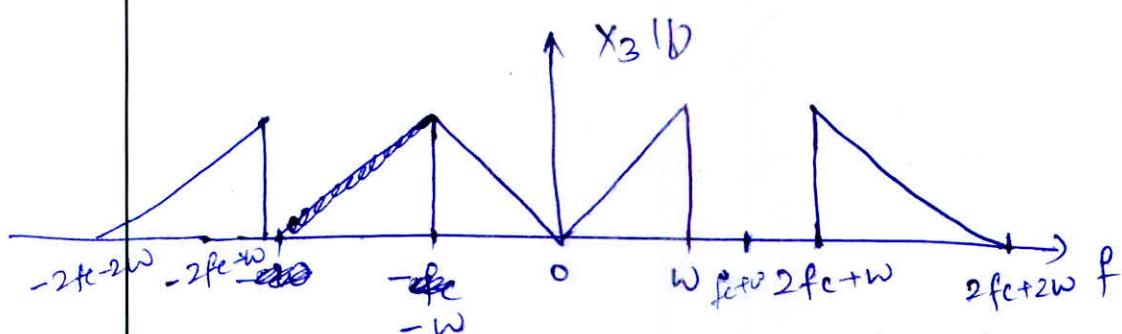
$$x_1(t) = m(t) \cdot \cos 2\pi f_c t$$



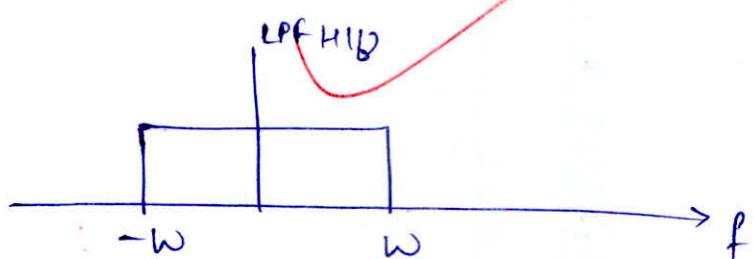
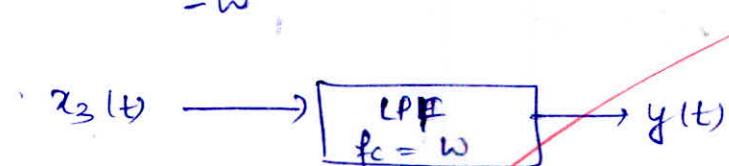
$$x_2(t) = x_1(t) * h(t)$$



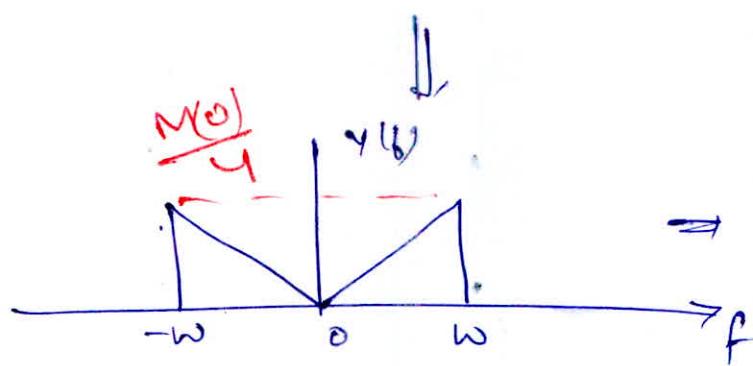
$$x_3(t) = x_2(t) \cdot \cos 2\pi (f_c + w) t$$



$f_c$	$\omega$
$-f_c - w$	$-f_c - \omega$
$-w$	$-f_c$



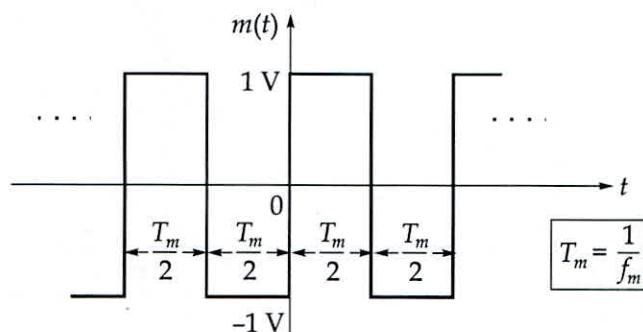
10



⇒ spectrum of  
output  
signal  $y(t)$

1.4 (a)

The periodic message signal  $m(t)$  shown in the figure below is applied to a phase modulator to modulate the carrier signal  $c(t) = \cos(2\pi f_c t)$ . If the phase sensitivity of the phase modulator is  $k_p = 1 \text{ rad/V}$ , then determine and sketch the spectrum of the modulated signal.



[25 marks]



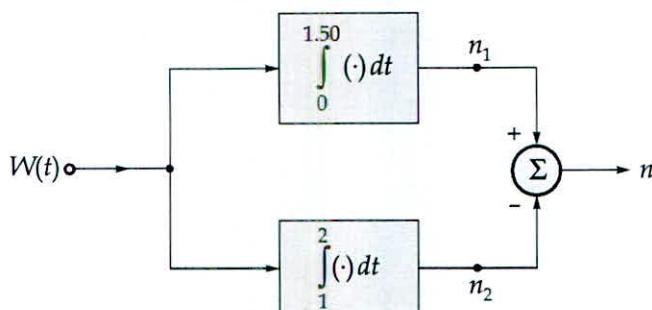


- Q.4 (b)**
- (i) A binary data is transmitted through an ideal AWGN channel with infinite bandwidth. The two sided power spectral density of the noise is  $\frac{N_0}{2}$ . If the average energy transmitted per bit is  $E_b$ , then derive the condition to be satisfied for error free transmission.
- (ii) A binary signal is transmitted through an ideal AWGN channel with infinite bandwidth. The two-sided PSD of the channel noise is  $7 \mu\text{W}/\text{Hz}$ . By using the condition obtained in part (i), determine the minimum average bit energy required for error-free transmission.

[12 + 3 marks]



- Q.4 (c) A zero mean white Gaussian noise  $W(t)$  is processed by the section of a receiver shown below.



If the two-sided noise power spectral density of the input white Gaussian noise  $W(t)$  is  $\frac{N_0}{2} = 1 \text{ W/Hz}$ , then determine the variance of the corresponding output random variable "n".

[20 marks]



**Section B : Network Theory-1 + Microprocessors and Microcontroller-1  
+ Digital Circuits-2 + Control Systems-2**

Q.5 (a)

Design a J-K flip-flop using a D flip-flop and a  $4 \times 1$  MUX. Write various steps involved in the process.

Ans: To design : JK flip flop using D-flip flop [12 marks] and  $4 \times 1$  MUX.

Steps : ① Write the characteristic Table of Required flip flop.

- ② From the characteristic Table, write excitation Table for D-flip flop i.e. D-inputs
- ③ Solve K-map for D-inputs  
we get  $D = \sum m(1, 4, 5, 6)$
- ④ Now Design  $D = \sum m(1, 4, 5, 6)$  using  $(4 \times 1)$  MUX using Implementation Table.
- ⑤ Draw the circuit.

Characteristic Table : Excitation Table

J	K	Q	$Q^+$	D
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

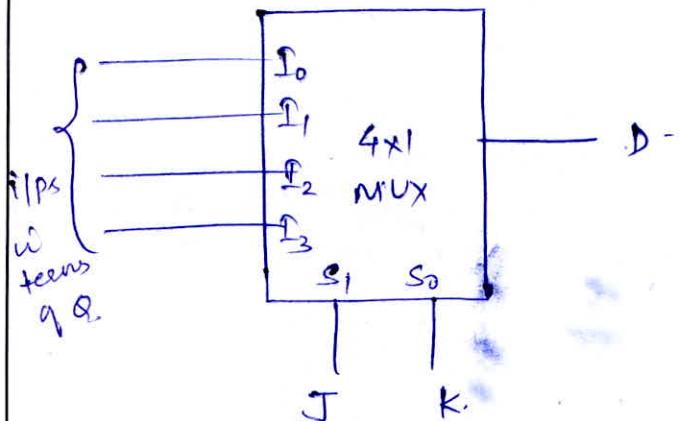
Characteristic  
Table -

Excitation Table for D-FF		
$Q$	$Q^+$	D
0	0	0
0	1	1
1	0	0
1	1	1

∴  $Q^+ = D$

Now :  $D = \sum m(1, 4, 5, 6)$

Now Designing  $D = \sum m(1, 4, 5, 6)$  using  $4 \times 1$  MUX



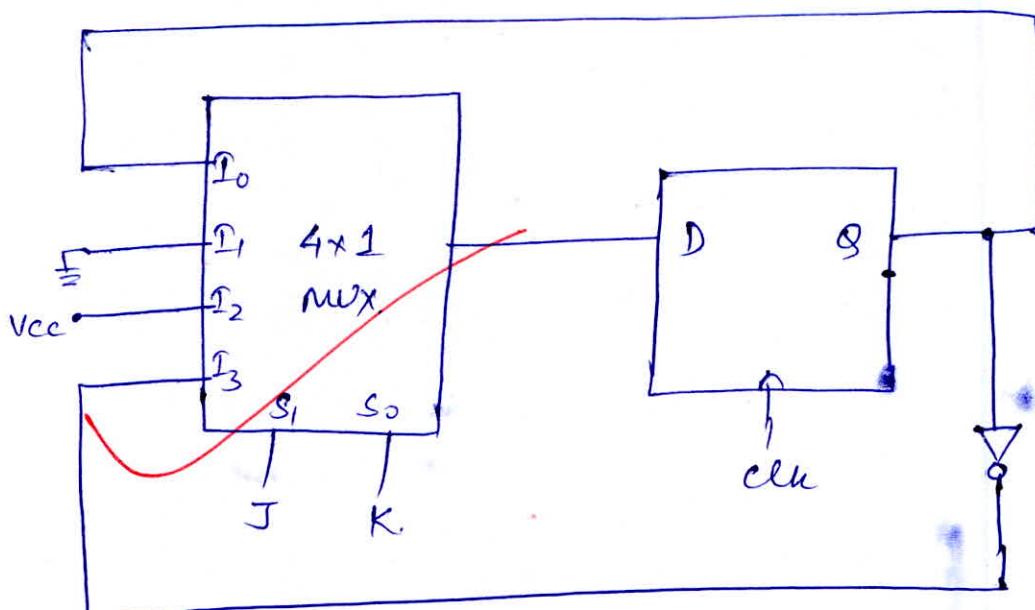
Implementation Table :

$$\begin{aligned} I_0 &= Q \\ I_1 &= 0 \\ I_2 &= 1 \\ I_3 &= \bar{Q} \end{aligned}$$

$I_0$	$I_1$	$I_2$	$I_3$
$\bar{Q}$	0	2	4
Q	1	3	5
	0	1	7

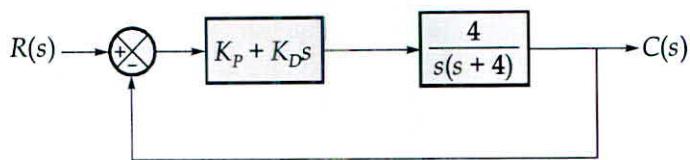
12

Circuit Diagram :



Q.5 (b)

A control system with PD controller is shown below:



Determine the value of  $K_p$  and  $K_D$  such that the damping ratio of the system will be 0.75 and the steady state error for unit ramp input will be 0.25.

Ans

The transfer function of given system is [12 marks]

$$\frac{C(s)}{R(s)} = \frac{(K_p + K_D s) \left( \frac{4}{s(s+4)} \right)}{1 + (K_p + K_D s) \left( \frac{4}{s(s+4)} \right)}$$

$$\frac{C(s)}{R(s)} = \frac{4(K_p + K_D s)}{s(s+4) + 4(K_p + K_D s)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{4(K_p + K_D s)}{s^2 + (4 + 4K_D)s + 4K_p}} \quad \textcircled{1}$$

Required  $\xi$  of system = 0.75.

Comparing denominator of Transfer function with standard characteristic eq<sup>n</sup>

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad \textcircled{2}$$

To eq we get

$$\boxed{\begin{aligned} \omega_n^2 &= 4K_p & \textcircled{A} \\ 2\xi\omega_n &= 4 + 4K_D \end{aligned}}$$

Now put  $\xi = \frac{3}{4}$  in eq<sup>n</sup> B and eq<sup>n</sup> A in eq<sup>n</sup> B

$$2 \times \frac{3}{4} \times 2\sqrt{K_p} = 4 + 4K_D$$

$$\boxed{3\sqrt{K_p} = 4 + 4K_D} \quad \textcircled{C}$$

Given  $\text{ess}$  for unit Ramp input =  $1/4$ .

We know  $\text{ess} = \frac{1}{K_V}$ ,  $K_V = \lim_{s \rightarrow 0} s \cdot G(s) H(s)$

$$G(s) H(s) = \frac{4(K_p + K_D s)}{s^2 + (4 + 4K_D)s + 4K_p - 4K_p - 4K_D s} = \frac{4(K_p + K_D s)}{s(s+4)}$$

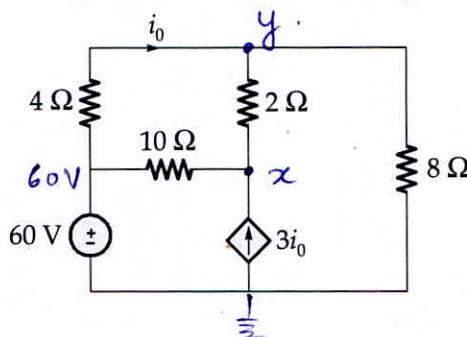
$$\therefore K_V = \lim_{s \rightarrow 0} s \cdot \frac{4(K_p + K_D s)}{s(s+4)} = K_p \quad \boxed{K_p = 4} \quad \text{--- (D)}$$

$$\therefore \text{ess} = \frac{1}{K_p} = \frac{1}{4} \Rightarrow \boxed{K_D = 1/2} \quad \text{--- (D)}$$

put (D) in (C)

$$3\sqrt{K_p} = 4 + 4K_D \Rightarrow K_D = \frac{3\sqrt{4} - 4}{4} = \frac{1}{2} \Rightarrow \boxed{K_D = 1/2}$$

5(c) Find the current  $i_0$  in the circuit shown below using nodal analysis.



[12 marks]

Assume Nodes x and y :

KCL at x :  $\frac{x-60}{10} + \frac{x-y}{2} - 3i_0 = 0$

$$\frac{x-60 + 5x - 5y - 30i_0}{10} = 0$$

$$\Rightarrow \boxed{6x - 5y - 30i_0 = 60} \quad \text{--- (1)}$$

KCL at y :  $\frac{y}{8} + \frac{y-x}{2} + \frac{y-60}{4} = 0$

$$\frac{y + 4y - 4x + 2y - 120}{8} = 0$$

$$\boxed{7y - 4x - 120 = 0} \quad \text{--- (2)}$$

from eqn:

$$i_0 = \frac{60-y}{4} \quad \text{--- (3)}$$

put (3) in eqn-①,  $6x - 5y - 30 \left( \frac{60-y}{4} \right) = 60$

$$12x - 10y - 900 + 15y = 120$$

$$\Rightarrow \boxed{12x + 5y = 1020} \quad \text{--- (4)}$$

from eqn ②

$$\boxed{4x - 7y = -120} \quad \text{--- (2) } \times 3$$

solving eqn's ② and ④

$$12x + 5y = 1020$$

$$\begin{array}{r} 12x - 21y = -360 \\ \hline \end{array}$$

$$26y = 1380$$

$$\boxed{y = \frac{1380}{26} = 53.07 \text{ V}}$$

$$x = \frac{7 \times \frac{1380}{26} - 120}{4}$$

$$\boxed{x = 62.87 \text{ V}}$$

(2)

$$\therefore i_0 = \frac{60-y}{4} = \frac{60-53.07}{4}$$

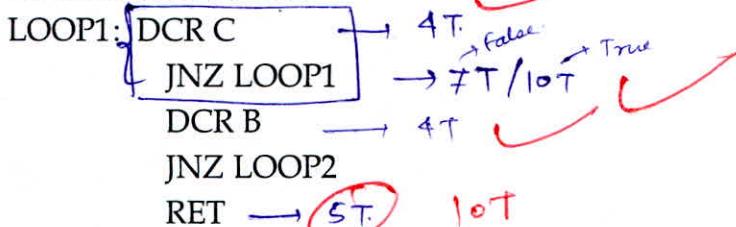
$$\boxed{i_0 = 1.7325 \text{ A}}$$

Ans.

5 (d) Calculate the delay produced by the following subroutine program of an 8085 microprocessor, which is operating with a clock frequency of 2 MHz.

DELAY : MVI B, 02H

LOOP2: MVI C, FFH



Delay Produced by Loop 1 :

[12 marks]

DCR C      }    executed 255 times.  
JNZ Loop1

DCR B

JNZ Loop2

Total Delay for  $B = 2$   
 $\xrightarrow{\text{times}} \text{Loop 1} \rightarrow 255 \text{ times True} + 1 \text{ time False.}$   
 $\xrightarrow{\text{2 times}} \frac{B=1}{4T} : ?$

Total Delay =

$$= 2 \times 255 (14T) + 2 (11T) + 14T + 7T + 5T + 8T + 4T.$$

$$= (510 \times 14T) + 22T + 21T + 17T$$

$$= 7140T + 60T$$

$$= 7200T \quad 7190T$$

$$= 7200 \times \frac{1}{2 \times 10^6}$$

$$= 36 \times 10^{-6} \text{ sec}$$

$$= \underline{\underline{3.6 \text{ msec}}}$$

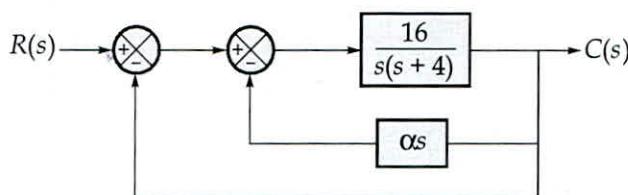
Q.5 (e)

Sketch the internal block diagram of an 8086 microprocessor.

[12 marks]

5(a)

The following figure shows a unity feedback control system with rate feedback loop.



Determine:

- The peak overshoot of the system for unit step input and the steady state error for unit ramp input in the absence of rate feedback.
- The rate feedback constant 'α' which will decrease the peak overshoot of the system for unit step input to 1.25%. What is the steady state error to unit ramp input with this setting.
- Illustrate how in the system with rate feedback, the steady state error to unit ramp input can be reduced to the same level as in part (i) while the peak overshoot to unit step input is maintained at 1.25%.

UN)

i) Absence of Rate Feedback: it becomes [7 + 8 + 10 marks]  
standard 2nd order system  
w/o FB

$$\frac{C(s)}{R(s)} = \frac{\frac{16}{s(s+4)}}{1 + \frac{16}{s(s+4)}} = \frac{16}{s^2 + 4s + 16} \quad \text{--- (1)}$$

Comparing with transfer function of std. 2nd order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{--- (2)}$$

we get  $\boxed{\omega_n = 4 \text{ rad/sec}}$ ,  $\boxed{2\zeta\omega_n = 4}$

$$\zeta = \frac{4}{2 \times 4} = 0.5$$

Peak overshoot :

$$\begin{aligned} M_p &= e^{-\pi \zeta / \sqrt{1-\zeta^2}} \\ &= e^{-\pi \times 0.5 / \sqrt{1-(1/2)^2}} \\ &= e^{-1.0813} = 0.163 \end{aligned}$$

P

Steady state error for unit Ramp Input

$$e_{ss} = \frac{1}{K_V}, \quad K_V = \lim_{s \rightarrow 0} s \cdot G(s) H(s)$$

$$G(s) H(s) = \frac{16}{s^2 + 4s} \quad \therefore K_V = \lim_{s \rightarrow 0} s \left( \frac{16}{s(s+4)} \right)$$

$\boxed{K_V = 4}$

$$\therefore \boxed{e_{ss} = \frac{1}{K_v} = \frac{1}{4} = 0.25}$$

Q8

with Rate feedback present :-

$$\frac{C(s)}{R(s)} = \frac{\left( \frac{16/s(s+4)}{1 + \frac{16}{s(s+4)} (\alpha s)} \right)}{1 + \left[ \frac{\frac{16/s(s+4)}{1 + \frac{16}{s(s+4)} (\alpha s)}}{} \right]} = \frac{\frac{16}{s^2 + 4s + 16\alpha s}}{1 + \frac{16}{s^2 + 4s + 16\alpha s}}$$

$$\boxed{\frac{e(s)}{R(s)} = \frac{16}{s^2 + 4s + 16\alpha s + 16} = \frac{16}{s^2 + (16\alpha + 4)s + 16}}$$

comparing with std. 2nd order system equation

$$\boxed{\omega_n = 4 \text{ rad/sec}} \quad \boxed{2\xi' \omega_n = 16\alpha + 4} \rightarrow \textcircled{A}$$

(B)

Now  $\because \% M_p$  reduced to  $1.25\%$   
 $M_p = 0.0125 \Rightarrow e^{-\pi \xi' \sqrt{1-(\xi')^2}}$

$$-4.382 = \frac{-\pi \xi'}{\sqrt{1-(\xi')^2}}$$

$$(\xi')^2 = (1-(\xi')^2) 1.945$$

$$(\xi')^2 = 1.945 - 1.945(\xi')^2$$

$$2.945(\xi')^2 = 1.945$$

$$\boxed{\xi' = 0.812}$$

put  $\xi'$  in eqn  $\textcircled{A}$ 

$$\therefore 2 \times 0.812 \times 4 = 16\alpha + 4$$

$$\Rightarrow \boxed{\alpha = 0.156} //$$

$$e_{ss} = \frac{1}{K_V}$$

$$K_V = \lim_{s \rightarrow 0} s \cdot \frac{16}{s + (16\zeta + 4)} = \frac{16}{16\zeta + 4}$$

$$e_{ss} = \frac{16(0.25 + 4)}{16} = \frac{16 \times 4.25}{16} = 4.25$$

$$= \frac{16 \times 0.156 + 4}{16} = 0.406$$

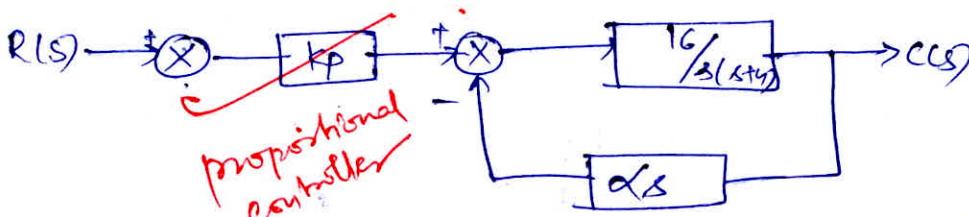
$$K_V = \frac{16}{(16\zeta + 4)}$$

$$\Rightarrow e_{ss} = 0.406$$

(8)

ii) Maintaining peak overshoot same at  $1.25\%$  but reducing  $e_{ss}$  to same as that in part i) ie  $e_{ss} = 0.25$ , we can use a

P-D controller : Gain of the system has to be increased.



(1)

$$\therefore e_{ss} = \frac{1}{K_V} = \frac{16\zeta + 4}{16} = 0.25$$

$$\zeta = 0$$

Derivative feedback has to be removed but can't be removed  $\because$  it improves  $\zeta$ ,

thereby reducing  $M_p$ .

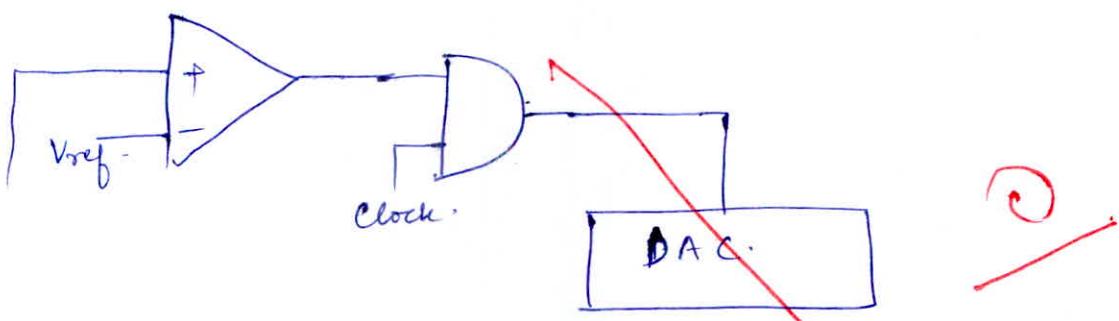
So Increase Gain to reduce  $e_{ss}$ .

$\therefore$  P+D controller is used.

- Q.6 (b) (i) Explain with a block diagram, the working principle of a dual-slope A/D converter. Derive the expression for the output and maximum conversion time of the circuit.
- (ii) A dual-slope A/D converter has a resolution of 4 bits. If the clock rate is 3.2 kHz, then calculate the maximum sampling rate with which the samples can be applied to the A/D converter.

[15 + 5 marks]

Dual Slope A/D Converter :





Q8

$$\text{Resolution} = 4 \text{ bits} = n$$

$f_{\text{clk}} = \text{clock rate} = 3.2 \text{ kHz}$   
To find Max<sup>m</sup> sampling Rate

$$T_{\text{conversion}} = 2^{n+1} T_{\text{clk}} \leq T_s.$$

↳ Sampling Time

$$\Rightarrow 2^5 \times \frac{1}{3.3 \times 10^{-3}} \leq T_s$$

$$\Rightarrow T_s \geq \frac{3.2 \times 10^{-3}}{3.3}$$

$T_s \geq 9.696 \text{ msec}$

Max<sup>m</sup> Sampling Rate: ( $f_s$ )

$$\frac{1}{f_s} \geq 9.696 \text{ msec}$$

$$\Rightarrow f_s \leq \frac{1}{9.696} \times 10^3 \text{ Hz}$$

(H)

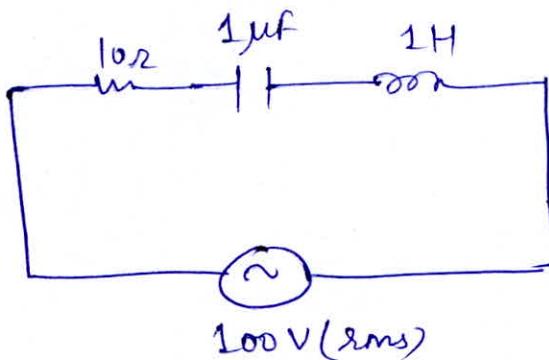
$$\Rightarrow f_{s \text{ max}} = 0.103 \text{ kHz}$$

$f_{s \text{ max}} = 103 \text{ Hz}$

6(c)

A circuit is made up of a  $10\ \Omega$  resistance, a  $1\ \mu F$  capacitance and  $1\ H$  inductance all connected in series. A sinusoidal voltage of  $100\ V$  (rms) at varying frequencies is applied to the circuit. Find the frequency at which the circuit would consume only  $10\%$  of the power it consumed at resonance?

[15 marks]



To find :  $f$  at which  $10\%$  of Power consumed at resonance.

$$\text{At resonance} : I = \frac{V}{R} = \frac{100}{10} = 10\ A$$

$$\begin{aligned} \text{Power in circuit} &= I^2 R \\ &= \cancel{10^2 \times 10} = \underline{\underline{1000\ W}} \end{aligned}$$

Power reqd =  $10\%$  of Power at Resonance

$$= \frac{10}{100} \times 1000$$

$$= \underline{\underline{100\ W}}$$

$$P = I^2 (R + X_L + X_C)$$

$$X_L = 2\pi f \quad , \quad X_C = \frac{1}{2\pi f} \times 10^6$$

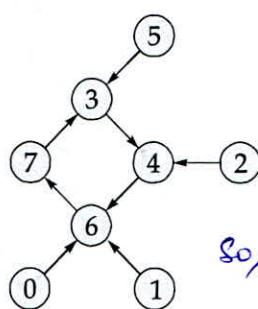
~~$$Z = 10 + j(2\pi f) - \frac{j}{2\pi f} \times 10^6$$~~

$$I^2(Z) = 100\ W$$



7(a)

Design a synchronous counter, whose sequence diagram is shown below, using D flip-flops.



States are from 0 to 7

So, Required No. of flip-flops =  $\log_2 8 = 3$

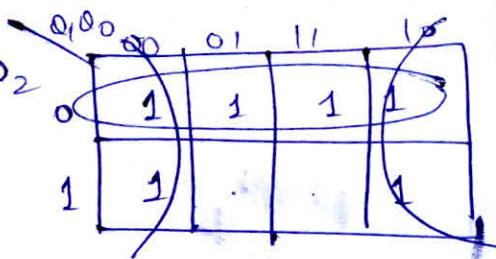
[20 marks]

11)

State Table :

Present State			Next State			flip flop inputs			
$Q_2$	$Q_1$	$Q_0$	$Q_2^+$	$Q_1^+$	$Q_0^+$	$= Q_2^+ = Q_1^+ = Q_0^+$	$D_2$	$D_1$	$D_0$
0	0	0	1	1	0	1	1	0	
0	0	1	1	1	0	1	1	0	
0	1	0	1	0	0	1	0	0	
0	1	1	1	0	0	1	0	0	
1	0	0	1	1	0	1	1	0	
1	0	1	0	1	1	0	1	1	
1	1	0	1	1	1	1	1	1	
1	1	1	0	1	1	0	1	1	

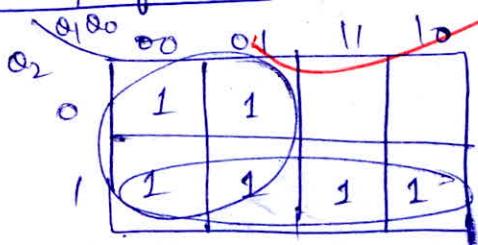
Now Kmap for  $D_2$



$$D_2 = \overline{Q_2} + \overline{Q_0}$$

$$= \overline{Q_2 \cdot Q_0}$$

K-map for  $D_1$ :



$$D_1 = Q_2 + \overline{Q_1}$$

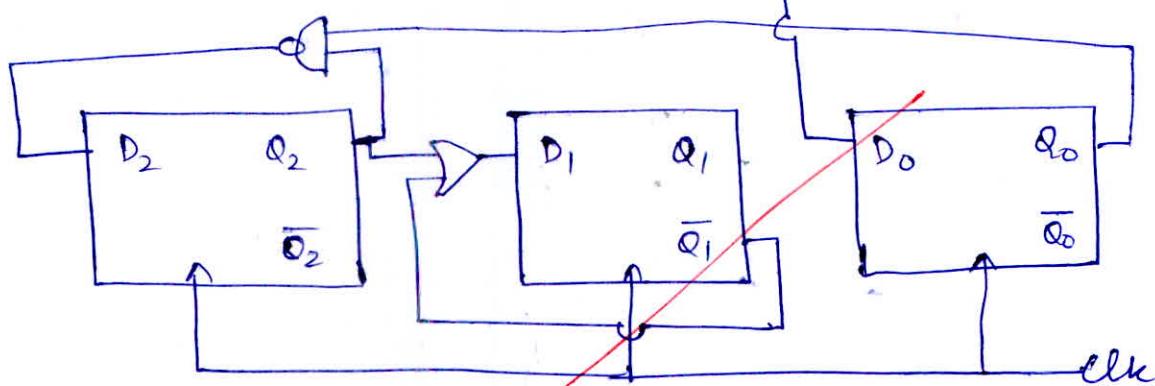
K-map for  $D_0$

$Q_1 Q_0$	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$D_0 = Q_2 Q_0 + Q_2 Q_1$$

$$D_0 = Q_2 (Q_0 + Q_1)$$

circuit diagram :



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7 (b)

A linear time invariant system is characterised by the homogeneous state equation,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i) Compute the solution of the homogeneous equation assuming the initial state vector

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(ii) Consider now the system has a forcing function and is represented by the following non-homogeneous state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where  $u$  is a unit step input function. Compute the solution of this equation assuming initial conditions of part (i).

$\rightarrow$  (i)  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad [10 + 10 \text{ marks}]$

Initial state vector  $\stackrel{A}{=} x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (\text{Given})$

Solution of Homogeneous state equation is given by

$$x(t) = \phi(t) \cdot x(0)$$

where  $\phi(t) = L^{-1}[(\delta I - A)^{-1}]$

so From given state equation, A matrix is

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$(\delta I - A) = \begin{bmatrix} \delta & 0 \\ 0 & \delta \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \delta-1 & 0 \\ -1 & \delta-1 \end{bmatrix}$$

$$\cancel{\phi(t)} = (\delta I - A)^{-1} = \frac{1}{(\delta-1)^2} \begin{bmatrix} \delta-1 & 0 \\ 1 & \delta-1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{(\delta-1)} & 0 \\ \frac{1}{(\delta-1)^2} & \frac{1}{(\delta-1)} \end{bmatrix}$$

$$\therefore \Phi(s) = L^{-1} \left[ (sI - A)^{-1} \right] = L^{-1} \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)} \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix},$$

$\therefore$  Solution of Homogeneous equation is :

$$x(t) = \Phi(t)x(0) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(t) = \boxed{\begin{bmatrix} 0 \\ e^t \end{bmatrix}}$$

Ans.

ii)

Now  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}_{2 \times 2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2 \times 1} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2 \times 1} u_{(1 \times 1)}$

initial condn :  $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Solution of Non-Homogeneous state equation is given by

$$x(t) = \underbrace{\Phi(t) \cdot x(0)}_{\Phi} + L^{-1} \left[ \Phi(s) B U(s) \right]$$

$\Phi$  same as computed in part i)

$$\Phi(s) = \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, U(s) = \begin{bmatrix} \frac{1}{s} \end{bmatrix}$$

$$\Phi(s) B U(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{(s+1)^2} \end{bmatrix}_{2 \times 2} \begin{bmatrix} 0 \\ \frac{1}{s} \end{bmatrix}_{2 \times 1}$$

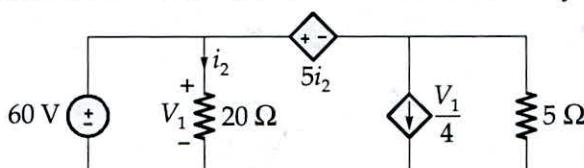
$$= \begin{bmatrix} 0 \\ \frac{1}{s(s+1)} \end{bmatrix}$$

(Q)

Soln is :

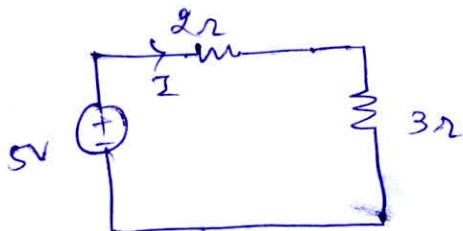
$$\begin{aligned} X(t) &= \cdot \begin{bmatrix} 0 \\ e^t \end{bmatrix} + L^{-1} \begin{bmatrix} 0 \\ \frac{1}{s(s+1)} \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ e^t \end{bmatrix} + \begin{bmatrix} 0 \\ (e^t - 1) \end{bmatrix} = \begin{bmatrix} 0 \\ 2e^t - 1 \end{bmatrix} \text{ Ans.} \end{aligned}$$

- (c) (i) State and explain the Tellegen's theorem.  
(ii) For the network shown below, show that it will satisfy Tellegen's theorem.



[8 + 12 marks]

Tellegen's Theorem states that in any network ~~the~~, total power delivered by the system is equal to the ~~the~~ total power absorbed/consumed by the system i.e. sum <sup>Total</sup> of all the powers absorbed/delivered in any network is zero.

Ex:

Consider a Network shown.

$$\therefore I = \frac{5}{2+3} = 1A_{\parallel}$$

$$\text{Power delivered by } 5V \text{ battery} = -5 \times 1 = -5W_{\parallel}$$

$$\text{Power absorbed by } 2\Omega \text{ resistance} = I^2 R$$

$$= (1)^2 \times 2 = \underline{\underline{2W}}$$

$$\text{Power absorbed by } 3\Omega \text{ resistance} = I^2 R$$

$$= (1)^2 \times 3 = \underline{\underline{3W}}$$

~~Total sum of Powers absorbed/delivered~~

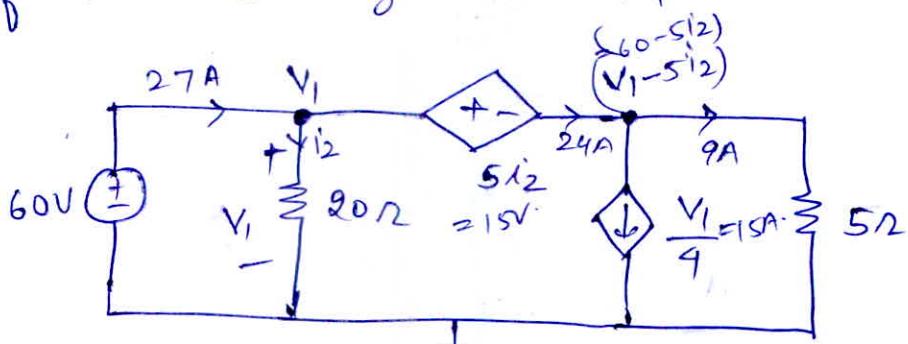
~~$= -5 + 2 + 3$~~

~~$= \underline{\underline{0W}}$~~

]} Tellegen's Theorem verified

Note:

Tellegen's Theorem is applicable for all kind of Networks viz active, passive, etc.



To show : given Network satisfies Tellegen's

Theorem :

Apply Nodal at  $V_1 = 60V$

$$i_2 = \frac{60}{20} = 3A$$

$$\text{Current in } 5\Omega = \frac{V_1 - 5i_2}{5} = \frac{60 - 5 \times 3}{5} = 9A$$

$$\therefore \text{Power delivered by } 60V = -60 \times 27A$$

$$= \underline{\underline{-1620W}}$$

$$\text{Power in } 20\Omega = V_1 \times i_2$$

$$= 60 \times 3 = \underline{\underline{180W}}$$

$$\text{Power in } S_2 = 9 \times 45 \\ = \underline{\underline{405 \text{ W}}}$$

$$\text{Power in } \frac{V_1}{4} \text{ current source} = 15 \times 45 \\ = \underline{\underline{675 \text{ W}}}$$

$$\text{Power in } S_{12} \text{ battery} = 24 \times 5f_2 \\ = 24 \times 15 = \underline{\underline{360 \text{ W}}}$$

$$\Sigma \text{ Power} = -1620 + 180 + 405 + 675 + 360 \\ = -1620 + 1620$$

$\underline{\underline{= 0 \text{ W}}}$  Hence proved  
Tellegens Theo satisfied

(II)

**Q.8 (a)**

Two 8-bit numbers are stored in the memory locations 2000H and 2001H. Write 8085 assembly language programs to multiply these two numbers using,

(i) Successive addition method      (ii) Shift and add method

The final result should be stored at the memory locations 3000H and 3001H.

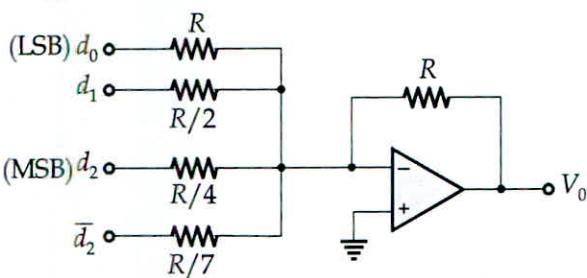
[10 + 10 marks]



Q.8 (b)

Consider the circuit shown in the figure below:

$$d_i = \begin{cases} +\frac{1}{2} V & \text{for logic } 0 \\ -\frac{1}{2} V & \text{for logic } 1 \end{cases}$$

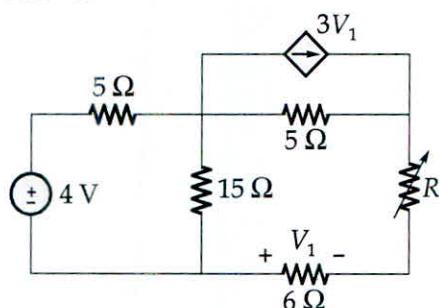


- Derive an expression for output voltage,  $V_0$  in terms of input logic values.
- Using the result obtained in part (i), determine the value of  $V_0$  for all the possible binary combinations of input and comment on the operation performed by the circuit.

[12 + 8 marks]



- Q.8 (c)**
- (i) State and prove the maximum power transfer theorem for purely resistive source circuit with variable load resistance.
  - (ii) Determine the maximum power that can be delivered to the variable resistor  $R$  in the circuit shown below.

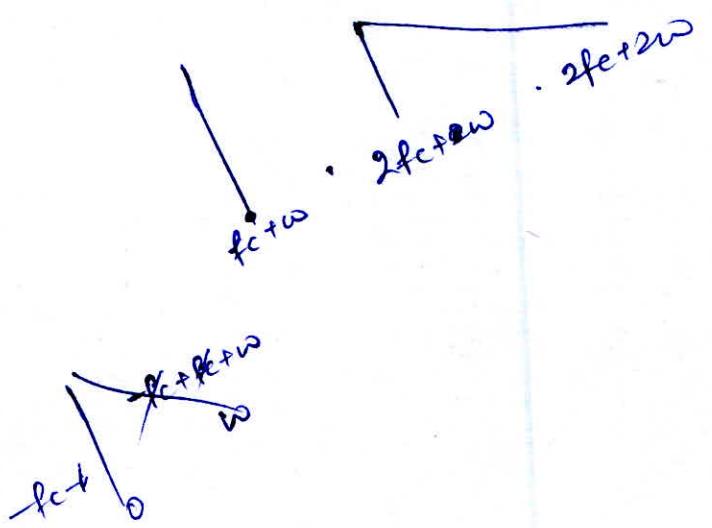


[10 + 10 marks]

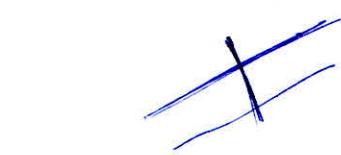


OOOO

## Space for Rough Work



## Space for Rough Work

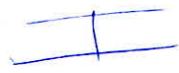


w<sub>0</sub>

$$n_1 = \int_0^{1.5} w_0 dt$$

$$n_2 = \int_{-1}^2 w_0 dt$$

mean = 0



$$ess = \frac{R}{t \rightarrow \infty} e(t)$$

$$= R - E(R)$$

t → 0

$$\frac{SR(s)}{E[G(s)H(s)]}$$

$$n_{1H} = (n_1 - n_2)$$

$$n_{2H} = (n_2 - n_1)$$

$$E[n^2(t)] = E[(n_1 - n_2)^2]$$

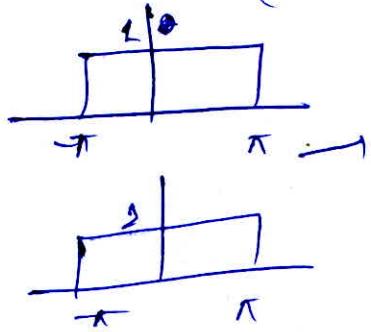
$$= E[n^2(t)] + E[n^2(t)] - 2E[n^2(t)]$$

vae = E

$$\frac{s - (\delta - 1)}{\delta(\delta - 1)}$$

$$\frac{1}{(\delta - 1)} - \frac{1}{\delta}$$

[Now]



1 + 2π

$$\begin{matrix} & & s \\ & 26 & 1380 \\ 26 & 1380 & 1380 \end{matrix}$$

$$\begin{matrix} & 9 & 5 \\ 2 & 3 & 5 \end{matrix}$$

$$\begin{matrix} & 2 & 5 \\ 13 & 26 & 3 \end{matrix}$$

$$\begin{matrix} & 2 & 0 \\ 4 & 2 & 0 \\ 2 & 0 & 0 \\ (2w) \times c & 3 \times 10 & 5 \times 27 \\ 2w & 2w & 2w \end{matrix}$$