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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-3: Analog and Digital Communication Systems
Network Theory-1 + Microprocessors and Microcontroller-1
Digital Circuits-2 + Control Systems-2

Name : ....

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	38
Q.2	39
Q.3	—
Q.4	—
Section-B	
Q.5	45
Q.6	42
Q.7	53
Q.8	—
Total Marks Obtained	217

Signature of Evaluator: Sumeet

Cross Checked by: [signature]

Remarks : Excellent presentation as well as accuracy ! Keep it up !

Section A : Analog and Digital Communication Systems

Q.1 (a) Let $X(t)$ be a real WSS process and another process $Y(t) = \hat{X}(t)$. i.e., $Y(t)$ is the Hilbert transform of $X(t)$. $R_X(\tau)$ and $R_Y(\tau)$ denote the auto-correlation function of $X(t)$ and $Y(t)$ respectively, and $R_{XY}(\tau)$ denotes the cross-correlation function of $X(t)$ and $Y(t)$. Then prove that the following two relations are true.

R_1: $R_Y(\tau) = R_X(\tau)$

R_2: $R_{XY}(-\tau) = -R_{XY}(\tau)$

[12 marks]

Ans: Given: $Y(t) = \hat{X}(t)$

• R_1: $R_X(\tau) = E[X(t) \cdot X(t+\tau)]$

$R_Y(\tau) = E[Y(t) \cdot Y(t+\tau)]$

$= E[\hat{X}(t) \cdot \hat{X}(t+\tau)]$

$\because$ $\hat{X}(\omega) = -j\,sgn(\omega) \cdot X(\omega)$

$\Rightarrow R_Y(\tau) = E[X(t) \cdot -j\,sgn(\omega) \cdot -j\,sgn(\omega) \cdot X(t+\tau)]$

$sgn^2(\omega) = 1$

$\Rightarrow$ $\boxed{R_Y(\tau) = E[X(t) \cdot X(t+\tau)] = R_X(\tau)}$

– Hence proved.

How can you write f-domain expressions inside E(·)

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• R_2: We know, $R_{XY}(\tau) = E[X(t_1) \cdot Y(t_1+\tau)]$

$= \int_{-\infty}^{\infty} X(\tau) \cdot Y(t+\tau) \, d\tau$

$= E[X(t) \cdot \hat{X}(t+\tau)]$

Q.1 (b) Consider a single-tone AM signal as follows:

$$s(t) = [1 + \mu\cos\omega_m t]\cos\omega_c t$$

If $\mu = \frac{1}{2}$ and the upper sideband component is attenuated by a factor of 2, then determine the expression for the envelope of the resulting modulated signal.

[12 marks]

Ans: Given: $S(t) = (1 + \mu\cos\omega_m t)\cos\omega_c t$

$\mu = \frac{1}{2}$

upper side-band is attenuated by a factor of 2.

$\Rightarrow S(t) = \cos\omega_c t + \mu\cos\omega_m t \cos\omega_c t$

$= \cos\omega_c t + \frac{1}{2} \times \frac{1}{2}[\cos(\omega_c+\omega_m)t] + \frac{1}{2} \times \frac{1}{2}[\cos(\omega_c-\omega_m)t]$

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As upper side-band is attenuated by a factor of 2

$\Rightarrow$ Expression for AM signal will be:

$S(t) = \cos\omega_c t + \frac{1}{2}\cos(\omega_c+\omega_m)t + \frac{1}{4}\cos(\omega_c-\omega_m)t$

Q.1 (c) Over the interval $|t| \le 1$, an angle modulated signal is given by, $s(t) = 10\cos 13000t$. Carrier frequency $\omega_c = 10000$ rad/s.

(i) If it is a PM signal with $k_p = 1000$ rad/V, then determine $m(t)$ over the interval $|t| \le 1$.

(ii) If it is an FM signal with $k_f = 1000$ rad/s/V, then determine $m(t)$ over the interval $|t| \le 1$.

[6 + 6 marks]

Ans: Given: Angle modulated signal: $s(t) = 10\cos 13000t$

$\omega_c = 10000$ rad/s.

(i) $K_p = 1000$ rad/s ; Given signal is a PM signal.

We know, standard equation for PM signal:

$$S_{PM}(t) = A_c \cos[\omega_c t + K_p m(t)]$$

On comparing with given $s(t)$; we get:

$A_c = 10V$, $\omega_c t + K_p m(t) = 13000t$

Substituting the values,

$$10000t + 1000\, m(t) = 13000t$$

$$1000\, m(t) = 3000t$$

$\Rightarrow$ $$\boxed{m(t) = 3t}$$

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(ii) If Given signal is an FM signal, given $K_f = 1000$ rad/s

We know, standard equation of FM signal:

$$S_{FM}(t) = A_c \cos\left[\omega_c t + K_f \int_{-\infty}^{t} m(\tau)\,d\tau\right]$$

comparing with given signal:

$$\omega_c t + K_f \int_{-\infty}^{t} m(\tau)\,d\tau = 13000t$$

Substituting the values:

$$10000t + 1000\int_{-\infty}^{t} m(\tau)\,d\tau = 13000t$$

$$\int_{-\infty}^{t} m(\tau)\,d\tau = 3t$$

differentiating wrt 't':

$$\boxed{m(t) = 3}$$

6

Q.1 (d) Two continuous random variables X and Y are related as, $Y = aX + b$. If 'a' and 'b' are positive constants, then derive the relation between the differential entropies of the two random variables.

[12 marks]

Ans:

Given: $Y = aX + b$

'a, b → Positive constants.

Differential entropy ⇒ Entropy of continuous Random variable.

~~Entropy of X:~~

~~$H(X) = \int_{-\infty}^{\infty} P_i \log_2 \frac{1}{P_i} \times f_X(x)\,dx$~~

~~Where P_i → Probabi~~

let Entropy of $X = H(x)$

Entropy of $Y = H(Y)$

$$H(Y) = \int_{-\infty}^{\infty} f_Y(y) \log_2 \frac{1}{f_Y(y)}\,dy$$

$f_Y(y)$ = pdf of Y.

$$H(X) = \int_{-\infty}^{\infty} f_X(x) \cdot \log_2 \frac{1}{f_X(x)} \, dx$$

$f_X(x) \rightarrow$ pdf of X.

Given : $Y = ax + b$

$$f_Y(y) = \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{x = \frac{y-b}{a}} \quad \Rightarrow dy = a\,dx \quad \text{—(1)}$$

$f_X(x) = f_X(x) \rightarrow$ given , $\frac{dy}{dx} = a$

$$f_Y(y) = \frac{f_X(x)}{a} \quad \text{—(2)}$$

$$\Rightarrow \quad H(Y) = \int_{-\infty}^{\infty} \frac{f_X(x)}{a} \log_2 \frac{1}{\left(\frac{f_X(x)}{a}\right)} \, dy$$

$$= \int_{\infty}^{\infty} \frac{f_X(x)}{a} \left[\log_2 \frac{1}{f_X(x)} + \log_2 a\right] a\,dx$$

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$$= \frac{a}{a}\int_{-\infty}^{\infty} f_X(x) \log_2 \frac{1}{f_X(x)} \, dx$$

$$+ \frac{a}{a}\int_{-\infty}^{\infty} \log_2 a \cdot f_X(x) \, dx$$

$$\Rightarrow \quad \boxed{H(Y) = H(X) + \log_2 a}$$

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Q.1 (e) What are the advantages and disadvantages of delta modulation compared to PCM? With the help of a sketch, mention various noises associated with delta modulation. How will you overcome these noises?

[12 marks]

Ans:

Advantages of Delta Modulation over PCM:

- The quantizer is simple as only one bit is used.
- Bandwidth requirement is less.
- Noise is less, as bandwidth is less.

Disadvantages of Delta Modulation:

- Magnitude of slope must be constant for proper modulation and demodulation.
- Not flexible for different types of signals.
- Slope overload and Granular error can occur.

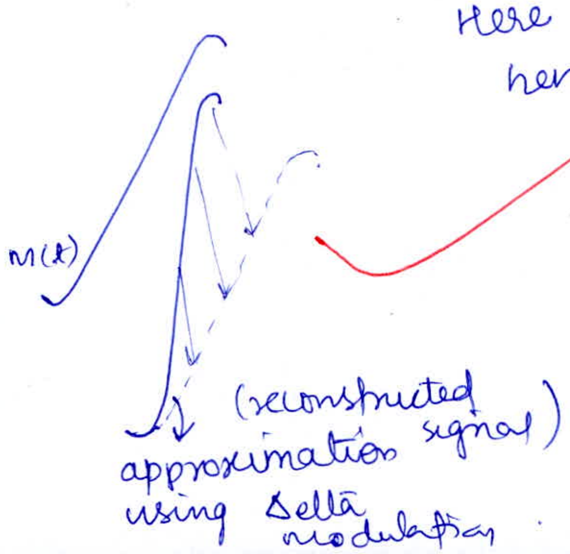
Various Noises in Delta Modulation:

(i) Granular Noise (ii) Slope Overload Error.

- Granular Noise:

This noise occurs if the step size used for approximation is more.

Here, step size is more, hence signal is not properly reconstructed



$$\frac{\Delta}{T_s} > \left|\frac{d}{dt} m(t)\right|$$

- It occurs generally at high frequencies.

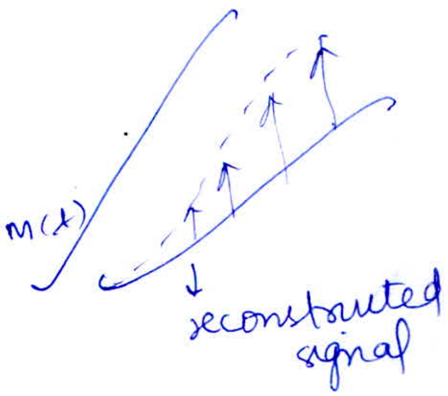
- To overcome granular noise, step size should be decreased.

$$\Rightarrow \boxed{\frac{\Delta}{T_s} \leq \left|\frac{d}{dt} m(t)\right|}$$

$\Delta \rightarrow$ step size
$T_s \rightarrow$ sampling period.
$m(t) \rightarrow$ message

- **Slope overload Error.**

This error occurs when step size is kept smaller.



- This occurs generally at low frequencies, thus more dangerous as most of the information is at low frequencies only.

- This type of noise can be avoided by increasing the step size.

$$\Rightarrow \boxed{\frac{\Delta}{T_s} \geq \left|\frac{d}{dt} m(t)\right|}$$

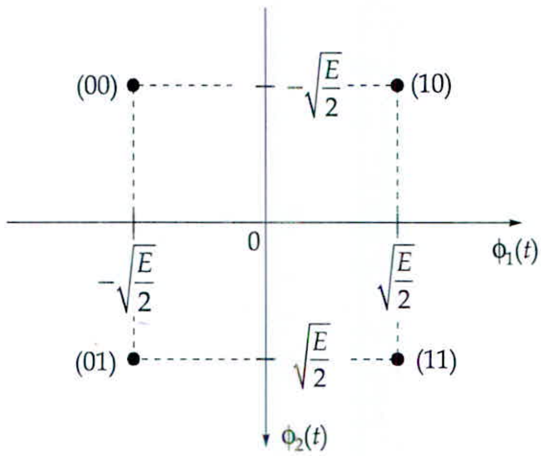
Q.2 (a) Two random variables X and Y are independent and identically distributed, each with a Gaussian density function with mean equal to zero and variance equal to σ^2. If these two random variables denote the coordinates of a point in the plane, find the probability density function of the magnitude and the phase of that point in polar coordinates.

[20 marks]

Q.2 (b) A double conversion superheterodyne receiver is designed with $f_{IF(1)}$ = 30 MHz and $f_{IF(2)}$ = 3 MHz. Local oscillator frequency of each mixer stage is set at the lower of the two possible values. When the receiver is tuned to a carrier frequency of 300 MHz, insufficient filtering by the RF and first IF stages results in interference from three image frequencies. Determine those three image frequencies.

[15 marks]

Q.2 (c) Consider the signal-space diagram of a coherent QPSK system as shown in the figure below:



$\phi_1(t)$ and $\phi_2(t)$ are two orthonormal basis functions, which are represented as,

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)\,;\ 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)\,;\ 0 \le t \le T$$

All the four message symbols are occurring with equal probability and they are transmitted through an AWGN channel with two-sided noise power spectral density of $\frac{N_0}{2}$. Suggest a receiver model to reproduce the symbols at channel output and derive an expression for the probability of symbol error.

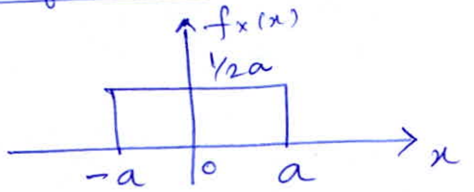
[25 marks]

Q.3 (a) The samples of a stationary random process $X(t)$, whose amplitude is uniformly distributed in the range $[-a, a]$, are applied to an n-bit uniform mid-riser quantizer. Derive an expression for the signal-to-quantization noise ratio at the output of the quantizer, with suitable assumptions. Using the expression obtained, find the signal-to-quantization noise ratio for an 8-bit quantizer.

[20 marks]

Ans :

Pdf of amplitude $X(t)$:



Signal power: $S = \int_{-\infty}^{\infty} x^2 \cdot f_X(x)\, dx$

$$= \int_{-a}^{a} x^2 \times \frac{1}{2a}\, dx$$

$$= \frac{2}{2a} \int_{0}^{a} x^2\, dx$$

$$= \frac{1}{a}\left[\frac{x^3}{3}\right]_0^a$$

$$\boxed{S = \frac{a^2}{3}} \quad \text{--- (1)}$$

Now, Noise power ; $N_Q = \frac{\Delta^2}{12}$

where, Δ = step size = $\frac{\text{Peak to peak voltage}}{2^n}$

$n \rightarrow$ Number of bits.

$$\Rightarrow \quad \Delta = \frac{2a}{2^n}$$

proof?

$$N_Q = \frac{\Delta^2}{12} = \frac{4a^2}{2^{2n}} \times \frac{1}{12} = \frac{a^2}{3 \times 2^{2n}}$$

⇒ Signal to Noise power ratio :

$$\frac{S}{N_Q} = \frac{\left(\frac{a^2}{3}\right)}{\left(\frac{a^2}{3 \times 2^{2n}}\right)} = 2^{2n}$$

in dB : $\frac{S}{N_Q} = 10 \log_{10} 2^{2n} = 6.02n$

⇒ $$\boxed{\frac{S}{N_Q} = 2^{2n} = 6.02n \text{ dB}}$$

- For 8 bit quantizer :

$n = 8$

⇒ $\frac{S}{N_Q} = 2^{2 \times 8} = 2^{16} = 6.02 \times 8 \text{ dB}$

(19)

$$\boxed{\frac{S}{N_Q} = 48.16 \text{ dB}}$$

Q.3 (b) A binary channel matrix is given by,

Outputs

		y_1	y_2
Inputs	x_1	$\frac{2}{3}$	$\frac{1}{3}$
	x_2	$\frac{1}{10}$	$\frac{9}{10}$

If $P(x_1) = 1/3$ and $P(x_2) = 2/3$, then determine: $H(x)$, $H(x \mid y)$, $H(y)$, $H(y \mid x)$ and $I(x ; y)$

[20 marks]

Ans: Given:

$$P(y|x) = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$$

$$P(x) = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

To determine: $H(x)$, $H(x|y)$, $H(y)$, $H(y|x)$ and $I(x;y)$.

~~We know $P(x|y) = P(y|x)$~~

• H(x): We know, $H(x) = -\sum P_x \log_2 P_x$

substituting the values:

$$H(x) = \frac{1}{3}\log_2 3 + \frac{2}{3}\log_2 \frac{3}{2}$$

$$\boxed{H(x) = 0.918} \text{ bits / symbols}$$

- Calculation of $H(x|y)$:

We know ; $P(x,y) = [P(x)]_d \cdot [P(y|x)]$

$$= \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$$

$$P(x,y) = \begin{bmatrix} \frac{2}{9} & \frac{1}{9} \\ \frac{2}{30} & \frac{18}{30} \end{bmatrix}$$

We know, $P(x|y)$ can be obtained by dividing the columns with $P(y)$.

$$P(y) = \begin{bmatrix} \frac{13}{45} & \frac{32}{45} \end{bmatrix}$$

$$\Rightarrow P(x|y) = \begin{bmatrix} \frac{10}{13} & \frac{5}{32} \\ \frac{3}{13} & \frac{27}{32} \end{bmatrix}$$

We know, $H(x|y) = -\sum P(x,y) \cdot \log_2 P(x|y)$

substituting the values ;

$$H(x|y) = -\left[\frac{2}{9}\log_2\left(\frac{10}{13}\right) + \frac{1}{9}\log_2\left(\frac{5}{32}\right) + \frac{2}{30}\log_2\left(\frac{3}{13}\right) + \frac{18}{30}\log_2\left(\frac{27}{32}\right) \right]$$

$$\boxed{H(x|y) = 0.876} \text{ bits/symbol.}$$

- Calculation of $H(Y|X)$:

We know, $H(Y|X) = -\sum P(x,y) \log_2 P(y|x)$

substituting the values:

$$H(Y|X) = -\left[\frac{2}{9}\log_2\left(\frac{2}{3}\right) + \frac{1}{9}\log_2\left(\frac{1}{3}\right) + \frac{2}{30}\log_2\left(\frac{1}{10}\right) + \frac{18}{30}\log_2\left(\frac{9}{10}\right) \right]$$

$$\boxed{H(Y|X) = 0.618} \text{ bits/symbol}$$

- Calculation of $H(Y)$:

$\because$ $H(Y) = \sum P(y) \log_2 \frac{1}{P(y)}$

substituting values:

$$H(Y) = \frac{13}{45}\log_2\left(\frac{45}{13}\right) + \frac{32}{45}\log_2\left(\frac{45}{32}\right)$$

$$\boxed{H(Y) = 0.867} \text{ bits/symbol}$$

- Calculation of $I(X;Y)$:

$\because$ $I(X;Y) = H(Y) - H(Y|X)$ ~~$H(X) - H(X|Y)$~~

$\Rightarrow$ $I(X;Y) = 0.867 - 0.618$

$$\boxed{I(X;Y) = 0.249} \text{ bits/symbol}$$

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Q.3 (c) **(i)** In a DSBSC system, the message signal $m(t)$ is multiplied with the carrier signal $c(t) = 4\cos(2\pi f_c t)$ to form a modulated signal $s(t)$. If $m(t) = 2\text{sinc}(2t) - \text{sinc}^2(t)$ and $f_c = 100$ Hz, then determine and sketch the spectrum of the modulated signal $s(t)$. Assume that, $\text{sinc}(t) = (\sin \pi t)/\pi t$.

(ii) The spectrum of the message signal $m(t)$ is shown below in Figure (a). This signal is processed by the system shown below in Figure (b).

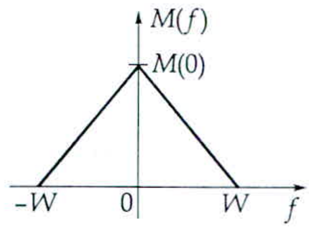


Figure (a)

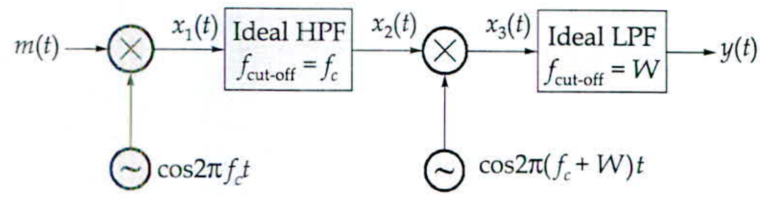


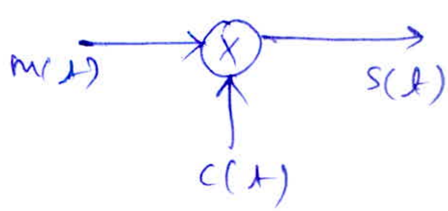
Figure (b)

If each filter has a passband gain of 1, then determine and sketch the spectrum of the output signal $y(t)$. Assume that $f_c >> W$.

[8 + 12 marks]

Ans :

(i)



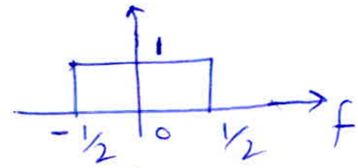
$S(t) = m(t) \cdot c(t)$

$\Rightarrow \quad S(t) = \left[2\,\text{sinc}\,2t - \text{sinc}^2 t\right] 4\cos\left[2\pi \times 100 t\right]$

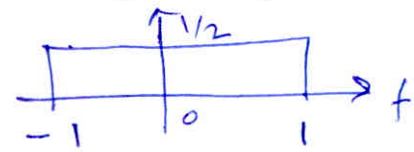
$= \left\{ 2\dfrac{\sin 2\pi t}{2\pi t} - \left(\dfrac{\sin \pi t}{\pi t}\right)^2 \right\} \times 4\cos\left[2\pi \times 100 t\right]$

As multiplication in one domain results in convolution in another domain.

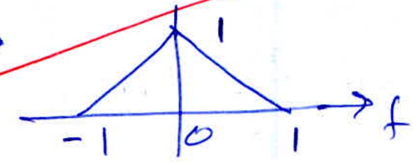
$\text{sinc}\, t \xrightarrow{\mathcal{F}}$



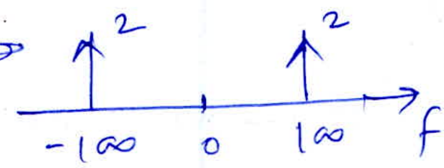
$\text{sinc}\, 2t \xrightarrow{\mathcal{F}}$



$\text{sinc}^2 t \xrightarrow{\mathcal{F}}$

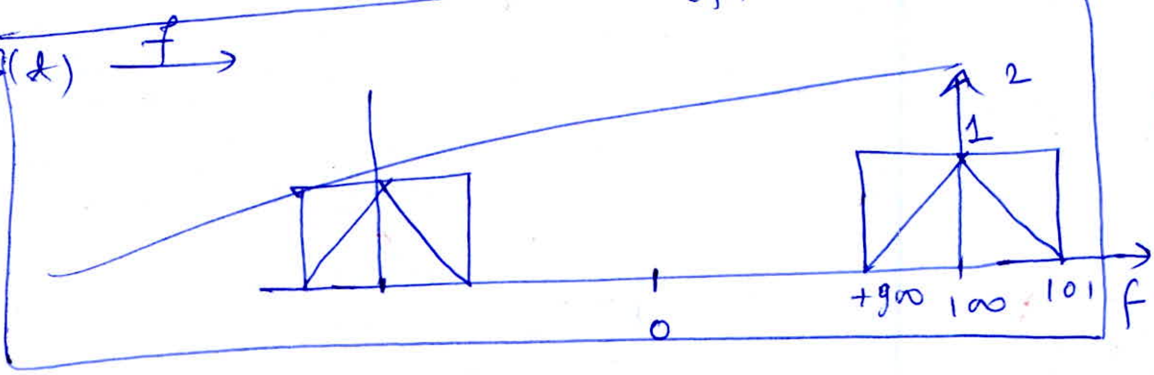


$4 \cos 2\pi \times 100 t \xrightarrow{\mathcal{F}}$

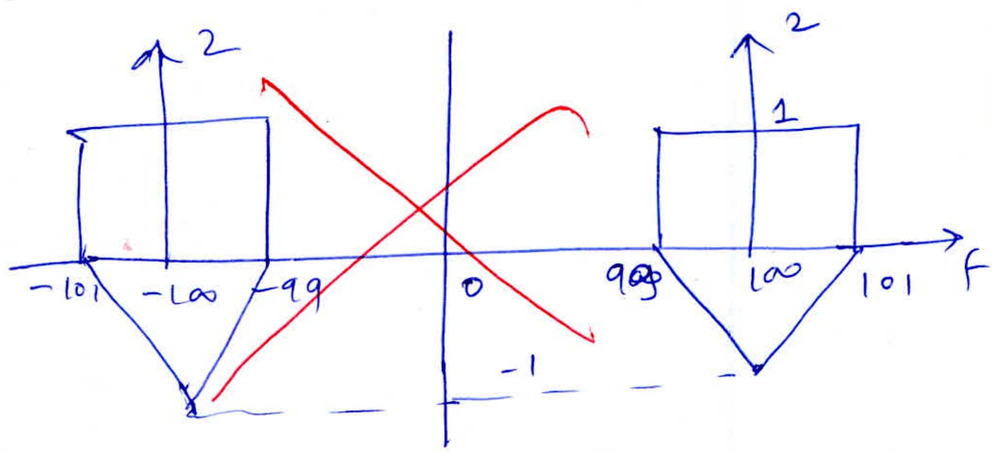


After multiplication, spectrum will be shifted to right and left by carrier frequency.

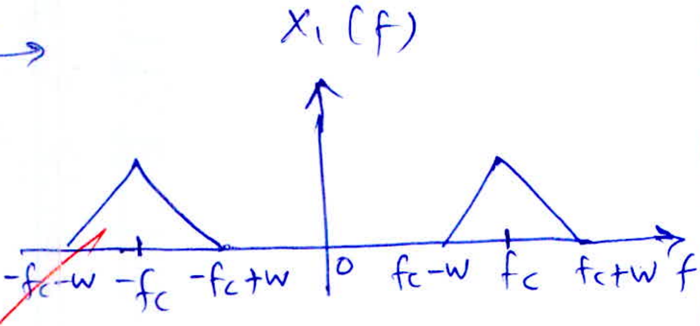
$s(t) \xrightarrow{\mathcal{F}}$ S(f)



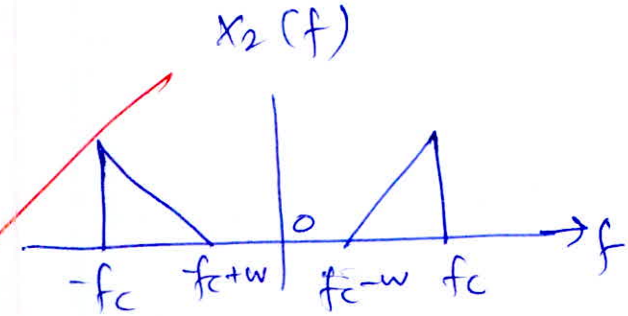
$s(t) \xrightarrow{\mathcal{F}}$ S(f)



(ii) $x_1(t) = m(t) \cdot c(t) \xrightarrow{\mathcal{F}}$

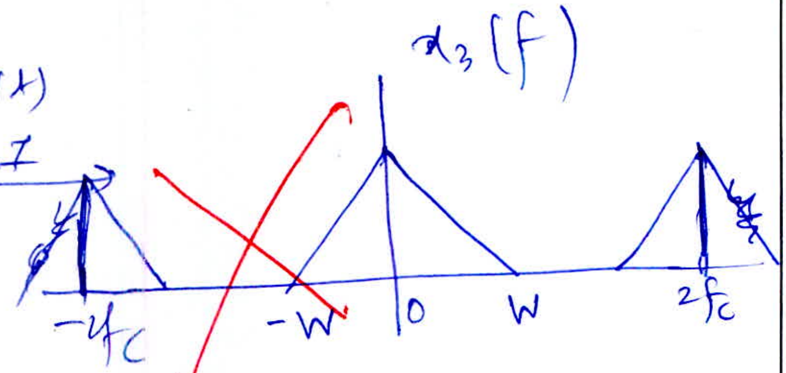


$x_2(t) = (LPF)_{output} \xrightarrow{\mathcal{F}}$

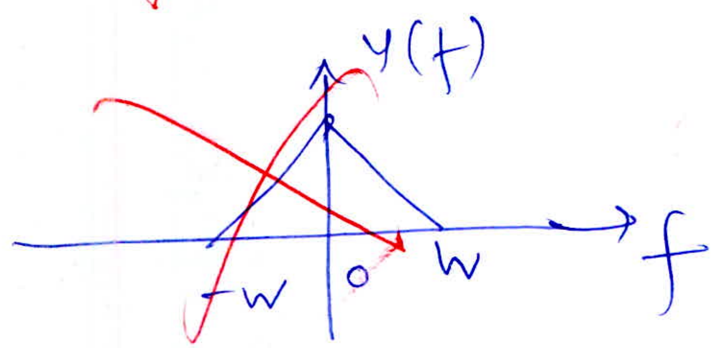


$x_3(t) = x_2(t) \cdot c_2(t) \xrightarrow{\mathcal{F}}$

4



$y(t) \xrightarrow{\mathcal{F}}$



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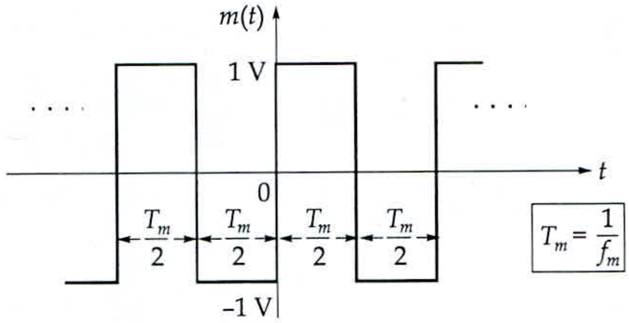
Q.4 (a) The periodic message signal $m(t)$ shown in the figure below is applied to a phase modulator to modulate the carrier signal $c(t) = \cos(2\pi f_c t)$. If the phase sensitivity of the phase modulator is $k_p = 1$ rad/V, then determine and sketch the spectrum of the modulated signal.



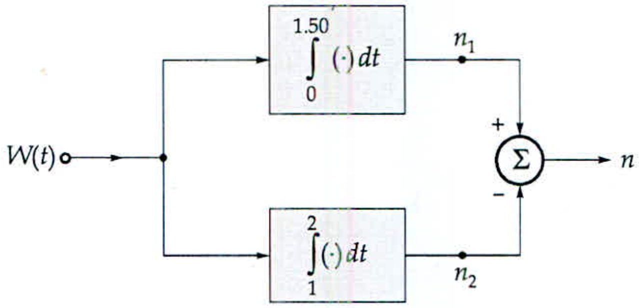
[25 marks]

Q.4 (b) **(i)** A binary data is transmitted through an ideal AWGN channel with infinite bandwidth. The two sided power spectral density of the noise is $\frac{N_0}{2}$. If the average energy transmitted per bit is E_b, then derive the condition to be satisfied for error free transmission.

(ii) A binary signal is transmitted through an ideal AWGN channel with infinite bandwidth. The two-sided PSD of the channel noise is 7 μW/Hz. By using the condition obtained in part (i), determine the minimum average bit energy required for error-free transmission.

[12 + 3 marks]

Q.4 (c) A zero mean white Gaussian noise $W(t)$ is processed by the section of a receiver shown below.



If the two-sided noise power spectral density of the input white Gaussian noise $W(t)$ is $\frac{N_0}{2} = 1\ \text{W/Hz}$, then determine the variance of the corresponding output random variable "n".

[20 marks]

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Section B : Network Theory-1 + Microprocessors and Microcontroller-1 + Digital Circuits-2 + Control Systems-2

Q.5 (a) Design a *J-K* flip-flop using a *D* flip-flop and a 4 × 1 MUX. Write various steps involved in the process.

[12 marks]

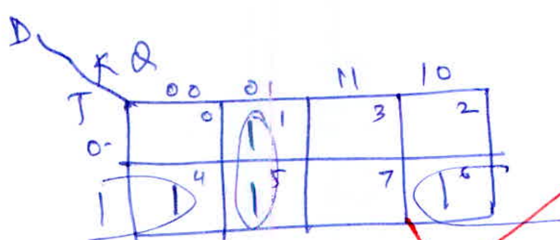
Ans: JK flip-flop using D-flip flop and a 4x1 mux

- Excitation table

J	K	Q	Q^+	$D=Q^+$
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

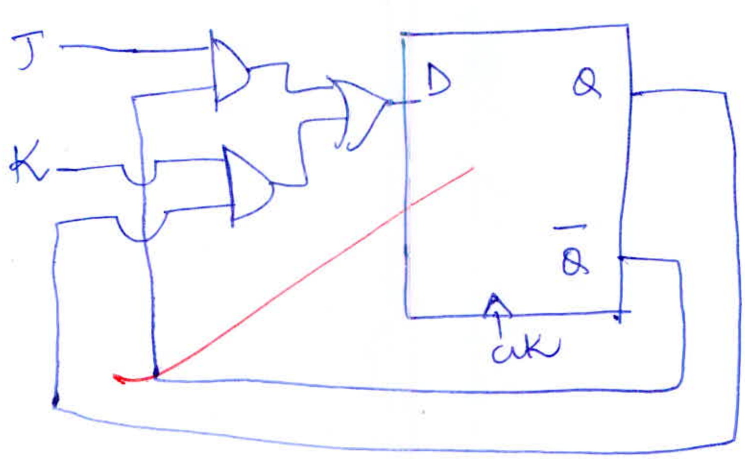
Question should be solved by using MUX

Minimizing using K-maps



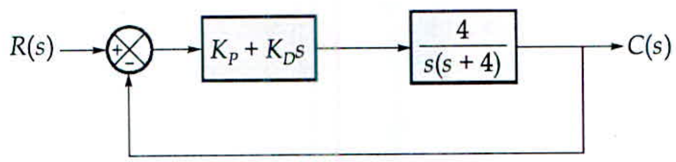
$D = J\bar{Q} + \bar{K}Q$

4



MUX ?

Q.5 (b) A control system with PD controller is shown below:



Determine the value of K_P and K_D such that the damping ratio of the system will be 0.75 and the steady state error for unit ramp input will be 0.25.

[12 marks]

Ans: Given : Damping ratio $(\xi) = 0.75$

Steady state error for unit ramp input $(e_{ss}) = 0.25$.

To find : K_p and K_D.

→ ~~note~~ from the given block diagram, forward path transfer function : $G(s) = \frac{4(K_p + K_D s)}{s(s+4)}$

Feedback : $H(s) = 1$

characteristic equation : $1 + G(s)H(s) = 0$

$$\Rightarrow 1 + \frac{4(K_p + K_D s)}{s(s+4)} = 0$$

$$s^2 + 4s + 4K_p + 4K_D s = 0$$

$$s^2 + (4 + 4K_D)s + 4K_p = 0$$

comparing with standard second order characteristic equation:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\Rightarrow \omega_n = 2\sqrt{K_p} \quad ; \quad 2\xi\omega_n = 4 + 4K_D$$

$$\Rightarrow 2 \times 0.75 \times 2\sqrt{K_p} = 4 + 4K_D$$

$$\frac{3}{4}\sqrt{K_p} = 1 + K_D \quad \text{——— (1)}$$

∵ $e_{ss} = 0.25$

Velocity error coefficient:

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$K_v = \lim_{s \to 0} s \cdot \frac{4(K_p + K_D s)}{s(s+4)} = \frac{4K_p}{4} = K_p$$

$\because$ $e_{ss} = 0.25 \Rightarrow K_v = \frac{1}{0.25} = 4 = K_p$

$\Rightarrow$ $\boxed{K_p = 4}$

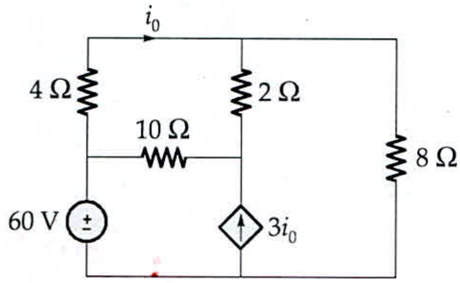
Substituting in ①:

$$\frac{3}{4}\sqrt{4} = 1 + K_D$$

$\Rightarrow$ $\boxed{K_D = 0.5}$

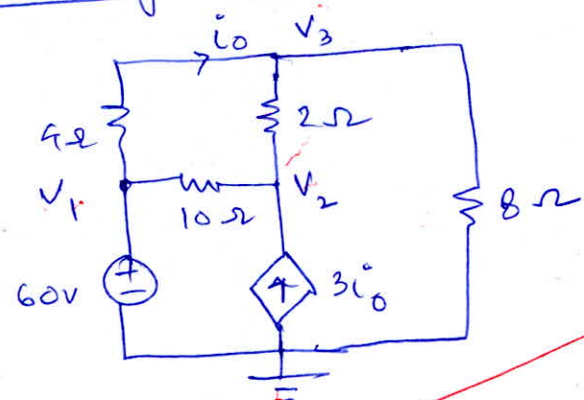
$\Rightarrow$ $\boxed{K_p = 4, \quad K_D = 0.5}$

5 (c) Find the current i_0 in the circuit shown below using nodal analysis.



[12 marks]

Ans: Re-drawing the circuit:



$\boxed{V_1 = 60V}$

Applying KCL at node 2:

$$\frac{V_2 - V_1}{10} + \frac{V_2 - V_3}{2} = 3i_0$$

$$V_2\left(\frac{1}{10} + \frac{1}{2}\right) + V_3\left(-\frac{1}{2}\right) = 3i_0 + 6$$

$$0.6V_2 - 0.5V_3 - 3i_0 = 6 \quad \text{——— ①}$$

KCL at node 3:

$$\frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{2} + \frac{V_3}{8} = 0$$

$$\Rightarrow V_2\left(-\frac{1}{2}\right) + V_3\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{8}\right) = 15 \quad —②$$

$$\because \quad i_0 = \frac{V_1 - V_3}{4} = \frac{60 - V_3}{4} = 15 - \frac{V_3}{4}$$

substituting in ①:

$$0.6V_2 - 0.5V_3 - 3\left(15 - \frac{V_3}{4}\right) = 6$$

$$\Rightarrow 1.35V_2 - 0.5V_3 = 51 \quad —③$$

On solving ② and ③:

$$V_2 = 55.98\,V, \quad V_3 = 49.12\,V$$

Calculation mistake

Thus,

$$i_0 = 15 - \frac{V_3}{4} = 15 - \frac{49.12}{4}$$

$$\boxed{i_0 = 2.72\,A}$$

7

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5 (d) Calculate the delay produced by the following subroutine program of an 8085 microprocessor, which is operating with a clock frequency of 2 MHz.

```
DELAY  : MVI B, 02H
LOOP2: MVI C, FFH
LOOP1: DCR C
       JNZ LOOP1
       DCR B
       JNZ LOOP2
       RET
```

[12 marks]

Ans:

Given: clock frequency : $f_{clk} = 2\,MHz$

$\Rightarrow$ clock-time period : $T_{clk} = \frac{1}{2}\,\mu s = 0.5\,\mu s$

T states :

DELAY : MVI B, 02 H → 7T

LOOP 2 : MVI C, FF H → 7T

LOOP 1 : DCR C → 4T

JNZ LOOP 1 → 10T / 7T

DCR B → 4T

JNZ LOOP2 → 10T / 7T

RET → 10T

Total Delay = $7T + 7T + 254[4T + 10T] + 11T + 4T + 10T + 7T + 254[14T] + 11T + 4T + 7T + 10T$

$= 7190\,T$

substituting the value of clock period.

Delay = $7190 \times 0.5 \times 10^{-6}$

Total Delay = 3595 μ seconds

= 3.595 m seconds

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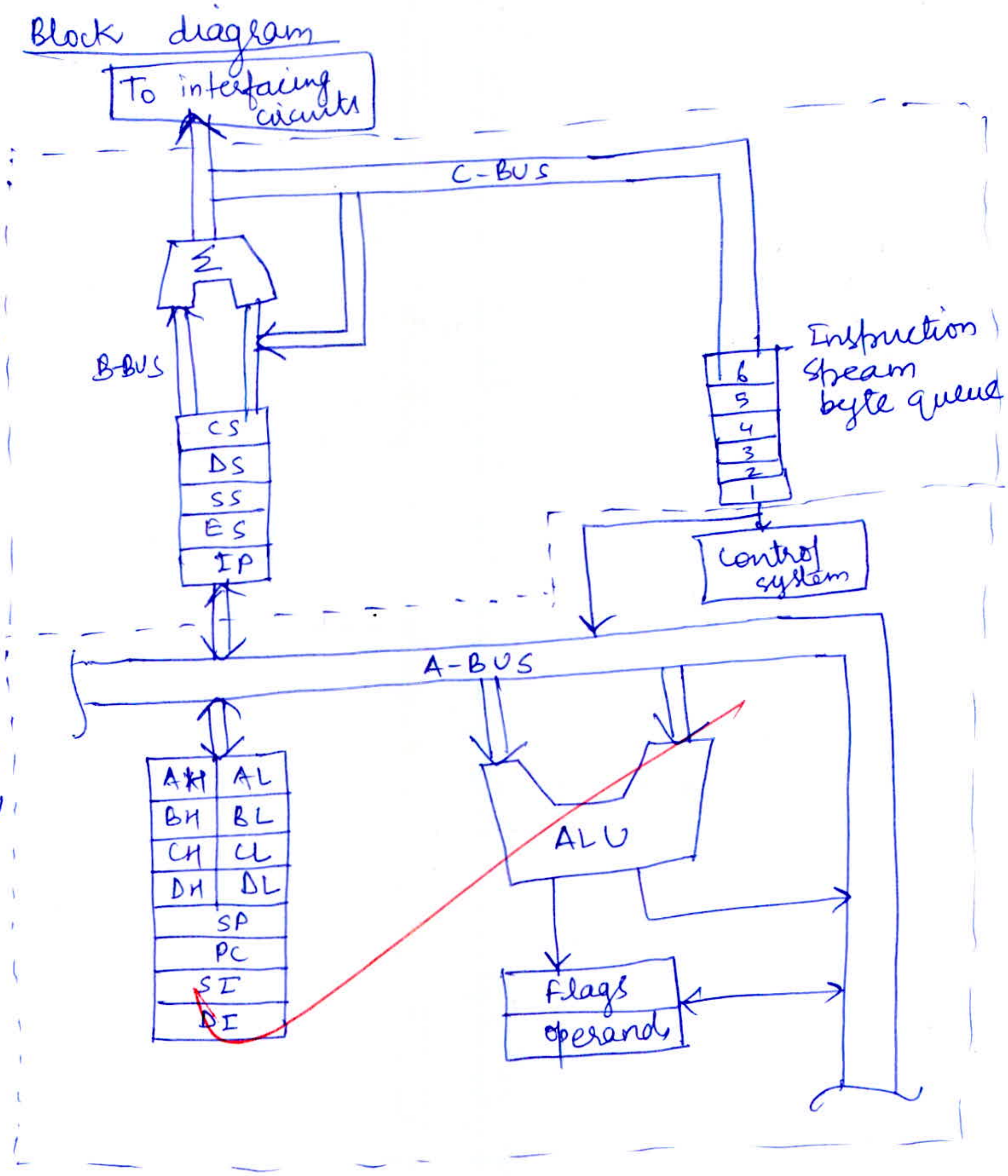
Q.5 (e) Sketch the internal block diagram of an 8086 microprocessor.

[12 marks]

Ans:

Block diagram



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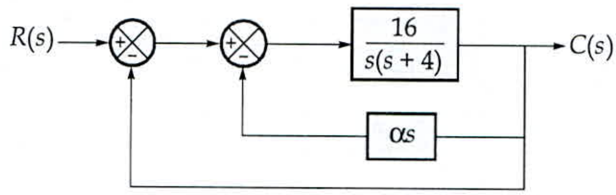
BIU: Bus interface unit: It interfaces the outer devices with the execution unit so that it can bring the data for execution unit.
- It contains segment registers and ready queue to increase speed.

EU: Execution unit: It tells BIU what to do and what not to do.
- It contains some general purpose registers and other special registers.
- It contains ALU to perform operations.

5 (a) The following figure shows a unity feedback control system with rate feedback loop.



Determine:

(i) The peak overshoot of the system for unit step input and the steady state error for unit ramp input in the absence of rate feedback.

(ii) The rate feedback constant 'α' which will decrease the peak overshoot of the system for unit step input to 1.25%. What is the steady state error to unit ramp input with this setting.

(iii) Illustrate how in the system with rate feedback, the steady state error to unit ramp input can be reduced to the same level as in part (i) while the peak overshoot to unit step input is maintained at 1.25%.

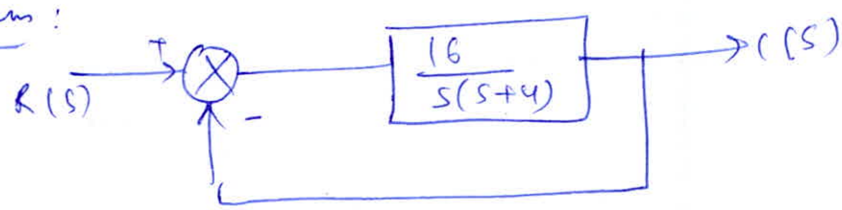
[7 + 8 + 10 marks]

Ans:

(a)

In the absence of rate feedback :

Block diagram :



Input : unit ramp.

characteristic equation : $1+G(s)H(s) = 0$

$\Rightarrow 1 + \frac{16}{s(s+4)} = 0$

$\Rightarrow s^2 + 4s + 16 = 0$

comparing with : $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\Rightarrow \omega_n = 4 \text{ rad/s}$; $\xi\omega_n = 2 \Rightarrow \xi = 0.5$

• We know, Maximum peak overshoot :

$\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

$= e^{-0.5\pi/\sqrt{1-(0.5)^2}} \times 100$

$\% M_p = 16.3\%$

Velocity error coefficient:

$$K_v = \lim_{s\to 0} s \cdot G(s) H(s)$$

$$= \lim_{s\to 0} s \cdot \frac{16}{s(s+4)} = 4$$

steady state error:

$$e_{ss} = \frac{1}{K_v} = \frac{1}{4} = 0.25$$

$$\boxed{e_{ss} = 0.25}$$

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(ii) Given: New input → unit step.

Peak overshoot = 1.25 %.

$\alpha = ?$

$e_{ss} = ?$

→ with rate feedback, forward path transfer function:

$$G(s) = \frac{16}{s(s+(16\alpha+4))}$$

characteristic equation: $1+G(s)H(s) = 0$

$$\Rightarrow s^2 + (16\alpha+4)s + 16 = 0$$

$$\omega_n = 4$$

$$2\xi\omega_n = 2(8\alpha+2)$$

$$\xi = \frac{8\alpha+2}{4} \quad \text{—— (1)}$$

3

Given: $\% M_p = 1.25\%$

$$\Rightarrow 0.0125 = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$-4.38 = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

$$\boxed{\text{New } \xi = 0.66} \quad \text{—— (2)}$$

From ① and ②

$$0.66 = \frac{.8\alpha + 2}{4}$$

$$\Rightarrow \boxed{\alpha = 0.08}$$

- steady state error

ii) Velocity ~~Position~~ error constant :

$$K_v = \lim_{s \to 0} s\,GH(s)$$

$$= \lim_{s \to 0} s \frac{16}{s(s + 5.28)} = 3.03$$

steady state error

$$\boxed{e_{ss} = \frac{1}{K_v} = \frac{1}{3.03} = 0.33}$$

iii) $e_{ss} = 0.25$

$\% M_p = 1.25 \%$

We need to adjust α for these values.

$\because \% M_p = 1.25 \% \Rightarrow \xi = 0.66$

$e_{ss} = 0.25 \Rightarrow K_v = \frac{1}{0.25} = 4$

$$\because K_v = \lim_{s \to 0} s \times \frac{16}{s(s + 16\alpha + 4)} = 4$$

$$\Rightarrow \frac{16}{16\alpha + 4} = 4$$

$$4 = 16\alpha + 4$$

$$\Rightarrow \boxed{\alpha = 0}$$

With $\alpha = 0$, $e_{ss} = 0.25$.

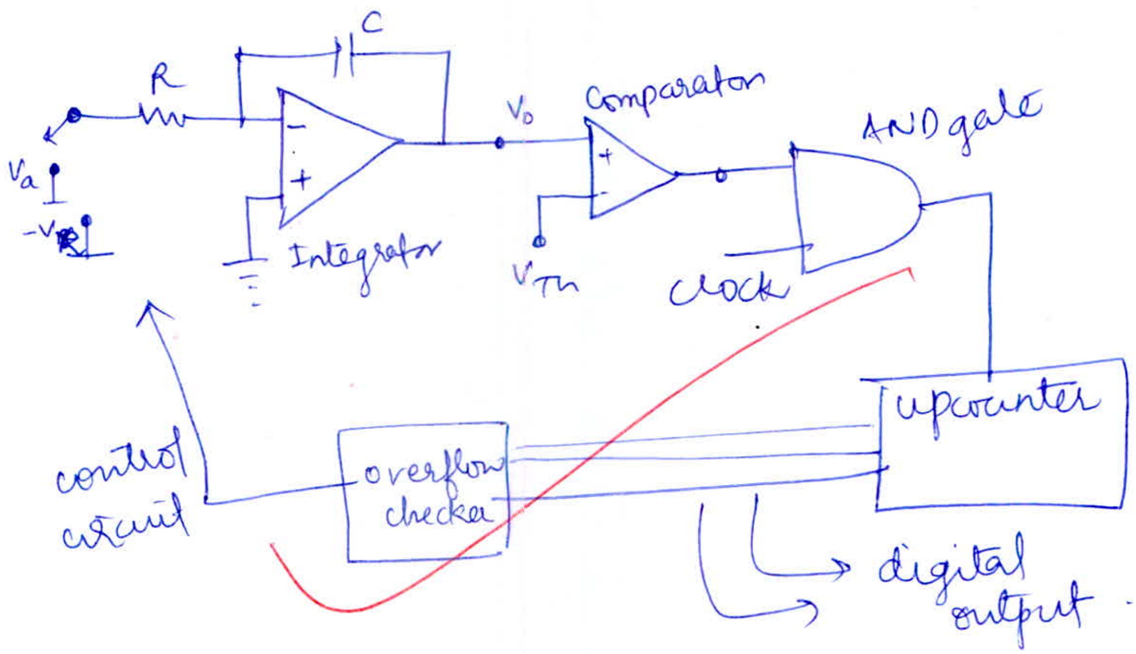
Q.6 (b) **(i)** Explain with a block diagram, the working principle of a dual-slope A/D converter. Derive the expression for the output and maximum conversion time of the circuit.

(ii) A dual-slope A/D converter has a resolution of 4 bits. If the clock rate is 3.2 kHz, then calculate the maximum sampling rate with which the samples can be applied to the A/D converter.

[15 + 5 marks]

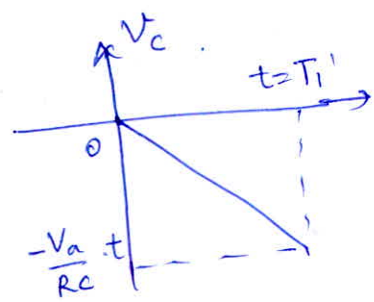
Ans :

(i) Dual slope ADC



Initially, let V_a is connected to the integrator input.

$$V_o = -\frac{1}{RC}\int_{-\infty}^{t} V_a \, dt$$



$$V_o = -\frac{V_a}{RC} \cdot t$$

counter runs from 00 to 00 →

integration time $T_1' = 2^n \cdot T_{clk}$ ↳ clock period

$$\Rightarrow V_o = -\frac{V_a}{RC} \cdot T_1' \quad \text{—— (1)}$$

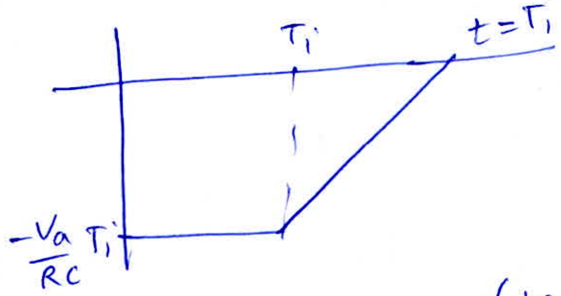
Now, after this the input is shifted to $-V_R$ by control circuit.

Thus,

$$V_o = -\frac{1}{RC}\int_{-\infty}^{t} (-V_R)\, dt$$

$$\boxed{V_o = -\frac{V_a}{RC}\cdot T_1^{\circ} + \frac{V_R}{R_c}\, t}$$ → Expression of output.

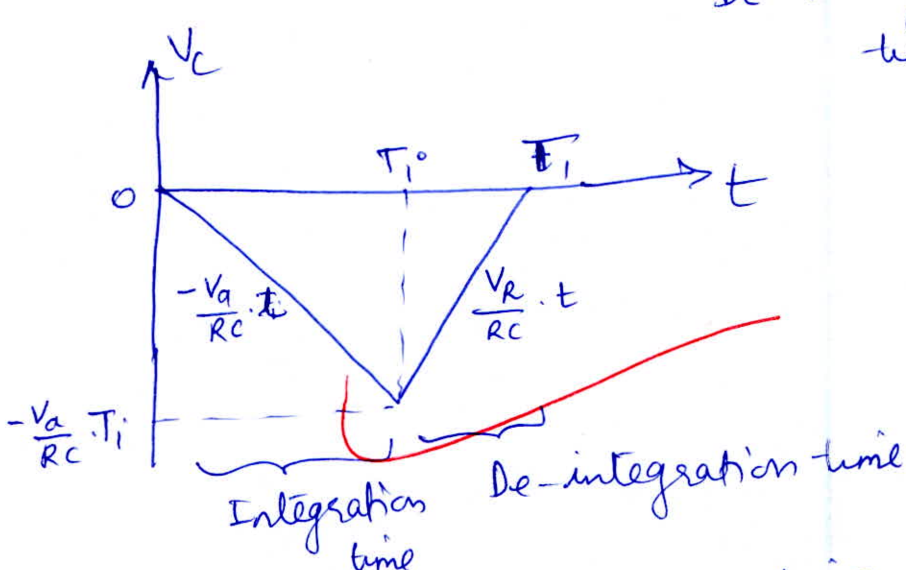
graph:



at $t = T_1$ → capacitor voltage (V_0) reaches zero

$$\Rightarrow 0 = -\frac{V_a}{RC}\cdot T_1^{\circ} + \frac{V_R}{RC}\cdot t$$

$$\Rightarrow \boxed{T_1 = t = +\frac{V_a}{V_R}\cdot T_1^{\circ}}$$ → De-integration time



Thus, conversion time (T_c) = Integration time + De-integration time

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$T_c = T_1 + T_1$

Conversion time : $T_c = 2^n \cdot T_{clk} + \left(\frac{V_a}{V_R}\right) \cdot 2^n \cdot T_{clk}$

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Maximum conversion time : $T_{conv} = 2^{n+1} \cdot T_{clk}$

(ii) Given : Number of bits = 4

clock rate : $f_{clk} = 3.2$ KHz

⇒ clock period : $T_{clk} = \frac{1}{3.2}$ ms

$T_{clk} = 312.5\ \mu$ seconds

Maximum sampling rate $= \frac{1}{T_{conversion}}$

$\because T_{conversion} = 2^{n+1} \cdot T_{clk}$

$= 2^{4+1} \times 312.5 \times 10^{-6}$

$= 10$ m seconds

5

⇒ Maximum sampling rate $= \frac{1}{10 \times 10^{-3}}$

Maximum Sampling rate = 100 Hz

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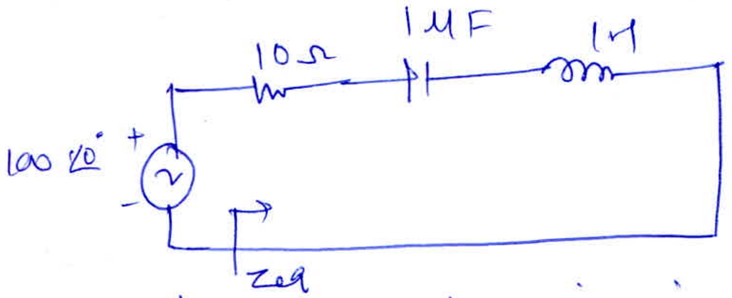
(c) A circuit is made up of a 10 Ω resistance, a 1 μF capacitance and 1 H inductance all connected in series. A sinusoidal voltage of 100 V (rms) at varying frequencies is applied to the circuit. Find the frequency at which the circuit would consume only 10% of the power it consumed at resonance?

[15 marks]



In frequency domain :

Equivalent impedance :

$$Z_{eq} = 10 + \frac{10^6}{j\omega} + j\omega$$

$$Z_{eq} = 10 - j\times\frac{10^6}{\omega} + j\omega$$

At resonance, imaginary part = 0

$$\Rightarrow \frac{10^6}{\omega_0} = \omega_0$$

$$\Rightarrow \boxed{\omega_0 = 10^3 \text{ rad/s}}$$

↳ Resonant frequency

current at resonance $= \frac{V}{R} = \frac{100}{10} = 10A$

⇒ Power consumed at resonance

$$P = I^2R = (10)^2 \times 10$$

$$\boxed{P = 1000 W}$$

• Given : at some different frequency, circuit consumes only 10 % of power consumed at resonance.

$$P = \frac{10}{100} \times 1000 = 100 W$$

We know, only resistance consumes power.

$P = I^2 R = 100\,W$

$\Rightarrow I^2 \times 10 = 100\,W$

$\Rightarrow \boxed{I = 3.16\,A}$

We know, $I = \frac{V}{Z}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{100 + \left(\omega - \frac{10^6}{\omega}\right)^2}$

$\Rightarrow 3.16 = \frac{100}{\sqrt{100 + \left(\omega - \frac{10^6}{\omega}\right)^2}}$

Squaring and cross multiplying

$\Rightarrow 10\left(100 + \left(\omega - \frac{10^6}{\omega}\right)^2\right) = 10^4$

$100 + \left(\omega - \frac{10^6}{\omega}\right)^2 = 1000$

$\left(\omega - \frac{10^6}{\omega}\right)^2 = 900$

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$\omega - \frac{10^6}{\omega} = 30$

$\omega^2 - 10^6 - 30\omega = 0$

$\Rightarrow \omega^2 - 30\omega - 10^6 = 0$

$\omega = 1015.11, -985.11$

Frequency can't be negative

$\Rightarrow \boxed{\omega = 1015.11\ rad/s}$

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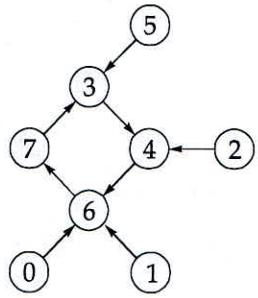
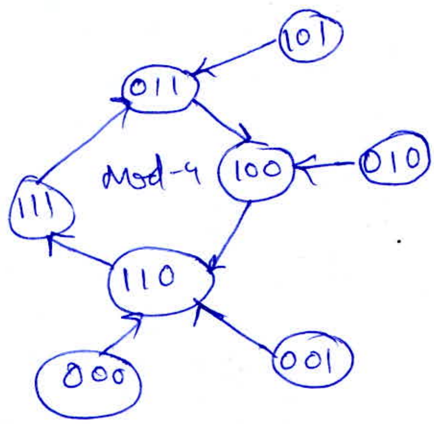
7 (a) Design a synchronous counter, whose sequence diagram is shown below, using D flip-flops.



[20 marks]

Ans:

Re-drawing state diagram:



- It is a self-starting Mod-4 counter.
- As 8 states are there, hence 3 flip flops will be required.

Excitation table

Present state Q_2	Q_1	Q_0	Next state Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
0	0	0	1	1	0	1	1	0
0	0	1	1	1	0	1	1	0
0	1	0	1	0	0	1	0	0
0	1	1	1	0	0	1	0	0
1	0	0	1	1	0	1	1	0
1	0	1	0	1	1	0	1	1
1	1	0	1	1	1	1	1	1
1	1	1	0	1	1	0	1	1

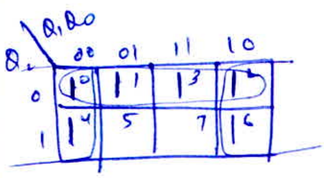
$D_2 = \Sigma m(0, 1, 2, 3, 4, 6)$

$D_1 = \Sigma m(0, 1, 4, 5, 6, 7)$

$D_0 = \Sigma m(5, 6, 7)$

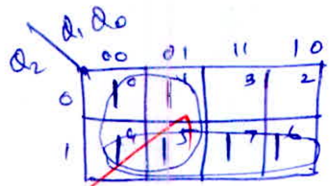
Minimizing using K-maps

For D_2



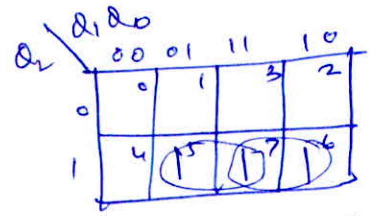
$D_2 = \overline{Q_2} + \overline{Q_0}$

For D_1



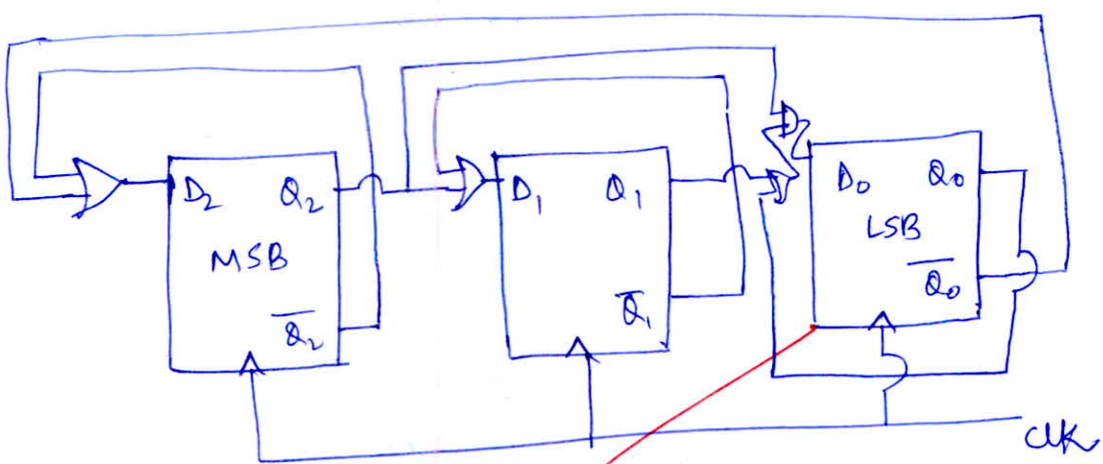
$D_1 = \overline{Q_1} + Q_2$

For D_0



$D_0 = Q_2 Q_0 + Q_2 Q_1$
$= Q_2 (Q_0 + Q_1)$

circuit :



It is a synchronous counter with positive edge triggering.

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7 (b) A linear time invariant system is characterised by the homogeneous state equation,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i) Compute the solution of the homogeneous equation assuming the initial state vector $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(ii) Consider now the system has a forcing function and is represented by the following non-homogeneous state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

where u is a unit step input function. Compute the solution of this equation assuming initial conditions of part (i).

[10 + 10 marks]

Ans:

i) Given : $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\Rightarrow$ system matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

We know,

$[\dot{X}] = A[X] + B[U]$; here $[B] = 0$

$\Rightarrow$ Taking laplace transform:

$sX(s) - x(0) = AX(s)$

$X(s)[sI - A] = x(0)$

$\Rightarrow X(s) = [sI-A]^{-1} \cdot x(0)$ —— ①

$x(t) = L^{-1}\{[sI-A]^{-1} \cdot x(0)\}$

$[sI-A] = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$

$[sI-A]^{-1} = \frac{1}{(s-1)^2} \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$

Substituting in ①:

$$X(S) = \begin{bmatrix} \frac{1}{S-1} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{S-1} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{S-1} \\ \frac{1}{(S-1)^2} \end{bmatrix}$$

Taking inverse Laplace transform:

$$x(t) = \begin{bmatrix} e^t \\ t \cdot e^t \end{bmatrix} = x_1(t) \text{ (let)}$$

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(ii) Given: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\because \quad X(S) = [SI - A]^{-1} \{ x(0) + B\,U(S) \}$$

$$\Rightarrow x(t) = \underbrace{\mathcal{L}^{-1}\{[SI-A]^{-1} \cdot x(0)\}}_{I} + \underbrace{\mathcal{L}^{-1}\{[SI-A]^{-1} \cdot B \cdot U(S)\}}_{II}$$

As I part is already calculated in part (i), we will calculate part II:

let $X_2(S) = [SI-A]^{-1} \cdot B \cdot U(S)$

$$= \begin{bmatrix} \frac{1}{S-1} & 0 \\ \frac{1}{(S-1)^2} & \frac{1}{S-1} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot U(S)$$

$$= \begin{bmatrix} 0 \\ \frac{1}{S-1} \end{bmatrix} U(S)$$

Given: $u \rightarrow$ unit step function $\Rightarrow U(S) = \frac{1}{S}$

$$X_2(S) = \begin{bmatrix} 0 \\ \frac{1}{S(S-1)} \end{bmatrix}$$

$$X_2(s) = \begin{bmatrix} 0 \\ \frac{A}{s} + \frac{B}{s-1} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-1}{s} + \frac{1}{s-1} \end{bmatrix}$$

Taking inverse Laplace transform:

$$x_2(t) = \begin{bmatrix} 0 \\ (-1+e^{t})\, u(t) \end{bmatrix}$$

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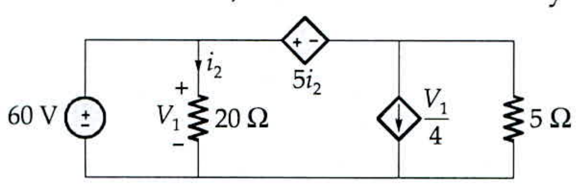
$$\Rightarrow \quad x(t) = x_1(t) + x_2(t)$$

$$x(t) = \begin{bmatrix} e^{t} \\ t \cdot e^{t} \end{bmatrix} + \begin{bmatrix} 0 \\ -1+e^{t} \end{bmatrix} u(t)$$

(c) (i) State and explain the Tellegen's theorem.

(ii) For the network shown below, show that it will satisfy Tellegen's theorem.



[8 + 12 marks]

1:
)

Tellegen's Theorem:

In Any linear, non-linear, unidirectional, bidirectional, Active, passive circuit, the sum of powers = 0.

i.e. $\sum_{K} V_K I_K = 0$ → statement 1

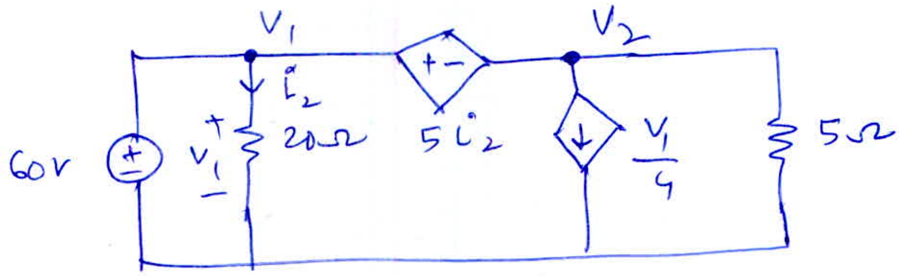
Statement 2:

If two graphs have identical graphs, then

$$\sum V_{1K} I_{2K} = \sum V_{2K} I_{1K} = 0$$

- Tellegen's theorem follows law of conservation of energy. i.e., energy can neither be created, nor be destroyed, it can be only changed from one form to another form.
- It is valid for all kinds of circuits.

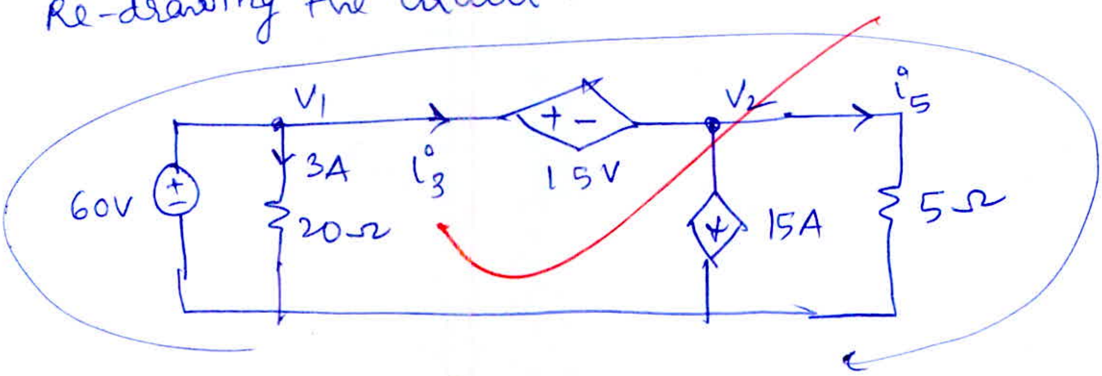
(ii) We will find voltages and currents in each branch and then prove $\Sigma V_K I_K = 0$



$V_1 = 60V$, $i_2 = \frac{V_1}{20} = 3A$,

$5i_2 = 15V$, $\frac{V_1}{4} = 15A$.

Re-drawing the circuit:



Applying KVL in outer loop:

$$-60 + 15 + 5i_5 = 0 \Rightarrow i_5 = 9A$$

$\Rightarrow$ current through 15V source $= 15 + 9 = 24A$
$\Rightarrow$ absorbing $= i_3$

$V_2 = 45V$

$V_1 = 60V$

current through 60V source

$= 3 + 24 = 27\ A$

- let delivering → +ve , absorbing → −ve

$$\Rightarrow \sum V_K I_K = 60 \times 27 - 3 \times 60 - 15 \times 24 - 45 \times 15 - 45 \times 9$$

$$= 0$$

$\sum V_K I_K$ = Power delivered through 60 V
+ Power absorbed by 20 Ω
+ Power absorbed by 15 V
+ power absorbed by 15 A
+ power absorbed by 5 Ω

$$= 60 \times 27 - 3 \times 60 - 15 \times 24 - 45 \times 15 - 45 \times 9$$

$$= 0$$

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Hence, Tellegen's theorem in satisfied.

Q.8 (a) Two 8-bit numbers are stored in the memory locations 2000H and 2001H. Write 8085 assembly language programs to multiply these two numbers using,

(i) Successive addition method **(ii)** Shift and add method

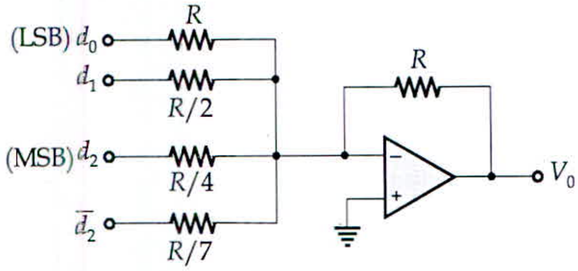
The final result should be stored at the memory locations 3000H and 3001H.

[10 + 10 marks]

Do not write in this ma

Q.8 (b) Consider the circuit shown in the figure below:

$$d_i = \begin{cases} +\frac{1}{2}\text{ V}; & \text{for logic}-0 \\ -\frac{1}{2}\text{ V}; & \text{for logic}-1 \end{cases}$$



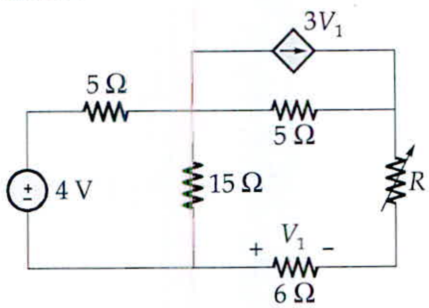
(i) Derive an expression for output voltage, V_0 in terms of input logic values.

(ii) Using the result obtained in part (i), determine the value of V_0 for all the possible binary combinations of input and comment on the operation performed by the circuit.

[12 + 8 marks]

Q.8 (c) **(i)** State and prove the maximum power transfer theorem for purely resistive source circuit with variable load resistance.

(ii) Determine the maximum power that can be delivered to the variable resistor R in the circuit shown below.



[10 + 10 marks]

OOOO

Space for Rough Work

Space for Rough Work

$$\frac{\frac{16}{s(s+4)}}{1+\frac{16}{s(s+4)}\times \alpha(s)}$$

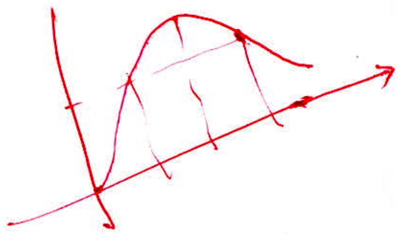
$$= \frac{16}{s^2+4s+16\alpha s} = \frac{16}{s(s+(4+16\alpha))}$$

$\overset{0}{X} = AX + BU$ $\Rightarrow sX(s) - x(0) = AX(s)+B$

$Y = CX + DU$

$X(s)[sI-A] = x[0]+BU($

$X(s) = [sI-A]^{-1}\{x(0)+B$



$\frac{1}{2}\times 2\times h = 1$

$$\frac{1}{s(s-1)} = \frac{A}{s}+\frac{B}{s-1}$$

$A=-1, \; B=1$

$-t x(t) \Rightarrow \frac{1}{s^2}$

$t\cdot u(t) \rightarrow -\frac{d}{ds}\times\frac{1}{s}$

$= \frac{+}{s^2}$

$A\tau \,\text{sinc}\, t\tau$

$\boxed{\tau = 2}$

$A = \frac{1}{2}$