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## ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

### Electrical Engineering

#### Test-3 : Power Systems

Electrical Circuits-1 + Microprocessors-1

Digital Electronics-2 + Control Systems-2

Name : .....

Roll No : 

E	E	I	9	M	T	H	Y	A	G	I	I
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#### Test Centres

Delhi  Bhopal  Noida  Jaipur  Indore   
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Hyderabad

#### Student's Signature

#### Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

#### FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	25
Q.2	
Q.3	52
Q.4	55
Section-B	
Q.5	38
Q.6	45
Q.7	
Q.8	
<b>Total Marks Obtained</b>	<b>215</b>

Signature of Evaluator

Cross Checked by

*Saikat Kumar* ..... *K. Sudhakar*

Handwritten marks or scribbles at the top of the page.

**Section A : Power Systems**

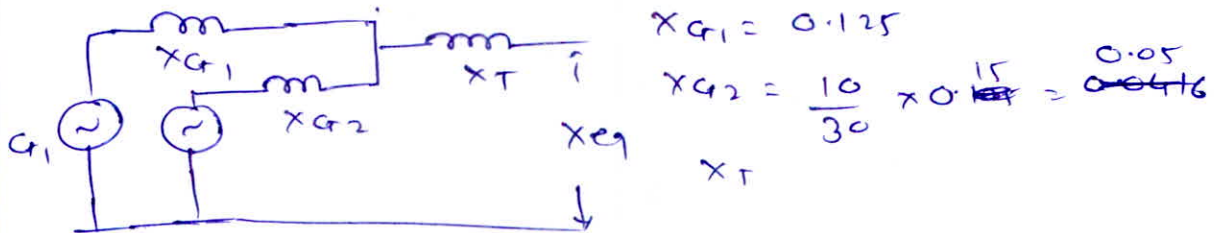
- 1 (a) Estimate the corona loss for a three-phase, 110 kV, 50 Hz, 150 km long transmission line consisting of three conductors each of 10 mm diameter and spaced 2.5 m apart in an equilateral triangle formation. The temperature of air is 30° C and the atmospheric pressure is 750 mm of mercury. Take the irregularity factor as 0.85. Ionization of air may be assumed to take place at a maximum voltage gradient of 30 kV/cm.

[12 marks]

- Q.1 (b) A power plant has three generators feeding a common bus:  
 2 generators, 30 MVA, 15% reactance each  
 1 generator, 10 MVA, 12.5% reactance

A 10 MVA transformer steps up the voltage and feeds a 33 kV circuit. Find the safe minimum reactance of transformer so that fault level on the secondary bus of the transformer may not exceed 100 MVA.

consider a base mva  $10 \phi$  mva [12 marks]  
 Per phase equivalent circuit is



For fault on secondary bus of Transformer,

$$SC \text{ mva} = \frac{\text{Base mva}}{|X_{eq}|}$$

$$100 = \frac{10}{|X_{eq}|}$$

$$|X_{eq}| = 0.1$$

2

$$|X_{eq}| = |X_T| + |(X_{G1} || X_{G2})|$$

$$0.1 = X_T + (0.125 || 0.05)$$

$$0.1 = X_T + \frac{0.125 \times 0.05}{0.125 + 0.05}$$

$$0.1 = X_T + 0.0357$$

$$X_T = 0.0643 \text{ pu}$$

$X_T$  in ohms will be

$$(X_T) = 0.0643 \times \frac{(kV_b)^2}{(MVA_b)}$$

$$= 0.0643 \times \frac{(33)^2}{10}$$

$$X_T = 7.0227 \Omega$$

You forgot  
one generator  
[missed]



$$\begin{aligned} \text{Transformer reactance } & \text{is} \\ & = 0.0643 \text{ pu} \\ & = 7.00227 \Omega \end{aligned}$$

1 (c) A power plant has 3 units with the input output curves.

$$Q_1 = 0.002 P_1^2 + 0.86 P_1 + 20 \text{ tons/hour}$$

$$Q_2 = 0.004 P_2^2 + 1.08 P_2 + 20 \text{ tons/hour}$$

$$Q_3 = 0.0028 P_3^2 + 0.64 P_3 + 36 \text{ tons/hour}$$

Fuel cost is Rs 500 per ton. Maximum and minimum generation level for each unit is 120 MW and 36 MW. Find optimum scheduling for a total load of 200 MW.

Let  $P_1, P_2, P_3$  be the power outputs of plants. then [12 marks]

Power generation = power demand

$$P_1 + P_2 + P_3 = 200 \text{ MW} \quad \text{--- (1)}$$

For optimum scheduling,

$$I_{c1} = I_{c2} = I_{c3} = \lambda \quad \text{--- (2)}$$

where  $I_{c1}, I_{c2}, I_{c3}$  are incremental cost of generating units.

$$I_{c1} = \frac{\partial \theta_1}{\partial P_1} = 0.004 P_1 + 0.86 \quad \text{--- (3)}$$

$$I_{c2} = \frac{\partial \theta_2}{\partial P_2} = 0.008 P_2 + 1.08 \quad \text{--- (4)}$$

$$I_{c3} = \frac{\partial \theta_3}{\partial P_3} = 0.0056 P_3 + 0.64 \quad \text{--- (5)}$$

From eqn (2), (3) & (4)

$$0.004 P_1 + 0.86 = 0.008 P_2 + 1.08$$

$$0.004 P_1 - 0.008 P_2 = 0.22 \quad \text{--- (5)}$$

From eqn (2), (4) & (5)

$$0.004 P_1 + 0.86 = 0.0056 P_3 + 0.64$$

$$0.004 P_1 - 0.0056 P_3 = -0.22 \quad \text{--- (7)}$$

solving eqn (5), (6) & (7) we get

$$P_1 = 85 \text{ MW}$$

$$P_2 = 15 \text{ MW}$$

$$P_3 = 100 \text{ MW}$$

as  $P_2$  is violating limits

$$P_2 = P_{2\min} = 36 \text{ MW}$$

$$\text{now } P_1 + P_3 = 200 - 36$$

$$P_1 + P_3 = 164 \quad \text{--- (8)}$$

$$I_{c1} = I_{c3}$$

$$0.004 P_1 - 0.0056 P_3 = -0.22 \quad \text{--- (9)}$$

solving eqn (8) & (9) we get

$$P_1 = 72.76 \text{ MW}$$

$$P_3 = 91.25 \text{ MW}$$

thus  $P_1 = 72.76 \text{ MW}$

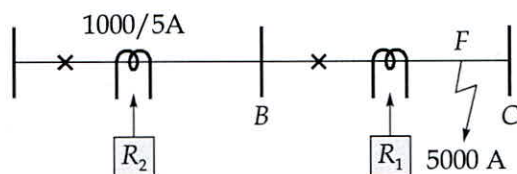
$$P_2 = 36 \text{ MW}$$

$$P_3 = 91.25 \text{ MW}$$



- (d) Two relays  $R_1$  and  $R_2$  are connected in two sections of a feeder as shown in figure below. CT's are of ratio 1000/5 A. The plug setting of relay  $R_1$  is 100% and  $R_2$  is 125%. The operating time characteristic of the relays is given in table. The TMS of relay  $R_1$  is 0.3. The time grading scheme has a discriminative margin of 0.5 s between the relays. A three phase short circuit at F results in a fault current of 5000 A. Find the actual operating time instants of  $R_1$  and  $R_2$ . What is the TMS of  $R_2$ ?

PSM	2	4	5	8	12	20
Operating time	10	5	4	3	2.8	2.4



[12 marks]

Given data - CT ratio = 1000/5

plug setting -  $R_1 \rightarrow 100\%$

$R_2 = 125\%$

TMS  $R_1 = 0.3$

margin = 0.5 sec

$I_{\text{fault}} = 5000 \text{ A}$

For  $R_1$  plug setting = 100%

$$I_{\text{pickup}} = 5 \text{ A} \times \frac{100}{100} = 5 \text{ A}$$

$$\text{PSM} = \frac{I_{\text{fault}}}{\text{CT ratio} \times I_{\text{pickup}}}$$

$$\text{PSM} = \frac{5000}{200 \times 5} = 5$$

from characteristics

top = 4 sec

$$\begin{aligned} \text{Actual top} &= \text{TMS} \times 4 \text{ sec} \\ &= 0.3 \times 4 = 1.2 \text{ sec} \end{aligned}$$

For Relay 2,

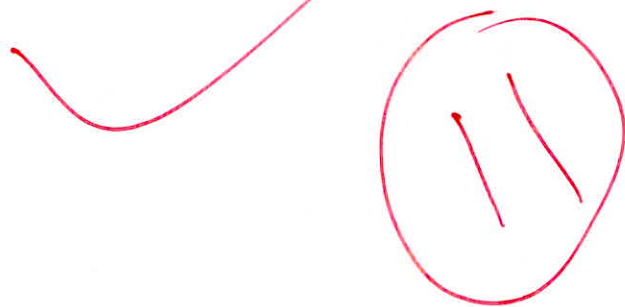
$$(\text{top})_{\text{Actual}} = 1.2 + 0.5 = 1.7 \text{ sec}$$

$$\text{PSM} = \frac{I_{\text{fault}}}{\text{CT ratio} \times I_{\text{pickup}}}$$

$$= \frac{5000}{200 \times 5 \times 1.25} = 4$$

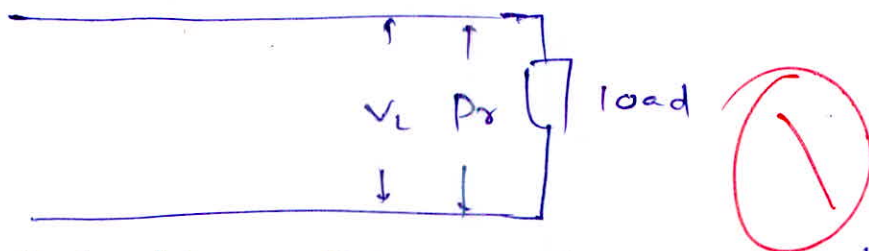
from characteristics,  
 $t_{op} = 5 \text{ sec}$   
 but for relay 2,  
 $(t_{op})_{Actual} = TMS \times 5 \text{ sec}$   
 $1.7 = TMS \times 5$

$$(TMS)_{R2} = 0.34$$



Q.1 (e) What is the percentage copper saving in feeder if the line voltage in a two-wire dc system be raised from 220 V to 500 V for the same power transmitted? State any assumption made.

[12 marks]



let the power transmitted be  $P$  watt.  
 at the voltage of 220V

① - Assume power factor of load is unity

then feeder c/n  $I = \frac{P}{V \cos \phi}$

$$I_1 = \frac{P_1}{220 \times (\frac{1}{1})} \quad \text{--- ①}$$

for the same power when vtg. is raised to 500V,

$$I_2 = \frac{P}{500 \times (1)}$$

② Assume the current density in both the cases is same



as  $I \propto Area$ .

$$\therefore \text{Saving of Cu} = \frac{(W_{Cu})_1 - (W_{Cu})_2}{(W_{Cu})_2} \times 100 \quad \text{--- (1)}$$

$W_{Cu}$  = wt. of copper

$W_{Cu} \propto Vol^m$  of Cu  $\propto Area$  of cs of Cu & current in copper feeder.

$$W_{Cu2} \propto I_2 \propto \frac{P}{500}$$

$$W_{Cu1} \propto I_1 \propto \frac{P}{220}$$

From (1)

$$\therefore \text{Saving} = \frac{\left(\frac{P}{500}\right) - \left(\frac{P}{220}\right)}{\left(\frac{P}{220}\right)} \times 100$$

$$= 0.56 \times 100$$

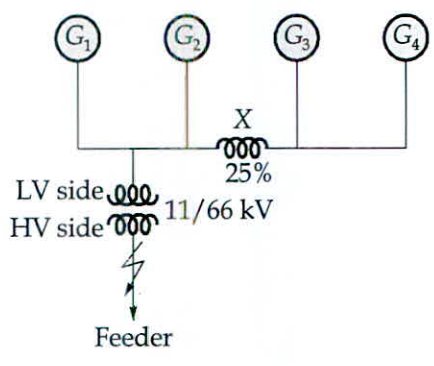
$\therefore$  saving = 56 %  
in copper

*wrong approach*



Q.2 (a)

A generating station has four identical generators,  $G_1, G_2, G_3$  and  $G_4$  each of 20 MVA, 11 kV having 20% reactance. They are connected to a bus bar which has a busbar reactor of 25% reactance on 20 MVA base, inserted between  $G_2$  and  $G_3$  as shown below. A 66 kV feeder is taken off from the bus bars through a 15 MVA, 11/66 kV transformer having 7.5% reactance. A symmetrical 3-phase fault occurs at the high voltage terminals of the transformers. Calculate the current feed into the fault.



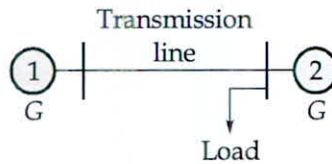
[20 marks]



- Q.2 (b) (i) A system consists of two plants connected by a transmission line as shown in figure below. The load is at plant-2. The transmission line loss calculations reveals that a transfer of 100 MW from plant-1 to plant-2 incurs a loss of 15 MW. Find the required generation at each plant for  $\lambda = 60$ . Assume that the incremental costs of the two plants are given by:

$$\frac{dC_1}{dP_1} = 0.2P_1 + 22 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_2} = 0.15P_2 + 30 \text{ Rs/MWh}$$



- (ii) Find the savings in Rs per hour by scheduling the generation by considering the transmission loss rather than neglecting the transmission loss in determining the outputs of the two generators. Assume a load of 285 MW connected at bus of plant-2.

[20 marks]



- Q.2 (c) A 50 Hz, 3 phase transmission line 300 km long has total series impedance of  $(40 + j125)$  ohm and a total shunt admittance of  $10^{-3}$  mho. The receiving end load is 50 MW at 220 kV with 0.8 lagging power factor. Calculate the sending end voltage, current, power and power factor using:
- (i) Short line approximation.
  - (ii) Nominal  $\pi$  method.
  - (iii) Exact transmission line equation of long line.
  - (iv) Approximation of long line.

[20 marks]





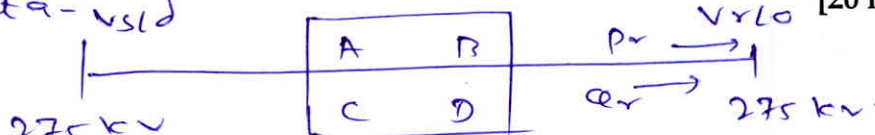


3 (a) A 275 kV transmission line has following line constants :

$$A = 0.85 \angle 5^\circ; B = 200 \angle 75^\circ$$

- (i) Determine the power at unity power factor that can be received if the voltage profile at each end is to be maintained at 275 kV.
- (ii) Calculate the value of compensation required for a load of 150 MW at unity power factor with the same voltage profile as in part (i).
- (iii) With the load as in part (ii), what would be the receiving end voltage if the compensation equipment is not installed?

Given data - vsld [20 marks]



$$A = 0.85 \angle 5^\circ \quad B = 200 \angle 75^\circ$$

i) For power at UPF, i.e.  $\phi_r = 0$   
 From power transfer equations,  

$$Q_r = \frac{|V_s||V_r|}{B} [\sin(\beta - \alpha)] - \frac{|A||V_r|^2}{|B|} \sin(\beta - \alpha)$$

$$Q_r = \frac{|V_s||V_r|}{B} [\sin(\beta - \alpha) - \frac{A}{|B|} \sin(\beta - \alpha)]$$

$$0 = \frac{(275)^2}{200} [\sin(75 - 5) - 0.85 \sin(75 - 5)]$$

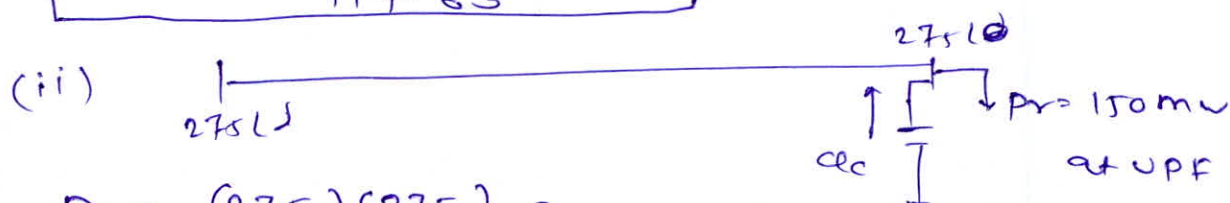
$$\sin(75 - \delta) = 0.85 \sin 70$$

$$\delta = 22^\circ$$

$$P_r = \frac{|V_s||V_r|}{|B|} \cos(\beta - \alpha) - \frac{|A||V_r|^2}{B} \cos(\beta - \alpha)$$

$$= \frac{(275)^2}{200} [\cos(75 - 22) - 0.85 \cos(70)]$$

$$P_r = \frac{200 \cdot 75}{117.63} \text{ mw}$$



$$P_r = \frac{(275)(275)}{200} [\cos(75 - \delta) - 0.85 \cos(70)]$$

$$150 = \frac{(275)^2}{200} [\cos(75 - \delta) - 0.85 \cos(70)]$$

$$\delta = 28.42^\circ$$

$Q_r$  in the line

$$Q_r = \frac{(V_s)(V_r)}{B} \sin(\beta - \delta) - \frac{A(V_s)^2}{B} \sin(\beta - \alpha)$$

$$= \frac{(275)^2}{200} \left[ \sin(75 - 28.42) - \frac{(0.85) 275^2 \sin(70)}{200} \right]$$

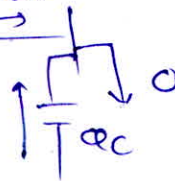
$$Q_r = -27.37 \text{ mVAR}$$

power balance at receiving end,

$$Q_r + Q_c = 0$$

$$Q_c = -Q_r$$

$$Q_c = 27.37 \text{ mVAR}$$



(iii) when load  $P = 150 \text{ mW}$  at VPF,

$$V_r = ?$$

$P_r$  = Receiving end power,

$$P = 150 \text{ mW}, \quad Q_r = 0$$

$$150 = \frac{(275)(V_r)}{200} \cos(75 - \delta) - \frac{(V_r)^2 (0.85) \cos(70)}{200}$$

$$0 = \frac{(275)(V_r)}{200} \sin(75 - \delta) - \frac{0.85(V_r)^2 \sin(70)}{200}$$

~~150.0014~~

$$150 + 1.453 \times 10^{-3} V_r^2 = \frac{(275)(V_r)}{200} \cos(75 - \delta)$$

$$0 + 3.9936 \times 10^{-3} V_r^2 = \frac{(275)(V_r)}{200} \sin(75 - \delta)$$

Squaring & adding

$$\frac{(150 + 1.453 \times 10^{-3} V_r^2)^2}{(150 + 1.453 \times 10^{-3})^2 V_r^2} + \frac{(3.9936 \times 10^{-3})^2 V_r^2}{(3.9936 \times 10^{-3})^2 V_r^2} = \frac{(275)^2 (V_r)^2}{(200)^2}$$



$$150^2 + 2 \times 150 \times 1.453 \times 10^{-3} V_r^2 + (1.453 \times 10^{-3})^2 V_r^4 - 1.89 V_r^2 = 0$$

$$(3.9936 \times 10^{-7}) V_r^4$$

$$- 1.89 V_r^2 = 0$$

$$\text{Solving above eqn } V_r^2 = 59,617.41$$

$$V_r = \sqrt{59,617.41} = 244.16 \text{ kV}$$

$$V_r = 244.16 \text{ kV}$$

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3 (b) A 100-MVA, 13.2 kV generator is connected to a 100 MVA, 13.2/132 kV transformer. The generator's reactances are  $X_d'' = 0.15$  p.u.,  $X_d' = 0.25$  p.u.,  $X_d = 1.25$  p.u. on a 100 MVA base, while the transformer reactance is 0.1 p.u. on the same base. The system is operating on no load, at rated voltage when a three phase symmetrical fault occurs at the HT terminals of the transformer. Determine:

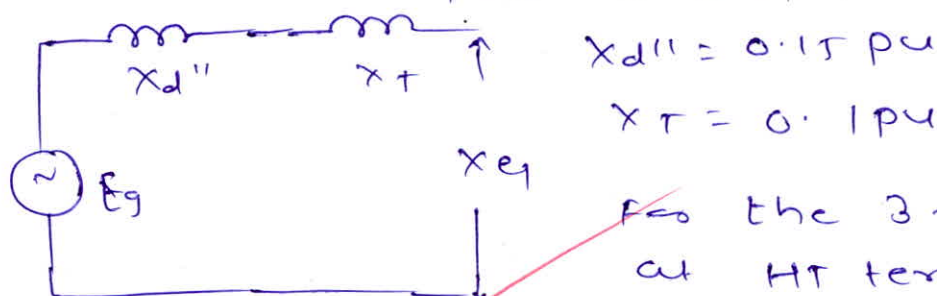
- The subtransient, transient and steady state symmetrical fault currents in p.u. and in amperes.
- The maximum possible dc component.
- Maximum value of instantaneous current.
- Maximum rms value of the asymmetrical fault current.

per phase equivalent circuit

[20 marks]

For 100 MVA base,

$$V_{\text{prefault}} = 1 \text{ pu}$$



$$X_d'' = 0.15 \text{ pu}$$

$$X_T = 0.1 \text{ pu}$$

For the 3- $\phi$  fault at HT terminal,

$$X_{eq} = X_d'' + X_T$$

$$= 0.15 + 0.1$$

$$X_{eq} = 0.25 \text{ pu}$$

$$(I_{\text{fault}})_{\text{sub-t}} = \frac{E_a}{X_{\text{sub-t}}} = \frac{1}{0.25} = 4 \text{ pu}$$



$$I_{base} = \frac{(mVA) \times 10^3}{\sqrt{3} \times (kV)} = 4373.86 \text{ Amp}$$

$$(I_{fault})_{sub-T} = 17.495 \text{ kA}$$

$$(I_{fault})_{trans} = \frac{E_{a1}}{X_{d'} + X_T} = \frac{1}{0.2 + 0.1} = 2.857 \text{ pu}$$

$$(I_{fault})_{trans} = 2.857 \times 4373.86 = 12.496 \text{ kA}$$

$$(I_{fault})_{SS} = \frac{E_{a1}}{X_d + X_T} = \frac{1}{1.2 + 0.1} = 0.740 \text{ pu}$$

$$(I_{fault})_{SS} = 0.740 \times 4373.8 = 3239.85 \text{ A}$$

② maximum DC component

$$\begin{aligned} (I_{dc})_{max} &= \sqrt{2} \times (I_{sub})_T \\ &= \sqrt{2} \times \left[ \frac{E_f}{X_{d''} + X_T} \right] \\ &= \sqrt{2} \times 17.495 \text{ kA} \end{aligned}$$

$$(I_{dc})_{max} = 24.741 \text{ kA}$$

③ maximum value of inst. cm.

$$(I_{inst})_{max} = (I_{dc})_{max} + (I_{sym})_{max}$$

$$\begin{aligned} &= \frac{2V_m}{X_{d''} + X_T} \\ &= \frac{2 \times \sqrt{2} \times V_{rms}}{(X_{d''} + X_T)} \\ &= 2 \times \sqrt{2} \times I_{sub-T} \\ &= 2\sqrt{2} \times 17.495 \end{aligned}$$

$$(I_{inst})_{max} = 49.48 \text{ kA}$$

④ maximum Asymmetrical current:

$$I_{Asymm} = 1.6 \times I_{symm}$$

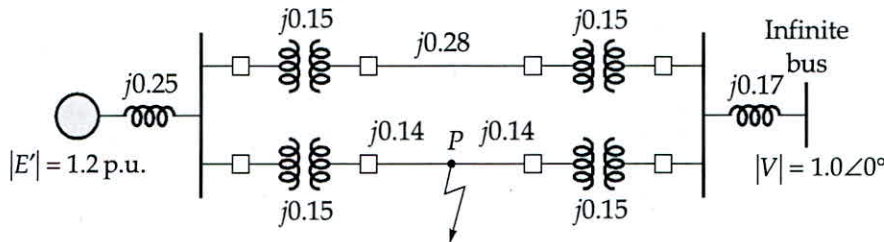
$$= 1.6 \times I_{sub-T}$$

$$= 1.6 \times 17.495 \text{ kA}$$

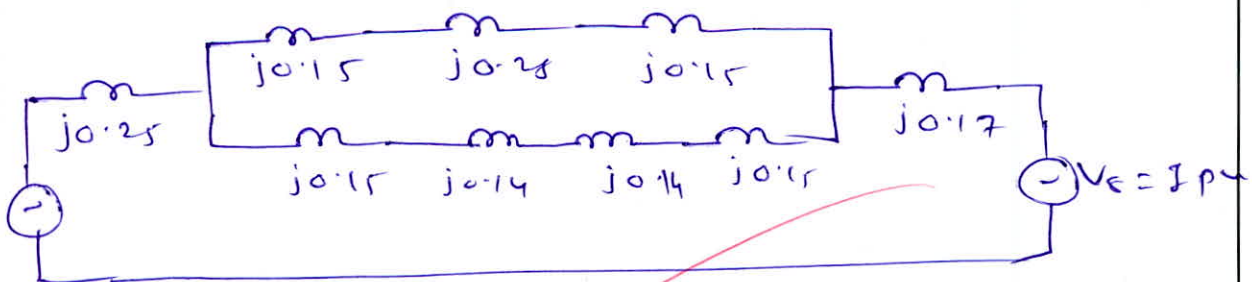
$$I_{Asymm} = 27.992 \text{ kA}$$

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1.3 (c) Find the critical clearing angle for the system in figure for a three phase fault at the point P. The generator is delivering 1.0 p.u. power under pre-fault conditions.



For the network, per phase circuit is [20 marks]



$P_{max}$  before fault,

$$P_{max} = \frac{|E_g| |V_c|}{X_{eq}}$$

$$E_g = 1.2 \text{ pu}$$

$$V_c = 1 \text{ pu}$$

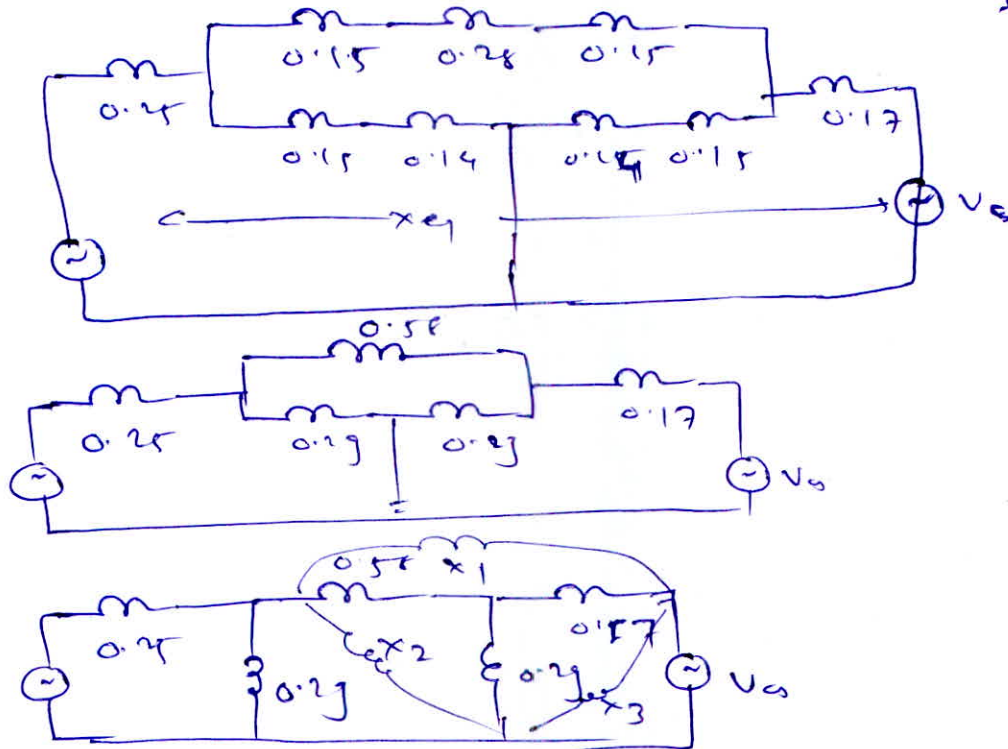
$$P_s = 1 \text{ pu}$$

$$X_{eq} = j0.25 + \frac{j(0.15 + 0.28 + 0.15)}{2} + j0.17$$

$$= 0.71$$

$$P_{max}' = \frac{(1.2)(1)}{0.71} = 1.690 \text{ pu.}$$

when fault occurs at middle of 2nd line

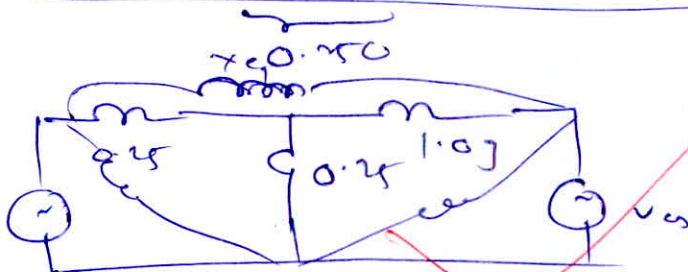
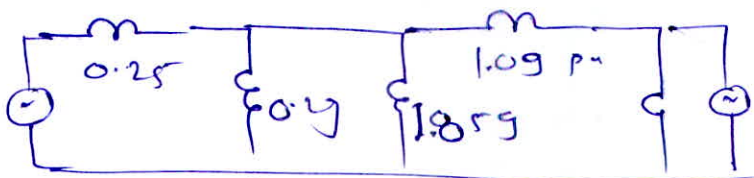


converting  $\gamma$  - to  $\Delta$

$$X_1 = 0.58 + 0.17 + \frac{(0.58)(0.17)}{0.29}$$

$$X_1 = 1.09$$

$$X_2 = 1.859 \text{ pu}$$



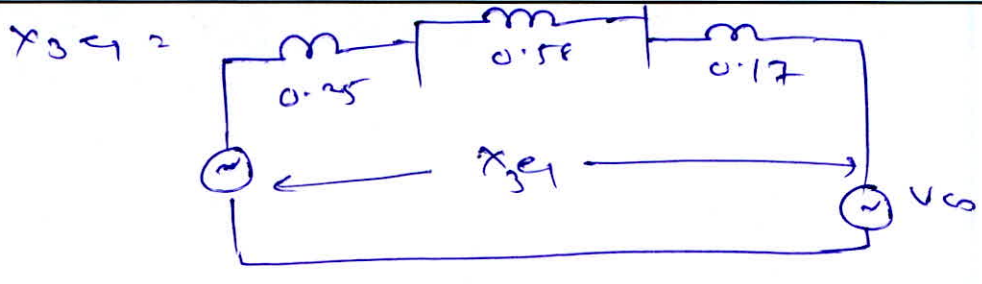
converting  $\gamma$  - to  $\Delta$

$$X_{e1} = 2.43 \text{ pu}$$

$$P_{2\text{max}} = \frac{(1.2)(1)}{2.43} = 0.493 \text{ pu}$$

when fault is cleared by opening 2nd line (faulty line)





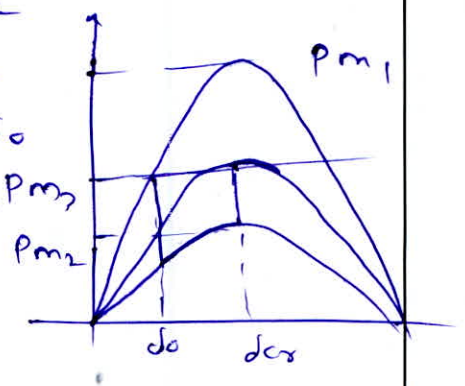
$X_{3\phi} = 1 \text{ pu}$

$P_{max3} = \frac{(1.2)(1)}{1} = 1.2 \text{ pu.}$

critical clearing angle,

$$\cos \delta_{cr} = \frac{P_s (\delta_m - \delta_0) + P_{m3} \cos \delta_m - P_{m2} \cos \delta_0}{P_{m3} - P_{m2}}$$

Initially  $P_s = P_{max1}$   
 $1 = 1.690 \sin \delta_0$   
 $\delta_0 = 36.27^\circ$



~~$\delta_m = \pi - \sin^{-1} \left( \frac{P_s}{P_{max3}} \right)$~~   
 ~~$= \pi - \sin^{-1} \left( \frac{1}{1.2} \right) = 123.55^\circ$~~   
 ~~$\frac{\pi}{2} = 90^\circ$~~

$$\cos \delta_{cr} = \frac{(1) \left[ \frac{123.55}{90} - 36.87 \right] \times \frac{\pi}{180} + (1.2) \cos \frac{123.55}{90} - 0.493 \times \cos(36.27)}{1.2 - 0.493}$$

~~$\cos \delta_{cr} > +0.45$  as  $P_s = P_{max3}$~~   
~~System will not be in stable state after fault.~~

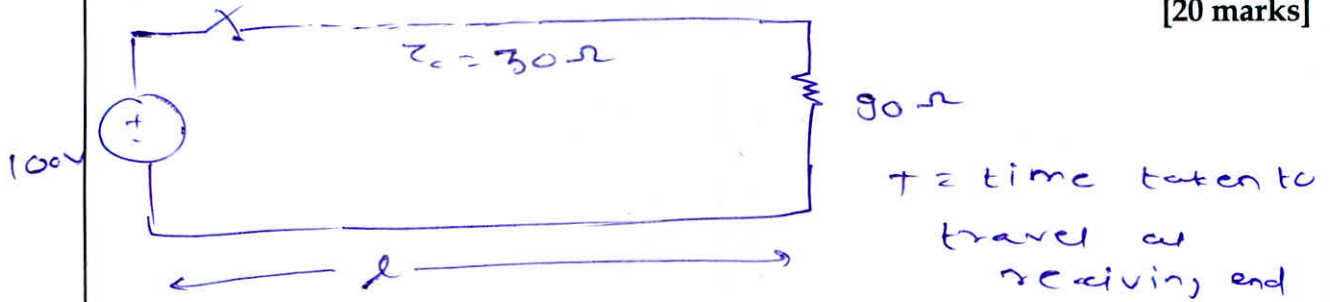
$\cos \delta_{cr} = 0.654$

$\delta_{cr} = 49.12^\circ$

18

Q.4 (a) A dc source of 100 V with negligible resistance is connected to a lossless line ( $Z_C = 30 \Omega$ ), through a switch S. If the line is terminated in a resistance of  $90 \Omega$ , on closing the switch at  $t = 0$ , plot the receiving end voltage ( $V_R$ ) w.r.t. time until  $5T$ . Where,  $T$  is the time for voltage wave to travel the length of the line. What will be the steady state voltage at receiving end? Also find the voltage at  $t = 3.25T$  on the mid length of the line.

[20 marks]

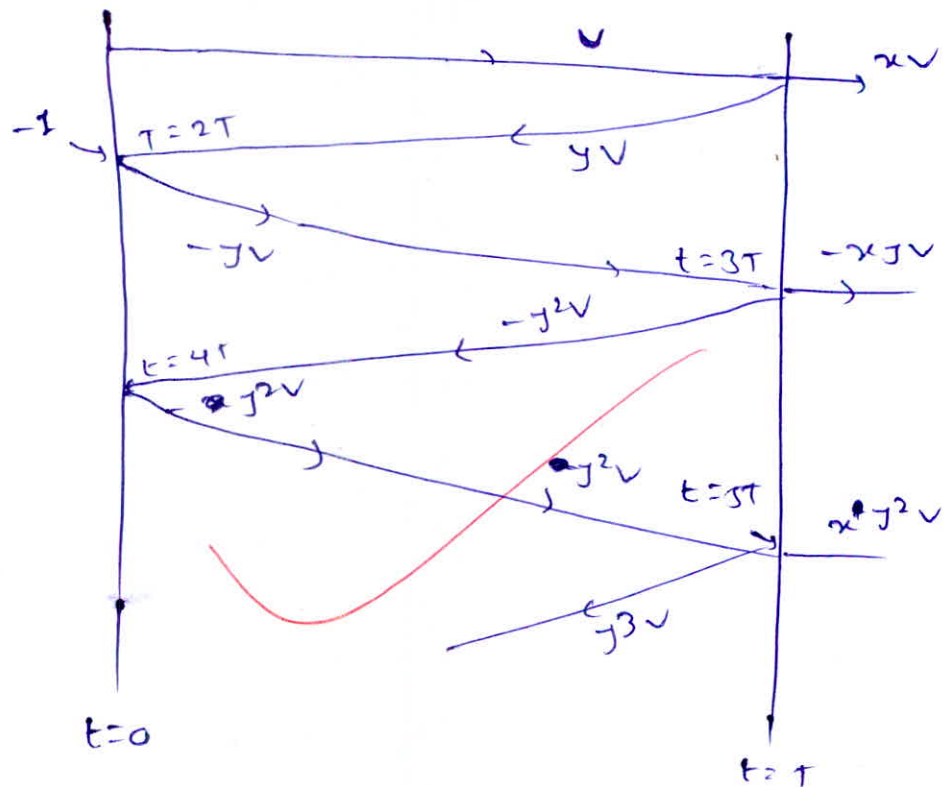


From refraction & reflection coefficients of voltage, At receiving end

$$\frac{V_{\text{refraction}}}{V_{\text{incident}}} = V_r = \frac{2Z_L}{Z_L + Z_C} = \alpha$$

$$V_r = \frac{2 \times 90}{90 + 30} = 1.5$$

$$\beta = \frac{V_{\text{reflection}}}{V_{\text{incident}}} = V_{\text{refl.}} = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{90 - 30}{90 + 30} = 0.5$$



where  $\alpha$  &  $\beta$  are refraction & reflection coefficients respectively.



at  $t = T$ ,

$$V_r = \alpha V = (1.5)(100) = 150 \text{ V}$$

at  $t = 3T$ ,

$$V_r = \alpha V - \alpha j V = \alpha [1 - j] V$$
$$= 1.5 [1 - 0.5j] \times 100$$

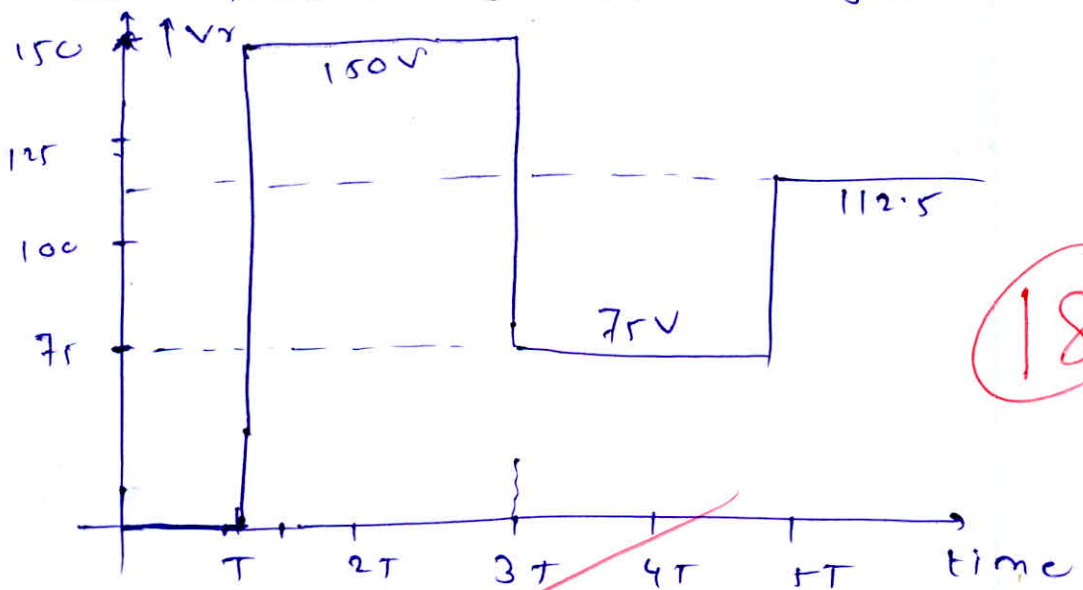
$$V_r \text{ at } 3T = 75 \text{ V}$$

$V_r$  at  $5T$

$$(V_r)_{5T} = \alpha V - \alpha j V + \alpha j^2 V$$
$$= \alpha [1 - j + j^2] V$$
$$= 1.5 [1 - 0.5j + 0.5^2] \times 100$$

$$(V_r)_{5T} = 112.5 \text{ V}$$

So, Receiving end voltage



at steady state,  $V_r =$

$$(V_r) = \alpha V - \alpha j V + \alpha j^2 V - \alpha j^3 V$$
$$= \alpha V [1 - j + j^2 + j^3 + \dots]$$
$$= \alpha V [1 + j]^{-1}$$

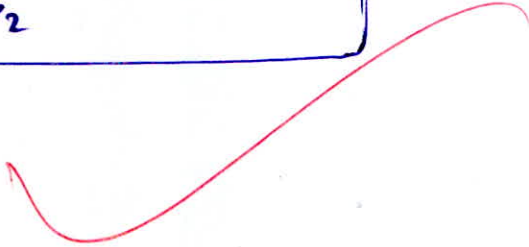
$$V_r = \left[ \frac{\alpha V}{1 + j} \right] = \frac{(1.5)(100)}{1 + 0.5j}$$

$$\boxed{V_r = 100 \text{ V}}$$

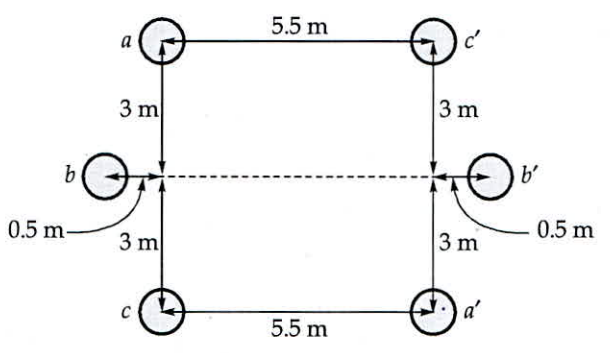
At the midlength of the line,  
at  $t = 3.25T$

$$(\text{Voltage})_{l/2} = V + \cancel{yV} - \cancel{yV}$$
$$= V$$

$(\text{Voltage})_{l/2} = 100 \text{ V}$	at $T = 3.25T$
--	----------------



2.4 (b) Determine the inductance of the double circuit fully transposed line shown in figure, if the self GMD of each conductor is 0.0069 m.



[20 marks]

Inductance per phase is given by

$$L_{ph} = 0.2 \ln \left[ \frac{G_{m d s y s t e m}}{G_{m R s y s t e m}} \right] \text{ mH/km}$$

$$G_{m d s y s t e m} = \sqrt[3]{D_{m(c a b)} D_{m(b c a)} D_{m(c a c)}}$$

$$G_{m R s y s t e m} = \sqrt[3]{G_{m R a} G_{m R b} G_{m R c}}$$

For, mutual GMD betn a & b

$$(D_m)_{ab} = [d_{ab} d_{ab} d_{ab1} d_{ab1}]^{1/4}$$

$$d_{ab} = \sqrt{3^2 + 0.5^2} = 3.041 \text{ m}$$

$$d_{ab1} = \sqrt{(5.5 + 0.5)^2 + 3^2} = 6.264 \text{ m}$$

$$(D_{ab})_m = [3.041 \times 6.264 \times 3.041 \times 6.264]^{1/4}$$

$$(D_m)_{ab} = 4.364 \text{ m}$$

For mutual GMD betn b & c

$$(D_m)_{bc} = [d_{bc} d_{bc} d_{bc1} d_{bc1}]^{1/4}$$

$$= (3.041 \times 6.264 \times 3.041 \times 6.264)^{1/4}$$

$$(D_m)_{bc} = 4.364 \text{ m}$$

For mutual GMD betn a & c

$$(D_m)_{Ac} = (d_{ac} d_{ac} d_{ac1} d_{ac1})^{1/4}$$

$$= (6 \times 5.5 \times 5.5 \times 6)^{1/4}$$

$$(D_m)_{Ac} = 5.74 \text{ m}$$

$$(Gmd)_{system} = \sqrt[3]{4.364 \times 4.364 \times 5.744}$$

$$(Gmd)_{system} = 4.782 \text{ m}$$

For GMR<sub>a</sub>

$$(GMR)_a = (r' \times daa)^{1/2} = (0.0069 \times 8.139)^{1/2}$$

$$(GMR)_a = 0.236 \text{ m}$$

$$GMR_b = (r' \times dbb)^{1/2} = (0.0069 \times 6.5)^{1/2}$$

$$(GMR)_b = 0.211 \text{ m}$$

$$(GMR)_c = (r' \times dcc)^{1/2} = (0.0069 \times 8.139)^{1/2}$$

$$(GMR)_c = 0.236 \text{ m}$$

$$(GMR)_{system} = \sqrt[3]{GMR_a \times GMR_b \times GMR_c}$$

$$= \sqrt[3]{0.236 \times 0.211 \times 0.236}$$

$$(GMR)_{system} = 0.227 \text{ m}$$

$$L_{ph} = 0.2 \ln \left[ \frac{4.782}{0.227} \right] \text{ mH/km}$$

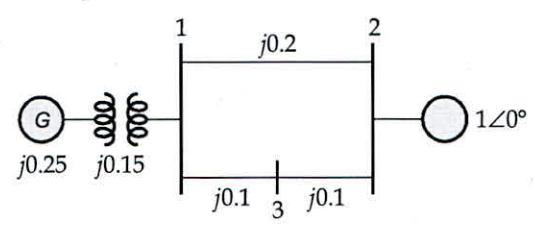
$$L_{ph} = 0.609 \text{ mH/km}$$

very good

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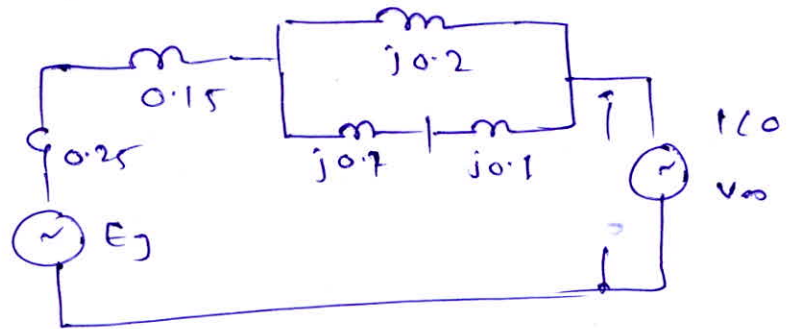


2.4 (c) A single line diagram of a system is shown below:



All the values are in per unit on a common base. The power delivered into bus-2 is 1.0 p.u. at 0.8 p.f. lagging. Obtain the power angle equation and swing equation of the system. Neglect all losses.

per phase network of the system is [20 marks]



$$P = V_r I_r \cos \phi$$

$$I_r = \frac{(1)}{(1) \cos 0.8} = 1.25 \angle -36.87^\circ$$

$$E_g = V_{\infty} + (I_{bus})_2 [j(0.2 || 0.2) + j0.4]$$

$$= 1 \angle 0 + 1.25 \angle -36.87^\circ [j0.1 + j0.4]$$

$$E_g = 1.46 \angle 19.98^\circ$$

$$P_{max \text{ transferred}} = \frac{|E_g| V_{\infty}}{x_{eq}}$$

$$= \frac{(1.46)(1)}{(0.5)}$$

$$P_{max} = 2.92 \text{ pu}$$

power angle eqn,

$$P_r = \frac{E V}{x} [\sin(\delta_0 - \alpha)] - \frac{V^2 \cos \delta_0}{x}$$

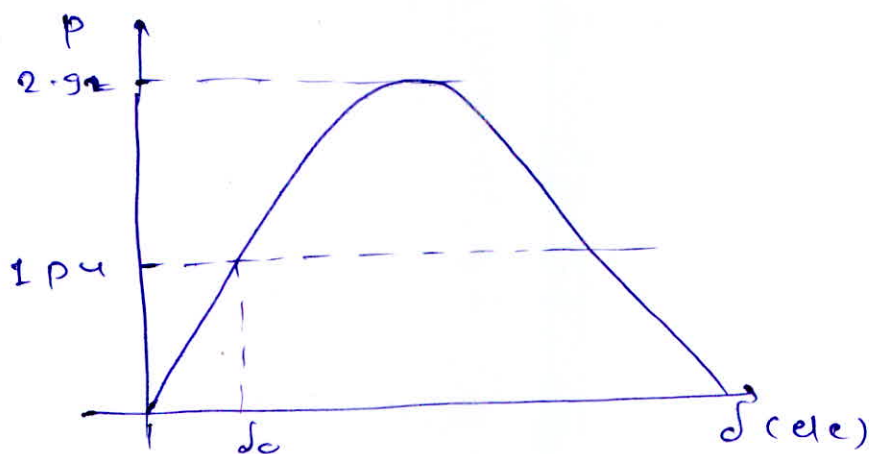
$$= \frac{E V}{x} \sin \delta$$

$$P_r = 2.92 \sin \delta$$

———— (1)



By neglecting all the losses



swing eqn is

$$P_s - P_e = m \frac{d^2 \theta}{dt^2}$$

$$P_s - P_e = m \frac{d^2 \delta}{dt^2} \quad \because \left[ \frac{d^2 \theta}{dt^2} = \frac{d^2 \delta}{dt^2} \right]$$

$$1 - 2.92 \sin \delta = m \frac{d^2 \delta}{dt^2} \quad \because \delta \rightarrow \text{ele. angle}$$

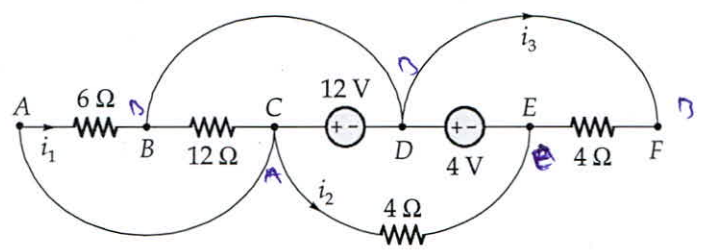
where  $m$  is the angular momentum  
in pu-mw-sec<sup>2</sup>/ele-rad.

$$\boxed{m \frac{d^2 \delta}{dt^2} = 1 - 2.92 \sin \delta}$$

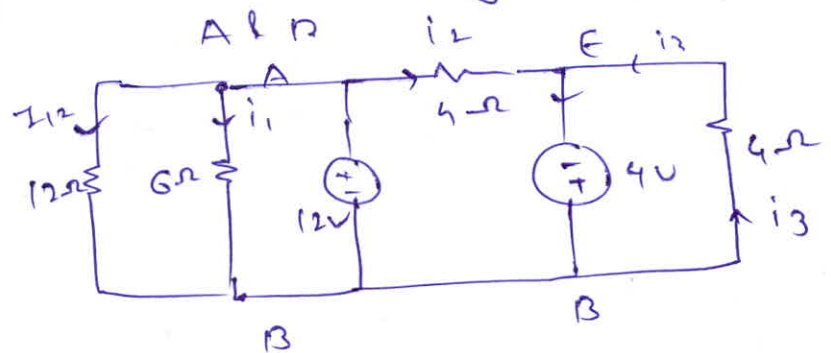
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Section B : Electrical Circuits-1 + Microprocessors-1 + Digital Electronics-2 + Control Systems-2

2.5 (a) Find the current  $i_1, i_2, i_3$  and power delivered by the sources of the network shown in figure.



By Rearranging the network betn [12 marks]



$i_1 = \frac{12}{6\Omega} \rightarrow$  by ohm's law

$i_1 = 2 \text{ Amp}$

$i_2 = \frac{12+4}{4} = \frac{16}{4} = 4 \text{ Amp}$

$i_2 = 4 \text{ Amp}$

$i_3 = \frac{4}{4} = 1 \text{ Amp}$

$i_3 = 1 \text{ Amp}$



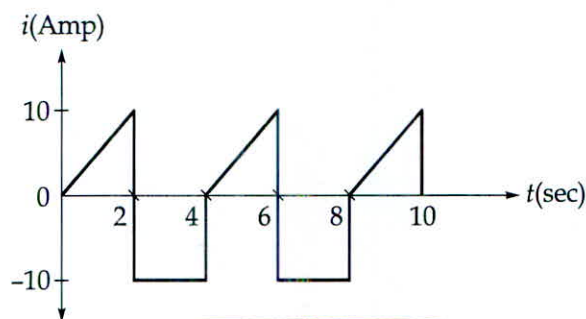
power delivered by 12V source =  $V(i_1 + i_2 + i_3)$   
 $= 12(2 + 4 + 1)$

$P_{12V} = 84 \text{ watt}$

power delivered by 4V source =  $V(i_2 + i_3)$   
 $= 4(4 + 1)$

$P_{4V} = 20 \text{ watt}$

- Q.5 (b) Determine the rms value of the waveform. If the current is passed through a  $9\ \Omega$  resistor. Find the average power absorbed by the resistor.



$$I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T I(t)^2 \cdot dt}$$

[12 marks]

$$I(t) = \begin{cases} \frac{10}{2}t & 0 \leq t < 2 \\ -10 & 2 \leq t < 4 \end{cases} \quad T = 4 \text{ sec}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{4} \int_0^2 (5t)^2 \cdot dt + \int_2^4 (-10)^2 \cdot dt}$$

$$= \sqrt{\frac{1}{4} \left\{ \left[ \frac{25t^3}{3} \right]_0^2 + 100 [t]_2^4 \right\}}$$

$$= \sqrt{\frac{1}{4} \left[ \frac{25}{3} (8-0) + 100 (4-2) \right]}$$

$$I_{\text{rms}} = 8.164 \text{ Amp}$$

Power loss in resistor,

$$P_{\text{avg}} = I_{\text{rms}}^2 \times R$$

$$= (8.164)^2 \times 9$$

$$P_{\text{avg}} = 600.59 \text{ watt}$$

2.5 (c) A third order system has state space model matrices as follows:

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = [1 \ 1 \ 0]$$

Determine whether the system is state controllable or not. What will be the output controllability of the system?

[12 marks]

For the controllability of the system

$$M = [B \ AB \ A^2B]$$

$$|M| \neq 0$$

$$AB = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 3 & 7 & 7 \\ 0 & 1 & 0 \\ 14 & -22 & 38 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 38 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 7 \\ 0 & 0 & 0 \\ 1 & 6 & 38 \end{bmatrix} = 0 \quad |M| = 0$$

System is not completely controllable,

For o/p controllability,

$$N = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} \quad N \neq 0$$

$$[CA] = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix}$$

$$CA = [1 \ 4 \ 1]$$

$$CA^2 = [1 \ 1 \ 0] \begin{bmatrix} 3 & 7 & 7 \\ 0 & 1 & 0 \\ 14 & -22 & 38 \end{bmatrix} = [3 \ 3 \ 7]$$

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O/P controllability matrix

$$N = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 1 \\ 3 & 3 & 7 \end{bmatrix}$$

$$\det |N| = 1(28-3) - 1(7-3)$$

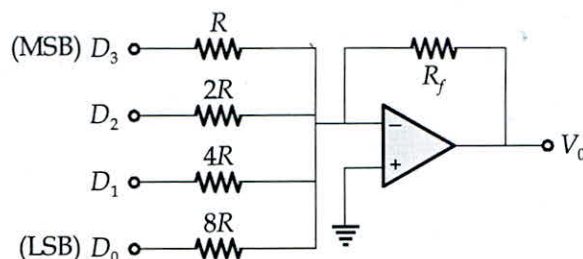
$$= 25 - 4$$

$$= 21 \neq 0$$

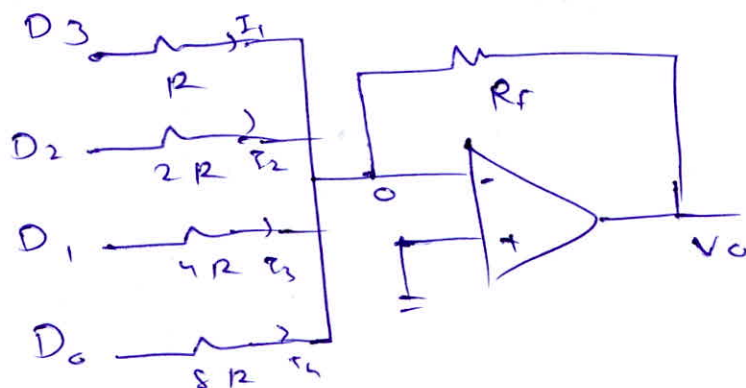
System is O/P controllable.



- 2.5 (d) (i) With a neat block diagram, explain the operation of 4-bit weighted-resistor type DAC.
- (ii) For the 4-bit weighted-resistor DAC shown in below figure. Determine the weight of each input bit if the inputs are 0 V and 5 V and also determine the full-scale output, if  $R_f = R = 1 \text{ k}\Omega$ . Also, find the full scale output if  $R_f$  is changed to  $500 \Omega$ .



i) Four bit weighted resistor type DAC [12 marks]



It requires 4 different resistors,  
all at inverting terminal,

$$\left[ \frac{D_3 V_{ref}}{R} + \frac{D_2 V_{ref}}{2R} + \frac{D_1 V_{ref}}{4R} + \frac{D_0 V_{ref}}{8R} \right] = -\frac{V_o}{R_f}$$

$$\frac{V_{ref}}{8R} [(2)^3 D_3 + (2)^2 D_2 + (2)^1 D_1 + D_0] = -\frac{V_o}{R_f}$$

$$V_o = -\frac{V_{ref} R_f}{2^3 R} [4 \text{ bit equivalent of digital I/P}]$$

$$\text{Resoln} = \frac{-V_{ref} R_f}{2^{n-1} R}$$

$$V_o = \text{Resoln} \times 4 \text{ bit equivt. of digital I/P}$$

- (ii) weight of  $D_0 = 2^0 = 1$   
 weight of  $D_1 = 2^1 = 2$   
 weight of  $D_2 = 2^2 = 4$   
 weight of  $D_3 = 2^3 = 8$

$$(V_{FS})_{o/p} \quad D_3 = D_2 = D_1 = D_0 = 1$$

$$(V_{FS})_{o/p} = \frac{-5 (1k)}{2^3 \times (1k)} [2^3(1) + 2^2(1) + 2^1(1) + 2^0(1)]$$

$$= \frac{-5}{2^3} \times [15]$$

$$(V_{FS})_{o/p} = -9.375 V$$

$$\text{when } R_F = 500 = 0.5 k$$

$$(V_{FS})_{o/p} = \frac{(-5) (0.5 k)}{(1) (1k)} [2^3 + 2^2 + 2^1 + 2^0]$$

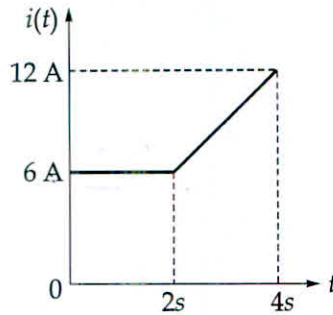
$$V_{FS o/p} = -4.6875 V$$

Q.5 (e) Explain the generation of control signals for memory and I/O devices of 8085 microprocessors?

[12 marks]



- Q.6 (a) (i) Figure below shows the waveform of the current passing through an inductor of resistance  $2 \Omega$  and inductance  $2 \text{ H}$ . Find the energy absorbed by the inductor in the first four seconds.



Energy absorbed =  $\int_0^4 v_L \cdot i \cdot dt + \int_0^4 v_R \cdot i \cdot dt$  [12 marks]

For  $0 < t < 2$ ,  $v = L \frac{di}{dt} = (2)(0)$

$v_L = 0$

For  $2 < t < 4$   $v = (2) \left[ \frac{12-6}{4-2} \right] = (2) \left( \frac{6}{2} \right) = 6v$

Energy absorbed =  $\int_0^2 v_L \cdot i \cdot dt + \int_2^4 (6) \cdot 3t \cdot dt$

=  $0 + \int_2^4 18 \left( \frac{t^2}{2} \right) \cdot dt$

=  $9 \left( \frac{t^3}{3} \right) \Big|_2^4$

=  $9(16 - 4) = 96 \text{ J}$

Energy dissipated =  $\int_0^2 v_R \cdot i \cdot dt + \int_0^4 v_R \cdot i \cdot dt$

=  $\int_0^2 i^2 \cdot R \cdot dt + \int_0^4 (3t)^2 \cdot 2 \cdot dt$

=  $\int_0^2 i^2 \cdot R \cdot dt + \int_0^4 18 t^2 \cdot dt$

=  $6^2 (2) [t]_0^2 + 18 \left( \frac{t^3}{3} \right) \Big|_0^4$

=  $36 \times 2 \times 2 + 6 [64]$

=  $144 \text{ J} + 384 \text{ J}$

Energy dissipated =  $528 \text{ J}$

Energy absorbed by inductor in 4 sec

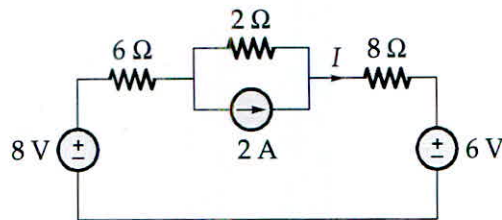
$$E_{\text{absorbed}} = E_{\text{stored}} + E_{\text{dissipated}}$$

$$= 63 + 144 + 385$$

$$E_{\text{absorbed}} = 591 \text{ J}$$

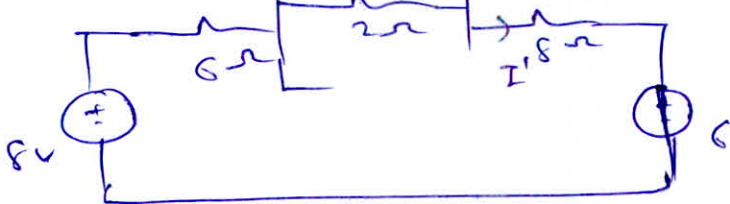


Q.6 (a) (ii) Find the current  $I$  in the circuit shown below using the superposition theorem.



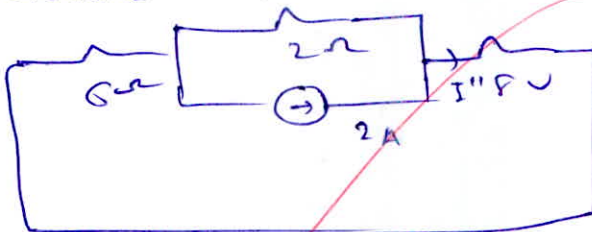
consider 8V source

[8 marks]



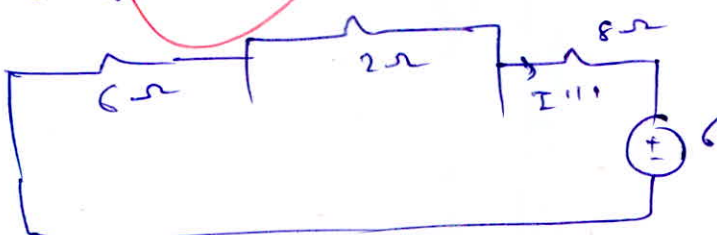
$$I' = \frac{8}{16} = 0.5 \text{ Amp}$$

consider 2A source



$$I'' = 2 \times \frac{2}{16} = 0.25 \text{ Amp}$$

consider 6V source



$$I''' = \frac{-6}{16} = -0.375 \text{ Amp}$$

By superposition theorem,

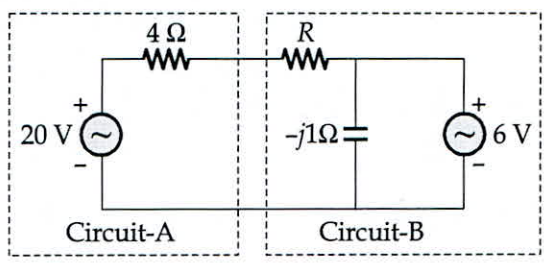
$$I = (I') + (I'') + (I''')$$

$$= 0.5 + 0.25 - 0.375$$

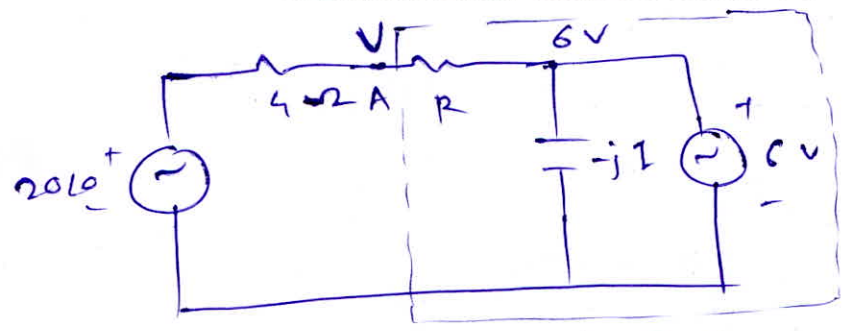
$$I = 0.375 \text{ Amp}$$

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6 (b) (i) Assuming both the voltage sources are in phase, find the value of  $R$  for which maximum power is transferred from circuit A to circuit B.



[12 marks]



For  $P_{max}$  to be transfer from ckt A to ckt B, 50% of source  $V_{tg}$  should be dropped across ckt. B  
 So,  $V = \frac{20}{2} = 10V$

now KCL at terminal A,

$$\frac{20 - V}{4} = \frac{V - 6}{R}$$

$$\frac{20 - 10}{4} = \frac{10 - 6}{R}$$

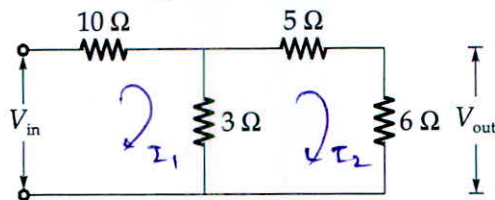
$$R = \frac{10}{\frac{(4)(4)}{10}}$$

$$R = 1.6 \Omega$$

10

good

Q.6 (b) (ii) Determine the voltage ratio  $V_{out}/V_{in}$  for the circuit shown below:



For  $V_{out} = (I_2) 6\Omega$  — (A) [8 marks]

kvl in loop ①

$$V_{in} = 10I_1 + 3(I_1 - I_2)$$

$$V_{in} = 13I_1 - 3I_2 \quad \text{--- (1)}$$

For kvl in loop ②

$$-I_1(3) + 14I_2 = 0$$

$$3I_1 = 14I_2$$

$$I_1 = \frac{14}{3} I_2 \quad \text{--- (2)}$$

$$V_{in} = 13 \left[ \frac{14}{3} \right] I_2 - 3I_2$$

$$V_{in} = 57.66 I_2 \quad \text{--- (3)}$$

$$V_{out} = 6I_2 \quad \text{--- from (A)}$$

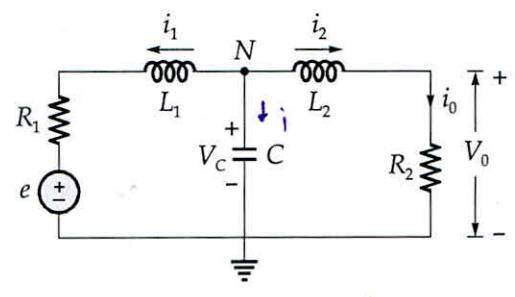
$$\frac{V_{out}}{V_{in}} = \frac{\cancel{57.66} I_2 \cdot 6I_2}{57.66 I_2}$$

$$\frac{V_{out}}{V_{in}} = 0.1040$$



6 (c)

Obtain the state space model considering current through the two inductors and voltage across the capacitor as state variables. Consider the output variables as current through  $R_2$  and voltage across  $R_2$  and input variable as 'e' as shown in figure. (Consider initial conditions to be zero)



[10 marks]

state variables be  $i_{L1}, i_{L2}$

KEC at  $v_c$

$$C \frac{dv_c}{dt} = -(i_{L1} + i_{L2})$$

$$\frac{dv_c}{dt} = \left(-\frac{1}{C}\right) i_{L1} + \left(-\frac{1}{C}\right) i_{L2} \quad \text{--- (1)}$$

$$L_1 \frac{di_{L1}}{dt} = \frac{v_c - e}{R_1} \quad \rightarrow \text{voltage across } L_1$$

$$\left(\frac{di_{L1}}{dt}\right) = \left(\frac{1}{R_1 L_1}\right) v_c - \left(\frac{e}{R_1 L_1}\right) \quad \text{--- (2)}$$

$$L_2 \frac{di_{L2}}{dt} = \frac{v_c - i_{L2} R_2}{L_2} \quad \rightarrow \text{voltage across } L_2$$

$$\left(\frac{di_{L2}}{dt}\right) = \left(\frac{v_c}{L_2}\right) - \left(\frac{R_2}{L_2}\right) i_{L2} \quad \text{--- (3)}$$

$$V_0 = (i_{L2}) R_2 \quad \text{--- (4)}$$

from (1), (2) & (3)

$$\begin{bmatrix} \frac{dv_c}{dt} \\ \frac{di_{L1}}{dt} \\ \frac{di_{L2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} & -\frac{1}{C} \\ \frac{1}{R_1 L_1} & 0 & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} v_c \\ i_{L1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{R_1 L_1} \\ 0 \end{bmatrix} e$$

$$[V_0] = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} v_c \\ i_{L1} \\ i_{L2} \end{bmatrix}$$



thus,

$$\begin{bmatrix} v_c \\ i_{L1} \\ i_{L2} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} & -\frac{1}{L} \\ \frac{1}{R_{L1}} & 0 & 0 \\ \frac{1}{L} & 0 & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v_c \\ i_{L1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{R_{L1}} \\ 0 \end{bmatrix} e$$

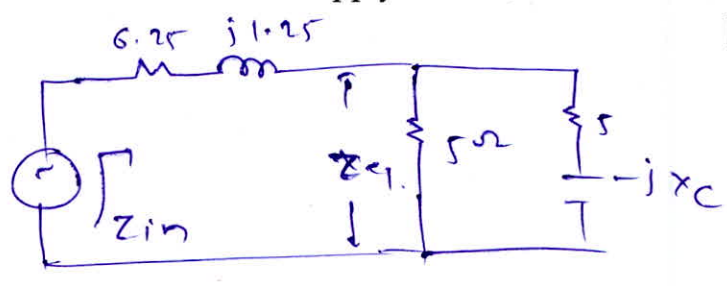
$$V_o = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} v_c \\ i_{L1} \\ i_{L2} \end{bmatrix}$$

8



6 (d) Two impedances  $Z_1 = 5 \Omega$  and  $Z_2 = (5 - jX_C)\Omega$  are connected in parallel and this combination is connected in series with  $Z_3 = (6.25 + j1.25)\Omega$ . Determine the value of capacitance of  $X_C$  to achieve resonance if the supply is 100 V, 50 Hz.

[10 marks]



to achieve resonance

$$Z_{in} = 6.25 + j1.25 + Z_p$$

$$\text{let } Z_p = R_p + jX_p$$

$$Z_{in} = 6.25 + j1.25 + (R_p + jX_p)$$

$$\text{Imj} [Z_{in}] = 0$$

$$\text{i.e. } j1.25 + jX_p = 0$$

$$X_p = -1.25$$

~~$$Z_p = 5 \parallel (5 - jX_c)$$~~

~~$$\frac{1}{-j1.25} = \frac{1}{5} + \frac{1}{5 - jX_c} \Rightarrow \frac{1}{Z_p} = \frac{1}{5} + \frac{1}{5 - jX_c}$$~~

~~$$\left[ \frac{1}{5 - jX_c} \right] = \frac{1}{5} + \frac{1}{j1.25} \Rightarrow \frac{1}{5 - jX_c} = \frac{1}{5} + \frac{j}{1.25}$$~~

8

~~$$\left( \frac{1}{5 - jX_c} \right)^2 = 0.824 (-75 - 96) = \frac{25 - j5X_c}{10 - jX_c}$$~~

~~$$\frac{1}{5 - jX_c} = 0.824 (104.4) = \frac{1}{5} + \frac{jX_c}{5^2 + X_c^2}$$~~

C = ?

~~$$\frac{1}{5 - jX_c} = \frac{1}{-0.301 - 1.75j}$$~~

~~$$\frac{1}{Z_p} = 0.25$$~~

~~$$Z_p = \frac{5(5 - jX_c)}{10 - jX_c} = \frac{(25 - j5X_c)(10 + jX_c)}{(10 - jX_c)(10 + jX_c)}$$~~

~~$$Z_p = \frac{(j25X_c - j50X_c) + (\text{Real part})}{(10^2 + X_c^2)}$$~~

~~$$-1.25 = \frac{-25X_c}{10^2 + X_c^2} \Rightarrow X_c = 10 \Omega$$~~

Q.7 (a) (i) Clearly differentiate between latches and flip-flops.

[8 marks]

1.7 (a)

(ii) Realize *T*-flip flop using *D*-flip flop.

**[12 marks]**

- Q.7 (b) (i) For a control system, the input output transfer function is given below:

$$\frac{Y(s)}{U(s)} = \frac{b_0s^n + b_1s^{n-1} + \dots + b_{n-1}s + b_n}{s^n + a_1s^{n-1} + a_2s^{n-2} \dots a_{n-1}s + a_n}$$

Construct a state diagram by direct decomposition using the first companion form. Use state diagram to obtain dynamic equations and state space model.

- (ii) Obtain the second companion form for the system whose input output transfer function is given by,

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 4s + 1}{s^3 + 3s^2 + 4s + 8}$$

Draw corresponding state diagram for above form and derive state space model for above system.

[20 marks]





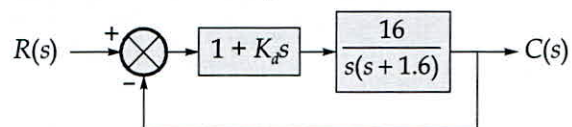


- 2.7 (c) (i) Write an assembly language program for an 8085 microprocessor, to find 2's complement of a 16-bit number. Write comments for selected instructions.
- (ii) Enumerate all internal registers present in 8259 programmable interrupt controller? Write short notes on their individual functionality?

[12 + 8 marks]



- 1.8 (a) A control system employing proportional and derivative control as shown below, has damping ratio equal to 0.8. Find the time instant at which the step response of system attains the peak value. Also find the percent maximum overshoot of system.



[20 marks]







Q.8 (b) Design a 3-bit gray UP/DOWN synchronous counter using  $T$ -flip flops with a control for UP/DOWN counting.

[20 marks]





8 (c) A control system is represented by the state equation given below,

$$\dot{x}(t) = Ax(t)$$

If the response of the system is  $x(t) = \begin{pmatrix} e^{-t} \\ -2e^{-t} \end{pmatrix}$  when  $x(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$  and  $x(t) = \begin{pmatrix} e^{-2t} \\ -e^{-2t} \end{pmatrix}$

when  $x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ .

Calculate the system matrix A and the state transition matrix for the system.

[20 marks]





**Space for Rough Work**

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**Space for Rough Work**

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