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India's Best Institute for IES, GATE & PSUs

# ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

## Electrical Engineering

### Test-3 : Power Systems

### Electrical Circuits-1 + Microprocessors-1

### Digital Electronics-2 + Control Systems-2

Name : ....

Roll No : | E | E | 1 | 9 | M | B | D | L | A | 7 | 3 | 2 |

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**Student's Signature**

**Instructions for Candidates**

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

**FOR OFFICE USE**

| Question No. | Marks Obtained |
|---|---|
| Section-A | |
| Q.1 | 44 |
| Q.2 | 41 |
| Q.3 | |
| Q.4 | |
| Section-B | |
| Q.5 | 43 |
| Q.6 | 54 |
| Q.7 | |
| Q.8 | 35 |
| **Total Marks Obtained** | 217 |

Signature of Evaluator: Sourabh kumar

Cross Checked by

## Section A : Power Systems

**(a)** Estimate the corona loss for a three-phase, 110 kV, 50 Hz, 150 km long transmission line consisting of three conductors each of 10 mm diameter and spaced 2.5 m apart in an equilateral triangle formation. The temperature of air is 30° C and the atmospheric pressure is 750 mm of mercury. Take the irregularity factor as 0.85. Ionization of air may be assumed to take place at a maximum voltage gradient of 30 kV/cm.

**[12 marks]**

Given data $V_{ph} = \frac{110}{\sqrt{3}} \times 10^3$ volts

$l = 150$ km

$f = 50$ Hz

$g = \frac{30}{\sqrt{2}}$ kV/cm (rms)

$r = 0.5 \times 10^{-2}$ cm.

$d = 2.5$ m

$m = 0.85$

pressure of atmosphere

$b = 750$ mm

$b = 75$ cm.

$t = 30°C$

~~$T = 273+30$~~ =

$$\delta = \frac{3.92b}{273+t} = \frac{3.92 \times 75}{273+30} = 0.9703$$

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Critical disruptive voltage

$$V_{c\,rms} = m g r \delta \ln(d/r)$$

$$= .85 \times \frac{30}{\sqrt{2}} \times 0.5 \times 0.9703 \ln\left(\frac{2.5}{5 \times 10^{-3}}\right)$$

$$= 54.36 \text{ kV/phase (rms)}$$

Corona loss

$$P_c = 242.2 \times 10^{-5} \frac{(f+25)}{\delta} \sqrt{\frac{r}{d}} (V_{ph} - V_c)^2 \text{ kW/phase/km.}$$

$$= 242.4 \times 10^{-5} \times \frac{(75)}{.9703} \sqrt{\frac{5 \times 10^{-3}}{2.5}} \left(63.5 - 54.36\right)^2$$

$$= 0.701 \text{ kW/phase/km}$$

Total corona loss for whole line $= 150 \times .7 = 104.8$ kW/phase

~~Total corona loss = $P_c \times l$ = $0.701 \times 150$ = 105 kW~~

Total Corona loss = 104.8 × 3 = ??

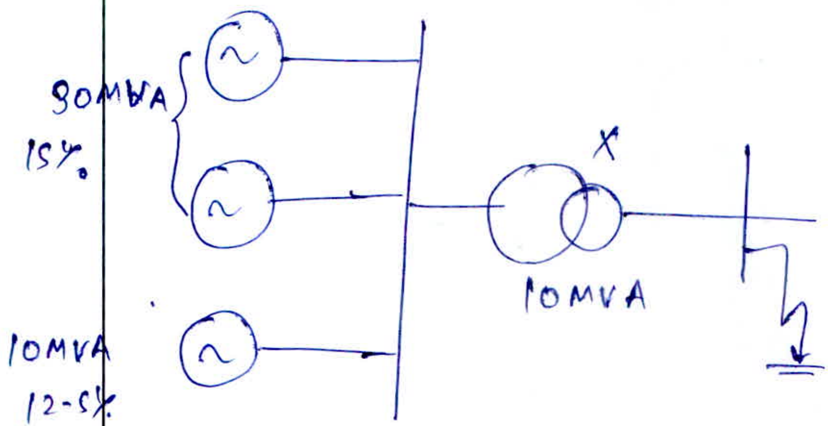
**Q.1 (b)** A power plant has three generators feeding a common bus:

2 generators, 30 MVA, 15% reactance each

1 generator, 10 MVA, 12.5% reactance

A 10 MVA transformer steps up the voltage and feeds a 33 kV circuit. Find the safe minimum reactance of transformer so that fault level on the secondary bus of the transformer may not exceed 100 MVA.

**[12 marks]**

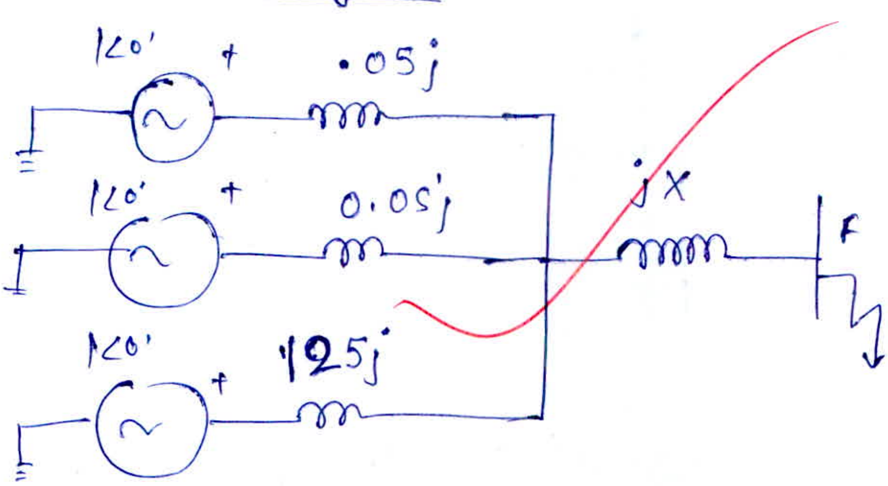



Let MVA base is 10 MVA

hence bigger generator 'pu' reactance of 10 MVA base

$$X_{g_1} = X_{g_2} = .15 \times \frac{10}{30} = .05 \text{ pu}$$

Reactance diagram




Thevenin reactance at fault point

$X_{Th} = X + .05 || .05 || .125$

$X_{Th} \quad X + 0.02083$

SC MVA or Fault level at secondary bus in pu

$$= \frac{100}{10} = \frac{1}{X_{Th}} \Rightarrow X_{Th} = 0.1$$

$$X + 0.02083 = 0.1$$

$X_{pu} = 0.07917 pu$

hence X (reactance of transfer) in ohms

$= X_{pu} \times$ Base on secondary side

$= 0.07917 \times \frac{33^2}{10}$

$= 8.62125 \Omega$

(1)

good.

**1 (c)** A power plant has 3 units with the input output curves.

$Q_1 = 0.002 P_1^2 + 0.86 P_1 + 20$ tons/hour
$Q_2 = 0.004 P_2^2 + 1.08 P_2 + 20$ tons/hour
$Q_3 = 0.0028 P_3^2 + 0.64 P_3 + 36$ tons/hour

Fuel cost is Rs 500 per ton. Maximum and minimum generation level for each unit is 120 MW and 36 MW. Find optimum scheduling for a total load of 200 MW.

**[12 marks]**

Incremental Cost of 3 units

$IC_1 = \frac{dQ_1}{dP_1} \times 500 = (0.004 P_1 + 0.86) 500$ Rs/MW-hour.

$IC_2 = \frac{dQ_2}{dP_2} \times 500 = 500(0.008 P_2 + 1.08)$ Rs/MW-hour

$IC_3 = \frac{dQ_3}{dP_3} \times 500 = 500[5.6 \times 10^{-3} P_3 + .64]$ Rs/MW hour

for optimum scheduling -

$IC_1 = IC_2 = IC_3$

$0.004 P_1 + .86 = 0.008 P_2 + 1.08 = 5.6 \times 10^{-3} P_3 + .64$

taking (I) and (II)

$0.004 P_1 - 0.008 P_2 = 0.22$ ——— (1)

Take (2) and (3)

$0.008 P_2 - 5.6 \times 10^{-3} P_3 = -0.44$ — (ii)

and total load is $P_1 + P_2 + P_3 = 200 MW$ — (iii)

Solving eq (i), (ii), (iii)

$P_1 = 85 MW$

$P_2 = 15 MW$ ⟶ out of range (lower limit voilated)

$P_3 \doteq 100 MW$

$\boxed{\text{hence take } P_2 = 36 MW}$

Now $P_1 + P_3 = 200 - 36 = 164 MW$ — (iv)

and ~~0.004 P1 − 0.008 P2 = .22~~ —

$0.004 P_1 - 5.6 \times 10^{-3} P_3 = 0.64 - 0.86$ — (v)

Solving (iv) and (v)

$$\boxed{\begin{aligned} P_1 &= 72.75 MW \\ P_3 &= 91.25 MW. \end{aligned}}$$

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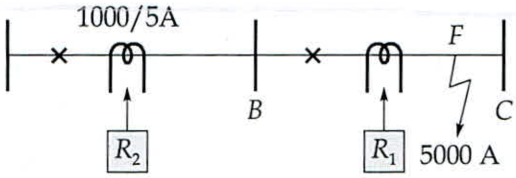
good

**1 (d)** Two relays $R_1$ and $R_2$ are connected in two sections of a feeder as shown in figure below. CT's are of ratio 1000/5 A. The plug setting of relay $R_1$ is 100% and $R_2$ is 125%. The operating time characteristic of the relays is given in table.
The TMS of relay $R_1$ is 0.3. The time grading scheme has a discriminative margin of 0.5 s between the relays. A three phase short circuit at $F$ results in a fault current of 5000 A. Find the actual operating time instants of $R_1$ and $R_2$. What is the TMS of $R_2$?

| PSM | 2 | 4 | 5 | 8 | 12 | 20 |
|---|---|---|---|---|---|---|
| Operating time | 10 | 5 | 4 | 3 | 2.8 | 2.4 |

$$PSM = \frac{\text{Fault current}}{\text{CT Ratio} \times PS.}$$




**[12 marks]**

fault current = 5000 A

$$\text{PSM of Relay 1} = \frac{5000 \times 5}{1000 \times \left(5 \times \frac{100}{100}\right)}$$

$$= \frac{25}{5}$$

$$= 5$$

Operating time from characteristics = 4 (sec) for TMS = 1

for TMS = 0.3, $\boxed{\text{actual } T_{op_1} = 4 \times 0.3 = 1.2 \text{ sec}}$

Operating time of Relay 2 = $T_{op_1} + T_{margin}$

$$= 1.2 + 0.5$$

$$\boxed{T_{op_2} = 1.7 \text{ sec}}$$

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$$\text{PSM of Relay 2} = \frac{5000 \times 5}{1000 \times \left(5 \times \frac{125}{100}\right)} = 4$$

Operating time from characteristics = 5 sec (for TMS = 1)

hence $\boxed{\text{TMS of } R_2 = \frac{1.7}{5} = 0.34}$

**Q.1 (e)** What is the percentage copper saving in feeder if the line voltage in a two-wire dc system be raised from 220 V to 500 V for the same power transmitted? State any assumption made.

**[12 marks]**

for same power transmitted $P =$ Constant.

$$P = VI\cos\phi$$

$$I = \frac{P}{V\cos\phi} \quad \text{(assume pf also ~~constact~~ constant)}$$

$$I \propto \frac{1}{V}$$

1

Now $\frac{I_1}{I_2} = \frac{V_2}{V_1} \Rightarrow I_2 = I_1 \times \frac{220}{500}$

assuming same current density in both cases

$$J_1 = J_2$$

$$I_1 A_1 = I_2 A_2$$

$$\frac{A_2}{A_1} = \frac{I_1}{I_2} = \frac{220}{500}$$

% Copper saving $= \dfrac{\text{Volume 1} - \text{Volume 2}}{\text{Volume 1}} \times 100$

$$= \frac{A_1 l_1 - A_2 l_2}{A_1 l_1} \times 100$$

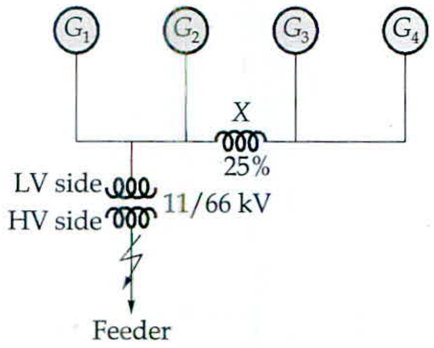
assuming same length for both cases $l_2 = l_1$

% Cu saving $= \left(1 - \dfrac{A_2}{A_1}\right) \times 100$

$$= \left(\frac{-220}{500} + 1\right) \times 100$$

$$= \underline{\underline{56\%}}$$

**Q.2 (a)** A generating station has four identical generators, $G_1$, $G_2$, $G_3$ and $G_4$ each of 20 MVA, 11 kV having 20% reactance. They are connected to a bus bar which has a busbar reactor of 25% reactance on 20 MVA base, inserted between $G_2$ and $G_3$ as shown below. A 66 kV feeder is taken off from the bus bars through a 15 MVA, 11/66 kV transformer having 7.5% reactance. A symmetrical 3-phase fault occurs at the high voltage terminals of the transformers. Calculate the current feed into the fault.



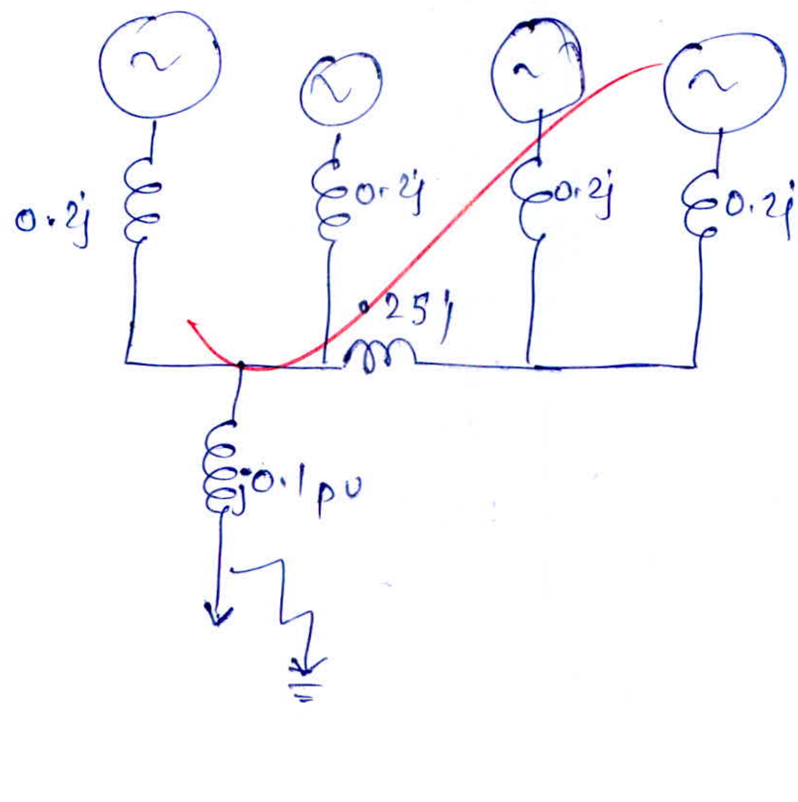

**[20 marks]**

Let the base MVA is 20 MVA

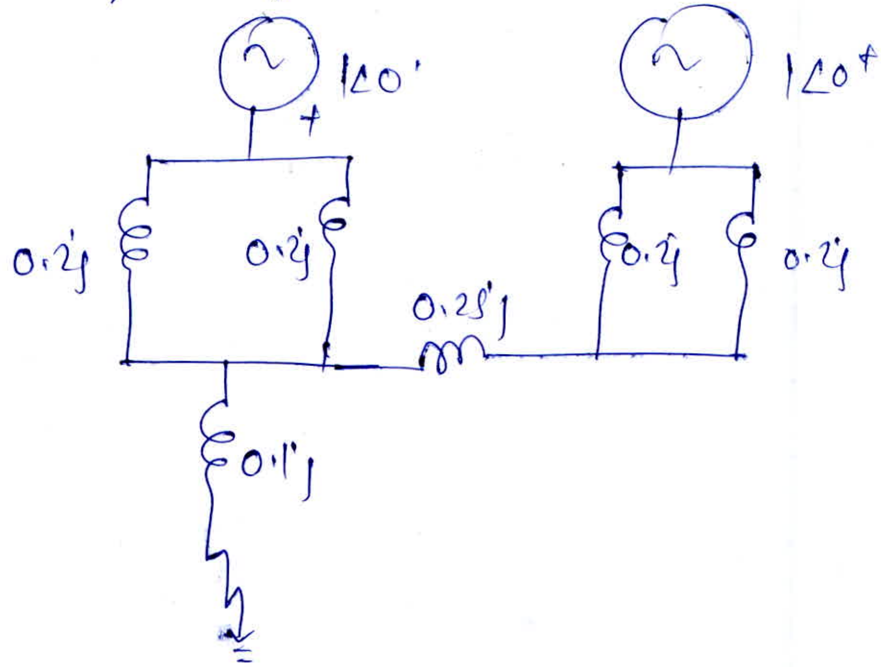
hence reactance of transformer on 20 MVA base

$$X_T = \frac{7.5}{100} \times \left[\frac{20}{15}\right] = 0.1 \text{ pu}$$

Drawing reactance diagram,



Simplifying we get :




$$X_{Th} = j\left[0.1 + (.2 \| .2) \| (.25 + (.2 \| .2))\right]$$

$$X_{Th} = j\left[0.1 + 0.1 \| (.25 + .1)\right]$$

$$= j\left[0.1 + \frac{0.1 \times .35}{.1 + .35}\right]$$

good.

$$X_{Th} = +j.17777 \text{ pu}$$

hence $I_f = \frac{V_{Th}}{X_{Th}} = \frac{1\angle 0°}{j\,.1777} = -j5.625 \text{ pu}$

$I_{base}$ on HV side $= \frac{20 \times 10^3}{\sqrt{3} \times 66} = 174.95$ A

$I_f(\text{amp}) = 174.95 \times 5.625$.

$I_f(\text{amps}) = 984.11$ amp

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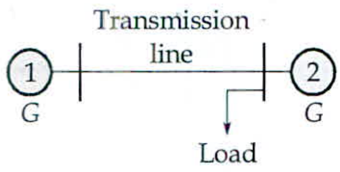
**Q.2 (b)** **(i)** A system consists of two plants connected by a transmission line as shown in figure below. The load is at plant-2. The transmission line loss calculations reveals that a transfer of 100 MW from plant-1 to plant-2 incurs a loss of 15 MW. Find the required generation at each plant for $\lambda = 60$. Assume that the incremental costs of the two plants are given by:

$$\frac{dC_1}{dP_1} = 0.2P_1 + 22 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_2} = 0.15P_2 + 30 \text{ Rs/MWh}$$




**(ii)** Find the savings in Rs per hour by scheduling the generation by considering the transmission loss rather than neglecting the transmission loss in determining the outputs of the two generators. Assume a load of 285 MW connected at bus of plant-2.

**[20 marks]**

Load in one substation 2

$B_{22} = 0, \quad B_{21} = B_{12} = 0 \quad P_L = B_{11} P_1^2$

Now given when $P_1 = 100\,MW$ then $P_L = 15\,MW$

$$B_{11} = \frac{15}{(100)^2}$$

Now penalty factors

$$L_2 = 1, \quad L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 2B_{11} \times P_1} = \frac{1}{1 - \frac{2 \times 15 P_1}{10000}}$$

Now

$$\lambda = L_1 IC_1 = L_2 IC_2$$

$$60 = \frac{1}{1 - \frac{30 P_1}{10000}} (0.2P_1 + 22) = 1 \times (.15P_2 + 30)$$

from above equation

$$\boxed{P_2 = \frac{60 - 30}{.15} = 200\,MW}$$

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$$60 = \frac{0.2P_1 + 22}{1 - \frac{3}{1000}P_1}$$

$$60 - \frac{180}{1000}P_1 = .2P_1 + 22$$

$$60 - 22 = (.2 + .18)P_1$$

$$\boxed{P_1 = 100 MW}$$

Losses $= B_{11}P_1^2$

$$P_L = \frac{.15}{(100)^2} \times (100)^2$$

$$\boxed{P_L = 15 MW}$$

$$\boxed{\text{Total Cost} = 0.1 \times 100^2 + 22 \times 100 + \frac{.15}{2} \times (200)^2 + 30 \times 200 = 12200 \text{ Rs/hour}}$$

(ii) If we ~~consider~~ neglect losses, for optimal schedule

$$IC_1 = IC_2$$

$$0.2P_1 + 22 = 0.15P_2 + 30$$

$$0.2P_1 - .15P_2 = 8 \quad \text{--- (i)}$$

and $P_1 + P_2 = 285$ — (ii)

Solving (i) and (ii)

$$\boxed{\begin{aligned} P_1 &= 145 MW \\ P_2 &= 140 MW \end{aligned}}$$

~~Total Cost~~ Total Cost

$$\boxed{0.1(145)^2 + 22 \times 145 + \frac{.15}{2}(140)^2 + 140 \times 30 = 10962.5 \text{ Rs/hour}}$$

(iii) If we ~~neglect~~ Consider the losses

$$\cancel{\frac{0.2P_1 + 22}{1 - 3 \times 10^{-3}P_1} = .15P_2 + 30}$$

$P_1 = 100 MW$
$P_2 = 200 MW$ } as above determined

→ Saving by consider losses -

Total cost when neglecting losses

− Total Cost when considering losses

$$= \boxed{12200 - 10962.5 = 1237.5 \text{ Rs/hour}}$$

**Q.2 (c)** A 50 Hz, 3 phase transmission line 300 km long has total series impedance of (40 + *j*125) ohm and a total shunt admittance of $10^{-3}$ mho. The receiving end load is 50 MW at 220 kV with 0.8 lagging power factor. Calculate the sending end voltage, current, power and power factor using:

**(i)** Short line approximation.

**(ii)** Nominal π method.

**(iii)** Exact transmission line equation of long line.

**(iv)** Approximation of long line.

**[20 marks]**

$I_R = \frac{50 \times 10^3}{\sqrt{3} \times 220 \times .8} \angle -36.86$

$= 164.01 \angle -36.86$ amp

(i) considering short line

$Z = 40 + j125$ ohm

$V_s = V_R + I_R Z$

$= \frac{220 \times 10^3}{\sqrt{3}} + 164.01 \angle -36.86 \times (40 + j125) = 145.1 \angle 4.929$ KV (phase)

$V_s$ (line) = 251.32 KV

$I_s = I_R = 164.01 \angle -36.86.$

$Pf = \cos(4.929 + 36.86) = 0.745$ lagging

(ii) Nominal π

$A = D = 1 + \frac{YZ}{2} = 1 + \frac{10^{-3} j \times (40 + j125)}{2} = 0.9377 \angle 1.22°$

$B = Z = 40 + j125$

$C = Y\left(1 + \frac{YZ}{4}\right) = 9.688 \times 10^{-4} \angle 90.59$

hence

$V_s = AV_R + BI_R = .9377 \angle 1.22 \times \frac{220 \times 10^3}{\sqrt{3}} + 164.01 \angle -36.86 \times (40 + j125)$

$= 137.445 \angle 6.26$ KV (per phase)

$V_s$ (line) = 238.06 KV

$I_s = CV_R + DI_R = 9.688 \times 10^{-4} \angle 90.59 \times \frac{220 \times 10^3}{\sqrt{3}} + .9377 \angle 1.22 \times 164.01 \angle -36.86$

$= 128.17 \angle 15.1128$ Amp.

$P.f = \cos\phi = \cos(15.1128 - 6.26)$

$= \cancel{\cos}\ 0.988$ lagd

)

Consider Long line

$Z_c = \sqrt{Z/y} = \sqrt{\dfrac{40+j125}{10^{-3}j}} = 362.276 \angle{-8.872}\ \Omega$

$\gamma l = \sqrt{Zy} = \sqrt{(40+j125) \times 1 \times 10^{-3} j} = 0.3622 \angle 81.2^\circ$

$\alpha l + j\beta l = 0.0554 + j\,0.358$

Now $\cosh\gamma l = \cosh\alpha l \cos\beta l + j \sinh\alpha l \sin\beta l$

$= 0.9382 \angle 0.0207^{rad} = 0.9382 \angle 1.186^\circ$

$\sinh\gamma l = \sinh\alpha l \cos\beta l + j \cosh\alpha l \sin\beta l$

$= 0.3547 \angle 1.4239^{rad}$

$= 0.3547 \angle 81.58^\circ$

Now $A = D = \cosh\gamma l = .9382 \angle 1.186^\circ$

$B = Z_c \sinh\gamma l = 128.499 \angle 72.7^\circ$

$C = \dfrac{\sinh\gamma l}{Z_c} = 9.79 \times 10^{-4} \angle 90.45^\circ$

$V_s = AV_R + BI_R = .9382 \angle 1.186 \times \dfrac{220 \times 10^3}{\sqrt{3}} + 128.5 \angle 72.7 \times 164.01 \angle{-36.86}$

$V_s = 137.028 \angle 6.2$ KV (perphase)

$V_s(\text{line}) = 237.34$ KV

$I_s = CV_R + DI_R = 9.79 \times 10^{-4} \angle 90.45 \times \dfrac{220 \times 10^3}{\sqrt{3}} + .9382 \angle 1.186 \times 164.01 \angle{-36.86}$

$\boxed{I_s = 128.76 \angle 15.59^\circ \text{ amps}}$

Sending end Pf $= \cos(15.59 - 6.2)$

$= 0.986$ leading

(iv) approximation of long line

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**3 (a)** A 275 kV transmission line has following line constants :

$$A = 0.85\angle 5°;\ B = 200\angle 75°$$

**(i)** Determine the power at unity power factor that can be received if the voltage profile at each end is to be maintained at 275 kV.

**(ii)** Calculate the value of compensation required for a load of 150 MW at unity power factor with the same voltage profile as in part (i).

**(iii)** With the load as in part (ii), what would be the receiving end voltage if the compensation equipment is not installed?

**[20 marks]**

.3 (b) A 100-MVA, 13.2 kV generator is connected to a 100 MVA, 13.2/132 kV transformer. The generator's reactances are $X_d'' = 0.15$ p.u., $X_d' = 0.25$ p.u., $X_d = 1.25$ p.u. on a 100 MVA base, while the transformer reactance is 0.1 p.u. on the same base. The system is operating on no load, at rated voltage when a three phase symmetrical fault occurs at the HT terminals of the transformer. Determine:

**(i)** The subtransient, transient and steady state symmetrical fault currents in p.u. and in amperes.

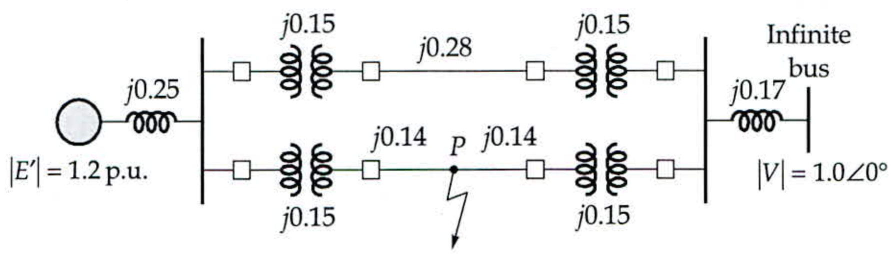
**(ii)** The maximum possible dc component.

**(iii)** Maximum value of instantaneous current.

**(iv)** Maximum rms value of the asymmetrical fault current.

**[20 marks]**

.3 (c) Find the critical clearing angle for the system in figure for a three phase fault at the point $P$. The generator is delivering 1.0 p.u. power under pre-fault conditions.




**[20 marks]**

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**Q.4 (a)** A dc source of 100 V with negligible resistance is connected to a lossless line ($Z_C = 30\ \Omega$), through a switch S. If the line is terminated in a resistance of 90 Ω, on closing the switch at $t = 0$, plot the receiving end voltage ($V_R$) w.r.t. time until 5T. Where, $T$ is the time for voltage wave to travel the length of the line. What will be the steady state voltage at receiving end? Also find the voltage at $t = 3.25$T on the mid length of the line.

**[20 marks]**

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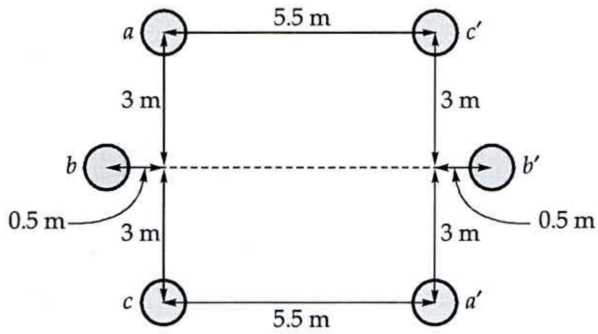
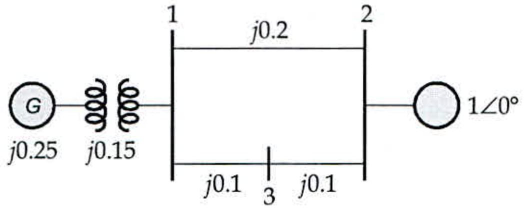
**4 (b)** Determine the inductance of the double circuit fully transposed line shown in figure, if the self GMD of each conductor is 0.0069 m.




**[20 marks]**

.4 (c) A single line diagram of a system is shown below:




All the values are in per unit on a common base. The power delivered into bus-2 is 1.0 p.u. at 0.8 p.f. lagging. Obtain the power angle equation and swing equation of the system. Neglect all losses.

**[20 marks]**

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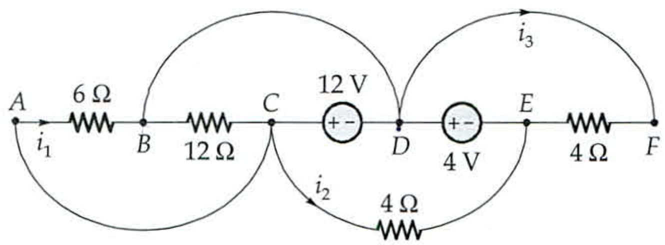
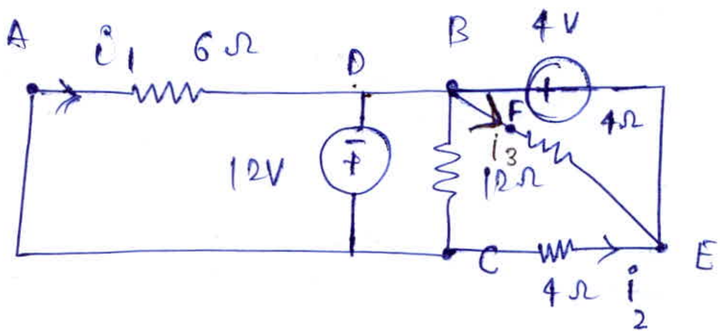
**Section B : Electrical Circuits-1 + Microprocessors-1 + Digital Electronics-2 + Control Systems-2**

**Q.5 (a)** Find the current $i_1$, $i_2$, $i_3$ and power delivered by the sources of the network shown in figure.

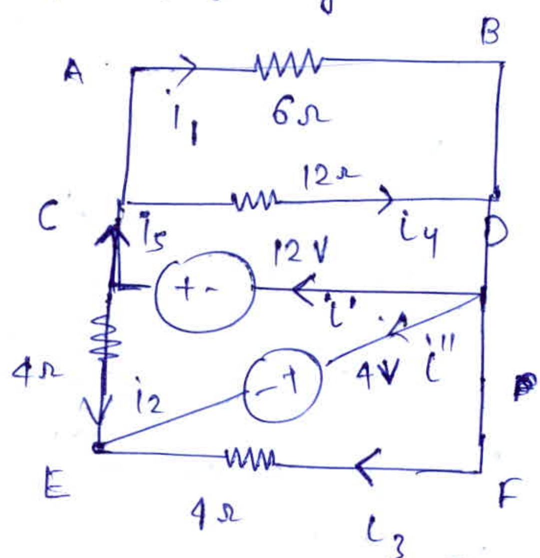



**[12 marks]**




Simplifying diagram.

good




$i_4 = \frac{12}{12} = 1A$

$i_5 = i_1 + i_4 = 1+2 = 3A$

$i' = i_5 + i_2$

$= 4 + 3 = 7$ amp.

hence power delivered by 12 V source

$= 12 \times 7$

$= 84$ W

$i_1 = 12/6 = 2A$

$i_3 = 4/4 = 1A$

$i_2 = \frac{12+4}{4} = 4A$

$i'' = i_4 + i_1 - i_3 - i'$

$= 2 + \cancel{1} - 7 - \cancel{1}$

$= -5A$

Power delivered by 4V source

$= 4 \times 5$

$= 20$ W

11

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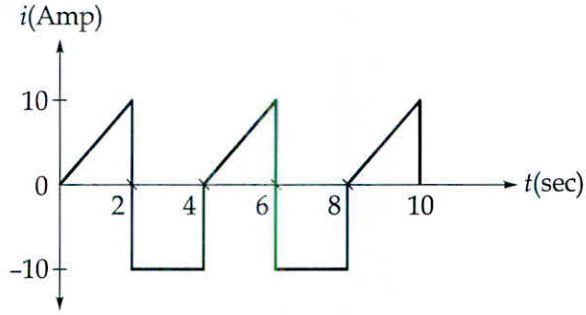
**Q.5 (b)** Determine the rms value of the waveform. If the current is passed through a 9 Ω resistor. Find the average power absorbed by the resistor.




**[12 marks]**

RMS value of Any signal

$$f_{RMS} = \left[\frac{1}{T}\int^{T} f(t)^2\,dt\right]^{1/2}$$

$$f(t) = \begin{cases} \frac{10}{2}t = 5t & 0 \le t < 2 \\ -10 & 2 \le t < 4 \end{cases}$$

Time period $T = 4$ sec

hence

$$I_{rms} = \left\{\frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt\right]\right\}^{1/2}$$

$$= \left\{\frac{1}{4}\left[25 \times \left(\frac{t^3}{3}\right)_0^2 + 100 \times (4-2)\right]\right\}^{1/2}$$

$$= \left\{\frac{1}{4}\left[\frac{25 \times 8}{3} + 200\right]\right\}^{1/2}$$

$$= \left(200/3\right)^{1/2} = 8.165 \text{ amp.}$$

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Average power loss in a resistance 9Ω

$$= I_{rms}^2 \times 9$$

$$= (8.165)^2 \times 9 = 600W$$

Do not write in this margin

.5 (c) A third order system has state space model matrices as follows:

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

Determine whether the system is state controllable or not. What will be the output controllability of the system?

**[12 marks]**

Check controllibility by Kalman's test

$$AB = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ 0 \\ 38 \end{bmatrix}$$

5

Controllibility matrix $Q_c = \begin{bmatrix} 0 & 1 & 7 \\ 0 & 0 & 0 \\ 1 & 6 & 38 \end{bmatrix}$

Since $|Q_c| = 0$

hence system is not controllable

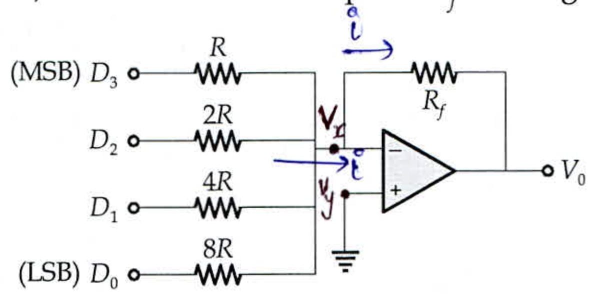
rank of ~~system~~ $Q_c$ matrix $= 2$

hence out of three state only 2 are controllable and output is not controllable.

output Controlability??

**5 (d)** **(i)** With a neat block diagram, explain the operation of 4-bit weighted-resistor type DAC.

**(ii)** For the 4-bit weighted-resistor DAC shown in below figure. Determine the weight of each input bit if the inputs are 0 V and 5 V and also determine the full-scale output, if $R_f = R = 1\ k\Omega$. Also, find the full scale output if $R_f$ is changed to 500 Ω.




**[12 marks]**

Diagram is already given.–

assume $V_x = V_y = 0$ (virtual ground concept)–

$$\frac{D_3 V_R}{R} + \frac{D_2 V_R}{2R} + \frac{D_1 V_R}{4R} + \frac{D_0 V_R}{8R} = i$$

$$\frac{V_R}{8R}\left[8D_3 + 4D_2 + 2D_1 + D_0\right] = i$$

8

hence output voltage $= -R_f i$

$$= \frac{-V_R R_f}{8\ R}\left[8D_3 + 4D_2 + 2D_1 + D_0\right]$$

where $D_3, D_2, D_1, D_0$ represent the bits of digital I/P and $V_R = 5V$ (say) (reference).

$$\boxed{V_0 = \frac{-V_R}{2^{n-1}} \cdot \frac{R_f}{R} \times \text{decimal equivalent}}$$

Now weight of each input bit

Weight of $D_3$ (MSB) $= \boxed{-\frac{V_R}{8} \cdot \frac{R_f}{R} \times 8} = -\frac{5}{8} \times 8 \times \frac{1}{1} = -5V$

Weight of $D_2$ $= \boxed{-\frac{V_R}{8}\ \frac{R_f}{R} \times 4} = -\frac{5}{8} \times 4 \times \frac{1}{1} = -2.5V$

Weight of $D_1$ $= \boxed{-\frac{V_R}{8} \frac{R_f \times 2}{R}} = -\frac{5}{8} \times 2 \times 1 = -1.25\,V$

Weight of $D_0$ $= \boxed{-\frac{V_R}{8} \frac{R_f \times 1}{R}} = -\frac{5}{8} \times 1 = -.625\,V$

full scale O/p corresponds to 1111 ($R_f = 1\,k\Omega$)

$V_o = -\frac{V_R}{8} \frac{R_f}{R} [8+4+2+1] = -\frac{5}{8} \times 1 \times 15 = -9.375$ volts

full scale O/p for $R_f = 500\,\Omega$

$V_o = -\frac{V_R}{8} \times \frac{R_f}{R} \times (8+4+2+1) = -\frac{5}{8} \times \frac{500}{1000} \times 15$

$= -4.6875$ Volts

**Q.5 (e)** Explain the generation of control signals for memory and I/O devices of 8085 microprocessors?

**[12 marks]**

There are following control signale which are used in the memory / I/O device operations

1) $IO/\overline{M}$

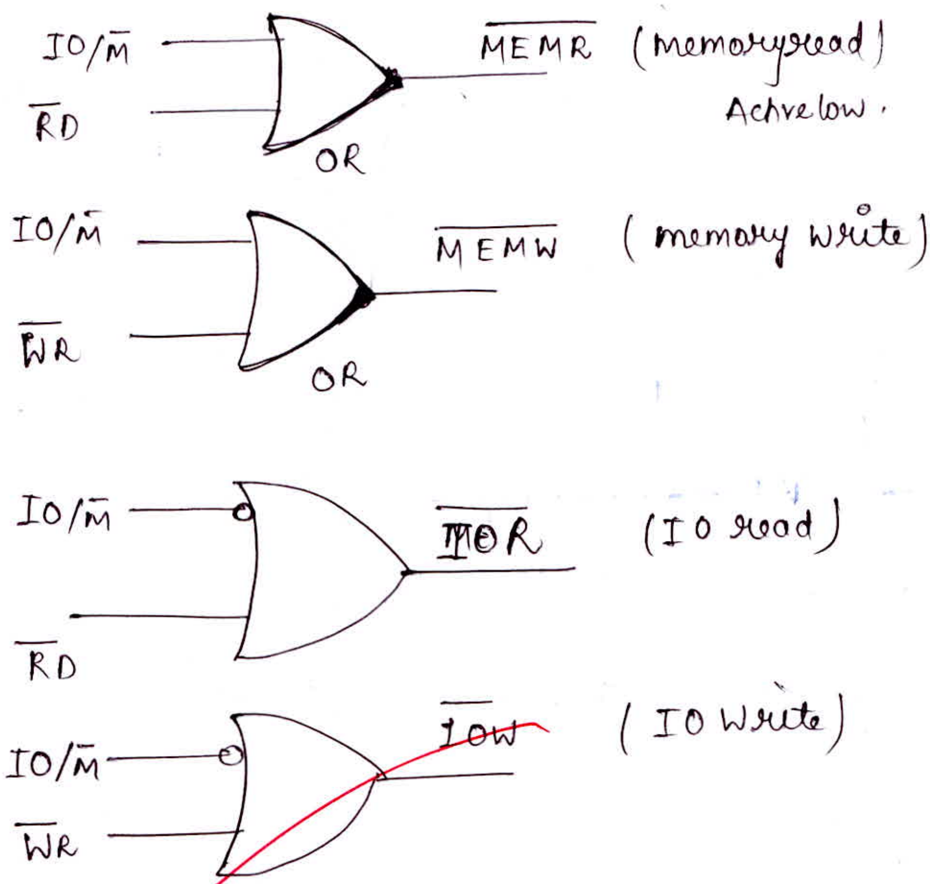
2) $\overline{RD}$

3) $\overline{WR}$

Now

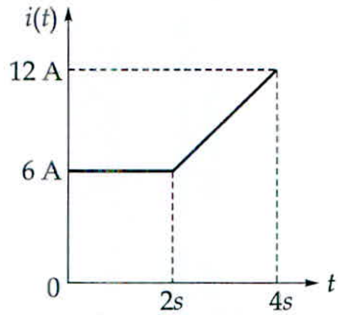
| $IO/\overline{M}$ | $\overline{RD}$ | $\overline{WR}$ | |
|---|---|---|---|
| 0 | 0 | 1 | Memory Read |
| 0 | 1 | 0 | Memory write |
| 1 | 0 | 1 | I/O read |
| 1 | 1 | 0 | I/O write |

can be generated as follows

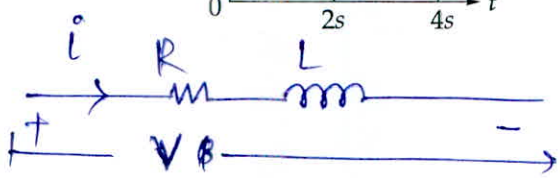



8

**Q.6 (a)** **(i)** Figure below shows the waveform of the current passing through an inductor of resistance 2 Ω and inductance 2 H. Find the energy absorbed by the inductor in the first four seconds.




3t $\frac{6}{2} t$




**[12 marks]**

Valtage $V = Ri + L\frac{di}{dt}$.

Energy absorbed by the inductor in 4sec

$$= \int_0^4 Vi\,dt$$

$$= \int_0^4 \left(Ri + L\frac{di}{dt}\right) i\,dt.$$

$$= \int_0^4 Ri^2\,dt + \int_0^4 Li\,di$$

$$= R\left[\int_0^2 36\,dt + \int_2^4 \left(\left(\frac{12-6}{4-2}\right)t\right)^2 dt\right] + \frac{L}{2}\left(i^2\right)_{i(0)}^{i(4)}$$

$$= 2\left[36\times 2 + \int_2^4 9t^2\,dt\right] + \frac{2}{2}\left[i(4)^2 - i(0)^2\right]$$

$$= 2\left[72 + \frac{9}{3}\times\left(t^3\right)_2^4\right] + \left(12^2 - 6^2\right)$$

$$= 2\left[72 + 3\,(64-8)\right] + (144-36)$$

$E = 480 + 144 - 36$

$E = 588$ Joules

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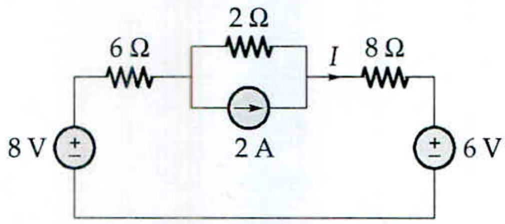
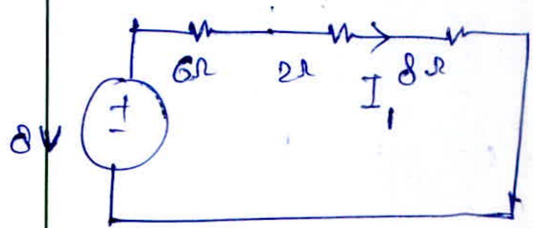
**Q.6 (a)** **(ii)** Find the current $I$ in the circuit shown below using the superposition theorem.

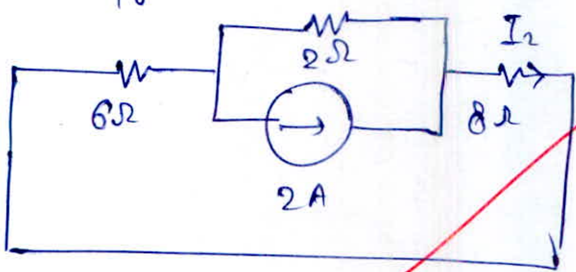



**[8 marks]**

Consider source of 8 V, openckting 2A and shorting 6 V



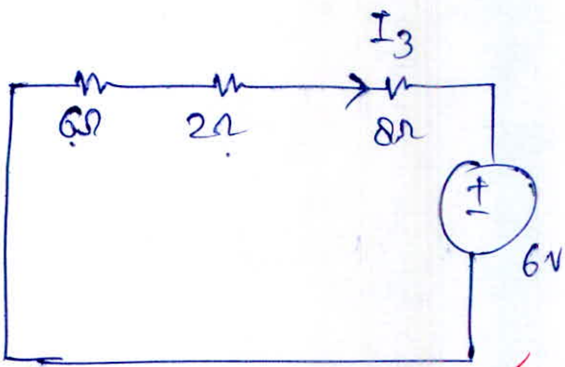

$$I_1 = \frac{8}{16} = 0.5\text{ A}$$




Consider 2A, shorting 8V & 6 V

$$I_2 = 2 \times \frac{2}{(2+6+8)}$$

$$I_2 = 0.25\text{ A}$$




Consider 6V source shorting other 8V and open 2A

$$I_3 = \frac{-6}{16} = -\frac{3}{8}\text{ Amp}$$

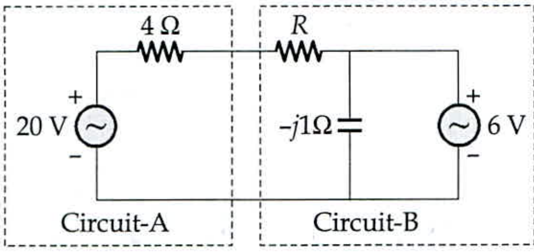
8

hence $I = I_1 + I_2 + I_3$

$$I = 0.5 + 0.25 - 3/8$$

$$I = 0.375\text{ A}$$

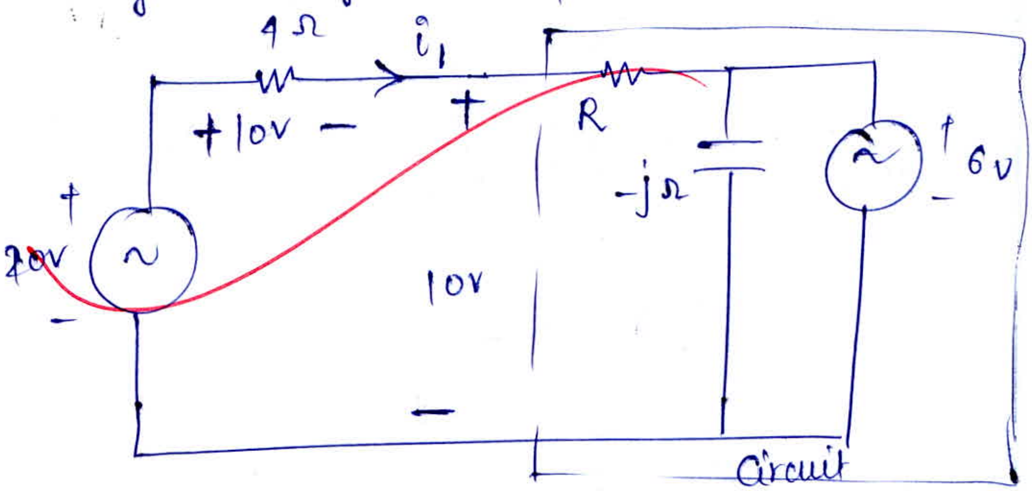
6 (b) (i) Assuming both the voltage sources are in phase, find the value of $R$ for which maximum power is transferred from circuit $A$ to circuit $B$.




[12 marks]

~~Circuit B andcan be represented Circuit A as thevenin equivalent~~

$4\Omega$ resistance and circuit B share half of input voltage during max. power transfer.




$$i_1 = \frac{10}{4} = 2.5 \text{ amp.}$$

Now applying KVL on circuit B.

$$10 = i_1 R + 6$$

$$4 = i_1 R$$

$$R = \frac{4}{i_1} = \frac{4}{2.5} = \underline{\underline{1.6\,\Omega}}$$

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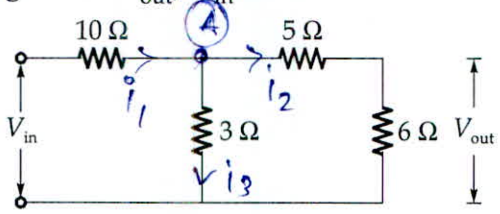
**Q.6 (b)** **(ii)** Determine the voltage ratio $V_{out}/V_{in}$ for the circuit shown below:




**[8 marks]**

Nodal-Analysis

assuming Node A voltage is $V_A$

hence KCL at Node A

$$i_1 = i_3 + i_2$$

$$\frac{V_{in} - V_A}{10} = \frac{V_A}{3} + \frac{V_A}{11}$$

$$\frac{V_{in}}{10} = V_A\left(\frac{1}{10} + \frac{1}{3} + \frac{1}{11}\right) = \frac{173 V_A}{330}$$

$$V_A = \frac{3 \times 11 \, V_{in}}{173} \quad \text{—— (1)}$$

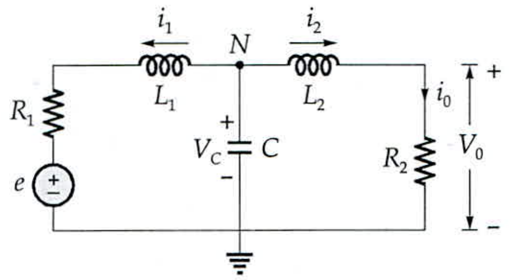
Now $V_{out} = \dfrac{6}{6+5} V_A$

$$V_{out} = \frac{6}{\cancel{11}} \times \frac{3 \times \cancel{11} \, V_{in}}{173} \quad (\text{from eq (1)})$$

$$\frac{V_{out}}{V_{in}} = \frac{18}{173} = \underline{\underline{0.104046}}$$

8

6 (c) Obtain the state space model considering current through the two inductors and voltage across the capacitor as state variables. Consider the output variables as current through $R_2$ and voltage across $R_2$ and input variable as '$e$' as shown in figure. (Consider initial conditions to be zero)




**[10 marks]**

KVL in loop ①

$$-e = R_1 i_1 + L_1 \frac{di_1}{dt} - V_C \quad \text{——— (I)}$$

KVL in loop ②

$$-V_C + L_2 \frac{di_2}{dt} + R_2 i_2 = 0 \quad \text{——— (II)}$$

Voltage across capacitor

$$-V_C = \frac{1}{C}\int (i_1 + i_2)\, dt$$

$$\frac{dV_C}{dt} = -\frac{1}{C}(i_1 + i_2) \quad \text{——— (III)}$$

In ~~from~~ eq (I), (II), (III) consider $x_1 = i_1$, $x_2 = i_2$ and $x_3 = V_C$

$$\frac{di_1}{dt} = -\frac{e}{L_1} + \frac{V_C}{L_1} - \frac{R_1 i_1}{L_1} \Rightarrow \dot{x}_1 = -\frac{e}{L_1} + \frac{x_3}{L_1} - \frac{R_1}{L_1} \cdot x_1$$

$$\frac{di_2}{dt} = \frac{V_C}{L_2} - \frac{R_2 i_2}{L_2} \Rightarrow \dot{x}_2 = \frac{x_3}{L_2} - \frac{R_2}{L_2} x_2$$

$$\frac{dV_C}{dt} = -\frac{1}{C}(i_1 + i_2) \Rightarrow \dot{x}_3 = -\frac{x_1}{C} - \frac{x_2}{C}$$

outputs $y_1 = i_0 = i_2 = x_2$

$y_2 = V_0 = i_0 R_2 = i_2 R_2 = x_2 R_2$

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drawing state space model.

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} -R_1/L_1 & 0 & 1/L_1 \\ 0 & -R_2/L_2 & 1/L_2 \\ -1/C & -1/C & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} -1/L_1 \\ 0 \\ 0 \end{bmatrix} e$$

$$\begin{matrix} i_0 \rightarrow \\ V_0 \rightarrow \end{matrix} \underset{2\times1}{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = \underset{2\times3}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & R_2 & 0 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$$

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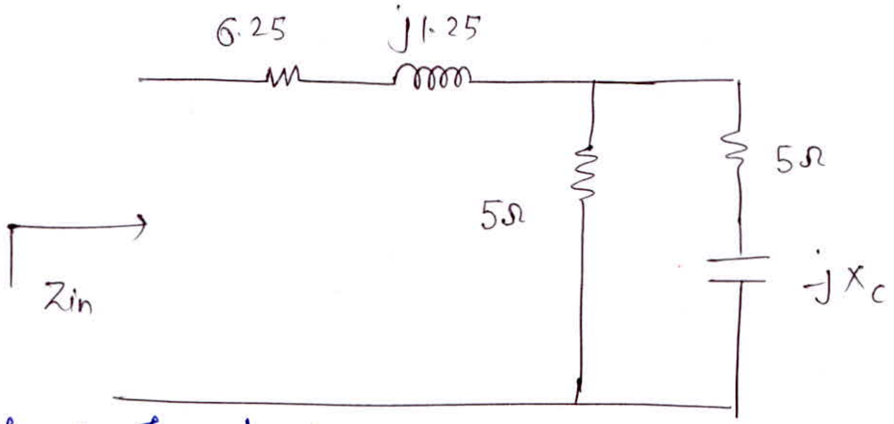
5 (d) Two impedances $Z_1 = 5\ \Omega$ and $Z_2 = (5 - jX_C)\Omega$ are connected in parallel and this combination is connected in series with $Z_3 = (6.25 + j1.25)\Omega$. Determine the value of capacitance of $X_C$ to achieve resonance if the supply is 100 V, 50 Hz.

**[10 marks]**




Input Impedance

$$Z_{in} = 6.25 + j1.25 + \frac{5 \times (5 - jX_c)}{5 + 5 - jX_c}$$

$$Z_{in} = 6.25 + j1.25 + \frac{5(5 - jX_c)}{10 - jX_c}$$

$$Z_{in} = 6.25 + j1.25 + \frac{5(5 - jX_c)(10 + jX_c)}{100 + X_c^2}$$

$$Z_{in} = 6.25 + j1.25 + \frac{5\left[50 + j5X_c - j10X_c + X_c^2\right]}{100 + X_c^2}$$

$$Z_{in} = 6.25 + \frac{5(50 + X_c^2)}{100 + X_c^2} + j\left[1.25 - \frac{5 \times 5X_c}{100 + X_c^2}\right]$$

for resonance imaginary part of Zin must be zero,

$$1.25 = \frac{25X_c}{100 + X_c^2}$$

8

$$X_c^2 - 20X_c + 100 = 0$$

$C = ?$

$$(X_c - 10)^2 = 0$$

$$\underline{X_c = 10\Omega}$$

**Q.7 (a)** **(i)** Clearly differentiate between latches and flip-flops.

**[8 marks]**

7 (a) **(ii)** Realize $T$-flip flop using $D$-flip flop.

**[12 marks]**

**Q.7 (b)** **(i)** For a control system, the input output transfer function is given below:

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \ldots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} \ldots a_{n-1}s + a_n}$$

Construct a state diagram by direct decomposition using the first companion form. Use state diagram to obtain dynamic equations and state space model.

**(ii)** Obtain the second companion form for the system whose input output transfer function is given by,

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 4s + 1}{s^3 + 3s^2 + 4s + 8}$$

Draw corresponding state diagram for above form and derive state space model for above system.

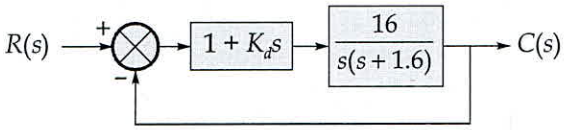
**[20 marks]**

(c) **(i)** Write an assembly language program for an 8085 microprocessor, to find 2's complement of a 16-bit number. Write comments for selected instructions.

**(ii)** Enumerate all internal registers present in 8259 programmable interrupt controller? Write short notes on their individual functionality?

**[12 + 8 marks]**

**(a)** A control system employing proportional and derivative control as shown below, has damping ratio equal to 0.8. Find the time instant at which the step response of system attains the peak value. Also find the percent maximum overshoot of system.




**[20 marks]**

Close loop TF is

$$\frac{C(s)}{R(s)} = \frac{\frac{(1+K_d s)16}{s(s+1.6)}}{1+\frac{(1+K_d s)\times 16}{s(s+1.6)}} = \frac{16(1+K_d s)}{s(s+1.6)+16(1+K_d s)}$$

$$\frac{C(s)}{R(s)} = \frac{16(1+K_d s)}{s^2+(1.6+16K_d)s+16}$$

CE is $s^2+(1.6+16K_d)+16 = 0$

comparing with standard CE $s^2+2\xi\omega_n s+\omega_n^2$

$\omega_n = 4$ rad/sec

$2\xi\omega_n = 1.6+16K_d$

$2\times 0.8\times 4 = 1.6+16K_d$

$\underline{\underline{K_d = 0.3}}$

5

Hence $\frac{C(s)}{R(s)} = \frac{16(1+0.3s)}{s^2+(1.6+.3\times 16)s+16} = \frac{16(1+0.3s)}{s^2+6.4s+16}$

Now for step input $R(s) = 1/s$

$$C(s) = \frac{16(1+0.3s)}{s(s^2+6.4s+16)} = \underbrace{\frac{A}{s}+\frac{Bs+C}{s^2+6.4s+16}}_{\text{partial fractions.}}$$

$16(1+.3s) = A(s^2+6.4s+16)+(Bs+C)s$

$16+4.8s = (A+B)s^2+(6.4A+C)s+16A$

comparing both sides

$A = 1$

$B \cancel{+} A = 0 \Rightarrow B = -A = -1$

$6.4A + C = 4.8 \Rightarrow C = -6.4 + 4.8 = -1.6$

hence

$$C(s) = \frac{1}{s} - \frac{(s+1.6)}{s^2+6.4s+16}$$

$$= \frac{1}{s} - \frac{(s+1.6)}{(s+3.2)^2+5.76}$$

$$= \frac{1}{s} - \frac{(s+1.6)}{(s+3.2)^2+(2.4)^2}$$

$$= \frac{1}{s} - \frac{(s+3.2)}{(s+3.2)^2+2.4^2} + \frac{1.6}{(s+3.2)^2+(2.4)^2}$$

applying laplace inverse.

$$C(t) = 1 - e^{-3.2t}\left[\cos 2.4t \quad \frac{1.6}{2.4}\sin 2.4t\right]$$

$$= 1 - e^{-3.2t}\left[\cos 2.4t \quad \frac{2}{3}\sin 2.4t\right]$$

Now for max. value of $C(t)$, $dc/dt = 0$

$$\frac{dc(t)}{dt} = 0 + e^{-3.2t}\left[\cos 2.4t \quad \frac{2}{3}\sin 2.4t\right]$$
$$- e^{-3.2t}\left[-2.4\sin 2.4t \quad \frac{2}{3}\times 2.4\cos 2.4t\right]$$

$$e^{-3.2t}\left[\cos 2.4t \quad \frac{2}{3}\sin 2.4t + 2.4\sin 2.4t + \frac{2}{3}\times 2.4\cos 2.4t\right]$$

$= 0$

$$\frac{26}{15}\sin 2.4t + \frac{13}{5}\cos 2.4t = 0$$

$$\cancel{\tan 2.4t = \frac{0.6 \times 15}{46} = 0.1956}$$

$$\cancel{2.4t = \tan^{-1}(.1956)}$$

$$\cancel{2.4t = \frac{11.06 \times \pi}{180}}$$

$$\cancel{t_p =}$$

$$\tan 2.4t = -\frac{13}{5} \times \frac{15}{26} = -\frac{3}{2}$$

$$\tan 2.4t = -3/2$$

$$2.4t = \pi - \frac{56.31 \times \pi}{180}$$

$$= 0.9827$$

$$\boxed{t_p = 0.409 sec}$$ Peak time

$$C(t)|_{max} = C(t_p).$$

$$= 1 - e^{-3.2 \times .41}\left[\cos(2.4 \times .41) - \frac{2}{3}\sin(2.4 \times .41)\right]$$

$$= 1.001137$$

$$\% \text{ overshoot} = \frac{C(t)|_{max} - C(t)|_{final}}{C(t)|_{final}} \times 100$$

$$= \frac{1.001137 - 1}{1} \times 100$$

$$= \underline{\underline{.1137\%}}$$

**Q.8 (b)** Design a 3-bit gray UP/DOWN synchronous counter using $T$-flip flops with a control for UP/DOWN counting.

**[20 marks]**

gray counter

| Binary code ~~gray code~~ | gray code |
|---|---|
| 0 0 0 | 0 0 0 |
| 0 0 1 | 0 0 1 |
| 0 1 0 | 0 1 1 |
| 0 1 1 | 0 1 0 |
| 1 0 0 | 1 1 0 |
| 1 0 1 | 1 1 1 |
| 1 1 0 | 1 0 1 |
| 1 1 1 | 1 0 0 |

Let a mod input M ⇒ M=0 upcounting.
M=1 downcounting.

State table

| M | $Q_2$ $Q_1$ $Q_0$ | $Q_2^+$ $Q_1^+$ $Q_0^+$ | $T_2$ | $T_1$ | $T_0$ |
|---|---|---|---|---|---|
| 0 | 0 0 0 | 0 0 1 | 0 | 0 | 1 |
| 0 | 0 0 1 | 0 1 1 | 0 | 1 | 0 |
| 0 | 0 1 1 | 0 1 0 | 0 | 0 | 1 |
| 0 | 0 1 0 | 1 1 0 | 1 | 0 | 0 |
| 0 | 1 1 0 | 1 1 1 | 0 | 0 | 1 |
| 0 | 1 1 1 | 1 0 1 | 0 | 1 | 0 |
| 0 | 1 0 1 | 1 0 0 | 0 | 0 | 1 |
| 0 | 1 0 0 | 0 0 0 | 1 | 0 | 0 |
| 1 | 1 0 0 | 1 0 1 | 0 | 0 | 1 |
| 1 | 1 0 1 | 1 1 1 | 0 | 1 | 0 |
| 1 | 1 1 1 | 1 1 0 | 0 | 0 | 1 |
| 1 | 1 1 0 | 0 1 0 | 1 | 0 | 0 |
| 1 | 0 1 0 | 0 1 1 | 0 | 0 | 1 |
| 1 | 0 1 1 | 0 0 1 | 0 | 1 | 0 |
| 1 | 0 0 1 | 0 0 0 | 0 | 0 | 1 |
| 1 | 0 0 0 | 1 0 0 | 1 | 0 | 0 |

Kmaps for

$T_2$

| $Q_1Q_0$ \ $MQ_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | 1 |
| 01 | 1 | | | |
| 11 | | | | 1 |
| 10 | 1 | | | |

$T_1$

| $MQ_1$ \ $Q_1Q_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | | |
| 01 | | | 1 | [illegible] |
| 11 | | 1 | | |
| 10 | | | 1 | |

$$T_2 = \bar{Q}_1\bar{Q}_0 M\bar{Q}_2 + \bar{M}\bar{Q}_2\bar{Q}_1Q_0 + M\bar{Q}_2Q_1Q_0 + \bar{M}\bar{Q}_2Q_1\bar{Q}_0$$

$$T_2 = \bar{M}Q_2(Q_1 \odot Q_0) + MQ_2(Q_1 \oplus Q_0)$$

$$T_2 = M\bar{Q}_2(Q_1 \odot Q_0) + \bar{M}\bar{Q}_2(Q_1 \oplus Q_0)$$

$T_0$

| $Q_1Q_0$ \ $MQ_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

Kmaps

for $T_2$

$\Sigma(2,4,8,14)$

| $MQ_2$ \ $Q_1Q_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | 1 |
| 01 | 1 | | | |
| 11 | | | | 1 |
| 10 | 1 | | | |

$T_2 = \cancel{M\bar{Q}_2(Q}$

$= \bar{Q}_1\bar{Q}_0(M \oplus Q_2)$

$+ Q_1\bar{Q}_0(M \odot Q_2)$

Let this is input X

$T_1 = \sum m(1, 7, 13, 11)$

$T_1 = \bar{Q}_1 Q_0 (M \odot Q_2) + Q_1 Q_0 (M \oplus Q_2)$

Let this is input Y

| $MQ_2 \backslash Q_1Q_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | 1 | | |
| 01 | | | 1 | |
| 11 | | 1 | | |
| 10 | | | 1 | |

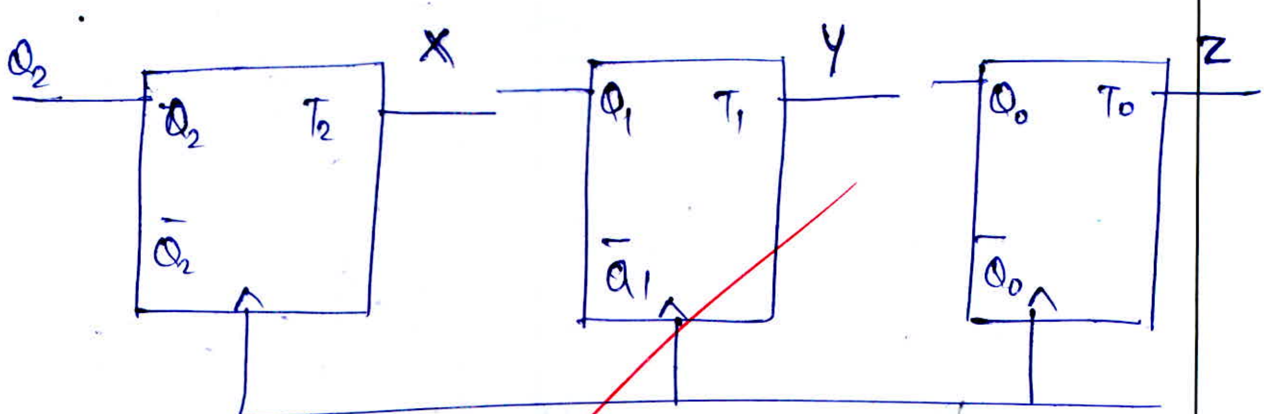
$T_0 = \sum m(0, 3, 6, 5, 12, 15, 10, 9)$

$T_0 = (\bar{Q}_1 \bar{Q}_0 + Q_1 Q_0)(M \odot Q_2) + (\bar{Q}_1 Q_0 + \bar{Q}_0 Q_1)(M \oplus Q_2)$

$T_0 = (M \odot Q_2)(Q_1 \odot Q_0) + (M \oplus Q_2)(Q_1 \oplus Q_0)$

Let this is input Z

| $MQ_2 \backslash Q_1Q_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $\phi$ | | 1 | |
| 01 | | 1 | | 1 |
| 11 | 1 | | 1 | |
| 10 | | 1 | | 1 |




X, Y, Z are ~~main~~ mentioned above

logic circuit diagram ??

15

Do not write in this margin

(c) A control system is represented by the state equation given below,

$$\dot{x}(t) = Ax(t)$$

If the response of the system is $x(t) = \begin{pmatrix} e^{-t} \\ -2e^{-t} \end{pmatrix}$ when $x(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $x(t) = \begin{pmatrix} e^{-2t} \\ -e^{-2t} \end{pmatrix}$ when $x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

Calculate the system matrix A and the state transition matrix for the system.

**[20 marks]**

~~Response to initial condition is~~

$\cancel{\dot{x}(t) = A\,x(t)}$

$$\cancel{\begin{bmatrix} e^{-t} \\ 2e^{-t} \end{bmatrix} = e^{At}\begin{bmatrix} 1 \\ -2 \end{bmatrix}}$$

~~and also~~

$$\cancel{\begin{bmatrix} e^{-2t} \\ e^{-2t} \end{bmatrix} = e^{At}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$\cancel{X(s) = (sI - A)^{-1} x(0)}$

$\cancel{(sI - A) X(s) = x(0)}$

$\cancel{(sI - A) = x(0)}$

$\dot{x}(t) = A\,x(t)$

applying Laplace transform

$sX(s) - x(0) = AX(s)$

$$s\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} - \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = A\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

for condition (1)

$$\begin{bmatrix} \frac{s}{s+1} \\ \frac{-2s}{s+1} \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \end{bmatrix} = A\begin{bmatrix} \frac{1}{s+1} \\ \frac{-2}{s+1} \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} \frac{s}{s+1} - 1 \\ \frac{-2s}{s+1} + 2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1/s+1 \\ -2/s+1 \end{bmatrix}$$

$$\begin{bmatrix} -1/s+1 \\ 2/s+1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/s+1 \\ -2/s+1 \end{bmatrix}$$

$-1 = a - 2b$ —— (I)

$2 = c - 2d$ —— (II)

for Condition (II)

$$\begin{bmatrix} \frac{s}{s+2} - 1 \\ -\frac{s}{s+2} + 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/s+2 \\ -\frac{1}{s+2} \end{bmatrix}.$$

$$\begin{bmatrix} -\frac{2}{s+2} \\ 2/s+2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \frac{1}{s+2} \\ -1/s+2 \end{bmatrix}.$$

$a - b = -2$ —— (III)

$c - d = 2$ —— (IV)

solving eq (III) & (I) solving eq (IV) and (II)

$a = -3$

$b = -1$

$c = 2$

$d = 0$

$$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

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Now state transition matrix $= L^{-1}\left((sI - A)^{-1}\right)$

OOOO

Space for Rough Work

Continued

$$SI-A = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$(SI-A)^{-1} = \frac{1}{s^2+3s+2}\begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} = \begin{bmatrix} \frac{s}{(s+1)(s+2)} & \frac{-1}{(s+1)(s+2)} \\ \frac{2}{(s+1)(s+2)} & \frac{s+3}{(s+1)(s+2)} \end{bmatrix}$$

&

$$(SI-A)^{-1} = \begin{bmatrix} \frac{-1}{(s+1)(1)} + \frac{-2}{(s+2)(-1)} & \frac{1}{s+2} - \frac{1}{s+1} \\ \frac{2}{s+1} - \frac{2}{s+2} & \frac{2}{(s+1)(1)} + \frac{1}{(s+2)(-1)} \end{bmatrix}$$

taking laplace inverse

STM

$$e^{At} = \begin{bmatrix} 2e^{-2t} - e^{-t} & e^{-2t} - e^{-t} \\ 2e^{-t} - 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix}$$

0

DONOT write in Rough Space

Space for Rough Work