



# MADE EASY

India's Best Institute for IES, GATE & PSUs

## ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

### Electrical Engineering

#### Test-3 : Power Systems

Electrical Circuits-1 + Microprocessors-1

Digital Electronics-2 + Control Systems-2

Name : .....

Roll No : 

E	E	1	9	M	T	D	L	A	0	1	0
---	---	---	---	---	---	---	---	---	---	---	---

#### Test Centres

Delhi  Bhopal  Noida  Jaipur  Indore   
Lucknow  Pune  Kolkata  Bhubaneswar  Patna   
Hyderabad

#### Student's Signature

.....

#### Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

#### FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	52
Q.2	45
Q.3	56
Q.4	
Section-B	
Q.5	39
Q.6	45
Q.7	
Q.8	
<b>Total Marks Obtained</b>	<b>237</b>

Signature of Evaluator

*[Handwritten Signature]*

Cross Checked by

*[Handwritten Signature]*



Section A : Power Systems

Q.1 (a) Estimate the corona loss for a three-phase, 110 kV, 50 Hz, 150 km long transmission line consisting of three conductors each of 10 mm diameter and spaced 2.5 m apart in an equilateral triangle formation. The temperature of air is 30° C and the atmospheric pressure is 750 mm of mercury. Take the irregularity factor as 0.85. Ionization of air may be assumed to take place at a maximum voltage gradient of 30 kV/cm.

[12 marks]

Solution for corona

$$\text{critical disruptive voltage } V_c = m_0 g_0 \delta \ln \left( \frac{D}{r} \right) \text{ kV/cm (RMS)}$$

where  $g = 21.1 \text{ kV/cm (RMS)}$

$$V_c = 0.85 \times 21.1 \times \frac{10 \times 10^{-3}}{2} \times \frac{3.92 \times 75}{273 + 30} \times \ln \left( \frac{2.5}{\frac{10 \times 10^{-3}}{2}} \right)$$

$$V_c = 54.074 \text{ kV/cm (RMS)}$$

$$\text{Now } V_{ph} = \frac{110}{\sqrt{3}} \text{ kV} = 63.50 \text{ kV (RMS)}$$

∴  $V_{ph} > V_c$

∴ corona loss will occur

$$P_{loss} = 242.2 \times 10^{-5} + \left( \frac{f+25}{\delta} \right) \times \sqrt{\frac{\delta}{d}} \times (V_{ph} - V_c)^2 \text{ (kW/ph/km)}$$

$$= 242.2 \times 10^{-5} \times \frac{75}{0.97029} \times \sqrt{\frac{5 \times 10^{-3}}{2.5}} (63.50 - 54.074)^2$$

$$P_{loss} = 0.7438 \text{ kW/ph/km}$$

kW?

total corona loss ??

9

Q.1 (b) A power plant has three generators feeding a common bus:  
 2 generators, 30 MVA, 15% reactance each  
 1 generator, 10 MVA, 12.5% reactance

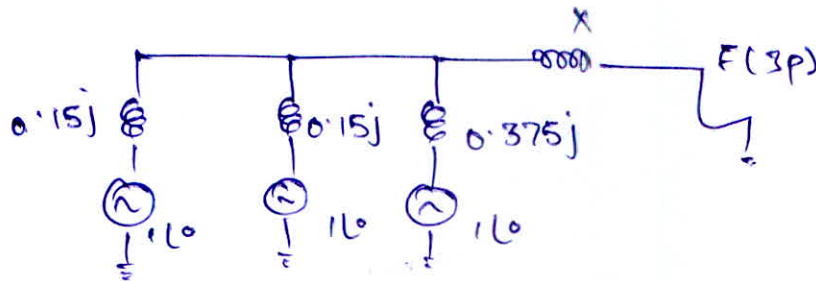
A 10 MVA transformer steps up the voltage and feeds a 33 kV circuit. Find the safe minimum reactance of transformer so that fault level on the secondary bus of the transformer may not exceed 100 MVA.

[12 marks]

Solution



Reactance diagram :  $S_b = 30 \text{ MVA}$



Now Fault level = 100 MVA =  $\frac{1}{X_{TH}(\text{PU})} \times \text{Base MVA}$

$\Rightarrow X_{TH}(\text{PU}) = \frac{30}{100} = 0.3 \text{ PU}$

Now Applying thevenin's across Fault to Neutral

$X_{TH} = (0.15j \parallel 0.15j \parallel 0.375j) + Xj$

$X_{TH} = (0.0625 + X)j$

$\therefore 0.0625 + X = 0.3$

$X = 0.2375$

Now on T/F Base :  $S_b = 10 \text{ MVA}$

$X = 0.2375 \times \frac{10}{30} = 0.079167 \text{ PU}$

$\therefore X_{\min} = 0.079167j$  Ans.

minimum reactance of T/F

10

mention p.u



Q.1 (c) A power plant has 3 units with the input output curves.

$$Q_1 = 0.002 P_1^2 + 0.86 P_1 + 20 \text{ tons/hour}$$

$$Q_2 = 0.004 P_2^2 + 1.08 P_2 + 20 \text{ tons/hour}$$

$$Q_3 = 0.0028 P_3^2 + 0.64 P_3 + 36 \text{ tons/hour}$$

Fuel cost is Rs 500 per ton. Maximum and minimum generation level for each unit is 120 MW and 36 MW. Find optimum scheduling for a total load of 200 MW.

[12 marks]

Solution  $IC_1 = (0.004P_1 + 0.86) 500 \text{ Rs/MWHr}$

$$IC_1 = (2P_1 + 430) \text{ Rs/MWHr}$$

$$IC_2 = (0.008P_2 + 1.08) 500 \text{ Rs/MWHr}$$

$$= (4P_2 + 540) \text{ Rs/MWHr}$$

$$IC_3 = (0.0056P_3 + 0.64) 500 \text{ Rs/MWHr}$$

$$= (2.8P_3 + 320) \text{ Rs/MWHr}$$

Now  $P_1 + P_2 + P_3 = 200 \text{ MW} \quad \text{---(i)}$

$$IC_1 = IC_2 \Rightarrow 2P_1 + 430 = 4P_2 + 540$$

$$\Rightarrow 2P_1 - 4P_2 = 110 \quad \text{---(ii)}$$

$$\text{Similarly } I_2 = I_3 \Rightarrow 4P_2 + 540 = 2.8P_3 + 320$$

$$\Rightarrow 4P_2 - 2.8P_3 = -220 \quad \text{---(iii)}$$

$$\begin{aligned} \text{Solving (i), (iii), (iii)} \quad P_1 &= 85 \text{ MW} \\ P_2 &= 15 \text{ MW} \\ P_3 &= 100 \text{ MW} \end{aligned}$$

now  $P_2 < P_{2\text{min}}$   $\therefore P_2$  will be equal to its minimum

$$\text{limit i.e. } \boxed{P_2 = 36 \text{ MW}}$$

$$\text{now } P_1 + P_3 = 164 \text{ MW} \quad \text{---(iv)}$$

$$I_1 = I_3 \Rightarrow 2P_1 + 430 = 2.8P_3 + 320$$

$$\Rightarrow 2P_1 - 2.8P_3 = -110 \quad \text{---(v)}$$

Solving (iv), (v)

$$\left. \begin{aligned} P_1 &= 72.75 \text{ MW} \\ P_3 &= 91.25 \text{ MW} \end{aligned} \right\} \text{ True within limits}$$

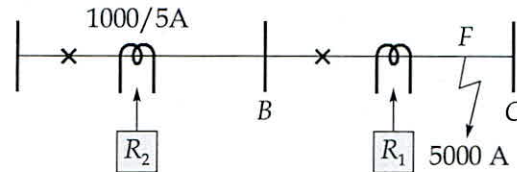
Finally

$$\left. \begin{aligned} P_1 &= 72.75 \text{ MW} \\ P_2 &= 36 \text{ MW} \\ P_3 &= 91.25 \text{ MW} \end{aligned} \right\} \text{ An.}$$

(11)

- Q.1 (d) Two relays  $R_1$  and  $R_2$  are connected in two sections of a feeder as shown in figure below. CT's are of ratio 1000/5 A. The plug setting of relay  $R_1$  is 100% and  $R_2$  is 125%. The operating time characteristic of the relays is given in table. The TMS of relay  $R_1$  is 0.3. The time grading scheme has a discriminative margin of 0.5 s between the relays. A three phase short circuit at F results in a fault current of 5000 A. Find the actual operating time instants of  $R_1$  and  $R_2$ . What is the TMS of  $R_2$ ?

PSM	2	4	5	8	12	20
Operating time	10	5	4	3	2.8	2.4



[12 marks]

Solution For  $R_1$ 

~~PSM =~~  $I_{PK} = \% \text{ PSM} \times \text{CT Secondary Current}$

$$I_{PK} = 1 \times 5 = 5 \text{ A}$$

$$\text{Now PSM} = \frac{I_{\text{fault}}}{I_{PK} \times \text{CTRatio}} = \frac{5000}{5 \times 1000/5} = 5$$

$$\therefore \text{For PSM} = 5, \text{ TMS} = 1 \rightarrow t_{op} = 4 \text{ sec}$$

$$\text{Now TMS}_{R_1} = 0.3$$

$$t_{\text{operation}} = (\text{TMS}) \times (t_{\text{operating for TMS}=1})$$

$$t_{\text{operating } R_1} = 0.3 \times 4 = 1.2 \text{ sec} \quad \text{Ans}$$

$$\text{Now } t_{\text{operating } R_2} = t_{\text{delay}} + t_{\text{operating } R_1}$$

$$t_{\text{operating } R_2} = (0.5 + 1.2) = 1.7 \text{ sec} \quad \text{Ans}$$

$$\underline{\text{For } R_2} \quad I_{PK} = 1.25 \times 5 = 6.25 \text{ A}$$

$$\text{PSM} = \frac{5000}{6.25 \times 1000/5} = 4$$

$$\text{Now } (I_{\text{top}})_{R_2} = (TMS)_{R_2} \times (I_{\text{top}})_{R_2 \text{ for PMSM} = 4}$$

$$1.7 = (TMS)_{R_2} \times 5$$

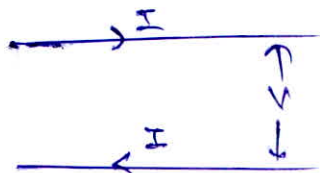
$$\Rightarrow (TMS)_{R_2} = 1.7/5 = 0.34 \text{ Ans}$$



- Q.1 (e) What is the percentage copper saving in feeder if the line voltage in a two-wire dc system be raised from 220 V to 500 V for the same power transmitted? State any assumption made.

[12 marks]

Solution  
For 2 wire dc system



$V =$  Line Voltage

$P =$  Power Transmitted

clearly  $P = VI$

$$\text{Power loss} = 2I^2R = W$$

$$\therefore R = \frac{W}{2I^2}$$

$$\text{Now } P_L = W = 2 \left( \frac{P}{V} \right)^2 R \propto \frac{1}{V^2}$$

Also volume of conductor used =  $AL$

Assumption: Length of Transmission is same in Both the cases



$$\therefore \text{Volume of conductor used} \propto \frac{1}{V^2}$$

$$\therefore \text{C.S. Area} \propto \frac{1}{V^2}$$

$$\therefore \text{Copper Savings} \rightarrow \text{Avoid striking between answer.}$$

$$\therefore \frac{(C.S.A)_1}{(C.S.A)_2} = \frac{V_2^2}{V_1^2} = 5.165$$

$$\therefore (C.S.A)_2 = (0.1936) (C.S.A)_1$$

Now copper comparison

$$\% \text{ copper savings} = \left( \frac{(C.S.A)_2 - (C.S.A)_1}{(C.S.A)_1} \right) \times 100$$

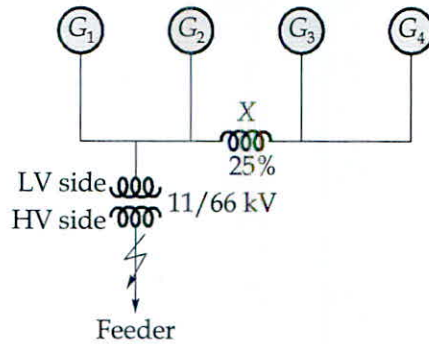
$$= \frac{(C.S.A)_1 - 0.1936(C.S.A)_1}{(C.S.A)_1} \times 100$$

$$\% \text{ copper savings} = 80.64\%$$

Ans.

11

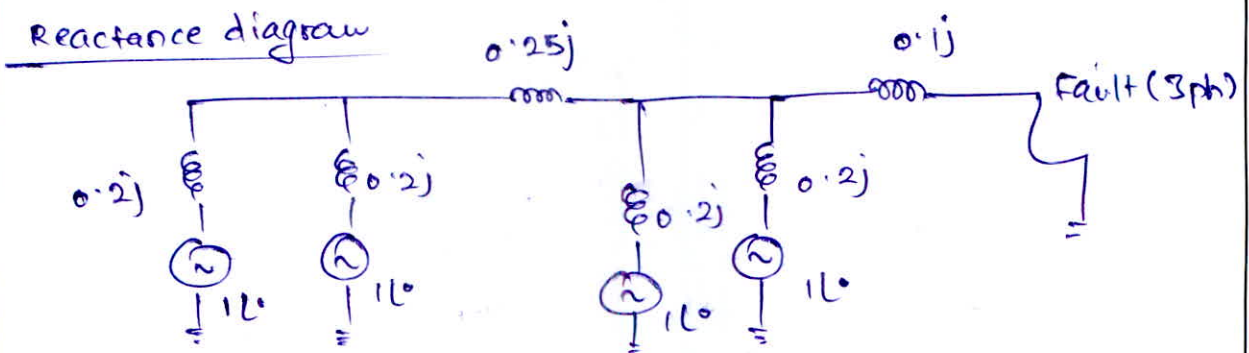
Q.2 (a) A generating station has four identical generators,  $G_1, G_2, G_3$  and  $G_4$  each of 20 MVA, 11 kV having 20% reactance. They are connected to a bus bar which has a busbar reactor of 25% reactance on 20 MVA base, inserted between  $G_2$  and  $G_3$  as shown below. A 66 kV feeder is taken off from the bus bars through a 15 MVA, 11/66 kV transformer having 7.5% reactance. A symmetrical 3-phase fault occurs at the high voltage terminals of the transformers. Calculate the current feed into the fault.



[20 marks]

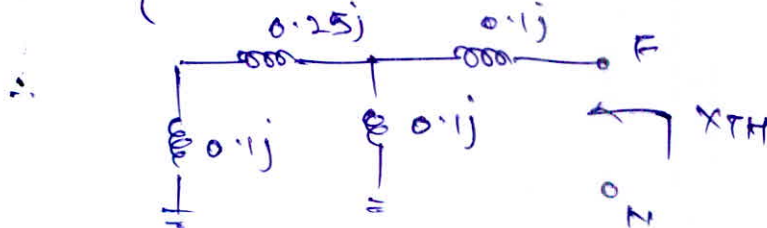
Solution

Take  $S_b = 20 \text{ MVA}$   
 $V_b = 11 \text{ kV}$  ('G' side)



∴ Applying thevenin's across Fault to Neutral

$$X_{TH} = \left\{ \left[ (0.2j \parallel 0.2j) + 0.25j \right] \parallel 0.2j \parallel 0.2j \right\} + 0.1j$$



$$= X_{TH} = (0.35j \parallel 0.1j) + 0.1j$$

$$X_{TH} = 0.17798j$$

$$∴ I_{\text{fault}} = \frac{10}{X_{TH}} = (-5.625j) \text{ pu}$$

$$\text{Now } I_{\text{base}} = \frac{20 \times 10^6}{\sqrt{3} \times 66 \times 10^3} = 174.95 \text{ A}$$

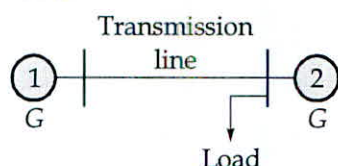
$$\therefore I_{\text{fault}} = 984.12 \text{ A} \quad \text{Ans.}$$

18

- Q.2 (b) (i) A system consists of two plants connected by a transmission line as shown in figure below. The load is at plant-2. The transmission line loss calculations reveals that a transfer of 100 MW from plant-1 to plant-2 incurs a loss of 15 MW. Find the required generation at each plant for  $\lambda = 60$ . Assume that the incremental costs of the two plants are given by:

$$\frac{dC_1}{dP_1} = 0.2P_1 + 22 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_2} = 0.15P_2 + 30 \text{ Rs/MWh}$$



- (ii) Find the savings in Rs per hour by scheduling the generation by considering the transmission loss rather than neglecting the transmission loss in determining the outputs of the two generators. Assume a load of 285 MW connected at bus of plant-2.

[20 marks]

Solution (i) For the given single line diagram

$$P_{\text{Loss}} = B_{11}P_1^2$$

given.  $P_L = 15 \text{ MW}$  for  $P_1 = 100 \text{ MW}$

$$\therefore 15 = B_{11}(100)^2 \Rightarrow \boxed{B_{11} = 1.5 \times 10^{-3} (\text{MW})^{-1}}$$

now given  $\lambda = 60 = L_i I_i$

For  $i=1$   $L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 2B_{11}P_1}$

$$\therefore \frac{0.2P_1 + 22}{1 - 2 \times 1.5 \times 10^{-3} P_1} = 60 \Rightarrow 0.2P_1 + 22 = 60 - 0.18P_1$$

$$\Rightarrow 0.38P_1 = 38$$

$$\Rightarrow \boxed{P_1 = 100 \text{ MW}}$$

For  $i=2$   $L_2 I_2 = 60 \therefore L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = 1$

$$\therefore 0.15P_2 + 30 = 60 \Rightarrow \boxed{P_2 = 200 \text{ MW}} \quad \text{Ans}$$



(11) Without considering Transmission loss (Assuming  $\lambda = 60$ )

~~$P_1 + P_2 = 285 \text{ MW}$~~  — (i)

$IC_1 = IC_2 \Rightarrow 0.2P_1 + 22 = 0.15P_2 + 30 = 60$

~~$\Rightarrow 0.2P_1 = 0.15P_2 + 8$~~  (ii)

Solving (i) & (ii)

$P_1 = 190 \text{ MW}$  ;  $P_2 = 200 \text{ MW}$

NOW

Saving in Rs/Hr by scheduling the generation by considering

T/L loss

= Savings in unit 1 + Savings in unit 2

$P_1 = 190 \text{ MW}$

$P_2 = 200 \text{ MW}$

$= \int_{100 \text{ MW}}^{190 \text{ MW}} (0.2P_1 + 22) dP_1 + \int_{170 \text{ MW}}^{200 \text{ MW}} (0.15P_2 + 30) dP_2$

$= \frac{0.2P_1^2}{2} + 22P_1 \Big|_{100}^{190} + \frac{0.15P_2^2}{2} + 30P_2 \Big|_{170}^{200}$

~~$= 2092.5$~~  + 4590 Rs/Hr

$\therefore \text{Savings in Rs/Hr} = 4590 \text{ Rs/Hr}$

12

Q.2 (c) A 50 Hz, 3 phase transmission line 300 km long has total series impedance of  $(40 + j125)$  ohm and a total shunt admittance of  $10^{-3}$  mho. The receiving end load is 50 MW at 220 kV with 0.8 lagging power factor. Calculate the sending end voltage, current, power and power factor using:

- (i) Short line approximation.
- (ii) Nominal  $\pi$  method.
- (iii) Exact transmission line equation of long line.
- (iv) Approximation of long line.

[20 marks]

given  $Z = 131.244 \angle 72.25^\circ \Omega$

$Y = 10^{-3} \angle 90^\circ \text{ S}$

$$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 220 \times 10^3 \times 0.8} \angle -\cos^{-1}(0.8) = 164.02 \angle -36.87^\circ \text{ A}$$

(i) Short line Approximation

$$I_S = I_R = 164.02 \angle -36.87^\circ \text{ A (m)}$$

$$V_S = V_R + Z I_R = 145.104 \angle 4.927^\circ \text{ KV (L-N)}$$

$$\therefore \begin{cases} V_S \text{ (L-L)} = 205.2 \angle 4.927^\circ \text{ (KV)} \\ I_S = 164.02 \angle -36.87^\circ \text{ A} \end{cases}$$

$$V_S = 251.32 \angle 4.927^\circ \text{ KV (L-L)}$$

$$\text{pf} = \cos(4.927 + 36.87) \text{ lagg}$$

$$\boxed{\text{pf} = 0.7455 \text{ lagg}}$$

power at sending ~~potu~~ end =  $\sqrt{3} V_S I_S \cos \phi_S$

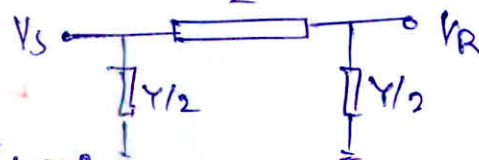
$$= \cancel{53.46 \text{ MW}} \quad \text{②} \quad 53.22 \text{ MW (m)}$$

(ii) Nominal  $\pi$  method :

$$A = 1 + \frac{YZ}{2} = D = 0.9377 \angle 1.22^\circ$$

$$B = Z = 131.244 \angle 72.25^\circ$$

$$C = Y \left(1 + \frac{YZ}{4}\right) = 9.68 \times 10^{-4} \angle 90.6^\circ$$



$$\text{Now } V_s = AV_R + BI_R = 137.45 \angle 6.26^\circ \text{ KV (L-N)}$$

$$\therefore V_s(L-L) = \cancel{237.38 \text{ KV}} \quad 238.07 \text{ KV (L-L)} \quad \text{Ans}$$

$$\text{Now } I_s = CI_R + DI_R = 128.1 \angle 15.07^\circ \text{ A} \quad \text{Ans}$$

$$\text{Pf} = \cos(6.26 - 15.07) \quad \text{lead}$$

$$\text{Pf} = 0.988 \text{ lead} \quad \text{Ans}$$

$$\text{Power} = \sqrt{3} V_s I_s \cos \phi_s = 52.188 \text{ MW} \quad \text{Ans}$$

(ii) Exact long line equation

$$\gamma l = \sqrt{ZY} = 0.3622 \angle 81.125^\circ = 0.0558 + 0.3578i$$

$$Z_c = \sqrt{\frac{Z}{Y}} = 362.27 \angle -8.875^\circ$$

$$\text{Now } A = \cosh \gamma l = \cosh(\alpha l + j\beta l) = \cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l$$

$$A = 0.9383 \angle 1.2^\circ = D$$

$$B = Z_c \sinh \gamma l = Z_c [\sinh(\alpha l + j\beta l)]$$

$$= Z_c [\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l] = 128.47 \angle 72.64^\circ$$

$$C = \frac{\sinh \gamma l}{Z_c} = \frac{B}{Z_c^2} = 9.789 \times 10^{-4} \angle 90.39^\circ$$

$$\text{Now } V_s = AV_R + BI_R = 137.053 \angle 6.2^\circ \text{ KV (L-N)}$$

$$V_s(L-L) = 237.38 \text{ KV}$$

$$I_s = CI_R + DI_R = 128.9 \angle 15.56^\circ \text{ A} \quad \text{Ans}$$

$$\text{Now pf} = \cos(15.56 - 6.2) \quad \text{lead}$$

$$= 0.9866 \text{ lead}$$

$$\text{Power} = \sqrt{3} V_s I_s \cos \phi_s = 52.28 \text{ MW} \quad \text{Ans}$$

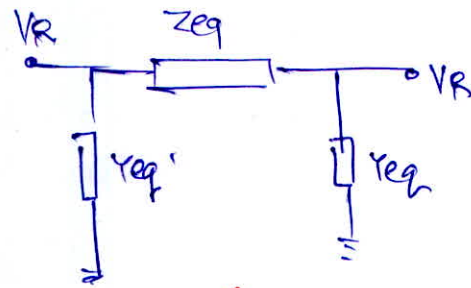
(iv) Approximation of long line

$$Z_{eq} = Z_c \sinh \gamma l$$

$$= 128.47 \angle 72.64^\circ$$

$$Y_{eq} = \frac{\tanh(\gamma l/2)}{Z_c} = \frac{\sinh(\gamma l/2)}{Z_c \cosh(\gamma l/2)}$$

$Y_{eq} =$



(15)



Q.3 (a) A 275 kV transmission line has following line constants :

$$A = 0.85 \angle 5^\circ; B = 200 \angle 75^\circ$$

- (i) Determine the power at unity power factor that can be received if the voltage profile at each end is to be maintained at 275 kV.
- (ii) Calculate the value of compensation required for a load of 150 MW at unity power factor with the same voltage profile as in part (i).
- (iii) With the load as in part (ii), what would be the receiving end voltage if the compensation equipment is not installed?

[20 marks]

Solution (i)  $\therefore$  given at UPF  $\therefore Q_R = 0$

$$V_S = V_R = 275 \text{ kV}$$

$$\therefore Q_R = \left| \frac{V_S V_R}{B} \right| \sin(\beta - \delta) - \left| \frac{A V_R^2}{B} \right| \sin(\beta - \alpha)$$

$$\therefore 0 = \left| \frac{275 \times 275 \times 10^6}{200} \right| \sin(75 - \delta) - \left( \frac{0.85 \times 275^2 \times 10^6}{200} \right) \sin(70)$$

$$\Rightarrow \frac{0.85 \times 275^2 \times 10^6}{200} \sin 70 = \frac{275 \times 275 \times 10^6}{200} \sin(75 - \delta)$$

$$\Rightarrow \boxed{\delta \approx 22^\circ}$$

Now  $P_R = \left| \frac{V_S V_R}{B} \right| \cos(\beta - \delta) - \left| \frac{A V_R^2}{B} \right| \cos(\beta - \alpha)$

$$\therefore P_R = \left| \frac{275 \times 275}{200} \right| \cos(75 - 22) - \left| \frac{0.85 \times 275^2}{200} \right| \cos(70)$$

MW

$$P_R = 117.63 \text{ MW} \quad \text{Ans.}$$

Power at UPF that can be received

(ii) Now given  $P_R = 150 \text{ MW}$

$$\therefore 150 \times 10^6 = \left| \frac{275 \times 275 \times 10^6}{200} \right| \cos(\beta - \delta) - \left( \frac{275^2 \times 10^6 \times 0.85}{200} \right) \cos(70)$$

Solving  $\delta = 28.42^\circ$

$$\text{Now } Q_R = \left| \frac{275^2 \times 10^6}{200} \right| \sin(75 - 28.42) - \left| \frac{0.85 \times 275^2 \times 10^6}{200} \right| \sin(70)$$

$$Q_R = -27.37 \text{ MVAR}$$

$\therefore$  compensation Required is shunt capacitor at Receiving end with  $Q_c = 27.37 \text{ MVAR}$  Ans.

(iii) Now with  $P_R = 150 \text{ MW}$ , compensation equipment not installed at UPF

again

$$Q_R = 0 \quad ; \quad P_R = 150 \text{ MW} \quad ; \quad V_R = ?$$

$$0 = \left| \frac{275 V_R \times 10^3}{200} \right| \sin(75 - \delta) - \left| \frac{0.85 V_R^2}{200} \right| \sin(70)$$

$$\Rightarrow \frac{0.85 V_R^2}{200} \sin 70 = \frac{275 \times 10^3 V_R}{200} \sin(75 - \delta)$$

$$\Rightarrow V_R = \frac{275 \times 10^3 \sin(75 - \delta)}{0.85 \sin 70} \quad \text{--- (1)} \quad \text{Volts.}$$

$$\text{Also, } P_R = 150 \times 10^6 = \left| \frac{275 V_R \times 10^3}{200} \right| \cos(75 - \delta) - \left| \frac{0.85 V_R^2}{200} \right| \cos 70$$

$$\begin{aligned} \Rightarrow 150 &= \frac{275}{200} \times 344.3 \sin(75 - \delta) \cos(75 - \delta) \\ &\quad - \frac{0.85}{200} \times 40542.2 \sin^2(75 - \delta) \end{aligned}$$

$$\Rightarrow 150 = \frac{473.4125}{2} \sin(150 - 2\delta) - 172.3 \sin^2(75 - \delta)$$

Solving  $\delta = 29.816^\circ$

From eqn (i)

$$V_R = \frac{275}{0.85 \sin 70} \sin(75 - \delta) \text{ kV}$$

$$V_R = 244.23 \text{ kV}$$

Ans

good

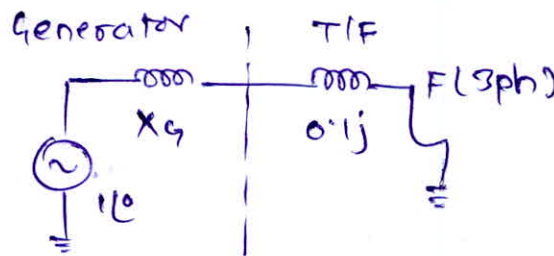
19

Q.3 (b) A 100-MVA, 13.2 kV generator is connected to a 100 MVA, 13.2/132 kV transformer. The generator's reactances are  $X_d'' = 0.15 \text{ p.u.}$ ,  $X_d' = 0.25 \text{ p.u.}$ ,  $X_d = 1.25 \text{ p.u.}$  on a 100 MVA base, while the transformer reactance is  $0.1 \text{ p.u.}$  on the same base. The system is operating on no load, at rated voltage when a three phase symmetrical fault occurs at the HT terminals of the transformer. Determine:

- (i) The subtransient, transient and steady state symmetrical fault currents in p.u. and in amperes.
- (ii) The maximum possible dc component.
- (iii) Maximum value of instantaneous current.
- (iv) Maximum rms value of the asymmetrical fault current.

[20 marks]

Solution Reactance diagram



$$S_b = 100 \text{ MVA}$$

$$V_b = 13.2 \text{ kV (G' side)}$$

$$V_b = 132 \text{ kV (HV side of T/F)}$$

$I_b$

(i) Subtransient current

$$I_F'' (\text{pu}) = \frac{1 \angle 0}{X_d'' + 0.1j} = \frac{1}{(0.15 + 0.1j)} = (-4i) \text{ pu}$$

$$\therefore I_F'' = 4 \times \frac{100 \times 10^6}{\sqrt{3} \times 132 \times 10^3} \text{ A} = 1749.54 \text{ A rms}$$



Transient Current

$$I_F' (\text{pu}) = \frac{1 \angle 0}{x_d' + 0.1j} = \frac{1}{0.25j + 0.1j} = (-2.857i) \text{ pu}$$

$$I_F' = 1249.61 \text{ A} \quad \text{Ans}$$

steady state Symmetrical Fault current

$$I_F (\text{pu}) = \frac{1 \angle 0}{x_d + 0.1j} = \frac{1 \angle 0}{1.25j + 0.1j} = (-0.7467i) \text{ pu}$$

$$I_F = 823.97 \text{ A} \quad \text{Ans}$$

(ii) Maximum possible DC component =  $\sqrt{2} I_F''$   
 $= 2474.22 \text{ A} \quad \text{Ans}$

(iii) Maximum value of instantaneous current  
 $= (\text{DC component max}) + (\text{max value of AC current})$   
 $= \sqrt{2} I_F'' + \sqrt{2} I_F''$   
 $= 2\sqrt{2} I_F''$   
 $= 4948.44 \text{ A} \quad \text{Ans}$

(iv) maximum rms value of Asymm. Fault current

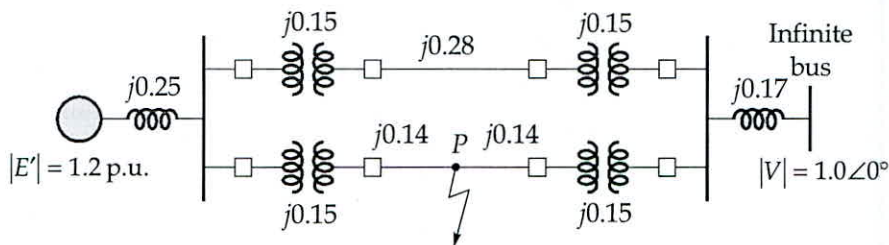
$$= \sqrt{(\text{DC component})^2 + (\text{AC fault current})^2}$$

$$= \sqrt{(\sqrt{2} I_F'')^2 + (I_F'')^2}$$

$$= \sqrt{3} I_F'' = 3030.3 \text{ A} \quad \text{Ans}$$

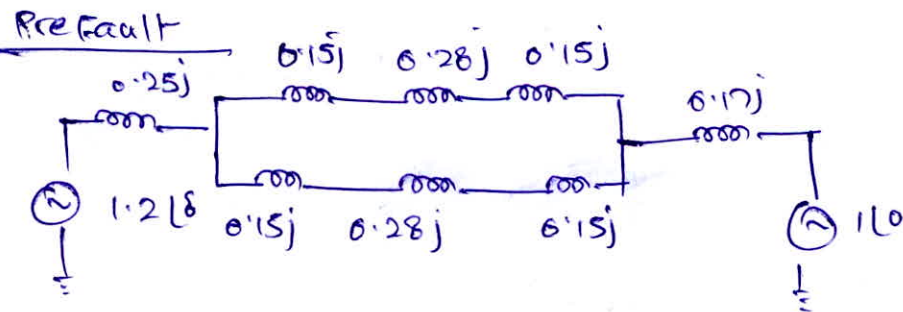


Q.3 (c) Find the critical clearing angle for the system in figure for a three phase fault at the point P. The generator is delivering 1.0 p.u. power under pre-fault conditions.



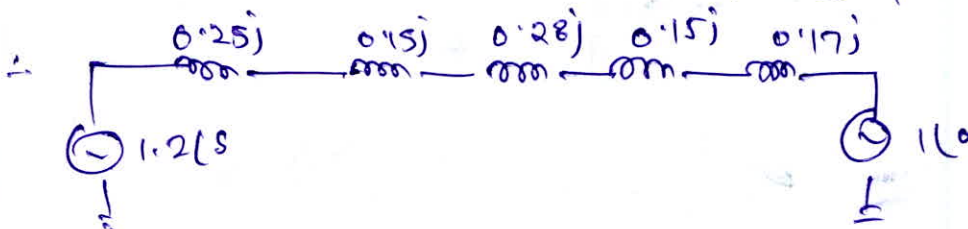
[20 marks]

Solution

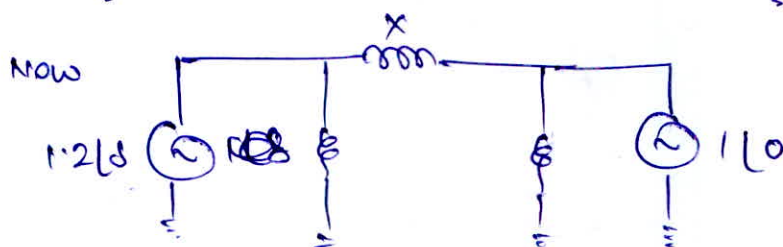
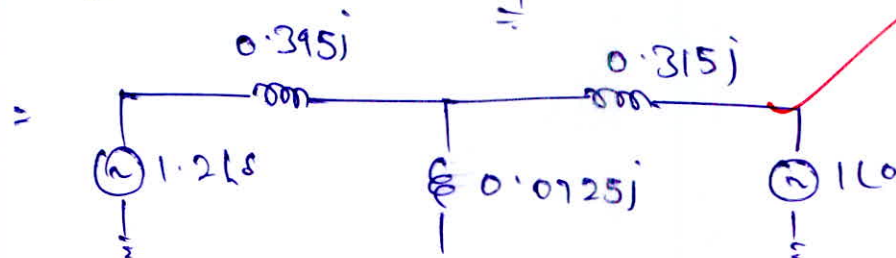
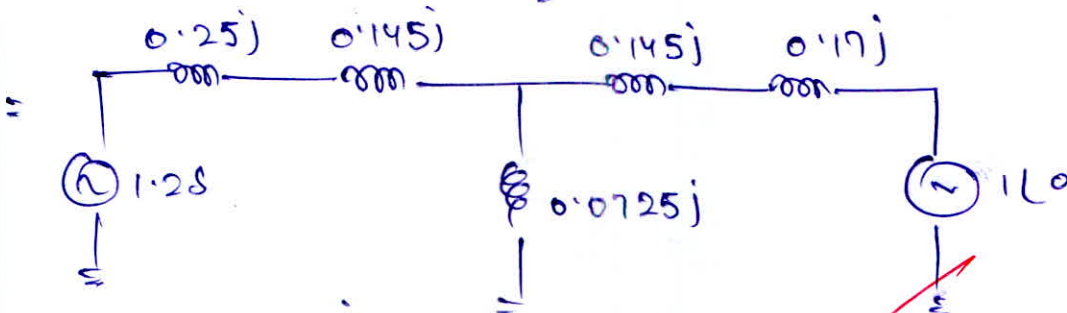
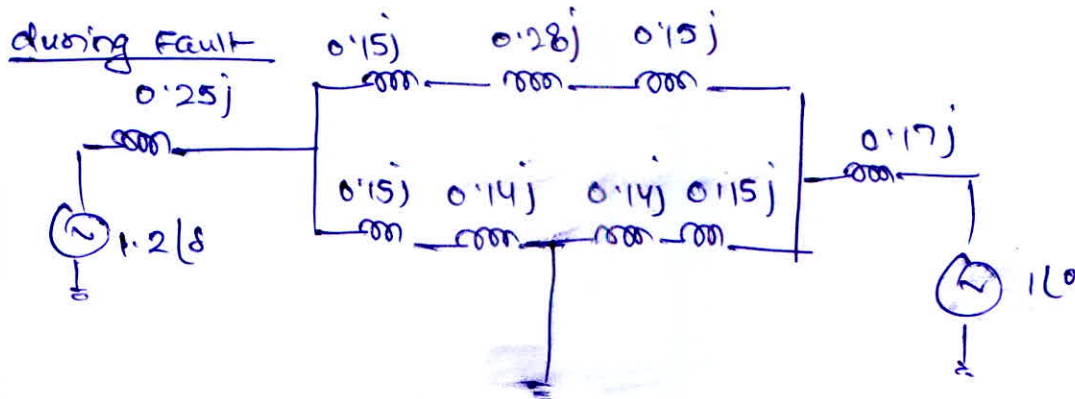


$$\therefore P_{max} = \frac{1.2 \times 1}{0.25j + 0.17j + \frac{0.58j}{2}} = 1.69 \text{ pu}$$

Post Fault : Assume simultaneous opening of CB poles



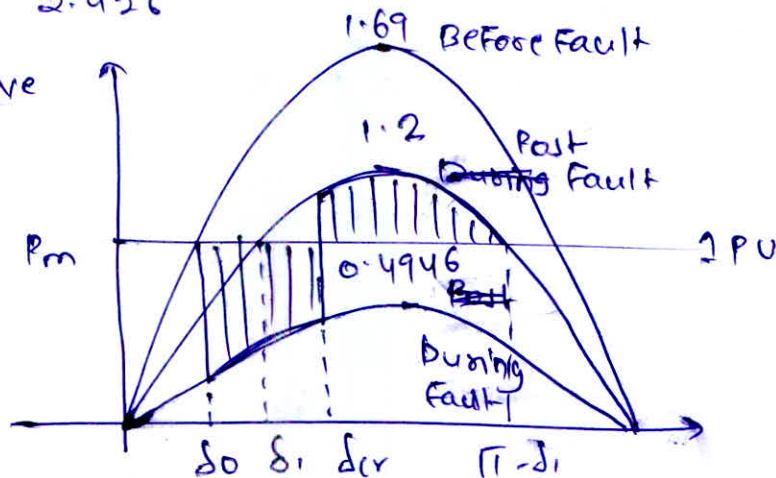
$$P_{max} = \frac{1.2 \times 1}{0.25j + 0.17j + 0.58j} = 1.2 \text{ PU}$$



$$X = 2.426j$$

$$\therefore P_{max} = \frac{1.2 \times 1}{2.426} = 0.4946 \text{ PU}$$

∴ Power angle Curve



$$\text{Now } \delta_0 = \sin^{-1}(1/1.69) = \delta_1 = \sin^{-1}(1/1.2) = 56.44^\circ$$

$$= 36.278^\circ \quad \pi - \delta_1 = 123.55^\circ$$

Now by equal Area criterion

$$\int_{36.278^\circ}^{\delta_{cr}} (1 - 0.4946 \sin \delta) d\delta = \int_{\delta_{cr}}^{123.55^\circ} (1.2 \sin \delta - 1) d\delta$$

$$\Rightarrow \delta_{cr} - 36.278 \times \frac{\pi}{180} + 0.4946 \cos \delta_{cr} - 0.4946 \cos(36.278)$$

$$= 1.2 \cos \delta_{cr} - 1.2 \cos 123.55 - 123.55 \times \frac{\pi}{180} + \delta_{cr}$$

$$\Rightarrow 0.46126 = 0.7054 \cos \delta_{cr}$$

$$\Rightarrow \boxed{\delta_{cr} = 49.16^\circ}$$

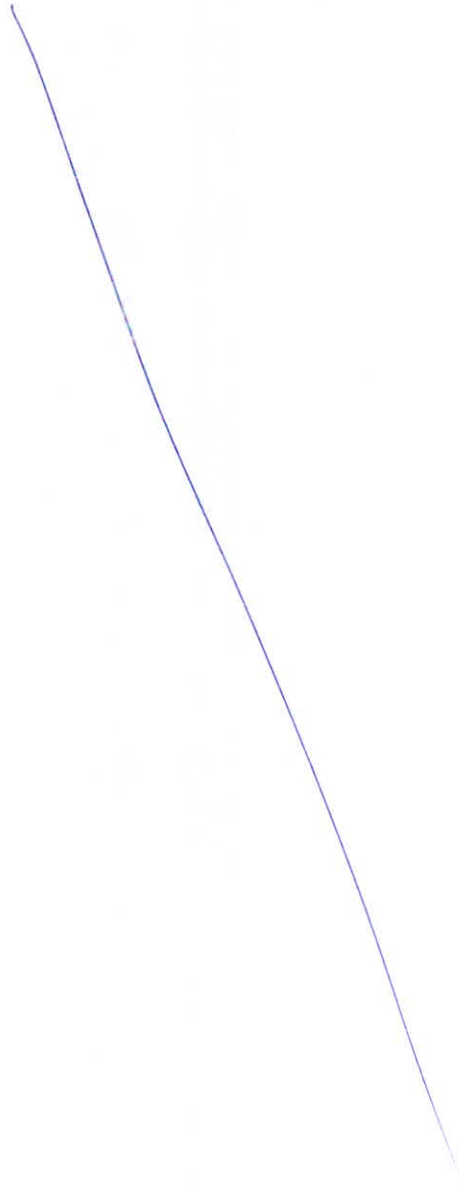
critical clearing angle Ans

good

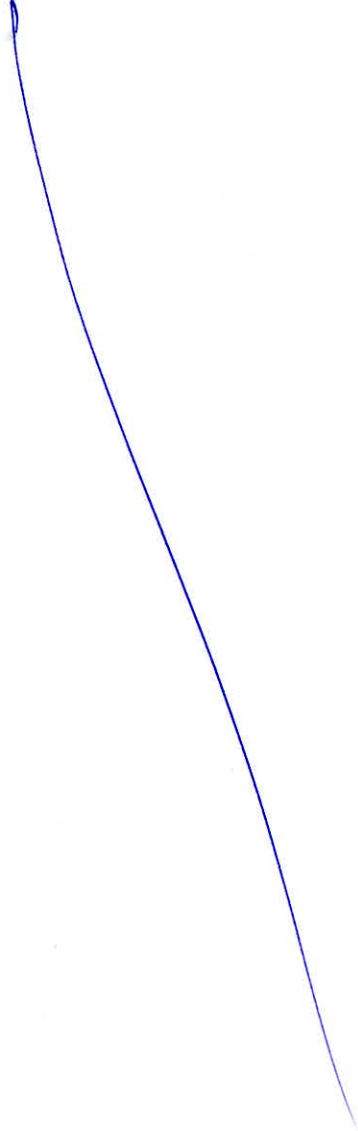
19

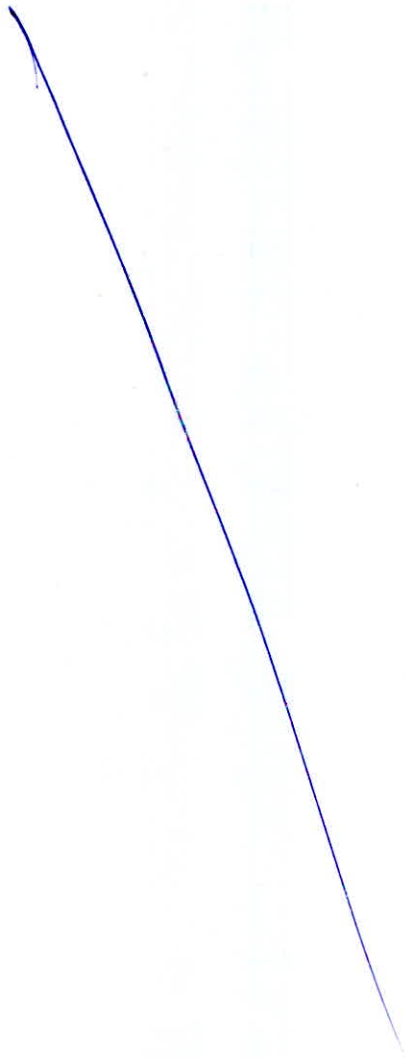
- Q.4 (a) A dc source of 100 V with negligible resistance is connected to a lossless line ( $Z_C = 30 \Omega$ ), through a switch S. If the line is terminated in a resistance of  $90 \Omega$ , on closing the switch at  $t = 0$ , plot the receiving end voltage ( $V_R$ ) w.r.t. time until  $5T$ . Where,  $T$  is the time for voltage wave to travel the length of the line. What will be the steady state voltage at receiving end? Also find the voltage at  $t = 3.25T$  on the mid length of the line.

[20 marks]

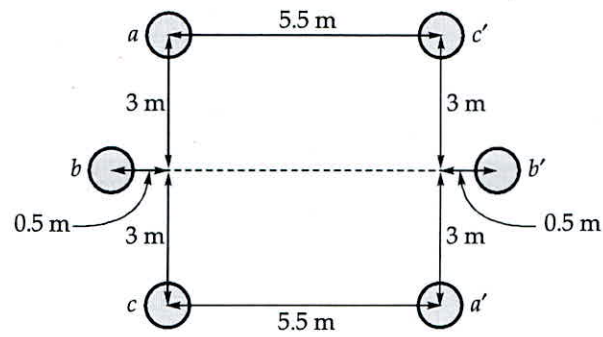




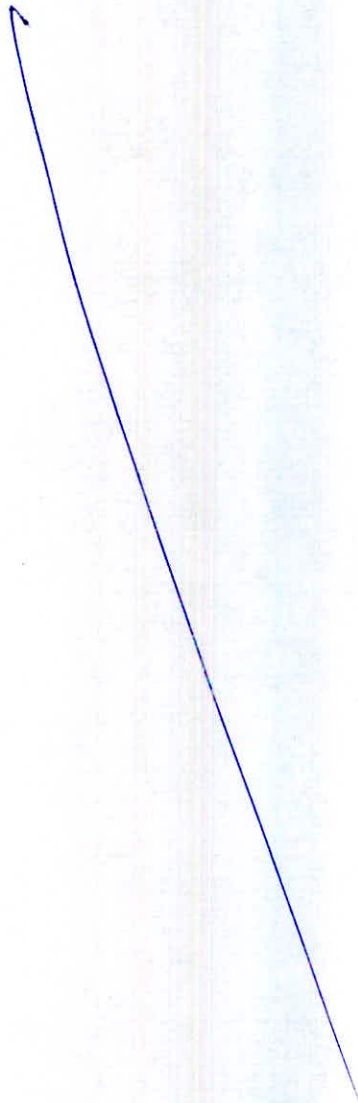




- Q.4 (b) Determine the inductance of the double circuit fully transposed line shown in figure, if the self GMD of each conductor is 0.0069 m.

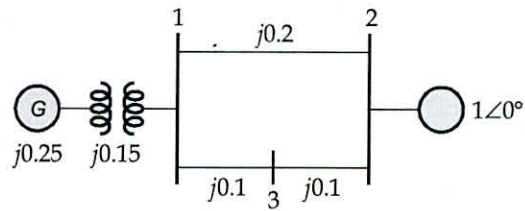


[20 marks]



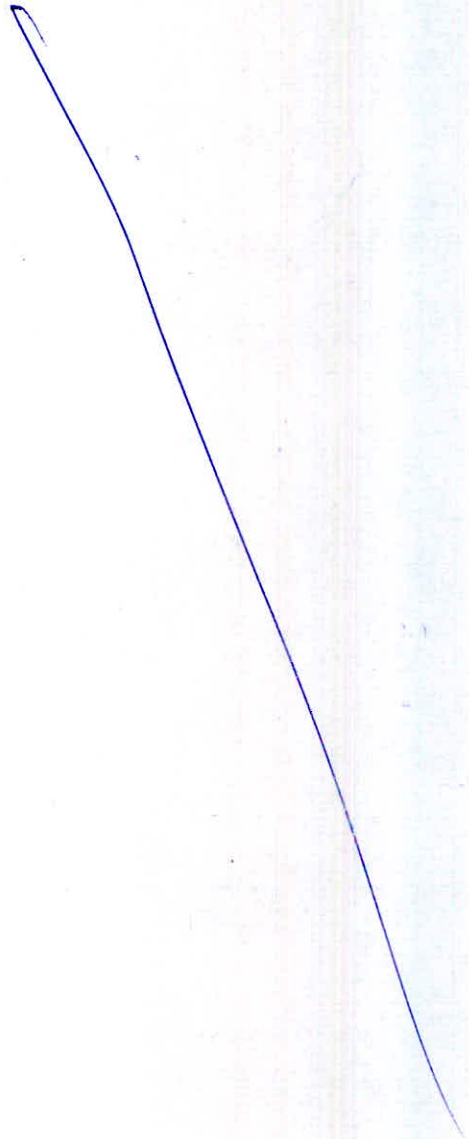


Q.4 (c) A single line diagram of a system is shown below:



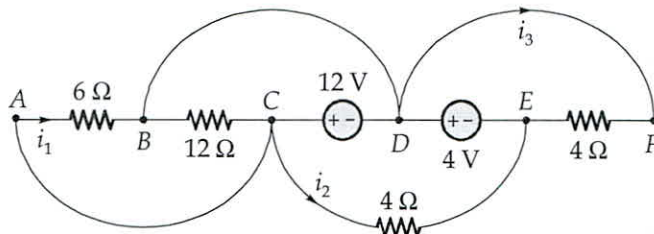
All the values are in per unit on a common base. The power delivered into bus-2 is 1.0 p.u. at 0.8 p.f. lagging. Obtain the power angle equation and swing equation of the system. Neglect all losses.

[20 marks]



**Section B : Electrical Circuits-1 + Microprocessors-1  
+ Digital Electronics-2 + Control Systems-2**

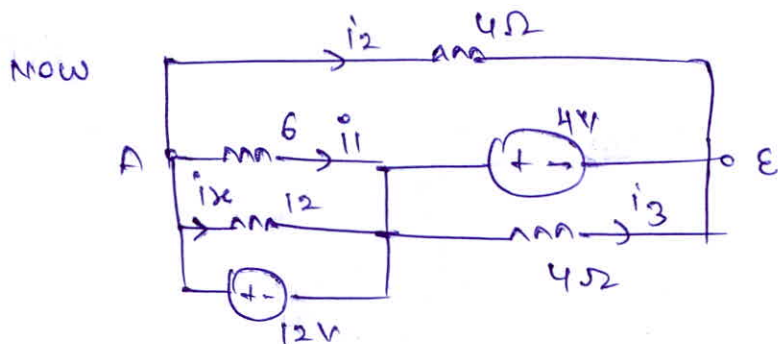
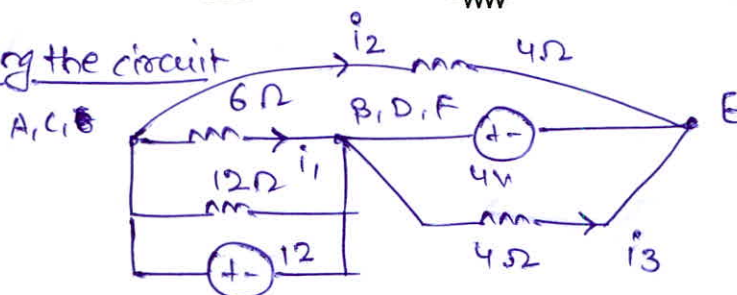
Q.5 (a) Find the current  $i_1, i_2, i_3$  and power delivered by the sources of the network shown in figure.



[12 marks]

Solution

Rearranging the circuit



power delivered by sources = ?

clearly  $4i_3 = 4 \Rightarrow i_3 = 1A$  Ans.

$$i_1 = \left(\frac{12}{6+12}\right) \times \frac{12}{6+12} = \frac{12}{6+12} \times \frac{12}{6+12} = 2A$$

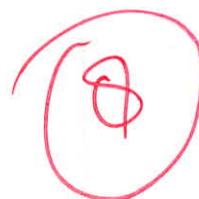
$\therefore i_1 = 2A$

$i_x = 1A$

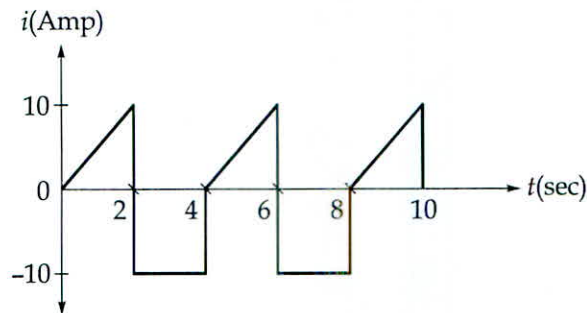
At Node A ~~KCL~~  $i_2 + i_1 + i_x$

now KVL  $4i_2 - 4 - 12 = 0 \Rightarrow i_2 = 16/4 = 4A$

$\therefore i_1 = 2A$   
 $i_2 = 4A$   
 $i_3 = 1A$  } Ans



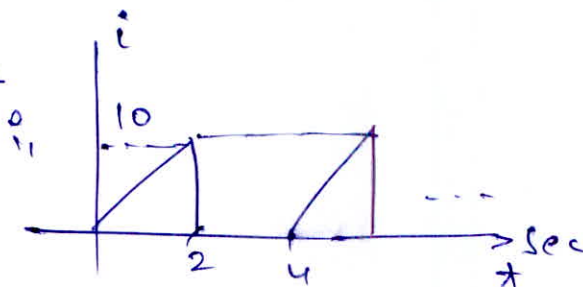
- Q.5 (b) Determine the rms value of the waveform. If the current is passed through a  $9\ \Omega$  resistor. Find the average power absorbed by the resistor.



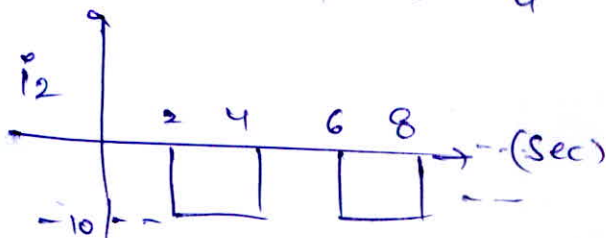
Solution

$$i = i_1 + i_2$$

where



[12 marks]



clearly  $i_1$  &  $i_2$  are  
orthogonal signals

$$\therefore P_{\text{total}} = P_{i_1} + P_{i_2} = \frac{(10)^2}{2 \times 3} + \frac{(10)^2}{2}$$

~~Power~~ Power total in w/f =  $200/3$

Now RMS value of waveform =  $\sqrt{\text{Power}} = \sqrt{200/3}$

RMS value of waveform =  $8.165\text{ A}$  Ans

$P_{\text{avg}}$  (absorbed by resistor) =  $(8.165)^2 \times 9$

$P_{\text{avg}} = 600\text{ W}$  Ans

11



Q.5 (c) A third order system has state space model matrices as follows:

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = [1 \ 1 \ 0]$$

Determine whether the system is state controllable or not. What will be the output controllability of the system?

[12 marks]

Solution

To check Controllability

Kalman's Test -  $Q = [B \ AB \ A^2B]$

$$AB = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$3 \times 3 \quad \quad \quad 3 \times 1$

$$A^2B = AAB = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 38 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 0 & 1 & 7 \\ 0 & 0 & 0 \\ 1 & 6 & 38 \end{bmatrix}$$

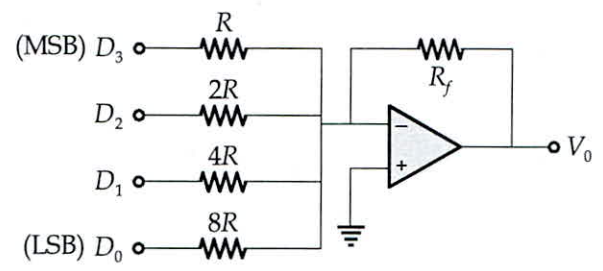
Clearly  $|Q| = 0$

$\therefore$  above system is Non-controllable system

(6)

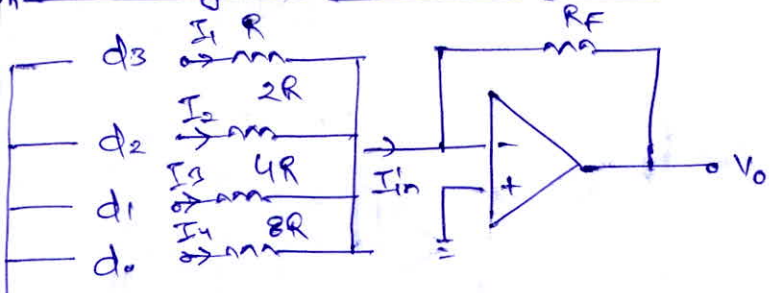


- Q.5 (d) (i) With a neat block diagram, explain the operation of 4-bit weighted-resistor type DAC.
- (ii) For the 4-bit weighted-resistor DAC shown in below figure. Determine the weight of each input bit if the inputs are 0 V and 5 V and also determine the full-scale output, if  $R_f = R = 1 \text{ k}\Omega$ . Also, find the full scale output if  $R_f$  is changed to  $500 \Omega$ .



[12 marks]

Solution 4 Bit Weighted Resistor Type DAC



$V_R$  if  $d_i = 1$   
 OR GND if  $d_i = 0$

$$\text{Now } I_{in} = I_1 + I_2 + I_3 + I_4$$

$$= \frac{V_R d_3}{R} + \frac{V_R d_2}{2R} + \frac{V_R d_1}{4R} + \frac{V_R d_0}{8R}$$

Now  $V_{output} = -I_{in} R_f$

$$V_{o/p} = \frac{-V_R}{R} (R_f) \left[ d_3 + \frac{d_2}{2} + \frac{d_1}{2^2} + \frac{d_0}{2^3} \right]$$

$$V_{o/p} = \frac{V_R}{2^3} \left( \frac{-R_f}{R} \right) \left[ d_3 \cdot 2^3 + d_2 \cdot 2^2 + d_1 \cdot 2^1 + d_0 \cdot 2^0 \right]$$

$$\therefore V_{o/p} = \frac{-V_R R_f}{(2^{n-1})R} \left( \text{decimal equivalent of digital i/p} \right)$$

where  $n$ : no. of bits in digital i/p

$(2^{n-1})R$ : LSB Resistor

(ii) if ip's are 0V and 5V

weight of D<sub>3</sub> bit =  $R_f \frac{V_R}{R} = \frac{5 \times 1K}{1K} = 5V$  Ans

D<sub>2</sub> bit =  $\frac{V_R}{2R} \times R_f = \frac{5 \times 1K}{2K} = 2.5V$  Ans

D<sub>1</sub> bit =  $\frac{V_R}{4R} \times R_f = 1.25V$  Ans

D<sub>0</sub> bit =  $\frac{V_R}{8R} \times R_f = 0.625V$  Ans

Now if  $R_f = 500\Omega$

(V<sub>o/p</sub>) Full scale =  $\frac{V_R}{2^3 R} (R_f)$  [decimal input equivalent]

=  $(10 + 5 + 2.5 + 1.25) = 18.75V$  Ans

??

6

Q.5 (e) Explain the generation of control signals for memory and I/O devices of 8085 microprocessors?

[12 marks]

Solution

Control Signals For memory and I/O

(i)  $\overline{MEMR}$  : memory Read

if  $\overline{MEMR} : 0 \equiv$  memory read operation takes place

$\overline{MEMR} : 1 =$  disabled

(ii)  $\overline{MEMW}$  : memory write operation

$\overline{MEMW} : 0 \equiv$  memory write operation takes place

$\overline{MEMW} : 1 =$  disabled

(iii)  $\overline{IOR}$  : I/O device read operation

$\overline{IOR} : 0$  : Enabled

1 : disabled



(iv)  $\overline{IOW}$  : I/O device write operation

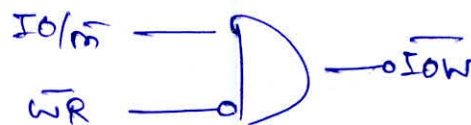
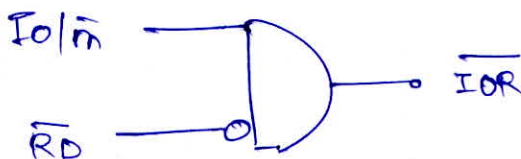
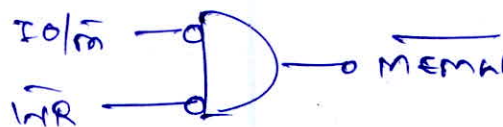
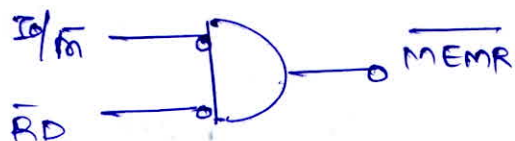
$\overline{IOW} : 0$  : enabled  
 $\overline{IOW} : 1$  : disabled

Now

$\overline{IO/\overline{M}}$  : Selecting I/O or memory

$\overline{IO/\overline{M}} : 0$  : Memory is selected

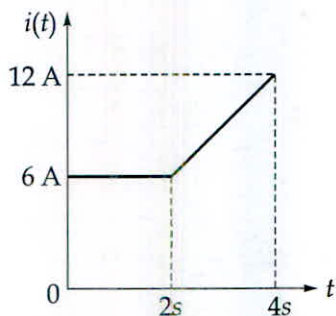
$\overline{IO/\overline{M}} : 1$  : ~~mem~~ I/O is selected



8

table and circuit

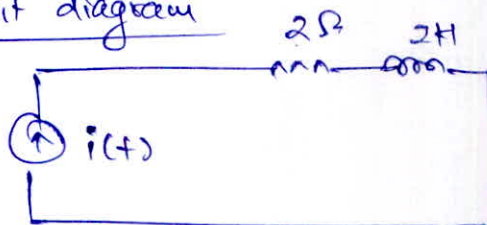
- Q.6 (a) (i) Figure below shows the waveform of the current passing through an inductor of resistance  $2\ \Omega$  and inductance  $2\ \text{H}$ . Find the energy absorbed by the inductor in the first four seconds.



[12 marks]

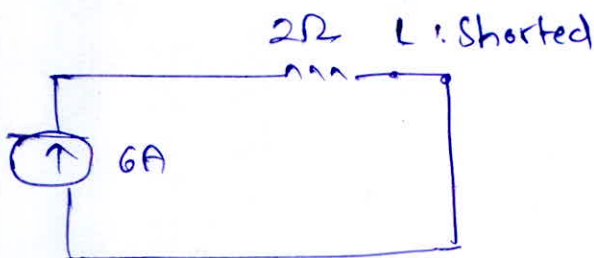
Solution

circuit diagram

Now From  $t=0$  to  $t=2\ \text{sec}$ 

$$i(t) = \text{constant} = 6\ \text{A}$$

∴ equivalent circuit :



∴ energy absorbed by inductor in first 2 seconds = 0

From  $t=2\ \text{sec}$  to  $t=4\ \text{sec}$ 

$$i_L(2^-) = i_L(2^+) = 6\ \text{A}$$

Now energy stored by inductor in (2 to 4) Sec.

$$\begin{aligned} \text{Energy} &= \int_2^4 v_L i_L dt \\ &= \int_2^4 L \frac{di_L}{dt} i_L dt \end{aligned}$$

$$\text{Energy} = \int_2^4 i_L di_L = L \frac{i_L^2}{2} \Big|_2^4$$

$$\therefore \text{energy stored} = \frac{1}{2} L (i(4)^2 - i(2)^2)$$

$$= \frac{1}{2} \times 2 [(12)^2 - (6)^2]$$

$$= 108 \text{ Joules} \quad \checkmark$$

$\therefore$  total energy stored in inductor from  $t=0$  to  $t=4$  sec

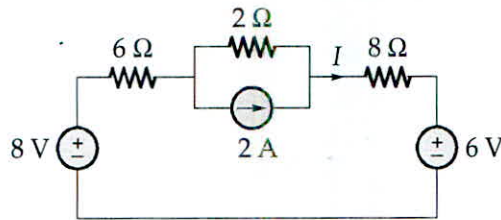
$$= E_1 + E_2$$

$$= 0 + 108$$

$$= 108 \text{ Joules} \quad \times$$

(4)

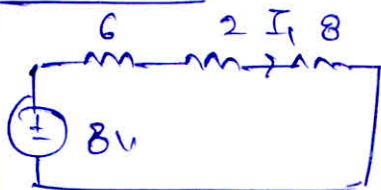
Q.6 (a) (ii) Find the current  $I$  in the circuit shown below using the superposition theorem.



[8 marks]

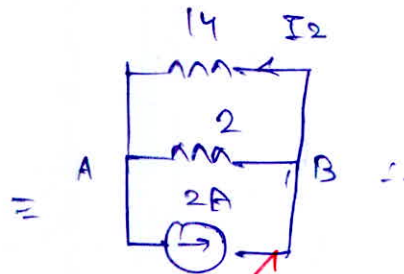
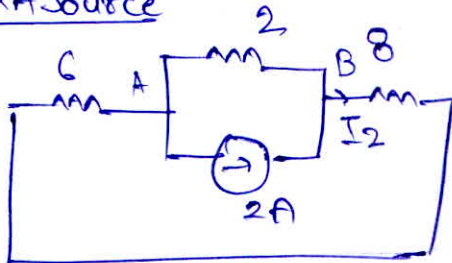
Solution

8V Source



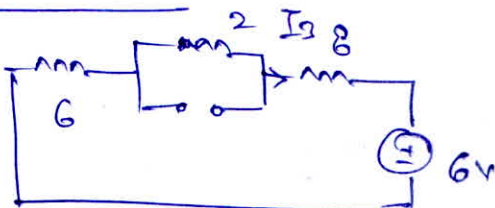
$$I_1 = \frac{8}{8+6+2} = 0.5A$$

2A Source



$$I_2 = 2 \times \frac{2}{16} = \frac{1}{4}A$$

6V Source



$$I_3 = \frac{-6}{6+2+8} = -3/8A$$

∴ By superposition theorem

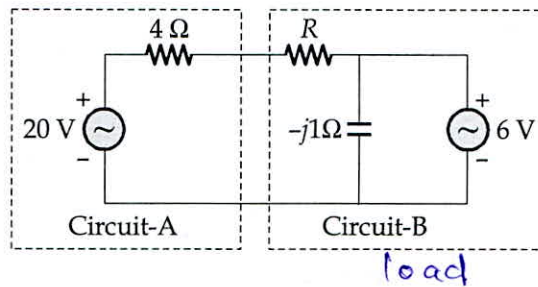
$$I = I_1 + I_2 + I_3 = 3/8 A \quad \text{Ans}$$

$$= 0.375A \quad \text{Ans}$$

7



Q.6 (b) (i) Assuming both the voltage sources are in phase, find the value of  $R$  for which maximum power is transferred from circuit A to circuit B.

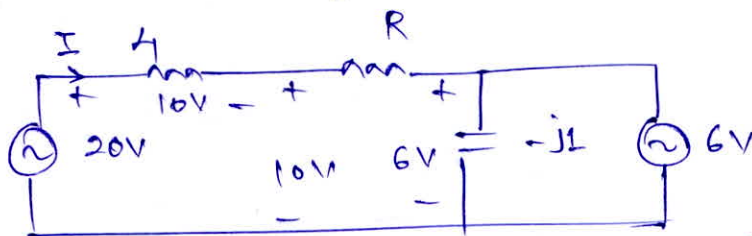


[12 marks]

Solution At Max power Transfer

$V_{th}$  gets divided equally b/w  $R_{th}$  & load

Here let circuit B : load



$$I = \frac{10}{4} \text{ A} = 2.5 \text{ A}$$

Now clearly voltage across  $R = 10 - 6 = 4 \text{ V}$

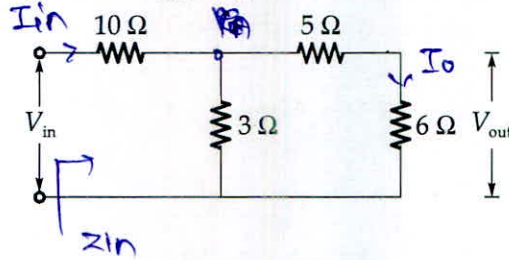
Current through  $R = 2.5 \text{ A}$

$$\therefore R = \frac{4}{2.5} = 1.6 \Omega \quad \text{Ans}$$

For maximum power Transfer

10

Q.6 (b) (ii) Determine the voltage ratio  $V_{out}/V_{in}$  for the circuit shown below:



[8 marks]

Solution  $Z_{in} = 10 + 3 \parallel 11 = 173/104 \Omega$

$$\therefore I_{in} = \frac{V_{in}}{Z_{in}} = \frac{14V_{in}}{173} \text{ A}$$

Now  $I_o = I_{in} \times \frac{3}{14}$

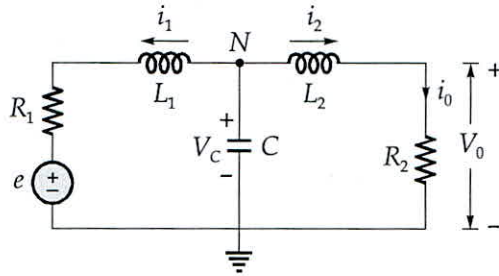
$$I_o = \frac{3V_{in}}{173} \text{ A}$$

Now  $\frac{V_{out}}{V_{in}} = \frac{6 \times 3V_{in}}{173} = \frac{18}{173}$

$$\frac{V_{out}}{V_{in}} = 0.104 = \frac{18}{173} \text{ Ans}$$

7

Q.6 (c) Obtain the state space model considering current through the two inductors and voltage across the capacitor as state variables. Consider the output variables as current through  $R_2$  and voltage across  $R_2$  and input variable as 'e' as shown in figure. (Consider initial conditions to be zero)



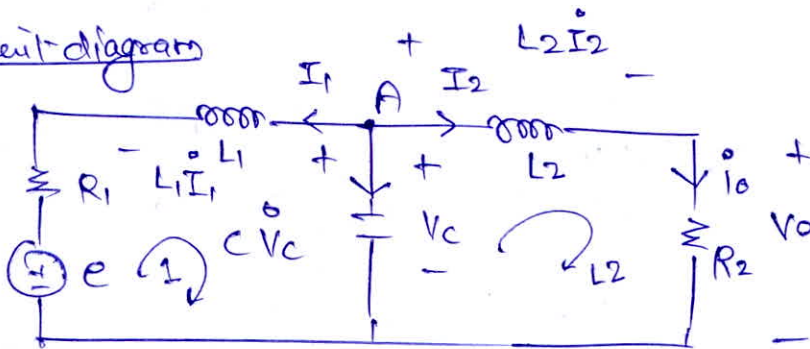
[10 marks]

Now  $u = e$

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} i_0 \\ V_0 \end{pmatrix}$$

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \\ V_c \end{pmatrix} ; \dot{X} = \begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{pmatrix} = \begin{pmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{V}_c \end{pmatrix}$$

Circuit diagram



At node A, KCL  $I_1 + I_2 + C \dot{V}_c = 0$

$$\therefore \dot{V}_c = -\frac{I_1}{C} - \frac{I_2}{C}$$

$$\dot{X}_3 = -\frac{X_1}{C} - \frac{X_2}{C} \quad \text{--- (1)}$$

KVL in loop 1

$$-L_1 \dot{I}_1 + V_c - e - I_1 R_1 = 0$$

$$\Rightarrow -L_1 \dot{I}_1 = e - V_c + I_1 R_1$$

$$\Rightarrow \frac{V_c - e - I_1 R_1}{L_1} = \dot{I}_1 \Rightarrow \dot{X}_1 = -\frac{R_1}{L_1} X_1 + \frac{X_3}{L_1} - \frac{u}{L_1}$$

$$\text{In Loop 2 KVL, } +L_2 \dot{I}_2 + I_2 R_2 - V_C = 0$$

$$\dot{I}_2 = -\frac{I_2 R_2}{L_2} + \frac{V_C}{L_2}$$

$$\therefore \boxed{\dot{X}_2 = -\frac{R_2}{L_2} X_2 + \frac{X_3}{L_2}}$$

$$\text{Now } Y_1 = i_0 = I_2 = X_2$$

$$Y_2 = V_0 = I_2 R_2 = R_2 X_2$$

Now state space model

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{pmatrix} = \begin{pmatrix} -R_1/L_1 & 0 & 1/L_1 \\ 0 & -R_2/L_2 & 1/L_2 \\ -1/C & -1/C & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} + \begin{pmatrix} -1/L_1 \\ 0 \\ 0 \end{pmatrix} U$$

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & R_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

Ans

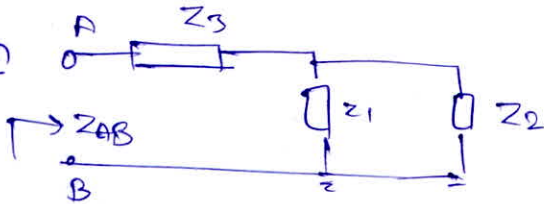
7



**Q.6 (d)** Two impedances  $Z_1 = 5 \Omega$  and  $Z_2 = (5 - jX_C)\Omega$  are connected in parallel and this combination is connected in series with  $Z_3 = (6.25 + j1.25)\Omega$ . Determine the value of capacitance of  $X_C$  to achieve resonance if the supply is 100 V, 50 Hz.

[10 marks]

Solution



$$\text{NOW } Z_{AB} = Z_3 + Z_1 || Z_2$$

$$= (6.25 + j1.25) + \frac{5(5 - jX_C)}{5 + 5 - jX_C}$$

$$= (6.25 + j1.25) + \frac{(25 - 5jX_C)(10 + jX_C)}{(10 - jX_C)(10 + jX_C)}$$

$$= 6.25 + j1.25 + \frac{1}{(100)^2 + X_C^2} [250 + j25X_C - 50X_Cj + 5X_C^2]$$

Now For Resonance

$$\text{Im}(Z_{AB}) = 0$$

$$\Rightarrow 1.25 + \frac{1}{(100)^2 + X_C^2} (-25X_C) = 0$$

$$\Rightarrow 1.25 = \frac{25X_C}{100^2 + X_C^2}$$

$$\Rightarrow 125 + 1.25X_C^2 - 25X_C = 0$$

s. Solving  $X_C = 10 \Omega$

$$\text{NOW } \frac{1}{\omega C} = 10 \Rightarrow C = \frac{1}{2\pi \times 50 \times 10}$$

$$C = 318.31 \mu\text{F} \quad \text{Ans}$$

10

Q.7 (a) (i) Clearly differentiate between latches and flip-flops.

[8 marks]

Q.7 (a) (ii) Realize  $T$ -flip flop using  $D$ -flip flop.

[12 marks]

Q.7 (b) (i) For a control system, the input output transfer function is given below:

$$\frac{Y(s)}{U(s)} = \frac{b_0s^n + b_1s^{n-1} + \dots + b_{n-1}s + b_n}{s^n + a_1s^{n-1} + a_2s^{n-2} \dots a_{n-1}s + a_n}$$

Construct a state diagram by direct decomposition using the first companion form. Use state diagram to obtain dynamic equations and state space model.

(ii) Obtain the second companion form for the system whose input output transfer function is given by,

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 4s + 1}{s^3 + 3s^2 + 4s + 8}$$

Draw corresponding state diagram for above form and derive state space model for above system.

[20 marks]







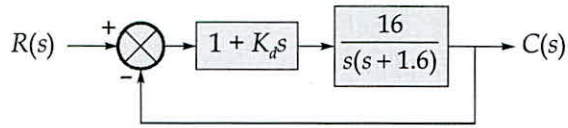
- Q.7 (c)
- (i) Write an assembly language program for an 8085 microprocessor, to find 2's complement of a 16-bit number. Write comments for selected instructions.
  - (ii) Enumerate all internal registers present in 8259 programmable interrupt controller? Write short notes on their individual functionality?

**[12 + 8 marks]**





- Q.8 (a) A control system employing proportional and derivative control as shown below, has damping ratio equal to 0.8. Find the time instant at which the step response of system attains the peak value. Also find the percent maximum overshoot of system.



[20 marks]

Solution Given System  $\frac{C(s)}{R(s)} = \frac{(1 + K_d s) 16}{s^2 + 1.6s}$

$$1 + \frac{(1 + K_d s) 16}{s^2 + 1.6s}$$

$$\frac{C(s)}{R(s)} = \frac{(1 + K_d s) 16}{s^2 + 1.6s + 16 + \frac{K_d s 16}{16}} = \frac{(1 + K_d s) 16}{s^2 + (1.6 + 16K_d) s + 16}$$

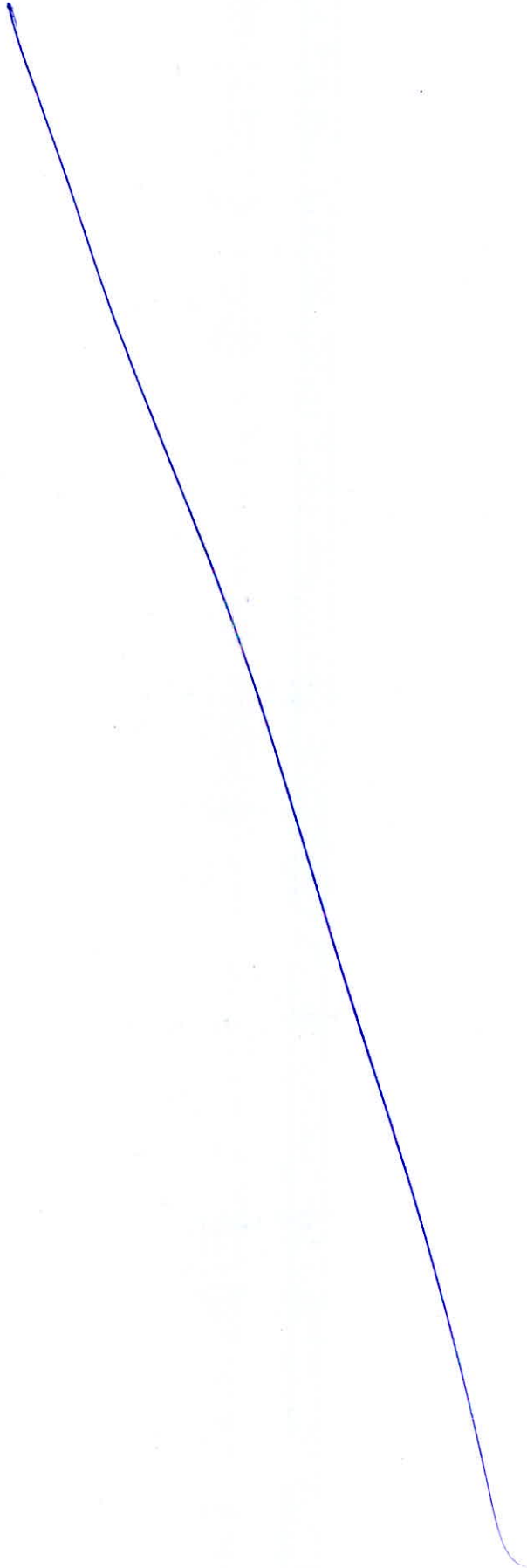
$$\frac{C(s)}{R(s)} = \frac{16(1 + K_d s)}{s^2 + s(1.6 + 16K_d) + 16}$$

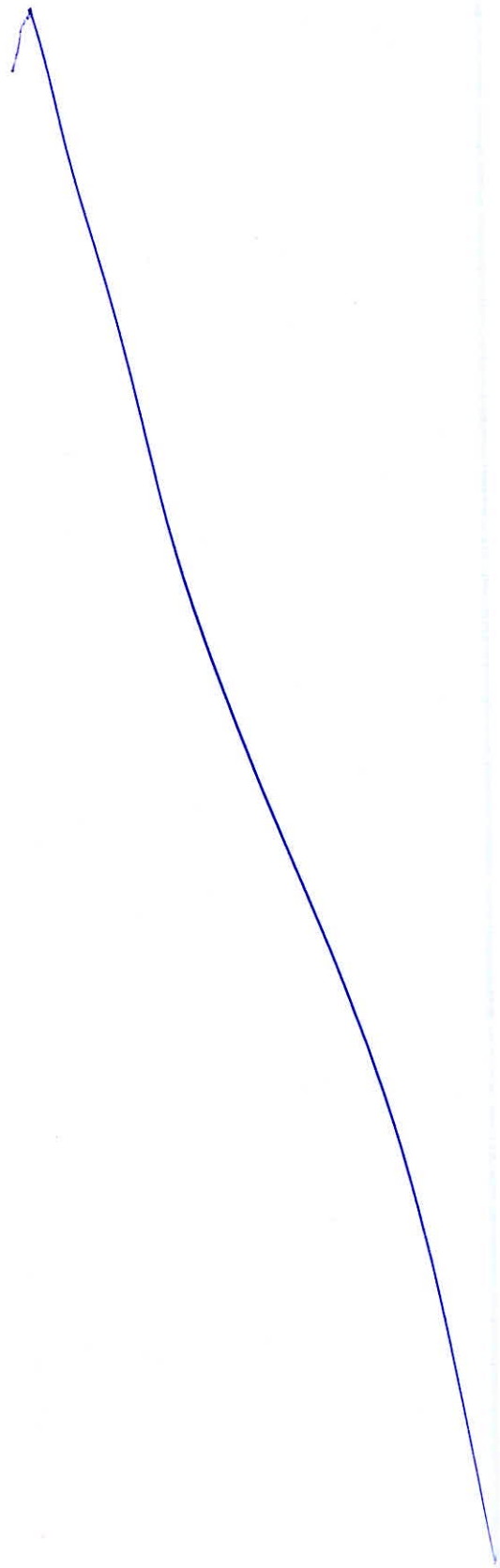
$$= \frac{16(1 + K_d s)}{\left(s + \frac{1.6 + 16K_d}{2}\right)^2 + 16 - \left(\frac{1.6 + 16K_d}{2}\right)^2}$$

Now  $C(s) = \frac{16(1 + K_d s)}{s \left\{ \left(s + \frac{1.6 + 16K_d}{2}\right)^2 + 16 - \left(\frac{1.6 + 16K_d}{2}\right)^2 \right\}}$

$$= \frac{1}{s} + \frac{16(1 + K_d s)}{\left(s + \frac{1.6 + 16K_d}{2}\right)^2 + 16 - \left(\frac{1.6 + 16K_d}{2}\right)^2}$$

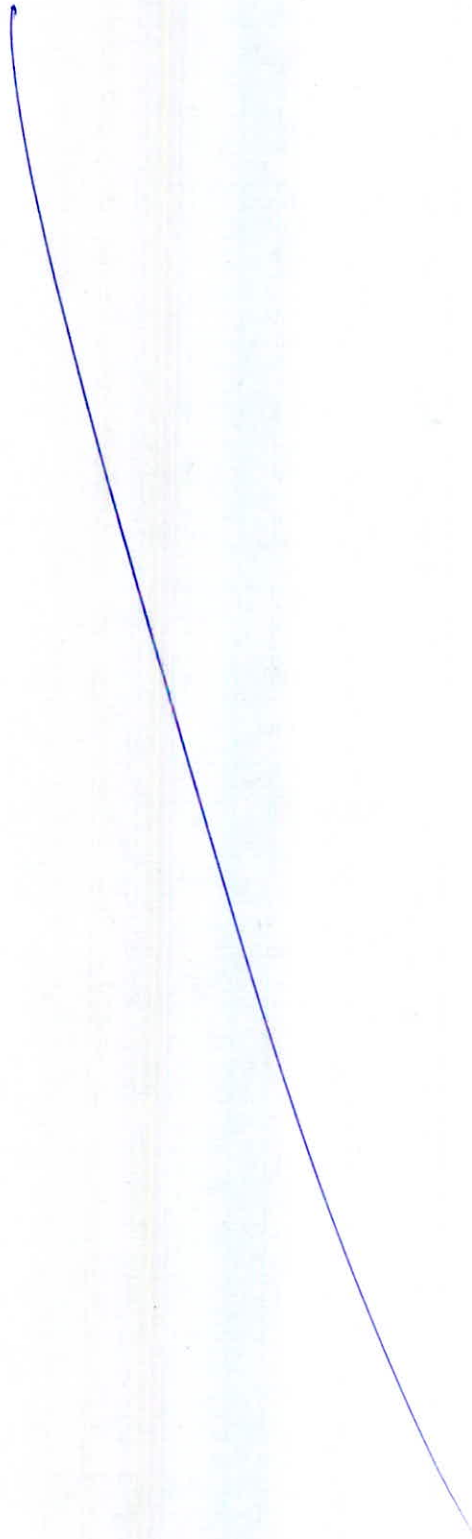
NOT ATTEMPTED





Q.8 (b) Design a 3-bit gray UP/DOWN synchronous counter using  $T$ -flip flops with a control for UP/DOWN counting.

[20 marks]



2.8 (c) A control system is represented by the state equation given below,

$$\dot{x}(t) = Ax(t)$$

If the response of the system is  $x(t) = \begin{pmatrix} e^{-t} \\ -2e^{-t} \end{pmatrix}$  when  $x(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$  and  $x(t) = \begin{pmatrix} e^{-2t} \\ -e^{-2t} \end{pmatrix}$

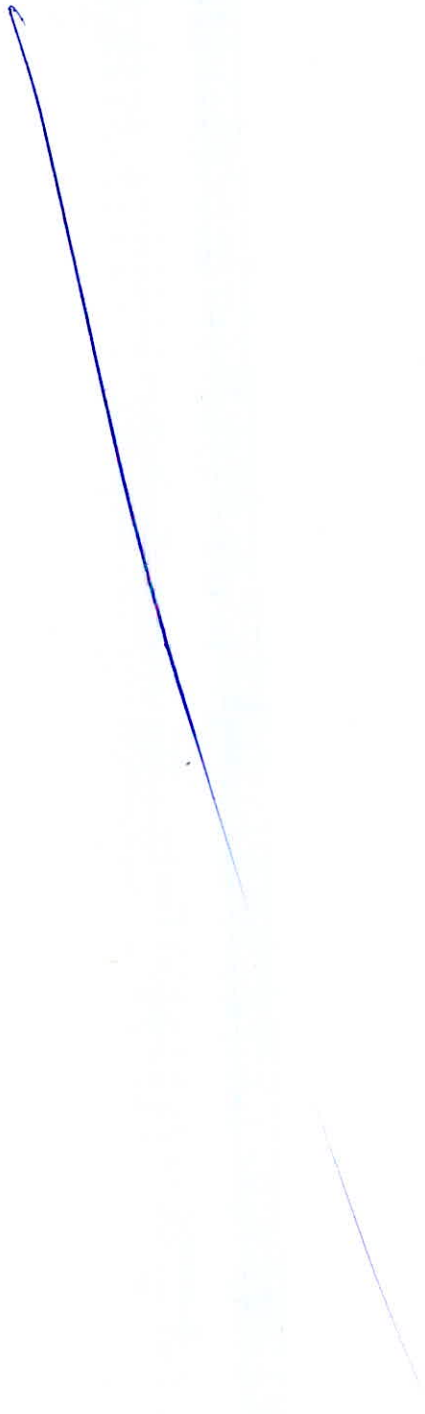
when  $x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ .

Calculate the system matrix A and the state transition matrix for the system.

[20 marks]







Space for Rough Work



$$P = VI$$

$$W = 2I^2R$$

$$\frac{P/A}{W} = \frac{2I^2}{W} = \frac{2}{W} \left(\frac{P}{V}\right)^2$$

$l \rightarrow \text{const.}$

$$A = \frac{P W V^2}{2P^2}$$

$$\text{volume} \propto \frac{l}{V^2}$$

$$W = \cancel{2I^2R} 2 \left(\frac{P}{V}\right)^2 R$$

$$P = VI$$

$$I = \frac{P}{V}$$

Space for Rough Work

$$x(t) = e^{At} x(0)$$

$$\mathcal{L}^{-1} \{ (sI - A)^{-1} \} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} e^{-t} \\ -2e^{-t} \end{pmatrix} = e^{At} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$4^2 - (0.8 + 8kd)^2$$

$$A = 1$$

$$(3.2 - 8kd) (4.8 + 8kd) \lambda = -2$$

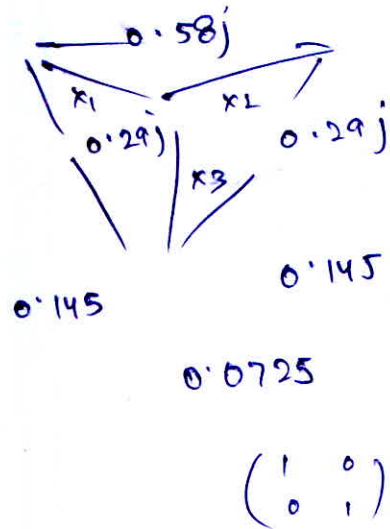
$$\lambda = -1$$

$$\lambda = -2$$

character

$$(\lambda + 1)(\lambda + 2)$$

$$|sI - A| = s^2 + 3s + 2 = 0$$



$$16(1 + kd)$$

$$s \left[ s + (0.8 + 8kd) \right]^2 + (3.2 - 8kd)(4.8 + 8kd)$$

$$\left\{ \frac{A}{s} + \frac{B}{(s + 0.8 + 8kd)^2 + (3.2 - 8kd)(4.8 + 8kd)} \right\}$$

$$16(1 + kd)s = Bs + A \left( s^2 + (0.8 + 8kd)^2 + (1.6 + 16kd)s \right) + A(3.2 - 8kd)(4.8 + 8kd)$$

16

$$16kd = B +$$

190

