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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Mechanical Engineering

**Test-3: Fluid Mechanics and Turbo Machinery, Heat Transfer-1 + TOM-1,
Thermodynamics-2 + Refrigeration and Air-conditioning-2**

Name :

Roll No :

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Test Centres

Student's Signature

Delhi Bhopal Noida Jaipur Indore
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	24
Q.2	30
Q.3	—
Q.4	—
Section-B	
Q.5	39-3-36
Q.6	53 50
Q.7	—
Q.8	50-2=48
Total Marks Obtained	202 188 (188)

Signature of Evaluator

Sumitk...

Cross Checked by

G.S.

Section A : Fluid Mechanics and Turbo Machinery

(a) Define degree of Reaction. Derive the expression of degree of reaction for an axial flow compressor in terms of inlet and outlet blade angles, blade and flow velocity.

Degree of reaction is defined as the ^{ratio of} enthalpy drop occurs due to pressure change in the turbine to the total enthalpy drop that occurs in the turbine. [12 marks]

$R = \frac{\text{enthalpy drop due to increase in pressure}}{\text{total enthalpy drop}}$

$$H = \frac{v_{w1} u_1}{g} = \frac{v_1^2 - v_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} + \frac{v_{r2}^2 - v_{r1}^2}{2g}$$

↓ due to kinetic energy → due to pressure drop.

$$R = \frac{\frac{u_1^2 - u_2^2}{2g} + \frac{v_{r2}^2 - v_{r1}^2}{2g}}{\frac{v_1^2 - v_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} + \frac{v_{r2}^2 - v_{r1}^2}{2g}}$$

$$R = 1 - \frac{\frac{v_1^2 - v_2^2}{2g}}{\frac{v_1^2 - v_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} + \frac{v_{r2}^2 - v_{r1}^2}{2g}}$$

In this question they put a question about degree of reaction of compressor not turbine

09

$$v_1^2 = v_{w1}^2 + v_{f1}^2$$

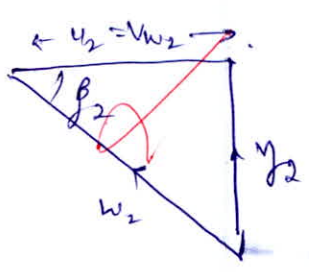
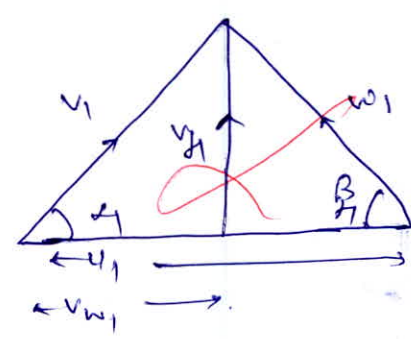
$$v_2 = v_{f2} = v_{f1}$$

$$v_1^2 = v_{w1}^2 + v_2^2$$

$$[v_1^2 - v_2^2 = v_{w1}^2]$$

$$H = \frac{v_{w1} u_1}{g}$$

$$H = \frac{v_{w1} u_1}{g}$$



$$R = 1 - \frac{\left(\frac{v_{uy}^2}{2g} \right)}{\left(\frac{v_{uy} u_1}{g} \right)}$$

$$R = 1 - \frac{v_{uy}}{2u_1}$$

$$R = 1 - \frac{(u_1 - v_{y1} \tan \beta)}{2u_1}$$

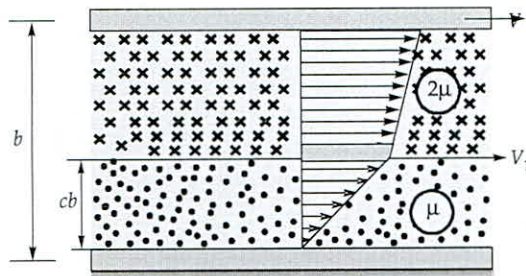
$$R = 1 - \left(\frac{u_1 - v_{y1} \tan \beta}{2u_1} \right)$$

$$R = 1 - \frac{1}{2} - \frac{v_{y1} \tan \beta}{2u_1}$$

$$\boxed{R = \frac{1}{2} - \frac{v_{y1} \tan \beta}{2u_1}} =$$

(b) Two flat plates are oriented in parallel configuration above a fixed lower plate as shown in figure. The top plate, located a distance, b above the fixed plate, is pulled along with speed V . The other thin plate is located a distance (cb) where $0 < c < 1$, above the fixed plate. This plate moves with speed V_1 which is determined by the viscous shear forces imposed on it by the fluids on its top and bottom. The fluid on the top is twice as viscous as that on the bottom, then obtain the ratio $\left(\frac{V_1}{V}\right)$ corresponding to value of c as given in table.

c	0	0.2	0.5	0.7	1.0
V_1/V	?	?	?	?	?



[12 marks]

At the interface ;

Net force on downward fluid = force on upward fluid

$$\mu \frac{A v}{h} = \frac{\mu A v}{h}$$

$$\mu \cdot \frac{V_1}{cb} = \frac{2 \cdot \mu (v - V_1)}{(b - cb)}$$

$$\frac{V_1}{cb} = \frac{2 (v - V_1)}{(b - cb)}$$

$$V_1 b - V_1 cb = 2 (v - V_1) cb.$$

$$V_1 b - V_1 cb = 2 v cb - 2 V_1 cb.$$

$$V_1 b + 2 V_1 cb = 2 v cb.$$

$$V_1 (b + 2cb) = 2 v cb.$$

$$\frac{V_1}{v} = \frac{cb}{b + 2cb} \quad \text{Wrong}$$

$$\frac{v_1}{v} = \frac{cb}{b + 2cb} = \frac{c}{1+2c}$$

$$(c=0)$$

$$\boxed{\frac{v_1}{v} = 0}$$

$$\underline{\underline{c=0.2}}$$

$$\frac{v_1}{v} = \frac{0.2}{1+0.4} = 0.1428$$

$$\underline{\underline{c=0.5}}$$

$$\frac{v_1}{v} = \frac{0.5}{1+2(0.5)} = \frac{0.5}{2} = 0.25$$

$$\underline{\underline{c=0.7}}$$

$$\frac{v_1}{v} = \frac{0.7}{1+1.4} = 0.29166$$

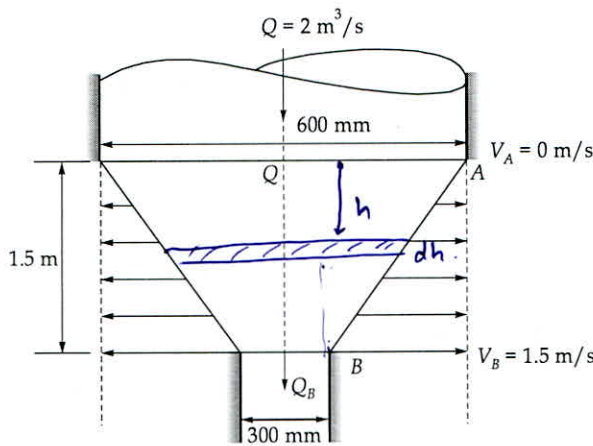
$$\underline{\underline{c=1}}$$

$$\frac{v_1}{v} = \frac{1}{1+2(1)} = \frac{1}{3} = 0.333$$

c	0	0.2	0.5	0.7	1.0
$\frac{v_1}{v}$	0	0.1428	0.25	0.29166	0.333

02

- (c) Water flow downward in a pipe of 600 mm diameter at the rate of $2 \text{ m}^3/\text{s}$. It then enters a conical duct with porous wall such that there is a radial outflow with flow velocity varying linearly from zero at A to 1.5 m/s at B. What is the rate of flow at B coming out from the conical duct.



[12 marks]

$$v_1 = \frac{Q}{A_1} = \frac{2}{\frac{\pi}{4} (0.6)^2} = 7.073 \text{ m/s.}$$

$$v_2 = \frac{Q}{A_2} = \frac{2}{\frac{\pi}{4} (0.3)^2} = 28.29 \text{ m/s.}$$

$$Q = Q_{\text{out}} + Q_B.$$

$$Q = A \times v.$$

$$Q_{\text{out}} = \int (B \cdot dh) \cdot v.$$

$$Q_{\text{out}} = \int B v dh$$

$$v = v_A + \frac{(v_B - v_A) h}{1.5} \quad \Rightarrow \quad v = \frac{1.5 h}{1.5} \quad [v = h]$$

$$B = 600 - \left(\frac{600 - 300}{1.5} \right) \times h.$$

$$B = 600 - \frac{0.3 h}{1.5}$$

$$B = 600 - 0.2 h$$

$$Q_{\text{out}} = \int_0^{1.5} (600 - 0.2 h) \cdot h dh.$$

$$Q = \int_0^{1.5} (0.6h - 0.2h^2) dh$$

$$Q = \left[0.6 \frac{h^2}{2} - 0.2 \frac{h^3}{3} \right]_0^{1.5}$$

$$Q = 0.3 (1.5)^2 - 0.2 \left(\frac{1.5}{3} \right)^3$$

$$Q_{\text{out}} = 0.675 - 0.225$$

$$Q_{\text{out}} = 0.45 \text{ m}^3/\text{sec.}$$

$$Q_B = 2 - 0.45$$

$$Q_B = 1.55 \text{ m}^3/\text{sec.}$$

- Q.1 (d) (i) Explain why there is a need of compounding of impulse steam turbine. Also mention types of compounding done.
- (ii) What are the differences between impulse and reaction turbine? Explain in a tabular form.

In order to increase the output from the turbine, we need to increase the input velocity on the blades. The velocity, sometimes becomes very high that it may reach to sonic velocity. This may cause wear and damage the blades suddenly. Hence, compounding is done to ensure that the velocity does not reach above a value at any stage. There are two types of compounding:

- 1) velocity compounding (also known as Curtis turbine)
- 2) Pressure Compounding (Rateau turbine)

In velocity compounding, the velocity is increased at every stage by installation of nozzle while in pressure compounding; the pressure is increased by the help of fixed vanes.

Impulse Turbine

Reaction turbine

1) The energy is in the form of kinetic energy only.

The energy is in the form of both kinetic energy and pressure energy.

2) All the energy is provided while striking the blades.

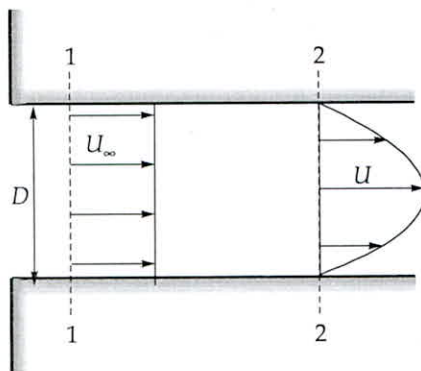
kinetic energy is lost while striking while pressure energy is provided while flowing through casing.

3) Very high head required.
 4) Produces less power output
 5) eg Pelton turbine

Comparatively low head required.
 produces more power output.
 eg Francis turbine

1 (e) In a steady entrance flow in a pipe of diameter D as shown in figure. The flow develops from uniform flow at section (1) to a parabolic profile at section (2). If the momentum correction factor at section (2) is $\frac{4}{3}$, then show that the wall drag force F is given by

$$F = \frac{\pi D^2}{4} \left(P_1 - P_2 - \frac{1}{3} \rho U_\infty^2 \right)$$



Where P_1 and P_2 are pressure at respective sections.

Assume that diameter remains same throughout;

$$A_1 = A_2 = \frac{\pi}{4} D^2$$

[12 marks]

Applying momentum eqⁿ;

Net force = Change in momentum.

$$F_x + P_1 A_1 - P_2 A_2 = \beta \dot{m} v_2 - \dot{m} v_1$$

$$F_x + (P_1 - P_2) A = \dot{m} (\beta v_2 - v_1)$$

$$F_x + (P_1 - P_2) A = (P A_2 v_2) \left[\beta v_2 - v_1 \right]$$

Momentum correction factor;

$$\beta = \frac{(\dot{m}v)_{\text{actual}}}{(\dot{m}v)_{\text{ideal}}} \quad (\dot{m}v)_{\text{act}} = \beta (\dot{m}v)_{\text{ideal}}$$

$$\boxed{\dot{m}v_{\text{act}} = \frac{4}{3} (\dot{m}v_1)}$$

$$F_x + (P_1 - P_2) A = \dot{m} v_2 - \dot{m} v_1$$

$$F_x + (P_1 - P_2) A = \frac{4}{3} \dot{m} v_1 - \dot{m} v_1$$

$$F_x + (P_1 - P_2) A = \frac{1}{3} \dot{m} v_1 \quad \left. \vphantom{F_x} \right\} \dot{m} = \rho A_1 v_1$$

$$F_x + (P_1 - P_2) A = \frac{1}{3} \rho \pi A_1 v_1^2$$

$$F_x = - \left[(P_1 - P_2) - \frac{1}{3} \rho U_{\infty}^2 \right] \frac{\pi}{4} D^2$$

Negative sign shows direction assumed is incorrect

∴ Drag free;

$$\boxed{F = \frac{\pi D^2}{4} \left[P_1 - P_2 - \frac{1}{3} \rho U_{\infty}^2 \right]}$$

(a) A model having scale ratio of $\frac{1}{10}$ is constructed to determine the best design of Kaplan turbine. The prototype Kaplan turbine develop 7355 kW under a net head of 10 m at a speed of 100 rpm. If the head available at the laboratory is 6 m and the model efficiency is 88% whereas the efficiency of prototype turbine is 4% better that of the model turbine. Find:

- (i) running speed of the model.
- (ii) the flow rate required in the laboratory.
- (iii) the specific speed in each case.

[20 marks]

$$L_p = \frac{1}{10} \Rightarrow \frac{L_m}{L_p} = \frac{1}{10}$$

Prototype
 $P = 7355 \text{ kW}$
 $H = 10 \text{ m}$
 $N = 100 \text{ rpm}$
 $\eta_p = 92\% = 0.92$

Model.
 $H = 6 \text{ m}$
 $\eta_m = 0.88$

effective head will be used

Using the relation

$$\left(\frac{H}{D^2 N^2} \right)_m = \left(\frac{H}{D^2 N^2} \right)_p$$

$$\frac{6}{D_m^2 N_m^2} = \frac{10}{D_p^2 (100)^2}$$

$$\frac{6}{10} (100)^2 \left(\frac{D_p}{D_m} \right)^2 = N_m^2$$

$$N_m^2 = \frac{6}{10} (100)^2 (10)^2$$

$$N_m = 774.596 \text{ rpm}$$

(ii) $\left(\frac{Q}{D^3 N} \right)_m = \left(\frac{Q}{D^3 N} \right)_p$

$\eta_o = \frac{P}{\rho g Q H}$ for prototype

$$0.92 = \frac{7355}{9.81 \times Q \times 10} \quad \left[Q = 81.494 \text{ m}^3/\text{sec} \right]$$

$$\left(\frac{Q}{D^3 N} \right)_m = \left(\frac{Q}{D^3 N} \right)_p$$

$$Q_m = Q_p \left(\frac{D_m}{D_p} \right)^3 \left(\frac{N_m}{N_p} \right)$$

$$Q_m = 81.494 \times \left(\frac{1}{10} \right)^3 \left(\frac{774.596}{100} \right)$$

$$Q_m = 0.6312 \text{ m}^3/\text{s}$$

$$\eta_m = \frac{P}{\rho g Q H}_m$$

$$P_m = 0.88 \times 9.81 \times 0.6312 \times 6$$

$$P_m = 32.696 \text{ kW}$$

procedure is ok

10

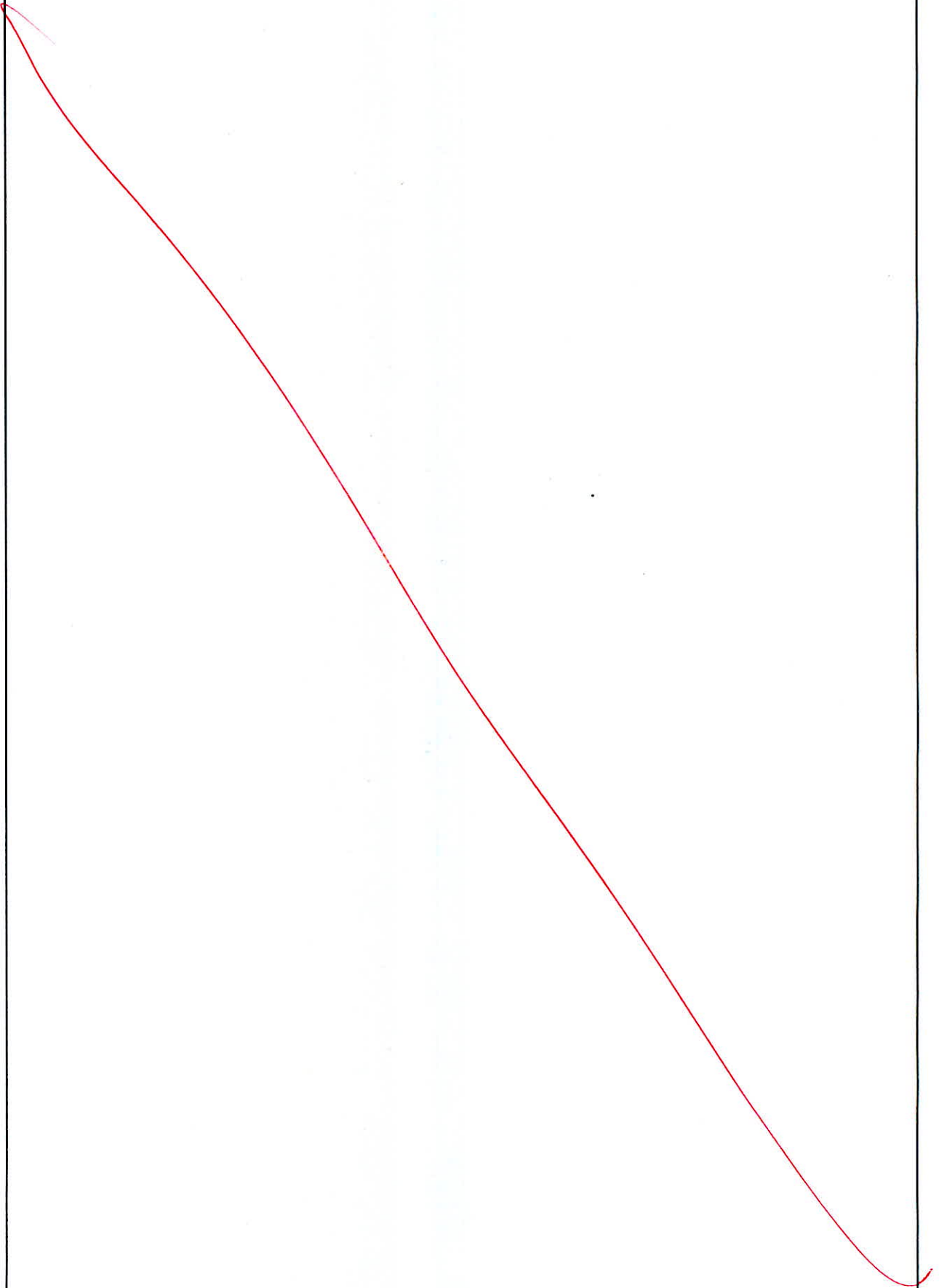
$$(ii) \quad N_s = \frac{N \sqrt{P}}{H_m^{5/4}} \quad \text{specific speed.}$$

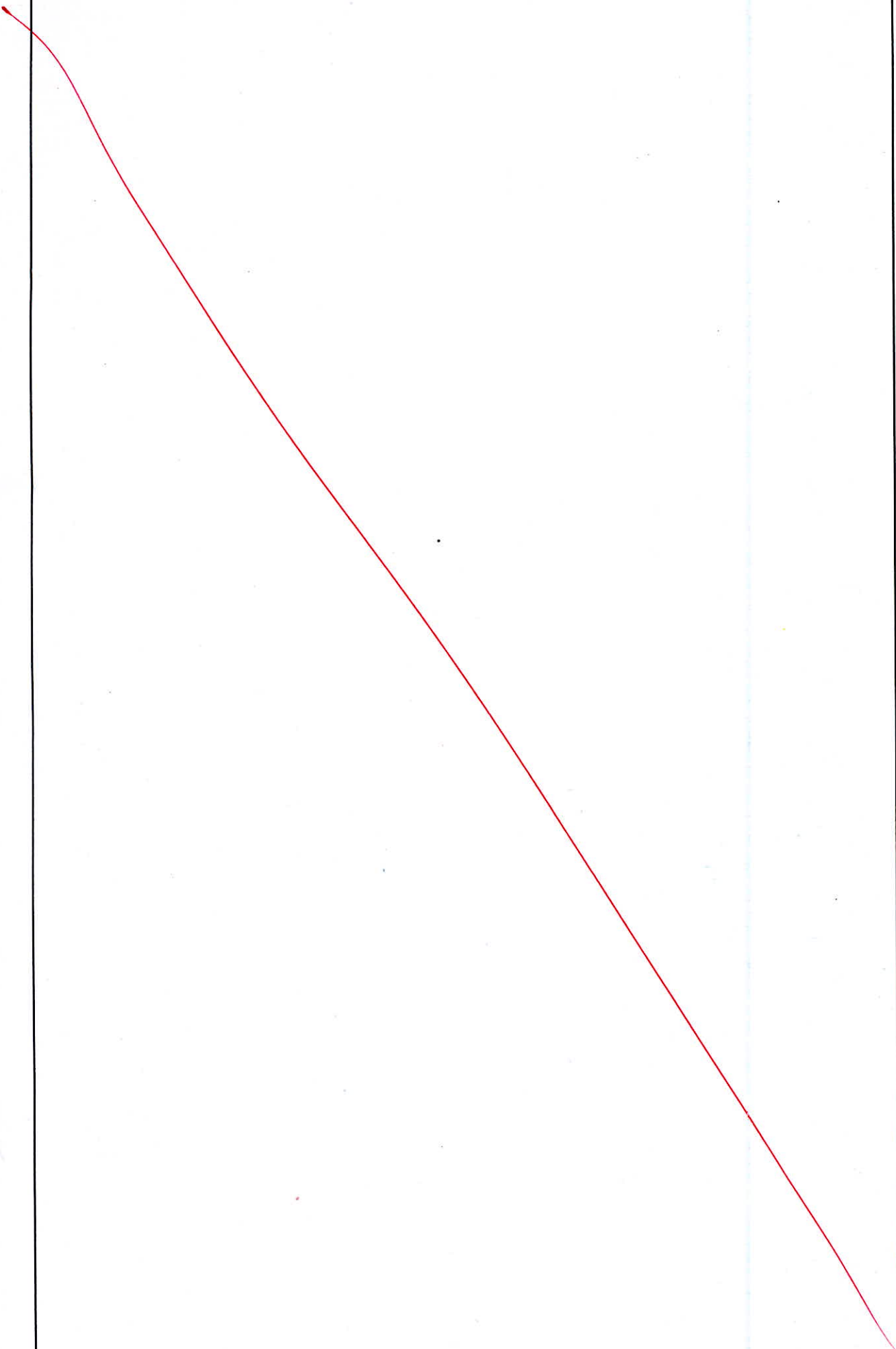
$$N_s/P = \frac{100 \sqrt{7355}}{(10)^{5/4}} = 482.2712$$

$$N_s/m = \frac{774.596 \sqrt{32.696}}{(6)^{5/4}} = 482.2712$$

- 2 (b) A centrifugal compressor develops a pressure ratio of 4 : 1. The inlet eye of the compressor impeller is 0.3 m in diameter. The axial velocity at inlet is 120 m/s and the mass flow rate is 10 kg/s. The velocity in the delivery duct is 110 m/s. The tip speed of the impeller is 450 m/s and runs at 16000 rpm with a total head isentropic efficiency of 80%. The inlet stagnation temperature and pressure are 300 K and 101 kPa.
(Take $c_p = 1.005$ kJ/kgK, $\gamma = 1.4$)
- the static temperature and pressure at inlet and outlet of the compressor
 - the static pressure ratio
 - the power required to drive the compressor
 - Mach number (based on relative velocity) at inlet

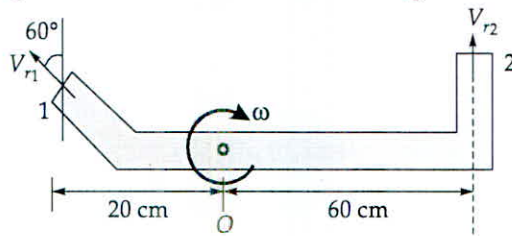
[20 marks]



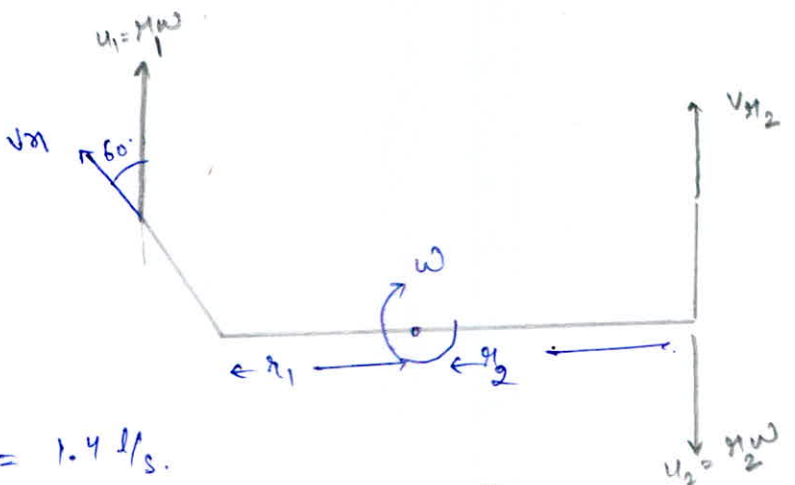


Q.2 (c) A sprinkler with unequal arms and jets of area 0.7 cm^2 is shown in figure. A flow of 1.4 l/s enters the assembly normal to the rotating arm.

- (i) Assuming the frictional resistance to be zero calculate its speed of rotation,
 (ii) What torque is required to hold it from rotating?



[20 marks]



$$Q = 1.4 \text{ l/s.}$$

$$v = \frac{(Q/2)}{0.7} = \frac{0.7 \times 10^{-3}}{0.7 \times 10^{-4}}$$

$$v = 10 \text{ m/s.}$$

$$T = \left(\begin{array}{l} \text{net momentum} \\ \text{due to outlet} \\ \text{of water} \end{array} \right) - \text{net momentum due to inlet of water}$$

$$T = \dot{m} [v_1 - v_2]$$

$$T = \dot{m} [(v_{r1} \cos 60^\circ + u_1)r_1 - (v_{r2} - r_2 \omega)r_2]$$

$$T = \dot{m} [(10 \cos 60^\circ + r_1 \omega)r_1 - (10 - r_2 \omega)r_2]$$

$$T = \dot{m} [(5 + 0.2\omega)(0.2) - (10 - 0.6\omega)(0.6)]$$

$$T = \dot{m} [1 + 0.04\omega - 6 + 0.36\omega]$$

$$= \dot{m} [0.4\omega - 5]$$

Since junctional resistance is zero; hence free vortex flow. $(T=0)$.

$$0.4 \omega = 5$$

$$\omega = \frac{5}{0.4}$$

$$\omega = 12.5 \text{ rad/s.}$$

(ii) To hold it from rotating; $\omega = 0$.

$$T = \dot{m} (0.4 \omega - 5)$$

$$T = -5 \dot{m}$$

$$T = 5 \rho Q$$

$$T = 5 \times 1000 \times 0.7 \times 10^{-3}$$

$$T = (-) 3.5 \text{ Nm}$$

20

Q.3 (a) An impulse steam turbine has a number of pressure stages, each having a row of nozzles and a single ring of blades. The nozzle angle in the first stage is 20° and the blade exit angle is 30° with reference to the plane of rotation. The mean blade speed is 125 m/s and the velocity of steam leaving the nozzles is 350 m/s.

- (i) Taking the blade friction factor as 0.9 and nozzle efficiency of 0.85, determine the work done in the stage per kg of steam and the stage efficiency.
- (ii) If the steam supply to the first stage is at 20 bar, 250°C and the condenser pressure is 0.07 bar, estimate the number of stages required, assuming that the stage efficiency and the work done are the same for all stages and the reheat factor is 1.05.

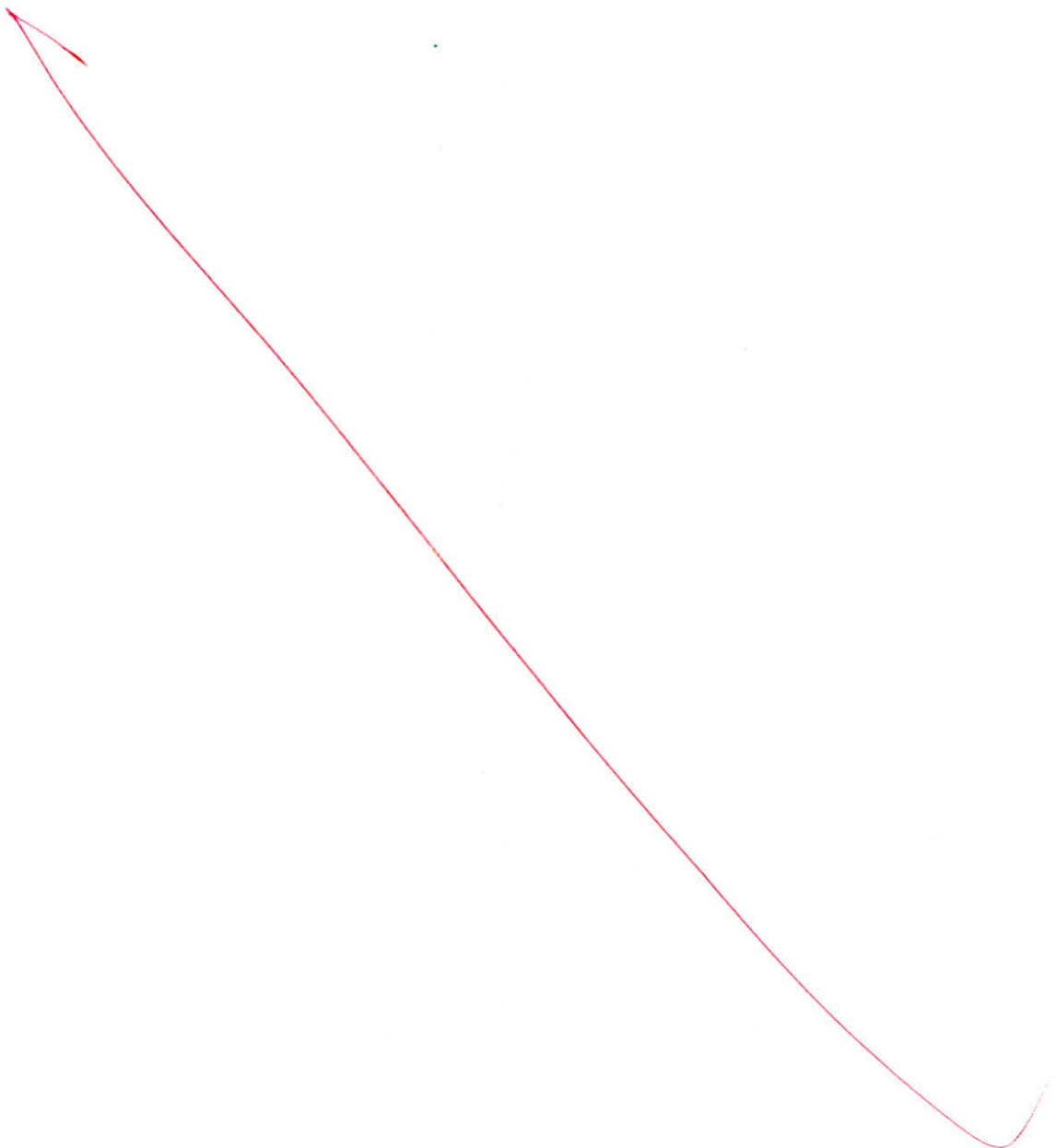
at 20 bar, 250°C ,

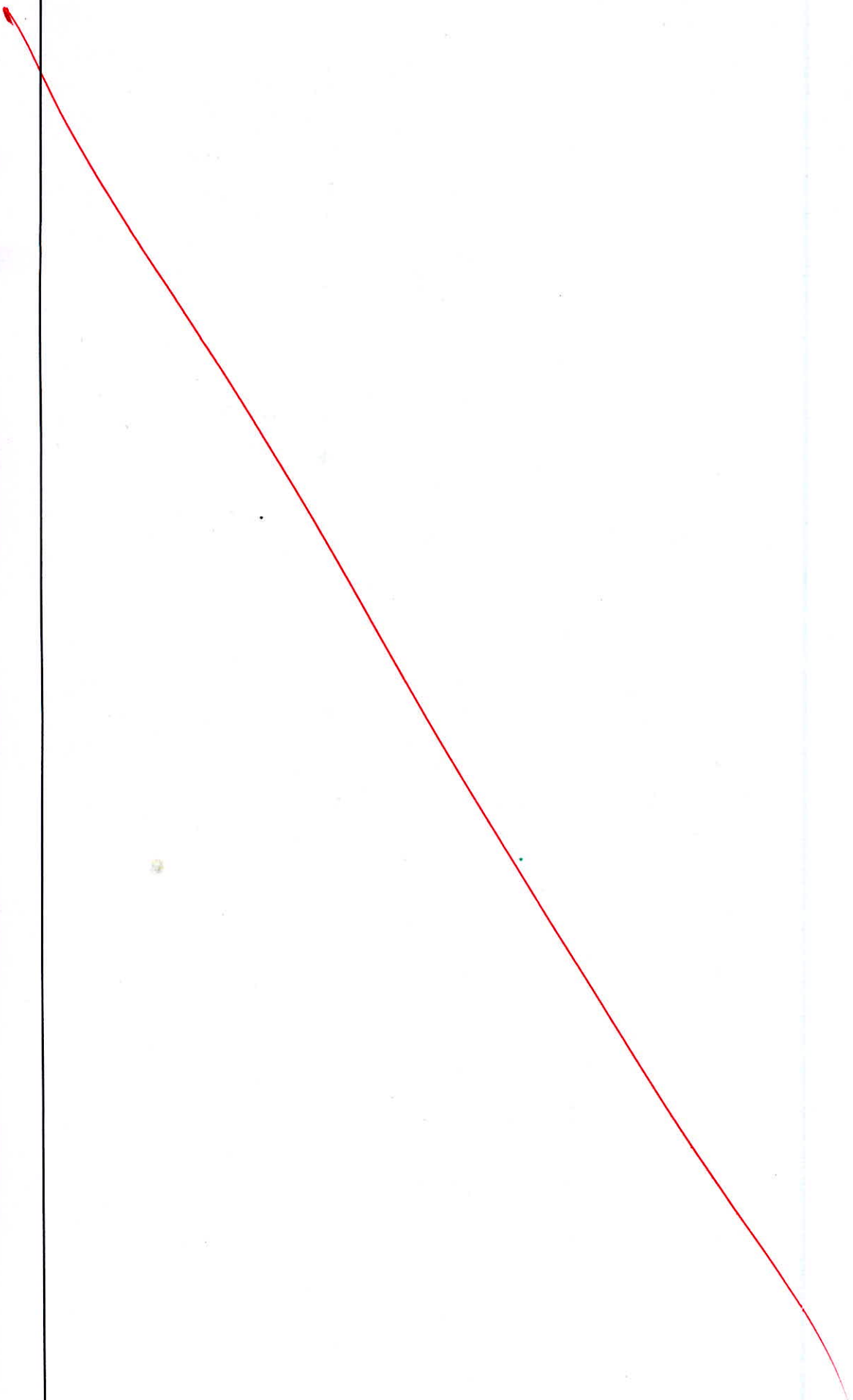
$$h = 2902.5 \text{ kJ/kg}, s = 6.5453 \text{ kJ/kgK}$$

at 0.07 bar,

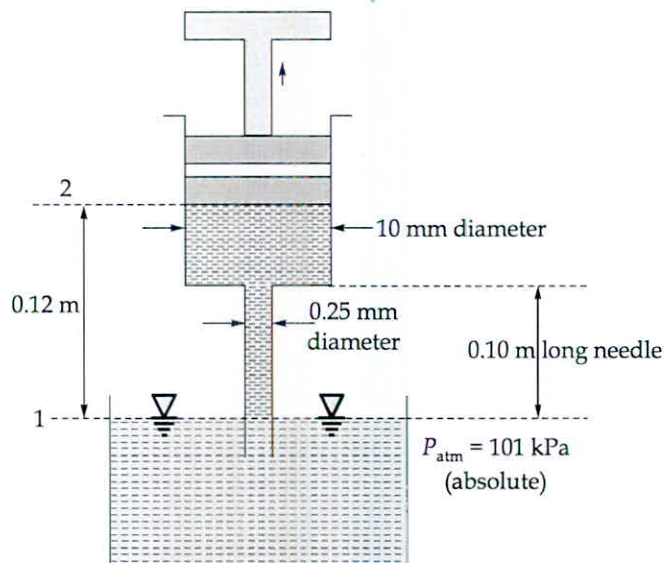
h_f (kJ/kg)	h_{fg} (kJ/kg)	s_f (kJ/kgK)	s_{fg} (kJ/kgK)
163.16	2409.54	0.5582	7.7198

[20 marks]

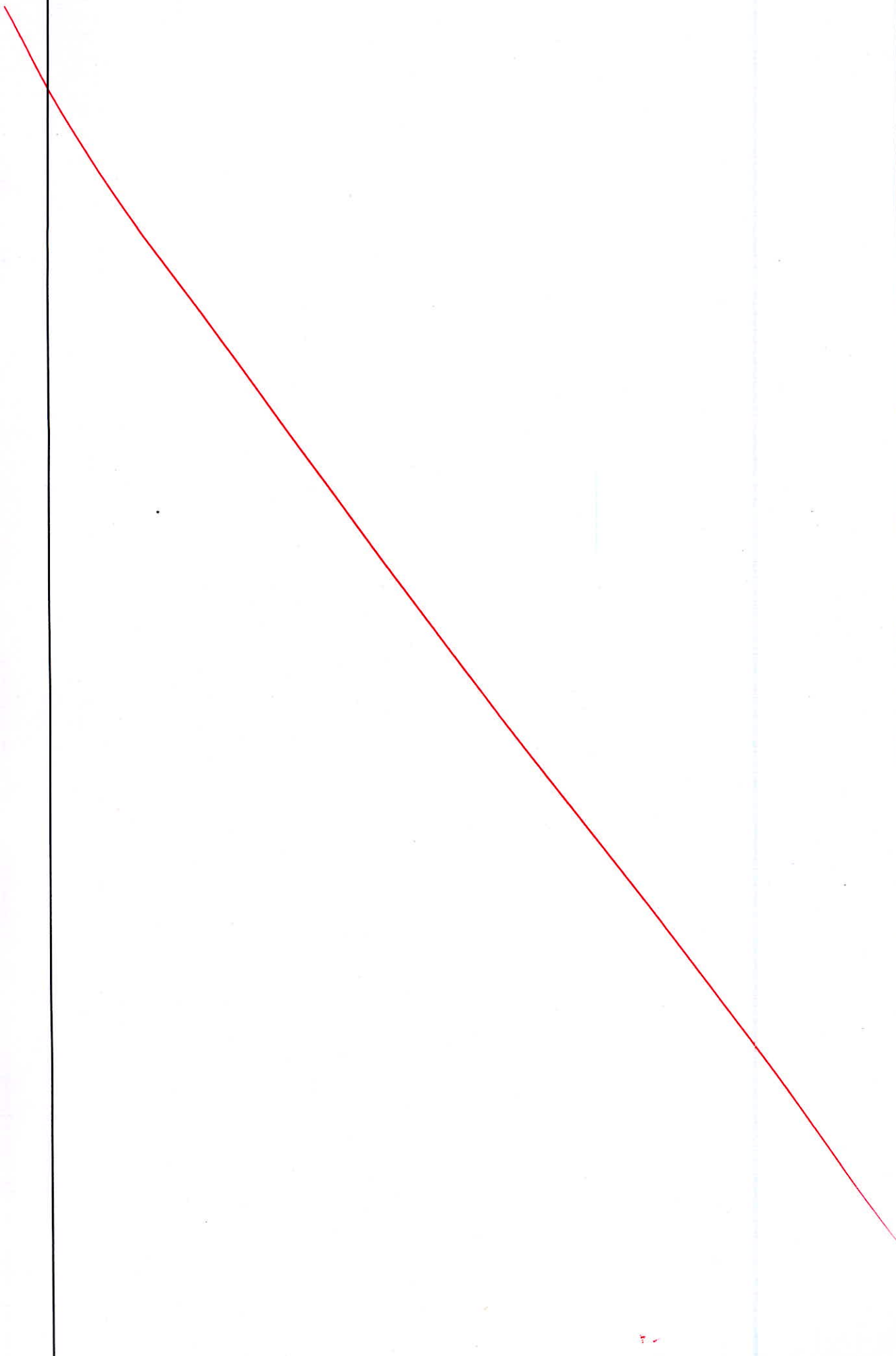


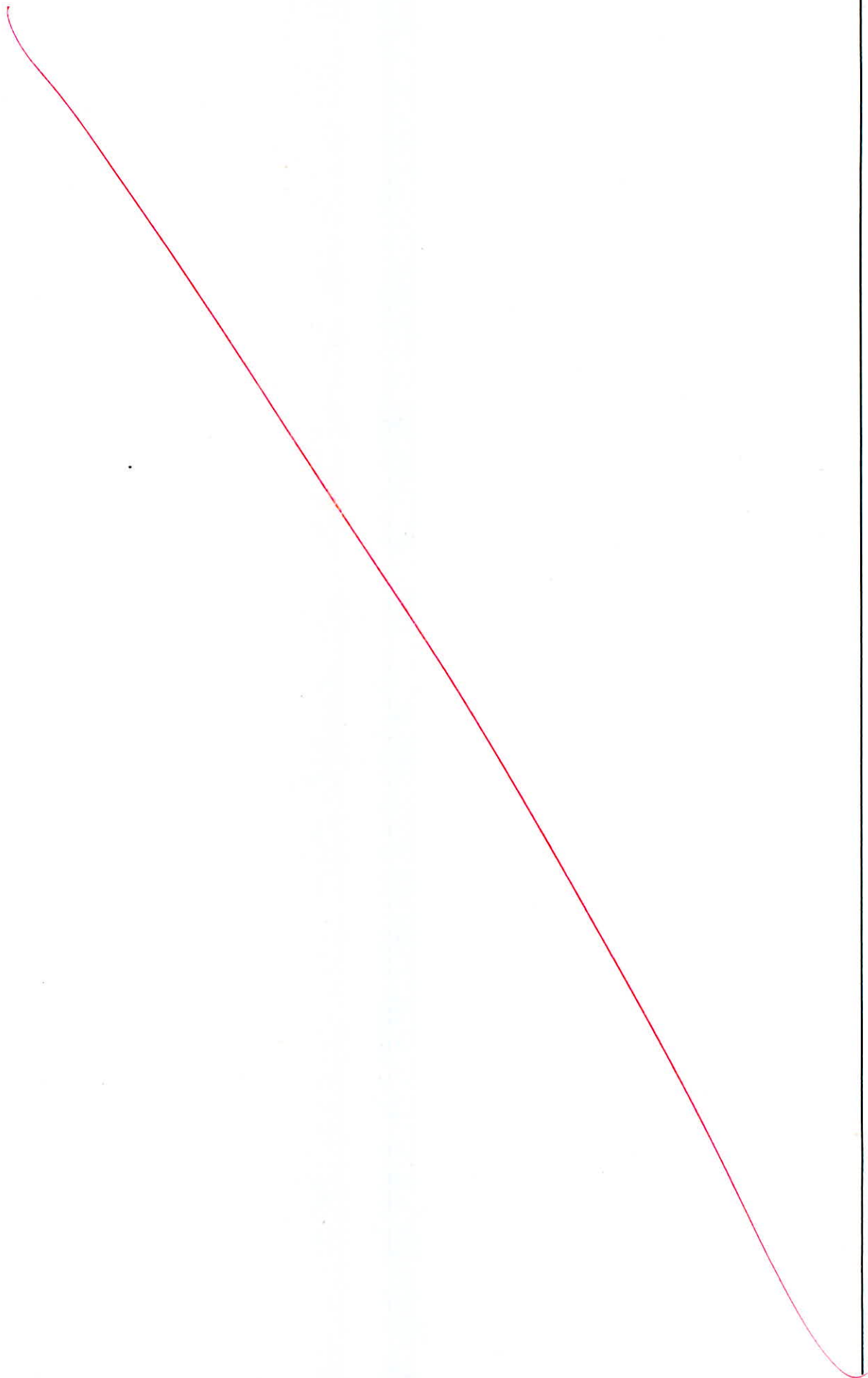


- Q.3 (b) A liquid with specific gravity of 0.96, dynamic viscosity $9.2 \times 10^{-4} \text{ Ns/m}^2$ and vapor pressure (P_v) = $1.2 \times 10^4 \text{ N/m}^2$ (absolute) is drawn into the syringe as indicated in figure. What is the maximum flow rate if cavitation is not to occur in the syringe? Assume that the flow corresponding to the small diameter is laminar and support your answer with the necessary calculations.

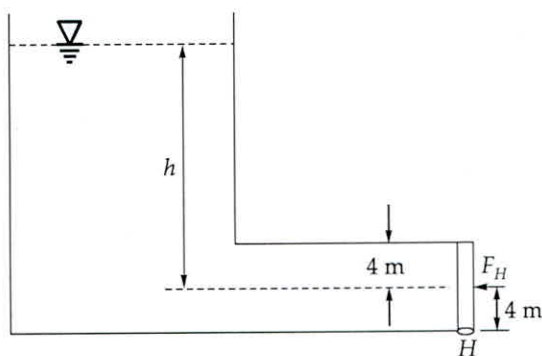


[20 marks]



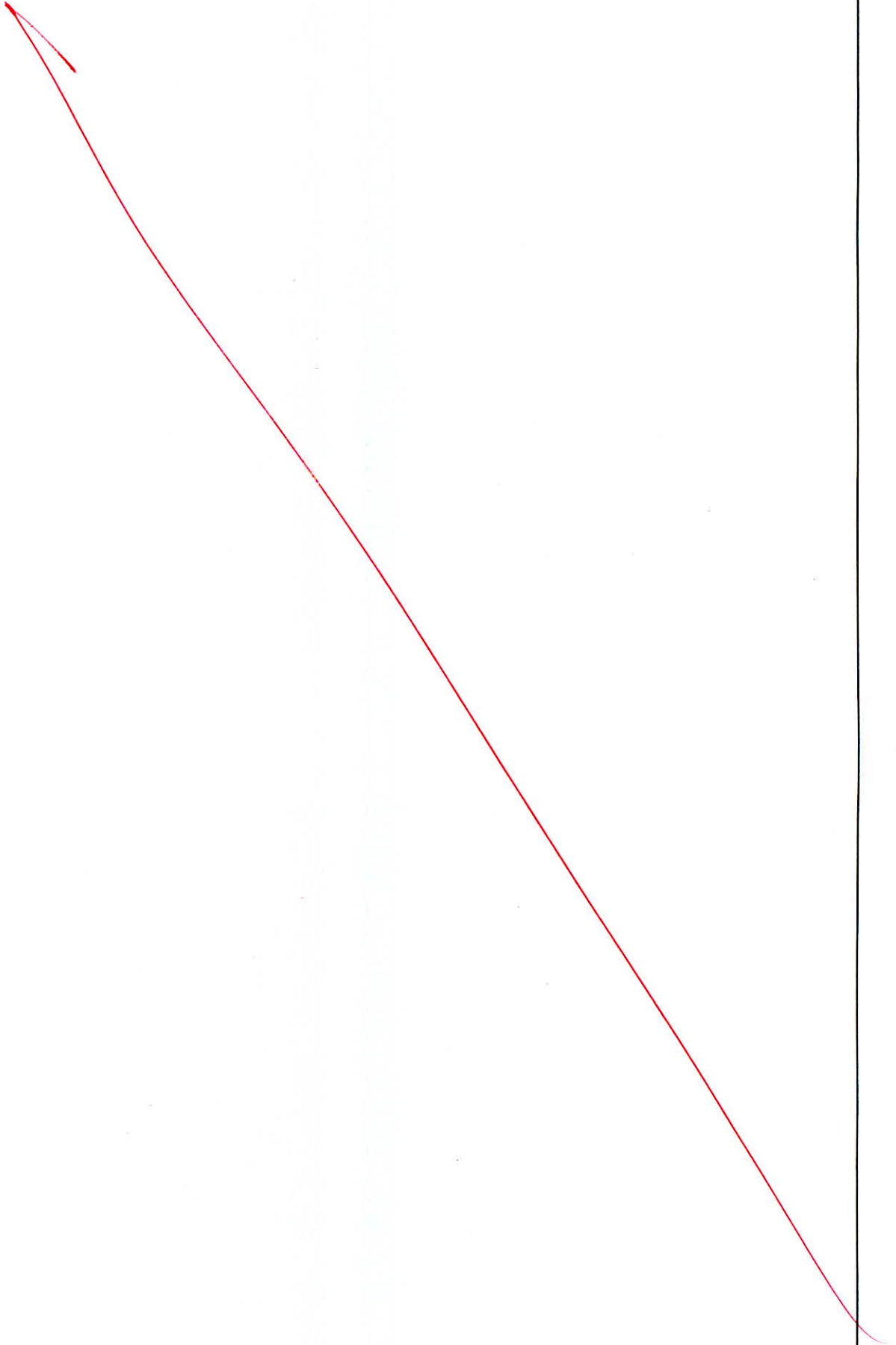


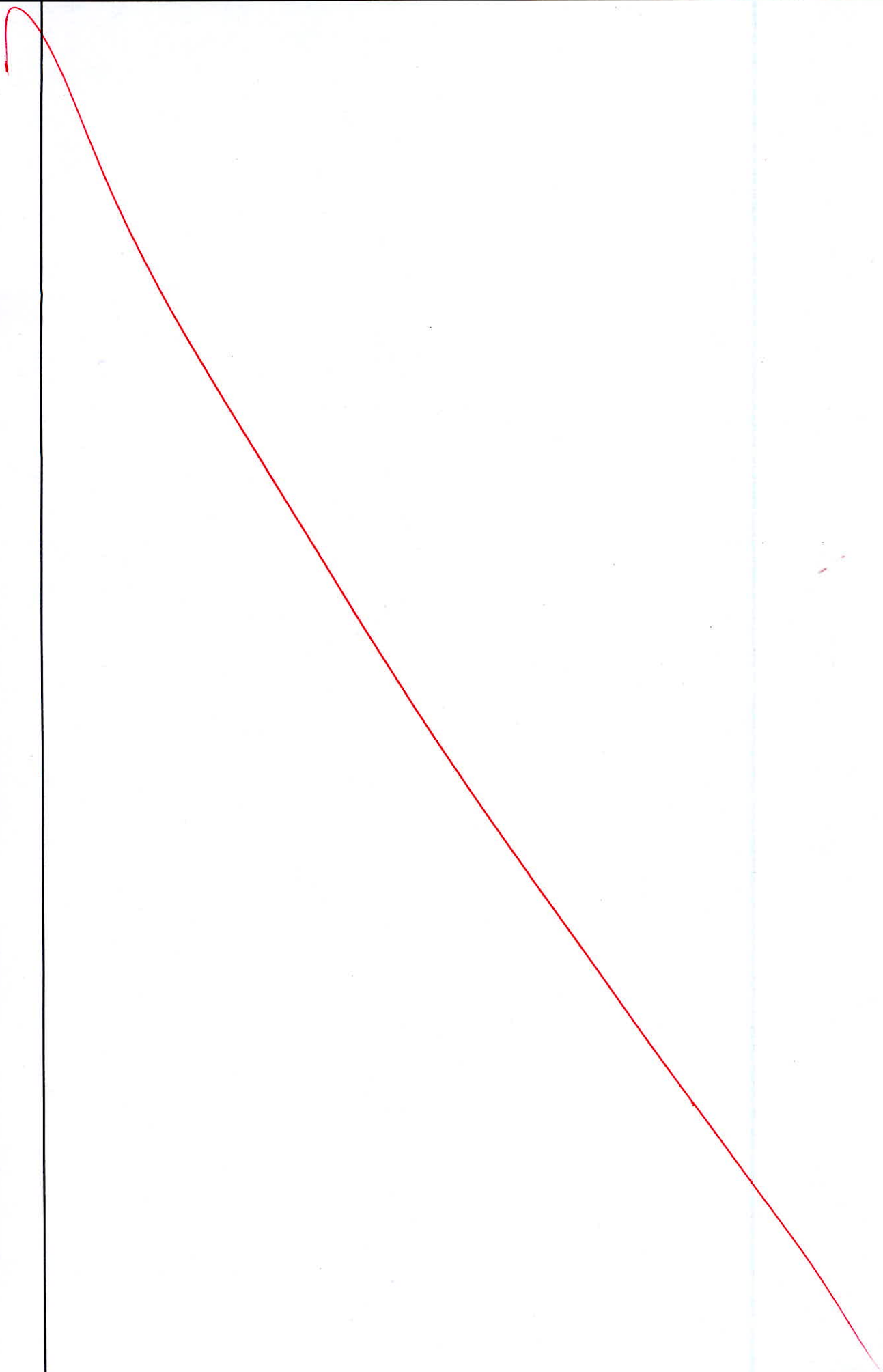
- 3 (c) A 3 m wide, 8 m high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in figure. The gate is hinged at its bottom and held closed by a horizontal force, F_H located at the centre of the gate. The maximum value for F_H is 3500 kN.



- (i) Determine the maximum water depth above the centre of the gate that can exist without the gate opening.
- (ii) Will the answer be same, if the gate is hinged at the top? Explain your answer.

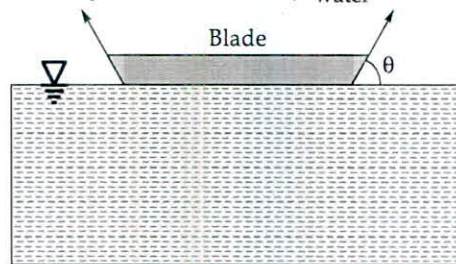
[20 marks]



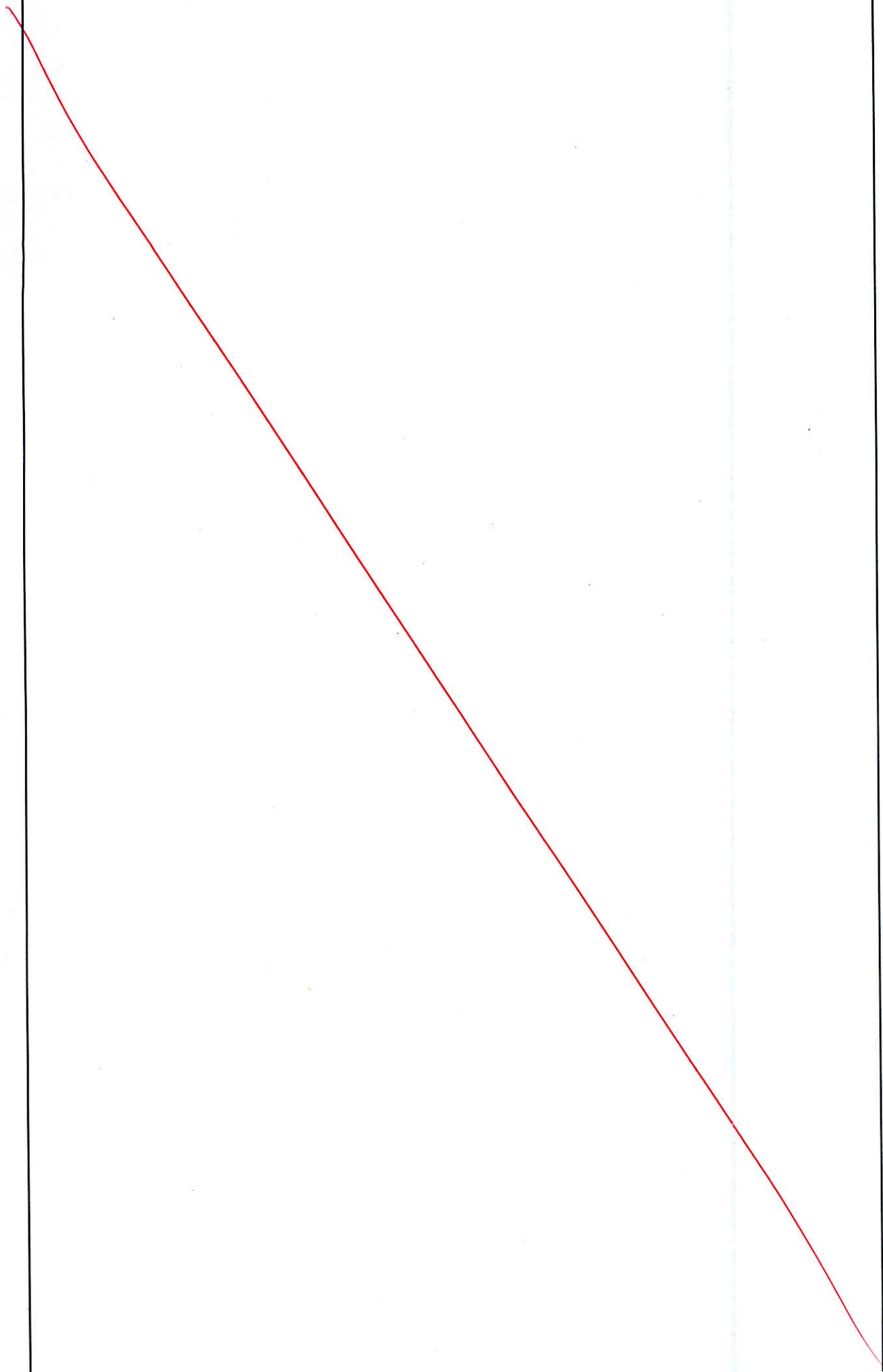


Q.4 (a) As surface tension forces can be strong enough to allow a double edge steel razor blade to 'float' on water. But a single edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in figure.

- The mass of the double edge blade is 0.64×10^{-3} kg and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and resultant surface tension force.
- The mass of the single edge blade is 2.61×10^{-3} kg and the total length of its side is 154 mm. Explain why this blade sink.
- If suppose one bug having weight of 10^{-4} N stays on the upper (air side) surface of steel razor, then what changes you expect in value of (θ) for case (a) and support your answer with the necessary calculations ($\sigma_{\text{water}} = 7.34 \times 10^{-2}$ N/m)?



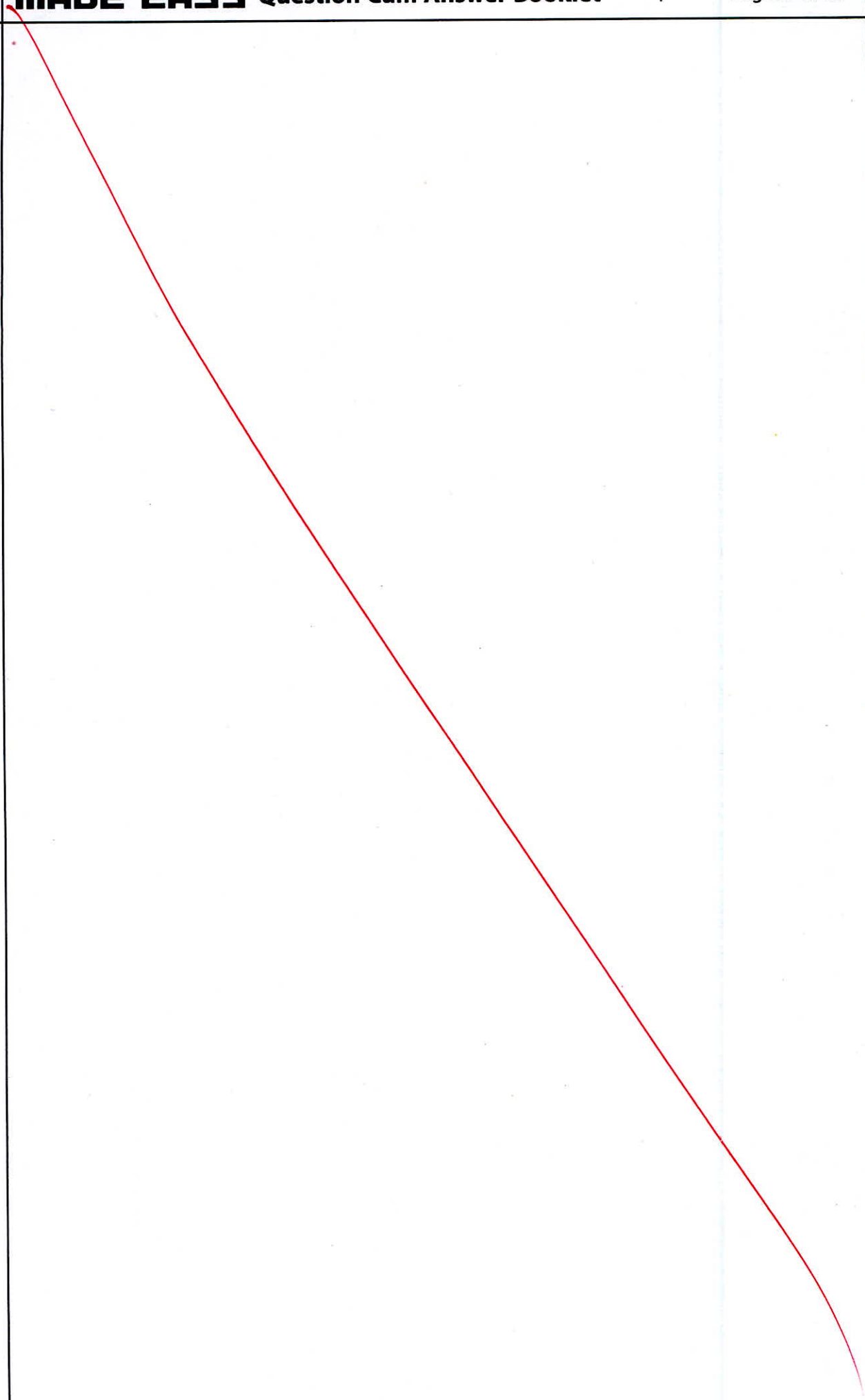
[5+5+10 = 20 marks]

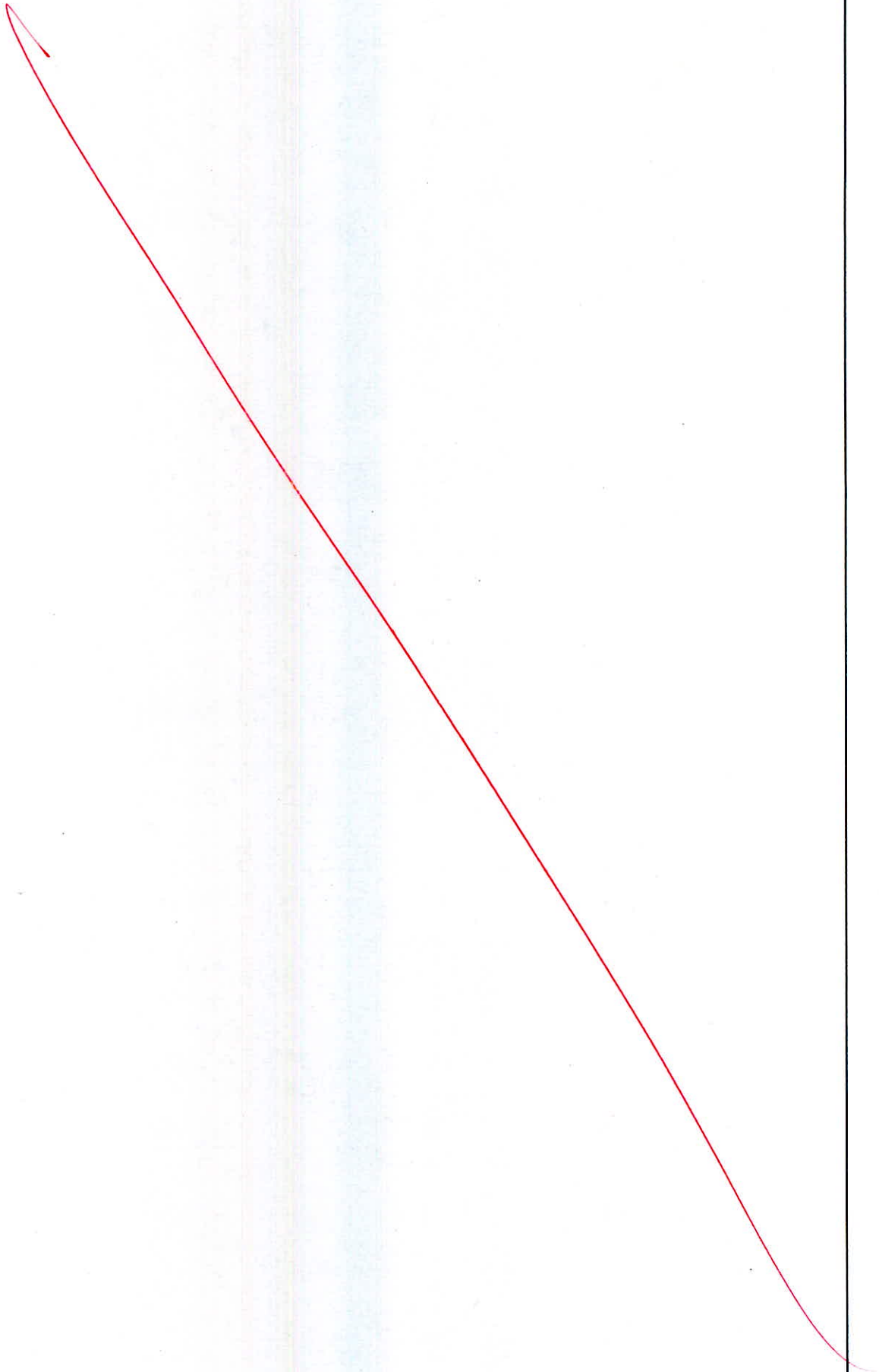


Q.4 (b) A steam turbine plant works between the limit of 150 bar, 600°C and 0.1 bar. The mean blade velocity is 220 m/s. The average nozzle efficiency is 0.91. The nozzle (fixed blade) angle is 20° . All stages operate at the condition of maximum efficiency. The total isentropic enthalpy drop is 1400 kJ/kg. Determine the number of stages required for the following cases.

- (i) All simple impulse stages.
- (ii) All 50% impulse-reaction stages.
- (iii) A two-row Curtis stage followed by simple impulse stages.
- (iv) A two row Curtis stage followed by 50% impulse reaction stages.

[20 marks]

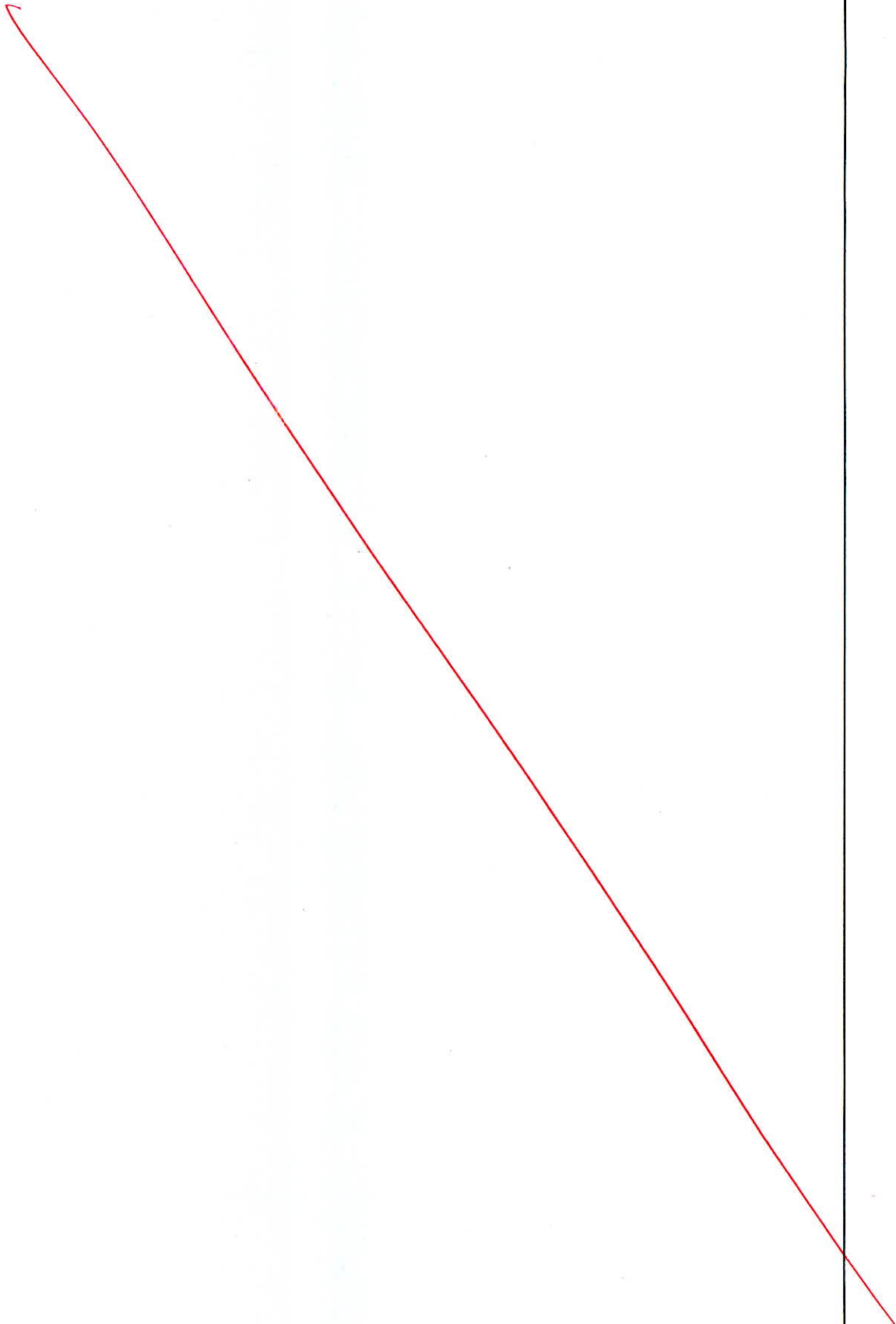


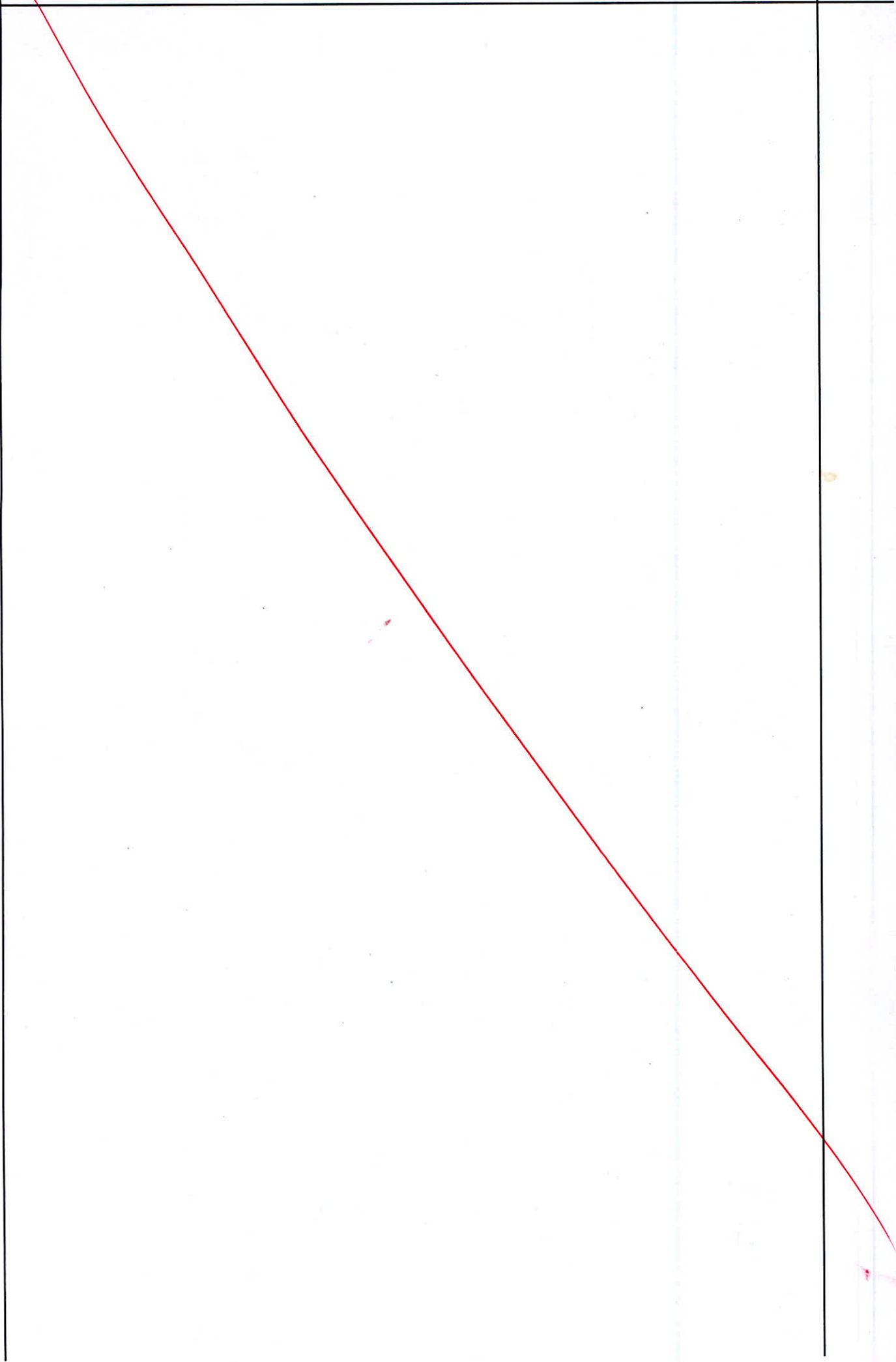




- 4 (c) (i) Explain the purpose of installing draft tube at the exit of reaction turbine.
- (ii) The draft tube of a Kaplan turbine has inlet diameter 2.8 m and inlet is set at 3 m above the tail race. When the turbine develops 1500 kW power under a net head of 6 m, it is found that the vacuum gauge fitted at inlet to draft tube indicates a negative head of 4 m. If the turbine overall efficiency is 88%, determine the draft tube efficiency. If the turbine output is reduced to half with the same head, speed and draft tube efficiency, what would be the reading of the vacuum gauge? (Neglect minor losses).

[5 +15 = 20 marks]





Section B : Heat Transfer - 1 + TOM - 1, Thermodynamics - 2 + RAC - 2

- Q.5 (a) For a sphere of radius R having a surface temperature of T_s in which heat is generated at a uniform rate of q_G W/m³, derive the following expression

$$T = T_\infty + \frac{q_G R}{3h} + \frac{q_G R^2}{6k} \left(1 - \frac{r^2}{R^2} \right)$$

where, T_∞ = Ambient temperature.

Assumptions?

[12 marks]

General eqⁿ for sphere;

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 k \frac{\partial T}{\partial r} \right) + q_{gen} = \rho c_p \frac{\partial T}{\partial t}$$

Assume process as steady ; $\frac{\partial T}{\partial t} = 0$

$k = \text{const.}$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{q_{gen}}{k} = 0$$

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{q_{gen}}{k} r^2 = 0$$

Integrating both sides;

$$r^2 \frac{\partial T}{\partial r} + \frac{q_{gen}}{k} \left(\frac{r^3}{3} \right) = C_1$$

$$\frac{\partial T}{\partial r} + \frac{q_{gen}}{k} \left(\frac{r}{3} \right) = \frac{C_1}{r^2} \quad \text{--- (1)}$$

Integrating again;

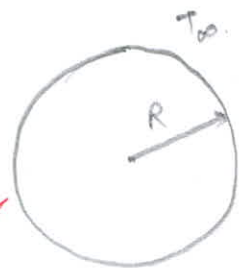
$$T + \frac{q_{gen}}{k} \left(\frac{r^2}{6} \right) = \frac{-2C_1}{r} + C_2$$

$$T + \frac{q_{gen}}{6k} r^2 = -\frac{2C_1}{r} + C_2 \quad \text{--- (2)}$$

Applying Boundary conditions;

at $r = R$; $k \frac{\partial T}{\partial r} = h \Delta T$

$$\frac{\partial T}{\partial r} = \frac{h}{k} (T_s - T_\infty)$$



$$\frac{h}{k} (T_s - T_\infty) + q_{gen} \frac{R}{3k} = \frac{C_1}{R^2}$$

$$q = \frac{hR^2}{k} (T_s - T_\infty) + q_{gen} \frac{R^3}{3k}$$

Putting q ;

$$T + q_{gen} \frac{r^2}{6k} = \frac{-2}{r^3} \left[\frac{hR^2}{k} (T_s - T_\infty) + q_{gen} \frac{R^3}{3k} \right] + C_2$$

$$T + \frac{q_{gen} r^2}{6k} + \frac{2}{3k} q_{gen} \frac{R^3}{r^3} = \frac{-2}{r^3} \left(\frac{hR^2}{k} \right) (T_s - T_\infty) + C_2$$

at $r = R$; $T_0 = T_\infty$

$$(C_2 = T_\infty)$$

$$T + \frac{q_{gen} r^2}{6k} + \frac{2}{3k} q_{gen} \frac{R^3}{r^3} = \frac{-2}{r^3} \left(\frac{hR^2}{k} \right) (T_s - T_\infty) + T_\infty$$

On simplifying ;

$$T = T_\infty + \frac{q_{gen} R}{3h} + \frac{q_{gen} R^2}{6k} \left(1 - \frac{r^2}{R^2} \right)$$

10

Q.5 (b) The barometer for atmospheric air reads 750 mm of Hg, the dry bulb temperature is 33°C , wet bulb temperature is 23°C . Determine:

- the relative humidity.
- the humidity ratio.
- the dew point temperature.
- density of atmospheric air.

Use the following relation,

$$\text{Partial pressure of vapour, } P_v = (P_s)_{WB} - \frac{(P_t - (P_s)_{WB})(t_{DB} - t_{WB})}{1527.4 - 1.3t_{WB}}$$

$P_t \rightarrow$ Barometric pressure

$(P_s)_{WB} \rightarrow$ Saturation pressure corresponding to WBT

$t_{WB} \rightarrow$ Wet bulb temperature (in $^\circ\text{C}$)

$t_{DB} \rightarrow$ Dry bulb temperature (in $^\circ\text{C}$)

Use following table:

P_s (mm of Hg)	t_s ($^\circ\text{C}$)
16.19	18.7
21.06	23
37.72	33

At 33°C density of Hg, $\rho_{\text{Hg}} = 13600 \text{ kg/m}^3$

Assume v_g (Specific volume of saturated vapour) at 37.72 mm of Hg is $28.05 \text{ m}^3/\text{kg}$.

[12 marks]

Solⁿ:

$$P_{\text{atm}} = 750 \text{ mm Hg} = 100 \text{ kPa}$$

$$\text{DBT} = 33^\circ\text{C} \quad ; \quad (P_v)_{\text{sat}} = 37.72 \text{ mm Hg}$$

$$\text{WBT} = 23^\circ\text{C} = t_{\text{wb}} \quad ; \quad (P_v)_{\text{WB}} = 21.06 \text{ mm Hg}$$

$$P_v = (P_v)_{\text{WB}} - \frac{(P_t - (P_s)_{\text{WB}})(t_{\text{DB}} - t_{\text{WB}})}{(1527.4 - 1.3t_{\text{WB}})}$$

$$\Rightarrow 21.06 - \frac{(750 - 21.06)(33 - 23)}{1527.4 - (1.3(23))}$$

$$P_v = 16.1922 \text{ mm Hg}$$

(i) Relative humidity $\phi = \frac{P_v}{P_{v,s}} = \frac{16.1922}{37.72}$

$$\phi = 42.927\%$$

(ii) Humidity Ratio

$$w = 0.622 \frac{P_v}{P_t - P_v}$$

$$w = 0.622 \times \frac{16.1922}{750 - 16.1922}$$

$$w = 0.01372 \text{ kg vap. / kg d.a.}$$

(iii) Dew pt. temperature corresponds to the saturation temperature at P_v (D.P.T.).corresponding to $P_v = 16.1922 \text{ mm Hg}$

$$T_{DPT} = 18.7^\circ \text{C}$$

$$P_a = \frac{P_a}{RT_a}$$

$$P_a = P_t - P_v$$

$$= 750 - 16.1922$$

$$P_a = 733.8 \text{ mm Hg} = 97.901 \text{ kPa}$$

Density of atmospheric air;

$$\rho_a = \frac{97.901}{0.287 \times (33 + 273)}$$

$$\rho_a = 1.114 \text{ kg/m}^3$$

$$\underline{\underline{1.13}}$$

+ Stmo

(iv)

Q.5 (c) What is the mobility of mechanism? Explain the Kutzbach equation for planar mechanism and in what way is the Gruebler's criterion different from it.

Mobility of mechanism is defined as the number of independent intensive variables ^(inputs) required to constrained the motion of the system. [12 marks]

According to kutzbach;

$$\text{Degree of freedom } F = 3(n-1) - 2j - h$$

where n = no. of links.

j = no. of ~~link~~ lower pairs.

h = no. of higher pairs.

Gruebler's assumed the degree of freedom of the mechanism to be 1; and derive the relation:

$$F = 1; \quad h = 0.$$

$$1 = 3(n-1) - 2j - 0.$$

$$3n - 3 - 2j = 1$$

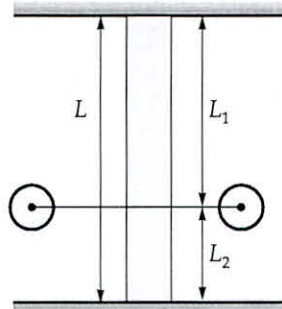
$$\boxed{3n = 2j + 4}$$

∴ According to Gruebler, in order to have a mechanism with degree of freedom 1 and no higher pair, minimum number of links should be 4.

$$n = 4; \quad (j = 4).$$

Hence, the first mechanism is a 4-bar mechanism.

A flywheel is mounted on a vertical shaft as shown in figure. The ends of the shaft being fixed. The shaft is having 20 cm diameter, the length L_1 is 0.9 m and the length L_2 is 0.6 m. The flywheel weighs 500 kg and its radius of gyration is 50 cm, then find the natural frequencies of the longitudinal, the transverse and torsional vibrations of the system. $E = 200 \text{ GPa}$, $G = 80 \text{ GPa}$.



$d = 20 \text{ cm}$; $L_1 = 0.9$; $L_2 = 0.6 \text{ m}$

[12 marks]

Longitudinal :

$$s_1 = \frac{EA}{L} = \frac{200 \times 10^9 \times \frac{\pi}{4} (0.2)^2}{1.5}$$

$[s_1 = 4.188 \times 10^9 \text{ N/m}]$

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{4.188 \times 10^9}{500}}$$

$\omega_n = 2894.4 \text{ rad/s}$

$[N = 27639.53 \text{ rpm}]$

Transverse :

$$\delta = \frac{Pa^3 b^3}{3EI L^3}$$

$$\delta = \frac{500 \times 9 \times (0.9)^3 (0.6)^3}{3 \times 200 \times 10^9 \times \frac{\pi}{64} \times (0.2)^4 \times (1.5)^3}$$

$\delta = 4.85629 \times 10^{-6} \text{ m}$

$$\omega_n = \sqrt{\frac{g}{\delta}}$$

\Rightarrow

$\omega_n = 1421.28 \text{ rad/s}$

$$N = 1357.292$$

Torsional :

$$S_T = \frac{GJ}{l}$$

$$S_T = \frac{80 \times \frac{\pi}{32} \times (0.2)^4 \times 10^9}{1.5}$$

$$S_T = 8.3775 \times 10^6 \text{ N/m}^2$$

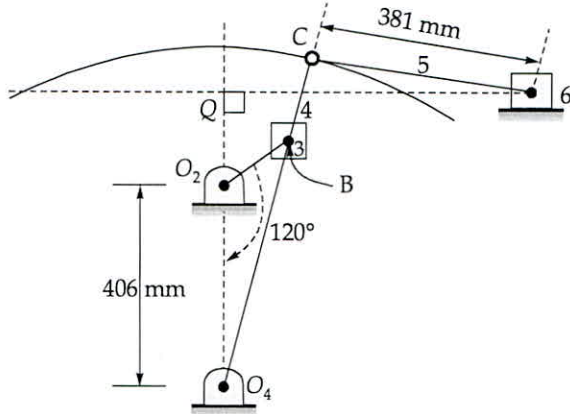
$$\omega_n = \sqrt{\frac{S_T}{M}} = \sqrt{\frac{S_T}{500}}$$

$$\omega_n = 129.44 \text{ rad/s}$$

$$N = 1236 \text{ rpm}$$

04

In order to design a crank-shaper mechanism as shown below, that will give a time ratio of 1.75:1 with a working stroke of 660 mm. Assumed that, point C as it moves along the arc of radius O_4C . The fixed dimensions are given in the figure and compute the required value of O_2B and O_4C . If the crank rotate at a constant speed of 40 rpm. Find the average speed of slider (in m/s) for the given working stroke and for the returning stroke.



[12 marks]

$$QRR = 1.75$$

$$QRR = \frac{\beta}{\alpha} \quad \beta = 1.75 \alpha$$

$$\beta + \alpha = 360^\circ$$

$$\alpha = 130.91^\circ$$

$$\beta = 229.09^\circ$$

$$\cos\left(\frac{\beta}{2}\right) = \frac{O_2B}{406}$$

$$O_2B = 168.65 \text{ mm.}$$

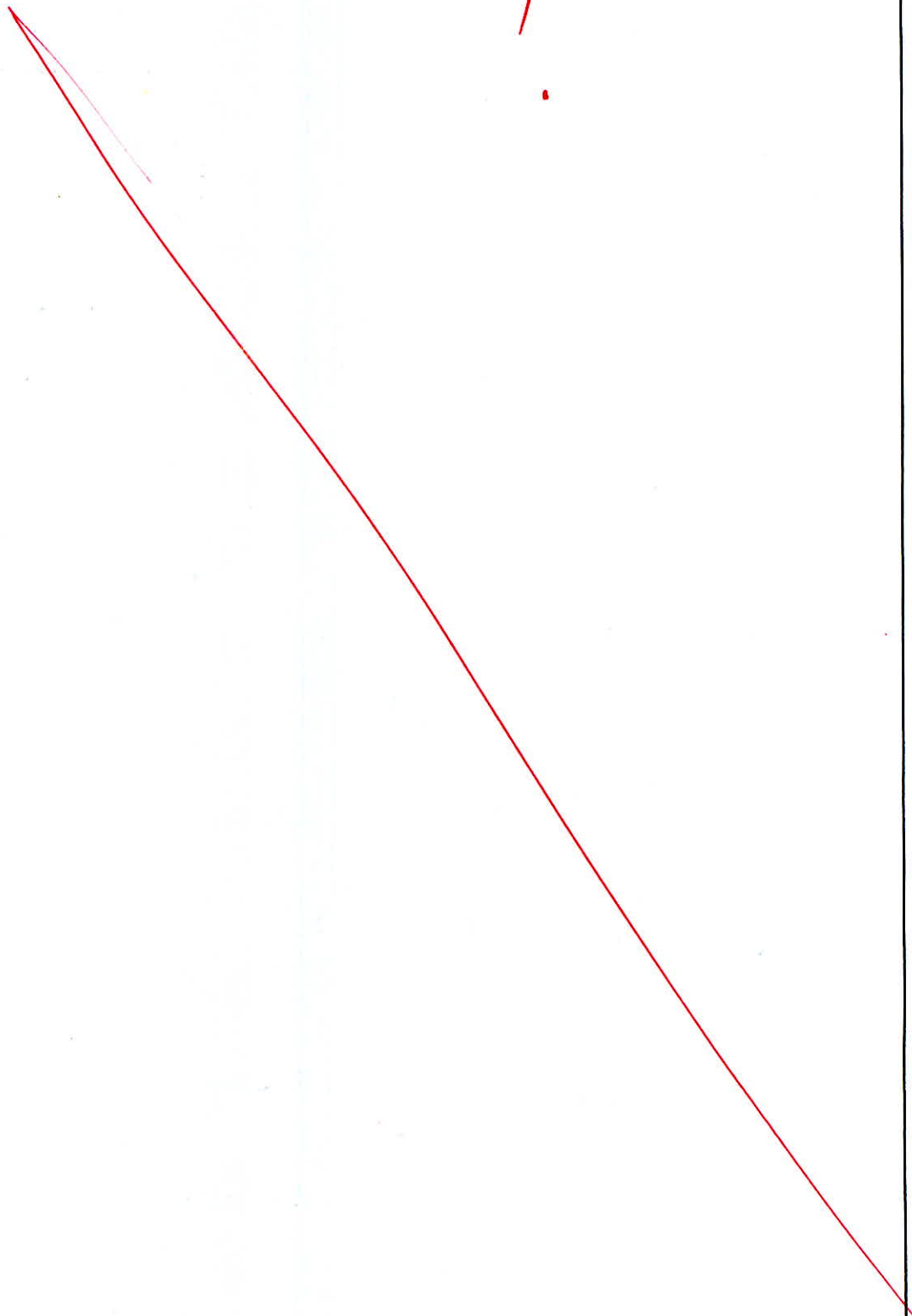
$$O_4C = \frac{2 \left(168.65 \right) \left(381 \right)}{406}$$

03

$$O_4C = 316.572 \text{ mm.}$$

$$O_4C = 633.08 \text{ mm}$$

9



A furnace is insulated with a firebrick lining of 200 mm thickness. The temperature of hot gases in the furnace is 1800 K and the temperature of the surroundings of the furnace is 300 K. The thermal conductivity of the firebricks is given by $k = k_0(1 + \beta T)$ where k_0 is equal to 0.85 W/m-K and β is equal to 7×10^{-4} per K. The heat transfer coefficient on the hot and cold sides of wall is 40 W/m²K and 10 W/m²K respectively. Determine the temperature at inner and outer surfaces of the wall. Also find out the heat lost per unit area of the wall.

[20 marks]

Let us take ;

$$Q = -k A \frac{dT}{dx}$$

$$\frac{Q}{A} = -k_0 (1 + \beta T) \int dT$$

Integrating

$$\frac{Q}{A} t = - \int_{T_1}^{T_2} k_0 \left(T + \beta \frac{T^2}{2} \right) dT$$

$$\frac{Q}{A} t = - \left[k_0 (T_2 - T_1) + \beta \left(\frac{T_2^2}{2} - \frac{T_1^2}{2} \right) \right]$$

$$\frac{Q}{A} t = (T_1 - T_2) \left[k_0 + \beta \frac{(T_1 + T_2)}{2} \right]$$

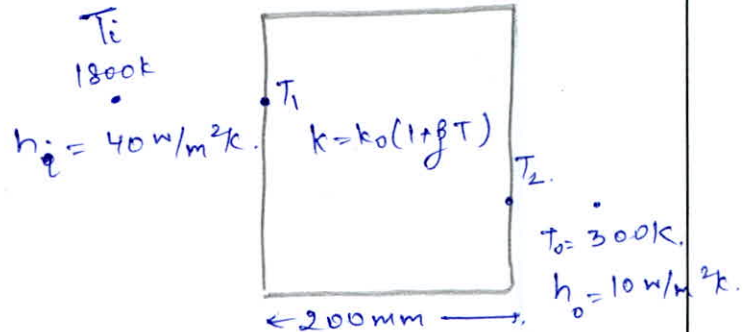
$$\frac{Q}{A} = \frac{(T_1 - T_2)}{\left\{ \frac{t}{k_0 + \beta \frac{(T_1 + T_2)}{2}} \right\}} = \frac{\Delta T}{R_{th}}$$

$$\left[R_{th} = \frac{t}{\left\{ k_0 + \beta \frac{(T_1 + T_2)}{2} \right\} A} \right]$$

Assumptions :

- 1) Steady flow.
- 2) Area of cross-section remains same throughout.
- 3) walls are at constant temperature.

$$\frac{Q}{A} = \frac{\Delta T}{R_{th}} = \frac{T_1 - T_0}{\left(\frac{1}{h} \right)} \quad (\text{for convection})$$



$$\frac{Q}{A} = \frac{1800 - T_1}{\left(\frac{1}{40}\right)} = \frac{(T_1 - T_2)}{\left\{k_0 + \frac{\beta}{2}(T_1 + T_2)\right\}} = \frac{T_2 - 300}{\left(\frac{1}{10}\right)}$$

$$4(1800 - T_1) = T_2 - 300$$

$$7200 - 4T_1 = T_2 - 300$$

$$4T_1 + T_2 = 7500 \quad \text{--- (2)}$$

from (i):

$$\frac{1800 - T_1}{\left(\frac{1}{40}\right)} = \frac{T_1 - T_2}{\left(\frac{0.2}{k_0 + \frac{\beta}{2}(T_1 + T_2)}\right)}$$

$$40(1800 - T_1) = \frac{T_1 - (7500 - 4T_1)}{\frac{0.2}{0.85 + \frac{7 \times 10^{-4}}{2}(T_1 + 7500 - 4T_1)}}$$

$$8(1800 - T_1) = \left[0.85 + 3.5 \times 10^{-4}(7500 - 3T_1)\right] (5T_1 - 7500)$$

$$14400 - 8T_1 = 4.25T_1 - 6375 + \left[3.5 \times 10^{-4}(7500 - 3T_1)(5T_1 - 7500)\right]$$

$$20775 - 12.25T_1 = (3.5 \times 10^{-4})(7500 - 3T_1)(5T_1 - 7500)$$

Solving by trial & error method;

$$T_1 = 1643.3 \text{ K}$$

from eqⁿ (2)

$$T_2 = 926.785 \text{ K}$$

$$\frac{Q}{A} = \frac{1800 - T_1}{\left(\frac{1}{40}\right)}$$

$$\frac{Q}{A} = 40(1800 - 1643.3)$$

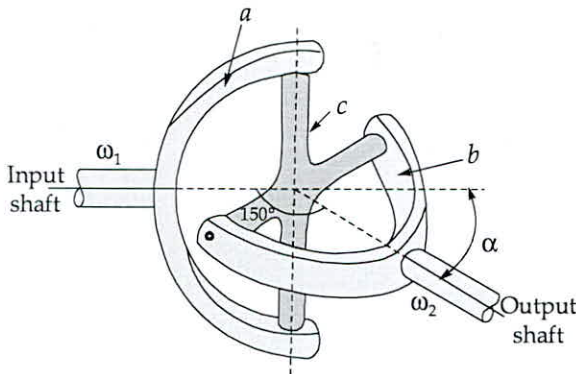
~~20~~

17

$$\frac{Q}{A} = 6268 \frac{w}{m^2}$$

✓

A Hooke's joint is to connect two shafts whose axes intersect at 150° . The driving shaft rotates uniformly at 120 rpm. Deduce a general expression for the angular velocity of the driven shaft. The driven shaft operates against a steady torque of 135 Nm and carries a flywheel whose weight is 45 kg and radius of gyration 0.15 m. What is the maximum value of the torque which must be exerted by the driving shaft?



Let α be the angle b/w the two shafts. [20 marks]

θ is the angle rotated by input shaft.

ϕ is the angle rotated by o/p shaft.

$$\omega_1 = \frac{d\theta}{dt} \quad ; \quad \omega_2 = \frac{d\phi}{dt}$$

By using displacement eqn;

$$\tan \theta = \tan \phi \cos \alpha \quad \text{--- (1)}$$

Differentiating both sides

$$\sec^2 \theta \frac{d\theta}{dt} = \sec^2 \phi \frac{d\phi}{dt} \cos \alpha$$

$$(1 + \tan^2 \theta) \omega_1 = (1 + \tan^2 \phi) \cdot \omega_2 \cos \alpha$$

$$\omega_2 = \frac{\omega_1 (1 + \tan^2 \theta)}{\cos \alpha (1 + \tan^2 \phi)}$$

from eqn (1); $\left[\tan \phi = \frac{\tan \theta}{\cos \alpha} \right]$.

$$\omega_2 = \frac{\omega_1 (1 + \tan^2 \theta)}{\cos \alpha \left(1 + \frac{\tan^2 \theta}{\cos^2 \alpha} \right)}$$

$$\omega_2 = \frac{\omega_1 (1 + \tan^2 \theta) \cos \alpha}{[\cos^2 \alpha + \tan^2 \theta]}$$

$$\omega_2 = \frac{\omega_1 \cos \alpha \left[\frac{1}{\cos^2 \theta} \right]}{\left[\cos^2 \alpha + \frac{\sin^2 \theta}{\cos^2 \theta} \right]}$$

$$\omega_2 = \frac{\omega_1 \cos \alpha}{\cos^2 \alpha \cdot \cos^2 \theta + \sin^2 \theta}$$

$$\omega_2 = \frac{\omega_1 \cos \alpha}{(1 - \sin^2 \alpha) \cos^2 \theta + \sin^2 \theta}$$

$$\omega_2 = \frac{\omega_1 \cos \alpha}{1 - \sin^2 \alpha \cos^2 \theta}$$

INPUT DATA :

$$\alpha = 30^\circ$$

$$N_1 = 120 \text{ rpm}$$

$$\omega_1 = \frac{2\pi N_1}{60} = 12.566 \text{ rad/s}$$

$$T_{\text{mean}} = 135 \text{ Nm}$$

$$m_{\text{flywheel}} = 45 \text{ kg} ; k = 0.15 \text{ m.}$$

$$I = mk^2 = 45 (0.15)^2 = 1.0125 \text{ kg m}^2.$$

$$\omega_2 = \frac{\omega_1 \cos \alpha}{1 - \sin^2 \alpha \cos^2 \theta} \quad \text{Differentiating w.r.t } \theta ;$$

$$\alpha_2 = \frac{-\omega_1^2 \cos \alpha \sin^2 \alpha \cdot \sin 2\theta}{(1 - \sin^2 \alpha \cos^2 \theta)^2}$$

For α_2 to be max/min;

$$\cos 2\theta = \frac{2 \sin^2 \alpha}{2 - \sin^2 \alpha} = \frac{2 \sin^2 30^\circ}{2 - \sin^2 30^\circ}$$

$$2\theta = 73.398 ; 286.6015$$

$$\Rightarrow \theta = 36.699^\circ, 143.3^\circ$$

At $\theta = 36.699^\circ$;

$$\alpha_2 = -46.51 \text{ rad/s}^2$$

$$\omega_2 = 12.966 \text{ rad/s.}$$

At $\theta = 143.3^\circ$

$$\alpha_2 = 46.51 \text{ rad/s}^2$$

$$\omega_2 = 12.966 \text{ rad/s.}$$

For Max value of torque;

$$T = T_{\text{mean}} + I \alpha_{\text{max}}$$

$$T = 135 + (1.0125) (46.51)$$

$$T_{\text{max}} = 182.09 \text{ Nm.}$$

\therefore Since power transmitted is equal.

$$\Rightarrow T_1 \omega_1 = T_2 \omega_2$$

$$T_1 = (182.09) \times \frac{12.966}{12.566}$$

$$T_1 = 187.8877 \text{ Nm}$$

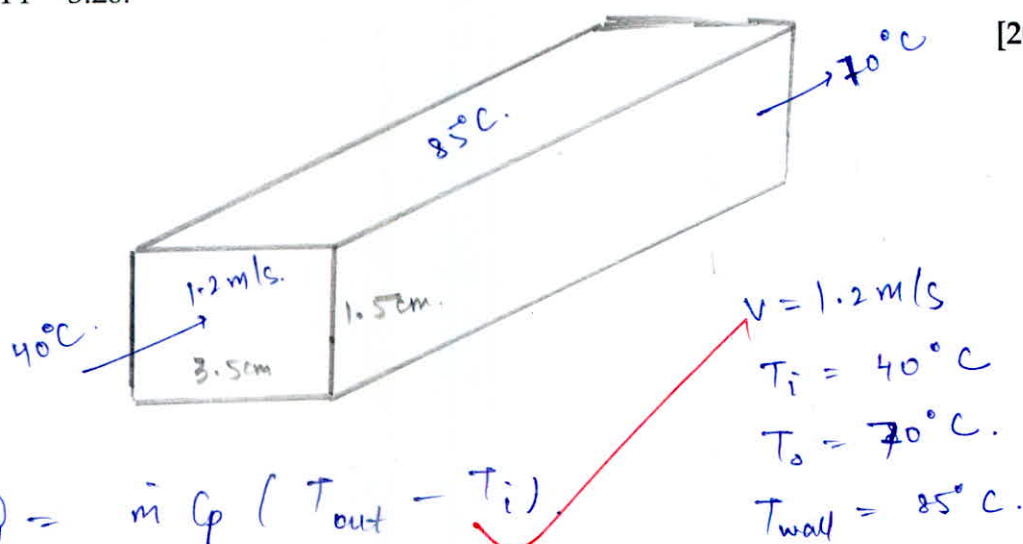
Max. value of torque by driving shaft = 187.8877 Nm



- Q.6 (c) Water flows through a 1.5 cm × 3.5 cm rectangular cross-section smooth tube at a velocity of 1.2 m/s. The inlet temperature of water is 40°C and tube wall is maintained at 85°C. Determine the length of tube required to raise the temperature of water to 70°C. Also find out the pumping power required if pump efficiency is 60%.

Properties of water at the mean bulk temperature of 55°C are:

$\rho = 985.5 \text{ kg/m}^3$, $c_p = 4.18 \text{ kJ/kgK}$, $\nu = 0.517 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.654 \text{ W/mK}$ and $Pr = 3.26$.



$$Q = \dot{m} c_p (T_{out} - T_i)$$

$$\dot{m} = \rho A v$$

$$= 985.5 \times 1.5 \times 3.5 \times 10^{-4} \times 1.2$$

$$\dot{m} = 0.62 \text{ kg/s.}$$

Let us assume $(c_p)_{\text{water}} = 4.187 \text{ kJ/kg K}$.

$$\dot{Q} = m_w (c_p)_w (T_o - T_i)$$

$$= 0.62 \times 4.187 \times (70 - 40)$$

$$\dot{Q} = 77.9843 \text{ kW.}$$

Hydraulic Diameter: $D_h = \frac{4A}{P} = \frac{4 \times 1.5 \times 3.5}{2(1.5 + 3.5)}$

$$D_h = \frac{4 \times 1.5 \times 3.5}{2 \times 5} \text{ cm}$$

$$D_h = 2.1 \text{ cm.}$$

$$Re = \frac{vD}{\nu} = \frac{1.2 \times 2.1 \times 10^{-2}}{0.517 \times 10^{-6}}$$

$$Re = 4.874 \times 10^4 \text{ (Turbulent)}$$

$$Nu = 0.023 (Re)^{0.8} (Pr)^n$$

$n = 0.3$ Cooling
 $n = 0.4$ Heating

$$Nu = 0.023 (Re)^{0.8} (Pr)^{0.4}$$

$$Nu = 0.023 (4.874 \times 10^4)^{0.8} (3.26)^{0.4}$$

$$Nu = 207.65$$

$$\frac{h D_h}{k} = 207.65$$

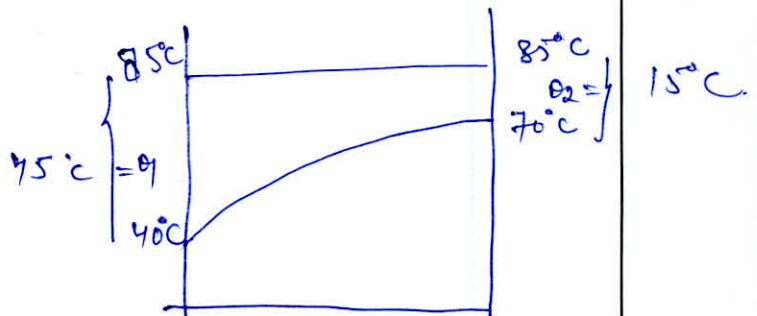
$$h = \frac{207.65 \times 0.654}{2.1 \times 10^{-2}}$$

$$h = 6466.97 \text{ W/m}^2 \text{ K.}$$

Now;

$$LMTD = \frac{\theta_1 - \theta_2}{\ln \left(\frac{\theta_1}{\theta_2} \right)}$$

$$= \frac{45 - 15}{\ln \left(\frac{45}{15} \right)}$$



$$\Delta m = \frac{30}{\ln 3}$$

$$\Delta m = 27.307^\circ\text{C}$$

$$Q = \rho A \Delta m$$

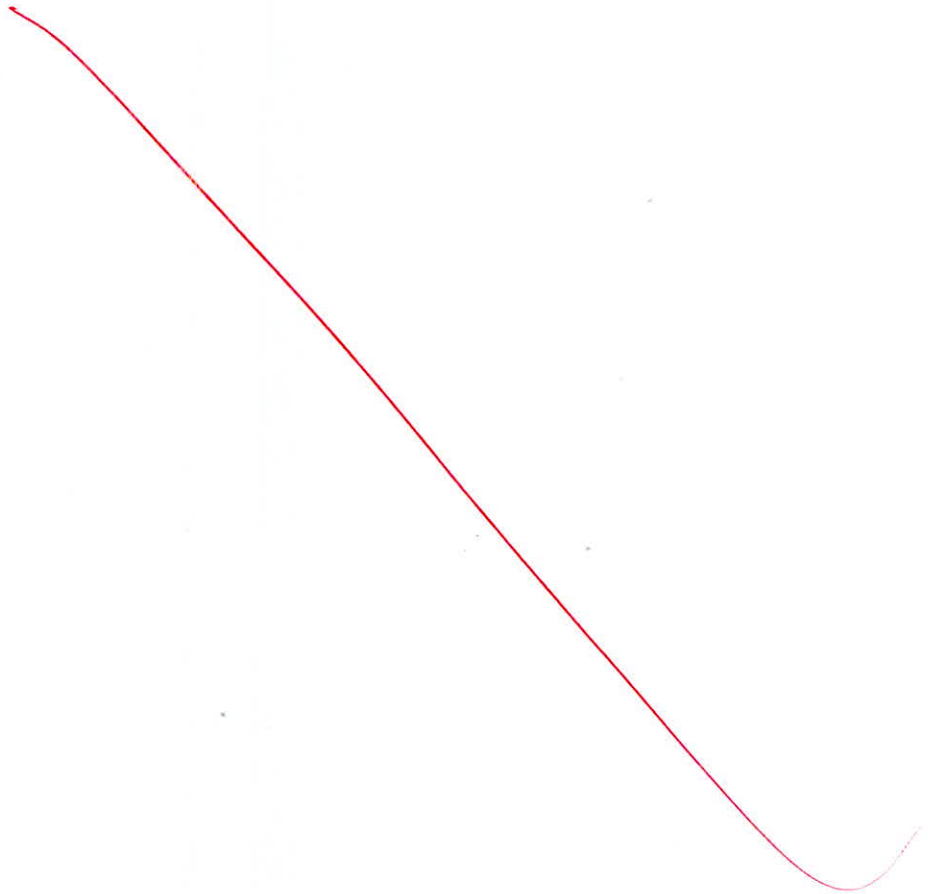
$$77.9843 \times 10^3 = 6466.97 \times A \times 27.307$$

$$A = 0.4481 \text{ m}^2 = P \times L$$

$$0.4481 = 2(3.5 + 1.5) \times 10^{-2} \times L$$

$$L = 4.416 \text{ m}$$

~~15-2=13~~ power required = 22



A punching machine punches 25 holes of 30 mm diameter and 20 mm thickness per minute. The actual punching operation is done in $\left(\frac{1}{15}\right)^{\text{th}}$ of a revolution of crank-shaft.

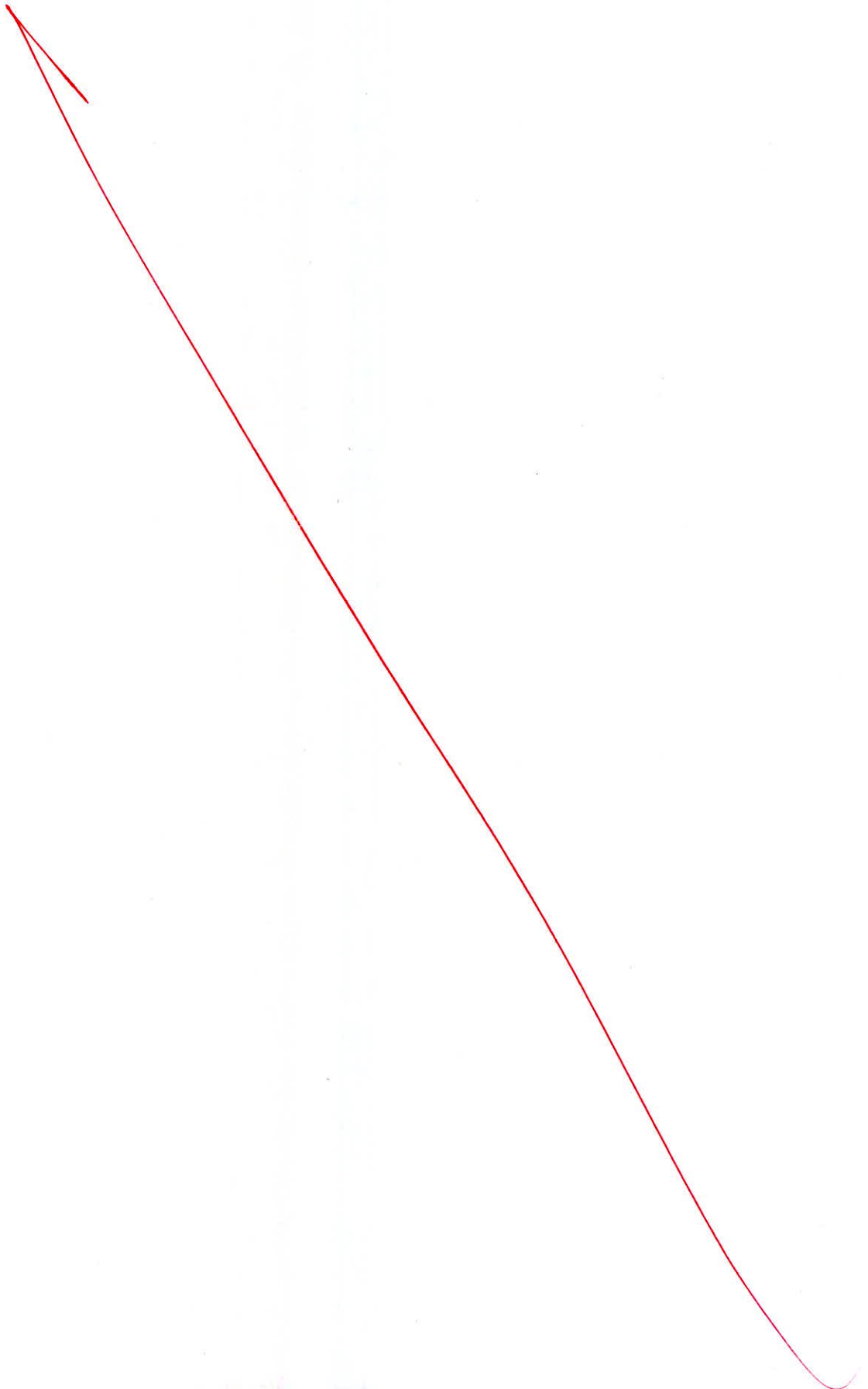
The ultimate shear strength of the steel plate is 300 MPa. The coefficient of fluctuation of speed is 0.12. The flywheel with a maximum diameter of 1.5 m rotate at 10 times the speed of the crank shaft.

Determine the following:

- (i) Power of motor assuming the mechanical efficiency to be 92%.
- (ii) Cross-section of the flywheel rim if width is twice the thickness of the flywheel. Flywheel is of cast iron with a working tensile stress 6 N/mm^2 and density of 7000 kg/m^3 .

Assume the hub and the spokes of the flywheel delivers 10% of the rotational inertia of the wheel.

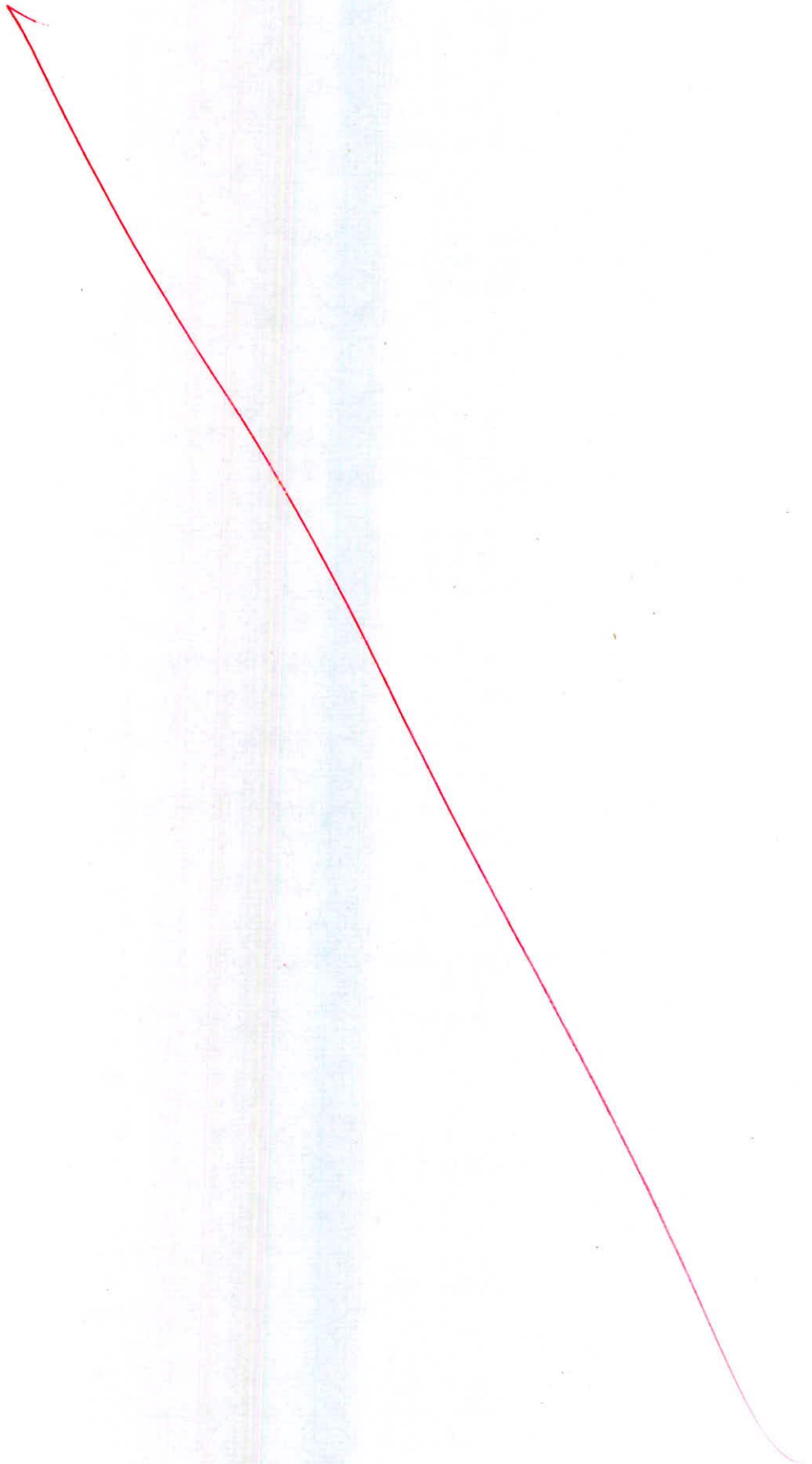
[20 marks]

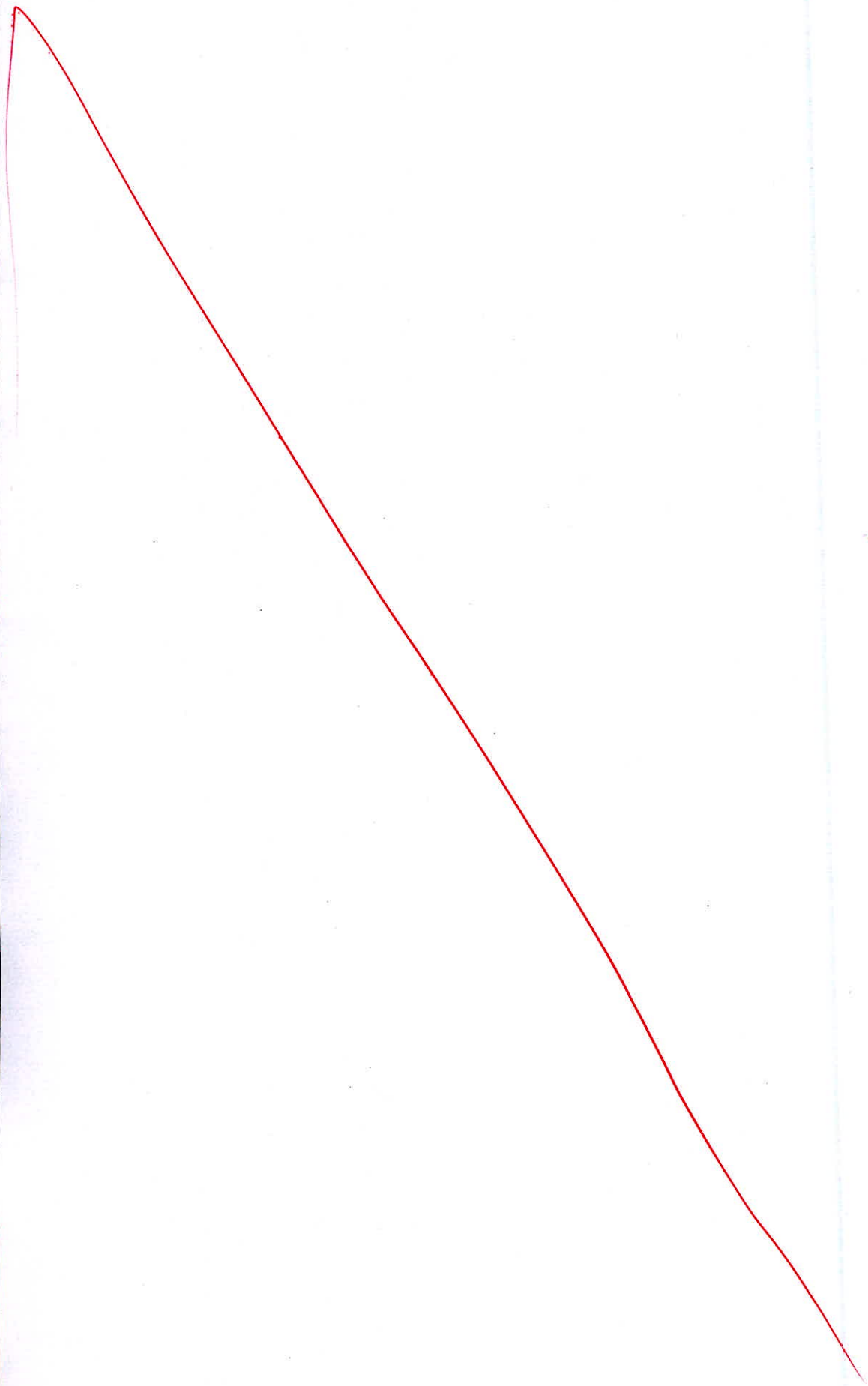


Derive an expression for temperature distribution in case of infinite fin.

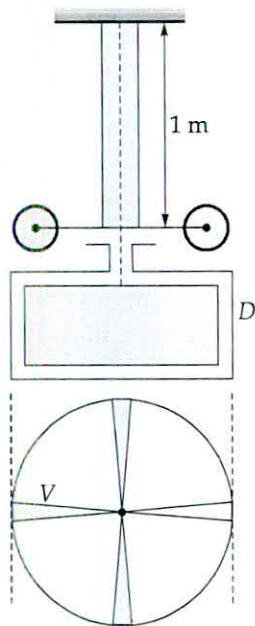
Two long slender rods A and B , made of different materials having same diameter of 12 mm and length 1 m, are attached to a surface maintained at a temperature of 100°C . The surfaces of the rods are exposed to ambient still air at 20°C . By traversing along the length of the rods with a temperature sensor, it is found that the surface temperatures of rods A and B are equal at positions 15 cm and 7.5 cm respectively away from the base surface. If material of A is carbon steel with thermal conductivity 60 W/mK , what is the thermal conductivity of rod B ? List the assumptions made. Assume that the average convection coefficient of air is $5 \text{ W/m}^2\text{K}$. Find the ratio of the rate of heat transfer for rods A and B .

[20 marks]

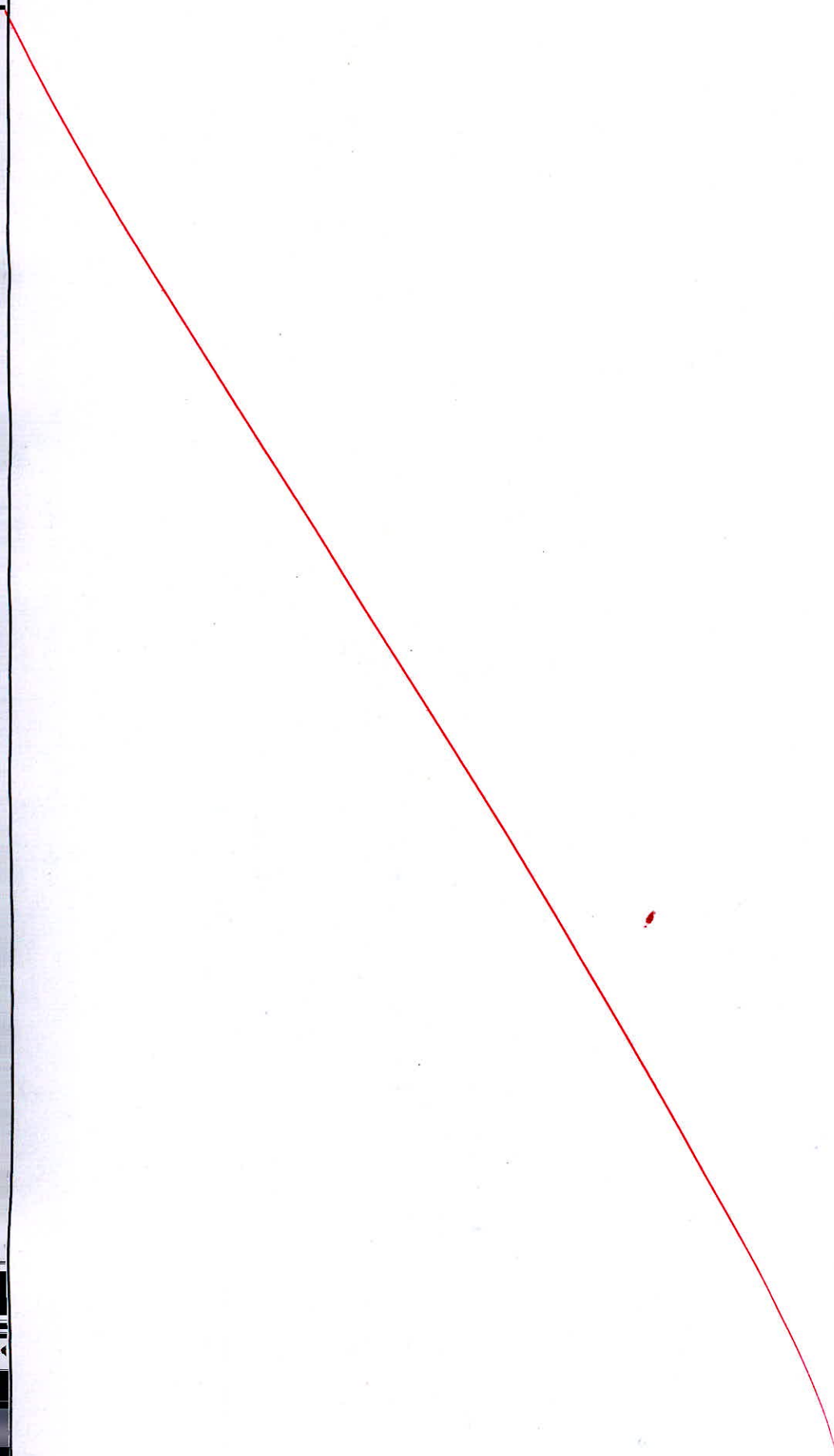




- Q.7(c) A flywheel of moment of inertia 25 kg.m^2 is fixed to one end of a vertical shaft diameter 2.54 cm and the length 1 m . The other end of the shaft is fixed. The torsional oscillations of the flywheel are damped by means of a vane as shown in figure, which moves in a dashpot D filled with oil. The amplitude of oscillations is found by experiment to diminish to $\left(\frac{1}{20}\right)^{\text{th}}$ of its initial value in three complete oscillations. Assuming the damping torque to be directly proportional to the angular velocity, find its magnitude at a speed of 1 rad/s . The modulus of rigidity of the shaft material is 85 GPa and compare later with the frequency of the free vibrations.



[20 marks]

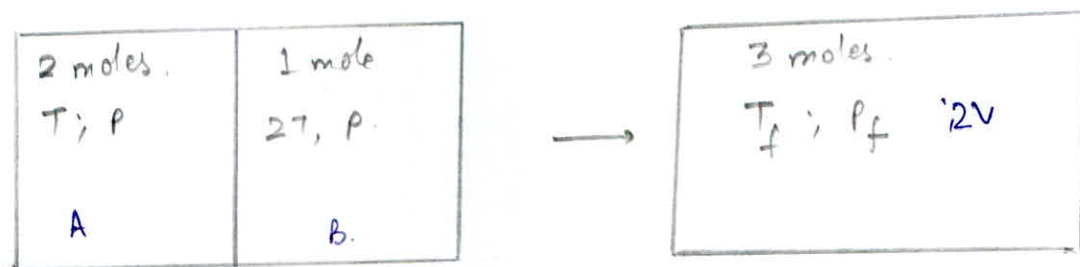


- Q.8 (a) Two moles of an ideal gas at temperature T and pressure P are contained in a compartment. In an adjacent compartment one mole of an ideal gas is at temperature $2T$ and pressure P . The gases mix adiabatically but do not react chemically when a partition separating the compartments is withdrawn. Show that the entropy increase due to the mixing process is given by:

$$\bar{R} \left(\ln \frac{27}{4} + \frac{\gamma}{\gamma-1} \ln \frac{32}{27} \right) \text{ where, } \bar{R} - \text{Universal gas constant}$$

provided that the gases are different and that the ratio of specific heat γ is the same for both gases and remains constant.

[20 marks]



Given : Ideal gas ; $PV = nRT$ or $PV = n\bar{R}T$ is valid.
 Constt. values of specific heat.
 Adiabatic ; $\therefore \Delta Q = 0$

Initial volume in A ; $V_A = \frac{n\bar{R}T}{P} = \frac{2\bar{R}T}{P}$

Initial volume in B ; $V_B = \frac{n\bar{R}T}{P} = \frac{1\bar{R}(2T)}{P}$

$$V_B = \frac{2\bar{R}T}{P}$$

Total final volume ; $V = V_A + V_B$.

$$V = \frac{3\bar{R}T}{P} \Rightarrow \frac{4\bar{R}T}{P}$$

Since the process is adiabatic ;

Net Heat interaction = 0.

Heat added to A = Heat lost by B.

$$n_A \bar{R} (T_f - T_A) = n_B \bar{R} (2T - T_f)$$

$$2(T_f - T) = 1(2T - T_f)$$

$$2T_f - 2T = 2T - T_f$$

$$3T_f = 4T$$

$$\left[T_f = \frac{4T}{3} \right]$$

$$\therefore P_f V = nRT_f$$

$$P_f (2V) = 3R \left(\frac{4T}{3} \right)$$

$$P_f = 2 \frac{RT}{V}$$

$$\Delta S = (\Delta S)_A + (\Delta S)_B$$

$$\Delta S)_A = n C_p \frac{T_2}{T_1} - n \cdot R \ln \frac{P_2}{P_1}$$

$$\Delta S)_A = n \left(C_p \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right)$$

$$\Delta S)_A = 2 \left[C_p \ln \left(\frac{4T/3}{T} \right) - R \ln \left(\frac{3}{2} \right) \right]$$

$$\Delta S)_A = \left[C_p \ln \left(\frac{16}{9} \right) - R \ln \left(\frac{9}{4} \right) \right] \quad \text{--- ①}$$

$$\Delta S)_B = 2 \left[C_p \ln \left(\frac{4T/3}{2T} \right) - R \ln \left(\frac{3}{2} \right) \right]$$

$$\Delta S)_B = \left[C_p \ln \left(\frac{2}{3} \right) - R \ln \left(\frac{3}{2} \right) \right] \quad \text{--- ②}$$

Total entropy increase;

$$\Delta S) = \Delta S)_A + \Delta S)_B$$

$$= \left[C_p \ln \left(\frac{16}{9} \right) - R \ln \left(\frac{9}{4} \right) \right] + \left[C_p \ln \left(\frac{2}{3} \right) - R \ln \left(\frac{3}{2} \right) \right]$$

$$\Delta S) = C_p \ln \left(\frac{32}{27} \right) - R \ln \left(\frac{27}{8} \right)$$

$$\Delta S)_{\text{total}} = C_p \ln \left(\frac{32}{27} \right) + R \ln \left(\frac{4}{27} \right)$$

for gas A;

$$P V)_A = nRT)_A$$

$$\frac{P_A V_A}{RT_A} = \frac{P_f V_f}{RT_f}$$

$$\left[\frac{P_f}{P_A} = \frac{2}{3} P_i \right]$$

for gas B;

$$\left[\frac{P_f}{P_B} = \frac{1}{3} P_i \right]$$

$$\Delta S = \Delta S_A + \Delta S_B$$

$$\Delta S_A = n \left[C_p \ln \frac{T_2}{T_1} - \bar{R} \ln \frac{P_f}{P_i} \right]$$

$$\Delta S_A = 2 \left[C_p \ln \left(\frac{47/3}{T} \right) - \bar{R} \ln \frac{2}{3} \right]$$

$$\Delta S_A = C_p \ln \left(\frac{16}{9} \right) - \bar{R} \ln \frac{4}{9} \quad \text{--- (1)}$$

$$\Delta S_B = n \left[C_p \ln \frac{T_1}{T_2} - \bar{R} \ln \frac{P_1}{P_2} \right]$$

$$= 1 \left[C_p \ln \left(\frac{47/3}{27} \right) - \bar{R} \ln \left(\frac{1}{3} \right) \right]$$

$$\Delta S_B = \left[C_p \ln \left(\frac{2}{3} \right) - \bar{R} \ln \left(\frac{1}{3} \right) \right] \quad \text{--- (2)}$$

net entropy change;

$$\Delta S = \Delta S_A + \Delta S_B$$

$$\Delta S = C_p \ln \left(\frac{32}{27} \right) - \bar{R} \ln \frac{4}{9} - \bar{R} \ln \left(\frac{1}{3} \right)$$

$$\Delta S = C_p \ln \frac{32}{27} - \bar{R} \ln \left(\frac{4}{27} \right)$$

$$\Delta S = \frac{\gamma \bar{R}}{\gamma - 1} \ln \left(\frac{32}{27} \right) - \bar{R} \ln \left(\frac{4}{27} \right)$$

$$\Delta S = \bar{R} \left[\frac{\gamma}{\gamma - 1} \ln \left(\frac{32}{27} \right) + \ln \left(\frac{27}{4} \right) \right]$$

$$\Delta S = \bar{R} \left[\ln \left(\frac{27}{4} \right) + \frac{\gamma}{\gamma - 1} \ln \left(\frac{32}{27} \right) \right]$$

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18

A steam turbine receives 600 kg/h of steam at 25 bar and 350°C. At a certain stage of the turbine, steam at the rate of 150 kg/h is extracted at 3 bar and 200°C. The remaining steam leaves the turbine at 0.2 bar and 0.92 dry. During the expansion process, there is heat transfer from the turbine to the surrounding at the rate of 10 kW. Evaluate per kg of steam entering the turbine:

- (i) the energy of steam entering and leaving the turbine,
- (ii) the maximum work,
- (iii) the irreversibility

The atmosphere is at 30°C.

Data given:

At 25 bar and 350°C, $h_1 = 3125.87$ kJ/kg; $s_1 = 6.8481$ kJ/kgK

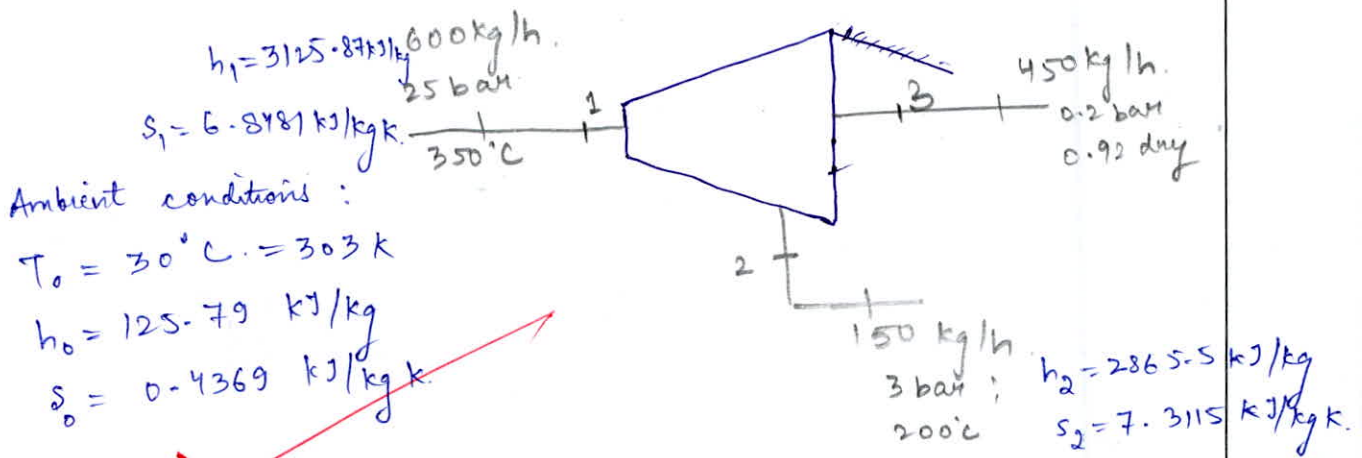
At 30°C, $h_0 = 125.79$ kJ/kg; $s_0 = s_{f30°C} = 0.4369$ kJ/kgK

At 3 bar and 200°C, $h_2 = 2865.5$ kJ/kg; $s_2 = 7.3115$ kJ/kgK

At 0.2 bar (0.92 dry), $h_f = 251.4$ kJ/kg; $h_{fg} = 2358.3$ kJ/kg

$s_f = 0.8320$ kJ/kgK; $s_g = 7.9085$ kJ/kgK

[20 marks]



(i) Energy of steam entering ;

$$\phi_1 - \phi_0 = (h_1 - h_0) - T_0 (s_1 - s_0)$$

$$= (3125.87 - 125.79) - 303 (6.8481 - 0.4369)$$

$$\phi_1 - \phi_0 = 1057.4864 \text{ kJ/kg}$$

Total energy entering = 176.24 kW

(ii) Energy of steam leaving ;

$$\phi_2 - \phi_0 = (h_2 - h_0) - T_0 (s_2 - s_0)$$

$$= (2865.5 - 125.79) - 303 (7.3115 - 0.4369)$$

$$\phi_2 - \phi_0 = 656.7062 \text{ kJ/kg}$$

$\phi_2 - \phi_0 = 27.36 \text{ kW}$

At section 3 (0.92 dry)

$$h_3 = h_f + x h_{fg} = 251.4 + x (2358.3) = 2421.036 \text{ kJ/kg}$$

$$s_3 = s_f + x (s_g - s_f) = 7.34238 \text{ kJ/kg K}$$

$$\phi_3 - \phi_0 = (h_3 - h_0) - T_0 (s_3 - s_0)$$

$$= (2421.036 - 125.79) - 303 (7.34238 - 0.4369)$$

$$\phi_3 - \phi_0 = 202.885 \text{ kJ/kg}$$

$$\dot{\phi}_3 - \dot{\phi}_0 = \dot{m}_3 (202.885) = 25.36 \text{ kW}$$

Total amount of energy leaving the system;

$$E_{\text{leaving}} = 27.36 + 25.36$$

$$E_{\text{leaving}} = 52.72 \text{ kW}$$

(ii) Maximum work;

$$W_{\text{max}} = \phi_1 - (\phi_2 + \phi_3) - \dot{Q}$$

$$= (\phi_1 - \phi_2) + (\phi_1 - \phi_3) - \dot{Q}$$

$$= 176.24 - 52.72 - 10$$

$$W_{\text{max}} = 113.519 \text{ kW}$$

(iii) Irreversibility:

$$S_{\text{gen}} = S_{\text{exit}} - S_{\text{entry}} - \frac{\dot{Q}}{T_0}$$

$$= \frac{450}{3600} \times \left(\frac{303}{303}\right) (7.3428 - 6.8481) + \left(\frac{150}{3600} \times \frac{303}{303}\right) (7.3115 - 6.8481) - \frac{10}{303}$$

$$= \frac{18.73676}{303} + 0.019308 - \frac{10}{303}$$

$$S_{\text{gen}} = 0.048142 \text{ kJ/kg K}$$

$$\text{Irreversibility } I = T_0 S_{\text{gen}}$$

$$I = 303 (0.048142)$$

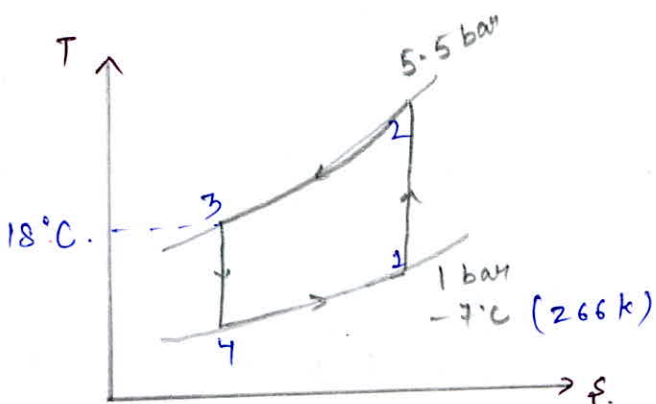
$$I = 14.58 \text{ kW}$$

An air refrigerator working on Bell-Coleman cycle takes the air into the compressor at 1 bar and -7°C and it is compressed isentropically to 5.5 bar and it is further cooled to 18°C at the same pressure. Find the COP of the system if:

- the expansion is isentropic
- the expansion follows the law $PV^{1.25} = \text{constant}$.

Take $\gamma = 1.4$ and $c_p = 1 \text{ kJ/kgK}$ for air.

[20 marks]



$$T_1 = -7^\circ\text{C} = 266 \text{ K.}$$

$$T_3 = 18^\circ\text{C} = 291 \text{ K.}$$

$$P_1 = P_4 = 1 \text{ bar}$$

$$P_2 = P_3 = 5.5 \text{ bar}$$

Assuming process 1-2 isentropic; $\gamma = 1.4$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 266 \left(5.5\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 432.926 \text{ K}$$

(i) Process 3-4 is isentropic;

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 291 \times \frac{1}{(5.5)^{0.4/1.4}}$$

$$T_4 = 178.79 \text{ K}$$

$$R.E. = C_p (T_1 - T_4) = 1 (266 - 178.79) = 87.202 \text{ kJ/kg}$$

$$W_c = C_p (T_2 - T_1) = 1 (432.926 - 266) = 166.926 \text{ kJ/kg}$$

$$W_T = C_p (T_3 - T_4) = 1 (291 - 178.79) = 112.21 \text{ kJ/kg}$$

$$W_{\text{net}} = W_c - W_T = 54.716 \text{ kJ/kg}$$

$$\text{COP} = \frac{R.E.}{W_{\text{net}}} = \frac{87.202}{54.716} = 1.5932$$

(ii) Expansion follows $PV^{1.25} = \text{const.}$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{1.25-1}{1.25}} \Rightarrow T_4 = 206.92 \text{ K}$$

$$W_T = \frac{n(P_3 V_3 - P_4 V_4)}{n-1} = \frac{n}{n-1} \cdot R (T_3 - T_4)$$

$$C_p = \frac{\gamma R}{\gamma - 1} = 1 \quad R = \frac{1 \times 0.4}{1.4} = 0.2857 \text{ kJ/kgK}$$

$$W_T = \frac{n}{n-1} \cdot 0.2857 (291 - 206.92)$$

$$W_T = 120.114 \text{ kJ/kg}$$

$$W_c = C_p (T_2 - T_1) = 166.926 \text{ kJ/kg}$$

$$R.E. = C_p (T_1 - T_4) = 1 (266 - 206.92) = 59.08 \text{ kJ/kg}$$

Net work input

$$W_{\text{in}} = W_c - W_T = 166.926 - 120.114 = 46.812 \text{ kJ}$$

$$\text{COP} = \frac{R.E.}{W_{in}} = \frac{59.08}{46.812}$$

$$\text{COP} = 1.262$$

20



Space for Rough Work



Rough

$$- P_1 A_1 - P_2 A_2 = \rho g (v_2 - v_1)$$

$$+ (P_1 A_1 - P_2 A_2) = \rho g \left(\frac{4}{3} v_1 - v_1 \right)$$

$$= \rho g v_1 \left(\frac{4}{3} - 1 \right)$$

$$F = (P_1 - P_2) \cdot \frac{1}{3} \rho A v_1^2$$

