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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Mechanical Engineering

Test-3: Fluid Mechanics and Turbo Machinery, Heat Transfer-1 + TOM-1, Thermodynamics-2 + Refrigeration and Air-conditioning-2

Name :

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	23
Q.2	45−1 = 44
Q.3	—
Q.4	—
Section-B	
Q.5	38
Q.6	—
Q.7	43+1 = 44
Q.8	40
Total Marks Obtained	189

Signature of Evaluator: H.G.

Cross Checked by: [illegible]

Section A : Fluid Mechanics and Turbo Machinery

1 (a) Define degree of Reaction. Derive the expression of degree of reaction for an axial flow compressor in terms of inlet and outlet blade angles, blade and flow velocity.

[12 marks]

Degree of Reaction: It is defined as the ratio of enthalphy drop in moving blades to the total enthalphy drop in a Stage (Fixed blades + moving blades)

$$R = \frac{\text{Enthalphy drop in moving part}}{\text{Enthalphy drop in stage}}$$

01

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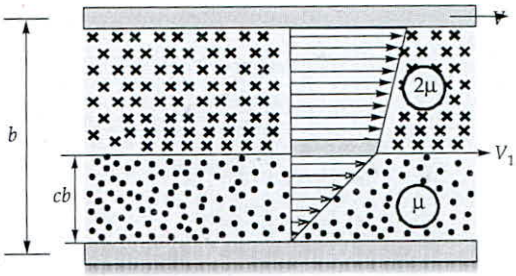
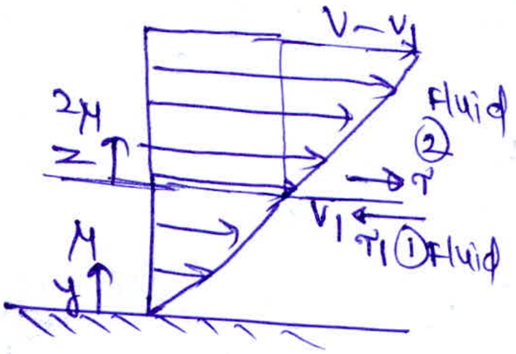
1 (b) Two flat plates are oriented in parallel configuration above a fixed lower plate as shown in figure. The top plate, located a distance, b above the fixed plate, is pulled along with speed V. The other thin plate is located a distance (cb) where $0 < c < 1$, above the fixed plate. This plate moves with speed V_1 which is determined by the viscous shear forces imposed on it by the fluids on its top and bottom. The fluid on the top is twice as viscous as that on the bottom, then obtain the ratio $\left(\frac{V_1}{V}\right)$ corresponding to value of c as given in table.

c	0	0.2	0.5	0.7	1.0
V_1/V	?	?	?	?	?



[12 marks]

Sol:



Assuming velocity profile to be linear

Velocity profile of ① $\Rightarrow u = \frac{V_1}{cb} \times y$

$$\left.\frac{du}{dy}\right|_{y=cb} = \frac{V_1}{cb}$$

$$\tau_{\text{at bottom of plate}} = \mu \left.\frac{du}{dy}\right|_{y=cb} = \frac{\mu V_1}{cb}$$

Velocity profile for fluid ② $= u = \frac{(V-V_1)}{(b-cb)} z + V_1$

$$\left.\frac{du}{dz}\right|_{z=0} = \frac{V-V_1}{b(1-c)}$$

$$\tau_{\text{at top of plate}} = 2\mu \left.\frac{du}{dz}\right)_{z=0}$$

$$= 2\mu\left(\frac{V-V_1}{b(1-c)}\right)$$

$$\sum \vec{F}_{net} = \cancel{m\vec{a}} \rightarrow 0$$

$$\tau_{\text{at bottom of plate}} = \tau_{\text{at top of plate}}$$

$$\mu\left(\frac{V_1}{cb}\right) = 2\mu\left(\frac{V-V_1}{b(1-c)}\right)$$

$$\frac{V_1}{c} = 2\left(\frac{V-V_1}{1-c}\right)$$

$$\frac{1-c}{2c} = \frac{V}{V_1} - 1$$

$$\frac{V_1}{V} = \frac{2c}{1+c}.$$

$$\boxed{\frac{V}{V_1} = \frac{1-c}{2c} + 1 = \frac{1+c}{2c}}$$

For $c = 0 \Rightarrow \frac{V_1}{V} = 0$

$c = 0.2 \Rightarrow \frac{V_1}{V} = \frac{2\times0.2}{1.2} = 1/3$

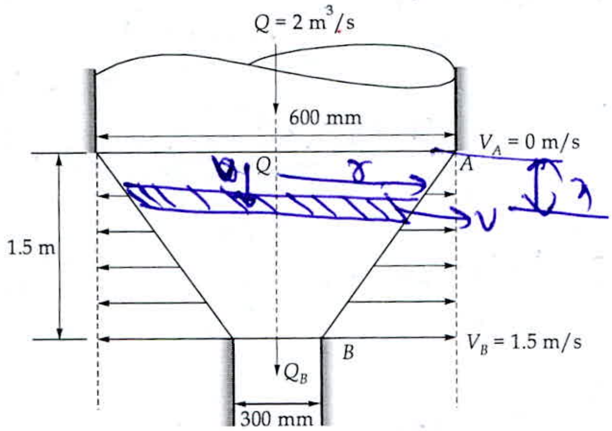
$c = 0.5 \Rightarrow \frac{V_1}{V} = \frac{2\times0.5}{1.5} = 2/3$

$c = 0.7 = \frac{V_1}{V} = \frac{2\times0.7}{1.7} = 14/17$

$c = 1.0 = \frac{V_1}{V} = \frac{2\times1}{2} = \underline{\underline{1}}$

11

1 (c) Water flow downward in a pipe of 600 mm diameter at the rate of 2 m^3/s. It then enters a conical duct with porous wall such that there is a radial outflow with flow velocity varying linearly from zero at A to 1.5 m/s at B. What is the rate of flow at B coming out from the conical duct.



[12 marks]

Sol:

$$Q = Q_B + Q_R$$

Flow rate of fluid coming out through small element

$$dQ_R = 2\pi r_x\, dx \times V$$

$$V_A = 0 \text{ m/s} \rightarrow 0$$

$$V = \rightarrow x$$

$$V_B = 1.5 \text{m/s} \rightarrow 1.5 \text{m}$$

Linear interpolation

$$\frac{V_B - V_A}{1.5 - 0} = \frac{V - V_A}{x - 0}$$

$$V = \frac{1.5}{1.5} x = x$$

$$\boxed{V = x}$$

$$r_x = 0.3 - \left(\frac{0.3 - 0.15}{1.5}\right) x$$

$$= 0.3 - \frac{x}{10}$$

$$dQ_R = 2\pi (0.3 - 0.1x)\, x\, dx$$

$$\int dQ_R = \int_0^{1.5} 2\pi\,(0.3r - 0.1r^2)\,dr$$

$$= 2\pi\left(0.3\frac{r^2}{2} - 0.1\frac{r^3}{3}\right]_0^{1.5}$$

$$= 2\pi\left(0.3\times\frac{1.5^2}{2} - 0.1\times\frac{1.5^3}{3}\right]$$

$$Q_R = 1.4137\ m^3/s$$

$$Q_A = Q_R + Q_B$$

11

$$Q_B = Q_A - Q_R$$
$$= 2 - 1.4137 = 0.5863\ m^3/s$$

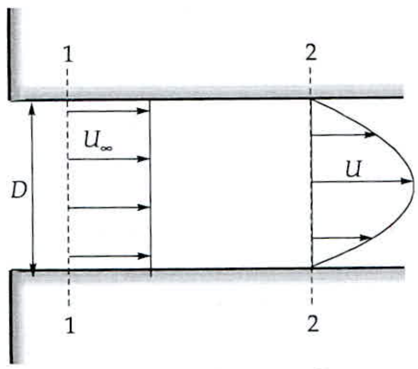
Q.1 (d) **(i)** Explain why there is a need of compounding of impulse steam turbine. Also mention types of compounding done.

(ii) What are the differences between impulse and reaction turbine? Explain in a tabular form.

[6 + 6 marks]

Q.1 (e) In a steady entrance flow in a pipe of diameter D as shown in figure. The flow develops from uniform flow at section (1) to a parabolic profile at section (2). If the momentum correction factor at section (2) is $\frac{4}{3}$, then show that the wall drag force F is given by

$$F = \frac{\pi D^2}{4}\left(P_1 - P_2 - \frac{1}{3}\rho U_\infty^2\right)$$



Where P_1 and P_2 are pressure at respective sections.

[12 marks]

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2 (a) A model having scale ratio of $\frac{1}{10}$ is constructed to determine the best design of Kaplan turbine. The prototype Kaplan turbine develop 7355 kW under a net head of 10 m at a speed of 100 rpm. If the head available at the laboratory is 6 m and the model efficiency is 88% whereas the efficiency of prototype turbine is 4% better that of the model turbine. Find:

(i) running speed of the model.

(ii) the flow rate required in the laboratory.

(iii) the specific speed in each case.

[20 marks]

Sol: Given $\frac{D_m}{D_p} = \frac{1}{10}$

Model

$H_m = 6m$

$\eta_m = 0.88$

Prototype

$P_p = 7355$ kW

$H_p = 10$ m

$N_p = 100$ rpm

$\frac{\eta_p - \eta_m}{\eta_m} \times 100 = 4$

$\boxed{\eta_p = 0.9152}$

Using similarity Laws

$$\left.\frac{gH}{N^2 D^2}\right|_{model} = \left.\frac{gH}{N^2 D^2}\right|_{Prototype}$$

$g_m = g_p$

$$\frac{H_m}{N_m^2 D_m^2} = \frac{H_p}{N_p^2 D_p^2}$$

$$N_m^2 = \frac{H_m}{H_p} \times \left(\frac{D_p}{D_m}\right)^2 \times N_p^2$$

$$N_m = \sqrt{\frac{H_m}{H_p}} \times \frac{D_p}{D_m} \times N_p$$

$$= 100 \times \sqrt{\frac{6}{10}} \times 10$$

$$= 774.596 \text{ rpm}$$

Consider effective head ?

Power of Prototype $P_p = \eta_p \times \rho \times g \times Q_p \times H_p$

$$Q_p = \frac{P_p}{\eta_p \times 9810 \times H_p}$$

$$= \frac{7355 \times 10^3}{0.9152 \times 9810 \times 10}$$

$$= 81.9214 \ m^3/s$$

$$\left.\frac{Q}{ND^3}\right|_{model} = \left.\frac{Q}{ND^3}\right|_{Prototype}$$

$$Q_m = Q_p \times \left(\frac{N_m}{N_p}\right) \times \left(\frac{D_m}{D_p}\right)^3$$

$$= 81.9214 \times \left(\frac{774.596}{100}\right) \times \left(\frac{1}{10}\right)^3$$

$$\boxed{Q_m = 0.6345 \ m^3/s}$$

Power developed by model. $P_m = \eta_m \times \rho g \times Q_m \times H_m = 32.868$ kW

Specific speed of model $= \dfrac{N_m \sqrt{P_m}}{H_m^{5/4}}$

$$= \frac{774.596 \sqrt{32.868}}{6^{5/4}}$$

$$= 472.905$$

Specific Speed of Prototype $= \dfrac{N_p \sqrt{P_p}}{H_p^{5/4}}$

$= 100 \times \sqrt{7355} / 10^{1.25} = 482.2711$

10

Q.2 (b) A centrifugal compressor develops a pressure ratio of 4 : 1. The inlet eye of the compressor impeller is 0.3 m in diameter. The axial velocity at inlet is 120 m/s and the mass flow rate is 10 kg/s. The velocity in the delivery duct is 110 m/s. The tip speed of the impeller is 450 m/s and runs at 16000 rpm with a total head isentropic efficiency of 80%. The inlet stagnation temperature and pressure are 300 K and 101 kPa.

(Take c_p = 1.005 kJ/kgK, γ = 1.4)

(i) the static temperature and pressure at inlet and outlet of the compressor

(ii) the static pressure ratio

(iii) the power required to drive the compressor

(iv) Mach number (based on relative velocity) at inlet

[20 marks]

Sol:

$$\frac{P_{02}}{P_{01}} = r_p = 4 \qquad d_1 = 0.3m$$

$$\frac{T_{02}}{T_{01}} = r_p^{\gamma-1/\gamma} = 4^{0.4/1.4}$$

$$T_{02} = 445.798\,K = T_{03}$$

$$W_{in} = c_p \frac{(T_{03}-T_{01})}{\eta_{iso}} = \frac{146.527}{\eta_{iso}} \frac{KJ}{kg}$$

$$= 183.16 \frac{KJ}{kg}$$

$$P = \dot{m} \times W_{in} = 1831.59\,kW$$

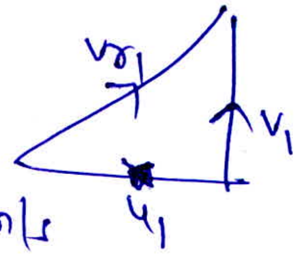
$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 16000}{60}$$

$$= 251.327 \text{ m/s}$$

Inlet velocity triangle

$$V_{r_1} = \sqrt{V_1^2 + u_1^2}$$

$$= 278.50577 \text{ m/s}$$



$$M_a = \frac{V_{r_1}}{\sqrt{\gamma R T_1}}$$

$$T_1 = T_{01} - \frac{C_1^2}{2000 c_p}$$

$$= \frac{278.50577}{\sqrt{1.4 \times 0.287 \times 292.836}}$$

$$= 300 - \frac{120^2}{2000 \times 1.005}$$

$$\boxed{T_1 = 292.836 \text{ K}}$$

$$\boxed{M_a = 0.812}$$

$$\left(\frac{P_1}{P_{01}}\right)^{\gamma-1/\gamma} = \left(\frac{T_1}{T_{01}}\right)$$

$$T_{03} = T_3 + \frac{C_3^2}{2000 c_p}$$

$$P_1 = P_{01} \times \left(\frac{T_1}{T_{01}}\right)^{\gamma/\gamma-1}$$

$$T_3 = 482.2475 - \frac{110^2}{2000 \times 1.005}$$

$$= 101 \times \left(\frac{292.836}{300}\right)^{\frac{1.4}{0.4}}$$

$$\boxed{T_3 = 476.2276}$$

$$\boxed{P_1 = 92.8074 \text{ kPa}}$$

15

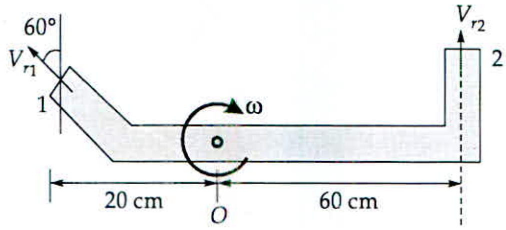
$$\frac{P_3}{P_1} = \left(\frac{T_3}{T_1}\right)^{\gamma/\gamma-1} = 5.4848$$

$$P_3 = 509.035 \text{ kPa} \qquad \boxed{\frac{P_3}{P_1} = 5.4848}$$

Q.2 (c) A sprinkler with unequal arms and jets of area 0.7 cm^2 is shown in figure. A flow of 1.4 l/s enters the assembly normal to the rotating arm.

(i) Assuming the frictional resistance to be zero calculate its speed of rotation,

(ii) What torque is required to hold it from rotating?



[20 marks]

Sol:

Flow rate coming out from nozzles 1 & 2

$$= \frac{Q}{2}$$

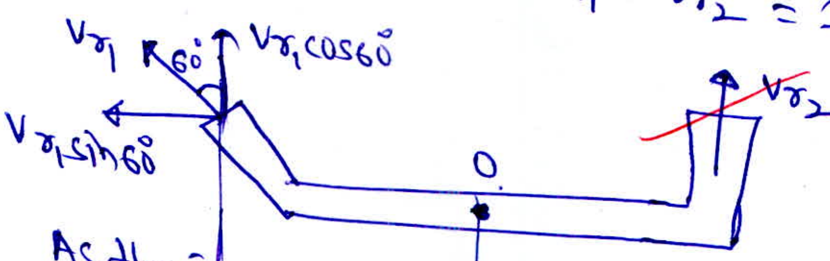
$$= \frac{1.4 \times 10^{-3}}{2}$$

$$= 0.7 \times 10^{-3}\ m^3/s$$

Velocity of Jet coming out from nozzles 1 & 2 $= \frac{0.7 \times 10^{-3}}{A}$

$$= \frac{0.7 \times 10^{-3}}{0.7 \times 10^{-4}}$$

$$V_{r_1} = V_{r_2} = 10\,m/s$$



As the jet leaves in upward directions then it will exert a force in opposite direction (Newton's third Law)

Torque exerted at O by Jet 1 (T_1) $= \dot{m}_1 V_{r_1} \cos 60° \times 0.2$ (Anticlockwise)

Torque exerted by Jet 2 at point O

$$T_2 = \dot{m}_2 V_{r_2} \times 0.6 \text{ (clockwise)}$$

$$\dot{m}_1 = \dot{m}_2 = \rho Q_{nozzle} = 10^3 \times 0.7 \times 10^{-3} = 0.7 \text{ kg/s}$$

Since $T_2 > T_1 \Rightarrow$ sprinkler will rotate in clockwise direction

Net torque $\Sigma T = T_1 + T_2$

$$= m V_{r_1} \cos 60^\circ \times 0.2 \text{ (CCW)} + m V_{r_2} \times 0.6 \text{ (CW)}$$

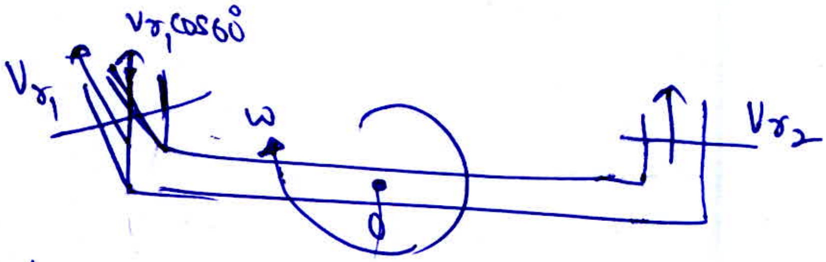
$$= m [V_{r_2} \times 0.6 - V_{r_1} \cos 60^\circ \times 0.2] \text{ (CW)}$$

$$= 0.7 \times [10 \times 0.6 - 10 \times \cos 60^\circ \times 0.2]$$

$$= 3.5 \text{ N-m (CW)}$$

So a torque of 3.5 Nm must be applied in CCW direction in order to hold it from rotating

When arm is free to rotation



19

Absolute velocity of water leaving jet 1

$$V_1 = V_{r_1} \cos 60^\circ + \omega \times 0.2$$

Absolute Velocity of water leaving jet 2

$$V_2 = V_{r_2} - \omega \times 0.6$$

$\omega = \frac{2\pi N}{60}$

$N = 119.366$ rpm

$T_1 = T_2$

$V_1 \times 0.2 = V_2 \times 0.6$ (3)

$(10 \cos 60^\circ + \omega \times 0.2) = 3 \times (10 - 0.6\omega) \Rightarrow 2\omega = 25 \Rightarrow \boxed{\omega = 12.5 \text{ rad/s}}$

Q.3 (a) An impulse steam turbine has a number of pressure stages, each having a row of nozzles and a single ring of blades. The nozzle angle in the first stage is 20° and the blade exit angle is 30° with reference to the plane of rotation. The mean blade speed is 125 m/s and the velocity of steam leaving the nozzles is 350 m/s.

(i) Taking the blade friction factor as 0.9 and nozzle efficiency of 0.85, determine the work done in the stage per kg of steam and the stage efficiency.

(ii) If the steam supply to the first stage is at 20 bar, 250°C and the condenser pressure is 0.07 bar, estimate the number of stages required, assuming that the stage efficiency and the work done are the same for all stages and the reheat factor is 1.05.

at 20 bar, 250°C,

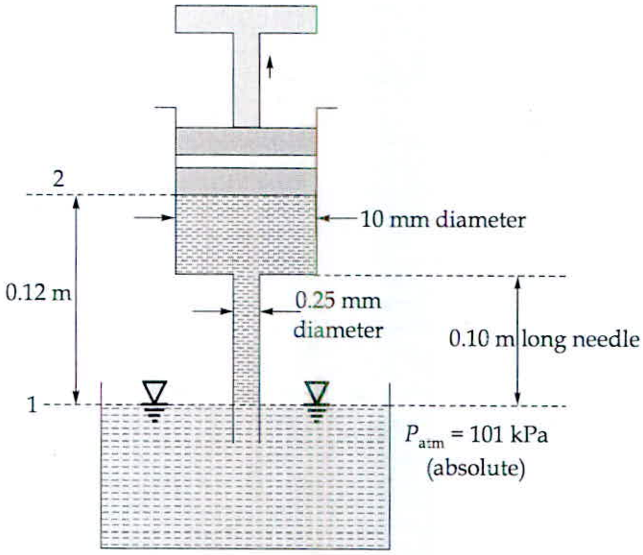
h = 2902.5 kJ/kg, s = 6.5453 kJ/kgK

at 0.07 bar,

h_f (kJ/kg)	h_{fg} (kJ/kg)	s_f (kJ/kgK)	s_{fg} (kJ/kgK)
163.16	2409.54	0.5582	7.7198

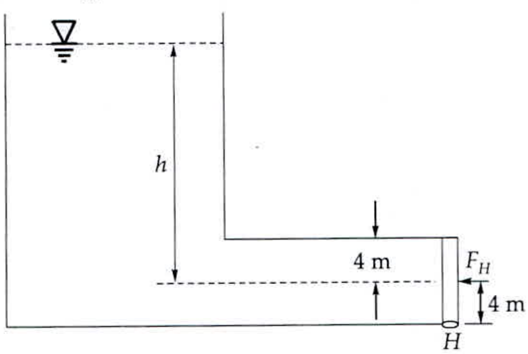
[20 marks]

Q.3 (b) A liquid with specific gravity of 0.96, dynamic viscosity 9.2×10^{-4} Ns/m^2 and vapor pressure (P_v) = 1.2×10^4 N/m^2 (absolute) is drawn into the syringe as indicated in figure. What is the maximum flow rate if cavitation is not to occur in the syringe? Assume that the flow corresponding to the small diameter is laminar and support your answer with the necessary calculations.



[20 marks]

Q.3 (c) A 3 m wide, 8 m high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in figure. The gate is hinged at its bottom and held closed by a horizontal force, F_H located at the centre of the gate. The maximum value for F_H is 3500 kN.



(i) Determine the maximum water depth above the centre of the gate that can exist without the gate opening.

(ii) Will the answer be same, if the gate is hinged at the top? Explain your answer.

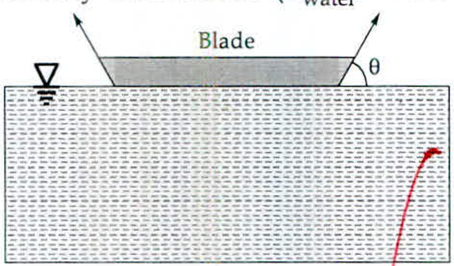
[20 marks]

Q.4 (a) As surface tension forces can be strong enough to allow a double edge steel razor blade to 'float' on water. But a single edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface as shown in figure.

(i) The mass of the double edge blade is 0.64×10^{-3} kg and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and resultant surface tension force.

(ii) The mass of the single edge blade is 2.61×10^{-3} kg and the total length of its side is 154 mm. Explain why this blade sink.

(iii) If suppose one bug having weight of 10^{-4} N stays on the upper (air side) surface of steel razor, then what changes you expect in value of (θ) for case (a) and support your answer with the necessary calculations ($\sigma_{water} = 7.34 \times 10^{-2}$ N/m)?



[5+5+10 = 20 marks]

Q.4 (b) A steam turbine plant works between the limit of 150 bar, 600°C and 0.1 bar. The mean blade velocity is 220 m/s. The average nozzle efficiency is 0.91. The nozzle (fixed blade) angle is 20°. All stages operate at the condition of maximum efficiency. The total isentropic enthalpy drop is 1400 kJ/kg. Determine the number of stages required for the following cases.

(i) All simple impulse stages.

(ii) All 50% impulse-reaction stages.

(iii) A two-row Curtis stage followed by simple impulse stages.

(iv) A two row Curtis stage followed by 50% impulse reaction stages.

[20 marks]

Q.4 (c) **(i)** Explain the purpose of installing draft tube at the exit of reaction turbine.

(ii) The draft tube of a Kaplan turbine has inlet diameter 2.8 m and inlet is set at 3 m above the tail race. When the turbine develops 1500 kW power under a net head of 6 m, it is found that the vacuum gauge fitted at inlet to draft tube indicates a negative head of 4 m. If the turbine overall efficiency is 88%, determine the draft tube efficiency. If the turbine output is reduced to half with the same head, speed and draft tube efficiency, what would be the reading of the vacuum gauge? (Neglect minor losses).

[5 +15 = 20 marks]

Section B : Heat Transfer - 1 + TOM - 1, Thermodynamics - 2 + RAC - 2

Q.5 (a) For a sphere of radius R having a surface temperature of T_s in which heat is generated at a uniform rate of q_G W/m^3, derive the following expression

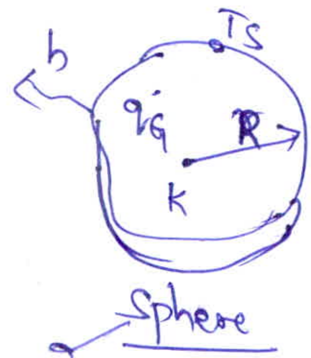
$$T = T_\infty + \frac{q_G R}{3h} + \frac{q_G R^2}{6k}\left(1 - \frac{r^2}{R^2}\right)$$

where, T_∞ = Ambient temperature.

[12 marks]

Sol:

1 dimensional heat conduction equation in case of sphere is



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{q_G}{k} = 0$$

Initial steps?

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{q_G}{k} r^2$$

— (Steady state) $dT/dt = 0$

— 1-D $T = f(r)$ only

$$r^2\, dT/dr = -\frac{q_G}{3k} r^3 + c_1$$

$$dT/dr = -\frac{q r}{3k} + \frac{c_1}{r^2} + c_2$$

$$T = -\frac{q r^2}{6k} - \frac{c_1}{r} + c_2$$

c_1 & c_2 are constants determined by boundary conditions

If $r \to 0$; $T \to \infty$ which is not possible

So $\boxed{c_1 = 0}$

This is obtained by $-k(4\pi R^2)\frac{dT}{dr} = \dot{q}_G \times \left(\frac{4}{3}\pi R^3\right)$

$$T = -\frac{q r^2}{6k} + c_2$$

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at $r = R$; $T = T_s$

$$T_s = \frac{-q R^2}{6K} + c_2$$

$$c_2 = T_s + \frac{q R^2}{6K}$$

$$T = -\frac{q r^2}{6K} + T_s + \frac{q R^2}{6K} \quad \text{—— (1)}$$

At the surface $(r = R)$

$$-KA \, dT/dr \big|_{r=R} = h(4\pi R^2)(T_s - T_\infty)$$

$$= \dot{q}_G \times \frac{4}{3}\pi R^3$$

$$h(4\pi R^2)(T_s - T_\infty) = \dot{q}_G \times \frac{4}{3}\pi R^3$$

$$T_s - T_\infty = \frac{\dot{q}_G R}{3h}$$

$$T_s = T_\infty + \frac{\dot{q}_G R}{3h} \quad \text{—— (2)}$$

09

Substitute (2) in (1)

$$\boxed{T = T_\infty + \frac{\dot{q}_G R}{3h} + \frac{\dot{q}_G}{6k}(R^2 - r^2)}$$

Q.5 (b) The barometer for atmospheric air reads 750 mm of Hg, the dry bulb temperature is 33°C, wet bulb temperature is 23°C. Determine:

(i) the relative humidity.
(ii) the humidity ratio.
(iii) the dew point temperature.
(iv) density of atmospheric air.

Use the following relation,

Partial pressure of vapour, $P_v = (P_s)_{WB} - \dfrac{\left(P_t - (P_s)_{WB}\right)(t_{DB} - t_{WB})}{1527.4 - 1.3t_{WB}}$

$P_t \rightarrow$ Barometric pressure

$(P_S)_{WB} \rightarrow$ Saturation pressure corresponding to WBT
$t_{WB} \rightarrow$ Wet bulb temperature (in °C)
$t_{DB} \rightarrow$ Dry bulb temperature (in °C)
Use following table:

P_s (mm of Hg)	t_s (°C)
16.19	18.7
21.06	23
37.72	33

At 33°C density of Hg, $\rho_{Hg} = 13600\ kg/m^3$
Assume v_g (Specific volume of saturated vapour) at 37.72 mm of Hg is 28.05 m^3/kg.

[12 marks]

Sol: $P_t = \dfrac{750}{760} \times 101.325 = 100\ kPa$

$t_{DB} = 33°C \longrightarrow (P_s)_{DB} = \dfrac{37.72}{760} \times 101.325 = 5.029\ kPa$

$t_{WB} = 23°C \longrightarrow (P_s)_{WB} = \dfrac{21.06}{760} \times 101.325$

$= 2.8077\ kPa$

Given

$P_v = (P_s)_{WB} - \dfrac{(P_t - (P_s)_{WB})(t_{DB} - t_{WB})}{1527.4 - 1.3t_{WB}}$

$= 2.8077 - \dfrac{(100 - 2.8077)(33-23)}{1527.4 - 1.3 \times 23}$

$= 2.15866\ kPa$

$$h = \frac{P_V}{\rho g_{Hg}} = 16.18 \text{ mm of Hg}$$

Dew Point temperature $\boxed{t \approx 18.7^0}$ ⟹ Saturation temperature corresponding to P_V

$$\boxed{\phi = \frac{P_V}{P_{VS}} = \frac{16.18}{37.72} = 42.89\%}$$

Humidity ratio $\omega = 0.622 \times \left(\frac{P_V}{P - P_V}\right)$

$$= 0.622 \times \left(\frac{16.18}{750 - 16.18}\right)$$

$$\boxed{\omega = 0.013714 \text{ kg/kg of d.A}}$$

09

density of air $\boxed{\rho_v = \omega \rho_a}$

$$\cancel{\rho_a = \frac{\omega}{v_g} = \frac{0.013714}{28.05}}$$

$$\rho_v = \omega \rho_a \qquad \rho_v = \frac{1}{v_g} = \frac{1}{28.05} \text{ kg/m}^3$$

$$\rho_a = \frac{\rho_v}{\omega} = \frac{1}{28.05 \times 0.013714}$$

$$= \cancel{2.6 \text{ kg/m}^3}$$

$\rho = \rho_{da} + \rho_{w.v.}$

Q.5 (c) What is the mobility of mechanism? Explain the Kutzback equation for planar mechanism and in what way is the Gruebler's criterion different from it.

[12 marks]

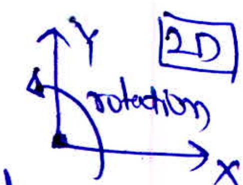
Mobility (or D.O.F): The no. of independent parameters required to define the position (or) motion of a link (mechanism) is known as Mobility.

(or)

The no. of inputs required to get a desired output in a mechanism.

For l links in a mechanism; j joints & h – higher pairs. Each link has 3 – doF in 2-D (2 translations + 1 rotation)

→ So no. of possible DOFs = $3 \times l$



Among l links; 1 link must be fixed

→ Maximum no. of possible DOFs = $3(l-1)$

Each link is connected to another link either by Lower pair (or) higher pair → Each Lower pair restricts 2 DOFs & Each Higher pair restricts 1 DOF

KutzBack equation

$$\boxed{DOF = 3(l-1) - 2j - h}$$

l – no. of links
j → No. of Lower pair
h → No. of higher pairs

For Grubler criterion

$DOF = 1$; $h = 0$

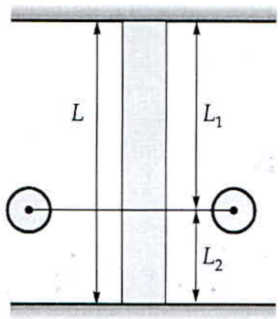
$1 = 3(l-1) - 2j - 0$

$3l - 2j = 4 \Rightarrow 3l = 2j + 4$

j → may be even (or) odd → $2j$ always even

RHS is even ⟹ so to match l must be even and in order to form a closed chain, minimum no. of links must be $\underline{4}$.

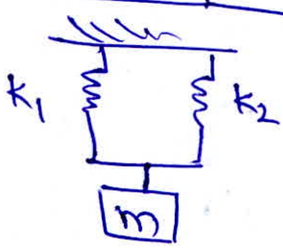
d) A flywheel is mounted on a vertical shaft as shown in figure. The ends of the shaft being fixed. The shaft is having 20 cm diameter, the length L_1 is 0.9 m and the length L_2 is 0.6 m. The flywheel weighs 500 kg and its radius of gyration is 50 cm, then find the natural frequencies of the longitudinal, the transverse and torsional vibrations of the system. $E = 200$ GPa, $G = 80$ GPa.



[12 marks]

Sol:

Longitudinal vibration



$A = \pi/4 \times 0.2^2 = \pi/100 \ m^2$

$k_1 = \frac{AE}{L_1}$ $\quad k_2 = \frac{AE}{L_2}$

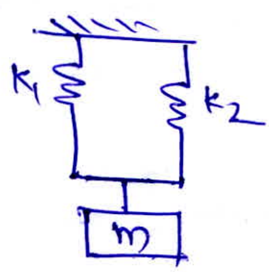
$k_1 = \frac{\frac{\pi}{100} \times 200 \times 10^9}{0.9}$ $\quad k_2 = \frac{\frac{\pi}{100} \times 200 \times 10^9}{0.6}$

$= 6.98 \times 10^9$ N/m $\quad k_2 = 1.0472 \times 10^{10}$ N/m

Combined stiffness $k_{eq} = k_1 + k_2 = 1.7452 \times 10^{10}$ N/m

$\omega_n = \sqrt{k/m} = \sqrt{\frac{1.7452 \times 10^{10}}{500}} = 5907.956$ rad/sec

Transverse Vibration



Deflection at the ~~shaft~~ flywheel

$= \frac{W a^3 b^3}{3EI L^3}$

Here $a = 0.9$ $b = 0.6$

$I = \pi/64 \times 0.2^4 = 7.854 \times 10^{-5} \ m^4$

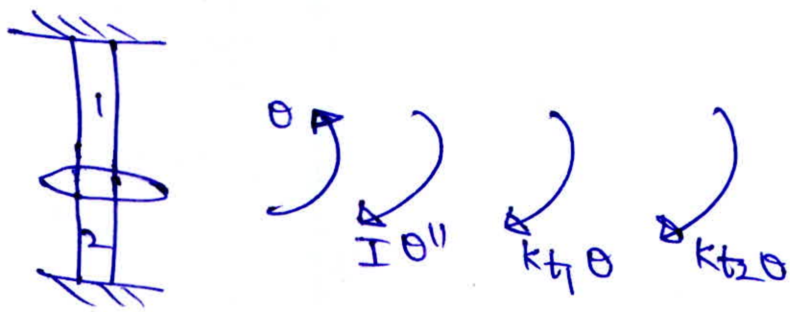
$\Delta = \frac{500 \times 9.81 \times 0.9^3 \times 0.6^3}{3 \times 200 \times 10^9 \times 7.854 \times 10^{-5} \times 1.5^3}$

$= 4.8563 \times 10^{-6}$ m

Do not write in this margin

$$\omega_n = \sqrt{g/\Delta} = \sqrt{\frac{9.81}{4.856\times10^{-6}}} = 1421.287 \ \underline{\underline{rad/s}}$$

<u>Torsional Vibration</u>



$$I\theta'' + k_{t_1}\theta + k_{t_2}\theta = 0$$

$$I\theta'' + (k_{t_1}+k_{t_2})\theta = 0$$

$$\theta'' + \left(\frac{k_{t_1}+k_{t_2}}{I}\right)\theta = 0$$

$\frac{N/m^2\times m^4}{m}$, N/m, $\frac{N\text{-}m}{kg\text{-}m^2}$

$$k_{t_1} = \frac{GJ_1}{L_1} = \frac{80\times10^9\times\frac{\pi}{32}\times0.2^4}{0.9} = 13.96\times10^6 \ \frac{N\text{-}m}{rad}$$

$$k_{t_2} = \frac{GJ_2}{L_2} = \frac{80\times10^9\times\pi/32\times0.2^4}{0.6} = 20.944\times10^6 \ \frac{N\text{-}m}{rad}$$

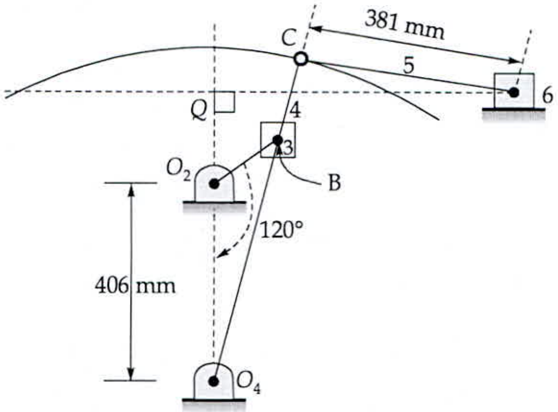
Good

12

$$\omega_n = \sqrt{\frac{k_{t_1}+k_{t_2}}{I}} = \sqrt{\frac{34.9065\times10^6}{500\times0.5^2}}$$

$$= 528.4436 \ \underline{rad/s}$$

(e) In order to design a crank-shaper mechanism as shown below, that will give a time ratio of 1.75:1 with a working stroke of 660 mm. Assumed that, point C as it moves along the arc of radius O_4C. The fixed dimensions are given in the figure and compute the required value of O_2B and O_4C. If the crank rotate at a constant speed of 40 rpm. Find the average speed of slider (in m/s) for the given working stroke and for the returning stroke.



[12 marks]

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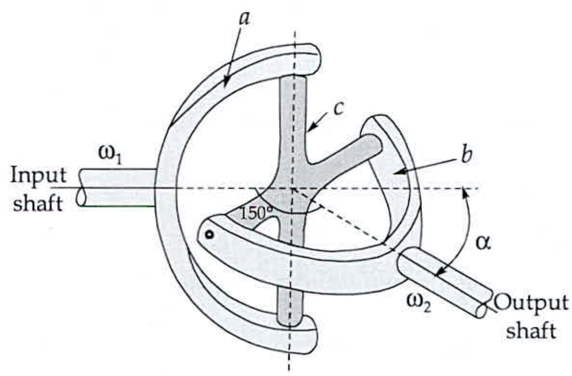
(a) A furnace is insulated with a firebrick lining of 200 mm thickness. The temperature of hot gases in the furnace is 1800 K and the temperature of the surroundings of the furnace is 300 K. The thermal conductivity of the firebricks is given by $k = k_0(1 + \beta T)$ where k_o is equal to 0.85 W/m-K and β is equal to 7×10^{-4} per K. The heat transfer coefficient on the hot and cold sides of wall is 40 W/m^2K and 10 W/m^2K respectively. Determine the temperature at inner and outer surfaces of the wall. Also find out the heat lost per unit area of the wall.

[20 marks]

b) A Hooke's joint is to connect two shafts whose axes intersect at 150°. The driving shaft rotates uniformly at 120 rpm. Deduce a general expression for the angular velocity of the driven shaft. The driven shaft operates against a steady torque of 135 Nm and carries a flywheel whose weight is 45 kg and radius of gyration 0.15 m. What is the maximum value of the torque which must be exerted by the driving shaft?



[20 marks]

Q.6 (c) Water flows through a 1.5 cm × 3.5 cm rectangular cross-section smooth tube at a velocity of 1.2 m/s. The inlet temperature of water is 40°C and tube wall is maintained at 85°C. Determine the length of tube required to raise the temperature of water to 70°C. Also find out the pumping power required if pump efficiency is 60%.

Properties of water at the mean bulk temperature of 55°C are:

$\rho = 985.5\ \text{kg/m}^3$, $c_p = 4.18\text{kJ/kgK}$, $\nu = 0.517 \times 10^{-6}\ \text{m}^2/\text{s}$, $k = 0.654\ \text{W/mK}$ and $\text{Pr} = 3.26$.

[20 marks]

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a) A punching machine punches 25 holes of 30 mm diameter and 20 mm thickness per minute. The actual punching operation is done in $\left(\frac{1}{15}\right)^{th}$ of a revolution of crank-shaft. The ultimate shear strength of the steel plate is 300 MPa. The coefficient of fluctuation of speed is 0.12. The flywheel with a maximum diameter of 1.5 m rotate at 10 times the speed of the crank shaft.

Determine the following:

(i) Power of motor assuming the mechanical efficiency to be 92%.

(ii) Cross-section of the flywheel rim if width is twice the thickness of the flywheel. Flywheel is of cast iron with a working tensile stress 6 N/mm^2 and density of 7000 kg/m^3.

Assume the hub and the spokes of the flywheel delivers 10% of the rotational inertia of the wheel.

[20 marks]

Sol:

25 holes ⟶ 1 minute

1 hole ⟶ $\frac{60}{25} = \frac{12}{5} = 2.4$ seconds

$T_c = 2.4$ seconds $\quad \omega = \frac{2\pi}{T_c} = \frac{2\pi}{2.4} \frac{rad}{sec}$

$S_{sy} = 300$ MPa $\quad d = 30$ mm ; $t = 20$ mm

$C_s = 0.12$;

Avg. Energy required for punching $= S_{sy} \times \pi \times d \times t \times \frac{t}{2}$

$T_a = \frac{1}{15} \times T_c$

$= \frac{2.4}{15}$ seconds

$= 300 \times \pi \times 30 \times 20 \times \frac{20}{2} \times 10^{-3}$

$E = 5654.866776$ J

Power of Motor $= \frac{\text{Energy required for Punching}}{\text{Cycle time}}$

$= \frac{5654.866776}{2.4}$

$= 750\pi$ W

Actual power of motor $= \frac{750\pi}{\eta_m} = 2.561$ kW

Energy fluctuation of flywheel $= P \times (T_c - T_a)$

$= \cancel{5054.867}\ 750\pi \left(2.4 - \frac{2.4}{15}\right)$

$I\omega^2 C_s = 5277.875$

$\omega_{flywheel} = 10 \times \omega_{crank}$

$= 10 \times \frac{2\pi}{2.4}$ rad/s

$I = \frac{5277.875}{\left(10 \times \frac{2\pi}{2.4}\right)^2 \times 0.12}$

$= 64.1712$ kg-m^2

$I_{rim} = 0.9 I = 57.754$ kg-m^2

$\underbrace{\rho \times (b \times t \times \pi \times D)}_{kg} \times \underbrace{R^2}_{m^2} = 57.754$ —— (1)

$\boxed{\sigma = \rho v^2} \Rightarrow \sigma = \rho \omega^2 R^2$

$R = \sqrt{\frac{\sigma}{\rho \omega^2}}$

$= 1.1182$

$D = 2R = 2.2365 > 1.5m$

$\Downarrow$

Not Possible

Take $D = 1.5m$

$\Downarrow$

$\omega = \sqrt{\frac{\sigma}{\rho R^2}}$

$= 123.44$ rad/s

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$$b = 2t$$

$$7000 \times 2t^2 \times \pi \times 2 \times 0.75^3 = 57.754$$

$$\boxed{t = 1.556 \times 10^{-3}\ m}$$

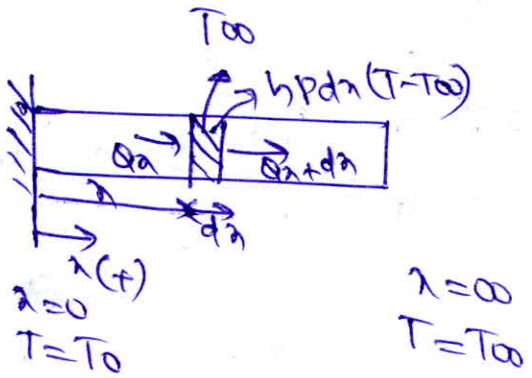
$$\boxed{b = 3.1125 \times 10^{-3}\ m}$$

11

) Derive an expression for temperature distribution in case of infinite fin.
Two long slender rods *A* and *B*, made of different materials having same diameter of 12 mm and length 1 m, are attached to a surface maintained at a temperature of 100°C. The surfaces of the rods are exposed to ambient still air at 20°C. By traversing along the length of the rods with a temperature sensor, it is found that the surface temperatures of rods *A* and *B* are equal at positions 15 cm and 7.5 cm respectively away from the base surface. If material of *A* is carbon steel with thermal conductivity 60 W/mK, what is the thermal conductivity of rod *B*? List the assumptions made. Assume that the average convection coefficient of air is 5 W/m²K. Find the ratio of the rate of heat transfer for rods *A* and *B*.

[20 marks]

Sol:



Temp of element (dx) = T

Energy balance for an element located at a distance x from base

$$Q_x = Q_{x+dx} + hPdx(T-T_\infty)$$

$$\cancel{Q_x = Q_x +} \frac{\partial Q_x}{\partial x}.dx + hPdx(T-T_\infty)$$

$Q_x = -KA_c\, dT/dx$

$\frac{\partial Q_x}{\partial x} = -KA_c\, \partial^2 T/\partial x^2$ ($\because$ K & A_c are constant)

& T is function of ~~temp~~ x only

$\frac{\partial^2 T}{\partial x^2} = d^2T/dx^2$

$KA_c\, \frac{d^2T}{dx^2}\, dx = hp\,dx\,(T-T_\infty)$

Let $T - T_\infty = \theta \Rightarrow dT/dx = d\theta/dx$

$KA_c \frac{d^2\theta}{dx^2} \cancel{dx} = hp \cancel{dx}\, \theta$

$\frac{d^2\theta}{dx^2} - \frac{hp}{KA_c}\theta = 0$

Let $m = \sqrt{\frac{hp}{KA_c}}$ $\quad \frac{d^2\theta}{dx^2} - m^2\theta = 0 \Rightarrow D^2 - m^2 = 0$

$D = \pm m$

Solution to the above eqn is of the form

$\theta = c_1 e^{mx} + c_2 e^{-mx}$

Where c_1 & c_2 are constants to be determined by boundary conditions

For <u>infinite Long fin</u>

at $x = 0$; $\theta = T_0 - T_\infty = \theta_0$

at $x \to \infty$; $\theta = 0$

$\theta_0 = c_1 e^0 + c_2 e^0 \Rightarrow \boxed{c_1 + c_2 = \theta_0}$ — (1)

$0 = c_1 e^{\infty} + \cancel{c_2 e^{-\infty}}^{\,0}$

c_2 $\quad c_1 e^{\infty} = 0$

$\boxed{c_1 = 0}$

$$\theta = \theta_0 e^{-mx}$$

$$\boxed{\theta/\theta_0 = e^{-mx}}$$

A

$d = 0.012m$
$L = 1m$
$T_0 = 100°c$
$\theta_0 = T_0 - T_\infty = \overset{100}{} - 20° = 80$
$K_A = 60 \ W/m\text{-}k$

$x_1 = 0.15 \ m \Rightarrow T$
$\theta_1 = \theta$

$$\frac{\theta_1}{\theta_0} = e^{-m_1 x_1} \quad \text{—— (2)}$$

(2) = (3)

B

$d = 0.012m$
$L = 1m$
$T_0 = 100°c$
$\theta_0 = 80°c$

$x_2 = 0.075 \Rightarrow T$
$\theta_1 = \theta$

$$\frac{\theta_1}{\theta_0} = e^{-m_2 x_2} \quad \text{—— (3)}$$

$$m_1 x_1 = m_2 x_2$$

$$m_1 \times 0.\cancel{15}^{2} = m_2 \times 0.\cancel{075}$$

$$2m_1 = m_2$$

$$4 \times \left(\frac{h_1 P_1}{K_1 A_c}\right) = \frac{hP}{K_2 A_c}$$

$$K_2 = K_1/4 = \frac{60}{4} = 15 \ W/m\text{-}k$$

Assumption:
1) Radiation effects neglected
2) Steady state
3) Fin is Infinitely Long

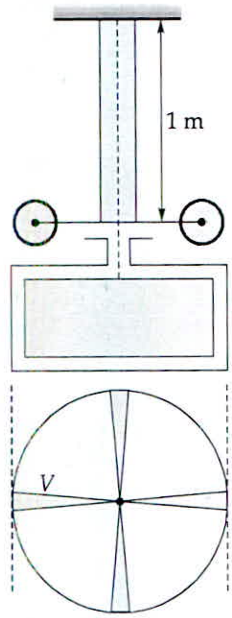
$$Q_A = \sqrt{hPK_1A_c}\ \theta_0 = \sqrt{5 \times \frac{\pi^2}{4} \times 0.012^3 \times 60} \times 80$$

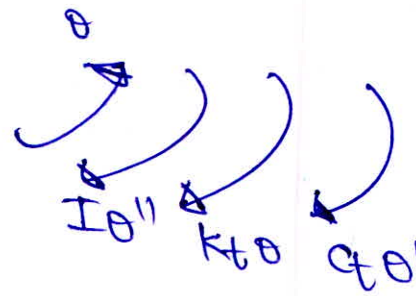
$$Q_B = \sqrt{hPK_2A_c}\ \theta_0 = \sqrt{5 \times \frac{\pi^2}{4} \times 0.012^3 \times 15} \times 80$$

$$\frac{Q_A}{Q_B} = \sqrt{4} = 2$$

8+01 = 19

Q.7(c) A flywheel of moment of inertia 25 kg.m^2 is fixed to one end of a vertical shaft diameter 2.54 cm and the length 1 m. The other end of the shaft is fixed. The torsional oscillations of the flywheel are damped by means of a vane as shown in figure, which moves in a dashpot D filled with oil. The amplitude of oscillations is found by experiment to diminish to $\left(\frac{1}{20}\right)^{th}$ of its initial value in three complete oscillations. Assuming the damping torque to be directly proportional to the angular velocity, find its magnitude at a speed of 1 rad/s. The modulus of rigidity of the shaft material is 85 GPa and compare later with the frequency of the free vibrations.





[20 marks]

Sol:

$I = 25\ kg\text{-}m^2$ $\quad d = 0.0254\ m$

$L = 1\ m$

$k_t = \frac{GJ}{L} = \frac{85\times10^9 \times \frac{\pi}{32}\times 0.0254^4}{1}$

$= \cancel{5.3838\times10^6}$ 3473.39 $\frac{N\text{-}m}{rad}$

$x_3 = \frac{1}{20}x_0$

$\frac{x_3}{x_0} = \frac{1}{20} \Rightarrow \frac{x_3}{x_2}\times\frac{x_2}{x_1}\times\frac{x_1}{x_0} = \frac{1}{20}$

$e^{-\delta}\times e^{-\delta}\times e^{-\delta} = \frac{1}{20}$

$3\delta = \ln(20)$

$\frac{3\times2\pi\xi}{\sqrt{1-\xi^2}} = \ln(20)$

$\xi = 0.15695$

$I\theta'' + c_t\theta' + k_t\theta = 0$

$\theta'' + \frac{c_t}{I}\theta' + \frac{k_t}{I}\theta = 0$

$\omega_n = \sqrt{\frac{k_t}{I}} = \sqrt{\frac{\cancel{\quad}\,c3473.39}{25}} = \cancel{\quad .06}\ 11.787$ rad/s

$\xi = \frac{c_t}{c_c} = 0.15695$

$c_t = 0.15695 \times 2 \times \sqrt{k_t \times I}$

$= \cancel{3641.7135}$ $\cancel{N\text{-}s}$ $\cancel{N\text{-}m\text{-}s^2/rad}$

92.499 N-m-s/rad

$\frac{N\text{-}m}{rad} \times \frac{kg\text{-}m^2}{s^2} \times s^2$

$\frac{kg^2\text{-}m^4}{s^2}$ $\frac{N^2\text{-}m^2\text{-}s^2}{rad}$

$\frac{kg\text{-}m^2}{s} \times s$

$T = \frac{\omega}{s^2}$

N-m-s²

$T = c_t \times \omega \Big|_{at\ \omega = 1\ rad/s}$

$\cancel{= 3641.7135 \times 1}$

$\boxed{T = 92.5\ N\text{-}m}$

$\cancel{= 3641.7135\ N\text{-}m}$

14

$T = c_t \times \omega_n \Big|\ \omega_n = \cancel{454.06}\ 11.787$ rad/s

$= \cancel{1.689\ MN\text{-}m}$

$\boxed{T = 1090.288\ N\text{-}m}$

Damping torque at 1 rad/s is asked.

$\omega_d = ?$

Q.8 (a) Two moles of an ideal gas at temperature *T* and pressure *P* are contained in a compartment. In an adjacent compartment one mole of an ideal gas is at temperature 2T and pressure *P*. The gases mix adiabatically but do not react chemically when a partition separating the compartments is withdrawn. Show that the entropy increase due to the mixing process is given by:

$$\bar{R}\left(\ln\frac{27}{4}+\frac{\gamma}{\gamma-1}\ln\frac{32}{27}\right)$$ where, $\bar{R}$ - Universal gas constant

provided that the gases are different and that the ratio of specific heat γ is the same for both gases and remains constant.

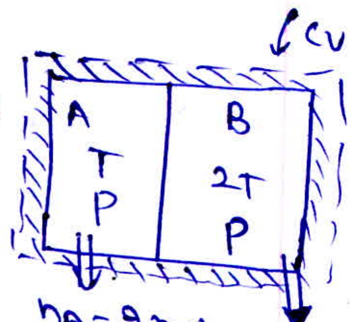
[20 marks]

Sol:

For I^{st} law of thermodynamics

$\cancel{\delta Q}^{\to 0} = dU + \cancel{\delta W}^{\to 0}$

(adiabatic) (No work)



$n_A = 2$ moles $n_B = 1$ mole

$dU_A + dU_B = 0$

$m_A c_{VA}(T_f - T) + m_B c_{VB}(T_f - 2T) = 0 \longrightarrow (1)$

$m_A c_{VA} = \frac{m_A R_A}{\gamma-1} = \frac{n_A \bar{R}}{\gamma-1}$

$\boxed{m_A R_A = n_A \bar{R}}$

$m_B c_{VB} = \frac{m_B R_B}{\gamma-1} = \frac{n_B \bar{R}}{\gamma-1}$

Substitute in (1)

$n_A(T_f - T) + n_B(T_f - 2T) = 0$

$2(T_f - T) + 1(T_f - 2T) = 0$

$2T_f - 2T + T_f - 2T = 0$

$3T_f = 4T \longrightarrow \boxed{T_f = \frac{4}{3}T}$

$V_A = \frac{n_A \bar{R} T_A}{P_A} = \frac{2\bar{R}T}{P}$ Let $\frac{\bar{R}T}{P} = x$

$= 2x$

$V_B = \frac{n_B \bar{R} T_B}{P_B} = \frac{1 \times \bar{R} \times (2T)}{P} = 2x$

$V_A + V_B = 4a$

Entropy change of fluid (gas) in A

$$(\Delta S)_A = m C_{vA} \ln\left(\frac{T_2}{T_A}\right) + m_A R_A \ln\left(\frac{V_A+V_B}{V_A}\right)$$

$T_2 = T_f$;

$$= m_A R_A \left[\frac{1}{\gamma-1} \ln\left(\frac{4/3\,T}{T}\right) + \ln\left(\frac{4a}{2a}\right)\right]$$

$$= m_A R_A \left[\frac{1}{\gamma-1} \ln(4/3) + \ln 2\right] \longrightarrow (1)$$

$$(\Delta S)_B = m_B C_{vB} \ln\left(\frac{T_2}{T_B}\right) + m_B R_B \ln\left(\frac{V_A+V_B}{V_B}\right)$$

$$= m_B R_B \left[\frac{1}{\gamma-1} \ln\left(\frac{4/3\,T}{2T}\right) + \ln 2\right] \longrightarrow (2)$$

Entropy change $= (\Delta S)_A + (\Delta S)_B$

$$= n_A \bar{R} \left[\frac{1}{\gamma-1} \ln(4/3) + \ln 2\right] + n_B \bar{R} \left[\frac{1}{\gamma-1} \ln(2/3) + \ln 2\right]$$

13

$$= \bar{R}\left[\frac{2}{\gamma-1} \ln(4/3) + 2\ln 2 + \frac{1}{\gamma-1} \ln(2/3) + \ln 2\right]$$

$$= \bar{R}\left[\frac{1}{\gamma-1}\left(\ln\left((4/3)^2 \times 2/3\right) + \ln(2^2 \times 2)\right)\right]$$

$$= \bar{R}\left[\frac{1}{\gamma-1} \ln\left(\frac{32}{27}\right) + \ln(8)\right]$$

$\rightarrow \left[\frac{\gamma}{\gamma-1} - \left(\frac{\gamma-1}{\gamma-1}\right)\right]$ and rearrange

Incomplete

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A steam turbine receives 600 kg/h of steam at 25 bar and 350°C. At a certain stage of the turbine, steam at the rate of 150 kg/h is extracted at 3 bar and 200°C. The remaining steam leaves the turbine at 0.2 bar and 0.92 dry. During the expansion process, there is heat transfer from the turbine to the surrounding at the rate of 10 kW. Evaluate per kg of steam entering the turbine:

(i) the energy of steam entering and leaving the turbine,
(ii) the maximum work,
(iii) the irreversibility

The atmosphere is at 30°C.

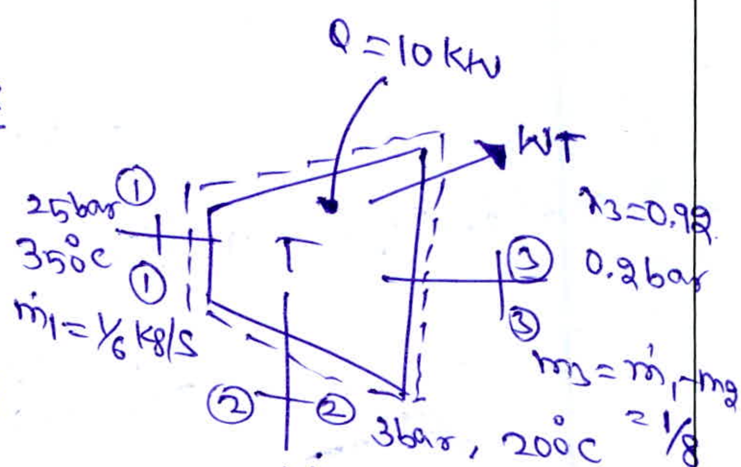
Data given:

At 25 bar and 350°C, $h_1 = 3125.87$ kJ/kg; $s_1 = 6.8481$ kJ/kgK
At 30°C, $h_0 = 125.79$ kJ/kg; $s_0 = s_{f30°C} = 0.4369$ kJ/kgK
At 3 bar and 200°C, $h_2 = 2865.5$ kJ/kg; $s_2 = 7.3115$ kJ/kgK
At 0.2 bar (0.92 dry), $h_f = 251.4$ kJ/kg; $h_{fg} = 2358.3$ kJ/kg
$s_f = 0.8320$ kJ/kgK; $s_g = 7.9085$ kJ/kgK

[20 marks]

Sol:

$\dot{m}_{steam} = \frac{1}{6}$ kg/s



Energy of Steam entering turbine $= h_1 = 3125.87$ kJ/kg

Energy of steam leaving turbine $h_3 = h_{f3} + x_3(\cancel{h_{g3} - h_{f3}})$

$\dot{m}_2 = \frac{1}{24}$ kg/s

$= 251.4 + 0.92 \times (2358.3)$

$= 2421.036$ kJ/kg

$h_2 = 2885.5$ kJ/kg

Applying S.F.E.E

$$\dot{m}_1 h_1 + Q = \dot{m}_2 h_2 + \dot{m}_3 h_3 + W_T$$

$$W_T = \frac{1}{6} \times 3125 + 10 - \frac{1}{24} \times 2885.5 - \frac{1}{8} \times 2421.036$$

$$= 107.9746 \text{ kW}$$

Maximum work:

Availability at 1 $= \phi_1 = (h_1 - h_0 - T_0(s_1 - s_0))\dot{m}_1$

$= 3125.87 - 303 \times 6.8481 - 125.? + 303 \times 0.4?$

$= (1057.8957) \times \frac{1}{6} = 149 kW$

$= 176.247 kW$

Availability at 2 $= \phi_2 = \dot{m}_2(h_2 - T_0 s_2) - \dot{m}_2(h_0 - T_0 s_0)$

$= \frac{1}{24} \times ((2885.5 - 125.79) - 303 \times (7.3115 - 0.4369))$

$= 28.1961 kW$

Availability at 3 $= \phi_3 = \dot{m}_3 (h_3 - h_0 - T_0(s_3 - s_0))$

$= \frac{1}{8} \times [2421.036 - 125.79 - 303 \times (0.8320 + 0.99 \times (7.9085 - 0.8320) - 0.4369)]$

$= 25.360695 kW$

$\phi_1 = \phi_2 + \phi_3 + W_{T(max)}$

$\boxed{W_{T(max)} = 122.69 kW}$

Irreversibility $= W_{T(max)} - W_{T(actual)}$

$= 14.7156 kW$

08

An air refrigerator working on Bell-Coleman cycle takes the air into the compressor at 1 bar and –7°C and it is compressed isentropically to 5.5 bar and it is further cooled to 18°C at the same pressure. Find the COP of the system if:

(i) the expansion is isentropic

(ii) the expansion follows the law $PV^{1.25}$ = constant.

Take $\gamma = 1.4$ and $c_p = 1$ kJ/kgK for air.

[20 marks]

Sol:

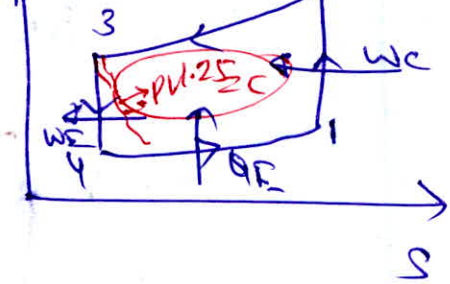
Assumption: Air is ideal gas $PV = mRT$ is valid

$T_1 = 266K$

$P_1 = 1 \text{ bar} = P_4$

$P_2 = 5.5 \text{ bar}$

$T_3 = 291K$



For process 1-2

$$\frac{T_2}{T_1} = \left(P_2/P_1\right)^{\gamma-1/\gamma} \Rightarrow T_2 = 432.926 K$$

Expansion is isentropic

$$COP = \frac{1}{r_p^{\gamma-1/\gamma} - 1} = \frac{1}{(5.5)^{0.4/1.4} - 1} = 1.59$$

35

Expansion follows $PV^{1.25} = C$

$$\boxed{n = 1.25}$$

$$W_E = \frac{n}{n-1}\left[P_3V_3 - P_4V_4\right] = \frac{n}{n-1}\left[mRT_3 - mRT_4\right]$$

$$= \frac{n}{n-1}\, \dot{m}RT_3\left[1 - T_4/T_3\right]$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$R = C_p\left(\frac{\gamma-1}{\gamma}\right) = 1 \times \left(\frac{0.4}{1.4}\right)$$

$$= 2/7 \ \frac{kJ}{kg\text{-}K}$$

$$\frac{T_3}{T_4} = \left(P_3/P_4\right)^{n-1/n}$$

$$= 5.5^{\frac{0.25}{1.25}}$$

$$= 1.40628$$

$$W_E = \frac{1.25}{0.25} \times \frac{2}{7} \times 291 \times \left[1 - \frac{1}{1.40628}\right]$$

$$= 120.1026 \ \frac{kJ}{kg}$$

$$W_C = \frac{\gamma}{\gamma-1}\left[P_1V_1 - P_2V_2\right]$$

$$\cancel{Tds}^{0} = dH - Vdp$$

$$dH = Vdp$$

$$= \frac{\gamma}{\gamma-1}\, mRT_1\left[1 - \frac{T_2}{T_1}\right]$$

$$= \frac{1.4}{0.4} \times \frac{2}{7} \times 266 \times \left[1 - (5.5)^{\frac{0.4}{1.4}}\right]$$

$$= -166.926 \ kJ/kg$$

(-ve means input)

$$Q_E = C_p(T_1 - T_4)$$

$$= 1 \times \left(266 - \frac{291}{1.40623}\right)$$

$$= 59.071 \text{ KJ/kg}$$

$$COP = \frac{\text{Desired effect}}{\text{Work input}} = \frac{59.071}{W_c - W_E}$$

$$= \frac{59.071}{166.926 - 120.1026}$$

$$= 1.26157$$

19

Space for Rough Work

Space for Rough Work

$$P_f (V_A + V_B) = T_f \times n_A \times \bar{R}$$

$$2 \times 4/3\, T \times R$$

$$: 4 \times \frac{RT}{P} \quad \boxed{2/3\, P}$$

$$\frac{d^2T}{dr^2} + \frac{q'}{k} = \cancel{0}\ 0$$

$$-\frac{q}{k}\frac{r^3}{3} + \frac{C_1}{r}$$

$$\frac{d}{dr}\left(r\, dT/dr\right)$$

$$-q$$

$$\frac{P_a}{RT_a} =$$

$$w = \frac{m_v}{m_a} = \frac{\frac{P_v}{RT_v}}{\frac{P_a}{R_a}} = w$$

$$\frac{P}{\rho} = RT$$

$$\frac{P}{RT} = \rho$$

$$\frac{P_v}{RT_v} = w\left(\frac{P_a}{R_aT_a}\right)$$

$$\rho_v =$$