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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-3: Strength of Materials

Transportation Engineering-1 + Surveying & Geology-1

Geo-technical & Foundation Engg-2 + Environmental Engg-2

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Student's Signature

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ndwriting

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	49 − 6 = 43
Q.2	
Q.3	60 − 6 = 54
Q.4	58
Section-B	
Q.5	46
Q.6	38 − 2 = 36
Q.7	
Q.8	
Total Marks Obtained	~~251~~ 237

Signature of Evaluator: Shweta

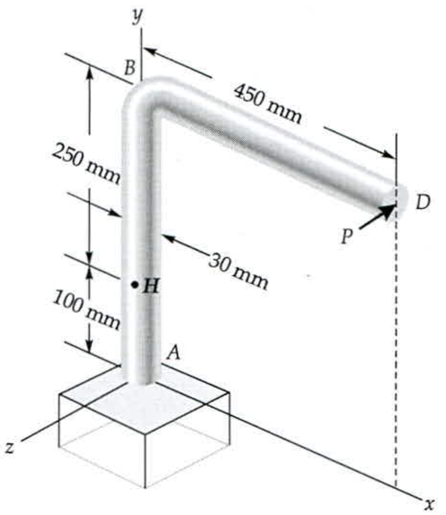
Cross Checked by

Section A : Strength of Materials

(a) A single horizontal force P of magnitude 600 N is applied to end D of lever ABD. The diameter of lever ABD is 30 mm. Determine :

(i) The normal and shearing stress on an element located at point H having sides parallel to x and y axis.

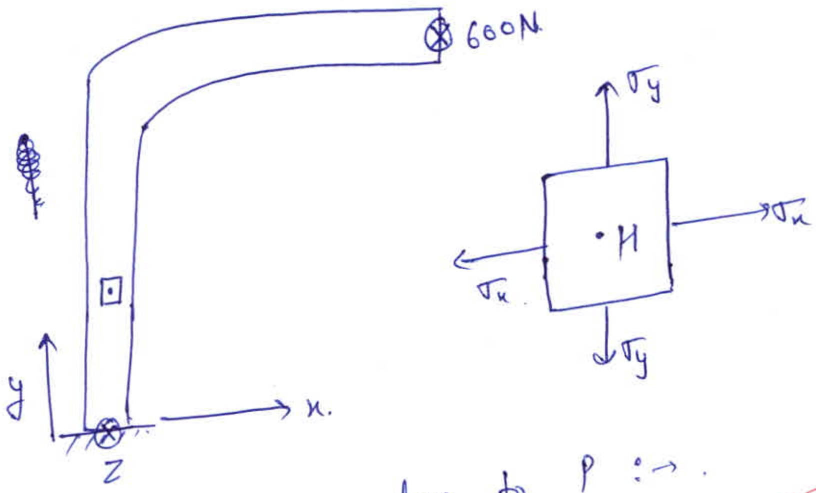
(ii) The directions of principal planes and principal stresses at point H.



[12 marks]

Assuming point H to be at surface.

(i)



Bending stresses due to P :→.

BM = 600 N × 250 mm.

$$\sigma_y = \frac{600 \times 250}{\frac{\pi}{64} \times 30^4} \times \frac{30}{2} \Rightarrow \cancel{3.56}\ 3.537\ \text{MPa} \times 16$$

$\boxed{56.592\ \text{MPa}}$

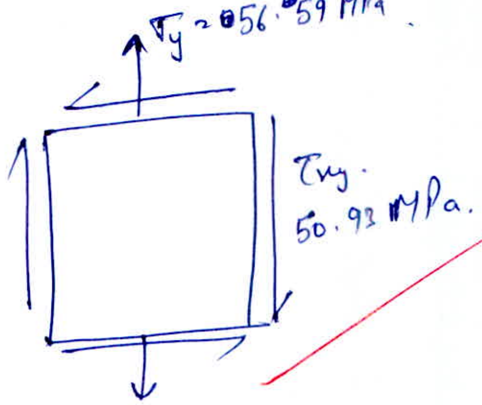
Tensile

Due to Direct shear stress →

No shear is experienced because Point 'H' is at surface.

Due to Torsion Produced.

$$\tau_{xy} = \frac{16T}{\pi D^3} = \frac{16 \times 600 \times 450}{\pi \times 30^3} = 50.93 \text{ MPa}$$



(ii) Principle stress :→

$$\sigma_{P_1}/\sigma_{P_2} = \frac{56.59}{2} \pm \sqrt{\left(\frac{56.59}{2}\right)^2 + 50.93^2}$$

$$\Rightarrow \frac{56.59}{2} \pm 58.262.$$

$\sigma_{P_1} = 86.56$ MPa.

$\sigma_{P_2} = -29.97$ MPa.

Direction ⇒ $\tan 2\theta_p = \dfrac{2 \times (-50.93)}{0 - 56.59}$

$\theta_{P_1} \Rightarrow 30.47°$

$\theta_{P_2} \Rightarrow -59.52°$

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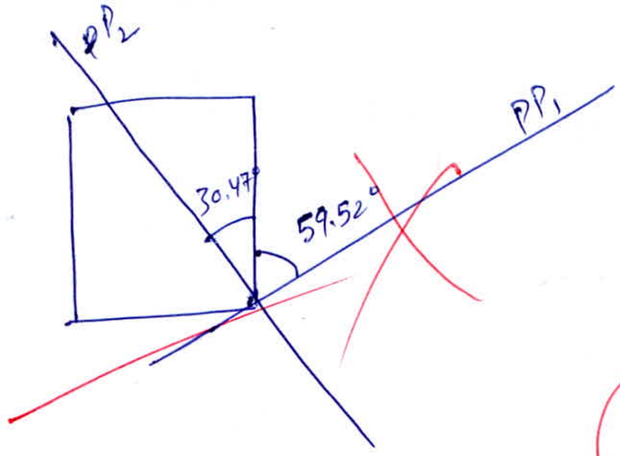

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(b) A steel specimen is subjected to the following principal stresses: (i) 120 N/mm² tensile (ii) 60 N/mm² tensile and (iii) 30 N/mm² compressive. The proportionality limit for the steel specimen is 250 N/mm². Find the factor of safety according to

(i) Maximum shear stress theory. *(ii)* Maximum principal strain theory.

(iii) Maximum strain energy theory.

Take Poisson's ratio = 0.3

[12 marks]

(i) Acc. to max shear stress theory.

$$(\tau_{max})_{abs.} = \frac{\sigma_y}{2 \times FOS}$$

$\sigma_{p_1} = 120$

$\sigma_{p_2} = 60$

$\sigma_{p_3} = -30.$

$$\frac{120 - 30}{2} = \frac{250}{2 \times FOS}$$

$$\boxed{FOS = 2.78}$$

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(ii) According to max principle strain theory.

$$\sigma_{P_1} - \mu\sigma_{P_2} - \mu\sigma_{P_3} = \frac{F_y}{FOS}$$

$$120 - 0.3\times60 + 0.3\times30 = \frac{250}{FOS.} \quad -(i)$$

$$-30 - 0.3\times60 - 0.3\times120 = -\frac{250}{FOS} \quad -(ii)$$

$$\left.\begin{array}{l}(FOS)_{i} = 2.252.\\ (FOS)_{ii} = 2.976.\end{array}\right\} \min = \boxed{2.252}$$

(iii) Maximum strain energy theory:-

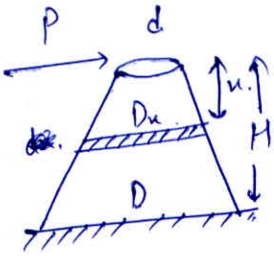
$$\frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)\right] = \frac{1}{2E}\left(\frac{\sigma_y}{FOS}\right)^2$$

$$120^2+60^2+30^2-2\times0.3\left(120\times60+60\times(-30)+120\times(-30)\right) = \left(\frac{250}{FOS}\right)^2$$

$$\boxed{FOS = 1.873}$$

(c) A uniformly tapering vertical post of height *H* having a diameter D at the base and a diameter *d* at the top is fixed at its base. A horizontal force P is applied at the top of the post. Determine the maximum bending stress for the post and state where it occurs.

[12 marks]



$$M_x = P.x.$$

$$D_x = d + \frac{(D-d)}{H}.x.$$

Assume $\frac{D-d}{H} = K.$

$$D_x = d + Kx.$$

$$\sigma_x = \frac{M_x}{I_x}.y_x. \Rightarrow \frac{P.x}{\frac{\pi}{64}\times D_x^4}\times\frac{D_x}{2} \Rightarrow \frac{32(P.x)}{\pi D_x^3}$$

$$\sigma_x = \frac{32Px}{\pi(d+Kx)^3}$$

For max $\frac{d\sigma_x}{dx} = 0.$ $\quad \frac{32P}{\pi}\frac{d}{dx}\left[\frac{x}{(d+Kx)^3}\right] = 0.$

$$\frac{32P}{\pi}\left[\frac{(d+Kx)^3 + 3(d+Kx)^2.K.x}{(d+Kx)^6}\right] = 0.$$

$$(d+Kx)^3 = 3(d+Kx)^2Kx.$$

$$d+Kx = 3Kx.$$

$$2Kx = d.$$

$$x = \frac{d}{2K} \Rightarrow \frac{dH}{2[D-d]}$$

location $\boxed{\frac{dH}{2(D-d)}}$ from top.

$$\text{Stress} = \frac{32P}{\pi}\cdot\frac{d}{2K}\times\frac{1}{\left(d+K.\frac{d}{2K}\right)^3}$$

$$\Rightarrow \frac{32P}{\pi}\times\frac{d}{2K}\times\frac{8}{3^3\times d^3} \Rightarrow \boxed{\sigma_{max} = \frac{128}{27\pi}\cdot\frac{P\times H.}{d^2\times(D-d)}}$$

12

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(d) A 10 mm diameter mild steel bar of length 1.50 metre is stressed by a weight of 120 N dropping freely through 20 mm before commencing to stretch the bar. Find the maximum instantaneous stress and the elongation produced in the bar.

[Take $E = 2 \times 10^5$ N/mm^2]

[12 marks]

Ans

Acc. to given condition :-

$$mg\,(h + \delta_{max}) = \frac{\sigma_{max}^2}{2E} \times 1.5 \times \frac{\pi}{4} \times 10^2 \times 1000$$

$$120\,\left(20 + \frac{\sigma_{max}}{E} \cdot L\right) = \frac{\sigma_{max}^2}{2E} \times \frac{1.5\pi}{4} \times 10^5$$

$$120\,\left(20 + \frac{\sigma_{max}}{2 \times 10^5} \cdot 1500\right) = \frac{\sigma_{max}^2}{2 \times 2 \times 10^5} \times \frac{1.5\pi}{4} \times 10^5$$

$$\sigma_{max} = 91.81 \text{ MPa}$$

Instantaneous elongation in the bar.

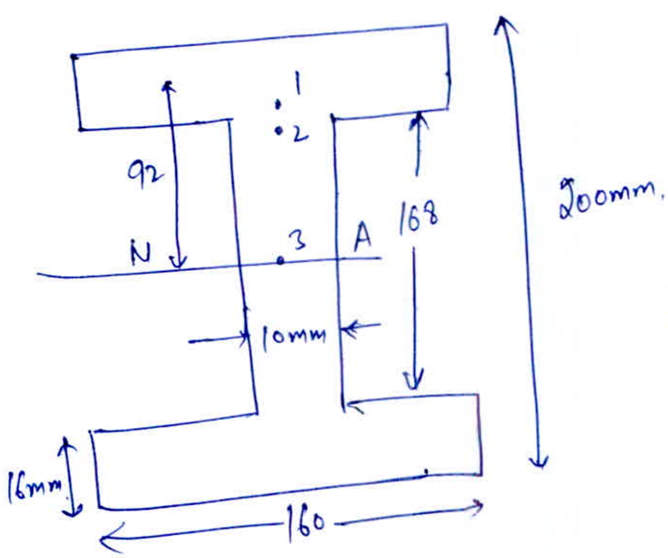
$$\Rightarrow \frac{91.81}{2 \times 10^5} \times 1500 = \boxed{0.688 \text{ mm}}$$

12

Q.1 (e) A steel beam of I-section, 200 mm deep and 160 mm wide has 16 mm thick flanges and 10 mm thick web. The beam is subjected to a shear force of 200 kN. Draw the shear stress distribution, if the web of the beam is kept horizontal.

[12 marks]

Ans



V = 200kN.

$$I = \frac{160 \times 200^3}{12} - \frac{150 \times 168^3}{12}$$

$$= 47396266.67\ mm^4$$

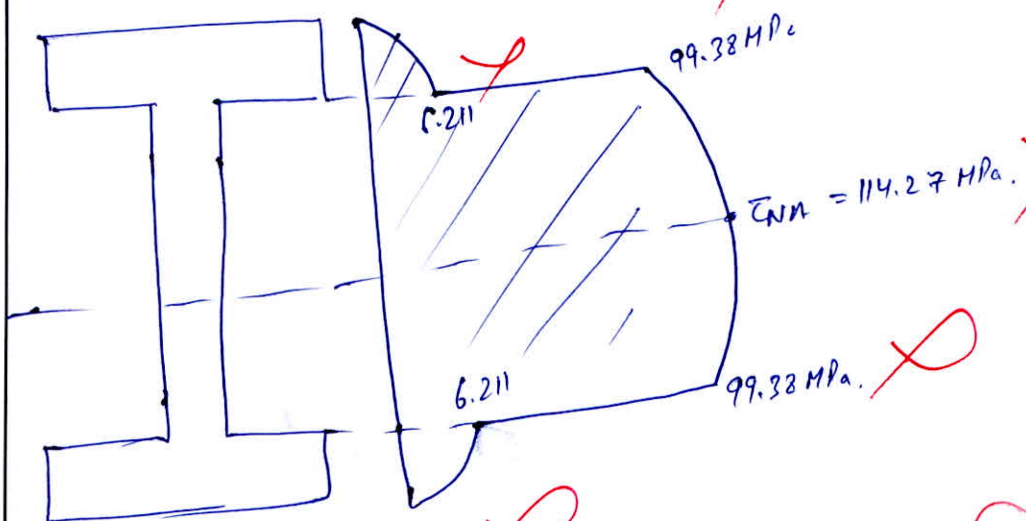
$$\tau_1 = \frac{200\times10^3 \times 160\times16\times92}{47396266.67\times160} = 6.211\ MPa.$$

$$\tau_2 = \tau_1 \times \frac{160}{10} = 99.38\ MPa.$$

$$\tau_{NA} = \frac{200\times10^3\left[160\times16\times92 + 84\times10\times42\right]}{I\times10}$$

$$\Rightarrow 114.27\ MPa.$$

Final Shear distribution.



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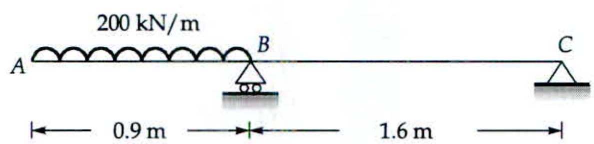
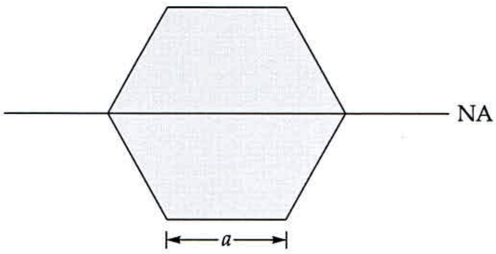
Q.2 (a) For the beam shown below, find the deflection at end A, using moment area method. [Take EI = constant]



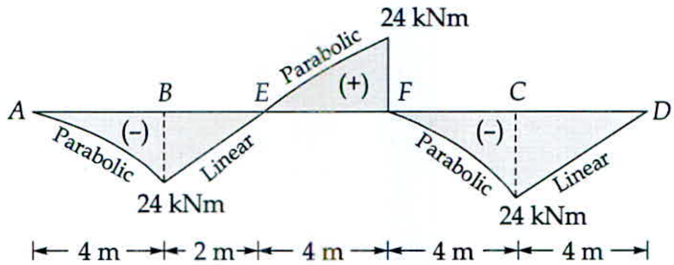
[20 marks]

2 (b) A beam section is a regular hexagon of side 'a' and is placed so that one diagonal is horizontal as shown below. If the beam section is subjected to a shear force S, obtain an expression for the shearing stress at any distance x from the horizontal diagonal.



[20 marks]

Q.2 (c) A beam ABCD is supported at B and C and has over-hangs AB and CD. Its bending moment diagram is shown below. Determine the loading diagram and the shear force diagram of the beam.



[20 marks]

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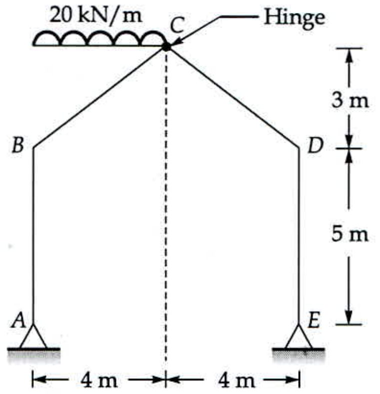
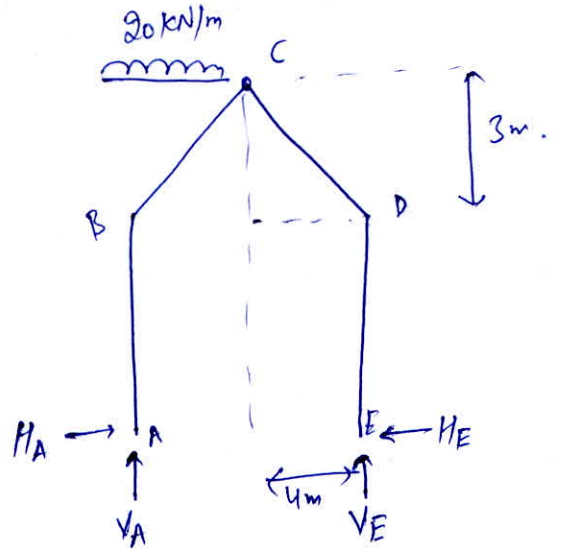
3 (a) Draw the bending moment diagram for the frame shown below.



[20 marks]

Ans



* $\Sigma M_A = 0$.

$V_E \times 8 = 20 \times 4 \times 2$

$V_E = 20 kN$

* $V_A + V_E = 20 \times 4$

$V_A = 80 - 20 = 60 kN$.

* $\Sigma M_C = 0$.

$H_E \times 8 = V_E \times 4$.

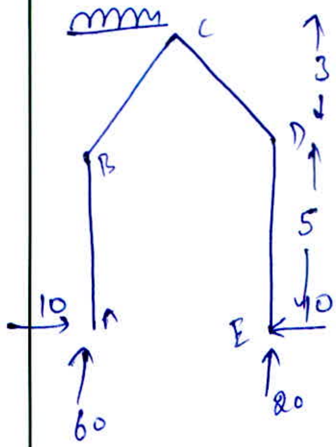
$H_E = \frac{4}{8} \times V_E$

$= \frac{1}{2} \times 20$

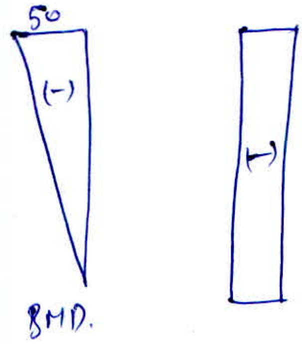
$H_E = 10$

* $\Sigma F_x = 0$.

$H_A = H_E = 10 kN$.

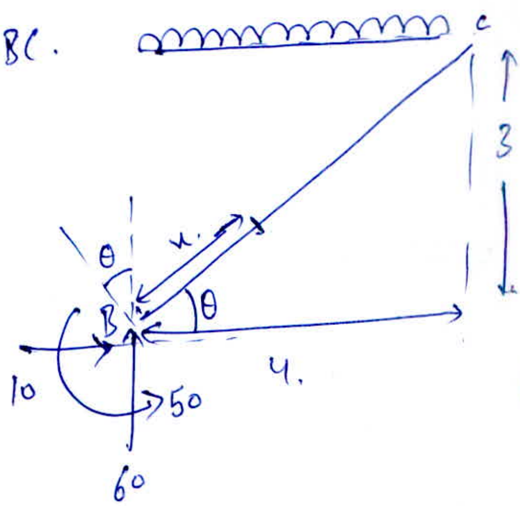


AB ⇒



$10 \times x = M_x.$ $\quad V_x + 10 = 0$

BC.

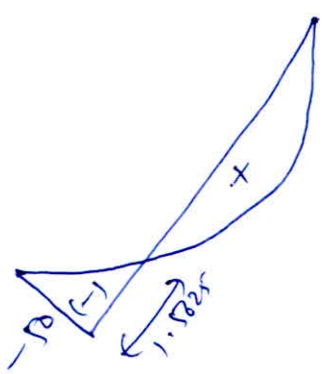


$M_x = 60 \times \cos\theta \times x - 10 \times \sin\theta \times x$
$- 20 \times x \cos\theta \times \frac{x \cos\theta}{2}$
$- 50$

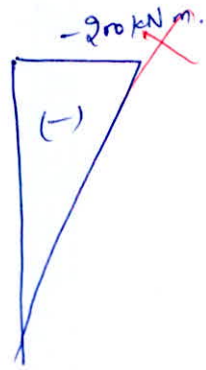
$M_x = 60 \times \frac{4}{5} \times x - 10 \times \frac{3}{5} \times x$
$- 20 \frac{\times x^2}{2} \times \frac{4^2}{5^2} - 50$

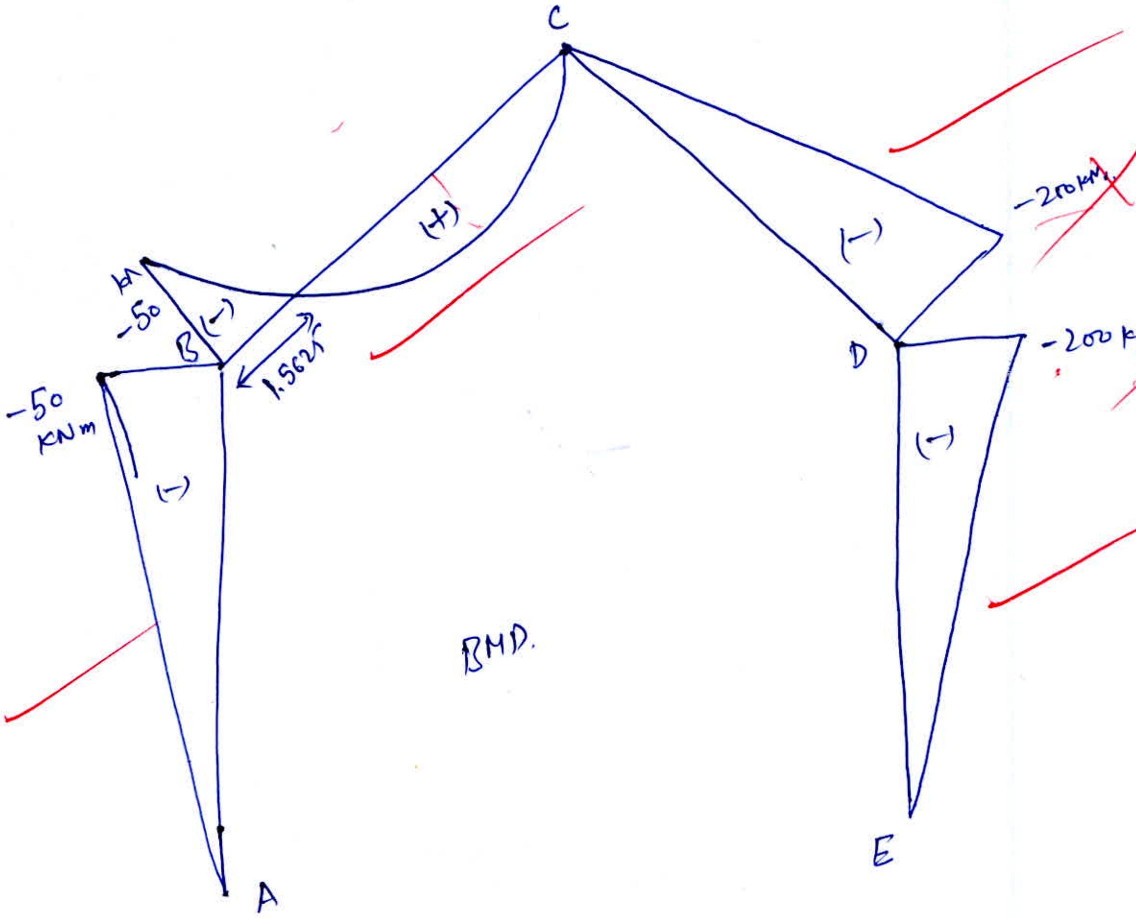
$\Rightarrow 48x - 6x - 6.4x^2 - 50$

At $x = 0$, $M_x = -50$. $\quad M_x = 0$ at $x = 5$
At $x = 5$, $M_x = 0$. $\quad x = 1.5625$



ED →





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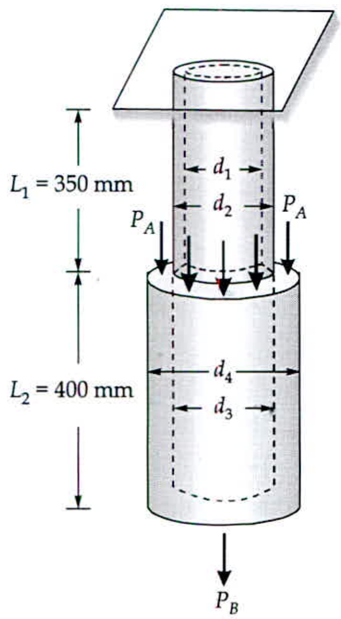
Q.3 (b) A hollow circular nylon pipe as shown in figure supports a load P_A = 7800 N, which is uniformly distributed around a cap plate at the top of lower pipe. A second load P_B is applied at the bottom. The inner and outer diameters of the upper and lower parts of the pipe are d_1 = 51 mm, d_2 = 60 mm, d_3 = 57 mm and d_4 = 63 mm respectively. The upper pipe has a length L_1 = 350 mm and lower pipe has a length L_2 = 400 mm. Neglect the self weight of the pipes.



(i) Find P_B so that the tensile stress in the upper pipe is 14.5 MPa. Also determine the resulting stress in lower pipe?

(ii) If P_A remains unchanged, find the new value of P_B so that upper and lower pipes have same tensile stress.

(iii) Find the tensile strains in the upper and lower pipe segments for the loads in part (ii) if the elongation of the upper pipe is 3.56 mm and downward displacement of bottom pipe is 7.63 mm?

[20 marks]

Ans

(i) Stress in upper pipe = 14.5 MPa.

$$\frac{(P_A + P_B) \times L_1}{A_1 E_n} = 14.5 \text{ MPa.}$$

$$\frac{(7800 + P_B) \times 350}{\frac{\pi}{4}(60^2 - 51^2) \times E_n} = 14.5 \text{ MPa.}$$

$$P_B = (32.5 E_n - 7800) \text{ KN.}$$

(ii) Resulting stress in lower pipe :-

32.5 E_n

(ii) -

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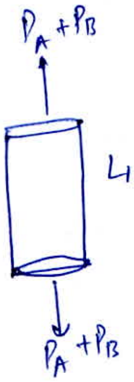
(i)

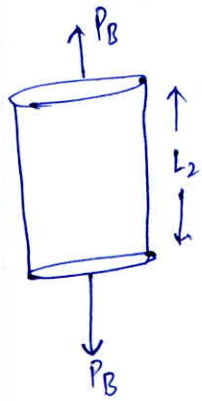
$$\frac{P_A + P_B}{A} = 14.5 \text{ MPa.}$$



$$\frac{7800 + P_B}{\frac{\pi}{4}(60^2 - 51^2)} = 14.5$$

$$\boxed{P_B = 3576.88 \text{ N.}}$$

Resultant stress on lower plate :→



$$\frac{P_B}{A_2} = \frac{3576.88}{\frac{\pi}{4}(63^2 - 57^2)} = 6.325 \text{ MPa.}$$

$$\boxed{\sigma_B = 6.325 \text{ MPa}}$$

(ii) for same tensile stress :→

$$\frac{P_A + P_B}{A_1} = \frac{P_B}{A_2}$$

$$\frac{7800 + P_B}{\frac{\pi}{4}(60^2 - 51^2)} = \frac{P_B}{\frac{\pi}{4}(63^2 - 57^2)}$$

$$\boxed{P_B = 20129 \text{ N}}$$

~~(iii) To find modulus of elasticity of nylon :→~~

$$\frac{(P_A + P_B) \times 350}{\frac{\pi}{4}(60^2 - 51^2) \times E} = 3.56 \text{mm.}$$

$$\frac{P_B \times 400}{\frac{\pi}{4}(63^2 - 57^2) \times E} = (7.63 \text{m.} - 3.56)$$

$$\Rightarrow \frac{P_A + P_B}{P_B} \times \frac{350}{400} \times \frac{63^2 - 57^2}{60^2 - 51^2} = \frac{3.56}{7.63 - 3.56}$$

$P_A =$

(iii) Strain in upward pipe = $\frac{3.56}{350} = 1.017 \times 10^{-2}$

Strain in lower pipe = $\frac{7.63 - 3.56}{400}$

$= 1.017 \times 10^{-2}$.

Same tensile strains in both pipes.

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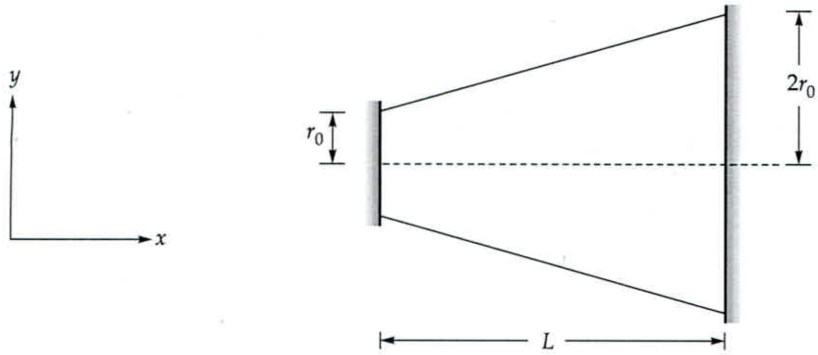
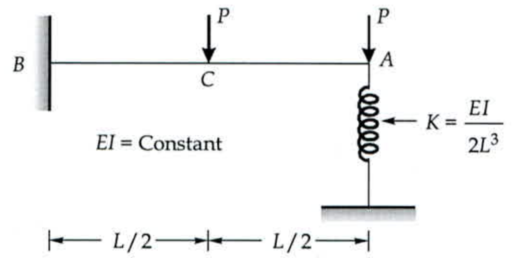
3 (c) **(i)** A bar as shown in figure below is in a shape of a solid, truncated cone of circular cross-section and is situated between two rigid supports. The temperature of the entire bar is then raised by ΔT. Assume that the cross-sections perpendicular to longitudinal axis of symmetry remain plane and neglect localised end effect due to the end supports. Determine the normal stress at any point in the bar.

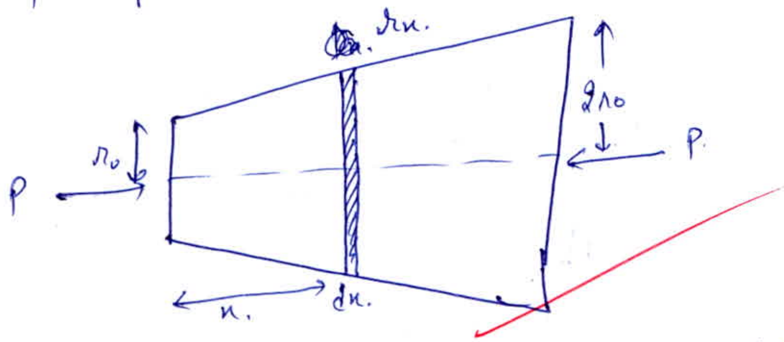


(ii) Determine the bending moment and shear force at support B in the uniform beam AB with flexural rigidity EI shown in the figure. Take spring constant $K = \dfrac{EI}{2L^3}$.



[10 + 10 marks]

(i) Assuming the cone to be free & applying load 'P' of the compressive natu.



$$r_x = r_0 + \frac{2r_0 - r_0}{L} \cdot x = r_0 + \frac{r_0}{L} \cdot x$$

Let $\frac{r_0}{L} = K$.

Deformation $\Rightarrow \int_0^L \frac{P.\,dx}{\pi \times r_x^2 \cdot E} = \delta.$

$\Rightarrow \frac{P}{\pi E} \int_0^L \frac{dx}{\cancel{ax}(r_0 + kx)^2} = \frac{P}{\pi E} \int_0^L (r_0 + kx)^{-2}\, dx.$

$\Rightarrow \frac{P}{\pi E} \left[\frac{(r_0 + kx)^{-1}}{-1 \times k} \right] dx. \Rightarrow \frac{P}{\pi E} \times \frac{1}{-k} \left[\frac{1}{r_0 + kx} \right]_0^L$

$\Rightarrow -\frac{P}{\pi k E} \times \left[\frac{1}{2r_0} - \frac{1}{r_0} \right]$

$\Rightarrow \frac{P}{\pi k E} \left[\frac{1}{r_0} - \frac{1}{2r_0} \right] \Rightarrow \frac{P}{\pi k E r_0} \times \frac{1}{2}]$

$\delta \Rightarrow \frac{P \times L}{\pi \times r_0 \times E \times r_0 \times 2} \Rightarrow \frac{PL}{2\,\cancel{a}\,\pi r_0^2 \times E}$

$\frac{4PL}{\pi \times 2x \times 4r_0}$

$\delta = \frac{PL}{2\pi r_0^2 \times E}$

Let ~~for~~ the same force P is developed between the supports due to increase in temperatur ΔT. And the restrainted deflection is δ.

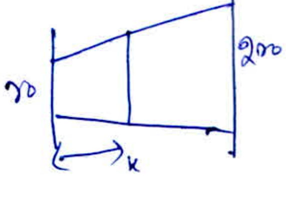
$\alpha L \Delta T = \delta.$

Hense. $\alpha L \Delta T = \frac{PL}{2\pi r_0^2 \times E}$

$P = \frac{(\alpha L \Delta T) \times 2\pi r_0^2 \times E}{L}$

$\boxed{P = \alpha \Delta T \times 2\pi r_0^2 \times E}$

Stress at any point 'x' in the ~~b~~ bar.



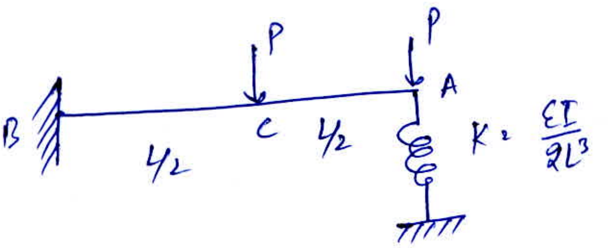
$\sigma_x = \frac{\alpha \Delta T\, 2\pi r_0^2 E}{\pi r_x^2} \Rightarrow \frac{2\pi \alpha \Delta T r_0^2 E}{\pi \left(r_0 + \frac{r_0}{L} x\right)^2}$

$$\boxed{\sigma_x = \frac{2\alpha \Delta T r_0^2 E}{\left(r_0 + \frac{r_0}{L}x\right)^2}}$$

Very Good

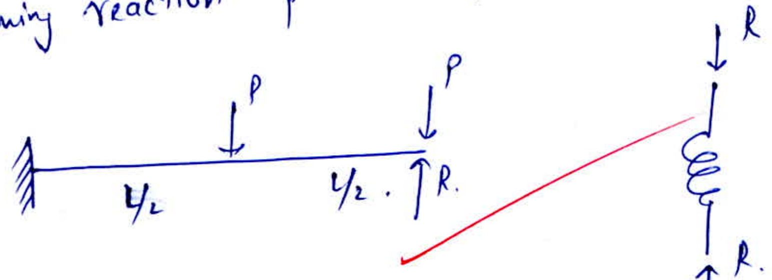
10

(ii)



$BM_D = ?$
$SF_D = ?$

Assuming reaction provided by spring = R.



Deflection in beam = compression in spring.

$$\frac{(P-R)L^3}{3EI} + \frac{P(L/2)^3}{3EI} + \frac{P(L/2)^2}{2EI}\times\frac{L}{2} = \frac{R}{K} = \frac{R\times 2L^3}{EI}$$

$$\frac{(P-R)L^3}{3EI} + \frac{PL^3}{24EI} + \frac{PL^3}{16EI} = \frac{2RL^3}{EI}$$

$$16(P-R) + 2P + 3P = 96R.$$

$$21P = 112R.$$

$$\boxed{R = \frac{21}{112}P}$$

BM at B = $-P\times\frac{L}{2} - P\times L + \frac{21}{112}P\times L \Rightarrow -\frac{21}{16}PL.$

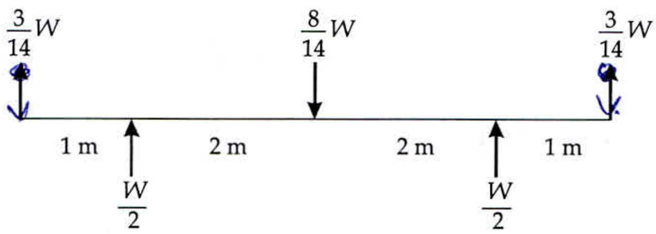
$$\boxed{BM_B = -\frac{21}{16}PL}$$

SF at B = $2P - \frac{21}{112}P = \frac{29}{16}P.$

$$\boxed{SF_B = \frac{29}{16}P}$$

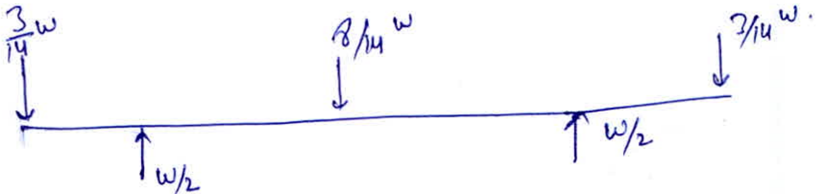
10

.4 (a) **(i)** A beam of T-section 6 m long supports the load system as shown below. The beam has a flange width of 100 mm and an overall depth of 120 mm. The flange and the web are 20 mm thick. The section is placed with flange at the bottom. Find the safe value of W if the stresses in compression and tension shall not exceed 90 N/mm^2 and 50 N/mm^2 respectively.



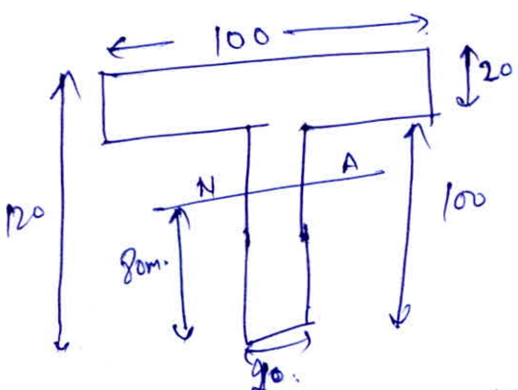
(ii) If a tension test bar is found to taper uniformly from (D – a) diameter to (D + a) diameter, prove that the error involved in using the mean diameter to calculate the Young's modulus is $\left(\frac{10a}{D}\right)^2$ percent.

[10 + 10 marks]



Max Hogg $\Rightarrow \frac{3}{14}\times w \times 1\,m = \frac{3}{14}W.$

Max Sagg $= \frac{W}{2}\times 2 - \frac{3}{14}\times 3 = \frac{5}{14}w.$

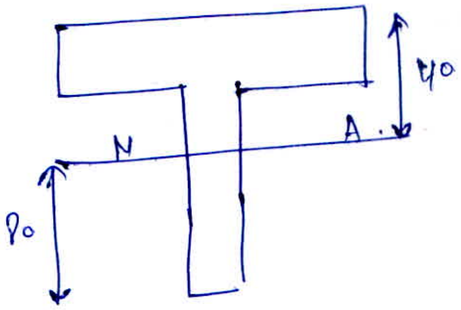


$$\bar{y}_{bottom} = \frac{100\times 20\times 110 + 100\times 20\times 50}{100\times 20\times 2}$$

$\Rightarrow$ 80 mm.

$$I_{NA} = \frac{20\times 100^3}{12} + 100\times 20\times 30^2 + \frac{100\times 20^3}{12} + 2000\times 30^2$$

$\Rightarrow 6933333.33\ mm^4.$



max hogg = $\frac{3}{14} w$

max sagg = $\frac{5}{14} w$.

$\sigma_c = 90$

$\sigma_t = 50$

Hogging →

$\sigma_{top} = 50$.

& $\sigma_{bottom} = 100$ Com.) Not per.

$\sigma_{bottom} = 90$ MPa) OK.

$\sigma_{top} = 45$ MPa

$$\frac{3 \times W \times 10^6 \times 80}{14 \times 6933333.33} = 90$$

$\boxed{W = 36.4 \text{ kN}}$

Sagging →

$\sigma_{top} = 90$ MPa.) Not Perm.

$\sigma_{bottom} = 180$ MPa.

$\sigma_{bottom} = 50$ MPa.

$\sigma_{top} = 25$ MPa.

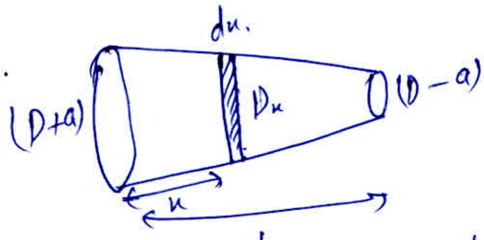
$$\frac{5 \times W \times 10^6 \times 80}{14 \times 6933333.33} = 50.$$

$\boxed{W = 12.133 \text{ kN}}$

So. Allowable load = $\boxed{12.133 \text{ kN}}$

8

(ii)



Actual Elongation $\rightarrow \int_0^L \frac{P.\,dx}{\frac{\pi}{4}(D_x)^2 .\,E} = \int_0^L P.\,dx$

$D_x = (D+a) - \frac{(D+a)-(D-a)}{L}.x.$

$\Rightarrow (D+a) - \left(\frac{2a}{L}\right)x.$

$\int_0^L \frac{P.\,dx}{\frac{\pi}{4}\left(D+a-\left(\frac{2a}{L}\right)x\right)^2 \cdot E} =$

Solving it we get $\Rightarrow \frac{4PL}{\pi(D+a)(D-a)\times E} \Rightarrow \delta_{actual}.$

For mean diameter :- $\frac{PL}{AE} \rightarrow$

$A_{mean} = \pi \times D_{mean}^2 \Rightarrow \frac{D+a+D-a}{2} \Rightarrow D.$

$\Rightarrow \pi \times D^2$

$$\delta_{m} = \frac{4PL}{\pi \times D^2 \times E}$$

$$\cancel{E_{actual} = \frac{4P \times L \times (D^2 - a^2)}{\pi D^2 \times 4PL}}$$

$$\cancel{E_{actual} = \frac{P}{\frac{\pi}{4} \times D^2 \times}}$$

$$E_{ac} = \frac{4PL}{\pi (D^2 - a^2)} \; ; \quad E_{app} = \frac{4PL}{\pi \times D^2}$$

$$\% \text{ error} = \frac{\left(\frac{4PL}{\pi (D^2 - a^2)} - \frac{4PL}{\pi D^2} \right)}{\frac{4PL}{\pi D^2}} \cancel{\text{percent}} \times 100$$

$$\Rightarrow \left[\frac{D^2 - (D^2 - a^2)}{(D^2 - a^2) D^2} \right] \times D^2 \times 100$$

$$\Rightarrow \frac{a^2}{D^2 - a^2}$$

$$\Rightarrow \left| \frac{1}{\frac{D^2}{a^2} - 1} \right| \Rightarrow$$

absolute value $\Rightarrow \left(\frac{1}{1 - \frac{a^2}{D^2}} \right) \times 100 \Rightarrow$

$$\Rightarrow \left(1 - \frac{a^2}{D^2} \right)^{-1}$$

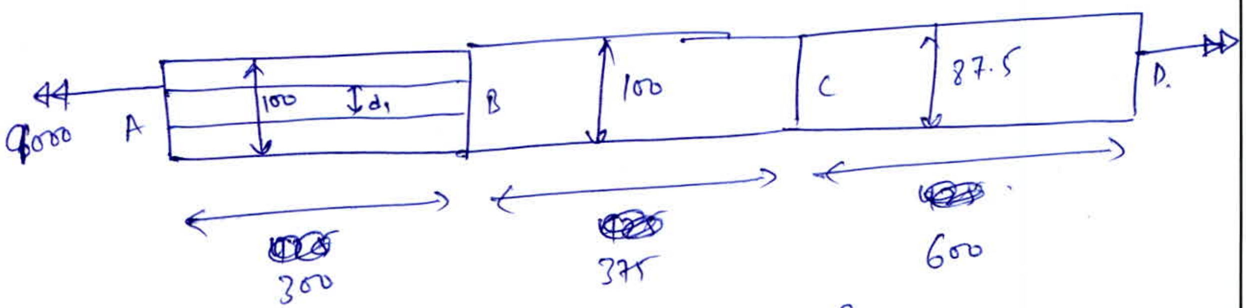
$$\Rightarrow \left(100 - \frac{100 a^2}{D^2} \right) \%$$

$$\Rightarrow \left(\frac{10a}{D} \right)^2 \text{ percent}$$

.4 (b) A steel shaft ABCD has a total length of 1275 mm and is made up as follows: AB = 300 mm, BC = 375 mm and CD = 600 mm. AB is hollow, its outside diameter being 100 mm and inside diameter d_1 mm. BC and CD are solid having diameters of 100 mm and 87.5 mm respectively. If equal and opposite torques are applied at the ends of the shaft, then find the maximum permissible value of d_1 for the maximum shearing stress in AB not to exceed that in CD. If the torque applied to the shaft is 9000 Nm, what is the total angle of twist? Take $G = 8 \times 10^4$ N/mm^2.

[20 marks]

Ans



Torque applied = 9000×10^3 Nmm.

Torque in AB = $T_{AB} = T_{BC} = T_{CD}$ = 9000 Nm.

τ_{max} in. AB = $\dfrac{T \times 16}{\pi \times D_1^3 (1-k^4)}$ where $k = \dfrac{d_1}{D_1}$

$\Rightarrow \dfrac{9000 \times 10^3 \times 16}{\pi \times 100^3 \times (1-k^4)}$

τ_{max} in CD = $\dfrac{9000 \times 10^3 \times 16}{\pi \times 87.5^3}$

Equating $\Rightarrow$

$$\frac{1}{100^3(1-k^4)} = \frac{1}{87.5^3}$$

$k = 0.7579.$

$d = kD = 79.797$ mm

$$\boxed{d_1 = 79.8 \text{ mm}}$$

Total angle of twist.

$$\frac{T}{G}\left[\frac{300}{\frac{\pi}{32}(100^4-79.8^4)}+\frac{375}{\frac{\pi}{32}\times 100^4}+\frac{600}{\frac{\pi}{32}\times 87.5^4}\right]$$

$\Rightarrow$ 1.249° – Twist.

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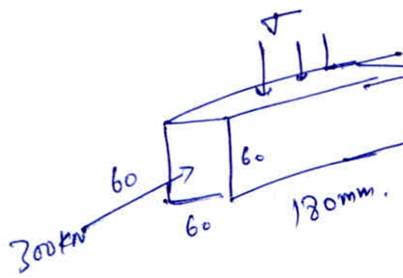
.4 (c) A steel bar of square section 60 mm × 60 mm and 180 mm long is subjected to an axial compressive load of 300 kN. The lateral strain is prevented by the application of uniform external pressure. If $\mu = 0.3$ and $E = 2 \times 10^5$ N/mm^2, find the alteration in the length of the bar.

If however, only one-half the lateral strain is prevented then what would be the alteration in the length of bar?

[20 marks]



$$\sigma_a = \frac{300 \times 10^3}{60 \times 60} = \frac{250}{3} \text{ MPa.}$$

As the lateral strain is prevented.

$$-\frac{\sigma}{\varepsilon} + \frac{\mu\sigma}{\varepsilon} + \mu \frac{250}{3 \times \varepsilon} = 0.$$

$$\sigma = \frac{-250 \times 0.3}{(0.3 - 1)3}$$

$$\sigma = \frac{250}{3} \times 0.3 \Big/ (1 - 0.3)$$

$$\boxed{\sigma \Rightarrow \frac{250}{7} \text{ MPa.}}$$

$$\text{length change} = \left[\frac{-\frac{250}{3} + \mu\frac{250}{7} + \mu\frac{250}{7}}{\varepsilon}\right] \times 180$$

$$\Rightarrow -\frac{39}{700} \text{ mm} \Rightarrow \boxed{-0.0557 \text{ mm}}.$$

$$\text{* free Lateral strain} \Rightarrow \frac{250 \times 0.3}{3 \times \varepsilon} = \epsilon.$$

$$\text{supposed lateral strain} \Rightarrow \frac{25}{\varepsilon \times 2}.$$

Do not write in this margin

$$\frac{25}{E\times 2} \Rightarrow -\frac{\sigma}{E} + \frac{4\sigma}{E} + \frac{4\times 250}{3\times E}$$

$$\sigma = \frac{125}{7}\ MPa.$$

Change in length of bar.

$$\Rightarrow \frac{\left[\frac{250}{3} + \frac{0.3\times 2\times 125}{7}\right]}{E}\times 180$$

$$\Delta L \Rightarrow \boxed{-0.06535\ mm.}$$

Section B : Transportation Engg-1 + Surveying and Geology-1
Geo-technical & Foundation Engg-2 + Environmental Engg-2

.5 (a) A footing 3 m × 2 m in size transmits a pressure of 140 kN/m² on a soil having $E = 5 \times 10^4$ kN/m² and $\mu = 0.50$. Find the immediate settlement for the footing at the centre. Assuming it to be (i) Flexible footing (ii) Rigid footing

For $L/B = 1.5$, Influence factor = 1.36 for flexible and 1.06 for rigid footing.

[12 marks]

Ans

(i) Flexible footing :-

$$S_i = \frac{qB(1-\mu^2)}{E} \cdot I_f$$

$$= \frac{140 \times 2(1-0.5^2) \times 1.36}{5 \times 10^4} \times 10^3$$

$$\Rightarrow \boxed{5.712\text{mm.}}$$

(ii) Rigid Footing :-

$$S_i = \frac{qB(1-\mu^2)}{E} I_f$$

$$\Rightarrow \frac{140 \times 2 \times (1-0.5^2)}{5 \times 10^4} \times 1.06 \times 10^3$$

$$\Rightarrow \boxed{4.452\text{mm.}}$$

12

Q.5 (b) A column footing of 1.8 m × 1.8 m is to be placed 1.5 m below ground level in a dry cohesionless soil. The unit weight of soil is 21 kN/m^3 and angle of internal friction, $\phi = 36°$. The footing is required to carry a total load of 1350 kN including column load, weight of footing and weight of soil surcharge. Determine the factor of safety against bearing capacity failure assuming:

(i) Ground water table well below the base of footing, and

(ii) Ground water table at ground level

Given for $\phi = 36°$, $N_c = 63.53$, $N_q = 47.16$, $N_\gamma = 51.7$

[Assume , $\gamma_{bulk} = \gamma_{saturated} = 21$ kN/m^3]

[12 marks]

Ans (i) Gr. W.T well below :→

As it is square footing.

As per terzaghi.

$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma$.

In this case 1350 kN → gross safe. ~~ultimate~~ load.

Gross safe load $= \left(\frac{q_{nu}}{FOS} + q\right) \times$ Area.

$$\frac{1350}{1.8^2} = \frac{21 \times 1.5 \times (47.16 - 1) + 0.4 \times 1.8 \times 21 \times 51.7}{FOS} + 1.5 \times 21$$

$\boxed{FOS = 5.71}$ 5.80

(ii) GWT is at ground level.

$\gamma' = \cancel{10.19}$ $11.19\ kN/m^3$

$$\frac{1350}{1.8^2} = \frac{11.19 \times 1.5 \times (46.16) + 0.4 \times 1.8 \times 11.19 \times 51.7}{F} + 1.5 \times 21$$

$\boxed{FOS = 3.69}$ 2.98

10

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Q.5 (c) **(i)** Define the processes involved in MBBR (Moving Bed Biofilm Reactor) used for secondary wastewater treatment?

(ii) One hundred cubic meters per day ($100\ m^3/d$) of mixed sludge at 4 percent solids is to be thickened to 8.0 percent solids. What is the approximate volume of the sludge after thickening and also comment on the result?

[6 + 6 marks]

(ii)

Assuming density $\cancel{\rho}$ = $1000 kg/m^3$.

Initially → Mass of sludge/day = 10^5 kg.

Mass of solids = 4000 kg.

Mass of water = 96000 kg.

final → Solids remain same = 4000kg.

Mass of water required = $\frac{4000}{0.08} \times 0.92$

⇒ 46000 kg.

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Volume of final sludge = $\frac{50000}{1000}$ = $50 m^3$

* Volume of sludge is halfed by thickening the sludge by 4%.
* Therefore thickening is necessary to decrease sludge transportation cost.

6

Q.5 (d) A 4-lane National Highway is passing through a built up area. Design the following geometric features for a horizontal circular curve of radius 350 m for this highway considering design speed as 80 kmph and the length of wheel base of largest truck as 6.0 m:

(i) Superelevation

(ii) Length of transition curve

Also suggest the most suitable shape of curve.

[12 marks]

(i) Super elevation :→

(i) Assuming 75% of design speed.

$$e_{max} = \frac{V^2}{225R} = \frac{80^2}{225 \times 350} = 0.081 > 0.07.$$

(ii) Assuming 100% friction.

Assume $f = 0.15$

$$f = \frac{80^2}{127 \times R} - 0.07$$

$$= 0.074 < 0.15$$

OK

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Provide super elevation of 7% to the highway.

(ii) Length of transition curve :-

$W = 7.5 \times 4 = 30m.$

Extra widening = $\frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}$

$\Rightarrow \frac{4 \times 6^2}{2 \times 350} + \frac{80}{9.5\sqrt{350}} = 0.656m.$

$W_E = 30.656m.$

(i) Acc. to change in centrifugal acceleration =>.

$C = \frac{80}{75+V} = \frac{80}{75+80} = 0.516$

$0.5 < 0.516 < 0.8$ OK

$L_T = \frac{(0.278 \times 80)^3}{0.516 \times 350} = 60.89m.$

(ii) Acc. to rate of superelevation. $\left(\frac{1}{N} = \frac{1}{100}\right)$ for plain terrain

built up area.

$\frac{N(ew+we)e}{2}$

$\Rightarrow \frac{100(30.656) \times 0.07}{2}$

$\Rightarrow$ ~~107.296m.~~

(iii) Min. length = $\frac{2.7V^2}{R} = \frac{2.7 \times 80^2}{350} = 69.12m.$

Length of Transition Curve = $\boxed{107.3m}$

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Most suitable curve is spiral Parabola

Spiral

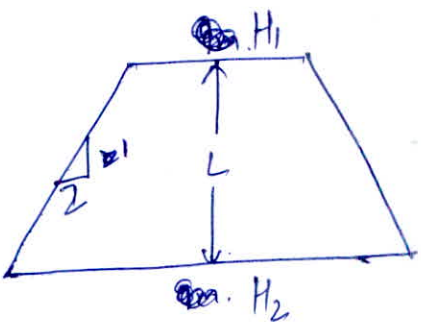
6

Q.5 (e) A railway embankment, 500 m long, has a width at formation level of 9 m with side slopes of 2 to 1. The ground levels at every 100 m distance along the centreline are as follows:

Distance, (*m*)	0	100	200	300	400	500
Ground level, (*m*)	107.8	106.3	110.5	111.0	110.7	112.2

The embankment has a rising gradient of 1.2 m per 100 m and the formation level is 110.5 m at zero chainage. Assume the ground to be level across the centerline, compute the volume of earthwork using trapezoidal method.

Ans



Distance	H_1	H_2	L	Area.
0	9m	19.8	2.7m.	38.88 m^2
100	9m	30.6	5.4m.	106.92 m^2
200	9m	18.6	2.4m.	33.12
300	9m	21.4	3.1m.	47.12
400	9m	27.4	4.6m.	83.72
500	9m.	26.2	4.3m.	75.68 m^2

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Volume reqd = $\frac{100}{2}\left[38.81 + 2\left(106.92 + 33.12 + 47.12 + 83.72\right) + 75.68\right]$

$\boxed{V. \Rightarrow 32812.5 m^3}$

12

[12 marks]

Q.6 (a) **(i)** For a railway track 7 m high embankment is required. The clay to be used for the embankment was found to have $c = 20\ kN/m^2$ and unit weight = $19\ kN/m^3$. Compute the critical maximum side slope angle for the embankment if a hard rocky stratum was found 3.5 m below the ground level. Assume ϕ for the clay equal to zero. The following values are given from Taylor's chart for depth factor D = 1.5 :

S_n	0.181	0.174	0.164	0.150
β	53°	45°	30°	20°

(ii) Using Terzaghi's method, determine the ultimate bearing capacity of a square footing of size 1.5 m with its base at a depth of 1 m below the ground level, resting on a dry sand stratum.

Take $\gamma_d = 17\ kN/m^3$, $\phi' = 38°$, $c' = 0$, $N_q = 60$, $N_\gamma = 75$.

[10 + 10 marks]

Ans (i) Depth factor for following condition.

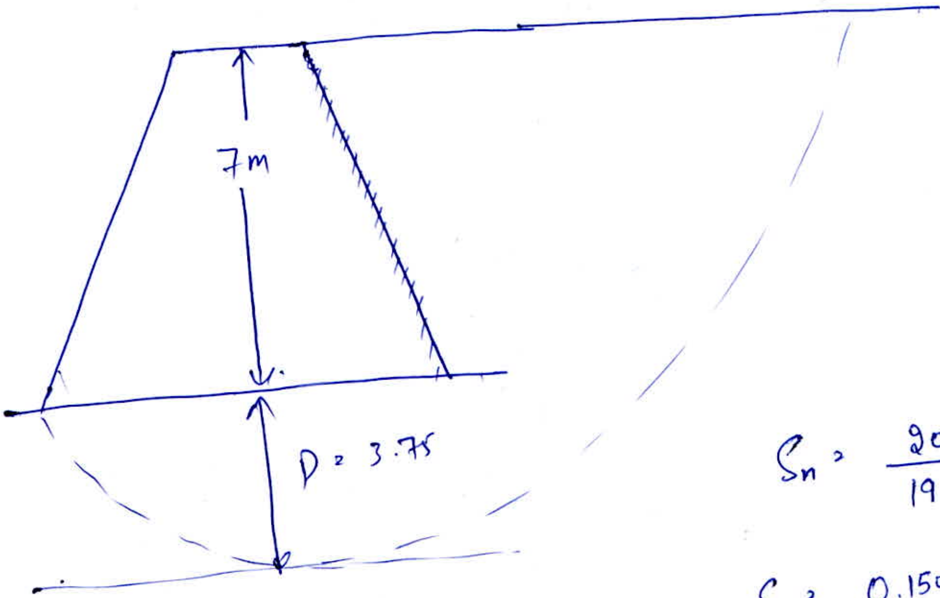
$$\Rightarrow \frac{H+D}{H} = \frac{7+3.5}{7} = 1.5$$

Acc. to Taylor →.

$$S_n = \frac{C}{\gamma H_c \cdot (FOS)}$$

Assuming FOS = 1

For the condition given :→



$S_n = \frac{20}{19 \times 7}$

$S_n = 0.1503$

Corresponding value of
$\beta = 20.268°$.

So, critical max side slope. $\boxed{\beta = 20.268°}$

$\boxed{1V : 2.71H}$

10

(ii) $B = 1.5m$. Dry state.

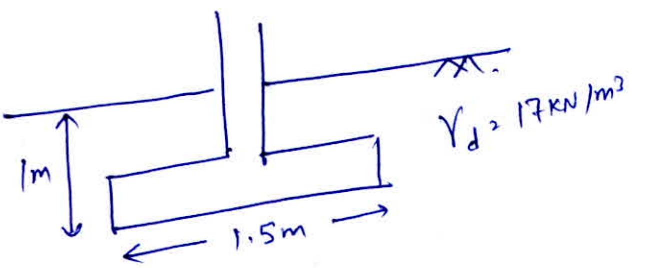
$D_f = 1m$.

$\phi' = 38°$

$c' = 0$

$N_q = 60$

$N_\gamma = 75$



According to Terzaghi, for a square footing. Ultimate bearing capacity is given by.:-

$$q_u = 1.3cN_c + qN_q + 0.4B\gamma N_\gamma.$$

$$q_u = 1.3 \times 0 \times N_c + qN_q + 0.4B\gamma N_\gamma.$$

For the given condition,

$$q = 17 \times 1 = 17 KN/m^2$$

$$q_u = 17 \times 60 + 0.4 \times 1.5 \times 17 \times 75$$

$$= 1785 KN/m^2$$

$$\boxed{q_u = 1785 KN/m^2}$$

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6 (b) Consider the following data for a completely mixed activated sludge system to treat wastewater from a community of 60000 persons:

Sewage flow, Q = 9000 m^3/day

BOD_5 = 360 mg/l (raw)

Assume 30% BOD removal in primary settling and 90% in biological treatment.

Winter temperature of mixed liquor = 10°C

Yield, y = 0.6

k_d = 0.07/day (BOD$_5$ basis at 15°C)

MLSS = 4000 mg/l, $\frac{VSS}{SS} = 0.8$

Adopt sludge age (θ_c) = 10 days

Determine F/M ratio and oxygen requirement uptake per day for this completely mixed activated sludge system.

[20 marks]

F/M ratio :-

$$\frac{1}{\theta_c} + k_d = \frac{Q(S_0 - S)}{VX} \cdot y$$

BOD inflow = 360 mg/l.

" " in PST = 360 mg/l.

" " " Biological treatment = 252 mg/l.

" outflow from " " = 25.2 mg/l.

k_d at 15° = 0.07/day.

~~at 10° = [1.047]~~

k_d at 20° ⟹ $0.07 = k_{20}[1.047]^{-5}$

$k_{20} = 0.0881$.

k_d at 10° ⟹ $0.0881[1.047]^{-10}$

⟹ 0.0556 / day.

$$\frac{1}{10} + 0.0556 = \frac{9000 \times 10^3 (252 - 25.2)}{4000 \times V} \cdot y$$

~~V = 3274 m³~~

V = 1967.7 m^3

$$\frac{F}{M} \text{ ratio} = \frac{9\times10^{9}\times 252}{4000\times 1967.7\times 10^{3}}$$

$\rightarrow$??

incomplete

6

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Q.6 (c) **(i)** What are the advantages of photogrammetric techniques in highway location and design? What are the various objectives of highway planning?

(ii) Following five alternate road plan development proposals with particulars as mentioned below are available:

Proposal	Number of towns and villages served along with population range					Total industrial products in thousand tonnes
	<2000	2001-5000	5001-10000	10001-20000	> 20000	
A	80	10	25	5	1	60
B	115	120	30	10	2	370
C	340	230	25	20	4	350
D	150	200	100	35	6	750
E	200	90	70	60	3	500

If the total road length of proposals A, B, C, D and E are respectively 200 km, 380 km, 605 km, 700 km and 400 km, calculate the utility rate per unit length of each road proposal and indicate the priority based on saturation system. Assume the utility units as follows:

For population :

Range		Unit
< 2000	:	0.25
2001 to 5000	:	0.50
5001 to 10000	:	1.00
10001 to 20000	:	2.00
> 20000	:	3.00

For products :

One unit for 1000 tonnes.

[8 + 12 marks]

Ans (ii)

	×(0.25) <2000	×(0.5) 2001-5000	×(1) 5001-10^4	×(2) 10^4-20000	×(3) >20000	×(1) Indu.	Utility
A	80	10	25	5	1	60	123
B	115	120	30	10	2	370	514.75
C	340	230	25	20	4	350	627
D	150	200	100	35	6	750	1075.5
E	200	90	70	60	3	500	794

Road	Utility per km.
A	0.615
B	1.354
C	1.036
D	1.536
E	1.985.

$\boxed{\text{Priority} \rightarrow E > D > B > C > A.}$

Show Calculations also

10

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Q.7 (a) **(i)** What are conditions which necessitate taking up of a realignment project of a highway? Discuss the general principles in the realignment of a highway and explain how the work is carried out.

(ii) Determine the extra width required for a road of carriageway 7.5 m on a horizontal curve of radius 300 m. The longest wheel base of vehicle using the road may be taken as 6.1 m. Design speed is 80 km/hr.

[16 + 4 marks]

(b) Design a septic tank with the following data :

(i) Number of users = 200

(ii) Rate of water supply = 150 l/head/day

(iii) Detention period = 18 hours

(iv) Percolating capacity of filter media of soak well = 1250 litres/m^3

(v) Rate of sludge accumulation = 40 litres/person/year

Also find the diameter of soak well. Assume reasonable data, if required.

[20 marks]

Q.7 (c) Two sets of tacheometric readings were taken from an instrument station A (RL of A = 100 m) to a staff station B as shown below.

Instruments	P	Q
Multiplying constant	100	95
Additive constant	0.30	0.45
Height of instrument	1.40 m	1.45 m
Staff held	Vertical	Normal

Instruments	Instruments station	Staff station	Vertical angle	Stadia readings
P	A	B	5°44′	1.090, 1.440, 1.795
Q	A	B	5°44′	?

Determine:

(i) The distance between instrument station and staff station.

(ii) The R.L. of staff station B.

(iii) Stadia readings with instrument Q.

[20 marks]

(a) The following whole circle bearings were observed in running a closed traverse:

Line	F.B.	B.B.
AB	71°05′	250°20′
BC	110°20′	292°35′
CD	161°35′	341°45′
DE	220°50′	40°05′
EA	300°50′	121°10′

Determine the correct magnetic bearings of the lines.

[20 marks]

Q.8 (b) **(i)** What are the desirable characteristics of grouting material in soils? List some of grouting methods adopted in practice.

(ii) A square pile group of 16 piles penetrates through a filled up soil of 3 m depth. The pile diameter is 250 mm and pile spacing is 0.75 m. The unit cohesion of the material is 18 kN/m^2 and the unit weight of soil is 15 kN/m^3. Draw plan and sectional elevation of the pile group and compute the negative skin friction on the group. [Take $\alpha = 0.7$]

[6 + 14 marks]

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Q.8 (c) **(i)** The driver of a vehicle requires 15 m less to stop after he applies the brakes while travelling up a grade than a driver travelling at the same initial speed down the same grade. Consider the coefficient of friction between tyres and pavement as 0.35 and initial speed to be 90 kmph. What is the percent grade?

(ii) Compute the moisture deficit in a landfill for each each cubic meter of waste if the parameters are :

Density of waste at time of deposit = 800 kg/m^3.

Field capacity = 60% by weight.

Water content of waste being deposited = 30% by weight.

Also discuss the result.

[10 + 10 marks]

Space for Rough Work

Space for Rough Work