MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Electrical Engineering Test No : 15

Section-A

Q.1 (a) Solution:

(i)
$$V_d = 295 - 325 \text{ V}$$

Peak of the fundamental frequency component in the output voltage is,

$$V_{01_{peak}} = m_a V_d = 1 \times 295 = 295 \text{ V}$$

$$V_{rms,\,01} = \frac{V_{01_{peak}}}{\sqrt{2}} = \frac{295}{\sqrt{2}} = 208.6 \text{ V}$$

(ii)
$$I_0 = \frac{2000}{208.6} = 9.6 \text{ A}$$

$$I_{0_{peak}} = \sqrt{2} \times I_0 = \sqrt{2} \times 9.6$$

$$= 13.57 \text{ A}$$

Switch voltage rating, $V_T = V_{d,\,max} = 325 \text{ V}$

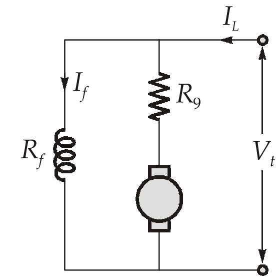
Switch current rating, $I_T = I_{0_{peak}} = 13.57 \text{ A} \simeq 13.6\text{A}$

Number of switches, $q = 4$

$\therefore$ Combined switch utilization ratio $= \dfrac{\text{Rated volt amperes}}{q V_T I_T}$

$$= \frac{2000}{4 \times 325 \times 13.6} = 0.113$$

Q.1 (b) Solution:



Given : V_t = 220 V, P = 8

Lap winding, Z = 1200 conductors,

R_{sh} = 220 Ω,

I_{L1} = 50 A,

$N_1 = N_{rated}$

Since winding is LAP, so number of parallel path in armature circuit is 8.

$$\text{Number of conductors per path} = \frac{Z}{A} = \frac{1200}{8} = 150$$

$$\text{Resistance of each conductor} = 50\text{ m}\Omega$$

$$\text{So, resistance of each parallel path} = \frac{Z}{A}\times r_{each} = 150 \times 50 \times 10^{-3} = 7.5\ \Omega$$

$$I_{a1} = I_L - I_{sh} = 50 - 1 = 49\text{ A}$$

$$E_{b1} = V_t - I_{a1}r_a = 220 - 49 \times 0.9375 = 174.06\text{ V}$$

Since torque is constant,

$$\frac{T_2}{T_1} = \frac{E_{b2}\cdot I_{a2}}{E_{b2}\cdot I_{a1}}\times\frac{N_1}{N_2}$$

$$(E_{b1}I_{a1})\times\frac{N_2}{N_1} = E_{b2}.I_{a2}$$

$$(174.06 \times 49) \times 1.4 = (V_t - I_{a2}.r_a)I_{a2}$$

$$11940.68 = (220 - I_{a2} \times 0.9375)I_{a2}$$

$$0.9375I_{a2}^2 - 220I_{a2} + 11940.68 = 0$$

On solving, I_{a2} = 149.43 A, 85.23 A

We will choose I_{a2} = 85.23 A, otherwise Cu-losses will increase and efficiency of machine will deteriorate.

As we know,

$$N \propto \frac{E_b}{\phi}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}\times\frac{\phi_1}{\phi_2}$$

$$\phi \propto I_{sh}$$

$$\frac{\phi_1}{\phi_2} = \frac{I_{sh1}}{I_{sh2}}$$

$$\frac{I_{sh2}}{I_{sh1}} = \frac{E_{b2}\times N_1}{E_{b1}\times N_2} = \frac{(220 - 0.9375\times 85.23)\times N_1}{174.06\times 1.4N_1}$$

$$\frac{I_{sh2}}{I_{sh1}} = 0.5749$$

$$I_{sh2} = I_{sh1} \times 0.5749 = 0.5749 \text{ A} \quad [\because I_{sh1} = 1\text{A}]$$

$$R_{ext} + R_{sh} = \frac{V_t}{I_{sh2}} = 382.66\ \Omega$$

So, required value of external resistance to be added in field winding is

$$R_{ext} = 382.66 - 220 = 162.66\ \Omega$$

to increase speed by 1.4 times of rated speed.

Q.1 (c) Solution:

Sending end phase voltage, $V_S = \frac{230}{\sqrt{3}} = 132.79$ kV

When the load is disconnected,

$$I_R = 0 \text{ A}$$

∴ sending end phase voltage,

$$V_S = AV_R + BI_R$$
$$= AV_R$$

$$\therefore \quad V_R = \frac{V_S}{A} = \frac{132.79}{0.938\angle 1.2} = 141.57\angle{-1.2°} \text{ kV}$$

Receiving end line voltage,

$$\therefore \quad V_{RL} = \sqrt{3}V_R = \sqrt{3}\times 141.57 = 245.2 \text{ kV}$$

Line charging current,

$$I_C = CV_R = 0.001\angle 90° \times 141.57\angle{-1.2°}$$
$$= 141.57\angle 88.8° \text{ A}$$

Maximum power that can be transmitted for $V_{RL} = 220$ kV and $V_S = 230$ kV

$$A = 0.938, \qquad B = 131.2,$$
$$\alpha = 1.2°, \qquad \beta = 72.3°$$

$$P_R = \frac{V_S V_R}{B}\cos(\beta - \delta) - \frac{AV_R^2}{B}\cos(\beta - \alpha)$$

For $P_R = P_{R\,max}$, $\delta = \beta = 72.3°$

$$P_{R\,(max)} = \frac{V_{SL}V_{RL}}{B} - \frac{A}{B}V_{RL}^2\cos(\beta - \alpha)$$

$$= \frac{(230)(220)}{131.2} - \frac{0.938}{131.2} \times 220^2 \cos(72.3 - 1.2)$$

$$= 272.58 \text{ MW}$$

Corresponding reactive power required at the receiving end,

$$Q_R = -\frac{A}{B} V_{RL}^2 \sin(\beta - \alpha) = \frac{-0.938}{131.2} \times 220^2 \sin(72.3° - 1.2°)$$

$$= -327.37 \text{ MVAR}$$

Q.1 (d) Solution:

(i) When a converter circuit is working in an inverter mode,

$$V_0 = -E + I_0 R$$

for 3-pulse converter, output voltage V_0 is,

$$V_0 = \frac{3V_{mL}}{2\pi} \cos\alpha$$

Therefore,
$$\frac{3V_{mL}}{2\pi} \cos\alpha = -230 + 10 \times 2$$

$$\frac{3V_{mL}}{2\pi} \cos\alpha = -210$$

$$\frac{3 \times 400\sqrt{2}}{2\pi} \cos\alpha = -210$$

$$\text{Firing angle, } \alpha = 141.033°$$

(ii) Source inductance is responsible for reduction in output voltage of converter.

Reduction in output voltage,

$$\Delta V_d = 3 f L_s I_0$$

$$\Delta V_d = 3 \times 50 \times 2 \times 10^{-3} \times 10 = 3\text{V}$$

Now,
$$V_0 = \frac{3V_{mL}}{2\pi} \cos\alpha - \Delta V_d = -E + I_0 R$$

$$\frac{3V_{mL}}{2\pi} \cos\alpha - \Delta V_d = -230 + 10 \times 2$$

$$\frac{3V_{mL}}{2\pi} \cos\alpha = -210 + 3$$

$$\frac{3 \times 400\sqrt{2}}{2\pi} \cos\alpha = -207$$

$$\text{Firing angle, } \alpha = 140.03°$$

For 3-pulse converter, reduction in output voltage in terms of overlap angle is given by,

$$\Delta V_d = \frac{3V_{mL}}{4\pi}[\cos\alpha - \cos(\alpha+\mu)]$$

$$\frac{3V_{mL}}{4\pi}\cos(\alpha+\mu) = \frac{3V_{mL}}{4\pi}\cos\alpha - \Delta V_{d0}$$

$$\frac{3\times 400\sqrt{2}}{4\pi}\cos(\alpha+\mu) = \frac{3\times 400\sqrt{2}}{4\pi}\cos 140.03° - 3 = -106.50$$

$$\cos(\alpha+\mu) = -0.7886$$

$$\alpha+\mu = 142.055°$$

$$\text{Overlap angle } \mu = 2.025°$$

Q.1 (e) Solution:

For phase lead controller,

$$G_c(s) = \frac{1+aTs}{1+Ts}, \quad \text{Put } s = j\omega$$

$$G_c(j\omega) = \frac{1+jaT\omega}{1+jT\omega}$$

$$\phi = \angle G_c(j\omega) = \tan^{-1}(a\,\omega T) - \tan^{-1}(\omega T) \quad \text{...(i)}$$

$$\tan\phi = \frac{a\omega T - \omega T}{1 + a\omega T\cdot\omega T}$$

$$\tan\phi = \frac{\omega T(a-1)}{1+a\omega^2T^2} \quad \text{...(ii)}$$

For getting maximum value of phase angle,

From equation (i), at $\omega = \omega_m$

$$\frac{d\phi}{d\omega} = 0 = \frac{aT}{1+(a\omega_m T)^2} - \frac{T}{1+(\omega_m T)^2}$$

$$a[1+(\omega_m T)^2] = 1 + (a\omega_m T)^2$$

$$a + a(\omega_m T)^2 = 1 + a^2(\omega_m T)^2$$

$$a - 1 = (\omega_m T)^2[a^2 - a]$$

$$(a-1) = (\omega_m T)^2\, a(a-1)$$

$$(\omega_m T)^2 = \frac{1}{a}$$

$$\omega_m^{\ 2} = \frac{1}{aT^2}$$

$$\omega_m = \frac{1}{T\sqrt{a}}$$

Now put this value in equation (ii),

$$\tan\phi_m = \left.\frac{\omega T(a-1)}{1+a\omega^2T^2}\right|_{\omega=\omega_m} = \frac{1\times T(a-1)}{T\sqrt{a}\left[1+a\times\frac{1}{a}\right]}$$

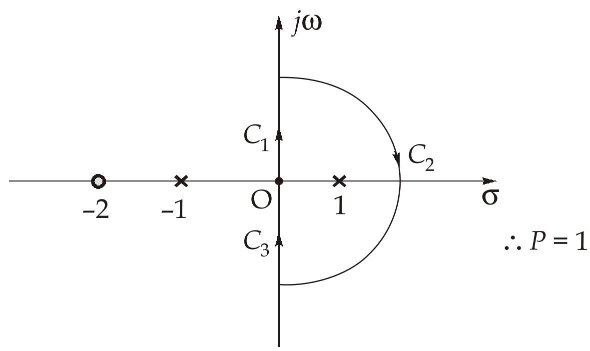
$$\tan\phi_m = \frac{(a-1)}{2\sqrt{a}}$$

Maximum phase, $$\phi_m = \tan^{-1}\left[\frac{(a-1)}{2\sqrt{a}}\right]$$

Q.2 (a) Solution:

(i) $$G(s)\,H(s) = \frac{s+2}{(s+1)\,(s-1)}$$

The Nyquist contour in s-plane is



Nyquist plot in $G(s)\,H(s)$ plane is obtained as below,

Corresponding to $C_1 : s = j\omega\,;\ \omega : 0 \to \infty$

$$G(j\omega)\,H(j\omega) = \frac{j\omega+2}{(j\omega+1)\,(j\omega-1)}$$

We have, $$M = \frac{\sqrt{\omega^2+4}}{\omega^2+1}$$

and $$\phi = \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega) - 180° + \tan^{-1}(\omega)$$

Angular frequency	M	$\angle G(j\omega)\,H(j\omega)$
$\omega = 0^+$	2	–180°
$\omega = \infty^+$	0	–90°

Corresponding to C_3 : $s = -j\omega$; $\omega: \infty \to 0$

$$G(-j\omega)\,H(-j\omega) = \frac{2-j\omega}{(1-j\omega)\,(-1-j\omega)}$$

Here, $$M = \frac{\sqrt{4+\omega^2}}{(\omega^2+1)}$$

and $$\phi = -\tan^{-1}\left(\frac{\omega}{2}\right)+\tan^{-1}(\omega)-180°-\tan^{-1}(\omega)$$

Angular frequency	M	$\angle G(j\omega)\,H(j\omega)$
$\omega = 0^-$	2	–180°
$\omega = \infty^-$	0	–270°

Corresponding to C_2 :

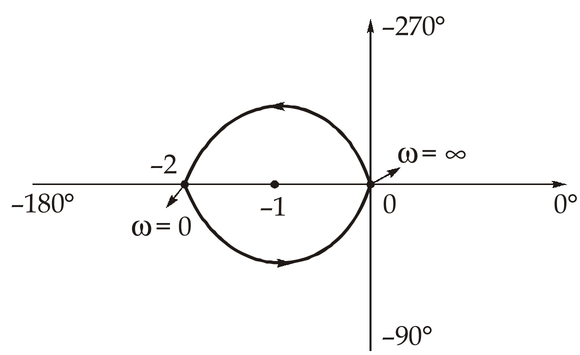
$$G(s)\,H(s) = \lim_{R\to\infty} R\,e^{j\theta};\ \theta \text{ varies from } +90° \text{ to } -90°$$

So, $$G(s)\,H(s) = \lim_{R\to\infty} \frac{(R\,e^{j\theta}+2)}{(R\,e^{j\theta}+1)\,(R\,e^{j\theta}-1)}$$

$$G(s)\,H(s) = \lim_{R\to\infty} \frac{1}{R}e^{-j\theta}$$

Thus, $M = 0$; $\phi = -\theta$ and thus, varies from –90° to 90°.

The Nyquist plot in $G(s)$ $H(s)$ plane is obtained as below:



Number of encirclement, of (–1 +j0) point in anticlockwise direction (N) = 1

Number of open loop poles on right half of s-plane (P) = 1

So, $$N = P - Z$$

or $$Z = 0$$

where, Z is the number of closed loop poles on the right hand side of s-plane. Thus, the closed loop system is stable.

(ii) The observability matrix for a control system is given by,

$$Q_0 = [C^T \; A^T C^T \; ...(A^T)^{n-1} C^T]$$

where n = number of states = 3 (given). We have,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$A^T(A^T C^T) = \begin{bmatrix} -6 \\ -11 \\ -5 \end{bmatrix}$$

$$\text{Observability Matrix, } Q_0 = \begin{bmatrix} 1 & 0 & -6 \\ 1 & 1 & -11 \\ 0 & 1 & -5 \end{bmatrix}$$

$$|Q_0| = 1[(1 \times -5) - (1 \times -11) - 0 + (-6)(1 - 0)]$$

$$= -5 + 11 - 6 = 0$$

Since determinant is zero, rank is not 3 and hence, the system is not completely observable.

The roots of the characteristic equation are given by $|\lambda I - A| = 0$ i.e.

$$\begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$

$$\Rightarrow \quad (\lambda + 1)(\lambda + 2)(\lambda + 3) = 0$$

Thus, Eigen values are –1, –2 and –3.

The Vandermonde matrix formed by the eigen values is,

$$M = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} = \text{Vandermonde matrix}$$

$$X = CM = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 \end{bmatrix}$$

$$y = CMZ = \begin{bmatrix} 0 & -1 & -2 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$z_1(t)$ is not observable, a mode due to eigen value $\lambda = -1$.

Q.2 (b) Solution:

Pre-fault, $z_{pre} = j0.3 + (j0.55) \,||\, (j0.55) + j0.15 = j0.725$ pu

Equivalent circuit during fault,

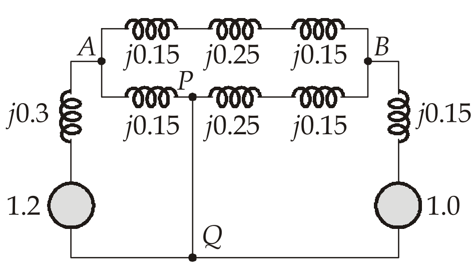


Fig. (i)

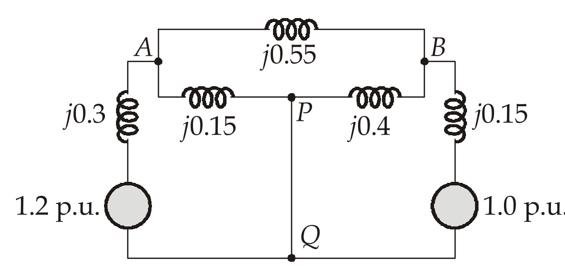
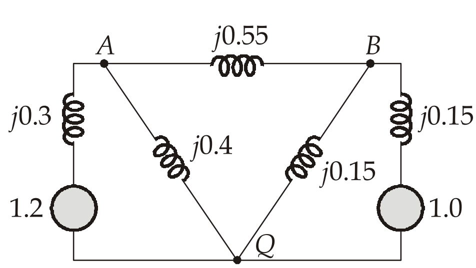


Fig. (ii)



(Δ - Y) Transformation

Fig. (iii)

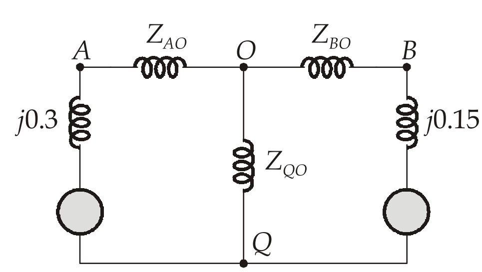
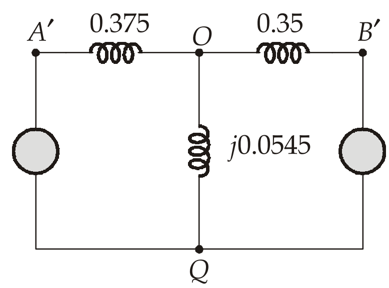


Fig. (iv)

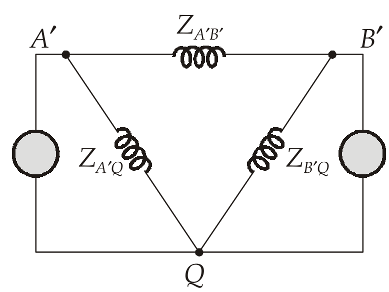
$$Z_{A0} = \frac{Z_{AB} \times Z_{AQ}}{Z_{AB} + Z_{AQ} + Z_{BQ}} = \frac{j0.15 \times j0.55}{j0.15 + j0.55 + j0.4} = j0.075$$



(Δ - Y) Transformation

$$Z_{BO} = \frac{Z_{AB} \times Z_{BQ}}{Z_{BA} + Z_{BQ} + Z_{QA}} = j0.2$$

$$Z_{QO} = \frac{Z_{QA} \times Z_{QB}}{Z_{QA} + Z_{QB} + Z_{AB}} = j0.0545$$



$$Z_{A'B'} = \frac{j0.375 \times j0.35 + j0.35 \times j0.0515 + j0.0545 \times i}{j0.0545}$$

$$= j3.133 \text{ p.u.}$$

During fault, z_f = $j3.133$ pu = $Z_{A'B'}$

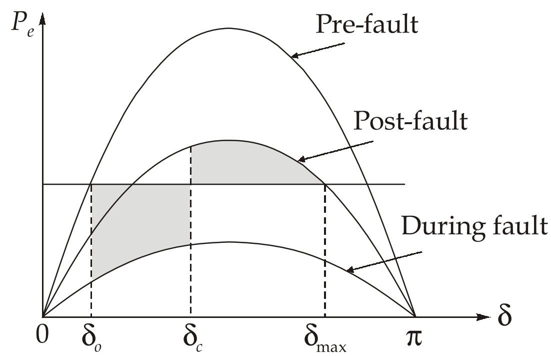
Post fault z_{post} = $j0.3 + j0.15 + j0.55 = j1.0$ pu

Maximum power output is $\dfrac{E_1 E_2}{X}$

$$\text{Before fault } P_{pre(max)} = \frac{1.2 \times 1.0}{0.725} = 1.66 \text{ pu}$$

$$\text{During fault } P_{f(max)} = \frac{1.2 \times 1.0}{3.133} = 0.383 \text{ pu}$$

$$\text{Post fault } P_{post(max)} = \frac{1.2 \times 1.0}{1} = 1.2 \text{ pu}$$



Using equal - area criterion, $\int_{\delta_o}^{\delta_{max}} P_e d\delta = 0$

$$\delta_o = \sin^{-1}\left[\frac{P_e}{P_{\text{pre(max)}}}\right] = \sin^{-1}\left[\frac{1}{1.66}\right]$$

$$= 37.17° \text{ or } 0.648 \text{ rad}$$

$$\text{Ratio, } r_1 = \frac{P_{f(\text{max})}}{P_{\text{pre(max)}}} = \frac{0.383}{1.66} = 0.23$$

$$r_2 = \frac{P_{\text{post(max)}}}{P_{\text{pre(max)}}} = \frac{1.2}{1.66} = 0.725$$

$$\delta_m = \pi - \sin^{-1}\left[\frac{P_e}{P_{\text{pre(max)}}}\right] = \sin^{-1}\left(\frac{1}{1.2}\right)$$

$$= 123.6° \text{ or } 2.157 \text{ rad}$$

$$\delta_c = \cos^{-1}\left[\frac{\sin\delta_o(\delta_m - \delta_o) + r_2 \times \cos\delta_m - r_1\cos\delta_o}{r_2 - r_1}\right]$$

$$= \cos^{-1}(0.66) = 48.7° \text{ or } 0.85 \text{ rad}$$

Q.2 (c) Solution:

(i) Using the open-loop poles and zeros, we represent the open-loop system whose root locus is given as

$$G(s)H(s) = \frac{k(s-3)(s-5)}{(s+1)(s+2)} = \frac{k(s^2 - 8s + 15)}{(s^2 + 3s + 2)}$$

But for all points along the root locus,

$$kG(s)H(s) = -1, \text{ and along the real axis, } s = \sigma$$

Hence,
$$\frac{k(\sigma^2 - 8\sigma + 15)}{(\sigma^2 + 3\sigma + 2)} = -1$$

$$k = \frac{-(\sigma^2 + 3\sigma + 2)}{(\sigma^2 - 8\sigma + 15)}$$

For break points,
$$\frac{dk}{d\sigma} = 0$$

$$\frac{dk}{d\sigma} = \frac{-\left[(2\sigma+3)(\sigma^2 - 8\sigma + 15) - (\sigma^2 + 3\sigma + 2)(2\sigma - 8)\right]}{(\sigma^2 - 8\sigma + 15)^2} = 0$$

$$\frac{dk}{d\sigma} = \frac{11\sigma^2 - 26\sigma - 61}{(\sigma^2 - 8\sigma + 15)^2} = 0$$

$$11\sigma^2 - 26\sigma - 61 = 0$$

$$\sigma = \frac{26 \pm \sqrt{26^2 + 4 \times 61 \times 11}}{2 \times 11}$$

$\therefore$ $\sigma = -1.45; 3.82$

$\therefore$ The breakaway point is,

$$\sigma_p = -1.45$$

The break-in point, $\sigma_z = 3.82$

(ii) Transfer function
$$= \frac{Y(s)}{U(s)} = \frac{10(s+1)}{s(s+1)(s+5)+10} = \frac{10(s+1)}{s^3 + 6s^2 + 5s + 10}$$

Transfer function decouposition,

$$0(s) \rightarrow \frac{10}{s^3 + 6s^2 + 5s + 10} X(s) \rightarrow (s+1) \rightarrow Y(s)$$

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3 + 6s^2 + 5s + 10}$$

and $Y(s) = (s+1)X(s)$

$$X(s)(s^3 + 6s^2 + 5s + 10) = 10\, U(s)$$

↓↑ I.L.T.

$$\frac{d^3x}{dt^3} + \frac{6d^2x}{dt^2} + 5\frac{dx}{dt} + 10x = 10u \quad \text{...(i)}$$

and $Y(s) = (s+1)X(s)$

↓↑ I.L.T.

and $$y = \frac{dx}{dt} + x \qquad \text{...(ii)}$$

Let, $$x = x_1$$

$$\dot{x}_1 = \dot{x} = x_2 \qquad \text{...(iii)}$$

$$\dot{x}_2 = \ddot{x} = x_3 \qquad \text{...(iv)}$$

$$\dot{x}_3 = \dddot{x}$$

Putting the above values in equation (i) and (ii), we get

$$\dot{x}_3 = -10x_1 - 5x_2 - 6x_3 + 104 \qquad \text{...(v)}$$

and $$y = x_2 + x_1 \qquad \text{...(vi)}$$

Now from equation (iii), (vi), (v) and (vi)

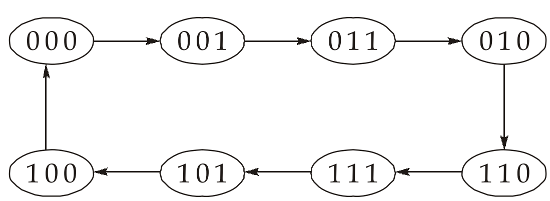
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q.3 (a) Solution:

Minimum no. of flip-flops required = 3

Sequence diagram:



Excitation table:

Present State			Next State			Required Excitations					
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	1	0	1	0	0	X	X	0	X	1
0	1	0	1	1	0	1	X	X	0	0	X
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	1	0	1	X	0	X	1	X	0
1	0	1	1	0	0	X	0	0	X	X	1
1	0	0	0	0	0	X	1	0	X	0	X

Minimization

K-Map for J_2

Q_2 \ Q_1Q_0	00	01	11	10
0				1
1	X	X	X	X

$$J_2 = Q_1\overline{Q}_0$$

K-Map for K_2

Q_2 \ Q_1Q_0	00	01	11	10
0	X	X	X	X
1	1			

$$K_2 = \overline{Q}_1\overline{Q}_0$$

K-Map for J_1

Q_2 \ Q_1Q_0	00	01	11	10
0		1	X	X
1			X	X

$$J_1 = \overline{Q}_2Q_0$$

K-Map for K_1

Q_2 \ Q_1Q_0	00	01	11	10
0	X	X		
1	X	X	1	

$$K_1 = Q_2Q_0$$

K-Map for J_0

Q_2 \ Q_1Q_0	00	01	11	10
0	1	X	X	
1		X	X	1

$$J_0 = \overline{Q}_2\overline{Q}_1 + Q_2Q_1$$

$$J_0 = Q_2 \odot Q_1$$

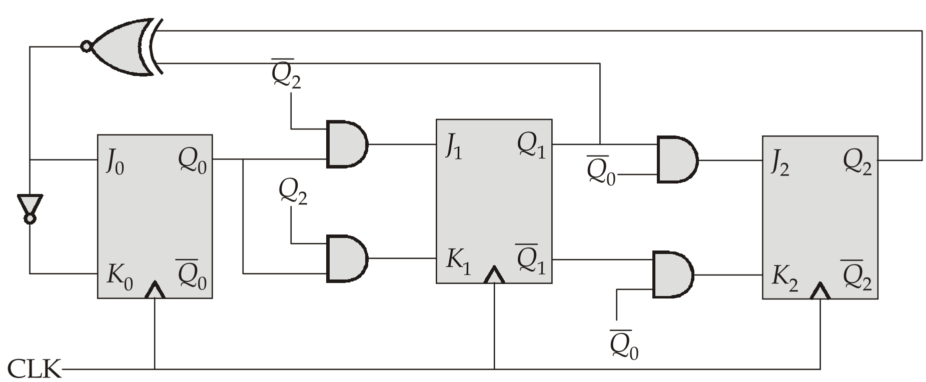
K-Map for K_0

Q_2 \ Q_1Q_0	00	01	11	10
0	X		1	X
1	X	1		X

$$K_0 = Q_1\overline{Q}_2 + Q_2\overline{Q}_1$$

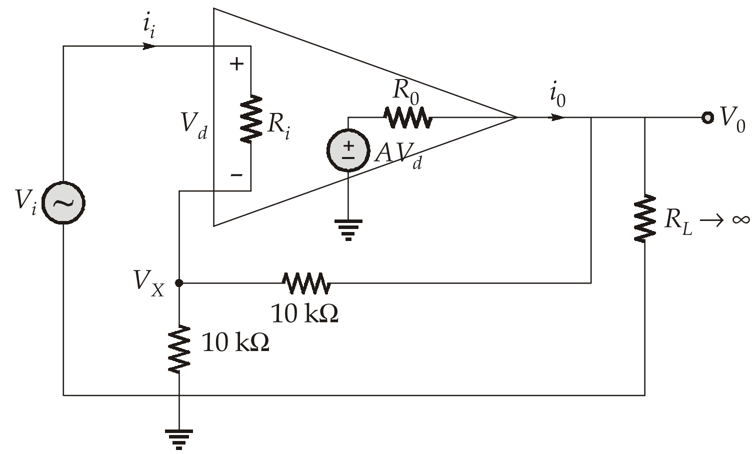
$$K_0 = Q_2 \oplus Q_1$$

Logic Circuit:



Q.3 (b) Solution:

(i)



$$V_X = V_0 \cdot \frac{10}{10+10} = \frac{V_0}{2}$$

$$V_d = V_i - V_X = V_i - \frac{V_0}{2} \quad \text{...(i)}$$

$$\therefore \quad V_0 = (A_v V_d) \cdot \frac{20}{20+R_0} = A_v\left(V_i - \frac{V_0}{2}\right) \cdot \frac{20}{21}$$

$$\frac{V_0}{A_v} = \left(V_i - \frac{V_0}{2}\right)\frac{20}{21}$$

Given : $A \to \infty$

$$0 = V_i - \frac{V_0}{2}$$

$$V_i = \frac{V_0}{2}$$

Hence, $$\frac{V_0}{V_i} = 2$$

i.e., $$A_f = 2$$

(ii) We have; $$V_0 = 11 \sin \omega t \text{ V}$$

$$\because \quad A_f = \frac{V_0}{V_i}$$

$$V_i = \frac{V_0}{A_f} = \frac{11 \sin \omega t}{2} = 5.5 \sin \omega t \text{ V}$$

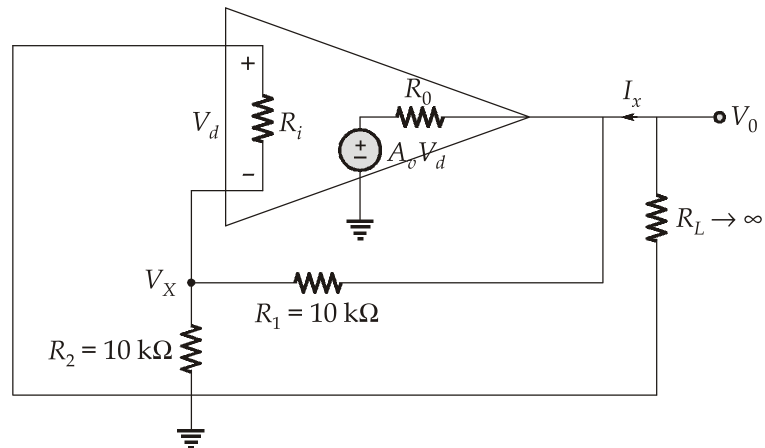
but, $$V_d = V_i - \frac{V_0}{2} = 5.5 \sin \omega t - 5.5 \sin \omega t$$

$$= 0$$

Hence, $$i_i = \frac{V_d}{R_i} = 0$$

$\therefore$ $$R_{if} = \frac{V_i}{i_i} = \text{Infinite}$$

(iii)



Here, $$V_d = 0 - V_x = -\frac{V_o}{2}$$

KCL at output Node:

$$I_x = \frac{V_o - A_v V_d}{R_o} + \frac{V_o}{R_1 + R_2}$$

$$I_x = \frac{V_o - A_v\left(-\frac{V_o}{2}\right)}{R_o} + \frac{V_o}{R_1 + R_2}$$

$$I_x = \frac{V_o\left(1 + \frac{A_v}{2}\right)}{R_o} + \frac{V_o}{R_1 + R_2}$$

$$I_x = V_o\left[\frac{1 + \frac{A_v}{2}}{R_o} + \frac{1}{R_1 + R_2}\right]$$

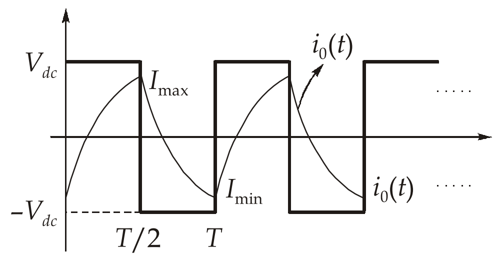
$$\frac{V_o}{I_x} = \left[\frac{R_o}{1 + A_v/2}\right] \Bigg\| \left[\frac{1}{R_1 + R_2}\right]$$

$\because$ $$A_v \to \infty$$

$$R_{of} = \frac{V_o}{I_x} = R_1 + R_2 = 10 + 10 = 20\ \text{k}\Omega$$

Q.3 (c) Solution:

(i) Waveform of steady state load current,



$$i_0(t) = \begin{cases} \frac{V_{dc}}{R} + \left(I_{min} - \frac{V_{dc}}{R}\right)e^{-t/\tau} & 0 < t < \frac{T}{2} \quad \text{...(i)} \\ \frac{-V_{dc}}{R} + \left(I_{max} + \frac{V_{dc}}{R}\right)e^{\frac{-(t-T/2)}{2}} & \frac{T}{2} < t < T \quad \text{...(ii)} \end{cases}$$

at $t = \frac{T}{2}$, $\quad i_0\left(\frac{T}{2}\right) = I_{max}$

$$I_{max} = \frac{V_{dc}}{R} + \left(I_{min} - \frac{V_{dc}}{R}\right)e^{-(T/2\tau)}$$

by symmetry, $\quad I_{min} = -I_{max}$

Substitute $-I_{max}$ for I_{min} and solving for

$$I_{max} = -I_{min} = \frac{V_{dc}}{R}\left[\frac{1-e^{-T/2\tau}}{1+e^{-T/2\tau}}\right]$$

From given data:

$$T = \frac{1}{f} = \frac{1}{60} = 0.0167 \text{ sec}$$

$$\tau = \frac{L}{R} = \frac{0.025}{10} = 0.0025 \text{ sec}$$

$$\frac{T}{2\tau} = 3.33$$

$$I_{max} = -I_{min} = \frac{100}{10}\left[\frac{1-e^{-3.33}}{1+e^{-3.33}}\right] = 9.31 \text{ A}$$

From equation (i),

$$i_0(t) = \frac{100}{10} + \left(-9.31 - \frac{100}{10}\right)e^{-t/0.0025}$$

$$= \left(10 - 19.31e^{-t/0.0025}\right)\text{A} \qquad 0 < t < \frac{1}{120}$$

$$= \left(-10 + 19.31e^{-\frac{(t-0.0085)}{0.0025}}\right)\text{A} \qquad \frac{1}{120} < t < \frac{1}{60}$$

(ii) Power is computed from $I_{\text{rms}}^2 R$, where I_{rms} is computed as

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i_0^2(t)\,dt}$$

$$= \sqrt{\frac{2}{T}\int_0^{T/2}\left[\frac{V_{dc}}{R} + \left(I_{\min} - \frac{V_{dc}}{R}\right)e^{-t/\tau}\right]^2 dt}$$

$$= \sqrt{\frac{1}{1/120}\int_0^{1/120}\left[(10 - 19.31)e^{-t/0.0025}\right]^2 dt}$$

$$= 6.64 \text{ A}$$

Power absorbed by the load is

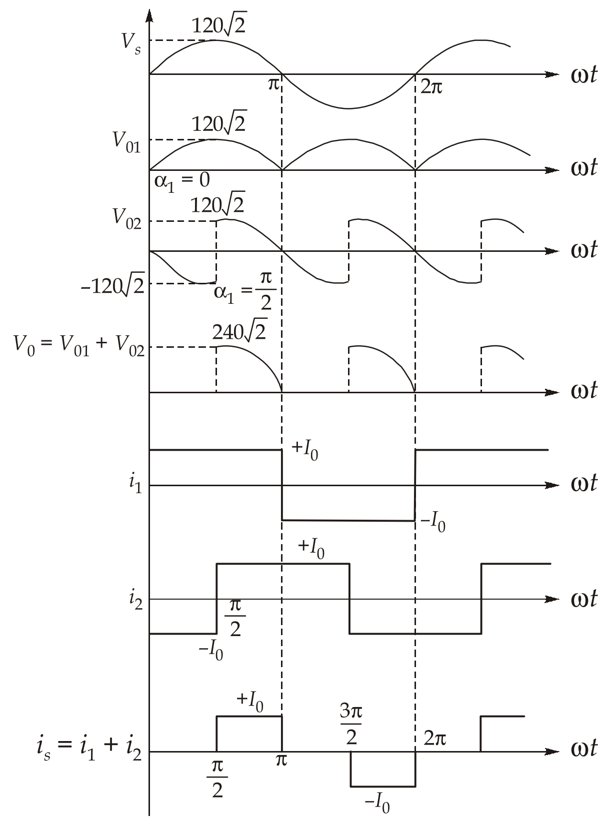
$$P = I_{\text{rms}}^2 R = (6.64)^2 \times (10) = 441 \text{ W}$$

(iii) Average source current can also be computed by equating source and load power

$$I_s = \frac{P_{dc}}{V_{dc}} = \frac{441}{100} = 4.41 \text{ A}$$

Q.4 (a) Solution:

(i)



(ii) Average output voltage, $V_o = \dfrac{2V_m}{\pi}(1+\cos\alpha_2) = \dfrac{2\times120\sqrt{2}}{\pi} = 108.04\text{ V}$

RMS output voltage, $V_{\text{rms}} = \sqrt{2}V_m\left[\dfrac{1}{\pi}\left(\pi-\alpha_2+\sin\dfrac{2\alpha_2}{2}\right)\right]^{1/2} = V_m = 169.71\text{ V}$

Fundamental current, $I_1 = I_o\dfrac{2\sqrt{2}}{\pi}\cos\dfrac{\pi}{4} = 0.6366I_o, \qquad \phi_1 = -\dfrac{\pi}{4}$

RMS current, $I_{\text{rms}} = 0.7071I_o$

$DF = \cos\phi_1 = 0.7071$

$PF = \dfrac{I_1}{I_{rms}}\times DF = 0.6366(\text{lagging})$

$HF = \left[\left(\dfrac{I_{\text{rms}}}{I_1}\right)^2 - 1\right]^{1/2} = 0.4835$

Q.4 (b) Solution:

(i) 1.

$$X(s) = \frac{1}{s^2}\frac{d}{ds}\left[\frac{e^{-3s}}{s}\right], \text{ROC}: \sigma > 0$$

$$= \frac{1}{s^2}\left[\frac{-3e^{-3s}s - 1e^{-3s}}{s^2}\right]$$

$$= \frac{-3}{s^3}e^{-3s} - \frac{e^{-3s}}{s^4};\ \sigma > 0 \quad \text{...(i)}$$

Using the Multiplication by power of 't' property,

$$t^n u(t) \rightleftharpoons \frac{n!}{s^{n+1}}, \sigma > 0$$

Put $n = 2$:

$$t^2 u(t) \rightleftharpoons \frac{2!}{s^3} = \frac{2}{s^3}, \sigma > 0$$

$$\frac{t^2}{2}\cdot u(t) \rightleftharpoons \frac{1}{s^3}, \sigma > 0$$

Using the time-shifting property of Laplace Transform,

$$\frac{-3(t-3)^2}{2}u(t-3) \rightleftharpoons \frac{-3e^{-3s}}{s^3}, \sigma > 0$$

Put $n = 3$:

$$t^3 u(t) \rightleftharpoons \frac{3!}{s^4},\ \sigma > 0$$

$$\frac{t^3}{6}u(t) \rightleftharpoons \frac{1}{s^4}, \sigma > 0$$

Using the time-shifting property of Laplace Transform,

$$\frac{(t-3)^3}{6}u(t-3) \rightleftharpoons \frac{e^{-3s}}{s^4}, \sigma > 0$$

By applying inverse Laplace transform on (i), we get

$$x(t) = \frac{-3}{2}(t-3)^2 u(t-3) - \frac{(t-3)^3}{6}\cdot u(t-3)$$

2.

$$X(s) = s\left[\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}\right], \text{ROC}: \sigma > 0$$

$$= \frac{1}{s} - \frac{e^{-s}}{s} - e^{-2s};\ \sigma > 0 \quad \text{...(ii)}$$

We know that, $u(t) \rightleftharpoons \frac{1}{s},\ \sigma > 0$

Using the time-shifting property of Laplace Transform,

$$u(t-1) \rightleftharpoons \frac{e^{-s}}{s}, \sigma > 0$$

Also, $\delta(t-2) \rightleftharpoons e^{-2s}$

By applying inverse Laplace transform on (ii), we get

$$x(t) = u(t) - u(t-1) - \delta(t-2)$$

(ii) $h(n) = \alpha^n u(n)$ with $|\alpha| < 1$

$$H(z) = \frac{1}{1-\alpha z^{-1}}, \text{ ROC: } |z| > |\alpha|$$

Considering the unit step input i.e.

$$x(n) = u(n)$$

$\therefore$ $$X(z) = \frac{1}{1-z^{-1}}, \text{ ROC: } |z| > 1$$

We have, $Y(z) = X(z) \cdot H(z)$

$\therefore$ $$Y(z) = \frac{1}{1-z^{-1}} \cdot \frac{1}{1-\alpha z^{-1}}, \text{ ROC: } |z| > 1$$

$$= \frac{A}{1-z^{-1}} + \frac{B}{1-\alpha z^{-1}}$$

$\therefore$ $$Y(z) = \frac{\frac{1}{1-\alpha}}{1-z^{-1}} - \frac{\frac{\alpha}{1-\alpha}}{1-\alpha z^{-1}}$$

$\Rightarrow$ $$y(n) = \frac{1}{1-\alpha} u(n) - \frac{\alpha}{1-\alpha} (\alpha)^n u(n)$$

$$= \frac{1}{1-\alpha} u(n) - \frac{\alpha^{n+1}}{1-\alpha} u(n)$$

$\therefore$ $$y(n) = \frac{1-\alpha^{n+1}}{1-\alpha} u(n)$$

When $n \to \infty$,

$$\alpha^{n+1} = 0 \qquad (\because |\alpha| < 1)$$

$\therefore$ Step response of the system

$$y(n) = \frac{1}{1-\alpha} \text{ when } n \to \infty$$

Q.4 (c) Solution:

Given, $$A(s) = \frac{1000}{\left(1+\frac{s}{10^4}\right)\left(1+\frac{s}{10^5}\right)^2}$$

and feedback factor β is independent of frequency

We have, $$\angle A(j\omega) = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$$

When phase shift is 180°, then

$$180° = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$$

Using: $$2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

We get, $$-180° = \tan^{-1}\left(\frac{\omega}{10^4}\right) + \tan^{-1}\left(\frac{2(\omega/10^5)}{1-(\omega/10^5)^2}\right)$$

Taking the tangent on both sides,

$$0 = \tan^{-1}\left(\frac{\frac{\omega}{10^4} + \frac{2\left(\frac{\omega}{10^5}\right)}{1-\left(\frac{\omega}{10^5}\right)}}{1-\left(\frac{\omega}{10^4}\right)\left(\frac{2\left(\frac{\omega}{10^5}\right)}{1-\left(\frac{\omega}{10^5}\right)^2}\right)}\right)$$

$$\frac{1}{10^4} + \frac{\frac{2}{10^5}}{1-\left(\frac{\omega}{10^5}\right)^2} = 0$$

$$1-\left(\frac{\omega}{10^5}\right)^2 = -0.2$$

$$\omega^2 = 1.2 \times 10^{10} \quad \Rightarrow \quad \omega = \omega_{pc} = 1.1 \times 10^5 \text{ rad/sec}$$

For oscillations to occurs

$$|A(j\omega_{pc})\beta| \geq 1$$

$$\frac{10^3\beta}{\sqrt{1+\left(\frac{\omega_{pc}}{10^4}\right)^2\left(1+\left(\frac{\omega_{pc}}{10^5}\right)^2\right)}} \geq 1$$

$$\frac{\beta\, 10^3}{\left(\sqrt{1+11^2}\right)\left(1+1.1^2\right)} \geq 1$$

$$\Rightarrow \qquad \beta \geq 0.0244$$

Section-B

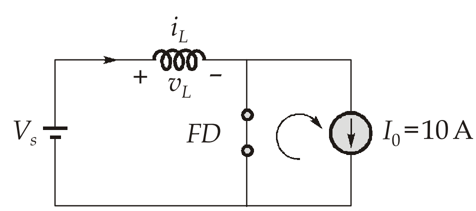
Q.5 (a) Solution:

(i) As switching frequency is in MHz range and desired ratings are small. MOSFET can be used as a switch because it has highest switching frequency and diode should be compatible. Therefore schottkey diode can be used.

(ii) Current through switch,

During $0 < t < t_1$

The freewheeling diode is in on state, so the circuit will be



Applying KVL, $\quad V_L = L\frac{di_L}{dt}$

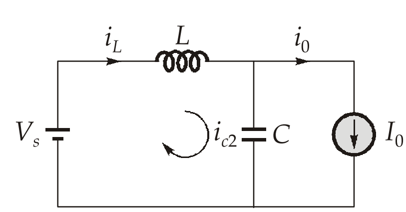
At $\quad t = t_1$

$i_L = I_0$

So freewheeling diode will stop conducting at

$$t = t_1 = \frac{LI_0}{V_s} = \frac{150\times10^{-6}\times10}{20} = 75\ \mu\text{s}$$

Now for $t > t_1$

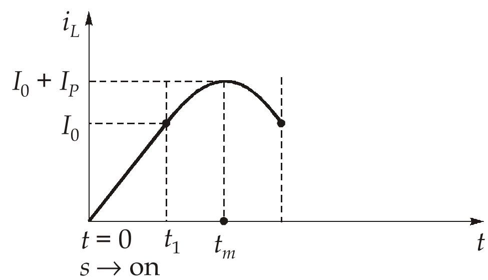


i_L will be,
$$i_L = I_0 + i_{C2}$$
$$i_L = I_0 + I_p \sin \omega t$$

The peak current through switch will be

$$I_m = I_0 + I_p$$
$$I_m = I_0 + V_s\sqrt{\frac{C}{L}} = 10 + 20\sqrt{\frac{0.05}{150}}$$
$$I_m = 10.37 \text{ A}$$

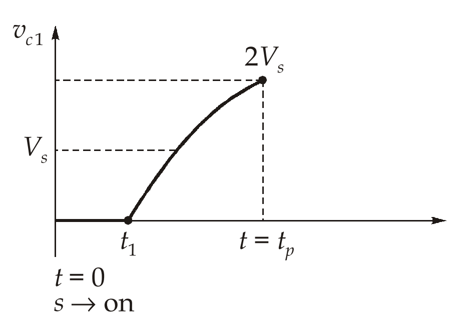
Current waveform will be



The peak value is obtained at $t = t_m$

$$t_m = t_1 + \frac{\pi}{2}\sqrt{LC} = \left(75 + \frac{\pi}{2}\sqrt{150 \times 0.05}\right) \times 10^{-6} = 79.3 \ \mu\text{s}$$

(iii) Capacitor voltage,



v_c is given by,

$$v_c = 0; \text{ for } t < t_1$$
$$v_c = V_s(1 - \cos \omega t); \text{ for } t_1 < t < t_p$$

v_c will be maximum at $t = t_p$

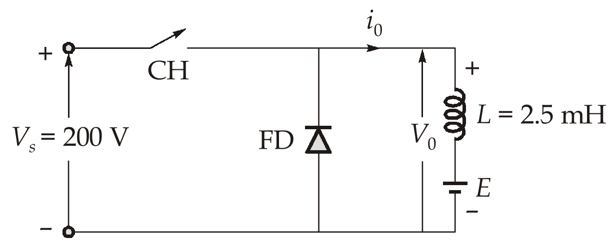
$$t_p = t_1 + \pi\sqrt{LC}$$
$$t_p = \left(75 + \pi\sqrt{150 \times 0.05}\right) \times 10^{-6} = 83.60 \ \mu\text{s}$$

Maximum capacitor voltage will be,

$$V_c \max = 2\ V_s = 40 \text{ V}$$

Q.5 (b) Solution:

Type-A chopper



Average output voltage is given by,

$$V_0 = \alpha V_s$$

$$\alpha = \text{Duty ratio} = 0.25$$

$$V_0 = 0.25 \times 200 = 50 \text{ V}$$

As the average value of voltage drop across L is zero

$$E = V_0 = \alpha V_s = 50 \text{ V}$$

During T_{on}, the difference in source voltage V_s and load emf E appears across inductance L. Also during T_{on}, the current through L rises from I_{min} to I_{max}.

Therefore,
$$v_L = L\frac{di}{dt}$$

$$V_s - E = L \cdot \frac{\Delta I}{T_{on}}$$

$$T_{on} = \frac{L \cdot \Delta I}{(V_s - E)}$$

$$T_{on} = \frac{2.5 \times 8 \times 10^{-3}}{200 - 50}$$

and,
$$\text{duty ratio, } \alpha = \frac{T_{on}}{T}$$

$$\frac{\alpha}{T_{on}} = f$$

$$\text{Chopping frequency, } f = \frac{\alpha}{T_{on}}$$

$$= \frac{0.25 \times 150}{2.5 \times 8 \times 10^{-3}} = 1.875 \text{ kHz}$$

Q.5 (c) Solution:

As we know efficiency of a transformer

$$\eta = \frac{P_{output}}{P_{input}} = \frac{VI\cos\phi}{VI\cos\phi + 2P_c}$$

$$0.985 = \frac{125\times 0.8}{125\times 0.8 + P_{cu} + P_c}$$

$$(P_{cu})_{\text{at max efficiency}} = P_c = 761 \text{ W}$$

We know $P_{cu} \propto S^2$

$$\Rightarrow \quad \frac{(P_{cu})_{rated}}{(P_{cu})_{max}} = \frac{S^2_{rated}}{S^2_{max}}$$

$$\Rightarrow \quad (P_{cu})_{rated} = \left(\frac{150}{125}\right)^2 \times 0.761 = 1.096 \text{ kW}$$

$$P_{h1} = P_{e1} = \frac{0.761}{2} = 0.3805 \text{ kW}$$

As we know $P_e \propto (f\beta_m)^2 \propto V^2$ (same)

$$P_h \propto f\beta_m{}^m = P_h \propto f\left(\frac{V}{f}\right)^n$$

$$P_h \propto \frac{1}{f^{n-1}} \Rightarrow P_h \propto \frac{1}{f^{1.6-1}} \Rightarrow P_h \propto \frac{1}{f^{0.6}}$$

$$\frac{P_{h1}}{P_{h2}} = \left(\frac{f_2}{f_1}\right)^{0.6}$$

$$\Rightarrow \quad \frac{P_{h2}}{P_{h1}} = \left(\frac{50}{40}\right)^{0.6}$$

$$\Rightarrow \quad P_{h2} = \left(\frac{50}{40}\right)^{0.6} \times 0.3805$$

$$\Rightarrow \quad P_{h2} = 0.4352 \text{ kW}$$

So, $P_{c2} = P_{h2} + P_{c2} = 0.4352 + 0.3805 = 0.816 \text{ kW}$

So, transformer efficiency, $\eta = \dfrac{150\times 1}{150\times 1 + P_{cu2} + P_{c2}} \times 100$

$$\eta = \frac{150\times 1}{150\times 1 + 1.096 + 0.816}\times 100 = 98.74\%$$

Q.5 (d) Solution:

The delay produced by the given subroutine program can be calculated by determining the total number of T-states required to execute the given program.

Total number of T-states required to execute the program can be determined by analyzing the given program as shown in table.

	Instruction	No. of times executed	No. of T-state for one time execution
Delay	MVI B, 02H	1	7
LOOP 2	MVI C, FFH	1 × 2 = 2	7
LOOP 1	DCR C	255 × 2 = 510	4
	JNZ LOOP 1	254 × 2 = 508 ⇒ True 1 × 2 = 2 ⇒ False	10 ⇒ True 7 ⇒ False
	DCR B	2	4
	JNZ LOOP 2	1 ⇒ True 1 ⇒ False	10 ⇒ True 7 ⇒ False
	RET	1	10

⇒ Total delay produced by the program in terms of T-state can be given by,

$$\begin{aligned}\text{Delay} &= (1 \times 7T) + (2 \times 7T) + (510 \times 4T) + (508 \times 10T) \\ &\quad + (2 \times 7T) + (2 \times 4T) + (10T + 7T) + 10T \\ &= 7T + 14\,T + 2040\,T + 5080\,T + 14T + 8T + 17T + 10T \\ &= 7190\,T\end{aligned}$$

Therefore, total delay produced $= 7190 \times \frac{1}{f} = \frac{7190}{2 \times 10^6} = 3595\ \mu\text{sec}$

Q.5 (e) Solution:

Given: Open loop transfer function (OLTF) as $G(s)H(s) = \frac{K}{(s+2)^2\,(s+3)}$

To determine the value of K:

(i) Position error constant $K_p \geq 2$

As we know, position error constant K_p is given by

$$K_p = \lim_{s \to 0} G(s)H(s)$$

So,
$$K_p = \lim_{s \to 0} \frac{K}{(s+2)^2\,(s+3)} = \frac{K}{12}$$

As given, $K_p \geq 2$

$$\Rightarrow \quad \frac{K}{12} \geq 2$$

So, $K \geq 24$

(ii) Gain margin ≥ 3

$$G(s)H(s) = \frac{K}{(s^2+4s+4)(s+3)} = \frac{K}{s^3+7s^2+16s+12}$$

Substituting $s = j\omega$,

$$\Rightarrow \quad G(j\omega)H(j\omega) = \frac{K}{-j\omega^3-7\omega^2+j16\omega+12} = \frac{K}{j(16\omega-\omega^3)+(12-7\omega^2)}$$

For imaginary part to be zero,

$$16\omega - \omega^3 = 0$$

$$\omega(16-\omega^2) = 0$$

$$\Rightarrow \quad \omega = 0, 4 \text{ rad/sec}$$

So, $\omega = 4$ rad/sec is phase cross-over frequency, ω_{pc}.

Now, $G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}=4\text{ rad/sec}} = \dfrac{K}{12-7(4)^2} = -\dfrac{K}{100}$

$$\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}} = \frac{K}{100}$$

Gain margin, $$\text{GM} = \frac{1}{\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}}} = \frac{1}{K/100} = \frac{100}{K}$$

As per question, $\text{GM} \geq 3$

$$\Rightarrow \quad \frac{100}{K} \geq 3 \quad \Rightarrow K \leq \frac{100}{3} = 33.33$$

$$\Rightarrow \quad K \leq 33.33$$

So, the value of K satisfying above both conditions is given by

$$24 \leq K \leq 33.33$$

Q.6 (a) Solution:

Since R_{EE} = 200 kΩ is very high, the current through R_{EE} can be neglected. Thus, the collector current in Q_1 and Q_2 are

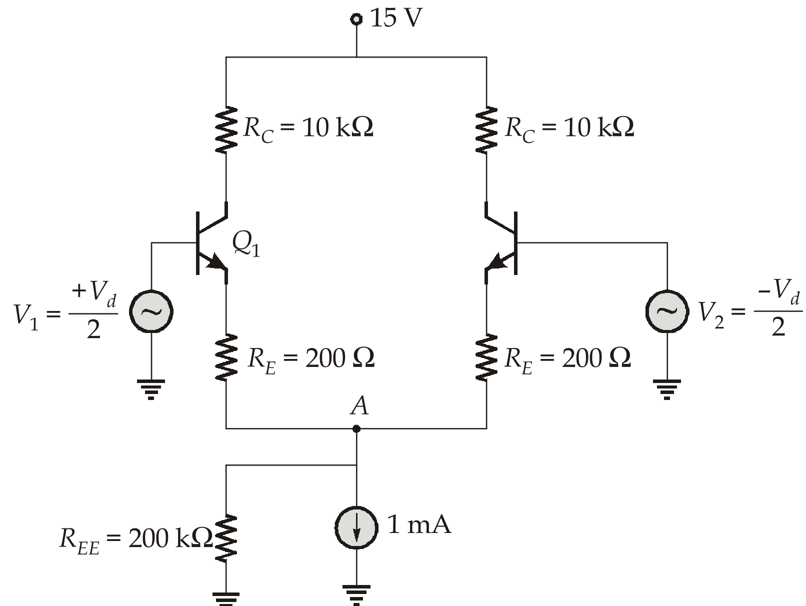
$$I_{C1} = I_{C2} = \frac{I}{2} = \frac{1}{2} = 0.5\text{ mA}$$

The small signal parameters of the transistors can be obtained as below,

$$g_m = \frac{I_C}{V_T} = \frac{0.5\times10^3}{25\times10^{-3}} = 20\text{ m℧}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5\text{ k}\Omega$$

(i) Differential mode Analysis



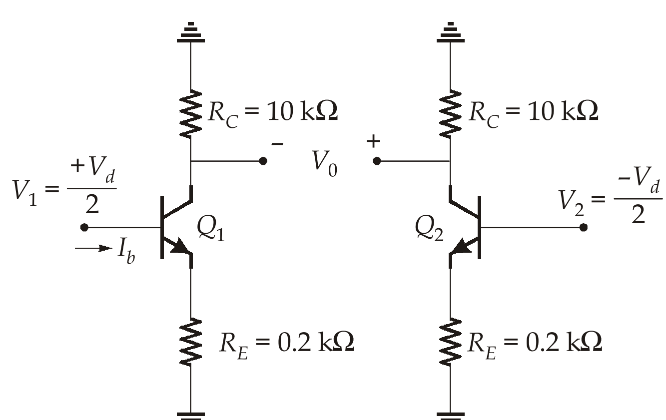
In differential mode analysis,

We assume input as

$$V_1 = \frac{V_d}{2} \text{ and } V_2 = \frac{-V_d}{2}$$

As the circuit is symmetric and $V_1 = -V_2$. In differential mode analysis, Node A can be assumed as AC ground.

For AC analysis, 1 mA current source is considered as open-circuit dc voltage source 15 V is considered as short circuit and since V_A = 0, R_{EE} would be dead resistor.



Replacing Q_1 with π-model.

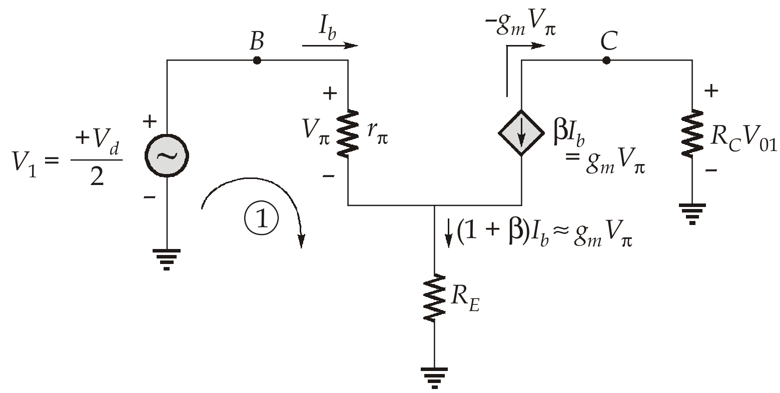


Fig. 1

KVL in base-emitter loop:

$$V_1 = I_b r_\pi + (1+\beta) I_b R_E$$

$$\frac{V_1}{I_b} = r_\pi + (1+\beta) R_E$$

We have,
$$V_1 = \frac{+V_d}{2}$$

$$\frac{\frac{V_d}{2}}{I_b} = r_\pi + (1+\beta) R_E$$

$$R_{id} = \frac{V_d}{I_b} = 2[r_\pi + (1+\beta) R_E]$$

R_{id}: Differential input resistance

Substituting the values

$$R_{id} = 2[5 + 101 \times 0.2]$$

$$R_{id} \cong 50 \text{ k}\Omega$$

(ii) Differential voltage gain $\left[A_{DM} = \frac{V_0}{V_d}\right]$

From fig. (1),

$$V_{01} = -g_m V_\pi R_C \quad ...(1)$$

and
$$V_1 = V_\pi + g_m V_\pi R_E \quad ...(2)$$

From (1) ÷ (2), we get
$$\frac{V_{01}}{V_1} = \frac{-g_m V_\pi R_C}{V_\pi (1 + g_m R_E)}$$

$$\Rightarrow \qquad \frac{V_{01}}{V_1} = \frac{-g_m R_C}{1+g_m R_E}$$

As $V_1 = \frac{V_d}{2}$,

$$\frac{V_{01}}{\frac{V_d}{2}} = \frac{-g_m R_C}{1+g_m R_E}$$

$$\frac{V_{01}}{V_d} = \frac{-g_m R_C}{2(1+g_m R_E)}$$

$\frac{V_{01}}{V_d}$: Differential gain when o/p is unbalanced

Similarly when $V_2 = \frac{-V_d}{2}$

$$\frac{V_{02}}{V_2} = \frac{-g_m R_C}{1+g_m R_E}$$

$$\frac{V_{02}}{V_d} = \frac{g_m R_C}{2(1+g_m R_E)}$$

$\frac{V_{01}}{V_d}$ and $\frac{V_{02}}{V_d}$ are differential mode gain for unbalanced output or single ended output.

$\frac{V_0}{V_d}$ is diffence mode gain for balanced output. We can write,

$$A_{DM} = \frac{V_0}{V_d} = \frac{V_{02}}{V_d} - \frac{V_{01}}{V_d}$$

Hence,

$$A_{DM} = \frac{g_m R_C}{1+g_m R_E}$$

Putting the values, we get

$$A_{DM} = \frac{20\times 10}{1+20\times 0.2} = 40$$

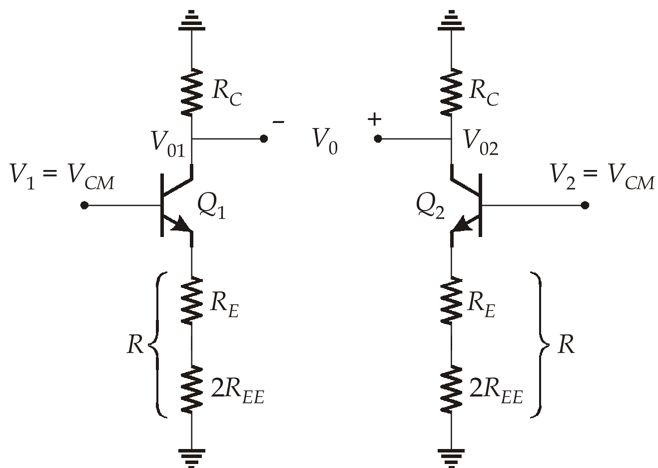
(iii) Common mode analysis:

We assume input as $V_1 = V_2 = V_{CM}$

and Common mode gain,

$$A_{CM} = \frac{V_0}{V_{CM}} = \frac{V_{02}-V_{01}}{V_{CM}}$$

In common mode analysis, we apply symmetry property



Q_1 and Q_2 are CE amplifier with unbypassed emitter resistance R.

$$\frac{V_{01}}{V_{CM}} = \frac{V_{02}}{V_{CM}} = \frac{-g_m R_C}{1+g_m R}$$

$$= \frac{-g_m R_C}{1+g_m(R_E+2R_{EE})}$$

$$\frac{V_{01}}{V_{CM}} \cong \frac{V_{02}}{V_{CM}} \cong \frac{-R_C}{R_E+2R_{EE}}$$

$$A_{CM} = \frac{V_0}{V_{CM}} = \frac{V_{02}-V_{01}}{V_{CM}}$$

$$A_{CM} = \frac{V_{02}}{V_{CM}} - \frac{V_{01}}{V_{CM}}$$

If R_C values are perfectly equal, then $A_{CM} = 0$.

In the question, it is given that there is some mismatch. Hence, common mode gain of differential amplifier becomes non-zero, because of mis-match in the value of R_C.

Now, $$\frac{V_{02}}{V_{CM}} = \frac{-R_{C2}}{R_E+2R_{EE}} \text{ and } \frac{V_{01}}{V_{CM}} = \frac{-R_{C1}}{R_E+2R_{EE}}$$

Common mode gain in worst case is equal to maximum possible common mode gain. A_{CM} is maximum if

$$R_{C1} = R_C + \Delta R$$
$$R_{C2} = R_C - \Delta R$$

$$A_{CM} = \frac{R_{C1} - R_{C2}}{R_E + 2R_{EE}}$$

$$A_{CM} = \frac{2\Delta R}{R_E + 2R_{EE}}$$

Given, $\Delta R = 1\%$ of R_C

$= 1\%$ of $10\ k\Omega$

$= 0.1\ k\Omega$

$$A_{CM} = \frac{2 \times 0.1}{0.2 + 2 \times 200}$$

$$A_{CM} \cong 0.5 \times 10^{-3}$$

(iv)

$$(\text{CMRR})_{\text{dB}} = 20\log_{10}\left|\frac{A_{DM}}{A_{CM}}\right|$$

$$= 20\log_{10}\left|\frac{40}{0.5 \times 10^{-3}}\right|$$

$$= 98\ \text{dB}$$

Hence, CMRR is 98 dB.

Q.6 (b) Solution:

(i) For sinusoidal signal $m(t) = A\cos(2\pi ft)$. To avoid slope overload distortion,

$$\frac{\sigma}{T_s} \geq \left|\frac{d}{dt}m(t)\right|_{\max} = 2\pi f A_{\max} = \omega A_{\max}$$

$$A_{\max} = \frac{\sigma f_s}{\omega}$$

or,

$$\sigma = \frac{\omega A_{\max}}{f_s} = \frac{2\pi \times 3.4 \times 10^3}{64000},$$

$$\sigma = 0.3337$$

(ii)

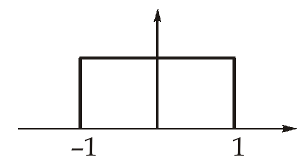
$$N_0 = \frac{\sigma^2 B}{3 f_s} = \frac{(0.3337)^2 (3.4 \times 10^3)}{3(64000)} = 0.00197$$

(iii) For sinusoidal signal,

$$S_0 = \frac{A^2}{2} = 0.5$$

$$\text{SNR} = \frac{S_0}{N_0} = \frac{0.5}{0.00197} = 253.80 = 24.04\ \text{dB}$$

(iv) For uniform distribution, $S_0 = \dfrac{(1-(-1))^2}{12} = \dfrac{1}{3}$



So, $\dfrac{S_0}{N_0} = \dfrac{0.333}{0.00197} = 22.28 \text{ dB}$

(v) The minimum transmission bandwidth BT for a baseband signal (using Nyquist criterion) is:

$$B_T = \frac{R_b}{2} = \frac{64 \text{ kHz}}{2} = 32 \text{ kHz}$$

Q.6 (c) Solution:

The sequence networks are shown in figure below:

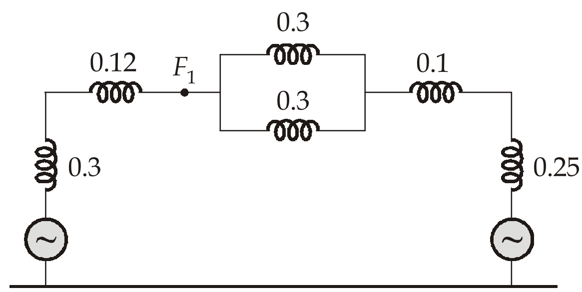


Fig. (a)

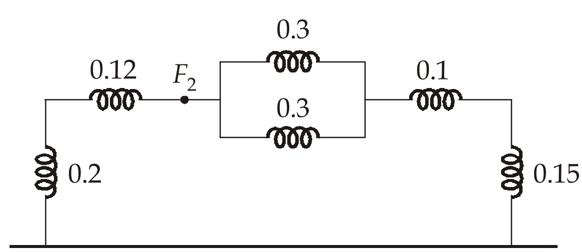


Fig. (b)

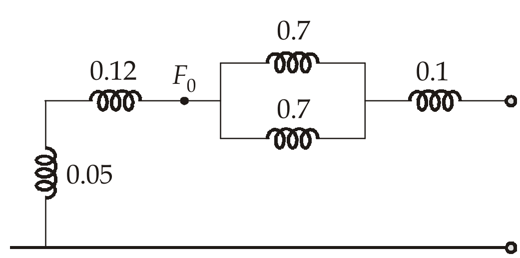


Fig. (c)

From the positive-sequence network of figure (a), the equivalent impedance upto the point of the fault is given by

$$Z_1 = j\left[(0.3+0.12) \,\Big\|\, \left(0.25+0.1+\frac{0.3}{2}\right)\right]$$

$$= j[(0.42) \,\|\, (0.5)] = j\frac{0.42\times 0.5}{0.42+0.5} = j0.22826 \text{ pu}$$

From the negative-sequence network of Fig. (b), the equivalent impedance upto F is given by

$$Z_2 = j\left[(0.2+0.12) \,\Big\|\, \left(0.15+0.1+\frac{0.3}{2}\right)\right]$$

$$= j(0.32) \,\|\, (0.4) = j\frac{0.32\times 0.4}{0.32+0.4} = j0.1778 \text{ pu}$$

From the zero-sequence network of Fig. (c), the equivalent impedance upto F is given by

$$Z_0 = j(0.05 + 0.12) = j0.17 \text{ pu}$$

(i) LLG Fault at *F*

If phase a is assumed to be the reference phasor and phases b and c are shorted at the fault.

$$I_{a1} = \frac{V_f}{Z_{a1} + \dfrac{Z_{a0}Z_{a2}}{Z_{a0}+Z_{a2}}} = \frac{1\angle 0^\circ}{j\left(0.22826 + \dfrac{0.17\times 0.1778}{0.17+0.1778}\right)}$$

$$= -j3.1729 \text{ pu} = 3.1729 \angle -90^\circ \text{ pu}$$

If we put $Z_f = 0$ and $Z_g = 0$ then by current division rule

$$I_{a0} = -I_{a1}\frac{Z_{a2}}{Z_{a0}+Z_{a2}} = j\left(3.1729\times\frac{0.1778}{0.17+0.1778}\right)$$

$$= j1.622 \text{ pu} = 1.622\angle 90^\circ \text{pu}$$

Check

$$I_a = I_{a0} + I_{a1} + I_{a2} = j1.622 - j3.1729 + j1.551 \simeq 0$$

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$

$$= j1.622 + (1\angle 240^\circ)(3.1729\angle -90^\circ) + (1\angle 120^\circ)(1.551\angle 90^\circ)$$

$$= j1.622 + 3.1729\angle 150° + 1.551\angle 210°$$
$$= j1.622 - 2.7478 + j1.5864 - 1.3432 - j0.7755$$
$$= -4.091 + j2.4729$$
$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$
$$= j1.622 + (1\angle 120°)(3.1729\angle -90°) + (1\angle 240°)(1.551\angle 90°)$$
$$= j1.622 + 2.7478 + j1.5864 + 1.3432 - j0.7755$$
$$= 4.091 + j2.4729$$
$$|I_b| = |I_c| = \sqrt{(4.091)^2 + (2.4729)^2} = 4.78 \text{ pu}$$

(ii) LL Fault at F

If the line-to-line fault is between phases b and c.

$$I_{a1} = \frac{V_f}{Z_{a1} + Z_{a2}} = \frac{1\angle 0°}{j(0.22826 + 0.1778)}$$
$$= -j2.4627 \text{ pu} = 2.4627\angle -90° \text{ pu}$$

The phase a negative-sequence current is given by

$$I_{a2} = -I_{a1} = +j2.4627 \text{ pu} = 2.4627\angle 90° \text{ pu}$$

The phase a fault current

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 - j2.4627 + j2.4627 = 0$$

The phase b fault current

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$
$$= 0 + (1\angle 240°)(2.4627\angle -90°) + (1\angle 120°)(2.4627\angle 90°)$$
$$= 2.4627\angle 150° + 2.4627\angle 210°$$
$$= -2.133 + j1.231 - 2.133 - j1.231 = -4.266 \text{ pu}$$

The phase c fault current

$$I_c = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}$$
$$= 0 + (1\angle 120°)(2.4627\angle -90°) + (1\angle 240°)(2.4627\angle 90°)$$
$$= 2.4627\angle 30° + 2.4627\angle 330°$$
$$= 2.133 + j1.231 + 2.133 - j1.231 = 4.266 \text{ pu}$$

Thus, it is calculated that

$$I_b = -I_c = -4.266 \text{ pu}$$

Q.7 (a) Solution:

Since the load is at the bus of plant 2, therefore, the line loss will not be affected by variation of P_2. Thus,

$$B_{12} = B_{21} = 0 \text{ and } B_{22} = 0$$

For a 2-plant system

$$P_L = P_1^2 B_{11} + P_2^2 B_{22} + 2P_1 P_2 B_{12}$$

When $P_1 = 125$ MW, $P_L = 12.5$ MW

we have $12.5 = (125)^2 B_{11} + 0 + 0$

or $$B_{11} = \frac{12.5}{(125)^2} = 8\times10^{-4}\ \text{MW}^{-1}$$

Therefore, $P_L = 8 \times 10^{-4}\ P_1^2$; $\dfrac{\partial P_L}{\partial P_1} = 16 \times 10^{-4}\ P_1$

We are also given $\lambda = 70$ Rs/MWh.

(i) Use of coordinates equation

The coordination equation for plant 1 is given by

$$\frac{dC_1}{dP_1} + \lambda \frac{\partial P_L}{\partial P_1} = \lambda$$

Substituting the values of $\dfrac{dC_1}{dP_1}$, λ and $\dfrac{\partial P_L}{\partial P_1}$ we have

$0.25P_1 + 40 + 70 \times 16 \times 10^{-4} P_1 = 70$

$$(0.25 + 0.112)P_1 = 30, \quad P_1 = 82.8729 \text{ MW}$$

The coordination equation for plant 2 is given by

$$\frac{dC_2}{dP_2} + \lambda \frac{\partial P_L}{\partial P_2} = \lambda$$

or $$0.20P_2 + 50 + 0 = 70, \quad P_2 = 100 \text{ MW}$$

The line loss is given by

$$P_L = 8\times10^{-4} P_1^2 = 8 \times 10^{-4} \times (82.8729)^2 = 5.494 \text{ MW}$$

Therefore, the total load, $P_R = P_1 + P_2 - P_L$

$= 82.8729 + 100 - 5.494$

$= 177.3789$ MW

(ii) Use of penalty factor

The penalty factor for plant 1 is given by

$$L_1 = \frac{1}{1-\frac{\partial P_L}{\partial P_1}} = \frac{1}{(1-16\times10^{-4}P_1)}$$

For optimal dispatch

$$L_1\frac{dC_1}{dP_1} = \lambda; \quad \frac{1}{(1-16\times10^{-4}P_1)}\times(0.25P_1+40)=70$$

or $$0.25P_1 + 40 = 70 - 70 \times 16 \times 10^{-4} P_1$$

or $$P_1 = \frac{30}{0.25+0.112} = 82.8729 \text{ MW}$$

Penalty factor for plant 2,

$$L_2 = \frac{1}{1-\frac{\partial P_L}{\partial P_2}} = \frac{1}{1-0} = 1$$

For optimal dispatch,

$$L_2\frac{dC_2}{dP_2} = \lambda; \quad 1 \times (0.20P_2 + 50) = 70$$

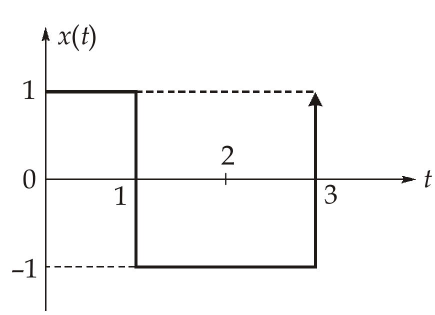
$$P_2 = \frac{20}{0.20} = 100 \text{ MW}$$

It is seen that both the methods give identical results. This is due to the fact that the penalty factor has been derived from the coordination equation.

Q.7 (b) Solution:

(i) Part 1:

Given waveform of $x(t)$ is



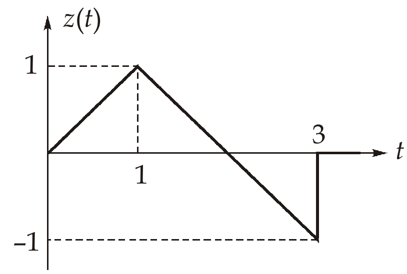
We can express $x(t)$ as:

$$x(t) = u(t) - 2u(t-1) + u(t-3) + \delta(t-3)$$

Let
$$z(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

then
$$z(t) = \int_{-\infty}^{t} u(\tau)d\tau - 2\int_{-\infty}^{t} u(\tau-1)d\tau + \int_{-\infty}^{t} u(\tau-3)d\tau + \int_{-\infty}^{t} \delta(\tau-3)d\tau$$

$$z(t) = r(t) - 2r(t-1) + r(t-3) + u(t-3)$$



Part 2:

We can express $y(t)$ as,

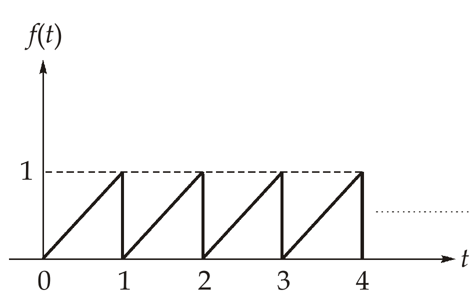
$$y(t) = u(t) - \sum_{n=1}^{\infty} \delta(t-n)$$

Let
$$f(t) = \int_{-\infty}^{t} y(\tau)d\tau$$

$$f(t) = \int_{-\infty}^{t} u(\tau)d\tau - \sum_{n=1}^{\infty}\left[\int_{-\infty}^{t} \delta(t-n)d\tau\right]$$

$$f(t) = r(t) - \sum_{n=1}^{\infty} u(t-n)$$

$$= r(t) - u(t-1) - u(t-2) - u(t-3) - \text{.........}$$



(ii) Given : $K_1 = \frac{1}{2}, K_2 = \frac{1}{4}$

The transfer function of a lattice filter of order 'm' is given by

$$A_m(z) = A_{m-1}(z) + K_m z^{-m} A_{m-1}(z^{-1})$$

with initial condition, $A_0(z) = 1$

We have,

$$A_1(z) = A_0(z) + K_1 z^{-1} A_0(z^{-1}) = 1 + \frac{1}{2}z^{-1} \times 1$$

$$= 1 + \frac{1}{2}z^{-1} \qquad (\because A_0(z) = 1; \text{ thus } A_0(z^{-1}) = 1)$$

$$A_2(z) = A_1(z) + K_2 z^{-2} A_1(z^{-1})$$

$$= \left(1 + \frac{1}{2}z^{-1}\right) + \frac{1}{4}z^{-2}\left(1 + \frac{1}{2}z\right)$$

$$\left(\because A_1(z^{-1}) = A_1(z)\Big|_{z \to z^{-1}} = 1 + \frac{1}{2}z\right)$$

$$= 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-1}$$

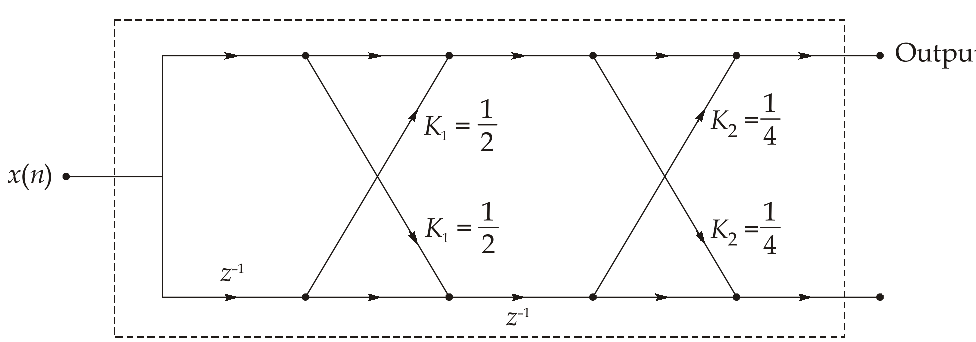
$$A_2(z) = 1 + \frac{5}{8}z^{-1} + \frac{1}{4}z^{-2}$$

We have,

$$A_2(z) = \sum_{i=0}^{2} a_2(i) z^{-1}$$

thus impulse response,

$$a_2(0) = 1,\ a_2(1) = \frac{5}{8},\ a_2(2) = \frac{1}{4}$$



Q.7 (c) Solution:

(i) When α is greater than 90°, then average output voltage is negative. If load circuit emf (DC source) is reserved, then this battery will feed power back to AC source.

$$V_0 = \frac{3V_{mL}}{\pi}\cos\alpha = I_0 R - E$$

$$\frac{3\times\sqrt{2}\times 4160}{\pi}\times\cos 120° = (I_0 \times 2) - 3000$$

$$I_0 = 95.5 \text{ A}$$

The power absorbed by the bridge and transferred back to the AC system

$$P_{ac} = I_0 V_0 = 95.5 \times (-2808.98) = 268.26 \text{ kW}$$

Power supplied by the DC source,

$$P_{dc} = I_0 V_{dc} = 95.5 \times 3000$$

$$P_{dc} = 286.5 \text{ kW}$$

(ii) Hence this is a six pulse converter, the normalized harmonic voltage is given,

$$\frac{V_6}{V_m} = 0.28$$

$$V_6 = 0.28 \times V_m = 0.28\times\sqrt{2}\times 4160 = 1647 \text{ V}$$

The peak to peak variation of 10 percent corresponds to a zero to peak amplitude of

$$0.05 \times 95.5 = 4.8 \text{ A}$$

$$Z_6 = \frac{V_6}{I_6} = \frac{1647}{4.8} = 343.125\ \Omega$$

$$Z_6 = \sqrt{R^2 + (6\omega L)^2}$$

$$\frac{(343.125)^2 - 4}{(6\times 2\pi\times 50)^2} = L^2$$

$$L = \sqrt{0.033135} = 0.18 \text{ H}$$

Q.8 (a) Solution:

(i) Using Routh's array table,

$$\begin{array}{lll} s^4 & b_0 & b_2 \quad b_4 \\ s^3 & b_1 & b_3 \\ s^2 & \dfrac{b_1 b_2 - b_0 b_3}{b_1} & b_4 \\ s^1 & \dfrac{\left[\dfrac{b_1 b_2 - b_0 b_3}{b_1}\right] b_3 - b_1 b_4}{\left[\dfrac{b_1 b_2 - b_0 b_3}{b_1}\right]} & \\ s^0 & b_4 & \end{array}$$

For system to be stable, there should be no sign change in the first column of Routh Array. Thus, the stability conditions are given by

$$b_0 > 0; b_1 > 0; b_1 b_2 - b_0 b_3 > 0;$$

$$(b_1 b_2 - b_0 b_3) b_3 - b_1^2 b_4 > 0$$

and $$b_4 > 0$$

(ii) For the given characteristic equation, Routh's Array can be drawn as below,

$$\begin{array}{llll} s^5 & 1 & 1 & 4 \\ s^4 & 1 & 1 & 4 \\ \boxed{s^3} & 0(4) & 0(2) & \\ s^2 & \dfrac{4-2}{4} = \dfrac{1}{2} & 4 & \\ s^1 & \dfrac{1-16}{0.5} = -30 & & \\ s^0 & 4 & & \end{array}$$

Using Auxiliary equation, we get

$$A(s) = s^4 + s^2 + 4 = 0$$

$$\frac{dA(s)}{ds} = 4s^3 + 2s$$

From the Routh-Hurtwiz table, since sign change occurs in the first column of Routh Array. Hence, the system is unstable. Further, we conclude that

- two poles are in right side of s-plane [since there are two sign changes in the first column of Routh Array].

- four poles are complex in nature [as s^3 row is zero].

Complex poles are given by:

$$s^4 + s^2 + 4 = 0 \Rightarrow s^2 = -0.5 \pm 1.936i = 2\angle \pm 104.48°$$

We get $s = \pm 1.414\angle \pm 52.24°$. Therefore,

$$s = (0.866 + 1.117i);\ (0.866 - 1.117i)$$

$$s = (-0.866 + 1.117i);\ (-0.866 - 1.117i)$$

Now,

the characteristic equation can be represented as

$$(s + a)(s^4 + s^2 + 4) = 0$$

$$s^5 + s^3 + 4s + as^4 + as^2 + 4a = 0$$

$$s^5 + as^4 + s^3 + as^2 + 4s + 4a = 0$$

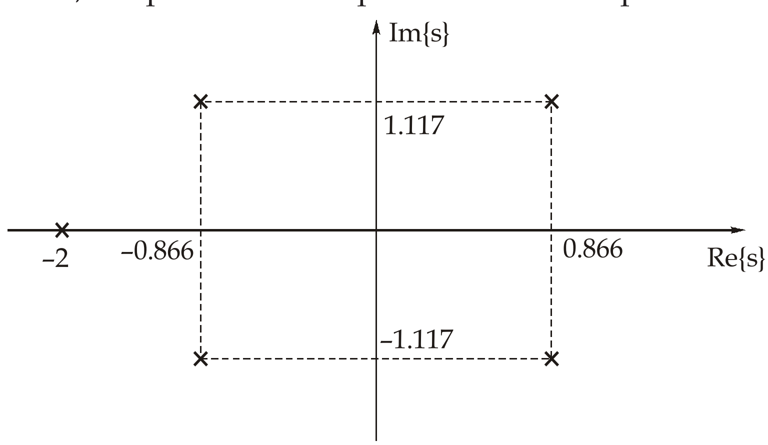
and on comparing with the given characteristic equation, we get, $a = 1$

Thus, Fifth pole lies at $s = -a$ i.e. at

$$s = -1$$

Hence, poles are at $s = (0.866 + 1.117i), (0.866 - 1.117i), (-0.866 + 1.117i), (-0.866 - 1.117i)$ and -1

Thus, the poles can be represented in the s-plane as below:



Q.8 (b) Solution:

(i) 1. Given information are:

(a) $x(n)$ is right-sided.

(b) $X(z)$ has single pole.

(c) $x(0) = 4,\ x(2) = \dfrac{1}{4}$

From (b), $X(z) = \dfrac{K}{1-az^{-1}}$

By applying inverse z-transform

$$x(n) = K(a)^n u(n) \quad \text{...(i)}$$

as $x(n)$ is right sided.

Put $n = 0$: $x(0) = K = 4$ ...from information (c)

$\Rightarrow$ $K = 4$

Put $n = 2$:

$$x(2) = K \cdot a^2 = \frac{1}{4}$$

$$a^2 = \frac{1}{4K} = \frac{1}{16}$$

$\Rightarrow$ $$a = \frac{1}{4}$$

From (i), $$x(n) = K(a)^n u(n)$$

$$= 4\left(\frac{1}{4}\right)^n u(n)$$

2. Given information are:

(a) Poles: $P_1 = \dfrac{1}{4}, P_2 = -1$

(b) ROC includes $|z| = \dfrac{1}{2}$

(c) $x(1) = 1, \quad x(-1) = 1$

From (a), $$X(z) = \frac{K}{(z-P_1)(z-P_2)}$$

$$= \frac{K}{\left(z-\frac{1}{4}\right)(z+1)} = \left[\frac{K \cdot z^2}{\left(z-\frac{1}{4}\right)(z+1)}\right] z^{-2}$$

$$= \left[\frac{K}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+z^{-1}\right)}\right] z^{-2}$$

$\Rightarrow$ $$X(z) = F(z)z^{-2} \quad \text{...(i)}$$

where, $$F(z) = \frac{K}{\left(1-\frac{1}{4}z^{-1}\right)(1+z^{-1})}$$

$$F(z) = K\left[\frac{A}{1-\frac{1}{4}z^{-1}} + \frac{B}{1+z^{-1}}\right]$$

Residue calculation:

$$A = \left(\frac{1}{1+z^{-1}}\right)\Bigg|_{z^{-1}=4} = \frac{1}{1+4} = \frac{1}{5}$$

$$B = \left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\Bigg|_{z^{-1}=-1} = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$$

Thus, $$F(z) = K\left[\frac{1}{5}\cdot\frac{1}{1-\frac{1}{4}z^{-1}} + \frac{4}{5}\cdot\frac{1}{1+z^{-1}}\right] \quad \text{...(ii)}$$

From information (b), since ROC includes $|z| = \frac{1}{2}$. Thus, ROC: $\frac{1}{4} < |z| < 1$

i.e., $x(n)$ will be a both sided signal.

By applying inverse z-transform on (ii)

$$F(z) = K\left[\frac{1}{5}\cdot\left(\frac{1}{4}\right)^n u(n) - \frac{4}{5}\cdot(-1)^n u(-n-1)\right]$$

From (i), $$X(z) = F(z)z^{-2}$$

By using time shifting property of z-transform,

$$x(n) = f(n-2)$$

$$\Rightarrow \quad x(n) = K\left[\frac{1}{5}\left(\frac{1}{4}\right)^{n-2} u(n-2) - \frac{4}{5}(-1)^{n-2} u(-(n-2)-1)\right]$$

$$x(n) = K\left[\frac{1}{5}\left(\frac{1}{4}\right)^{n-2} u(n-2) - \frac{4}{5}(-1)^{n-2} u(-n+1)\right]$$

Put $n = 1$: $$x(1) = K\left[\frac{-4}{5}(-1)^{-1}\right] = 1 \quad \text{.....from information (c)}$$

$$\Rightarrow \qquad K = \frac{5}{4}$$

Using the above value of K and substituting $n = -1$, we can verify,

$$x(-1) = K\left[\frac{-4}{5}(-1)^{-3}\right] = 1 \qquad \text{...from information (c)}$$

Thus,
$$x(n) = \frac{5}{4}\left[\frac{1}{5}\left(\frac{1}{4}\right)^{n-2} u(n-2) - \frac{4}{5}(-1)^{n-2} u(-n+1)\right]$$

$$\Rightarrow \qquad x(n) = \left(\frac{1}{4}\right)^{n-1} u(n-2) - (-1)^{n-2} u(-n+1)$$

(ii) 1. The complex Fourier series coefficient is given by

$$C_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}dt$$

The given signal is periodic with time period $T = 1$ and $f(t) = t$ for $0 \le t \le 1$

$$= \frac{1}{1}\int_0^1 te^{-jn\cdot\frac{2\pi}{1}t}dt = \left[t \times \frac{e^{-j2\pi nt}}{-j2\pi n}\right]_0^1 - \int_0^1 \frac{e^{-j2\pi nt}}{-j2\pi n}dt$$

Now, $\quad e^{-jn\pi} = (-1)^n$

$\therefore \quad e^{-j2n\pi} = (-1)^2 = 1$

$$\therefore \qquad C_n = \frac{1}{-j2\pi n} - \left[\frac{e^{-j2\pi nt}}{(-j2\pi n)^2}\right]_0^1 = \frac{1}{-j2\pi n} + \frac{1}{(2\pi n)^2} - \frac{1}{(2\pi n)^2}$$

$$C_n = \frac{j}{2\pi n}$$

but C_n becomes ∞ when we put $n = 0$, so again calculate C_n at $n = 0$

We have, $\quad C_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}dt$

putting, $\quad n = 0$

$$\therefore \qquad C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{1}\int_0^1 t\,dt = \left[\frac{t^2}{2}\right]_0^1$$

$$\therefore \qquad C_0 = \frac{1}{2}$$

$$\therefore \qquad C_n = \begin{cases} \frac{1}{2} & \text{when } n = 0 \\ \frac{j}{2\pi n} & \text{when } n \neq 0 \end{cases}$$

Thus, the complex Fourier series representation of a signal is given by

$$x(n) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt}$$

2. Magnitude, $C_0 = \frac{1}{2}$

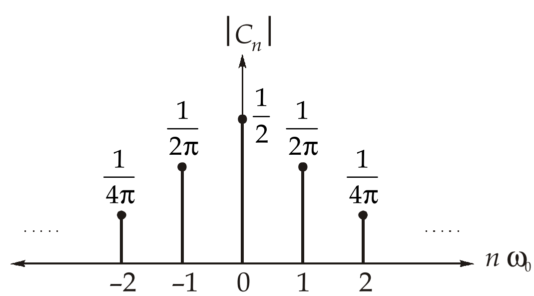
and $|C_n| = \left|\frac{j}{2\pi n}\right| = \frac{1}{2\pi n}$

Phase, $\angle C_n = +j = \frac{\pi}{2}$ when $n > 0$

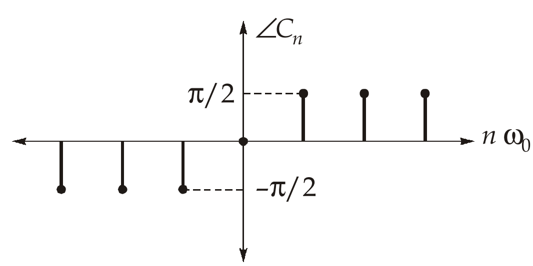
$= -j = \frac{-\pi}{2}$ when $n < 0$

$= 0$ when $n = 0$

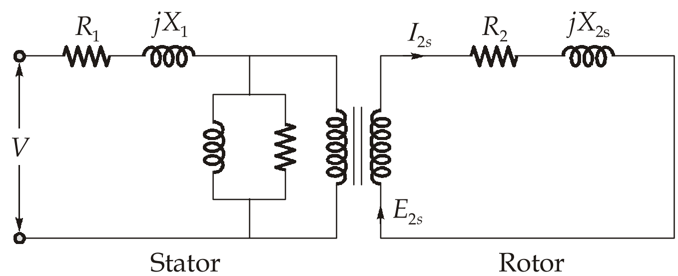
Magnitude plot:



Phase plot:



Q.8 (c) Solution:



Equivalent circuit of an induction motor

(i) Power generated in the rotor = 3 × 1 phase power

$$= 3E_{2s}\, I_{2s} \cos\phi_{2s} = 3E_{2s}\frac{E_{2s}}{Z_{2s}} \times \frac{R_2}{Z_{2s}} = 3\frac{E_{2s}^2 R_2}{Z_{2s}^2}$$

$$= 3s^2 \frac{E_{20}^2 R_2}{R_2^2 + (sX_{20})^2} \qquad \begin{bmatrix} E_{2s} = sE_{20} \\ X_{2s} = sX_{20} \end{bmatrix}$$

All this power will be lost as copper losses.

$$\Rightarrow \quad sP_g = \frac{3s^2 E_{20}^2 R_2}{R_2^2 + (sX_{20})^2} \quad [P_g = \text{Airgap power}]$$

$$\Rightarrow \quad (s2\pi n_s \tau) = \frac{3s^2 E_{20}^2 R_2}{R_2^2 + (sX_{20})^2} \quad \begin{bmatrix} n_s = \text{speed in rps} \\ \tau = \text{torque} \end{bmatrix}$$

$$\Rightarrow \quad \tau = \left(\frac{3}{2\pi n_s}\right)\frac{sE_{20}^2 R_2}{R_2^2 + (sX_{20})^2} = \underbrace{\frac{3E_{20}^2 R_2}{2\pi n_s}}_{\text{constant}} \times \frac{1}{\frac{R_2^2}{s} + sX_{20}^2}$$

Maximum torque will occur when,

$$\frac{R_2^2}{s} = sX_{20}^2$$

$$\Rightarrow \quad s = \frac{R_2}{X_{20}}$$

To have maximum starting torque, the maximum torque will have to occur at $s = 1$

$$\Rightarrow \quad \frac{R_2}{X_{20}} = 1$$

$$\Rightarrow \quad R_2 = X_{20}$$

⇒ Condition for maximum starting torque

(ii) Given, $P = 4$; $f = 50$ Hz

$$\therefore \quad \text{Synchronous speed,} \quad N_s = \frac{120f}{p}$$

$$\Rightarrow \quad N_s = 120 \times \frac{50}{4} = 1500 \text{ rpm}$$

Let, s_M = slip of maximum torque ; s_{fl} = slip at full load ; $s = 1$ at starting

$$\text{Torque,} \quad \tau = \frac{3E_{20}^2 R_2}{2\pi n_s} \times \frac{1}{R_2 X_{20}} \times \frac{1}{\frac{R_2}{sX_{20}} + \frac{sX_{20}}{R_{20}}}$$

$$\Rightarrow \quad \tau = k \times \frac{1}{\frac{s_M}{s} + \frac{s}{s_M}} \quad \left[k = \frac{3E_{20}^2}{2\pi X_{20} n}\right]$$

1. τ_{fl} = Full load torque

τ_s = Starting torque

$$\text{Now,} \quad \frac{\tau_{fl}}{\tau_s} = \frac{\dfrac{1}{\frac{s_M}{s_{fl}} + \frac{s}{s_M}}}{\dfrac{1}{\frac{s_M}{1} + \frac{1}{s_M}}}$$

$$\Rightarrow \quad \frac{1}{1.6} = \frac{1}{\dfrac{\frac{1}{2s_{fl}} + 2s_{fl}}{2/s}} \quad [S_M = 2 \text{ from part (b)}]$$

$$\Rightarrow \quad \frac{2}{84} = \frac{1}{\frac{1}{2s_{fl}} + 2s_{fl}}$$

$$\Rightarrow \quad 4 = 2s_{fl} + \frac{1}{2s_{fl}}$$

$$\Rightarrow \quad 8s_{fl} = 4s_{fl}^2 + 1$$

$$\Rightarrow \quad 4s_{fl}^2 - 8s_{fl} + 1 = 0$$

$$s_{fl} = 0.134$$

$$\therefore \quad \text{Full load speed} = N_{fl} = 1299 \text{ rpm}$$

2. Let,

$$\tau_M = \text{Maximum torque}$$

$$\frac{\tau_s}{\tau_M} = \frac{1}{\dfrac{\dfrac{s_M}{1} + \dfrac{1}{s_M}}{1/2}}$$

$$\Rightarrow \quad \frac{1.6}{2} = \frac{2}{s_M + \dfrac{1}{s_M}}$$

$$\Rightarrow \quad s_M + \frac{1}{s_M} = \frac{5}{2}$$

$$\Rightarrow \quad 2s_M^2 - 5s_M + 2 = 0$$

$$\Rightarrow \quad s_M = \frac{1}{2}$$

∴ Speed at maximum torque

$$= N_M = N_s(1 - s_M) = 750 \text{ rpm}$$

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