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Detailed Solutions

ESE-2026 Mains Test Series

Civil Engineering Test No : 15

Section - A

1. (a) (i) Solution:

Sewage sickness (also known as "soil sickness") is a condition that occurs when sewage is applied to land continuously for irrigation or disposal over a long period. This process leads to the clogging of soil pores, which prevents the natural aeration of the soil.““When the soil becomes anaerobic (oxygen-deficient), the aerobic bacteria die off, and anaerobic bacteria take over. This results in the decomposition of organic matter releasing foul gases like hydrogen sulfide, methane, and carbon dioxide, turning the land into a marshy, stinking area where crops cannot survive.

Methods to Prevent Sewage Sickness: To maintain the productivity of the land and prevent anaerobic conditions, the following methods are adopted:

- **Primary Treatment of Sewage:** Before applying sewage to the land, it should undergo primary treatment (screening and sedimentation) to remove suspended solids. This significantly reduces the load on the soil and prevents immediate clogging of pores.
- **Rotation of Crops:** Different crops require different amounts of nutrients and water. By rotating crops, the soil is not consistently saturated with the same sewage components, allowing the soil to recover its structure and fertility.
- **Intermittent Application:** Sewage should never be applied continuously. Giving the land "rest periods" allows the sewage to percolate down and lets air enter the soil pores, which reactivates aerobic bacteria to decompose the organic matter.

- **Provision of Under-drains:** Installing a network of open-jointed pipes (under-drains) at a depth of about 0.9 m to 1.2 m helps in quickly removing the effluent that has filtered through the soil. This prevents the water table from rising and ensures the soil remains aerated.
- **Shallow Plowing and Turning:** Periodically plowing the land helps break up the surface crust formed by dried sewage solids. This "turning" of the soil exposes deeper layers to air and improves permeability.
- **Choice of Land:** Using sandy or loamy soils is preferable as they have higher permeability compared to clayey soils, which clog very easily.
- **Dilution:** If the sewage is too concentrated, it can be diluted with fresh water before irrigation to reduce the organic load per unit volume.

1. (a) (ii) Solution:

1. **Procedure to Obtain SPT Value (N) :** The test is conducted inside a borehole at desired intervals (usually every 1.5 m or at change of strata).
   - **Apparatus:** A standard split-spoon sampler is used. It has an outside diameter of 50 mm and an inside diameter of 35 mm.
   - **Driving the Sampler:** The sampler is driven into the soil by a drop hammer weighing 63.5 kg falling from a height of 75 cm.
   - **Recording Blows:** The driving is performed in three stages of 15 cm penetration each (total 45 cm):
   - **Seating Drive:** The number of blows for the first 15 cm is recorded but ignored as it is considered to be in disturbed soil.
   - **Actual Test:** The number of blows required for the next two 15 cm intervals (total 30 cm) are added together.
   - **Result:** This sum is known as the Standard Penetration Number (N).
2. **Corrections to the Observed N-Value:** The observed value (N_{obs}) must be corrected before it can be used in design charts or empirical equations.

   A. Overburden Pressure Correction: In granular soils, the N-value depends on the confining pressure. At shallower depths, the confining pressure is low, leading to lower N-values even if the soil is dense. The corrected value (N_1) is given by:

   $$N_1 = N_{obs} \times C_N$$

   Where C_N is the correction factor. According to Gibbs and Holtz

   $$C_N = \left(\frac{350}{\bar{\sigma}_0 + 70}\right)$$

(Note: This is valid for $\bar{\sigma}_0 \le 280\ \text{kN/m}^2$, where $\bar{\sigma}_0$ is the effective overburden pressure).

B. Dilatancy Correction (Silty Fine Sands):This correction is applied only if the soil is fine sand or silt below the water table and the N_1 value obtained after overburden correction is greater than 15. The high resistance is due to negative pore water pressure. The corrected value (N_c) is:

$$N_c = 15 + \frac{1}{2}(N_1 - 15)$$

If $\quad N_1 = 15$

then, $\quad N_c = N_1$

1. (b) Solution:

Given:

$$P_g = 20\ \text{kPa} = 20\ \text{kN/m}^2$$
$$R = 2.5\ \text{m}$$
$$w = 3\ \text{m}$$
$$\rho = 1000\ \text{kg/m}^3$$
$$g = 9.81\ \text{m/s}^2$$
$$h_s = \frac{P_g}{\rho g} = \frac{20 \times 10^3}{9.81 \times 10^3} = 2.039\ \text{m}$$

The horizontal component of force is equal to the pressure force acting on the vertical projection of the curved surface. The vertical projection is a rectangle of height R and width *w*.Depth of centroid of vertical projection from equivalent free surface,

$$\bar{h} = h_s + \frac{R}{2}$$

$$\Rightarrow \quad \bar{h} = 2.039 + \frac{2.5}{2} = 3.289\ \text{m}$$

Area of vertical projection, $A = R \times w = 2.5 \times 3\ \text{m}^2$

Horizontal force, $F_x = 1000 \times 9.81 \times 3.289 \times (2.5 \times 3) = 241.988\ \text{kN}$

The vertical component of force is equal to the weight of the imaginary water above the curved surface up to the equivalent free surface.

Volume of rectangular portion, $V_1 = R \times h_s \times w = 2.5 \times 2.039 \times 3$

$$= 15.293\ \text{m}^3$$

Volume of quadrant portion, $V_2 = \frac{\pi R^2}{4} \times w = \frac{\pi \times 2.5^2}{4} \times 3 = 14.726\ \text{m}^3$

Total volume, $V = 15.293 + 14.726 = 30.019\ \text{m}^3$

$$\text{Vertical force, } F_y = \rho g V$$

$$\Rightarrow \quad F_y = 1000 \times 9.81 \times 30.019 = 294.486 \text{ kN}$$

The magnitude of the resultant force is given by,

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$\Rightarrow \quad F_R = \sqrt{241.988^2 + 294.486^2} = 381.156 \text{ kN}$$

The direction of the resultant force with the horizontal is,

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{294.486}{241.988}\right) = 50.589^\circ$$

1. (c) Solution:

Given that, pH = 10.2

$$[Ca^{2+}] = 90 \text{ mg}/l, \text{ Eq. wt} = 20.0$$

$$[Mg^{2+}] = 25 \text{ mg}/l, \text{ Eq. wt} = 12$$

$$[Na^{+}] = 65 \text{ mg}/l, \text{ Eq. wt} = 23$$

$$[K^{+}] = 8 \text{ mg}/l, \text{ Eq. wt} = 39$$

$$[Cl^{-}] = 110 \text{ mg}/l, \text{ Eq. wt} = 35.50$$

$$[SO_4^{2-}] = 180 \text{ mg}/l, \text{ Eq. wt} = 48$$

$$[HCO_3^{-}] = 150 \text{ mg}/l, \text{ Eq. wt} = 61$$

$$\text{Equivalent weight of } CaCO_3 = 50$$

At pH = 10.2,

$$pOH = 14 - 10.2 = 3.8$$

$$[OH^{-}] = 10^{-3.8} \times 17 \times 1000 = 2.694 \text{ mg}/l$$

Total alkalinity is contributed by bicarbonate and hydroxide ions.

$$\text{Alkalinity} = \left(\frac{[HCO_3^{-}]}{61} + \frac{[OH^{-}]}{17}\right) \times 50$$

$$= \left(\frac{150}{61} + \frac{2.694}{17}\right) \times 50 = 130.874 \text{ mg}/l$$

Total hardness is due to calcium and magnesium ions.

$$TH = \left(\frac{[Ca^{2+}]}{20} + \frac{[Mg^{2+}]}{12}\right) \times 50$$

$$\Rightarrow \quad TH = \left(\frac{90}{20} + \frac{25}{12}\right) \times 50 = 329.167 \text{ mg}/l$$

Carbonate hardness is the lesser of total hardness and alkalinity.

$$CH = \min(\text{Total Hardness, Total Alkalinity})$$

$$\Rightarrow \quad CH = \min(329.167, 130.874) = 130.874 \text{ mg}/l$$

Noncarbonate hardness is the difference between total hardness and carbonate hardness.

$$NCH = TH - CH$$

$$\Rightarrow \quad NCH = 329.167 - 130.874 = 198.293 \text{ mg}/l$$

Milliequivalents of cations, $Ca^{2+} = \frac{90}{20} = 4.5$

$$Mg^{2+} = \frac{25}{12} = 2.083$$

$$Na^{+} = \frac{65}{23} = 2.826$$

$$K^{+} = \frac{8}{39} = 0.205$$

$$\sum \text{Cations} = 4.5 + 2.083 + 2.826 + 0.205 = 9.614 \text{ meq}/l$$

Milliequivalents of anions, $Cl^{-} = \frac{110}{35.5} = 3.098$

$$SO_4^{2-} = \frac{180}{48} = 3.75$$

$$HCO_3^{-} = \frac{150}{61} = 2.459$$

$$\sum \text{Anions} = 3.098 + 3.75 + 2.459 = 9.307 \text{ meq}/l$$

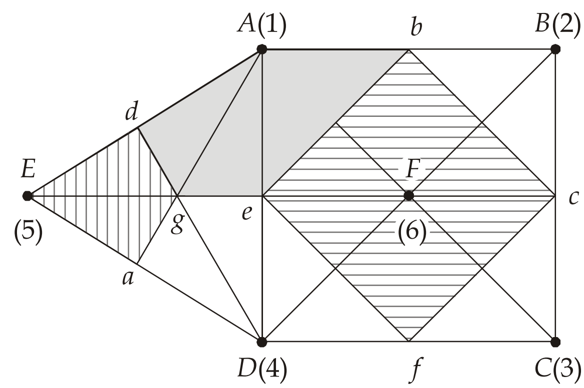
Percentage error in cation-anion balance ratio,

$$\%\text{error} = \frac{|\Sigma\text{Cations} - \Sigma\text{Anions}|}{\Sigma\text{Cations} + \Sigma\text{Anions}} \times 100$$

$$\%\text{error} = \frac{|9.614 - 9.307|}{9.614 + 9.307} \times 100 = 1.622\%$$

1. (d) Solution:

Let the stations 1, 2, 3, 4, 5, 6 be named as station A, B, C, D, E and F respectively, for convenience. Let the length of the sides of square ABCD be a (= 4 km). Then the length of each side of the equilateral triangular plot will be also a. Now for the triangular plot, draw perpendicular bisectors Aa, Dd and Ee, so that they meet in point g. Similarly draw the perpendicular bisectors eb, bc, cf and fe of the lines FA, FB, FC and FD respectively, shown in figure.



Evidently, station F (or station 6) will be fed by the square area bcfe, where length of its side

$$bc = \frac{1}{2}AC = \frac{1}{2}\sqrt{2}\,a = a/\sqrt{2}$$

Hence, $$\text{area } bcfe = A_6 = \frac{a}{\sqrt{2}}\times\frac{a}{\sqrt{2}} = \frac{a^2}{2} = \frac{(4)^2}{2} = 8 \text{ km}^2$$

Then each of the corner stations, say station A, will be fed by triangular area Abe and sectorial area Adge.

$$\text{Triangular area Abe} = \frac{1}{2}\times\frac{a}{2}\times\left(\frac{a}{2}\right) = \frac{a^2}{8}$$

$$\text{Sectiorial area, Adge} = \frac{1}{3}\text{ area of triangle ADE} = \frac{1}{3}\times\frac{1}{2}\times a\times\frac{\sqrt{3}}{2}a = \frac{a^2}{4\sqrt{3}}$$

Hence station A will be fed by area = Area Abe + area Adge

$$= \frac{a^2}{8}+\frac{a^2}{4\sqrt{3}} = \frac{(4)^2}{8}+\frac{(4)^2}{4\sqrt{3}} = 4.3094 \text{ km}^2$$

Hence, $$A_1 = A_4 = 4.3094 \text{ km}^2 \quad \text{...(ii)}$$

Also station E will be fed by sectorial area Edga = Area Adge = $\frac{a^2}{4\sqrt{3}}$

Hence, $$A_5 = \frac{a^2}{4\sqrt{3}} = \frac{(4)^2}{4\sqrt{3}} = 2.3094 \text{ km}^2 \quad ...(iii)$$

Station B will be fed by area bcB = area Abe = $\frac{a^2}{8} = \frac{(4)^2}{8} = 2 \text{ km}^2$

Hence $A_2 = A_3 = 2 \text{ km}^2$

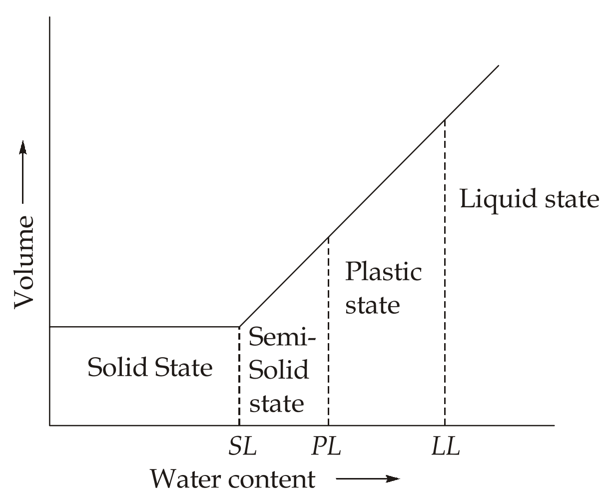
Hence, $$P_{avg} = \frac{P_1A_1 + P_2A_2 + \dots P_6A_6}{A_1 + A_2 + \dots A_6}$$

$$\Rightarrow \quad P_{avg} = \frac{5.8 \times 4.3094 + 14.0 \times 2 + 9.0 \times 2 + 6.4 \times 4.3094 + 4.2 \times 2.3094 + 10.4 \times 8}{4.3094 + 2 + 2 + 4.3094 + 2.3094 + 8}$$

$$\Rightarrow \quad P_{avg} = 8.35 \text{ cm}$$

1. (e) Solution:

The shrinkage limit is the maximum water content at which a further reduction in moisture will not cause a decrease in the volume of a soil sample. It represents the lowest moisture content, expressed as a percentage, at which the soil is still fully saturated. It marks the transition from a semi-solid to a solid state, where the soil volume stays constant.



State of Soils

Given that: $w_L = 55\% = 0.55$, $w_P = 25\% = 0.25$, $w_s = 15\% = 0.15$

$V_L = 35 \text{ cm}^3$

$V_s = 21.5 \text{ cm}^3$

The shrinkage ratio is defined as the ratio of volume change to the corresponding change in water content, expressed with respect to dry volume.

Since volume remains constant below shrinkage limit,

$$V_d = V_s$$

$$R = \frac{V_L - V_s}{V_s(w_L - w_s)} = \frac{35-21.5}{21.5\times(0.55-0.15)}$$

$$\Rightarrow \quad R = 1.569$$

The relation between shrinkage limit, shrinkage ratio, and specific gravity is,

$$w_s\% = \left(\frac{1}{R}-\frac{1}{G}\right)\times 100$$

$$\Rightarrow \quad \frac{1}{G} = \frac{1}{R}-\frac{w_s}{100}$$

$$\Rightarrow \quad \frac{1}{G} = \frac{1}{1.569}-\frac{15}{100}$$

$$\Rightarrow \quad G = 2.052$$

2. (a) Solution:

Footing size = 4 m × 4 m

Footing depth (D_f) = 2 m

Total Load (P) = 2500 × kN

$$\gamma_w = 9.81 \text{ kN/m}^3$$

Submerged unit weights, $\gamma' = \frac{G_s - 1}{1+e}\gamma_w$

$$\gamma_{sand'} = \frac{2.65-1}{1+0.62}\times 9.81 = 9.992 \text{ kN/m}^3$$

$$\gamma_{clay1'} = \frac{2.72-1}{1+1.15}\times 9.81 = 7.848 \text{ kN/m}^3$$

$$\gamma_{clay2'} = \frac{2.75-1}{1+1.30}\times 9.81 = 7.464 \text{ kN/m}^3$$

$$\gamma_{clay3'} = \frac{2.70-1}{1+1.05}\times 9.81 = 8.135 \text{ kN/m}^3$$

Compression Indices $(C_c = 0.009\times(LL-10))$:

$$C_{c1} = 0.009 \times (55 - 10) = 0.405$$

$$C_{c2} = 0.009 \times (62 - 10) = 0.468$$

$$C_{c3} = 0.009 \times (48 - 10) = 0.342$$

Effective Stress and Load Increment Calculations:

The vertical stress increase at depth z below footing is $\Delta\sigma = \dfrac{P}{(B+z)^2}$

Layer 1: (mid-depth $z = 3 + 1.5 = 4.5$ m below footing):

$$\sigma_{01'} = (5 \times 9.992) + (1.5 \times 7.848) = 61.732 \text{ kPa}$$

$$\Delta\sigma_1 = \frac{2500}{(4+4.5)^2} = 34.602 \text{ kPa}$$

Layer 2: (mid-depth $z = 4.5 + 3 = 7.5$ m below footing):

$$\sigma_{02'} = 61.732 + (1.5 \times 7.848) + (1.5 \times 7.464) = 84.7 \text{ kPa}$$

$$\Delta\sigma_2 = \frac{2500}{(4+7.5)^2} = 18.904 \text{ kPa}$$

Layer 3: (mid-depth $z = 7.5 + 3 = 10.5$ m below footing):

$$\sigma_{03'} = 84.7 + (1.5 \times 7.646) + (1.5 \times 8.135) = 108.37 \text{ kPa}$$

$$\Delta\sigma_3 = \frac{2500}{(4+10.5)^2} = 11.891 \text{ kPa}$$

Settlement calculation:

$$S_1 = \frac{0.405 \times 3}{1+1.15} \log_{10}\left(\frac{61.732 + 34.602}{61.732}\right) = 109.22 \text{ mm}$$

$$S_2 = \frac{0.468 \times 3}{1+1.30} \log_{10}\left(\frac{84.7 + 18.904}{84.7}\right) = 53.40 \text{ mm}$$

$$S_3 = \frac{0.342 \times 3}{1+1.05} \log_{10}\left(\frac{108.37 + 11.891}{108.37}\right) = 22.63 \text{ mm}$$

Total settlement:

$$S_t = 109.22 + 53.4 + 22.63 = 185.25 \text{ mm}$$

2. (b) (i) Solution:

The self-purification process of a river after the discharge of sewage leads to the formation of distinct ecological zones. These zones are primarily defined by the concentration of Dissolved Oxygen (DO) and the presence of specific aquatic life.

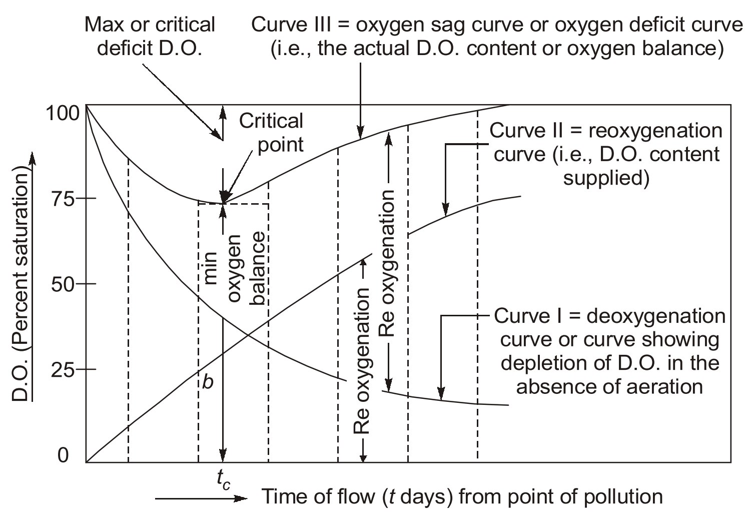
Zones of Pollution in a River Stream:

1. **Zone of Degradation:** This zone is located immediately below the point of sewage discharge.
   - **Characteristics:** The water becomes turbid and dark. Algae begin to die due to turbidity, but fish may still survive near the upper limits of this zone.
   - **DO Levels:** The dissolved oxygen starts falling rapidly from its saturation value, generally dropping to about 40% of the saturation level by the end of this zone.
2. **Zone of Active Decomposition:** This is the most heavily polluted section of the river, characterized by intense microbial activity.
   - **Characteristics:** The water is dark and grayish. Anaerobic conditions may set in, leading to the evolution of gases like methane (CH_4), hydrogen sulfide (H_2S), and carbon dioxide (CO_2). Fish life is generally absent.
   - **DO Levels:** DO concentration falls to zero or very near zero. In the latter half of this zone, as organic load decreases, DO begins to rise again.
3. **Zone of Recovery :** In this zone, the river begins to regain its original healthy state.
   - **Characteristics:** The organic matter has been largely stabilized. Turbidity decreases, and sunlight can penetrate deeper, promoting the growth of algae and fungi. Nitrates and sulfates start to form.
   - **DO Levels:** The DO content increases significantly, often rising above 40% of the saturation value.
4. **Zone of Clear Water:** This is the final stage where the river has successfully self-purified.
   - **Characteristics:** The water becomes clear and attractive again. Pathogenic organisms have largely disappeared, and normal aquatic life (fish) returns.
   - **DO Levels:** The dissolved oxygen level reaches its saturation value, though the concentration of dissolved salts (nitrates) may remain slightly higher than the original upstream levels.

The Oxygen Sag Curve (or Streeter-Phelps curve) represents the relationship between the Deoxygenation Rate (due to BOD exertion) and the Reoxygenation Rate (due to atmospheric aeration).

Key Components:

1. **Deoxygenation Curve:** Represents the depletion of DO as microorganisms consume oxygen to break down organic matter.
2. **Reoxygenation Curve:** Represents the absorption of oxygen from the atmosphere into the water body.
3. **Oxygen Deficit (D):** The difference between the saturation DO and the actual DO at any point.
4. **Critical Oxygen Deficit (D_c):** The point where the DO is at its lowest. This occurs when the rate of deoxygenation exactly equals the rate of reoxygenation.



2. (b) (ii) Solution:

Given dataDischarge of sewage, Q_s = 1.5 m^3/s

Discharge of river, Q_r = 12 m^3/s

BOD_5 of sewage, BOD_5 = 180 mg/L

DO_s = 1.5 mg/L

DO of river, DO_r = 8.2 mg/L

Saturation DO, DO_{sat} = 9.1 mg/L

Deoxygenation constant, K_D = 0.15 day^{-1} (base 10)

Reoxygenation constant, K_R = 0.45 day^{-1} (base 10)

River velocity, v = 0.25 m/s

Initial Mixed Conditions

The initial BOD_5 of the mixture, BOD_{mix} :

$$\text{BOD}_{5\text{mix}} = \frac{(Q_s \cdot \text{BOD}_s)+(Q_r \cdot 0)}{Q_s+Q_r} = \frac{(1.5\times180)+(12\times0)}{1.5+12} = 20 \text{ mg/L}$$

$$\text{Ultimate BOD, } L_0 = \frac{\text{BOD}_{\text{mix}}}{1-10^{-K_D\times5}} = \frac{20}{1-10^{-0.15\times5}} = 24.326 \text{ mg/L}$$

$$\text{Initial DO of the mixture, DO}_{\text{mix}} = \frac{(Q_s \cdot \text{DO}_s)+(Q_r \cdot \text{DO}_r)}{Q_s+Q_r}$$

$$= \frac{(1.5\times1.5)+(12\times8.2)}{1.5+12} = 7.456 \text{ mg/L}$$

$$\text{Initial Oxygen Deficit, D}_0 = \text{DO}_{\text{sat}} - \text{DO}_{\text{mix}}$$

$$= 9.1-7.456 = 1.644 \text{ mg/L}$$

Critical Time and Deficit

$$\text{Critical time, } t_c = \frac{1}{K_R-K_D}\log_{10}\left[\frac{K_R}{K_D}\left(1-D_0\frac{K_R-K_D}{K_DL_0}\right)\right]$$

$$t_c = \frac{1}{0.45-0.15}\log_{10}\left[\frac{0.45}{0.15}\left(1-1.644\frac{0.45-0.15}{0.15\times24.326}\right)\right] = 1.380 \text{ days}$$

$$\text{Critical Oxygen Deficit, } D_c = \frac{K_D}{K_R}L_0\times10^{-K_Dt_c}$$

$$D_c = \frac{0.15}{0.45}\times24.326\times10^{-0.15\times1.380} = 5.033 \text{ mg/L}$$

$$\text{Distance to Minimum DO, } X_c = v\,t_c = 0.25\times(1.380\times24\times3600)$$

$$= 29808 \text{ m} = 29.808 \text{ km}$$

2. (c) (i) Solution:

Given data

$$\text{Area ratio, } r = \frac{A_1}{A_2} = 4$$

Throat diameter, d_2 = 0.02 m

Specific gravity of oil, S_o = 0.85

Specific gravity of mercury, S_m = 13.6

Differential gauge reading, x = 0.15 m

Acceleration due to gravity, g = 9.81 m/s^2

$$\text{Area of throat, } A_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4}(0.02)^2 = 3.142\times10^{-4}\ \text{m}^2$$

$$\text{Area, } A_1 = r\cdot A_2 = 4\times3.142\times10^{-4}\ \text{m}^2 = 1.2568\times10^{-3}\ \text{m}^2$$

$$\text{Differential head of oil, } h = x\left(\frac{S_m}{S_o}-1\right) = 0.15\left(\frac{13.6}{0.85}-1\right) = 2.25\ \text{m}$$

$$\text{Discharge through venturi meter, } Q = \frac{A_1\cdot A_2\cdot\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$\Rightarrow \quad Q = \frac{0.0012568\times0.0003142\times\sqrt{2\times9.81\times2.25}}{\sqrt{(0.0012568)^2-(0.0003142)^2}} = 2.156\times10^{-3}\text{m}^3/\text{s}$$

$$\Rightarrow \quad Q = 2.156\ \text{L/s}$$

For horizontal orientation, the discharge remains the same because the differential gauge measures the difference in piezometric head and the same head difference produces the same flow rate.

2. (c) (ii) Solution:

By Von Karman momentum integral equation,

$$\frac{\tau_0}{\rho U^2} = \frac{\partial\theta}{\partial x}$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$$

Put $\frac{y}{\delta} = t,$ $\quad y = 0, t = 0$

and $\quad y = \delta, t = 1$

$\Rightarrow \quad dy = \delta\, dt$

$$\theta = \int_0^1 (2t - 2t^3 + t^4)(1 - 2t + 2t^3 - t^4)\delta\, dt$$

$$= \int_0^1 (2t - 4t^2 + 4t^4 - 2t^5 - 2t^3 + 4t^4 - 4t^6 + 2t^7 + t^4$$

$$-2t^5 + 2t^7 - t^8)\delta\, dt$$

$$= \int_0^1 (-t^8 + 4t^7 - 4t^6 - 4t^5 + 9t^4 - 2t^3 - 4t^2 + 2t)\delta\, dt = \frac{37\delta}{315}$$

$\therefore$
$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left(\frac{37\,\delta}{315}\right) \quad ...(i)$$

Also,
$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$= U\mu\left[\frac{2}{\delta} - 2\times 3\left(\frac{y}{\delta}\right)^2 \cdot \frac{1}{\delta} + 4\left(\frac{y}{\delta}\right)^3 \cdot \frac{1}{\delta}\right]_{y=0} = \frac{2U\mu}{\delta}$$

Putting value of τ_0 in equation (i),

$$\frac{2\mu U}{\rho U^2 \delta} = \frac{d}{dx}\left(\frac{37\,\delta}{315}\right)$$

$$\frac{2\,\mu U}{\rho U^2} \times \frac{315}{37} dx = \delta\; d\delta$$

Integrating on both sides,

$$\frac{630}{37}\frac{\mu}{\rho U}x = \frac{\delta^2}{2} + C$$

At $x = 0$, $\delta = 0$

$\Rightarrow$ $C = 0$

$$\Rightarrow \quad \delta^2 = \frac{1260 \cdot x^2}{37\dfrac{\rho U x}{\mu}}$$

$$\Rightarrow \quad \frac{\delta}{x} = \frac{5.836}{\sqrt{Re_x}} \qquad \left(\text{where, } Re_x = \frac{\rho U x}{\mu}\right) \quad ...(ii)$$

Shear stress:

Now,
$$\tau_0 = \frac{2\mu U}{\delta}$$

Substituting value of δ from (ii),

$$\Rightarrow \quad \tau_0 = \frac{2\mu U\sqrt{Re_x}}{5.836x} = \frac{0.3427\mu U}{x} \cdot \sqrt{\frac{\rho U x}{\mu}}$$

$$= \frac{0.3427\mu^{1/2}U^{3/2}\rho^{1/2}}{x^{1/2}} \cdot \frac{\rho^{1/2}U^{1/2}}{\rho^{1/2}U^{1/2}} \cdot \frac{2}{2}$$

$\therefore \quad \tau_0 = 0.6854 \dfrac{PU^2}{2}\left(\dfrac{\mu}{\rho U x}\right)^{1/2}$

or $\quad \dfrac{\tau_0}{\frac{1}{2}\rho U^2} = \dfrac{0.6854}{\sqrt{Re_x}} = C_f$

Force on one side:

Consider a plate of unit width and length L

$$F_D = \int_0^L \tau_0 (1 \cdot dx)$$

$$= 0.6854 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U}\right)^{1/2} \int_0^L x^{-1/2} dx$$

$$= 0.6854 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U}\right)^{1/2} \left[2x^{1/2}\right]_0^L$$

$$= 0.6854 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U}\right)^{1/2} 2\sqrt{L}$$

$$F_D = 1.3708 \frac{\rho U^2}{2}\left(\frac{\mu}{\rho U L}\right)^{1/2} L$$

$$F_D = \frac{1}{2}\left(\frac{1.3708}{\sqrt{Re_L}}\right)\rho U^2 L$$

or $\quad \dfrac{F_D}{\frac{1}{2}\rho U^2 L} = C_{DF} = \dfrac{1.3708}{\sqrt{Re_L}}$

3. (a) Solution:

The runway length is increased by 7% for every 300 m rise in elevation.

$$C_e = \frac{7}{100} \times \frac{420}{300} \times 2250 = 220.5 \text{ m}$$

$$L_1 = 2250 + 220.5 = 2470.5 \text{ m}$$

Correction for temperature

Airport reference temperature, $T_{ref} = T_a + \dfrac{T_m - T_a}{3} = 31 + \dfrac{38-31}{3} = 33.333^\circ\text{C}$

Standard temperature at elevation, $T_s = 15 - 0.0065 \times 420 = 12.27°C$

$$\text{Temperature correction, } C_t = \frac{1}{100} \times (33.333 - 12.27) \times 2470.5 = 520.361 \text{ m}$$

$$L_2 = 2470.5 + 520.361 = 2990.861 \text{ m}$$

Check for total ICAO correction, $C_e + C_t = 220.5 + 520.361 = 740.861$ m

$$\text{Percentage correction} = \frac{740.861}{2250} \times 100 = 32.927\%$$

Since the correction is less than 35%, it is acceptable.

Correction for gradient.

Calculation of reduced levels:

Segment (m)	l_i(m)	g_i(%)	RL change (m)
0 to 400	400	+1.20	+4.8
400 to 1000	600	–0.60	–3.6
1000 to 1600	600	+0.40	+2.4
1600 to 2000	400	+0.80	+3.2
2000 to 2400	400	–0.40	–1.6
2400 to 2990.866	590.866	–0.20	–1.182

Cumulative reduced levels:

RL at 0 = 0

RL at 400 = 4.8

RL at 1000 = 1.2

RL at 1600 = 3.6

RL at 2000 = 6.8

RL at 2400 = 5.2

RL at 2990.861 = 4.018

Maximum RL = 6.8 m, Minimum RL = 0 m

$$\text{Effective gradient, } G = \frac{6.8 - 0}{2990.861} \times 100 = 0.227\%$$

$$\text{Gradient correction, } C_g = \frac{20}{100} \times 0.227 \times 2990.861 = 135.785 \text{ m}$$

Final runway length, L = 2990.861 + 135.785 = 3126.646 m

Actual length of runway = 3126.646 m

3. (b) (i) Solution:

Given data

Thickness of pavement, h = 18 cm

Modulus of subgrade reaction, k = 8 kg/cm^3

Spacing of dowel bars, s = 25 cm

Wheel load, P = 4500 kg

Load transfer = 45%

Poisson's ratio, μ = 0.15

Modulus of elasticity, E = 2.1×10^5 kg/cm^2

$$\text{Radius of relative stiffness, } l = \left[\frac{Eh^3}{12k(1-\mu^2)}\right]^{0.25}$$

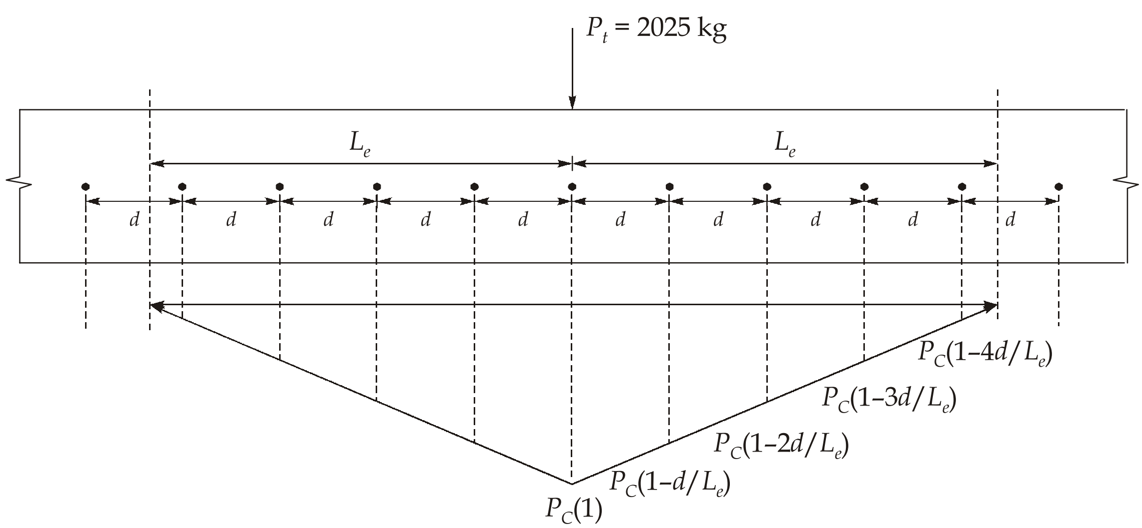
$$= \left[\frac{210000 \times 18^3}{12 \times 8 \times (1-0.15^2)}\right]^{0.25} = 60.105 \text{ cm}$$

Effective distance of load transfer, $L_e = 1.8 \times l = 1.8 \times 60.105 = 108.189$ cm

Total load transferred by joint, P_t = $0.45 \times P = 0.45 \times 4500 = 2025$ kg

Participation factors for dowels at distances, d = 0, 25, 50, 75, 100 cm:

$$= 1 - \frac{d}{L_e}$$



The load taken by any dowel bar at distance $x = P_c\left(1 - \frac{x}{L_e}\right)$ summing the loads of the center bar end all active bars on sides gives the total transferred load:

$$P_t = P_d\left[1+2\sum\left(1-\frac{x}{L_e}\right)\right]$$

$\Rightarrow$ $$2025 = P_d[1 + 2\,(0.769 + 0.538 + 0.307 + 0.076)]$$

$\Rightarrow$ $$P_d = \frac{2025}{4.38} = 462.33 \text{ kg}$$

Hence; maximum load carried by a single dowel bar, just below the wheel is 462.33 kg.

3. (b) (ii) Solution:

Given data:

Speed of car A (ascending), V_A = 80 km/h

Speed of car B (descending), V_B = 70 km/h

Gradient, n = 3%

$\Rightarrow$ S = 0.03

Reaction time, t = 2.5 s

Coefficient of friction, f = 0.35

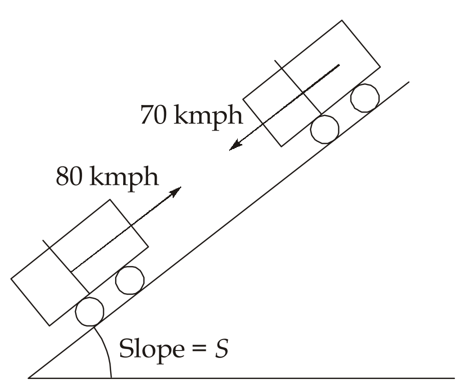
Acceleration due to gravity, g = 9.81 m/s^2

Sight distance is the length of road ahead visible to a driver at any instant. It is measured along the center line and depends on the heights of the driver's eye and the object.

The factors restricting sight distance include horizontal curves with obstructions, vertical summit curves, intersection obstructions, environmental conditions like fog or rain, and blockage due to other vehicles.

Stopping sight distance is the minimum distance required for a vehicle to stop safely after the driver perceives an obstruction. It ensures safety, governs geometric design, and maintains smooth traffic operation.

Stopping sight distance on a gradient consists of lag distance and braking distance.Lag distance, $d_1 = v \times t$



Braking distance on gradient, $d_2 = \frac{v^2}{2g(f \pm S)}$

Total stopping sight distance,SSD = $vt + \frac{v^2}{2g(f \pm S)}$

For ascending gradient, resistance increases, hence (f + S).

For descending gradient, resistance decreases, hence (f – S).

Conversion of speeds,

$$v_A = \frac{80}{3.6} = 22.222 \text{ m/s}, \quad v_B = \frac{70}{3.6} = 19.444 \text{ m/s}$$

Stopping sight distance for car A (ascending),

$$SSD_A = 22.222 \times 2.5 + \frac{22.222^2}{2 \times 9.81 \times (0.35 + 0.03)} = 121.789\text{m}$$

Stopping sight distance for car B (descending),

$$SSD_B = 19.444 \times 2.5 + \frac{19.444^2}{2 \times 9.81 \times (0.35 - 0.03)} = 108.827 \text{ m}$$

Minimum sight distance to avoid collision,

$$SD = SSD_A + SSD_B = 121.789 + 108.827 = 230.616 \text{ m}$$

Minimum sight distance required = 230.616 m

3. (c) (i) Solution:

Given data

Population, P = 150000

Area, A = 120 ha = 1.2×10^6 m^2

Mean sewage flow = 200 L/capita/day

Rainfall intensity, i = 0.05 m/hr

Runoff coefficient, k = 0.45

Peak factor = 2.5

Manning's coefficient, n = 0.013

Hazen-William's coefficient, C_H = 90

Velocity, v = 1.5 m/s

Average sewage flow,

$$= 150000 \times 200 = 30000000 \text{ L/day}$$

$$= \frac{30000000 \times 10^{-3}}{24 \times 3600} = 0.347 \text{ m}^3/\text{s}$$

$$\text{Peak Sewage flow, } Q_s = 2.5 \times 0.347 = 0.868 \text{ m}^3/\text{s}$$

$$\text{Storm water flow, } Q_w = k \times \left(\frac{i}{3600}\right) \times A$$

$$\Rightarrow \quad Q_w = 0.45 \times \left(\frac{0.05}{3600}\right) \times 1.2 \times 10^6 = 7.5 \text{ m}^3/\text{s}$$

$$\text{Total discharge, Q} = Q_s + Q_w = 0.868 + 7.5 = 8.368 \text{ m}^3/\text{s}$$

Diameter of sewer

$$Q = \frac{\pi}{4} D^2 v$$

$$\Rightarrow \quad 8.368 = \frac{\pi}{4} D^2 \times 1.5$$

$$\Rightarrow \quad D = \sqrt{\frac{8.368 \times 4}{\pi \times 1.5}} = 2.665 \text{ m}$$

$$\text{Hydraulic mean depth, } R = \frac{D}{4} = \frac{2.665}{4} = 0.666 \text{ m}$$

Gradient using Manning's equation

$$1.5 = \frac{1}{0.013} \times (0.666)^{2/3} \times S^{1/2}$$

$$\Rightarrow \quad S = \left(\frac{1.5 \times 0.013}{0.666^{2/3}}\right)^2 = 6.538 \times 10^{-4} = \frac{1}{1529.52}$$

Gradient using Hazen-William's expression

$$v = 0.85 C_H R^{0.63} S^{0.54}$$

$$\Rightarrow \quad 1.5 = 0.85 \times 90 \times (0.666)^{0.63} \times S^{0.54}$$

$$\Rightarrow \quad S = \left(\frac{1.5}{0.85 \times 90 \times 0.666^{0.63}}\right)^{\frac{1}{0.54}} = \frac{1}{904.108}$$

3. (c) (ii) Solution:

For a triangular channel, the top width is

$$T = 2zy$$

$$\text{Area of flow, } A = \frac{1}{2}\times T\times y = \frac{1}{2}\times(2zy)\times y = zy^2$$

$$z = \frac{A}{y^2}$$

Wetted perimeter is the sum of the two sloping sides,

$$P = 2\times\sqrt{y^2+(zy)^2} = 2y\sqrt{1+z^2}$$

$$\text{Substituting, } z = \frac{A}{y^2}$$

$$P = 2y\sqrt{1+\left(\frac{A}{y^2}\right)^2} = 2\sqrt{y^2+\frac{A^2}{y^2}}$$

For most efficient section, wetted perimeter is minimum for constant area.

$$\frac{dP}{dy} = 0$$

$$\Rightarrow \quad \frac{d}{dy}[2(y^2+A^2y^{-2})^{1/2}] = 0$$

$$\Rightarrow \quad (y^2+A^2y^{-2})^{-1/2}(2y-2A^2y^{-3}) = 0$$

$$\Rightarrow \quad 2y = \frac{2A^2}{y^3}$$

$$\Rightarrow \quad y^4 = A^2$$

$$\Rightarrow \quad A = y^2$$

Since $A = zy^2$,

$$zy^2 = y^2$$

$$\Rightarrow \quad z = 1$$

Thus, the side slope is 1 horizontal to 1 vertical.

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$

Hence, the included angle of the triangle is $2\theta = 90°$.

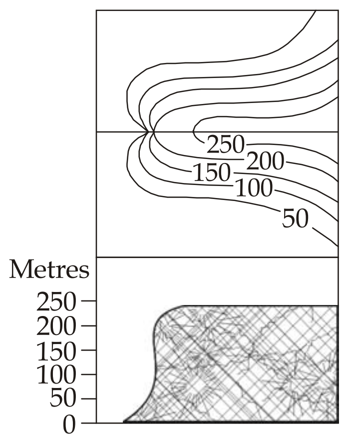
Therefore, the triangular section is a right-angled isosceles triangle, which is geometrically equal to half of a square cut along its diagonal.

Hence, the most efficient triangular channel is half of a square with its diagonal horizontal.

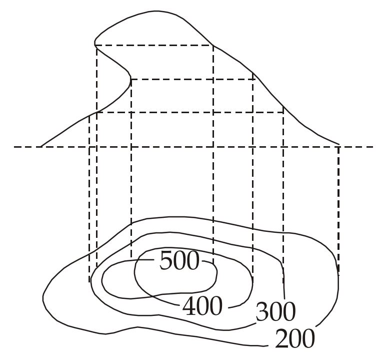
3. (c) (iii) Solution:

Characteristics of contour lines are listed below:

(i) All the points on a contour lines have the same elevation. The elevations of the contours are indicated either by inserting the figure in a break in the respective contour or printed close to the contour. When no value is present, it indicates a flat terrain. A zero meter contour line represents the coast line.



A cliff



Overhanging cliff

(ii) Two contour lines do not intersect each other except in the cases of an overhanging cliff or a cave penetrating a hillside.

(iii) A contour line must close onto itself, not necessarily within the limits of a map.

(iv) Equally spaced contour represent a uniform slope and contours that are well apart indicate a gentle slope.

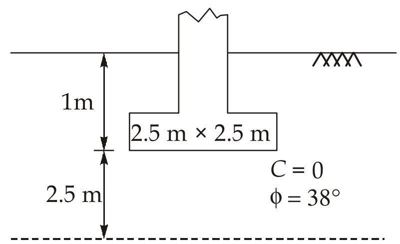
(v) A set of close contours with higher figures inside and lower figures outside indicate a hillock, whereas in the case of depressions, lakes, etc., the higher figures are outside and the lower figures are inside.

(vi) A watershed or ridge line (line joining the highest points of a series of hills) and the thalweg or valley line (line joining the lowest points of a valley) cross the contours at right angles.

(vii) Irregular contours represent uneven ground.

(viii) The direction of the steepest slope is along the shortest distance between the contours. The directions of the steepest slope at a point on a contour is, therefore, at right angles to the contour.

4. (a) Solution:



Given,

$$q_{safe} = \text{Safe load carrying capacity} = 400 \text{ kN/m}^2$$
$$\text{Size of footing } (B) = 2.5 \text{ m} \times 2.5 \text{ m}$$
$$\text{Depth of footing } (D_f) = 1 \text{ m}$$
$$\text{Saturated unit weight } (\gamma_{sat}) = 20 \text{ kN/m}^3$$
$$\text{Unit weight of soil above water table } (\gamma) = 17 \text{ kN/m}^3$$
$$N_q = 66.34$$
$$N_\gamma = 77.2$$
$$C = 0$$
$$\phi = 38°$$

(i) Factor of safety when water table is at 5 m below ground level:

Depth of influence zone below ground level

$$= D_f + B = 1 + 2.5 = 3.5 \text{ m}$$

Water table is below the depth of influence zone. Therefore its effect on bearing capacity is neglected.

Using Terzaghi's equation for square footing,

$$q_u = 1.3\, CN_C + qN_q + 0.4B\,\gamma N_\gamma$$

and

$$q_{safe} = \frac{q_{net}}{FOS} + \gamma \cdot D_f$$

$$\Rightarrow \quad q_{safe} = \frac{\gamma \cdot D_f \cdot (N_q - 1) + 0.4B \cdot \gamma N_\gamma}{\text{FOS}} + \gamma \cdot D_f$$

$$\Rightarrow \quad 400 = \frac{17 \times 1 \times 65.34 + 0.4 \times 2.5 \times 17 \times 77.2}{\text{FOS}} + 17 \times 1$$

$$\Rightarrow \quad \text{FOS} = 6.33$$

(ii) Factor of safety when water table is 1 m below the ground level:

$$q_{safe} = \frac{q_{net}}{FOS} + \gamma \cdot D_f$$

$$\Rightarrow \quad q_{\text{safe}} = \frac{\gamma \cdot D_f \cdot N_q + 0.4 \cdot \gamma' \cdot B \cdot N_\gamma}{\text{FOS}} + \gamma \cdot D_f$$

$$\Rightarrow \quad 400 = \frac{17 \times 1 \times 65.34 + 0.4 \times (20 - 9.81) \cdot 2.5 \times 77.2}{\text{FOS}} + 17 \times 1$$

$$\Rightarrow \quad \text{FOS} = 4.95$$

(iii) Factor of safety when water table is at ground and seepage is occurring.

Hydraulic gradient is given as 0.2

$$\therefore \quad q_{\text{safe}} = \frac{q_{\text{net}}}{\text{FOS}} + \gamma' \cdot D_f$$

$$\Rightarrow \quad q_{\text{safe}} = \frac{(\gamma' D_f - i \cdot z \cdot \gamma_w)(N_q - 1) + 0.4 \cdot \gamma' B N_\gamma}{\text{FOS}} + \left(\gamma' D_f - i \cdot z\gamma_w\right)$$

Here, $z = D_f = 1$ m

$$400 = \frac{[(20 - 9.81) \times 1 - 0.2 \times 1 \times 9.81] \times 65.34 + 0.4 \times (20 - 9.81) \times 2.5 \times 77.2}{\text{FOS}} + [(20 - 9.81) \times 1 - 0.2 \times 1 \times 9.81]$$

$$\Rightarrow \quad \text{FOS} = 3.38$$

4. (b) (i) Solution:

Level field book:

Distance	Readings			Rise	Fall	Reduced level	Remark
	BS	IS	FS				
0	0.780			–	–	180.750	A
30		1.535			0.755	179.995	
60		1.955			0.420	179.575	
90		2.430			0.475	179.100	
120		2.985			0.555	178.545	
150	1.155		3.480		0.495	178.050	CP
180		1.960			0.805	177.245	
210		2.365			0.405	176.840	
240	0.935		3.640		1.275	175.565	CP
270		1.045			0.110	175.455	
300		1.630			0.585	174.870	
330			2.545		0.915	173.955	

Formula used:

$$\text{Rise/fall} = (\text{Previous reading}) - (\text{Current reading}) \begin{Bmatrix} +\text{ve Rise} \\ -\text{ve Fall} \end{Bmatrix}$$

$$\text{RL of any point} = \text{RL of previous point} \pm \text{Rise/Fall} \begin{Bmatrix} +\text{ve Rise} \\ -\text{ve Fall} \end{Bmatrix}$$

Arithmetical checks:

$$\Sigma BS - \Sigma FS = 2.870 - 9.665 = -6.795$$

$$\Sigma\text{Rise} - \Sigma\text{Fall} = 0 - 6.795 = -6.795$$

$$\text{Last RL} - \text{First RL} = 173.955 - 180.750 = -6.795$$

There is a fall of 6.795 m in a distance of 330 m, gradient = $\frac{6.795}{330} = \frac{1}{48.565}$ (Falling)

4. (b) (ii) Solution:

Given data: Root zone depth, d = 80 cm

Field capacity, FC = 0.25

Wilting point, WP = 0.10

Apparent specific gravity, S_a = 1.6

Consumptive use, C_u = 12 mm/day

Irrigation efficiency, η = 0.7

Allowable depletion = 0.5

Available moisture, FC – WP = 0.25 – 0.10 = 0.15

Maximum depth of available water, $D_w = S_a \times d \times (FC - WP)$

$\Rightarrow \quad D_w = 1.6 \times 80 \times 0.15 = 19.2 \text{ cm} = 192 \text{ mm}$

Net depth of irrigation water, $d_{net} = 0.5 \times 192 = 96$ mm

Field depth of irrigation water,

$$d_{field} = \frac{d_{net}}{\eta} = \frac{96}{0.7} = 137.143 \text{ mm}$$

$$\text{Frequency of irrigation, } f = \frac{d_{net}}{C_u} = \frac{96}{12} = 8 \text{ days}$$

Depth of irrigation water = 137.143 mm

Frequency of irrigation = 8 days

4. (c) (i) Solution:

Given data: Wheel load, P = 4100 kg

Modulus of elasticity, $E = 3 \times 10^5$ kg/cm^2

Thickness of pavement, h = 20 cm

Poisson's ratio, μ = 0.15

Modulus of subgrade reaction, K = 6 kg/cm^3

Radius of contact area, a = 15 cm

$$\text{Radius of relative stiffness, } l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{1/4}$$

$$\Rightarrow \quad l = \left[\frac{(3\times 10^5)\times 20^3}{12\times 6\times (1-0.15^2)}\right]^{1/4} = 76.417 \text{ cm}$$

Equivalent radius of resisting section,

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$

$$\Rightarrow \quad b = \sqrt{1.6 \times 15^2 + 20^2} - 0.675 \times 20 = 14.068\,\text{cm}$$

$$\text{Interior stress, } S_i = \frac{0.316P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right]$$

$$\Rightarrow \quad S_i = \frac{0.316 \times 4100}{20^2}\left[4\log_{10}\left(\frac{76.417}{14.068}\right) + 1.069\right] = 12.984\ \text{kg/cm}^2$$

$$\text{Edge stress, } S_e = \frac{0.572P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right]$$

$$S_e = \frac{0.572 \times 4100}{20^2}\left[4\log_{10}\left(\frac{76.417}{14.068}\right) + 0.359\right]$$

$$\Rightarrow \quad S_e = 19.341\ \text{kg/cm}^2$$

$$\text{Corner stress, } S_c = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

$$\Rightarrow \quad S_c = \frac{3 \times 4100}{20^2}\left[1 - \left(\frac{15\sqrt{2}}{76.435}\right)^{0.6}\right] = 16.50\ \text{kg/cm}^2$$

4. (c) (ii) Solution:

Given data:

Gas flow rate for ESP, Q = 20 m^3/s

Drift velocity, w = 0.25 m/s

Efficiency, η = 0.95

Electrostatic precipitator plate area

$$\eta = 1 - e^{-\left(\frac{Aw}{Q}\right)}$$

$$A = -\frac{Q}{w}\ln(1-\eta)$$

$$A = -\frac{20}{0.25}\ln(1-0.95) = 239.659\ \text{m}^2$$

4. (c) (iii) Solution:

Air flow rate for baghouse, $Q = 15 \text{ m}^3/\text{s}$

Filtering velocity, $V_f = \frac{2.5}{60}$ m/s

Bag diameter, $d = 0.4$ m

Bag length, $l = 6$ m

Fabric filter design total filtering area required,

$$A_{total} = \frac{Q}{V_f} = \frac{15}{2.5/60} = 360 \text{ m}^2$$

Area of one bag, $A_{bag} = \pi dl = 3.14159 \times 0.4 \times 6 = 7.540 \text{ m}^2$

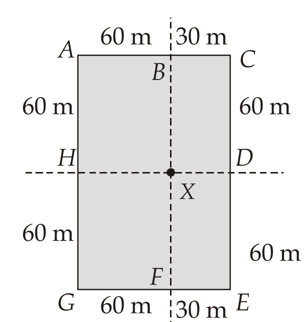
Number of bags, $N = \frac{A_{total}}{A_{bag}} = \frac{360}{7.540} = 47.746$

Number of bags required = 48

Section - B

5. (a) Solution:

The settlement at point '*X*' can be worked out as being the settlement caused by four rectangular loaded areas at their corner,



These rectangular areas are:

1. Rectangular *ABXH* having L = 60 m, B = 60 m, $\frac{L}{B} = 1$.
2. Rectangular *BCDX* having L = 60 m, B = 30, $\frac{L}{B} = 2$.
3. Rectangular *XDEF* having L = 60 m, B = 30 m, $\frac{L}{B} = 2$.
4. Rectangular *HXFG* having L = 60 m, B = 60 m, $\frac{L}{B} = 1$.

The values of influence factor (I_f) to be used for finding the immediate settlement caused by these four rectangular areas at corner are as follows:

For, $\frac{L}{B} = 2, \quad I_f = 0.77$

For, $\frac{L}{B} = 1, \quad I_f = 0.56$

Now, immediate settlement (S_i) is given by,

$$S_i = \frac{q \cdot B}{E}(1-\mu^2) \cdot I_f$$

Here,
$$q = 175 \text{ kN/m}^2$$
$$E = 13.5 \text{ MN/m}^2$$
$$\mu = 0.5$$

$$(S_i)_1 = \frac{q \times 60}{E} \times (1-\mu^2) \times 0.56$$

$$(S_i)_2 = \frac{q \times 30}{E} \times (1-\mu^2) \times 0.77$$

$$(S_i)_3 = \frac{q \times 30}{E} \times (1-\mu^2) \times 0.77$$

$$(S_i)_4 = \frac{q \times 60}{E} \times (1-\mu^2) \times 0.56$$

Now, total settlement at X,

$$(S_i) = (S_i)_1 + (S_i)_2 + (S_i)_3 + (S_i)_4$$

$$= \frac{q}{E}(1-\mu^2)[60 \times 0.56 + 30 \times 0.77 + 30 \times 0.77 + 60 \times 0.56]$$

$$= \frac{175}{13.5 \times 10^3} \times (1-0.5^2) \times 113.4 = 1.1025 \text{ m}$$

5. (b) Solution:

The commonly used samplers can be classified into three categories:

1. **Open-drive sampler:** The open-drive sampler is the simplest type of sampler for collection of samples. These are made up of seamless steel. The bottom of the tube is sharpened and beveled, which act as a cutting edge. The tube is connected through a head to the drill rod. The sampler head is provided with vents to permit water and air to escape during sampling and also a ball check valve to certain the sample during the withdrawal of sampler. The sampling tube may be thick walled or thin

walled. Thick walled samplers are used for obtaining disturbed but representative samples. They may be in the form of a solid tube or a split tube with or without a liner. The sample is collected by the thick walled sampler by the repeated blows of a falling weight.

Thin wall samplers are used for obtaining undisturbed samples. The area ratio is usually below 15 percent. The sampling tube for sampling of soil is pushed into the soil in a continuous rapid motion without impact or twisting.

2. **Piston sampler:** A piston sampler consists of two parts- (a) simpler cylinder (b) piston system. The piston rod fits easily inside the hollow drill rod. During the driving and upto the start of the sampling, the bottom of the piston is maintained flush with the cutting edge of the sampler. At the proposed sampling depth, the bottom of the piston is fixed in relation to the ground and the sampler cylinder forced in the soil independently, cutting a sample out of soil. As the sampler cylinder slides past the tight fitting portion during the sampling operation, a negative pressure develops above the sample which holds back the samples during withdrawal. After the cylinder is pushed to the required depth, both the cylinder and piston system are withdrawn with the sample inside the sample cylinder. Piston sampler is useful in sampling saturated sands and other soft and wet soils which cannot be sampled by open drive sampler.

3. **Rotary sampler:** A rotary sampler is a double-walled tube sampler with an inner removable liner. The outer tube or the rotating barrel is provided with a cutting bit. The bit cuts an annular ring when the barrel is rotated. The inner tube is stationary, slides over the cylindrical sample cut by the other rotating barrel. The sample is collected in the inner tube.

   Rotary sampler can be used for collection of undisturbed samples in stiff to hard clays, silts and sands with some cementation and also in rocks. The sampler is however undisturbed samples in stiff to hard clays, silts and sands with some cementation and also in rocks. The sampler is however unsuitable for gravelly soils and loose cohesionless soils.

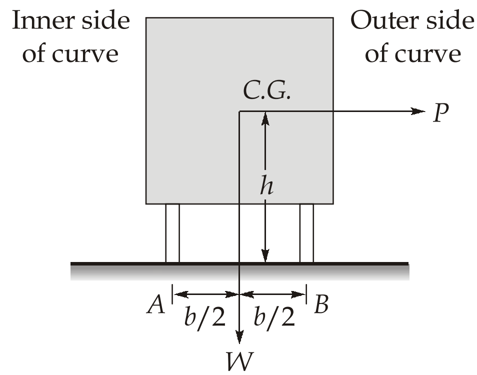
5. (c) Solution:

The centrifugal force acting on a vehicle negotiating a horizontal curve has the following two effects:

1. Tendency to overturn the vehicle about the outer wheels and
2. Tendency to skid the vehicle laterally, outwards.

The analysis of stability of these two conditions against overturning and transverse skidding of the vehicles negotiating horizontal curves without superelevation are as follows:

1. **Overturning effect:** The centrifugal force tends to overturn the vehicle about the outer-wheel B on the horizontal curve without superelevation.



Let 'h' be the height of the centre of gravity of the vehicle above the road surface and 'b' be the width of the wheel base of the vehicle.The overturning moment due to centrifugal force = P· h.

Restoring moment due to weight of the vehicle = $W \cdot \frac{b}{2}$

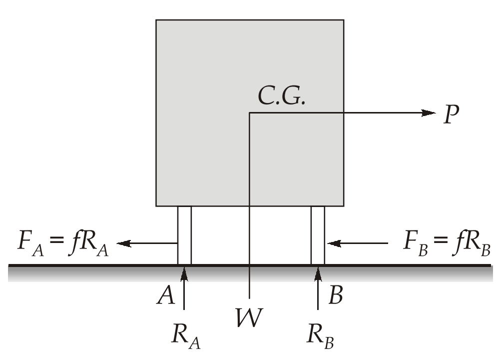
For vehicle to be on the verge of over-turning,

$$P.h = W \cdot \frac{b}{2}$$

$$\Rightarrow \quad \frac{P}{W} = \frac{b}{2h}$$

If the centrifugal ratio $\frac{P}{W}$ attains a value of $\frac{b}{2h}$, there is danger of overturning.

2. Transverse skidding effect: The centrifugal force developed has also the tendency to push the vehicle outwards in the transverse direction.



The equilibrium condition for the transverse skid resistance developed is given by,

$$P = F_A + F_B = f(R_A + R_B) = fW$$

where 'f' is coefficient of friction between the tyre and the pavement surface in the transverse direction, R_A and R_B are normal reaction of the wheels A and B such that $(R_A + R_B)$ is equal to the weight 'W' of the vehicle, as no superelevation has been provided.

$$\text{Centrifugal ratio, } \frac{P}{W} = f$$

When centrifugal ratio attains a value equal to coefficient of lateral friction, then there is a danger of lateral skidding.

Thus to avoid both overturning and lateral skidding on a horizontal curve, the centrifugal ratio should always be less than $\frac{b}{2h}$ and should also be less than transverse friction coefficient, 'f'.

5. (d) Solution:

Given data

Prototype:
$$D_p = 4 \text{ m}, Pp = 8000 \text{ kW}$$
$$N_p = 250 \text{ rpm}$$
$$H_p = 60 \text{ m}, \eta_o = 0.85$$

Model:
$$\frac{D_m}{D_p} = \frac{1}{10}$$
$$H_m = 10 \text{ m}$$
$$\eta_o = 0.85$$

$$\text{Size of model, } D_m = D_p \times \frac{1}{10}$$

$\Rightarrow$ $$D_m = 4 \times 0.1$$

$\Rightarrow$ $$D_m = 0.4 \text{ m}$$

Speed of model using head coefficient,

$$N_m = N_p \times \frac{D_p}{D_m} \times \sqrt{\frac{H_m}{H_p}}$$

$$\Rightarrow \qquad N_m = 250 \times 10 \times \sqrt{\frac{10}{60}}$$

$$\Rightarrow \qquad N_m = 1020.621 \text{ rpm}$$

Discharge of prototype,

$$P_p = \frac{\rho g Q_p H_p \eta_o}{1000}$$

$$\Rightarrow \qquad 8000 = \frac{1000 \times 9.81 \times Q_p \times 60 \times 0.85}{1000}$$

$$\Rightarrow \qquad Q_p = 15.990 \text{ m}^3/\text{s}$$

Discharge of model using discharge coefficient,

$$Q_m = Q_p \times \left(\frac{D_m}{D_p}\right)^3 \times \frac{N_m}{N_p}$$

$$\Rightarrow \qquad Q_m = 15.990 \times (0.1)^3 \times \frac{1020.621}{250}$$

$$\Rightarrow \qquad Q_m = 0.065 \text{ m}^3/\text{s}$$

Power developed by model using power coefficient,

$$P_m = P_p \times \left(\frac{D_m}{D_p}\right)^5 \times \left(\frac{N_m}{N_p}\right)^3$$

$$\Rightarrow \qquad P_m = 8000 \times (0.1)^5 \times \left(\frac{1020.621}{250}\right)^3$$

$$\Rightarrow \qquad P_m = 5.443 \text{ kW}$$

Specific speed,

$$N_s = \frac{N_p \sqrt{P_p}}{H_p^{1.25}}$$

$$\Rightarrow \qquad N_s = \frac{250 \times \sqrt{8000}}{60^{1.25}}$$

$$\Rightarrow \qquad N_s = 133.905 \text{ rpm}$$

5. (e) (i) Solution:

Given data

$$Q = 45 \text{ litre/sec} = 0.045 \text{ m}^3\text{/sec}$$

$$S = 6 \times 10^{-4}$$

$$T = 0.3 \text{ m}^2\text{/min} = 0.005 \text{ m}^2\text{/sec}$$

For radius $r = 10$ m, time, $t = 5$ hours $= 18000$ sec

$$u = \frac{r^2 S}{4Tt}$$

$$\Rightarrow \quad u = \frac{10^2 \times 6 \times 10^{-4}}{4 \times 0.005 \times 18000}$$

$$\Rightarrow \quad u = 0.000167$$

$$W(u) = -0.5772 - \ln(0.000167) + 0.000167$$

$$W(u) = 8.120$$

$$\text{Drawdown, } s = \frac{Q}{4\pi T} \times W(u)$$

$$\Rightarrow \quad s = \frac{0.045}{4 \times \pi \times 0.005} \times 8.120$$

$$\Rightarrow \quad s = 5.816 \text{ m}$$

5. (e) (ii) Solution:

Given data

$$\text{Wheelbase, B} = 6.6 \text{ m}$$

$$\text{Diameter of wheel, } D = 1500 \text{ mm} = 1.5 \text{ m}$$

$$\text{Radius of wheel, } R_w = \frac{D}{2} = 0.75 \text{ m}$$

$$\text{Depth of flange below rail, } h = 34 \text{ mm} = 0.034 \text{ m}$$

$$\text{Radius of curve, } R = 200 \text{ m}$$

Lap of flange,

$$L = 2\sqrt{h(D+h)}$$

$$\Rightarrow \quad L = 2\sqrt{0.034 \times (1.5 + 0.034)}$$

$$\Rightarrow \quad L = 0.457 \text{ m}$$

Extra width of gauge

$$W_e = \frac{13(B+L)^2}{R}$$

$$\Rightarrow \quad W_e = \frac{13(6.6+0.454)^2}{200}$$

$$\text{We} = 3.237 \text{ cm}$$

6. (a) Solution:

Given data: Hauling capacity, H = 22 tonnes

Speed on level track, V_1 = 80 kmph

Coefficient of friction, μ = 0.166

Gradient, G = $\frac{1}{400}$

D = 2°

1. Maximum permissible train load

Total train resistance, $R_t = 0.0016W + 0.00008WV + 0.0000006WV^2$

At maximum load on level track, $H = R_t$

$$\Rightarrow \quad 22 = 0.0016W + 0.00008W \times 80 + 0.0000006W \times 80^2$$

$$\Rightarrow \quad W = 1858.108 \text{ tonnes}$$

2. Reduction in speed on gradient 1 in 400

Gradient resistance, $R_g = W \times \frac{1}{400} = 1858.108 \times \frac{1}{400} = 4.645$ tonnes

$$22 = (0.0016 \times 1858.108 + 0.00008 \times 1858.108 \times V_2 + 0.0000006 \times 1858.108 \times V_2^2) + 4.645$$

$$\Rightarrow \quad V_2 = 65.033 \text{ kmph}$$

Reduction in speed = 80 – 65.033 = 14.967 kmph

3. Reduced speed with gradient and curve

Curve resistance, $R_c = 0.0004 \times W \times D$

$= 0.0004 \times 1858.108 \times 2 = 1.486$ tonnes

$$22 = (0.0016 \times 1858.108 + 0.00008 \times 1858.108 \times V_3 + 0.0000006 \times 1858.108 \times V_3^2) + 4.645 + 1.486$$

$$\Rightarrow \quad V_3 = 59.87 \text{ kmph}$$

6. (b) Solution:

Given data:

$$N = 250 \text{ rpm}$$
$$Q = 6 \text{ m}^3/\text{sec}$$
$$H = 45 \text{ m}$$
$$D_1 = 2 \text{ m}$$
$$D_2 = 2.5 \text{ m}$$
$$B_1 = B_2 = 0.2 \text{ m}$$

Discharge to be radial at outlet,

We know,

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

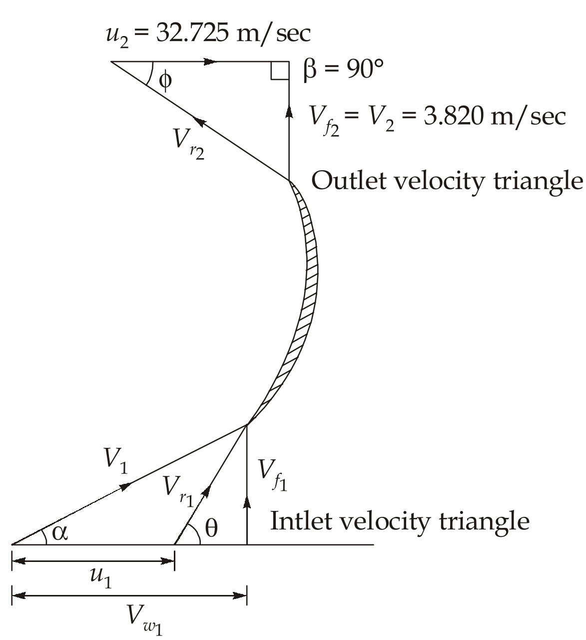
$$6 = \pi \times 2 \times 0.2 V_{f1} = \pi \times 2.5 \times 0.2 V_{f2}$$

$$V_{f1} = 4.775 \text{ m / sec}$$

$$V_{f2} = 3.820 \text{ m / sec}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2 \times 250}{60} = 26.180 \text{ m / sec}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 2.5 \times 250}{60} = 32.725 \text{ m / sec}$$



For radial outlet,

$$V_{w2} = 0 \text{ and } V_{f2} = V_2$$

Using equation,
$$H - \frac{V_2^2}{2g} = \frac{V_{w1}u_1}{g}$$

$$45 - \frac{3.820^2}{2 \times 9.81} = \frac{V_{w1} \times 26.180}{9.81}$$

$$V_{w1} = 16.583 \text{ m/sec}$$

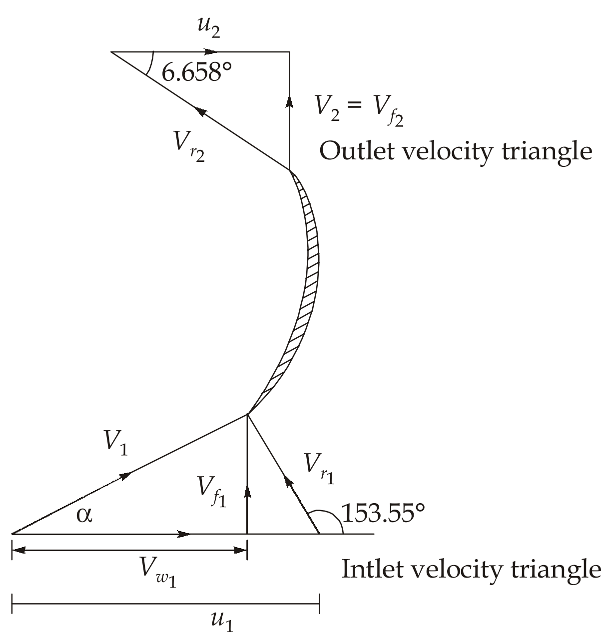
Vane angle at inlet and outlet:

From inlet velocity triangle, $\tan\theta = \dfrac{V_{f1}}{V_{w1} - u_1} = \dfrac{4.775}{16.583 - 26.180}$

$$\theta = 153.55°$$

From outlet velocity triangle, $\tan\phi = \dfrac{V_{f2}}{u_2} = \dfrac{3.820}{32.725} = 6.658°$

Inlet and outlet velocity triangle diagram shown as below



6. (c) (i) Solution:

Given:

$$BOD = 300\ \text{mg}/l$$
$$Q_o = 5000\ \text{m}^3/\text{day}$$
$$R_1 = R_2 = 1$$
$$f = 0.9\ \text{(treatability factor)}$$
$$V_1 = 7500\ \text{m}^3$$
$$V_2 = 5000\ \text{m}^3$$

$$\text{Total BOD load in first stage} = \frac{300\times10^3\times5000}{10^6} = 1500\ \text{kg/day}$$

$$\text{Recirculation factor, } F_1 = \frac{1+R_1}{\left[1+(1-f_1)R_1\right]^2} = \frac{1+1}{\left[1+(0.1)1\right]^2} = 1.653$$

$$\text{Efficiency of first stage, } \eta_1 = \frac{100}{1+0.44\sqrt{\dfrac{BOD_1}{V_1\times F_1}}}$$

$$\Rightarrow \quad \eta_1 = \frac{100}{1+0.44\sqrt{\dfrac{1500}{7500\times1.653}}}$$

$$\Rightarrow \quad \eta_1 = 86.73\%$$

$$\text{Total BOD load in second stage} = 1500\times(1-0.8673) = 199.05\ \text{kg/day}$$

$$\because \quad R_1 = R_2 = 1\ \text{(Recirculation ratio)}$$

$$\therefore \quad F_1 = F_2 = 1.653\ \text{(Recirculation factor)}$$

Organic loading rate on second stage filter,

$$OLR_2 = \frac{\text{BOD in kg/day}}{\text{Volume}} = \frac{199.05}{5000} = 0.0398\ \text{kg/m}^3/d$$

$$\therefore \quad \text{Efficiency of second stage, } \eta_2 = \frac{100}{1+\dfrac{0.44}{(1-0.8673)}\sqrt{\dfrac{0.0398}{1.653}}}$$

$$\Rightarrow \quad \eta_2 = 66.03\%$$

Overall efficiency

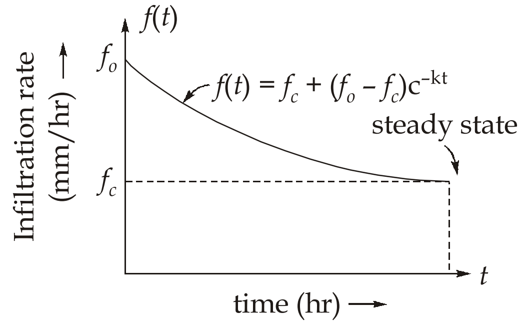
$$1-\eta_{overall} = (1-\eta_1)(1-\eta_2)$$

$$\Rightarrow \quad 1-\eta_{overall} = (1-0.8673)(1-0.6603)$$

$$\Rightarrow \quad \eta_{overall} = 0.9549 \simeq 95.5\%$$

6. (c) (ii) Solution:

Given: Horton's infiltration equation: $f(t) = 5 + 12\,e^{-2.5\,t}$ ...(i)



Infiltration rate at any time t is given by $f(t) = f_c + (f_o - f_c)e^{-kt}$ ...(ii)

Comparing equation (i) and (ii)

$$f_c = 5 \text{ mm/hr}$$

$$k = 2.5 \text{ per hr}$$

$$f_o - f_c = 12$$

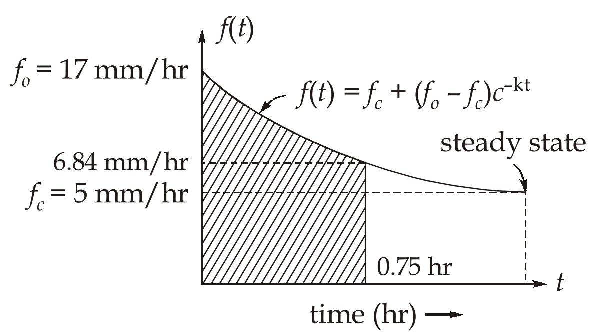
$$\Rightarrow \quad f_o = 12 + f_c = 1242*900 + 5 = 17 \text{ mm/hr}$$

Infiltration rate at first 45 min i.e. 0.75 hr is,

$$f\,(t = 0.75 \text{ hr}) = 5 + 12\,e^{-2.5 \times 0.75} = 6.84 \text{ mm/hr}$$

$$\because \quad f(t = 0.75 \text{ hr}) < f_o$$

$\therefore$ Steady state is not attained



Depth of infiltration for first 45 min is,

$$I_{45\text{ min}} = \int_0^{0.75} \left(5 + 12e^{-2.5t}\right) dt$$

$$\Rightarrow \quad I_{45\text{ min}} = \left(5t - \frac{12e^{-2.5t}}{2.5}\right)_0^{0.75}$$

$$\Rightarrow \quad I_{45\text{ min}} = 5 \times 0.75 - \frac{12}{2.5}\left(e^{-2.5\times0.75} - 1\right)$$

$$\Rightarrow \quad I_{45\text{ min}} = 7.81 \text{ mm}$$ **Ans.**

Average infiltration rate for first 75 min i.e. 1.25 hr

$$(i_{avg})_{1.25\text{ hr}} = \frac{\int_0^{1.25} f(t)\,dt}{\int_0^{1.25} 1\,dt} = \frac{\int_0^{1.25}\left(5+12e^{-2.5t}\right)dt}{1.25}$$

$$= \frac{1}{1.25}\left(5t - \frac{12}{2.5}e^{-2.5t}\right)_0^{1.25}$$

$$= \frac{1}{1.25}\left[5\times1.25 - \frac{12}{2.5}\left(e^{-2.5\times1.25} - 1\right)\right]$$

$$= 8.67 \text{ mm/hr}$$ **Ans.**

7. (a) (i) Solution:

Given data

Flow rate, Q = 12000 m^3/d

Influent BOD, S_o = 1.5 kg/m^3

Effluent BOD, S = 0.15 kg/m^3

Mean cell residence time, θ_c = 6 days

MLSS concentration, X = 4 kg/m^3

Yield coefficient, Y = 0.6 kg/kg

Decay coefficient, K_d = 0.04 d^{-1}

Underflow concentration, X_u = 12 kg/m^3

1. Volume of reactor

$$\frac{1}{\theta_c} = \frac{YQ(S_o - S)}{VX} - K_d$$

$$\Rightarrow \quad \frac{1}{6} = \frac{0.6\times12000\times(1.5-0.15)}{V\times4} - 0.04$$

$$\Rightarrow \quad V = 11758.064 \text{ m}^3$$

2. Mass of solids wasted per day

$$= \frac{VX}{\theta_c} = \frac{11758.064\times4}{6} = 7838.709 \text{ kg/d}$$

Volume of solids wasted per day

$$Q_w = \frac{7838.709}{12} = 653.225 \text{ m}^3/\text{d}$$

3. Sludge recirculation ratio $R = \frac{X}{X_u - X}$

$$\Rightarrow \quad R = \frac{4}{12-4} = 0.5$$

7. (a) (ii) Solution:

Given data:

$$\text{BOD}_{4,35} = 350 \text{ mg/l}$$
$$t_1 = 4 \text{ days}$$
$$T_1 = 35°\text{C}$$
$$k_{20} = 0.23 \text{ day}^{-1}$$
$$t_2 = 5 \text{ days}$$
$$T_2 = 20°\text{C}$$

Deoxygenation constant at 35°C,

$$k_{35} = k_{20} \times (1.047)^{(35-20)}$$

$$\Rightarrow \quad k_{35} = 0.23 \times (1.047)^{15} = 0.458 \text{ day}^{-1}$$

Ultimate BOD, $\text{BOD}_t = L_0(1-e^{-kt})$

$$350 = L_0(1-e^{-0.458\times 4})$$

$$\Rightarrow \quad L_0 = 416.716 \text{ mg}/l$$

5-day 20°C BOD, $\text{BOD}_{5,20} = L_0(1-e^{-k_{20}\times 5})$

$$\Rightarrow \quad \text{BOD}_{5,20} = 416.716(1-e^{-0.23\times 5}) = 284.768 \text{ mg}/l$$

7. (a) (iii) Solution:

Mass of glucose = 100 g

Molecular weight of carbon = 12.011 g/mol

Molecular weight of hydrogen = 1.008 g/mol

Molecular weight of oxygen = 15.999 g/mol

Balanced equation for oxidation,

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$$

Molecular weight of glucose, $M = 6\times12.0+12\times1.00+6\times16 = 180$ g/mol

Molecular weight of oxygen, $M_{O_2} = 2\times16 = 32$ g/mol

Oxygen required for 1 mole of glucose,

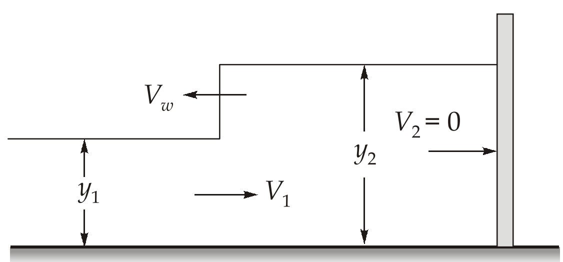
$$= 6\times32 = 192 \text{ g}$$

Maximum BOD, $\text{BOD}_{max} = \dfrac{192}{180}\times100 = 106.67$ g

7. (b) Solution:

Since gate is suddenly dropped

$$V_2 = 0$$
$$Q = 6 \text{ m}^3/\text{s}$$
$$y_1 = 1.3 \text{ m}$$
$$b = 3 \text{ m}$$
$$V_1 = \frac{Q}{A_1} = \frac{6}{3\times1.3} = 1.538 \text{ m/s}$$



Velocity of water at section 2 with respect to surge = V_w
Velocity of water at section 1 w.r.t. surge = $(V_1 + V_w)$
By continuity equation

$$V_w y_2 = (V_1 + V_w)y_1$$
$$y_1 = 1.3 \text{ m}$$
$$\Rightarrow \quad V_1 = 1.538 \text{ m/s}$$
$$\Rightarrow \quad V_w y_2 = (1.538 + V_w)\times1.3 \quad ...(i)$$

By momentum equation

$$\rho Q[V_w - (V_w + V_1)] = P_1 - P_2$$

$$Q = by_1(V_1 + V_w) = 3 \times 1.3(1.538 + V_w) = 3.9(1.538 + V_w)$$

$$P_1 = \gamma_w \frac{y_1}{2} \times 1.3 \times 3 = 1.95\gamma_w y_1 = 2.535\gamma_w$$

$$P_2 = \gamma_w \frac{y_2}{2} \times y_2 \times 3 = 1.5\gamma_w y_2^2 = 1.5\gamma_w y_2^2$$

$$\rho \times 3.9(1.538 + V_w)[-1.538] = \gamma_w(2.535 - 1.5y_2^2)$$

$$\Rightarrow \quad 6(1.538 + V_w) = 9.81(1.5y_2^2 - 2.535) \quad \text{...(ii)}$$

From (i)

$$V_w y_2 = (1.538 + V_w) \times 1.3$$

$$\Rightarrow \quad V_w(y_2 - 1.3) = 2$$

$$\Rightarrow \quad V_w = \frac{2}{y_2 - 1.3}$$

$$\Rightarrow \quad 6\left(\frac{2}{y_2 - 1.3} + 1.583\right) = 9.81(1.5y_2^2 - 2.535)$$

$$\frac{2}{y_2 - 1.3} = 1.635(1.5y_2^2 - 2.535) - 1.583$$

$$= 2.453y_2^2 - 5.728$$

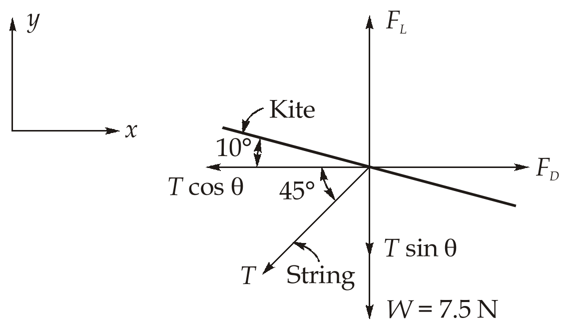
$$\frac{2}{y_2 - 1.3} = 2.453y_2^2 - 5.728$$

By trial and error $\quad y_2 \simeq 1.885 \text{ m}$

$$y_2 > 1.75 \text{ m}$$

Channel will overflow $\quad V_w = \dfrac{2}{1.885 - 1.3} = 3.419 \text{ m/s}$

7. (c) (i) Solution:



Given: $C_D = 0.6,\ C_L = 0.8$

For equilibrium

$$\Sigma F_x = 0 \Rightarrow F_D = T.\cos 45° \quad \text{...(i)}$$

$$\Sigma F_y = 0 \Rightarrow F_L = W + T.\sin 45° \quad \text{...(ii)}$$

$\because$
$$F_D = \frac{C_D}{2}\rho_{air} A V_0^2$$

$$F_L = \frac{C_L}{2}\rho_{air} A V_0^2$$

Now,

$$\frac{C_D}{2}\rho_{air} A V_0^2 = T.\cos 45°$$

$$\Rightarrow \frac{0.6}{2}\times 1.2\times(0.9\times 0.9)\times V_0^2 = T.\cos 45°$$

$$\Rightarrow V_0^2 = 2.4249\ T \quad \text{...(iii)}$$

$$\frac{C_L}{2}\rho_{air} A V_0^2 = W + T.\sin 45°$$

$$\Rightarrow \frac{0.8}{2}\times 1.2\times(0.9\times 0.9)V_0^2 = 7.5 + T\sin 45°$$

$$\Rightarrow 0.3888 \times 2.4249\ T = 7.5 + \frac{T}{\sqrt{2}}$$

$$\Rightarrow 0.23569\ T = 7.5$$

$$\Rightarrow T = 31.82\ \text{N}$$

From equation (iii)

$$V_0^2 = 2.4249 \times 31.82$$

$$\Rightarrow V_0^2 = 8.784\ \text{m/sec} = 31.62\ \text{kN/m}$$

7. (c) (ii) Solution:

Given:

Depth of flow, $y = 1.5$ m

Bed slope, $S = 0.0009$

Width of channel, $B = 3$ m

Manning's coefficient, $n = 0.015$

Hydraulic mean radius, $R = \frac{A}{P} = \frac{By}{B+2y}$

$\Rightarrow \quad R = \frac{3\times1.5}{3+2\times1.5} = 0.75$ m

Velocity,

$$v = \frac{1}{n}R^{2/3}S^{1/2}$$

$$\Rightarrow \quad v = \frac{1}{0.015}(0.75)^{\frac{2}{3}}(0.0009)^{\frac{1}{2}} = 1.65 \text{ m/sec}$$

Discharge per unit width

$$q = \frac{Q}{B} = yv = 1.5 \times 1.65 = 2.475 \text{ m}^3/\text{sec/m}$$

Critical depth,

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{2.475^2}{9.81}\right)^{\frac{1}{3}} = 0.855 \text{ m}$$

Maximum height of hump,

$$z_{max} = E_1 - E_2 = E_1 - E_C$$

$$\Rightarrow \quad z_{max} = \left(y_1 + \frac{v_1^2}{2g}\right) - \frac{3}{2}y_c$$

$$\Rightarrow \quad z_{max} = \left(1.5 + \frac{1.65^2}{2\times9.81}\right) - \frac{3}{2}\times0.855$$

$$\Rightarrow \quad z_{max} = 0.3563 \text{ m}$$

8. (a) Solution:

1. Bulk density of specimen by uncoated specimen procedure and immersion test. The bulk specific gravity G_{mb} is calculated as:

$$G_{mb} = \frac{1200.5}{1200.5-695.2} = 2.376$$

Bulk density, $\gamma_b = G_{mb} \times \gamma_w$ (taking $\gamma_w = 1\ g/cm^3$)

2. Bulk density of specimen by paraffin-coated sample procedure

Weight of paraffin:

$$W_p = 1248.3-1200.5 = 47.8\ g$$

Bulk specific gravity with paraffin coating:

$$G_{mb} = \frac{1200.5}{1248.3-680.8-\left(\frac{47.8}{0.90}\right)} = 2.334$$

Bulk density, $\gamma_b = 2.334\ g/cm^3$

3. Percent volume occupied by constituents

Percent volume of asphaltic cement (V_b):

$$V_b = \frac{P_b \times G_{mb}}{G_b} = \frac{5.5\times 2.334}{1.03} = 12.463\%$$

Percent volume of coarse aggregates (V_1),

$$V_1 = \frac{P_1 \times G_{mb}}{G_1}\ \frac{53.4\times 2.334}{2.62} = 47.571\%$$

Percent volume of fine aggregates, $V_2 = \frac{P_2 \times G_{mb}}{G_2} = \frac{33.6\times 2.334}{2.75} = 28.517\%$

Percent volume of mineral filler $V_3 = \frac{P_3 \times G_{mb}}{G_3} = \frac{7.5\times 2.334}{2.70} = 6.483\%$

4. Percent voids in mineral aggregate (VMA)

Total aggregate by weight:

$$P_s = 100 - P_b = 100 - 5.5 = 94.5\%$$

Bulk specific gravity of aggregate G_{sb}:

$$G_{sb} = \frac{P_1+P_2+P_3}{\frac{P_1}{G_1}+\frac{P_2}{G_2}+\frac{P_3}{G_3}} = \frac{53.4+33.6+7.5}{\frac{53.4}{2.62}+\frac{33.6}{2.75}+\frac{7.5}{2.70}} = 2.671$$

VMA:

$$\text{VMA} = 100-\left(\frac{G_{mb}\times P_s}{G_{sb}}\right) = 100-\left(\frac{2.334\times 94.5}{2.671}\right) = 17.423\%$$

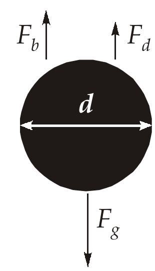
5. Percent aggregates voids filled with asphalt (VFA)
   Air voids (V_a):

$$V_a = \text{VMA} - V_b = 17.423 - 12.463 = 4.96\%$$

VFA:

$$\text{VFA} = \frac{V_b}{VMA}\times 100 = \frac{12.463}{17.423}\times 100 = 71.532\%$$

8. (b) (i) Solution:



Gravitational force:

$$F_g = \rho_p\, g\, V_p$$

Buoyant force:

$$F_b = \rho_w\, g\, V_p$$

Drag force:

$$F_d = C_d\, A\,\frac{\rho_w\, v^2}{2}$$

Where:

$$V_p = \frac{\pi}{6}d^3 \text{ (volume of spherical particle)}$$

$$A = A = \frac{\pi}{4}d^2 \text{ (projected area)}$$

At terminal velocity v_s, the net force is zero:

$$F_g - F_b - F_d = (\rho_p - \rho_w)g\frac{\pi}{6}d^3 = C_d\frac{\pi}{4}d^2\frac{\rho_w v_s^2}{2}$$

Rearranging for terminal settling velocity:

$$v_s = \sqrt{\frac{4g(\rho_p-\rho_w)d}{3C_d\rho_w}} = \sqrt{\frac{4(G-1)gd}{3C_d}}$$

8. (b) (ii) Solution:

The characteristic equation is given as

$$V(m/s) = 0.51\,N + 0.03$$

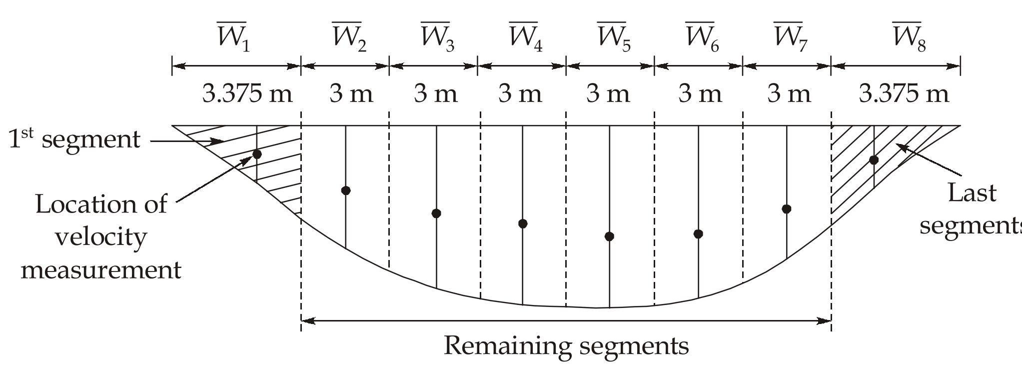
Where N is the number of revolutions per second.Assuming depth at a distance of 27 m from one bank to be zero.The total discharge is calculated by method of mid sections. For the first and last section average width,

$$\bar{W}_1 = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1} = \frac{\left(3 + \frac{3}{2}\right)^2}{2 \times 3} = 3.375 \text{ m}$$

For the rest of segments, $$\bar{W} = \left(\frac{3}{2} + \frac{3}{2}\right) = 3 \text{ m}$$

Distance from one bank (m)	Depth (m) (y)	Average width (m) $\bar{W}$	N	V	Segmental discharge $DQ = y \times V \times \bar{W}$
3.0	0.4	3.375	0.2	0.132	0.1782
6.0	0.8	3.0	0.385	0.226	0.543
9.0	1.2	3.0	0.7	0.387	1.393
12.0	2.0	3.0	1.25	0.667	4.005
15.0	3.0	3.0	2.5	1.305	11.745
18.0	2.5	3.0	4	2.07	15.525
21.0	2.2	3.0	3.25	1.687	11.137
24.0	1.0	3.375	0.69	0.382	1.288
27.0	0	–			

Total discharge, SQ = 45.816 m^3/sec



8. (c) Solution:

Given Data:Total height of wall, H= 5 m

Top layer: $h_1 = 2$ m,

$\gamma_1 = 18.5 \text{ kN/m}^3$

$c_1 = 10$ kPa

$\phi_1 = 20°$

Middle layer: $h_2 = 1$ m

$\gamma_2 = 17.2 \text{ kN/m}^3$

$c_2 = 0$

$\phi_2 = 30°$

Bottom layer: $h_3 = 2$ m

$\gamma_3 = 18.8 \text{ kN/m}^3$

$c_3 = 0$

$\phi_3 = 38°$

Coefficient of active earth pressure:

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45^\circ - \frac{\phi}{2}\right)$$

$$K_{a1} = 0.49, \quad K_{a2} = 0.333, \quad K_{a3} = 0.238$$

Active pressure at depth z is calculated as:

$$p_a = K_a\sigma_v - 2c\sqrt{K_a}, \sigma_v = \text{vertical stress at depth } z$$

For the top layer (sandy silt):

At base of top layer, z = 2 m:

$$\sigma_v = \gamma_1 h_1 = 18.5 \times 2 = 37 \text{ kPa}$$

$$p_2(\text{top layer}) = K_{a1}\sigma_v - 2c_1\sqrt{K_{a1}}$$

$$= 0.49 \times 37 - 2 \times 10\sqrt{0.49} = 4.13 \text{ kPa}$$

Tension crack depth in top layer:

$$z_c = \frac{2c_1}{\gamma_1\sqrt{K_{a1}}} = \frac{2 \times 10}{18.5\sqrt{0.49}} = 1.544 \text{ m}$$

For the middle layer (loose sand), $c_2 = 0$

At top of middle layer (z = 2 m):

$$p_{2(\text{mid})} = K_{a2}\sigma_v = 0.333 \times 37 = 12.32 \text{ kPa}$$

At bottom of middle layer (z = 3 m)

$$\sigma_v = 37 + \gamma_2 \times h_2 = 37 + 17.2 \times 1 = 54.2 \text{ kPa}$$

$$p_{3(\text{mid})} = K_{a2}\sigma_v = 0.333 \times 54.2 = 18.05 \text{ kPa}$$

For the bottom layer (dense sand), $c_3 = 0$:

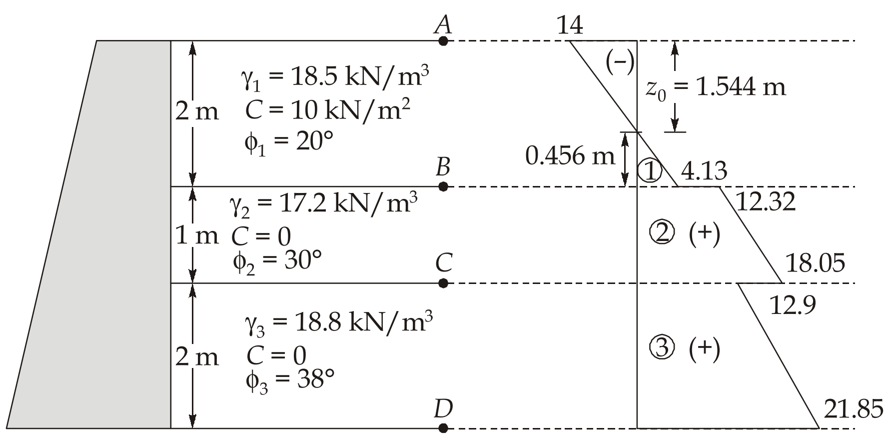
At top of bottom layer (z = 3 m):

$$p_{3(\text{bot})} = K_{a3}\sigma_v = 0.238 \times 54.2 = 12.9 \text{ kPa}$$

At bottom of bottom layer (z = 5 m):

$$\sigma_v = 54.2 + \gamma_3 \times h_3 = 54.2 + 18.8 \times 2 = 91.8 \text{ kPa}$$

$$p_4 = K_{a3}\sigma_v = 0.238 \times 91.8 = 21.85 \text{ kPa}$$



(Earth Pressure diagram)

Total active force is the area under the pressure diagram:

Top layer triangle (from z = 1.544 to 2 m):

$$P_1 = \frac{1}{2} \times 4.13 \times (2 - 1.544) = 0.942 \text{ kN/m}$$

Middle layer rectangle: $P_2 = 12.321 \times 1 = 12.321 \text{ kN/m}$

Middle layer triangle: $P_3 = \frac{1}{2} \times (18.05 - 12.32) \times 1 = 2.865 \text{ kN/m}$

Bottom layer rectangle: $P_4 = 12.9 \times 2 = 25.8 \text{ kN/m}$

Bottom layer triangle: $P_5 = \frac{1}{2} \times (21.85 - 12.9) \times 2 = 8.95 \text{ kN/m}$

Total active pressure: $P_{total} = 0.942 + 12.321 + 2.865 + 25.8 + 8.95 = 50.878$ kN/m

Point of application from base:

$$y = \frac{(0.942 \times 3.152) + (12.321 \times 2.5) + (2.865 \times 2.333) + (25.8 \times 1) + (8.95 \times 0.667)}{50.878}$$

$$y = 1.42 \text{ m}$$

The total active pressure is 50.878 kN/m, acting at 1.42 m above the base.

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