MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 14

Full Syllabus Test (Paper-I)

Section A

Q.1 (a) Solution:

(i) Power supplied to the motor,

$$P_{3\text{-}\phi} = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3}\times 11000\times 60\times 0.8 = 914.52 \text{ kW}$$

(ii) For star connection, $I_{ph} = I_L = 60\angle\cos^{-1}(0.8) = 60\angle 36.87° \text{ A}$

For star connection, phase voltage,

$$V_{ph} = V_L\sqrt{3} = 11000/\sqrt{3} = 6350.85 \text{ V}$$

Synchronous impedance per phase, $Z_S = (1 + j30)\Omega$.

The induced EMF per phase is given by,

$$E_{ph} = V_{ph} - I_{ph}Z_S$$

$$= (6350.85\angle 0°) - [(60\angle 36.87°)\times(1+j30)] \text{ V}$$

$$= 6350.85 - [(48 + j36)(1 + j30)] = 7382.85 - j1476$$

$$E_{ph} = 7528.95\angle -11.30° \text{ V}$$

For star connection, $|E_{L\text{-}L}| = \sqrt{3}E_{ph} = \sqrt{3}\times 7528.95 = 13040.52 \text{ V} = 13.04 \text{ kV}$

Q.1 (b) Solution:

(i) Given: $V = 240$ V; $V_R = 100$ V; $P_R = 300$ W; $f = 50$ Hz

Power dissipated in the resistor,

$$P_R = \frac{V_R^2}{R}$$

$$300 = \frac{(100)^2}{R}$$

$$R = 33.33\ \Omega$$

Also,

$$P_R = I^2R$$

$$300 = I^2 \times 33.33$$

$$I = 3\text{ A}$$

Impedance of the circuit,

$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$

Thus,

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(80)^2 - (33.33)^2} = 72.72\ \Omega$$

We have,

$$X_C = \frac{1}{2\pi fC}$$

$$72.72 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 43.77\ \mu\text{F}$$

Voltage across capacitor,

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(240)^2 - (100)^2} = 218.17\text{ V}$$

Maximum value of capacitor voltage,

$$V_{C\max} = 218.17 \times \sqrt{2} = 308.54\text{ V}$$

Maximum charge $Q_{max} = CV_{C\max} = 43.77 \times 10^{-6} \times 308.54 = 0.0135$ C

Maximum stored energy

$$E_{\max} = \frac{1}{2}C(V_{C\max})^2$$

$$= \frac{1}{2} \times 43.77 \times 10^{-6} \times (308.54)^2 = 2.08\text{ J}$$

(ii) Applying KCL at Node 1,

$$I_1 = \frac{V_1 - V_3}{1} + \frac{V_1 - V_2}{\frac{1}{s}} = (s+1)V_1 - sV_2 - V_3 \quad \text{...(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2 - V_3}{1} + \frac{V_2 - V_1}{\frac{1}{s}} = (s+1)V_2 - sV_1 - V_3 \quad \text{...(ii)}$$

Applying KCL at Node 3,

$$\frac{V_3}{\frac{1}{s}} + \frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{1} = 0$$

$$(s + 2)V_3 - V_1 - V_2 = 0$$

$$V_3 = \frac{1}{s+2}V_1 + \frac{1}{s+2}V_2 \quad \text{...(iii)}$$

Substituting the equation (iii) in the equation (i),

$$I_1 = (s+1)V_1 - sV_2 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= \left[\frac{(s+1)(s+2)-1}{(s+2)}\right]V_1 - \left[\frac{s(s+2)+1}{(s+2)}\right]V_2$$

$$= \left(\frac{s^2+3s+1}{s+2}\right)V_1 - \left(\frac{s^2+2s+1}{s+2}\right)V_2 \quad \text{...(iv)}$$

Substituting the Eq. (iii) in the Eq. (ii),

$$I_2 = (s+1)V_2 - sV_1 - \left(\frac{1}{s+2}V_1 + \frac{1}{s+2}V_2\right)$$

$$= -\left[\frac{s(s+2)+1}{(s+2)}\right]V_1 + \left[\frac{(s+1)(s+2)-1}{(s+2)}\right]V_2$$

$$= -\left(\frac{s^2+2s+1}{s+2}\right)V_1 + \left(\frac{s^2+3s+1}{s+2}\right)V_2 \quad \text{...(v)}$$

Comparing Eqs (iv) and (v) with Y-parameter equations,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \text{ and } I_2 = Y_{21}V_1 + Y_{22}V_2$$

we get

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{s+2} & \dfrac{-(s^2+2s+1)}{s+2} \\ \dfrac{-(s^2+2s+1)}{s+2} & \dfrac{s^2+3s+1}{s+2} \end{bmatrix}$$

Q.1 (c) Solution:

Given,

$$\text{Quantum efficiency, } \eta = 50\% = 0.5$$

$$\text{Wavelength, } \lambda = 900 \text{ nm} = 900 \times 10^{-9} \text{ m}$$

$$\text{Multiplied photocurrent, } I_M = 10\ \mu\text{A} = 10 \times 10^{-6} \text{ A}$$

$$\text{Multiplication factor, } M = 250$$

(i) Received optical power

Primary photocurrent,

$$I_p = \frac{I_M}{M}$$

$$I_p = \frac{10\times10^{-6}}{250} = 4\times10^{-8}\text{A}$$

$$I_p = 0.04\ \mu\text{A}$$

Responsivity at unity gain,

$$R_0 = \eta\frac{q\lambda}{hc} = 0.5\frac{(1.6\times10^{-19})(900\times10^{-9})}{(6.63\times10^{-34})(3\times10^{8})}$$

$$R_0 \approx 0.362 \text{ A/W}$$

Incident optical power,

$$P = \frac{I_P}{R_0} = \frac{4\times10^{-8}}{0.362}$$

$$P \approx 1.10 \times 10^{-7} \text{ W}$$

$$P \approx 0.11\ \mu\text{W}$$

(ii) Number of photons per second:

Energy of one photon,

$$E_{ph} = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{900\times10^{-9}}$$

$$E_{ph} \approx 2.21 \times 10^{-19} \text{ J}$$

Number of photons incident per second,

$$N = \frac{P}{E_{ph}} = \frac{1.10\times10^{-7}}{2.21\times10^{-19}}$$

$$N \approx 5 \times 10^{11} \text{ photons/s}$$

Q.1 (d) Solution:

(i) Junction to case Thermal resistance,

$$\theta_{JC} = \frac{T_J - T_C}{P_D} = \frac{180° - 50°}{50 \text{ W}} = 2.6°\text{C/W}$$

Temperature rise from junction to sink:

$$T_J - T_S = \theta_{JS} P_D$$

$$180° - T_S = (\theta_{JC} + \theta_{CS}) P_D$$

$$\Rightarrow \quad T_S = 180° - (2.6 + 0.6) \times 30 = 84°\text{C}$$

(ii)

$$T_S - T_A = \theta_{SA} P_D$$

$$84°\text{C} - 39°\text{C} = \theta_{SA} \times 30 \text{ W}$$

$\Rightarrow$ Required heat sink thermal resistance,

$$\theta_{SA} = 1.5°\text{C/W}$$

(iii) Given sink-to-air thermal resistance: 4.5°C/W per cm length. The size of heat sink needed is

$$= \frac{4.5°\text{C/W/cm}}{1.5°\text{C/W}} = 3 \text{ cm}$$

Q.1 (e) Solution:

The Truth Table and Excitation Table for J-K flip flop is as shown below,

Truth table

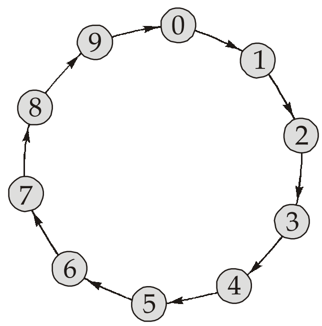
J	K	$Q(n+1)$
0	0	$Q(n)$
0	1	0
1	0	1
1	1	$\overline{Q(n)}$

Excitation table

$Q(n)$	$Q(n+1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Design of MOD-10 synchronous counter using J-K flip-flop: A MOD-10 counter counts from 0 to 9 and then return to 0.

State diagram



State assignment

0 – 0 0 0 0
1 – 0 0 0 1
2 – 0 0 1 0
3 – 0 0 1 1
4 – 0 1 0 0
5 – 0 1 0 1
6 – 0 1 1 0
7 – 0 1 1 1
8 – 1 0 0 0
9 – 1 0 0 1

	Present state				Next state				Flip-Flop Excitations							
	Q_3	Q_2	Q_1	Q_0	Q_3	Q_2	Q_1	Q_0	J_3	K_3	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	0	0	0	0	1	0	x	0	x	0	x	1	x
1	0	0	0	1	0	0	1	0	0	x	0	x	1	x	x	1
2	0	0	1	0	0	0	1	1	0	x	0	x	x	0	1	x
3	0	0	1	1	0	1	0	0	0	x	1	x	x	1	x	1
4	0	1	0	0	0	1	0	1	0	x	x	0	0	x	1	x
5	0	1	0	1	0	1	1	0	0	x	x	0	1	x	x	1
6	0	1	1	0	0	1	1	1	0	x	x	0	x	0	1	x
7	0	1	1	1	1	0	0	0	1	x	x	1	x	1	x	1
8	1	0	0	0	1	0	0	1	x	0	0	x	0	x	1	x
9	1	0	0	1	0	0	0	0	x	1	0	x	0	x	x	1

J_3

$Q_3 Q_2$ \ $Q_1 Q_0$	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	x	x	x	x
10	x	x	x	x

$J_3 = Q_2 Q_1 Q_0$

K_3

$Q_3 Q_2$ \ $Q_1 Q_0$	00	01	11	10
00	x	x	x	x
01	x	x	x	x
11	x	x	x	x
10	0	1	x	x

$K_3 = Q_0$

J_2

$Q_3 Q_2$ \ $Q_1 Q_0$	00	01	11	10
00	0	0	1	0
01	x	x	x	x
11	x	x	x	x
10	0	0	x	x

$J_2 = Q_1 Q_0$

K_2

$Q_3 Q_2$ \ $Q_1 Q_0$	00	01	11	10
00	x	x	x	x
01	0	0	1	0
11	x	x	x	x
10	x	x	x	x

$K_2 = Q_1 Q_0$

J_1

$Q_3Q_2 \backslash Q_1Q_0$	00	01	11	10
00	0	1	x	x
01	0	1	x	x
11	x	x	x	x
10	0	0	x	x

$J_1 = \overline{Q}_3 Q_0$

K_1

$Q_3Q_2 \backslash Q_1Q_0$	00	01	11	10
00	x	x	1	0
01	x	x	1	0
11	x	x	x	x
10	x	x	x	x

$K_1 = Q_0$

J_0

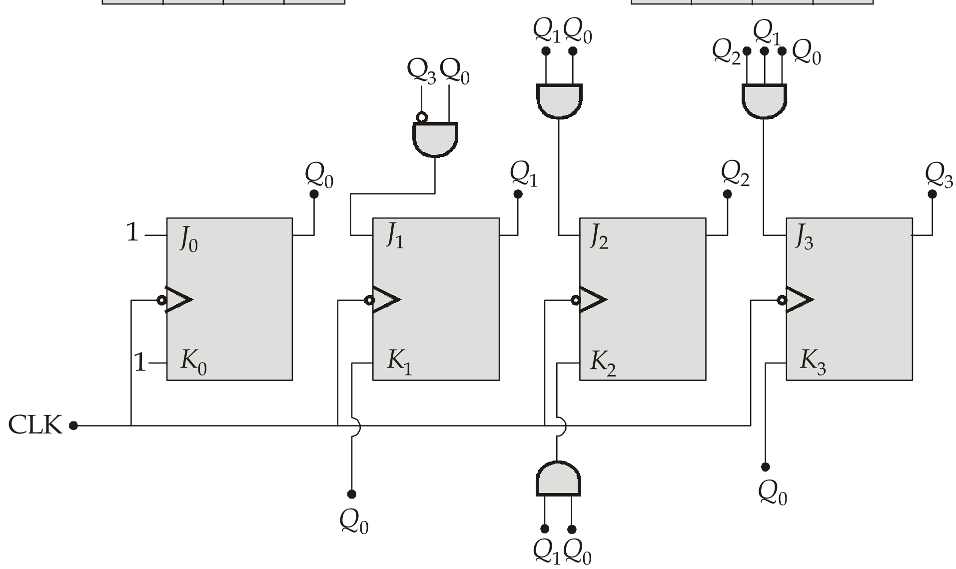
$Q_3Q_2 \backslash Q_1Q_0$	00	01	11	10
00	1	x	x	1
01	1	x	x	1
11	x	x	x	x
10	1	x	x	x

$J_0 = 1$

K_0

$Q_3Q_2 \backslash Q_1Q_0$	00	01	11	10
00	x	1	1	x
01	x	1	1	x
11	x	x	x	x
10	x	1	x	x

$K_0 = 1$



Q.2 (a) Solution:

Given:

$$W = 50\ \mu m = 5 \times 10^{-3}\ cm$$

$$L = 5\ \mu m = 5 \times 10^{-4}\ cm$$

$$t_{ox} = 0.05\ \mu m = 5 \times 10^{-6}\ cm$$

$$N_A = 10^{15}\ cm^{-3}$$

$$\mu_n = 800\ cm^2/Vs$$

Ignore body effect $\rightarrow V_{SB} = 0$

Constants: $\varepsilon_{ox} = 3.45 \times 10^{-13}$ F/cm, $\varepsilon_{si} = 1.04 \times 10^{-12}$ F/cm

(i) Oxide capacitance, $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-13}}{5\times10^{-6}} = 6.9\times10^{-8}\ \text{F/cm}^2$

$$\text{Fermi potential, } \phi_F = V_T \ln\left(\frac{N_A}{n_i}\right) = 0.026\ln\left(\frac{10^{15}}{10^{10}}\right) = 0.026\ln(10^5)$$

$$= 0.026 \times 11.51 = 0.30\ \text{V}$$

Flat-band voltage (n^+ poly),

$$V_{FB} = \phi_{ms} - \frac{Q_{ox}}{C_{ox}} \approx -\left(\frac{E_g}{2} + \phi_F\right) = -(0.56 + 0.30) = -0.86\ \text{V}$$

$$\text{Threshold voltage, } V_T = V_{FB} + 2\phi_F + \frac{\sqrt{2q\epsilon_{si} N_A (2\phi_F)}}{C_{ox}}$$

$$V_T = -0.86 + 0.60 + \frac{\sqrt{2\times1.6\times10^{-19}\times1.04\times10^{-12}\times10^{15}\times0.6}}{6.9\times10^{-8}}$$

$$V_T = -0.86 + 0.60 + 1.45$$

$$V_T = -0.056\ \text{V}$$

(ii) I_{Dsat} at $V_{GS} = 2$ V

$$I_{Dsat} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

From the given values, $\frac{W}{L} = 10$

$$\mu_n C_{ox}\frac{W}{L} = 800 \times 6.9 \times 10^{-8} \times 10 = 5.52 \times 10^{-4}\ \text{A/V}^2$$

and $V_{GS} - V_T = V_{ov} = 2 - (-0.056) = 2.056$ V

Thus, $I_{Dsat} = \frac{1}{2}(5.52\times10^{-4})(2.056)^2$

$$= 11.7 \times 10^{-4}\ \text{A}$$

$\therefore$ $I_{Dsat} = 1.17$ mA

(iii) $\frac{dI_D}{dV_{DS}}$ at $V_{GS} = 2$ V, $V_{DS} = 0$

For the given values: $V_{GS} > V_T$ and $V_{DS} < V_{GS} - V_T$, thus MOSFET is operating in linear region.

In linear region:

$$I_D = k\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right] \text{ where, } k = \mu_n C_{ox}\frac{W}{L}.$$

$$\frac{dI_D}{dV_{DS}} = k[(V_{GS} - V_T) - V_{DS}]$$

At $V_{DS} = 0$:

$$\frac{dI_D}{dV_{DS}} = k(V_{GS} - V_T)$$

$$= 5.52 \times 10^{-4} \times 2.056$$

$$= 1.13 \times 10^{-3} \text{ A/V} = 1.13 \text{ mA/V}$$

(iv) $\frac{dI_D}{dV_{GS}}$ at $V_G = 1$ V, $V_D = 2$ V

Since $V_D = 2 > V_G - V_T = 1.056$

Hence, device in saturation region,

$\therefore$
$$I_D = \frac{1}{2}k(V_{GS} - V_T)^2$$

$$g_m = \frac{dI_D}{dV_{GS}} = k(V_{GS} - V_T)$$

$$= 5.52 \times 10^{-4} \times 1.056$$

$$\frac{dI_D}{dV_{GS}} = 5.83 \times 10^{-4} \text{ A/V}$$

Q.2 (b) Solution:

Given

$$V_{CC} = 3 \text{ V}, V_{in} = 1 \text{ V}, V_{out} = 1.5 \text{ V}$$
$$\beta \to \infty, V_{BE} = 0.7 \text{ V}, r_0 = \infty$$

(i) Collector currents and Transconductances:

For transistor Q_1, $V_{E1} = V_{in} - V_{BE(on)} = 1 - 0.7 = 0.3$ V

For transistor Q_3, $I_{C3} = \frac{V_{CC} - V_{out}}{R_{C2}} = \frac{3 - 1.5}{375} = 4 \text{ mA} \approx I_{E3}$

Applying KCL at emitter of Q_3,

$$I_{E3} = \frac{V_{E3}}{200} + \frac{V_{E3} - V_{E1}}{300}$$

$$0.004 = \frac{V_{E3}}{200} + \frac{V_{E3} - 0.3}{300}$$

$$\Rightarrow \quad 2.4 = 3V_{E3} + 2V_{E3} - 0.6$$

$$V_{E3} = 0.6 \text{ V}$$

Thus, we can obtain

$$I_{E1} = \frac{V_{E1}}{150} + \frac{V_{E1} - V_{E3}}{300} = \frac{0.3}{150} + \frac{0.3 - 0.6}{300}$$

$$I_{E1} = 1 \text{ mA} \approx I_{C1}$$

We have, $V_{B3} = V_{E3} + V_{BE(on)} = 0.6 + 0.7 = 1.3 \text{ V} = V_{C2}$

$$I_{C2} = \frac{V_{CC} - V_{C2}}{R_{C2}} = \frac{3 - 1.3}{850} = 2 \text{ mA}$$

The transconductances of MOSFET can thus be obtained as below:

$$g_{m1} = \frac{I_{C2}}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mS}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ V}} = 80 \text{ mS}$$

$$g_{m3} = \frac{I_{C3}}{V_T} = \frac{4 \text{ mA}}{0.025 \text{ V}} = 160 \text{ mS}$$

(ii) The feedback resistor 300 Ω samples the output current at emitter of Q_3 (current sampling) and feeds back to the input emitter of Q_1 in series with input signal, $v_{be} = v_{in} - v_f$ (series mixing). Thus, the topology is Current-Series Feedback.

(iii) Feedback Factor,

$$\beta = \frac{V_f}{I_0} = \left(\frac{200}{300 + 150 + 200}\right) \times 150 = 46.15 \, \Omega$$

From the topology analysis:

Input-side loading, $R'_{E1} = 150 \,||\, 300 = 100 \, \Omega$

Output-side loading, $R'_{E3} = 200 \,||\, 300 = 120 \, \Omega$

(iv) Calculating open loop gain without feedback,

Stage-1: $$A_{v1} = \frac{v_{c1}}{v_{in}} = \frac{-g_{m1} R_{C1}}{1 + g_{m1} R'_{E1}} = -\frac{40 \times 2.3}{1 + 40 \times 0.1} = -18.4$$

Stage-2: $$A_{v2} = \frac{v_{c2}}{v_{c1}} = -g_{m2} \times R_{C2} = -80 \times 0.850 = -68$$

Stage-3: $$A_3 = \frac{i_{c3}}{v_{c2}} = \frac{1}{\frac{1}{g_{m3}} + R'_{E3}} = \frac{1}{\frac{1}{160 \times 10^{-3}} + 120} = 0.00792\, S$$

Thus, overall open-loop gain,

$$A = A_1 A_2 A_3 = (-18.4)(-68)(0.00792) = 9.91 \text{ S}$$

The open-loop voltage gain can be obtained as,

$$A_v = -A \times R_{C3} = -9.91 \times 375 = -3716.25$$

(v) Loop gain of the feedback network,

$$A\beta = 9.91 \times 46.15 = 457.35$$

(vi) The closed loop transconductance of the amplifier,

$$A_f = \frac{A}{1 + A\beta} = \frac{9.91}{1 + 457.35} = 0.02162\, S$$

The closed-loop voltage gain can be obtained as,

$$A = -A_f \times R_{C3} = -0.02162 \times 375 = -8.11$$

Q.2 (c) Solution:

(i) **1.** Given, Diffraction angle = 2θ = 27°. Using Bragg's Law to obtain the interplanar spacing for the (3 2 1) set of planes for rubidium (Rb) (n = 1 for first order reflection)

$$d_{321} = \frac{n\lambda}{2\sin\theta} = \frac{(1) \times 0.0711 \times 10^{-9}}{2 \times \left(\sin\frac{27°}{2}\right)} = 0.1523 \text{ nm}$$

2. The interplanar spacing for a set of planes with Miller indices (hkl) in a cubic crystal system is given by

$$d = \frac{a}{\sqrt{h^2 + l^2 + k^2}}$$

Lattice parameter, $$a = d_{321}\sqrt{(h)^2 + (k)^2 + (l)^2}$$

$$\Rightarrow \quad a = (0.1523 \times 10^{-9})\sqrt{(3)^2 + (2)^2 + (1)^2} \quad \Rightarrow \quad a = 0.57 \text{ nm}$$

∴ Atomic radius for BCC crystal structure is

$$R = \frac{a\sqrt{3}}{4} = \frac{0.57 \times \sqrt{3}}{4} \text{nm} = 0.2468 \text{ nm}$$

(ii) To extend the range of PMMC ammeter, the total shunt resistance R_{sh} is determined by

$$R_{sh} = \frac{R_m}{n-1}, \quad \text{where, } n = \frac{I}{I_m}$$

Given, $I_m = 100\ \mu A$

and $R_m = 1000\ \Omega$

Step-1 :

For 10 mA range:

$$n = \frac{I}{I_m} = \frac{10\text{ mA}}{100\ \mu\text{A}} = 100$$

$$R_{sh} = R_a + R_b + R_c = \frac{R_m}{n-1} = \frac{1000\,\Omega}{100-1} = 10.1\ \Omega$$

Step-2 :

When the meter is set on the 100 mA range, the resistance R_b and R_c provides the shunt. The shunt can be found from the equation

$$(R_b + R_c) = \frac{I_m(R_m + R_{sh})}{I}$$

$$= \frac{100\,\mu\text{A}(10.1 + 1000)}{100\text{ mA}} = 1.01\ \Omega$$

Step-3 :

When the meter is set on the 1A range, we have

$$(I - I)(R_b + R_c) = I_m(R_a + R_m)$$

$$I(R_b + R_c) = I_m(R_m + R_a + R_b + R_c)$$

$$R_c = \frac{I_m(R_m + R_{sh})}{I}$$

$$= \frac{100\,\mu\text{A}(10.1 + 1000)}{1000\text{ mA}} = 0.101\ \Omega$$

Step-4:

$$R_b + R_c = 1.01\ \Omega$$

$$R_b = 1.01 - R_c = 1.01\ \Omega - 0.101\ \Omega$$

$$= 0.909\ \Omega$$

Step-5:

Resistor R_a is found by

$$R_a = R_{sh} - (R_b + R_c) = 9.09\ \Omega$$

Q.3 (a) Solution:

(i) Relationship between polarizability and permittivity:

In a dielectric, the displacement flux density

$$D = \in_0 E + P \quad \text{...(i)}$$

where, E is the electric field strength and P is the polarization.

Also, we know that $$D = \in_0 \in_r E \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\in_0 \in_r E = \in_0 E + P$$

$$\Rightarrow \quad P = \in_0(\in_r - 1)E$$

But polarization, $P = N\alpha E_i$

where, N is the number of dipoles per unit volume, E_i is the internal electric field in the dielectric and α is called polarizability

So, $$\in_0(\in_r - 1)E = N\alpha E_i \quad \text{...(iii)}$$

This is the desired relationship between the polarizability and permittivity.

In a cubic crystal, the internal field seen by an atom is given by the Lorentz field,

$$E_i = E + \frac{P}{3\in_0}$$

or $$E_i = E + \frac{\in_0 (\in_r - 1)E}{3\in_0}$$

$$\Rightarrow \quad E_i = \left(\frac{\in_r + 2}{3}\right)E$$

Using equation (iii),

$$\in_0(\in_r - 1)E = N\alpha\left(\frac{\in_r + 2}{3}\right)E$$

$$\Rightarrow \quad \frac{\in_r - 1}{\in_r + 2} = \frac{N\alpha}{3\in_0} \quad \text{...(iv)}$$

This equation is the Clausius Mossotti equation.

(ii) Polarization, $$P = \varepsilon_0(\varepsilon_r - 1)E$$

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E} = 1 + \frac{4.3 \times 10^{-8}\,\text{C/m}^2}{(8.854 \times 10^{-12}\,\text{F/m})\,(1000\,\text{V/m})}$$

$$\varepsilon_r \approx 5.86$$

Q.3 (b) Solution:

(i) Serial input X : 001101101, Output Y : 000010010

From the given input and output sequence, it is observed that the circuit can detect the overlapping sequences of 0110. Thus, the Mealy state diagram can be drawn as below,

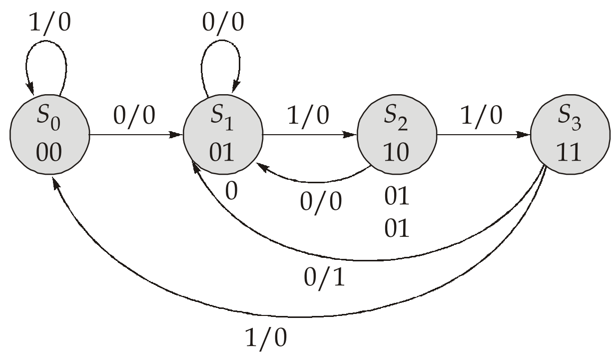


Table from State Diagram :

State	Next state		Output (y)	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
S_0	S_1	S_0	0	0
S_1	S_1	S_2	0	0
S_2	S_1	S_3	0	0
S_3	S_1	S_0	1	0

A simple state assignment can be assuming the outputs of flip flops Q_1 and Q_0 as given below :

State	Q_1	Q_0
S_0	0	0
S_1	0	1
S_2	1	0
S_3	1	1

State Table :

Present state			Next State		Output (Y)	Flip-Flop Excitations	
Q_1	Q_0	X	Q_1^+	Q_0^+		D_1	D_0
0	0	0	0	1	0	0	1
0	0	1	0	0	0	0	0
0	1	0	0	1	0	0	1
0	1	1	1	0	0	1	0
1	0	0	0	1	0	0	1
1	0	1	1	1	0	1	1
1	1	0	0	1	1	0	1
1	1	1	0	0	0	0	0

Minimization using K-Map:

D_1

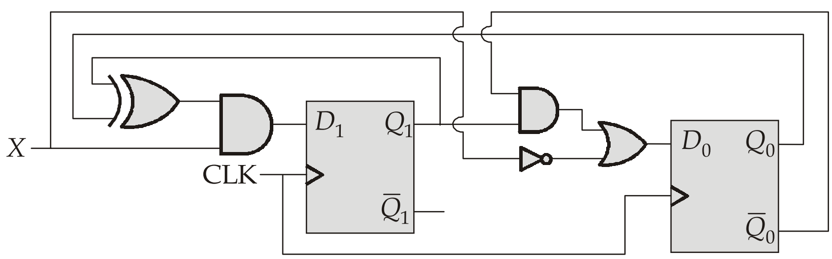
$Q_1 \backslash Q_0X$	00	01	11	10
0	0	0	1	0
1	0	1	0	0

D_0

$Q_1 \backslash Q_0X$	00	01	11	10
0	1	0	0	1
1	1	1	0	1

$$D_0 = \bar{X} + Q_1\bar{Q}_0$$

$$D_1 = Q_1\bar{Q}_0X + \bar{Q}_1Q_0X = (Q_1 \oplus Q_0)X$$



(ii) In a N-bit dual-slope ADC, the input voltage V_{in} is integrated for a fixed time period $T_1 = 2^N T_c$, where T_c is the clock period. Thereafter, a reference voltage V_{ref} of opposite polarity is integrated until the output of the integrator returns to zero. The time taken during this de-integration process is T_2 and the counter value 'n' is proportional to the input voltage. Thus, time for conversion is given by:

$$T = T_1 + T_2 = 2^N T_c + nTc$$

At full scale input, $n = 2^N$. Thus, maximum conversion time is given by

$$T_{max} = 2^{N+1} T_c$$

Here, $f_c = 1$ MHz

$$T_c = \frac{1}{10^6} \text{ sec} = 10^{-6} \text{ sec}$$

Here, $N = 12$

$$\therefore \quad T_{max} = 2^{13} \times 10^{-6} = 8192 \times 10^{-6}$$

$$= 8.192 \text{ msec}$$

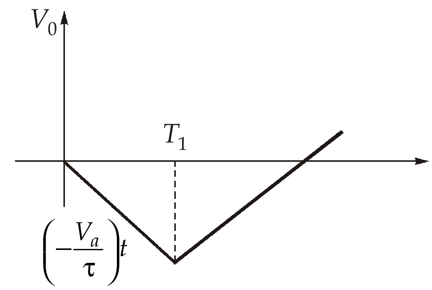
The output of the integrator is given by $V_0 = -\frac{V_a}{\tau}t$. Given that the peak voltage reached at the output of integrator is 10 V. Thus, considering peak input $V_a = -10$ V, we get

Also, $$\frac{V_a T_1}{\tau} = 10 \text{ V}$$

$$\Rightarrow \quad \frac{10 \times 2^N \times T_c}{\tau} = 10$$

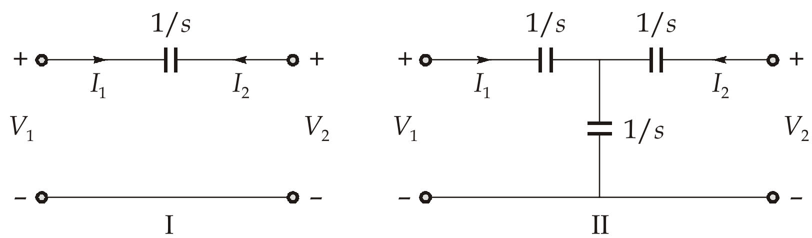
$$\tau = 2^{12} \times 10^{-6}$$

$$\tau = 4.096 \text{ msec}$$



Q.3 (c) Solution:

(i) The given circuit can be considered as two networks connected in parallel as shown below,



The y-parameters of the two-port network are given by,

$$I_1 = y_{11}V_1 + y_{12}V_2$$

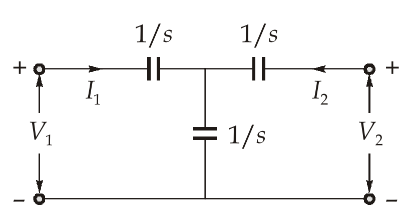
$$I_2 = y_{21}V_1 + y_{22}V_2$$

y-parameter of (I) network,

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = s = y_{22}$$

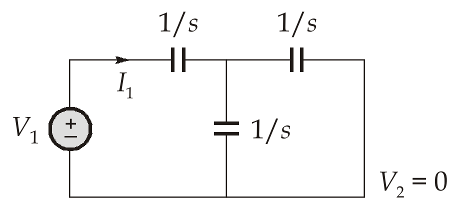
and $$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -s = y_{21}$$

y-parameter of (II) network,



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

Considering $V_2 = 0$, the circuit can be drawn as below.

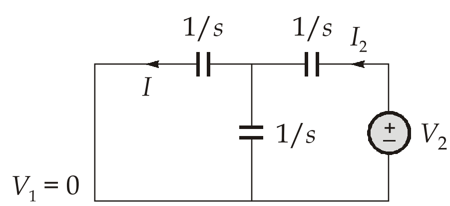


$$\frac{I_1}{V_1} = \frac{1}{\frac{1}{s} + \frac{\frac{1}{s} \times \frac{1}{s}}{\frac{1}{s} + \frac{1}{s}}} = \frac{2s}{3}$$

$$y_{11} = \frac{2s}{3} = y_{22} \qquad (\because \text{ Network is symmetrical})$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

Considering $V_1 = 0$, the circuit can be drawn as below.



$$\frac{I_2}{V_2} = \frac{1}{\frac{1}{s} + \frac{\frac{1}{s} \times \frac{1}{s}}{\frac{1}{s} + \frac{1}{s}}} = \frac{2s}{3} \Rightarrow I_2 = \frac{2s}{3} V_2$$

Using current division rule,

$$I = V_2 \frac{\frac{2s}{3} \times \frac{1}{s}}{\frac{1}{s} + \frac{1}{s}} = \frac{s}{3} V_2$$

We have, $I_1 = -I$

$$\therefore \qquad y_{12} = \frac{-s}{3} = y_{21} \qquad (\because \text{ Network is reciprocal})$$

From above, we obtain

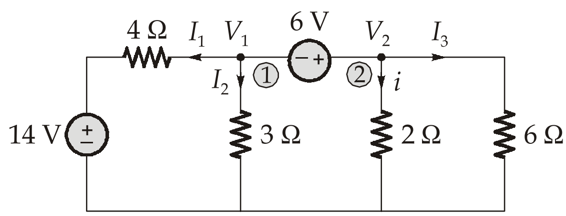
$$[Y_{(II)}] = \begin{bmatrix} \frac{2s}{3} & \frac{-s}{3} \\ \frac{-s}{3} & \frac{2s}{3} \end{bmatrix}$$

$$[Y_{(I)}] = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix}$$

For parallel connection, y-parameters can be added. Thus,

$$[y] = [Y_{(I)}] + [Y_{(II)}] = \begin{bmatrix} \frac{5s}{3} & \frac{-4s}{3} \\ \frac{-4s}{3} & \frac{5s}{3} \end{bmatrix}$$

(ii)



Applying KCL at node-1 and 2 (super node),

$$I_1 + i + I_2 + I_3 = 0$$

$$\frac{V_1 - 14}{4} + \frac{V_1}{3} + \frac{V_2}{2} + \frac{V_2}{6} = 0$$

Substituting $V_2 = V_1 + 6,$

$$\frac{V_1 - 14}{4} + \frac{V_1}{3} + \frac{V_1 + 6}{2} + \frac{V_1 + 6}{6} = 0$$

$$\frac{3V_1 - 42 + 4V_1 + 6V_1 + 36 + 2V_1 + 12}{12} = 0$$

$$15\,V_1 + 6 = 0$$

$$V_1 = \frac{-6}{15} = -0.4\text{ V}$$

$$V_2 = 6 - 0.4 = 5.6\text{ V}$$

Thus, $$i = \frac{V_2}{2} = \frac{5.6}{2} = 2.8\text{ A}$$

Q.4 (a) Solution:

(i) Given, voltmeter readings,

112.5, 112.0, 112.2, 112.3 and 112.4

Actual reading is 112.1 V

∴ Mean of the voltmeter readings,

$$\overline{X} = \frac{112.5 + 112.0 + 112.2 + 112.3 + 112.4}{5}$$

$$\therefore \qquad \overline{X} = \frac{561.4}{5} = 112.28 \text{ V}$$

Random error is defined as variability in measurements that results from fluctuations causing some measurements to be overestimated and others underestimated. The random error for each reading is calculated as below:

Reading (V_m)	Random Error = $\lvert V_m - \overline{V} \rvert$
112.5	0.22 V
112.0	0.28 V
112.2	0.08 V
112.3	0.02 V
112.4	0.12 V

The random error is usually expressed as the average absolute random error. Thus,

$$\text{Random Error} = \frac{0.22 + 0.28 + 0.08 + 0.02 + 0.12}{5} = 0.144 \text{ V}$$

(ii) For a multi-plate capacitive transducer,

$$\text{Capacitance, } C = \frac{(n-1)\in A}{d}$$

where,
n = number of plates
A = plate area
d = separation between plates
$\in = \in_0 \in_r$ (for air $\in_r = 1$)

Plate Area, A = 20 mm * 20 mm
$A = 4 \times 10^{-4}$ m^2
$d = 0.25$ mm $= 2.5 \times 10^{-4}$ m

Sensitivity of capacitance transducer,

$$S = \frac{\partial C}{\partial d}$$

$$S = \frac{\partial}{\partial d}\left[\frac{(n-1)\in A}{d}\right]$$

$$S = \frac{-(n-1)\in A}{d^2}$$

$$= \frac{-(5-1)\times 8.85\times 10^{-12}\times 4\times 10^{-4}}{(2.5\times 10^{-4})^2}$$

$$= -2.27 \times 10^{-7} \text{ F/m}$$

$\therefore$ $S \simeq 0.227$ µF/m (disregarding sign)

The negative sign indicates that the capacitance decreases as displacement increases.

Q.4 (b) Solution:

(i) Given: $I = 12$ A; $P = 1800$ W

Let $\bar{I} = 12\angle 0°$ A

We have, $|Z_2| = \frac{200}{12} = 16.667\ \Omega$

$$|Z_2| = \sqrt{R_2^2 + X_2^2}$$

$$16.67 = \sqrt{(10)^2 + X_2^2}$$

$$(16.67)^2 = 10^2 + X_2^2$$

$$277.88 = 100 + X_2^2$$

$$X_2 = 13.33\ \Omega$$

Thus, $\overline{V_2} = \bar{I}(R_2 + jX_2) = (12\angle 0°)(10 + j13.33)$

$$= (12\angle 0°)(16.666\angle 53.13°) = 200\angle 53.13° \text{ V}$$

We have, $P = VI\cos\phi$

$$1800 = 200 \times 12 \times \cos\phi$$

$$\cos\phi = 0.75$$

$$\phi = 41.41°$$

Applied voltage $\overline{V_{req}} = 200\angle 41.41°$ V

Voltage across parallel branches = $\overline{V}_{req} - \overline{V}_2$ = $200\angle 41.41° - 200\angle 53.13°$

$$= 30 - j27.71 = 40.84\angle -42.73° \text{ V}$$

Current through branches $= \frac{40.84\angle -42.73°}{20\angle -90°} = 2.04\angle 47.27°$ A

Current through R_1 and X_1 $= 12\angle 0° - 2.04\angle 47.27° = 10.616 - j1.5 = 10.72\angle -8.03°$ A

$$\overline{Z_1} = \frac{40.84\angle -42.73°}{10.72\angle -8.03°}$$

$$R_1 + jX_1 = 3.81\angle -34.7° \ \Omega = 3.13 - j2.17 \ \Omega$$

Thus, we get

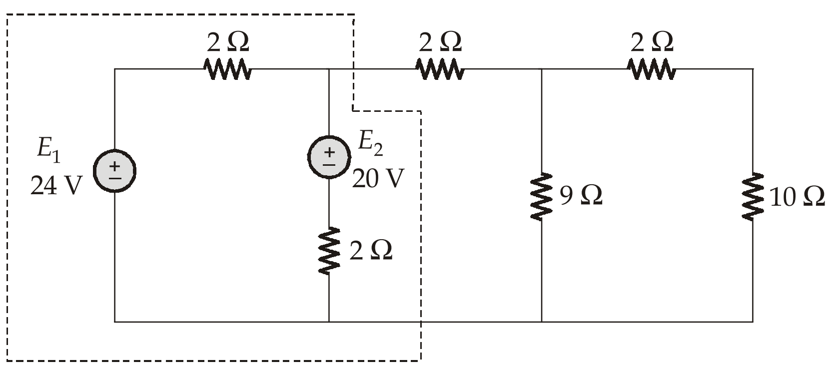
$$R_1 = 3.13 \ \Omega$$

$$X_1 = 2.17 \ \Omega$$

(ii) As per Millman's theorem , 'n' voltage sources (V_1, V_2, ... V_n) in parallel having internal resistances (R_1, R_2, ... R_n) respectively can be replaced by a single voltage source 'V_{eq}' and resistance 'R_{eq}' given by,

$$V_{eq} = \frac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

$$R_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}}$$



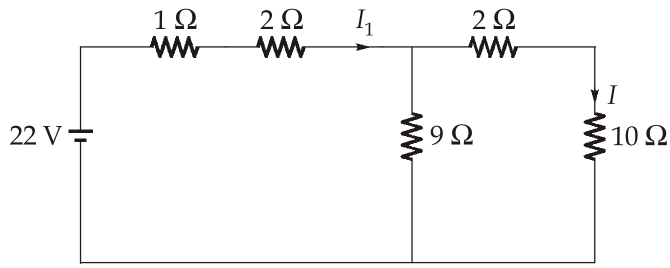
Millman's equivalent,

$$V_{eq} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{\frac{24}{2} + \frac{20}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{12 + 10}{1}$$

$$V_{eq} = 22 \text{ V}$$

$$R_{eq} = \frac{1}{\frac{1}{2} + \frac{1}{2}} = 1 \ \Omega$$

The equivalent circuit can be drawn as below,



$$I_1 = \frac{22}{(1+2)+\frac{12\times 9}{12+9}}$$

$$I_1 = \frac{22}{3+\frac{108}{21}} = 2.70\text{ A}$$

Using current division rule,

$$I = \frac{9I_1}{9+2+10}$$

$$I = \frac{9}{21} * 2.70$$

$$I = 1.15\text{ A}$$

Q.4 (c) Solution:

In an induction motor, at slip s, the rotor induced emf is given by

$$E_{2s} = sE_2$$

where E_2 is the standstill rotor emf per phase.

and rotor reactance becomes

$$X_{2s} = sX_2$$

Where X_2 is the Standstill rotor reactance

Rotor induced emf at s_2 = 0.02,

$$E_{r2} = s_2 E_2 = 0.02 \times 141 = 2.82\text{ V}$$

The torque of an induction motor is given by,

$$T_e = \frac{3}{\omega^2} \cdot \frac{I_2^2 R_2}{s} \Rightarrow T_e \propto \frac{I_2^2}{s}$$

Given in both the cases torque is equal. Considering the rotor current as I_2' at slip, s' = 0.02. We have,

$$\frac{I_2^2}{s_1} = \frac{I_1'^2}{s_2} \Rightarrow I_2' \sqrt{\frac{s_2}{s_1}}$$

$$I_2' = 31.3\sqrt{\frac{0.02}{0.035}} = 23.51 \text{ A}$$

At $s_2 = 0.02$, Rotor Impedance,

$$Z_2' = R_2 + js_2 X_2 = 0.16 + j(0.02 \times 0.27)$$

$$Z_2' = 0.16 + j0.0054\ \Omega = 0.16\angle 1.93°\ \Omega$$

Thus, the net voltage needed in the rotor loop at $s_2 = 0.02$ is:

$$V_r = I_2' Z_2' = 23.51 \times 0.16\angle 1.93°$$

$$V_r = 3.76\angle 1.93° \text{ V}$$

The natural induced EMF in the rotor at $s_2 = 0.02$ without injection is $E_{r2} = 2.82$ V

Thus, the emf to be injected to the rotor circuit is given by

$$E_{inj} = V_r - E_{r2} = 3.76\angle 1.93° - 2.82$$

$$E_{inj} = 0.94 + j0.127 \text{ V}$$

$$E_{inj} = 0.95\angle 7.7° \text{ V (per phase)}$$

Section B

Q.5 (a) Solution:

Given,strain gauge resistance, $R = 350\ \Omega$

Gauge factor, $G.F = 2$

Strain, $\in = 0.001$

Supply voltage, $V_s = 10$ V

Quarter bridge resistances, $R_1 = R_2 = R_3 = 350\ \Omega$

Resistance of strain gauge is,

$$R_4 = 350 + \Delta R$$

Change in resistance of strain gauge due to strain,

$$\frac{\Delta R}{R} = GF \times \in$$

$$\frac{\Delta R}{R} = 2 \times 0.001$$

$$\Delta R = 350 \times 2 \times 0.001$$

Change in resistance; $\Delta R = 0.7\ \Omega$

$\therefore$ New gauge resistance, $R_4 = 350 + 0.7 = 350.7\ \Omega$

Bridge output voltage, $V_0 = V_R - V_L$

where, $V_L = V_s \times \dfrac{R_3}{R_1 + R_3}$

$$= 10\times\frac{350}{350+350}$$

$$V_L = 5\text{ V}$$

and
$$V_R = V_s\times\frac{R_4}{R_2+R_4} = 10\times\frac{350.7}{350.7+350}$$

$\therefore$
$$V_R = 5.005\text{ V}$$

Thus, $V_0 = V_R - V_L = 5 - 5.005 = -0.005\text{ V} = -5\text{ mV}$

Q.5 (b) Solution:

Given,

Quantum efficiency, $\eta = 65\% = 0.65$

Wavelength, $\lambda = 900\text{ nm} = 900\times10^{-9}\text{ m}$

Optical power, $P = 0.5\text{ mW} = 5\times10^{-4}\text{ W}$

Multiplied photocurrent, $I_M = 10\text{ mA} = 0.01\text{ A}$

Responsivity (without multiplication),

$$R_0 = \eta\frac{q\lambda}{hc}$$

$$= 0.65\times\frac{(1.6\times10^{-19})(900\times10^{-9})}{(6.63\times10^{-34})(3\times10^{8})}$$

$$R_0 \approx 0.471\text{ A/W}$$

Primary photocurrent,

$$I_P = R_0P$$

$$I_P = 0.471\times5\times10^{-4}$$

$$I_P = 2.36\times10^{-4}\text{ A}$$

$$I_P = 0.236\text{ mA}$$

Multiplication factor,

$$M = \frac{I_M}{I_P}$$

$$M = \frac{10}{0.236} \approx 42.4$$

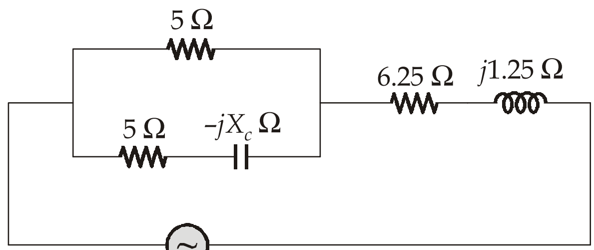
From above, we can see that $I_P \propto \eta$. Thus,

$$M = \frac{I_M}{I_P} \propto \frac{1}{\eta}$$

If quantum efficiency is lower than the given value, I_P decreases and therefore,multiplication factor M increase to maintain the same multiplied photocurrent. Whereas if quantum efficiency is higher than the given value, I_P increases and therefore,multiplication factor M decreases for the same multiplied photocurrent.

Q.5 (c) Solution:

From the given data, the circuit can be drawn as below:



Equivalent impedance of above circuit.

$$= \frac{(5)(5-jX_c)}{5+5-jX_c} + 6.25 + j1.25$$

$$= \frac{(25-j5X_c)(10+jX_c)}{100+X_c^2} + 6.25 + j1.25$$

$$= \frac{5X_c^2 + 250 - j25X_c}{100+X_c^2} + 6.25 + j1.25$$

For resonance, imaginary part of the equivalent impedance = 0

$$\frac{-25X_c}{100+X_c^2} + 1.25 = 0$$

On simplifying we get,

$$0.05X_c^2 - X_c + 5 = 0 \Rightarrow X_c^2 - 20X_c + 100 = 0 \Rightarrow (X_c - 10)^2 = 0$$

$$X_c = 10$$

We have, $$X_c = \frac{1}{\omega C}$$

and $$\omega = 2\pi f$$

$$f = 50 \text{ Hz} \quad \text{(given)}$$

Thus, $$\frac{1}{2\pi(50)*C} = 10$$

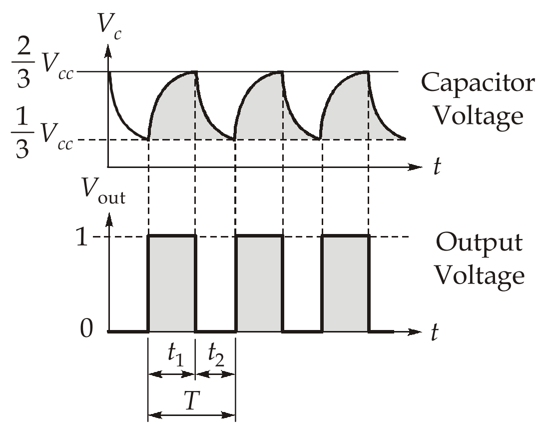
$$C = \frac{1}{1000\pi} = 318.3\ \mu\text{F}$$

Q.5 (d) Solution:

Given, Astable multivibrator,

$$R_A = 2.2\ \text{k}\Omega$$
$$R_B = 3.9\ \text{k}\Omega$$
$$C = 0.1\ \mu\text{F}$$

During the positive pulse width, the capacitor C charges through R_A and R_B upto $2/3\ V_{cc}$ after which the output switch from HIGH to LOW. During the negative pulse width, the capacitor C discharges through R_B upto $1/3\ V_{cc}$ which triggers the lower comparator of 555 IC switching the output from LOW to HIGH. Thus, the capacitor charges and discharges continuously between $2/3\ V_{cc}$ and $1/3\ V_{cc}$ as shown below:



During the charging phase, voltage across the capacitor

$$V_c = V_{cc} + \left(\frac{V_{cc}}{3} - V_{cc}\right)e^{-t(R_A+R_B)C}$$

At $t = t_1$, $V_c = 2V_{cc}/3$. Thus,

$$\frac{2V_{cc}}{3} = V_{cc} - \frac{2V_{cc}}{3}e^{-t_1(R_A+R_B)C}$$

$$t_1 = (R_A + R_B)C\ \ln 2 = 0.693(R_A + R_B)C$$

During the discharging phase, voltage across the capacitor

$$V_c = \frac{2V_{cc}}{3}e^{-tR_BC}$$

At $t = t_2$, $V_c = V_{cc}/3$. Thus,

$$t_2 = R_BC\ \ln 2 = 0.693R_BC$$

(i) Positive pulse width,

$$t_{PH} = t_1 = 0.693\,(R_A + R_B)C$$
$$= 0.693(2.2k + 3.9k) \times 0.1 \times 10^{-6}$$

$\therefore$ $$t_{PH} = 0.423 \text{ msec}$$

(ii) Negative pulse width,

$$t_{PL} = t_2 = 0.693\,R_B C$$
$$= 0.693 \times 3.9k \times 0.1 \times 10^{-6}$$

$\therefore$ $$t_{PL} = 0.27 \text{ msec}$$

(iii) Total time period, $T = t_{PH} + t_{PL}$

$\therefore$ $$T = 0.423 \text{ ms} + 0.27 \text{ msec}$$
$$T = 0.693 \text{ msec}$$

$\therefore$ Free running frequency,

$$f_0 = \frac{1}{T} = 1.44 \text{ kHz}$$

(iv) Duty cycle, $$D = \frac{t_{PH}}{T} \times 100\%$$

$$= \frac{0.423 \times 10^{-3} \text{ sec}}{0.693 \times 10^{-3} \text{ sec}} \times 100\% = 0.6104 \times 100\%$$

$\therefore$ Duty cycle, $$D = 61.04\%$$

Q.5 (e) Solution:

For a thermometer, using the Newton's Law of cooling,

$$\theta(t) = \theta_f(1 - e^{-t/\tau}) + \theta_i e^{-t/\tau}$$

where,
θ_f = final temperature (in °C)
$\theta(t)$ = temperature at time t (in °C)
θ_i = initial temperature (in °C)

Given,
$\theta(30)$ = 97°C
θ_f = 100°C
θ_i = 30°C
t = 30 sec

$\therefore$ $$97 = 100(1 - e^{-30/\tau}) + 30e^{-30/\tau}$$
$$= 100 - 100e^{-30/\tau} + 30e^{-30/\tau}$$
$$97 = 100 - 70e^{-30/\tau}$$

$$\therefore \qquad 70e^{-30/\tau} = 3$$

$$e^{-30/\tau} = \frac{3}{70} = 0.043$$

By taking 'ln' on both sides,

$$\frac{-30}{\tau} = \ln(0.043) = -3.15$$

$$\therefore \qquad \tau = \frac{30}{3.15} = 9.52 \text{ sec}$$

For $\theta(t) = 98°C$,

$$98 = 100(1 - e^{-t/9.52}) + 30e^{-t/9.52}$$

$$98 = 100 - 100e^{-t/9.52} + 30e^{-t/9.52}$$

$$98 = 100 - 70e^{-t/9.52}$$

$$70e^{-t/9.52} = 2$$

$$e^{-t/9.52} = \frac{2}{70} = 0.0285$$

By taking 'ln' on both sides,

$$\frac{-t}{9.52} = -3.55$$

$$\therefore \qquad t = 33.8 \text{ sec}$$

Q.6 (a) Solution:

(i) Let the three inputs are A, B, C and output, Y is 1 when two or more inputs are 1.

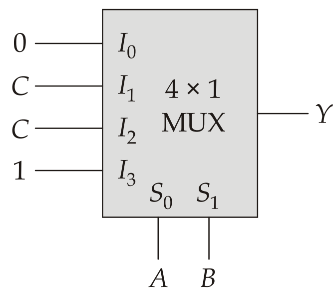
Truth table:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\therefore \qquad Y = \Sigma m(3, 5, 6, 7)$$

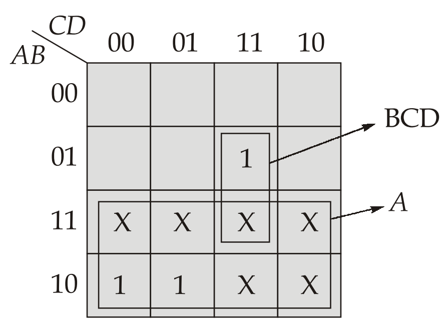
For 4 × 1 MUX: Consider A and B are select inputs and C is input

	I_0	I_1	I_2	I_3
$\overline{C}$	0	2	4	⑥
C	1	③	⑤	⑦
	0	C	C	1



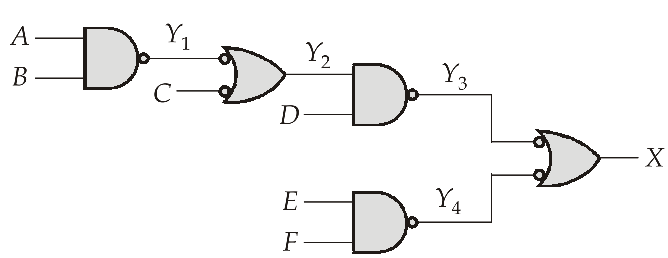
(ii) Using the four-variable K-Map for simplification,

$$F(A, B, C, D) = \Sigma m(7, 8, 9) + \Sigma d(10, 11, 12, 13, 14, 15)$$



$\therefore$ $$F = A + BCD$$

(iii) Given circuit,



We have,

$$Y_1 = \overline{AB}$$

$$Y_2 = \overline{Y_1} + \overline{C}$$

$$= \overline{\overline{AB}} + \overline{C}$$

$$Y_2 = \overline{C} + AB$$

$$Y_3 = \overline{Y_2 \cdot D}$$

$$= \overline{(\overline{C} + AB) \cdot D} = \overline{\overline{C} + AB} + \overline{D}$$

$$Y_4 = \overline{EF}$$

$$X = \overline{Y_3} + \overline{Y_4}$$

$$= \overline{\overline{Y_2 \cdot D}} + \overline{Y_4}$$

$$= Y_2 \cdot D + EF$$

$$= (\overline{C} + AB)D + EF$$

$$= ABD + \overline{C}D + EF$$

Q.6 (b) Solution:

Oxide capacitance, $$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$\varepsilon_{ox} = 3.45 \times 10^{-13}\ \text{F/cm},\ t_{ox} = 15\ \text{nm} = 1.5 \times 10^{-6}\ \text{cm}$$

$$C_{ox} = \frac{3.45 \times 10^{-13}}{1.5 \times 10^{-6}} = 2.3 \times 10^{-7}\ \text{F/cm}^2$$

Total gate oxide capacitance is given by $C = WLC_{ox}$. We have,

$$W = 10\ \mu\text{m} = 10^{-3}\ \text{cm},\ L = 1\ \mu\text{m} = 10^{-4}\ \text{cm}$$

$$WL = 10^{-7}\ \text{cm}^2$$

Thus, $$WLC_{ox} = 10^{-7} \times 2.3 \times 10^{-7} = 2.3 \times 10^{-14}\ \text{F}$$

$$WLC_{ox} = 23\ fF$$

For $N_A = 10^{17}\ \text{cm}^{-3}$,

$$\phi_F = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right) = 0.026 \ln\left(\frac{10^{17}}{1.5 \times 10^{10}}\right) = 0.42\ \text{V}$$

Work function difference,

$$\phi_{ms} = W_m - W_s = 0$$

Thus, $V_{FB} = 0$. The threshold voltage is given by

$$V_T = V_{FB} + 2\phi_F + \frac{\sqrt{2q\varepsilon_{si} N_A (2\phi_F)}}{C_{ox}}$$

$$V_T = 0.84 + \frac{\sqrt{2 \times 1.6 \times 10^{-19} \times 11.7 \times 8.854 \times 10^{-14} \times 10^{17} \times 0.84}}{2.3 \times 10^{-7}}$$

$$V_T = 0.84 + 0.72 = 1.56 \text{ V}$$

For $V_{GS} = 2.5$ V, $V_{ov} = V_{GS} - V_T = 2.5 - 1.56 = 0.94$ V

Thus, $V_{DSsat} = V_{ov} = 0.94$ V

Linear region ($V_{DS} < V_{DSsat}$)

At $V_{DS} = 0$:

$$C_{gsi} = C_{gdi} = \frac{1}{2}WLC_{ox}$$

Saturation region ($V_{DS} > V_{DSsat}$)

$$C_{gsi} = \frac{2}{3}WLC_{ox}, C_{gdi} \approx 0$$

(i) $V_{DS} = 0$

$$C_{gsi} = C_{gdi} = \frac{1}{2}\times 23 = 11.5 fF$$

$$C_{gsi} = C_{gdi} = 11.5 fF$$

(ii) $V_{DS} = 1 \text{ V} > V_{DSsat}$

From the graph for $V_{DS} > V_{DSsat}$,

$$C_{gsi} \approx \frac{2}{3} WLC_{ox} = \frac{2}{3}\times 23 = 15.3 \ fF$$

$$C_{gdi} = 0$$

(iii) $V_{DS} = 4 \text{ V} > V_{DSsat}$

$$C_{gsi} = \frac{2}{3}\times 23 = 15.3 \ fF$$

$$C_{gdi} \approx 0$$

$$C_{gsi} \approx 15.3 fF, C_{gdi} \approx 0$$

Q.6 (c) Solution:

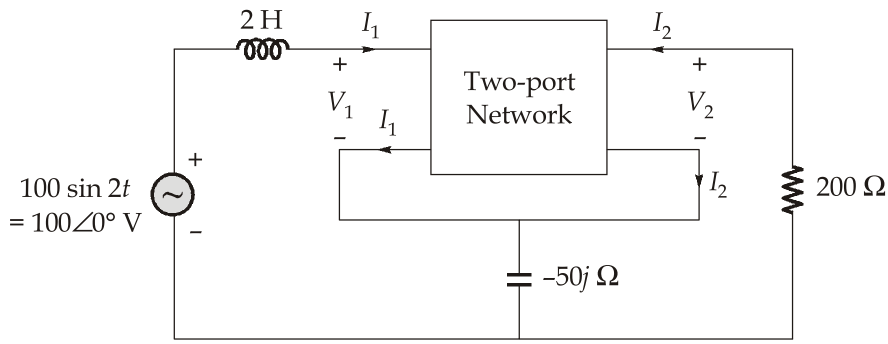
$$V_s = 100 \sin(2t) \text{ V}$$

Comparing with $V_m \sin(\omega t)$, we get

$$\omega = 2 \text{ rad/sec}$$

Thus, $X_L = j\omega L = j2 \times 2 = j4\ \Omega$

$$X_C = \frac{-j}{\omega C} = \frac{-j}{2\times 0.01} = -50 j\Omega$$



Y-parameter equations for two port network are given by:

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Given that, $Y_{11} = 3.3 \times 10^{-3}$ S

$$Y_{22} = 5 \times 10^{-3} \text{ S}$$

$$Y_{12} = Y_{21} = 0$$

Therefore, $I_1 = 3.3 \times 10^{-3}\ V_1$ ...(1)

$$I_2 = 5 \times 10^{-3}\ V_2 \quad ...(2)$$

Applying KVL at input loop, we get

$$100 = j4I_1 + V_1 + (-50j)(I_1 + I_2) \quad ...(3)$$

From equation (1), (2) and (3) we get

$$100 = j4I_1 + \frac{I_1 \times 10^3}{3.3} \times (-50j)I_1 - 50jI_2$$

$$100 = \left[j4 + \frac{10^3}{3.3} - 50j\right]I_1 - 50jI_2$$

$$100 = \left[\frac{10^3}{3.3} - 46j\right]I_1 - 50jI_2 \quad ...(4)$$

Applying KVL at output loop, we get

$$-200I_2 = V_2 - 50j(I_1 + I_2)$$

From equation (2),

$$V_2 = \frac{10^3}{5}I_2 = 200I_2$$

Thus, $-200I_2 = 200I_2 - 50jI_1 - 50jI_2$

$$50jI_1 = 400I_2 - 50jI_2$$
$$50jI_1 = (400 - 50j)I_2$$
$$I_1 = \left(\frac{400-50j}{50j}\right)I_2 \qquad \text{...(5)}$$

From equation (4) and (5), we get

$$100 = \left[\frac{10^3}{3.3} - 46j\right]\left[\frac{400-50j}{50j}\right]I_2 - 50jI_2$$

$$I_2 = \frac{100}{\left[\frac{10^3}{3.3} - 46j\right]\left[\frac{400-50j}{50j}\right] - 50j}$$

$$= \frac{100}{(303-46j)(-1-j8)-50j} = \frac{100}{-671-j2428}$$

$$I_2 = 0.03969\angle 105.44° \text{ A}$$

$\therefore$ Voltage across load $= -200I_2$

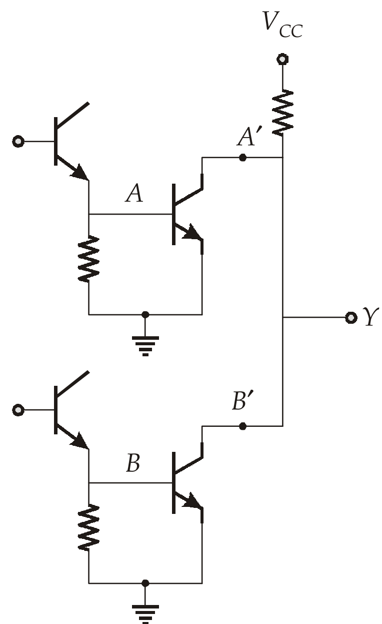
$= -200(0.03969\angle 105.44°)$

$= 7.9388\angle -74.55° \text{ V}$

$\therefore$ Voltage across load $= 7.9388 \sin(2t - 74.55°)$ V

Q.7 (a) Solution:

(i) An open collector TTL inverter has an output that behaves like a switch to ground (logic 0) when ON, and is left floating (open) when OFF. To get a valid logic output, a pull-up resistor is connected from the output to V_{cc}. Let the two open collector inverters have inputs A and B, and their respective outputs A' and B' are tied together at point Y with a pull-up resistor to V_{cc}. Either open collector output "Pulls down" the voltage at Y.

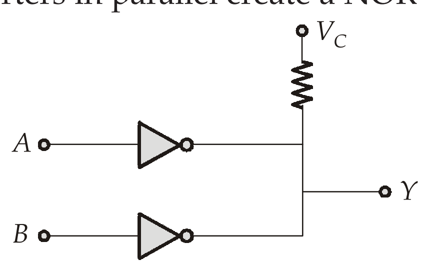


Truth table:

A'	B'	Y
0	0	0
0	1	0
1	0	0
1	1	1

$Y = A'B'$

From above, we see that the outputs of two open collector gates, when put in parallel, create an AND function. So, $Y = A'B'$. But by, DeMorgan's law, $A'B' = \overline{A+B}$. Thus, two open collector inverters in parallel create a NOR function.



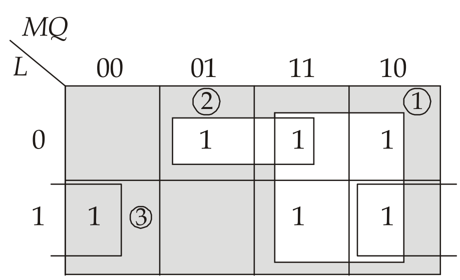
(ii) Excitation table of T flip-flop:

Present state $Q(t)$	**Next state** $Q(t+1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

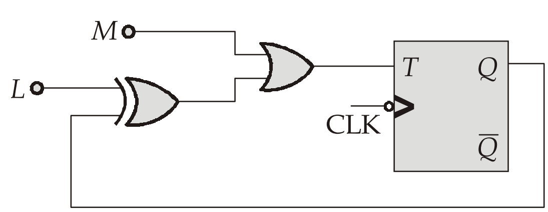
State Table for LM Flip flop:

		Present state	Next state	
L	M	$Q(t)$	$Q(t+1)$	T (Based on $Q(t)$ & $Q(t+1)$)
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	1

Getting the value of T in terms of L, M and $Q(t)$:



$$T = M + L\bar{Q} + \bar{L}Q = M + (L \oplus Q)$$



Q.7 (b) Solution:

(i) For MOS transistor to be in saturation region,

$$V_{DS} \geq V_{GS} - V_{Th}$$
$$V_D \geq V_G - V_{Th}$$

So, for maximum value of gate voltage, we have,

$$V_D = V_G - V_{Th} \qquad [\text{..... as } V_S = 0]$$

Now,
$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{Th})^2 \qquad \text{...(i)}$$

and from KVL in the output loop,

$$1.8 - I_D(R_D) - V_D = 0 \qquad \text{...(ii)}$$

Substituting equation (i) in equation (ii), we get,

$$1.8 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_G - V_{Th})^2 R_D - V_G + V_{Th} = 0$$

$$1.8 - \frac{1}{2}\times 10\times 10^{-6}\times 11.11(V_G - 0.4)^2\times 10^3 - V_G + 0.4 = 0$$

$$2.2 - 0.055\,(V_G - 0.4)^2 - V_G = 0$$

$$0.055V_G^2 + 0.956V_G - 2.1912 = 0$$

$\therefore$ $V_G = 2.05$ V, -19.43 V

Thus, $V_i = V_G = 2.05$ V (Since $V_{GS} > V_{Th}$)

Hence, the maximum value of V_i for which the MOS transistor will work in saturation region is 2.05 V.

(ii) Given, diffusion resistance,

$$r_d > 48\ \Omega$$

Diffusion conductance,

$$g_d < \frac{1}{r_d} = \frac{1}{48} = 0.0208\text{ S}$$

But, $$g_d = \frac{I_D}{\eta V_T}$$

Diode current, $I_D = g_d V_T$ (Assume η = 1)

$\therefore$ $I_D < 0.0208 \times 0.0259 = 0.539$ mA

We know that, $$I_D = I_s \exp\left(\frac{V_a}{V_T}\right)$$

where V_a = applied forward bias voltage.

$$\frac{I_D}{I_s} = \exp\left(\frac{V_a}{V_T}\right)$$

$$\frac{V_a}{V_T} = \ln\left(\frac{I_D}{I_s}\right)$$

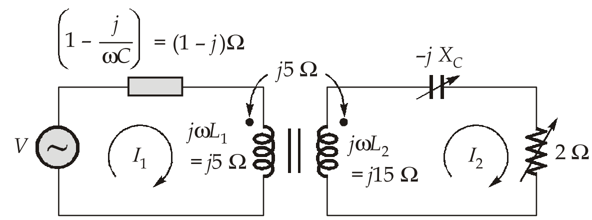
$\therefore$ $$V_a = V_T \ln\left(\frac{I_D}{I_s}\right)$$

Hence, $$V_a < 0.0259\ln\left(\frac{0.539\times 10^{-3}}{2\times 10^{-11}}\right) = 0.443\text{ V}$$

Therefore, the maximum forward-bias voltage that can be applied to meet the given specification is 0.443V.

Q.7 (c) Solution

(i) Redrawing the given network in resistive form, we get,



$$\because \qquad \omega = 5 \text{ rad/sec} \qquad \text{(given)}$$

Applying KVL in the loop (1), we get

$$V = (1-j)I_1 + j5I_1 - j5I_2 \qquad \text{...(i)}$$

Also, applying KVL in loop (2), we get

$$0 = -jX_C I_2 + j15I_2 + 2I_2 - j5I_1 \qquad \text{...(ii)}$$

From equation (i),

$$I_1 = \frac{V + j5I_2}{(1+4j)}$$

Putting the above value of I_1 in to equation (ii), we get,

$$0 = j(15 - X_C)I_2 + 2I_2 - \frac{j5[V + j5I_2]}{(1+j4)}$$

$$= (1 + j4)\,[j(15 - X_C) + 2]I_2 + 25I_2 - j5\text{ V}$$

or
$$j5\text{ V} = [j(23 - X_C)]I_2 + 4X_C - 33]I_2$$

$$I_2 = I_L = \frac{j5V}{j(23 - X_C) + 4X_C - 33}$$

The average power delivered to the load,

$$P_L = I_L^2 R_L = \frac{|j5\,V|^2 R_L}{|j(23 - X_C) + 4X_C - 33|^2}$$

$$P_L = \frac{5V^2 R_L}{\left[\sqrt{(4X_C - 33)^2 + (23 - X_C)^2}\right]^2}$$

For maximum power to be transferred

$$\frac{dP_L}{dX_c} = 0$$

or $\left[(4X_C-33)^2-(23-X_C)^2\right]\times 0-[2(4X_C-33)\times 4+(23-X_C)\times 2(-1)]\times 5V^2\times R_L=0$

or $[-8(4X_C-33)+2(23-X_C)]\times 5V^2\times R_L=0$

or $-32X_C+264+46-2X_C = 0$

$-34X_C = -310$

or $X_C = 9.117\ \Omega = 1/\omega C$

$\Rightarrow$ $C = 21.935$ mF

(ii) Loading of the transformer is as follows:

In first time slot : kVA load = $\frac{4}{0.6}=6.667$ kVA

Ohmic loss vary as the square of the kVA load i.e. $P_{cu} = x^2 P_{cu,FL}$, where x is the fractional load given by kVA load/rated kVA.

$\therefore$ Ohmic loss during first time slot = $400\times\left(\frac{6.667}{10}\right)^2=177.778$ W

In second time slot : kVA load = $\frac{6}{0.8}=7.5$ kVA

$\therefore$ Ohmic loss during second time slot = $400\times\left(\frac{7.5}{10}\right)^2=225$ W

In third time slot : kVA load = $\frac{10}{0.9}=11.11$ kVA

$\therefore$ Ohmic loss during third time slot = $400\times\left(\frac{11.11}{10}\right)^2=493.83$ W

In fourth time slot : Ohmic loss = 0

$\therefore$ Daily energy lost as ohmic loss

$$= \frac{1}{1000}[(177.778)\times 6+(225)\times 5+(493.83)\times 7]=5.649\text{ kWhr}$$

Energy lost as core loss on daily basis = $\frac{100\times 24}{1000}=2.4$ kWhr

Daily total energy loss = 2.4 kWhr + 5.649 kWhr = 8.049 kWhr

Daily total energy output = [4 × 6 + 6 × 5 + 10 × 7 + (0)] = 124 kWhr

All day efficiency, $\eta = 1-\frac{\text{Daily loss in kWhr}}{\text{(Daily output + Daily loss) in kWhr}}$

$$= 1-\frac{8.049}{124+8.049}=1-\frac{8.049}{132.049} = 0.939 \text{ or } 93.90\%$$

Q.8 (a) Solution:

(i) For the two-port network shown in Fig. (a), the *ABCD* parameters are given by,

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

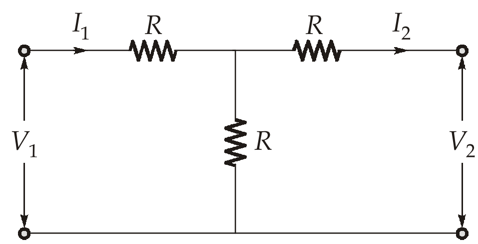


Fig. (a)

From Fig. (a), using KVL, we can write

$$-V_1 + I_1R + I_2R + V_2 = 0$$
$$V_1 = V_2 + I_1R + I_2R \quad \text{...(1)}$$

also, $$-V_2 - I_2R + (I_1 - I_2)R = 0$$
$$V_2 = -I_2R - I_2R + I_1R$$
$$I_1 = \frac{1}{R}V_2 + 2I_2 \quad \text{...(2)}$$

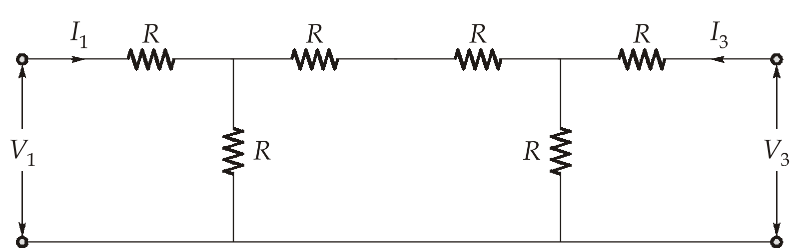
On putting the value of equation (2) in equation (1),

we get $$V_1 = 2V_2 + 3RI_2$$

On comparison we get

$A = 2$; $B = 3R$; $C = \dfrac{1}{R}$; $D = 2$

The equivalent *ABCD* parameters for two cascaded networks is the matrix multiplication of their individual *ABCD* parameters. Thus, for the below network,

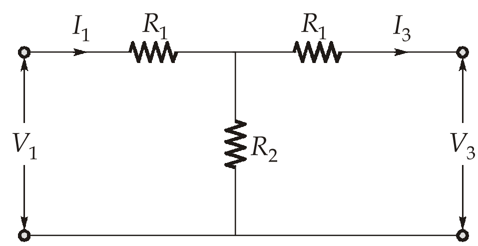


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 3R \\ \frac{1}{R} & 2 \end{bmatrix}\begin{bmatrix} 2 & 3R \\ \frac{1}{R} & 2 \end{bmatrix}$$

For $R = 1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

Let the equivalent T-network be,



We have,

- $A = \left.\dfrac{V_1}{V_3}\right|_{I_3=0} = \dfrac{R_1+R_2}{R_2} = 7$

$\Rightarrow \quad R_1 = 6R_2$

- $B = \left.\dfrac{V_1}{I_3}\right|_{V_3=0}$

For $V_3 = 0$, we have, $\quad I_3 = I_1\left(\dfrac{R_2}{R_1+R_2}\right)$

and, $\quad V_1 = I_1\left(R_1 + \dfrac{R_1R_2}{R_1+R_2}\right)$

$$\Rightarrow \quad 12 = (R_1+R_2)\frac{R_1}{R_2} + R_1$$

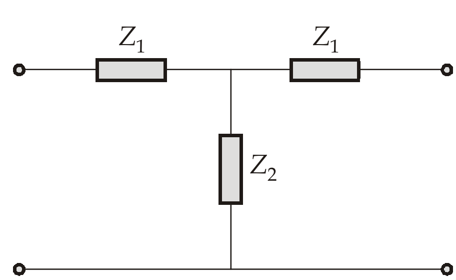
$$12 = 6(R_1 + R_2) + R_1$$

$$12 = 6(7R_2) + 6R_2$$

$$12 = 48R_2$$

$$R_2 = \frac{1}{4}\Omega; \; R_1 = \frac{3}{2}\Omega$$

Note: For a symmetrical T-Network



The *ABCD* parameters for this T-network are:

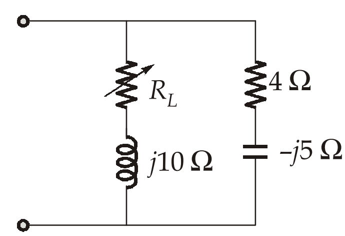
$$A = 1+\frac{Z_1}{Z_2}$$

$$B = 2Z_1+\frac{Z_1^2}{Z_2}$$

$$C = \frac{1}{Z_2}$$

$$D = 1+\frac{Z_1}{Z_2}$$

(ii) Input admittance for the given circuit,



$$Y_{in} = \frac{1}{R_L+j10}+\frac{1}{4-j5}$$

$$= \frac{R_L-j10}{R_L^2+100}+\frac{4+j5}{41}$$

$$= \frac{R_L}{R_L^2+100}+\frac{4}{41}+\frac{j5}{41}-\frac{j10}{R_L^2+100}$$

For resonance, Y_{in} must be real

So, $$\frac{5}{41} = \frac{10}{R_L^2+100}$$

$$5R_L^2+500 = 410$$

$$R_L^2 = -18$$

The equation is not satisfied for any real value of R_L. Thus, no value of R_L can make the circuit resonant.

Q.8 (b) Solution:

The moving coil instrument reads average value of current while hot wire reads rms value of current. The electrostatic voltmeters do not take any current for their operation and they read the rms value of voltage.

Let '*i*' be the instantaneous value of current,

$$i = I_0 + I_{1m}\sin\omega t + I_{2m}\sin 2\omega t$$

$$= 0.5 + 0.3\sin\omega t - 0.2\sin 2\omega t$$

Average value of i, I_{av} = 0.5 A

Hence, reading of moving–coil instrument = 0.5 A

RMS value of current i,

$$I_{rms} = \sqrt{I_0^2 + \left(\frac{I_{1m}}{\sqrt{2}}\right)^2 + \left(\frac{I_{2m}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{(0.5)^2 + \left(\frac{0.3}{\sqrt{2}}\right)^2 + \left(\frac{-0.2}{\sqrt{2}}\right)^2} = 0.56125 \text{ A}$$

Hence, reading of hot-wire instrument = 0.56125 A

Reading of electrostatic voltmeter across 1000 Ω resistance,

$$V_R = I_{rms}.R = 0.56125 \times 1000$$

$$= 561.25 \text{ V}$$

Instantaneous value of voltage across 1 mH inductor,

$$V_L = L\frac{di}{dt}$$

$$= (1\times10^{-3})\frac{d}{dt}[0.5 + 0.3\sin\omega t - 0.2\sin 2\omega t]$$

For $\omega = 10^6$ rad/sec, $V_L = (1\times10^{-3})[(0.3)\times\omega\times\cos\omega t - 0.2\times2\omega\times\cos 2\omega t]$

$$V_L = (300\cos\omega t - 400\cos 2\omega t) \text{ V}$$

Hence, reading of electrostatic voltmeter across 1 mH inductor,

$$V_L = \sqrt{\left(\frac{300}{\sqrt{2}}\right)^2 + \left(\frac{-400}{\sqrt{2}}\right)^2} = 353.55 \text{ V}$$

Q.8 (c) Solution:

(i) 1. We know that, drift current:

$$I_D = \mu_n W C_{ox}(x)[(V_{GS} - V_T) - V(x)]\frac{dV}{dx}$$

where, $$C_{ox}(x) = \frac{\in_{ox}}{T_{ox}} = \frac{\in_{ox}}{Ax^2 + B}$$

$\therefore$ $$I_D = \mu_n W\frac{\varepsilon_{ox}}{Ax^2 + B}[(V_{GS} - V_T) - V(x)]\frac{dV}{dx}$$

By rearranging above equation,

$$\frac{I_D}{\mu_n W\varepsilon_{ox}}(Ax^2 + B)dx = [(V_{GS} - V_T) - V(x)]\, dV$$

By integrating the left side from $x = 0$ to $x = L$ and the right side from $V(0) = 0$ to $V(L) = V_{DS}$:

$$\text{Left side} \quad \int_0^L (Ax^2 + B)dx = A\frac{x^3}{3}\Big]_0^L + BL = \frac{AL^3}{3} + BL$$

$$\text{Right side} \quad \int_0^{V_{DS}} [(V_G - V_T) - V(x)]dV = (V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}$$

$$\therefore \qquad I_D = \mu_n W \varepsilon_{ox} \frac{(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}}{\frac{AL^3}{3} + BL}$$

2. Saturation voltage V_{Dsat}

At saturation condition:

$$V_{Dsat} = V_{GS} - V_T$$

Saturation current,

$$I_{Dsat} = \frac{1}{2}\mu_n W \varepsilon_{ox} \frac{(V_G - V_T)^2}{\frac{AL^3}{3} + BL}$$

3. Interpretation of constant V_T

Threshold voltage depends on oxide thickness:

$$V_T \propto \frac{1}{C_{ox}} \propto T_{ox}$$

Given $\qquad T_{ox}(x) = Ax^2 + B$

- If variation is large $\Rightarrow$ V_T varies along channel.
- But assumption says V_T is nearly constant.

Therefore, the variation in T_{ox} must be small.

Therefore, small W_{dmax} (small oxide variation) is required for a nearly constant V_T.

(ii) Given:

Threshold current, I_{th} = 5 mA

Wavelength, λ = 1.33 μm = 1.33×10^{-6} m

Quantum efficiency, η = 0.8

Required optical power, P_0 = 9 dBm

Conversion of optical power from dBm to Watts

$$10 \log P_0 \text{ (mW)} = 9$$

$$\Rightarrow \quad P_0 = 10^{\frac{9}{10}} \text{ mW}$$

$$P_0 = 7.94 \times 10^{-3} \text{ W}$$

Above the Threshold current, Laser diode power-current relation is given by,

$$P_0 = \eta \frac{hv}{q}(I - I_{th})$$

Since, $$v = \frac{c}{\lambda}$$

$$P_0 = \eta \frac{hc}{q\lambda}(I - I_{th})$$

Evaluate constant, $$\frac{hc}{q\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 1.33 \times 10^{-6}}$$

$$\approx 0.934 \text{ V}$$

Thus, $$P_0 = 0.8 \times 0.934(I - I_{th})$$

$$P_0 = 0.747(I - I_{th})$$

Given, $$P_0 = 7.94 \times 10^{-3} \text{ W}$$

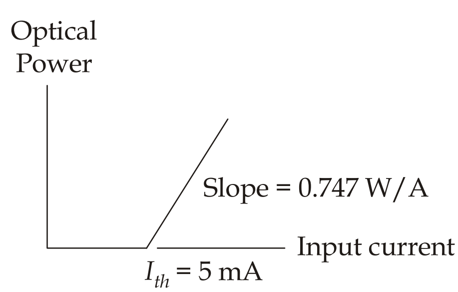
$$7.94 \times 10^{-3} = 0.747(I - 5 \times 10^{-3})$$

$$I - 5 \times 10^{-3} = 0.01063 \text{ A}$$

$$I = 0.01563 \text{ A}$$

$$\therefore \quad I \approx 15.6 \text{ mA}$$

The optical power vs input current characteristics for the Laser diode can be drawn as,



- Below threshold → very small spontaneous emission
- Above threshold → The output power increases linearly (stimulated emission) with a slope equal to 0.747 W/A.

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