MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Electrical Engineering Test No : 14

Section-A

Q.1 (a) Solution:

Before closing the switch,

$$\text{Energy stored} = \frac{1}{2}C_1V^2$$

$$= \frac{1}{2}\times 4\times 12^2 = 288 \text{ J}$$

and $\quad \text{Charge } Q = CV = 4 \times 12 = 48 \text{ C}$

After closing the switch,

$$\text{Charge } Q = (C_1 + C_2)\text{ V}$$

where V is the common terminal voltage.

$$48 = (4 + 2)\text{V}$$

$$V = 8 \text{ V}$$

and the energy stored after closing the switch

$$= \frac{1}{2}CV^2 = \frac{1}{2}(4+2)8^2 = 192 \text{ J}$$

Initial stored energy was 288 J and final energy stored is 192 J, so total 96 Joule is lost during redistribution.

Q.1 (b) Solution:

Given,

Area of hysteresis loop on graph:

$$A = 32 \text{ cm}^2$$

Scales: $1 \text{ cm} = 800 \text{ A/m and } 0.6 \text{ Wb/m}^2$

$$l = 1.2 \text{ m}; \quad A = 8 \text{ cm}^2; \quad f = 50 \text{ Hz}$$

(i) Energy lost per cycle per unit volume

$$= (\text{Area on graph}) \times (\text{H-scale}) \times (\text{B-scale})$$

$$= 32 \times 800 \times 0.6 = 15360 \text{ J/m}^3$$

(ii) Total hysteresis energy loss per cycle

$$= (\text{Volume of Magnetic Material}) \times (\text{Energy density})$$

$$= \text{Area} \times \text{length} \times 15360 \text{ J/cycle}$$

$$= 8 \times 10^{-4} \times 1.2 \times 15360 = 14.75 \text{ J/cycle}$$

(iii) Hysteresis power loss in the specimen

$$= (\text{frequency}) \times \text{Energy lost per cycle}$$

$$= 50 \times 14.75 = 737.28 \text{ Watts}$$

(iv) If $f = 75$ Hz,

Then, hystersis power loss,

$$P_L = \frac{75}{50} \times 737.28 = 1106 \text{ watts}$$

Q.1 (c) Solution:

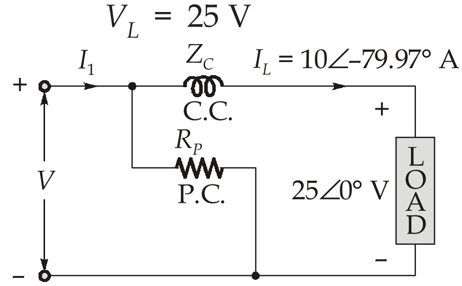
For case-I:

$$Z_C = (0.06 + j0.02)\Omega$$

$$I = I_C = 10 \text{ A}$$

$$\cos\phi_L = 0.174 \text{ lag}$$

$$V_L = 25 \text{ V}$$



$$\vec{V}_{PC} = \vec{V}_L + \vec{I}_L \cdot \vec{Z}_C$$

$$= 25\angle 0° + (0.06 + j0.02)(10\angle -79.97°)$$

$$\vec{V}_{PC} = 25.30\angle -1.25° \text{ volts}$$

$$\text{Measured power} = \left(\vec{V}_{PC} \cdot \vec{I}_{CC}^{\,*}\right)$$

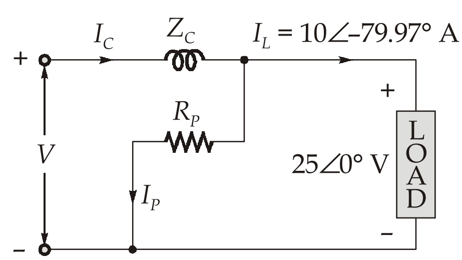
$$P_m = \text{Real}\{(25.30\angle -1.25)(10\angle 79.97)\} = 49.54 \text{ Watt}$$

True power P_T, $\text{Real}[V_L \cdot I_C^*] = 25 \times 10 \times 0.174 = 43.5$ Watt

$$\%\text{error} = \frac{P_m - P_T}{P_T} \times 100\%$$

$$= \frac{49.54 - 43.5}{43.5} \times 100 = 13.88\%$$

For case-II:



$$I_P = \frac{V_L}{R_P} = \frac{25}{6250} = 0.004 \text{ A}$$

$$I_C = \vec{I}_L + \vec{I}_P = (10\angle -79.97° + 0.004\angle 0°)$$

$$\vec{I}_C = 10\angle -79.94° \text{ A}$$

Measured power, $P_m = \text{Real}\left(V_L \cdot \vec{I}_{CC}^{\,*}\right) = \text{Real}\,[25 \times 10\angle 79.94°]$

$$P_m = 43.64 \text{ watt}$$

$$P_T = 25 \times 10 \times 0.174 = 43.50 \text{ watts}$$

$$\%\text{ error} = \frac{P_m - P_T}{P_T} \times 100 = \frac{43.64 - 43.50}{43.50} \times 100 = 0.32\%$$

Q.1 (d) Solution:

We know that, $H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$

Given: $1.6 \times 10^5 = H_0\left[1 - \left(\frac{16}{T_c}\right)^2\right]$

$$4 \times 10^5 = H_0\left[1-\left(\frac{14}{T_c}\right)^2\right]$$

$$\frac{16}{40} = \frac{1-\left(\frac{16}{T_c}\right)^2}{1-\left(\frac{14}{T_c}\right)^2}$$

$$0.4\left[1-\left(\frac{14}{T_c}\right)^2\right] = 1-\left(\frac{16}{T_c}\right)^2$$

$$\left(\frac{16}{T_c}\right)^2 - 0.4\left(\frac{14}{T_c}\right)^2 = 1 - 0.4 = 0.6$$

$$\frac{1}{T_c^2}[256-78.4] = 0.6$$

$$T_c^2 = 296$$

$$T_c = 17.205 \text{ K}$$

Now, $$1.6 \times 10^5 = H_0\left[1-\frac{256}{296}\right]$$

$$H_0 = 1.184 \times 10^6 \text{ A/m}$$

At 4.2 K, $$H_c = 11.84 \times 10^5\left[1-\frac{(4.2)^2}{296}\right]$$

$$H_c = 1.113 \times 10^6 \text{ A/m} = H_c \text{ at } 0 \text{ K}$$

Q.1 (e) Solution:

$$v(t) = At \text{ where } A \text{ is slope of ramp.}$$

Slope, $$A = \frac{100-0}{2-0} = 50$$

$\therefore$ $$v(t) = 50t;\ 0 \le t \le T$$

$\therefore$ $$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T v^2(t)\cdot dt\right]^{1/2}$$

$$= \left[\frac{1}{2}\int_0^T 50t^2 \cdot dt\right]^{1/2}$$

$$= \left[\frac{2500t^3}{6}\Bigg|_0^2\right]^{1/2}$$

$$= \sqrt{\frac{2500 \times 8}{6}} = 57.735 \text{ Volts}$$

While
$$V_{av} = \frac{1}{T}\int_0^T v(t) \cdot dt$$

$$= \frac{1}{2}\int_0^2 50t \cdot dt = \frac{1}{2} \times \frac{50t^2}{2}\Bigg|_0^2$$

$$V_{av} = 50 \text{ Volts}$$

∴ Form factor,
$$K_f = \frac{V_{\text{rms}}}{V_{av}} = \frac{57.73}{50} = 1.1547$$

For sine wave,
$$K_f = 1.11$$

Hence, the meter will read less by a factor.

$$\frac{K_f(\text{sine})}{K_f(\text{sawtooth})} = \frac{1.11}{1.1547} = 0.9613$$

∴
$$\%\text{ error} = \frac{\text{Measured} - \text{True}}{\text{True}} \times 100\%$$

$$= \frac{0.9613 - 1}{1} \times 100\%$$

$$\%\text{ error} = -3.87\%$$

Q.2 (a) (i) Solution:

We know that,

Hall voltage is given by
$$V_H = \frac{R_H I_x B_z}{d}$$

where R_H is Hall coefficient given by

$$R_H = -\frac{\mu_e}{\sigma}$$

where,

μ_e = Electron mobility of aluminium

σ = Electrical conductivity of aluminium

So, $$R_H = \frac{0.0012m^2/V-s}{3.8\times10^7(\Omega-\text{m})^{-1}} = -3.16 \times 10^{-11} \text{ V-m/A-T}$$

Given, $I_x = 25\text{A}$

$B_z = 0.6T$

$d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$

Using Hall voltage equation

$$V_H = \frac{(-3.16\times10^{-11})(25)(0.6)}{15\times10^{-3}} = -3.16 \times 10^{-8} \text{ V}$$

Q.2 (a) (ii) Solution:

We know that, $\in = \in_r \in_0$

Where, $\in$: permittivity of medium

$= (6) \times (8.85 \times 10^{-12} \text{ F/m})$

$= 5.31 \times 10^{-11} \text{ F/m}$

Capacitance, $C = \dfrac{\in \times A}{d}$

Given, Area of plates, $A = 6.45 \times 10^{-4} \text{ m}^2$

Seperation of plate, $d = 2 \times 10^{-3} \text{ m}$

$$\therefore \quad C = (5.31\times10^{-11})\frac{(6.45\times10^{-4})}{2\times10^{-3}} = 1.71\times10^{-11} \text{ F}$$

Charge stored in each plate,

$Q = CV$

$= (1.71 \times 10^{-11}) \times (10 \text{ V})$

$= 1.71 \times 10^{-10} \text{ C}$

1. Dielectric displacement, $$D = \in E = \in \frac{V}{d} = \frac{(5.31\times10^{-11})(10V)}{2\times10^{-3}}$$

$= 2.66 \times 10^{-7} \text{ C/m}^2$

2. Polarization , $P = D - \in_0 E$

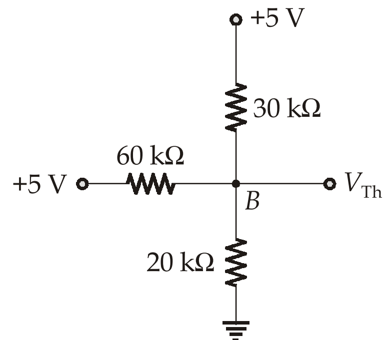
$$= D - \in_0 \frac{V}{d} = \left(2.66\times10^{-7}\right) - \frac{(8.85\times10^{-12})(10)}{2\times10^{-3}}$$

$= 2.66 \times 10^{-7} - 4.43 \times 10^{-8}$

$= 2.22 \times 10^{-7} \text{ C/m}^2$

Q.2 (b) Solution:

- To simply the circuit, the portion left side to be base terminal of the transistor can be replaced with it's Thevenin's equivalent circuit, which can be determined as follows:



By applying KCL at node B, we get

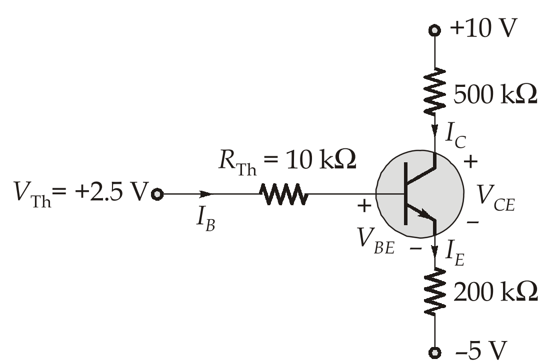
$$\frac{5-V_{\text{Th}}}{30}+\frac{5-V_{\text{Th}}}{60} = \frac{V_{\text{Th}}}{20}$$

$$6V_{\text{Th}} = 15$$

$$V_{\text{Th}} = 2.5\text{ V}$$

$$R_{\text{Th}} = (60\,||\,30\,||\,20)\text{ k}\Omega = 10\text{ k}\Omega$$

- So, the equivalent of the given circuit will be as shown below.



- By assuming that the transistor is in forward active mode, we get,

$$I_B = \frac{2.5-0.7+5}{R_{\text{Th}}+R_E(1+\beta)} = \frac{6.8}{10\text{k}+(200\times 100)} = \frac{6.8}{30}\text{ mA}$$

$$= 0.2267\text{ mA}$$

$$I_C = \beta I_B = 99 \times 0.2267 = 22.44\text{ mA}$$

$$I_E = (1+\beta)I_B = 22.67\text{ mA}$$

$$V_{CE} = 15 - I_C R_C - I_E R_E = -0.754\text{ V}$$

V_{CE} is negative. It indicates that the initial assumption of forward active mode operation of transistor is wrong. Hence the transistor is working in saturation mode.

- For saturation mode of operation,

$$I_{CQ} = I_{C(\text{sat})}$$
$$V_{CE} = V_{CE(\text{sat})} = 0.2 \text{ V}$$
$$V_{BEQ} = V_{BE(\text{sat})} = 0.8 \text{ V}$$
$$I_{EQ} = I_{C(\text{sat})} + I_{BQ}$$

By applying KVL in collector-emitter loop, we get,

$$500 I_{C(\text{sat})} + 200 I_{EQ} = 15 - V_{CE(\text{sat})} = 15 - 0.2$$
$$500 I_{C(\text{sat})} + 200(I_{C(\text{sat})} + I_{BQ}) = 14.8$$
$$700 I_{C(\text{sat})} + 200 I_{BQ} = 14.8 \quad \text{...(i)}$$

By applying KVL in base-emitter loop, we get,

$$10\text{k } I_{BQ} + 200 I_{EQ} = 7.5 - V_{BE(\text{sat})} = 7.5 - 0.8$$
$$10000 I_{BQ} + 200(I_{C(\text{sat})} + I_{BQ}) = 6.7 \quad \text{...(ii)}$$

By solving the equations (i) and (ii), we get,

$$I_{BQ} = 0.2437 \text{ mA}$$
$$I_{CQ} = I_{C(\text{sat})} = 21.073 \text{ mA}$$
$$I_{EQ} = I_{BQ} + I_{C(\text{sat})} = 21.317 \text{ mA}$$

Q.2 (c) Solution:

(i) $y - x \leq 2$ or $y \leq (x + 2)$ is in region-1

Let the surface of the plane be described by $f(x, y) = (y - x - 2)$; the unit vector normal to plane is given as

$$\hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\hat{a}_y - \hat{a}_x}{\sqrt{2}}$$

Magnetisation vector is

$$\vec{M}_1 = \chi_{m1}\vec{H}_1 = (\mu_{r1} - 1)\vec{H}_1$$
$$= (5-1)\left(-2\hat{i} + 6\hat{j} + 4\hat{k}\right)$$
$$= -8\hat{i} + 24\hat{j} + 16\hat{k} \text{ A/m}$$
$$\vec{B}_1 = \mu_1\vec{H}_1 = \mu_0\mu_{r1}\vec{H}_1$$
$$= 4\pi \times 10^{-7} \times 5 \times \left(-2\hat{i} + 6\hat{j} + 4\hat{k}\right)$$
$$= -12.5\hat{i} + 37.7\hat{j} + 25.13\hat{k} \ \mu\text{Wb/m}^2$$

(ii) In region-1, normal component of the field is

$$\vec{H}_{1n} = \left(\vec{H}_1 \cdot \hat{a}_n\right) \cdot \hat{a}_n$$

$$= \left[(-2\hat{i} + 6\hat{j} + 4\hat{k})\left(\frac{\hat{j}-\hat{i}}{\sqrt{2}}\right)\left(\frac{\hat{j}-\hat{i}}{\sqrt{2}}\right)\right]$$

$$= \left(\frac{2+6}{2}\right)(\hat{j}-\hat{i})$$

$$= (-4\hat{i} + 4\hat{j})$$

Tangential component is

$$\vec{H}_{1t} = \vec{H}_1 - \vec{H}_{1n}$$

$$= \left(-2\hat{i} + 6\hat{j} + 4\hat{k}\right) - \left(-4\hat{i} + 4\hat{j}\right)$$

$$= \left(2\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

Applying boundary conditions

$$\vec{H}_{2t} = \vec{H}_{1t} = \left(2\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

and $\vec{B}_{2n} = \vec{B}_{1n}$

$$\vec{H}_{2n} = \frac{\mu_1}{\mu_2}\vec{H}_{1n} = \frac{5}{2}(-4\hat{i} + 4\hat{j}) = (-10\hat{i} + 10\hat{j})$$

$\therefore$ $$\vec{H}_2 = \vec{H}_{2t} + \vec{H}_{2n} = \left(2\hat{i} + 2\hat{j} + 4\hat{k}\right) + \left(-10\hat{i} + 10\hat{j}\right)$$

$$= \left(-8\hat{i} + 12\hat{j} + 4\hat{k}\right) \text{A/m}$$

$\therefore$ $$\vec{B}_2 = \mu_2\vec{H}_2 = \mu_0\mu_{r2}\vec{H}_1$$

$$= 4\pi \times 10^{-7} \times 2 \times \left(-8\hat{i} + 12\hat{j} + 4\hat{k}\right)$$

$$= -20.11\hat{i} + 30.16\hat{j} + 10.05\hat{k} \ \mu\text{Wb/m}^2$$

Q.3 (a) (i) Solution:

High Performance CPU design

1. High performance computer exhibit the concurrency.
   Concurrency means two or more instruction execution at a time.
2. According to flynn's classification, computer Architecture is of 4 types:
   (i) SISD (Single Instruction Stream Single Data Stream microprocessor computer)

(ii) SIMD (Single Instruction Stream and Multiple Data Stream microprocessor)

(iii) MISD (Multiple Instruction Stream and Single Data Stream)

(iv) MIMD (Multiple Instruction Stream and Multiple Data Stream)

Shortcuts:

CU : Control Unit

PU : Processing Unit

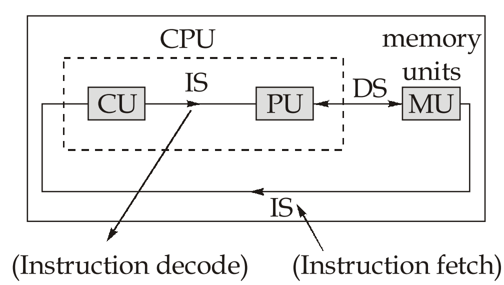
MU : Memory Unit

IS : Instruction Stream

DS : Data Stream

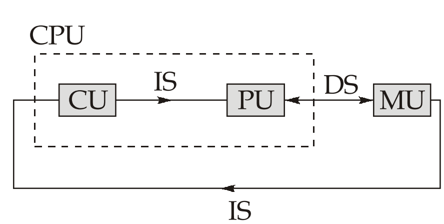
To increase the performance of CPU, concurrency is needed which depends upon CPU and Memory organization. No relation to the Input/Output device.

In Von-Neumann Architecture:



- No concurrency for instructions.
- Low performance hardware
- SISD Architecture
    - 8085 µP, Personal Computer

SISD Architecture:

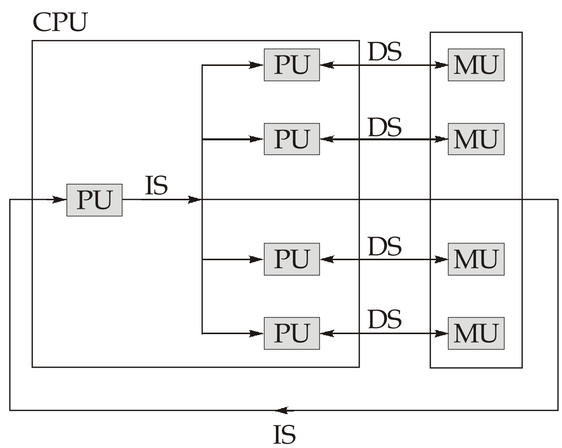


- Implemented as a uni-processor system design

  Ex: 8085 µP [Von-NeuMann Computer]

  8086 µP [Von-NeuMann Computer]

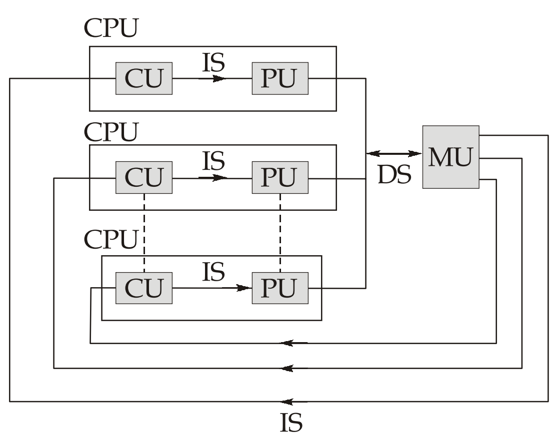
SIMD Architecture:



- Implemented as an Array processor system design (Vector processor)

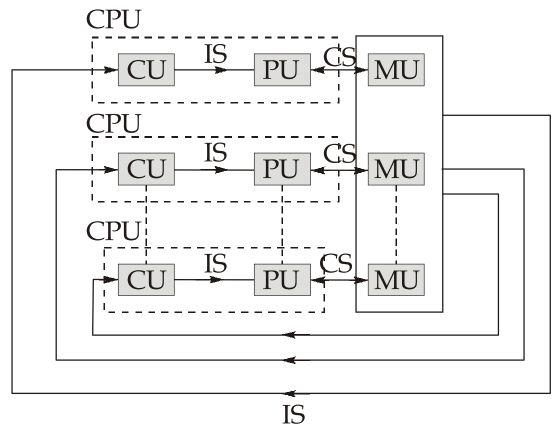
  Ex: Staran Processor, PEPE processor

MISD Architecture:



- Single instrument is executed on multiple CPU with single data stream.
- This architecture contains multiple processor but only one processor is in use at a time.
- It is not yet implemented.

MIMD Architecture:



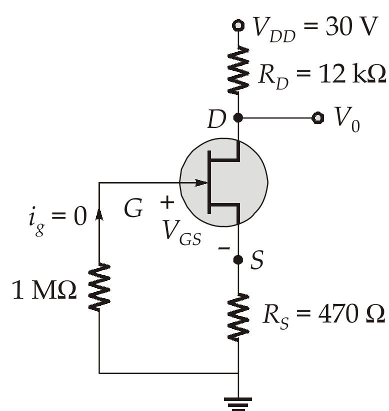
More than one instruction is executed at a time

- Implemented as a multiprocessor system design

  Ex: Cray processor, Cyber processor

Q.3 (a) (ii) Solution:

Simplified circuit for dc analysis



By applying KVL in the Gate source loop,

$$V_{GS} + I_D R_S = 0$$

$$V_{GS} = -I_D R_S$$

$$V_{GS} = -470\, I_D \quad \text{...(i)}$$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 \quad \text{...(ii)}$$

Substituting equation (i) in equation (ii), we get

$$I_D = I_{DSS}\left(1 - \frac{470 I_D}{V_P}\right)^2$$

$$I_D = 3 \times 10^{-3}(1 - I_D 195.83)^2$$

$$(38349.389)I_D^2 - 724.99 I_D + 1 = 0$$

On solving the quadratic equation we obtain two roots,

$$I_D = 17.4 \text{ mA} \qquad (\text{rejected because } I_D \not> I_{DSS})$$

and $\quad I_D = 1.5$ mA

By KVL in the drain source loop,

$$-V_{DD} + I_D R_D + V_{DS} + I_D R_S = 0$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$

Now calculating V_{DS} taking,

$$I_D = 1.5 \text{ mA}$$

$$V_{DS} = 30 - (1.5 \times 10^{-3})(12 \times 10^3 + 470)$$

$$V_{DS} = 11.3 \text{ V}$$

from equation (i),

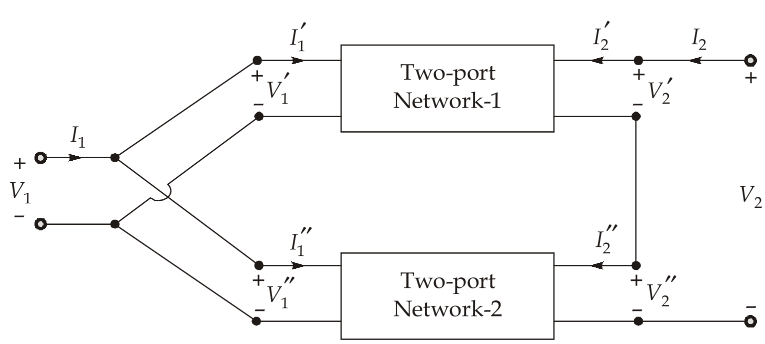
$$V_{GS} = -I_D R_S$$

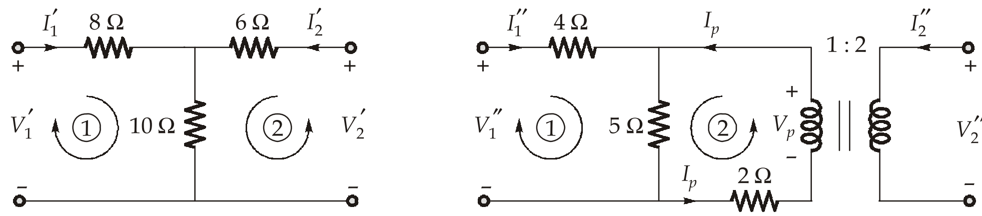
$$= -1.5 \times 10^{-3} \times 470$$

$\therefore \quad V_{GS} = -0.705$ V

Q.3 (b) Solution:

- The given two-port network can be viewed as a combination of two individual two-port networks with parallel connection at port-1 side and series connection at port-2 side as shown below.





- As the input side has parallel connection and output side has series connection, it is convenient to calculate the overall g-parameters (inverse hybrid parameters) from the g-parameters of individual two-port networks.

So, $$[g] = [g_1] + [g_2]$$

To determine the g-parameters of network-1 :

- By applying KVL in loop (1) of network-1, we get,

$$V_1' = 8I_1' + 10(I_1' + I_2') = 18I_1' + 10I_2'$$

$$I_1' = \left(\frac{1}{18}\right)V_1' - \left(\frac{10}{18}\right)I_2' \quad ...(i)$$

- By applying KVL in loop (2) of network-1, we get,

$$V_2' = 6I_2' + 10(I_1' + I_2') = 16I_2' + 10I_1'$$

$$= 16I_2' + 10\left(\frac{1}{18}V_1' - \frac{10}{18}I_2'\right)$$

$$V_2' = \left(\frac{10}{18}\right)V_1' + \left(\frac{188}{18}\right)I_2' \quad ...(ii)$$

- From equations (i) and (ii), we get,

$$[g_1] = \begin{bmatrix} \frac{1}{18}\,℧ & -\frac{10}{18} \\ \frac{10}{18} & \frac{188}{18}\,\Omega \end{bmatrix}$$

To determine the g-parameters of network-2:

- From the basic properties of the transformer,

$$V_p = \frac{V_2''}{2} \text{ and } I_p = 2I_2''$$

- By applying KVL in loop (1) of network - 2, we get,

$$V_1'' = 4I_1'' + 5(I_1'' + I_p) = 9I_1'' + 10I_2''$$

$$I_1'' = \left(\frac{1}{9}\right)V_1'' - \left(\frac{10}{9}\right)I_2'' \quad ...(iii)$$

- By applying KVL in loop (2) of network - 2, we get,

$$5(I_p + I_1'') + 2(I_p) - V_p = 0$$

$$5\left(\frac{1}{9}V_1'' - \frac{10}{9}I_2'' + 2I_2''\right) + 4I_2'' - \frac{V_2''}{2} = 0$$

$$V_2'' = \frac{10}{9}V_1'' + \left(8 - \frac{100}{9} + 20\right)I_2''$$

$$V_2'' = \left(\frac{10}{9}\right)V_1'' + \left(\frac{152}{9}\right)I_2'' \quad ...(iv)$$

- From equations (iii) and (iv), we get,

$$[g_2] = \begin{bmatrix} \frac{1}{9}\,℧ & -\frac{10}{9} \\ \frac{10}{9} & \frac{152}{9}\,\Omega \end{bmatrix}$$

To determine the *z*-parameters of the overall network:

- The *g*-parameters of the overall network can be given by,

$$[g] = [g_1] + [g_2]$$

$$= \begin{bmatrix} \frac{1}{18}\,℧ & -\frac{10}{18} \\ \frac{10}{18} & \frac{188}{18}\,\Omega \end{bmatrix} + \begin{bmatrix} \frac{2}{18}\,℧ & -\frac{20}{18} \\ \frac{20}{18} & \frac{304}{18}\,\Omega \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{18}\,℧ & -\frac{30}{18} \\ \frac{30}{18} & \frac{492}{18}\,\Omega \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 1\,℧ & -10 \\ 10 & 164\,\Omega \end{bmatrix}$$

- Which can be expressed as,

$$I_1 = \frac{V_1}{6} - \frac{10\,I_2}{6} \quad ...(v)$$

$$V_2 = \frac{10\,V_1}{6} + \frac{164\,I_2}{6} \quad ...(vi)$$

- From equation (v), we get,

$$V_1 = 6I_1 + 10I_2 \quad ...(vii)$$

- From equations (vi) and (vii), we get,

$$V_2 = \frac{10}{6}(6I_1 + 10I_2) + \frac{164I_2}{6}$$

$$V_2 = 10I_1 + 44I_2 \quad \text{...(viii)}$$

- From equations (vii) and (viii), the z-parameters of the overall network can be expressed as,

$$[z] = \begin{bmatrix} 6\,\Omega & 10\,\Omega \\ 10\,\Omega & 44\,\Omega \end{bmatrix}$$

Alternative Method:

- The standard equations of z-parameters can be given as,

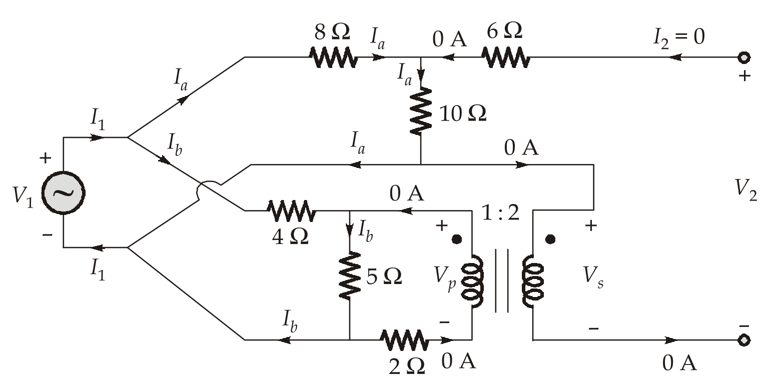
$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$\left.\begin{aligned} z_{11} &= \frac{V_1}{I_1}\bigg|_{I_2=0} \\ z_{21} &= \frac{V_2}{I_1}\bigg|_{I_2=0} \end{aligned}\right\} \text{When port-2 is open circuited}$$

$$\left.\begin{aligned} z_{12} &= \frac{V_1}{I_2}\bigg|_{I_1=0} \\ z_{22} &= \frac{V_2}{I_2}\bigg|_{I_1=0} \end{aligned}\right\} \text{When port-1 is open circuited}$$

When port-2 is open circuited:

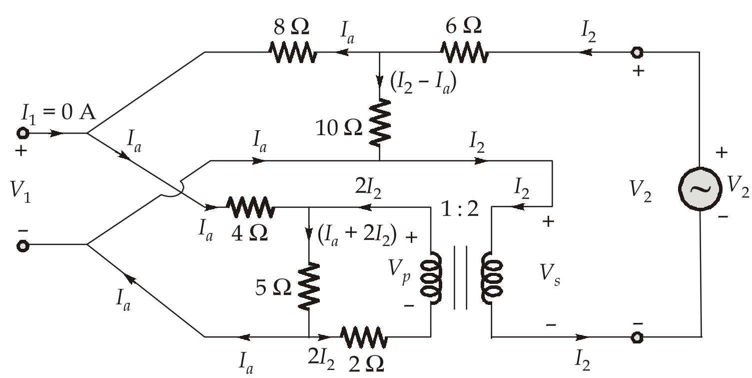


$$V_1 = 8I_a + 10I_a = 18I_a$$

Also, $$V_1 = 4I_b + 5I_b = 9I_b$$

So,
$$18I_a = 9I_b$$
$$I_b = 2I_a$$
$$I_1 = I_a + I_b = 3I_a$$
$$V_p = 5I_b = 10I_a$$
$$V_s = 2V_p = 20I_a$$
$$V_2 = 10I_a + V_s = 30I_a$$
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{18I_a}{3I_a} = 6\ \Omega$$
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{30I_a}{3I_a} = 10\ \Omega$$

When port-1 is open circuited:



By applying KVL in input side loop, we get,

$$8I_a + 4I_a + 5(I_a + 2I_2) - 10(I_2 - I_a) = 0$$
$$27I_a + 10I_2 - 10I_2 = 0$$
$$I_a = 0$$

So,
$$V_1 = -8I_a + 10(I_2 - I_a) = 10I_2$$
$$V_2 = 6I_2 + 10(I_2 - I_a) + V_s = 16I_2 + 2V_p$$
$$= 16I_2 + 2(5I_a + 10I_2 + 4I_2)$$
$$= 16I_2 + 28I_2 = 44I_2$$
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 10\ \Omega$$
$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 44\ \Omega$$

- By summarizing the results, we can express the z-parameters of the overall network as,

$$[z] = \begin{bmatrix} 6\,\Omega & 10\,\Omega \\ 10\,\Omega & 44\,\Omega \end{bmatrix}$$

Q.3 (c) (i) Solution:

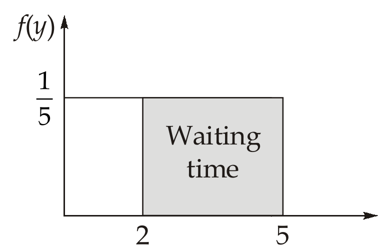
t be arrival time of vehicles of the junction is uniformly distributed in [0, 5].
Let y be the waiting time of the junction.

Then,
$$y = \begin{cases} 0 & t < 2 \\ 5 - t & 2 \le t < 5 \end{cases}$$

$$y \to [0, 5]$$

$$f(y) = \frac{1}{5-0} = \frac{1}{5}$$

$$E(y) = \int_{-\infty}^{0} y(y)\,dy = \int_{0}^{5} y\,f(y)\,dy$$



$$= \int_{2}^{5} y\left(\frac{1}{5}\right)dy = \frac{1}{5}\int_{2}^{5}(5-t)\,dt = \frac{1}{5}\left(5t - \frac{t^2}{2}\right)\Bigg|_{2}^{5}$$

$$= \frac{1}{5}\left\{\left(25 - \frac{25}{2}\right) - \left(10 - \frac{4}{2}\right)\right\}$$

$$= \frac{1}{5}\left(\frac{25}{2} - 8\right) = \frac{1}{5}\left(\frac{9}{2}\right) = 0.9 \text{ minutes}$$

Q.3 (c) (ii) Solution:

Since the total probability is unity,

$$\therefore \quad \int_{0}^{6} f(x)dx = 1$$

$$\int_{0}^{2} Kx\,dx + \int_{2}^{4} 2K\,dx + \int_{4}^{6}(-Kx + 6K)dx = 1$$

$$K\left|\frac{x^2}{2}\right|_0^2 + 2K|x|_2^4 + \left(-\frac{Kx^2}{2} + 6Kx\right)_4^6 = 1$$

$$2K + 4K + (-10K + 12K) = 1$$

$$K = \frac{1}{8}$$

$$\text{Mean of } x = \int_0^6 x\, f(x)dx = \int_0^2 Kx^2 dx + \int_2^4 2Kx\, dx + \int_4^6 x(-Kx + 6K)dx$$

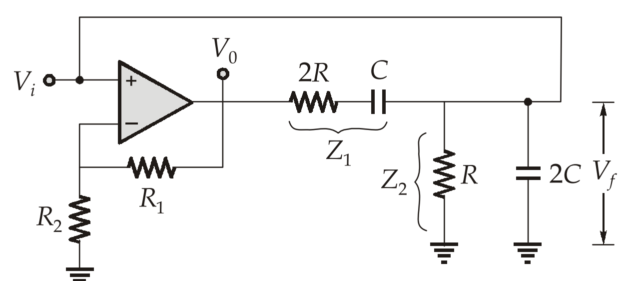
$$= K\left|\frac{x^3}{3}\right|_0^2 + 2K\left|\frac{x^2}{2}\right|_2^4 + \left(-K\left|\frac{x^3}{3}\right|_4^6 + 6K\left|\frac{x^2}{2}\right|_4^6\right)$$

$$= K\left(\frac{8}{3}\right) + K(12) - K\left(\frac{152}{3}\right) + 3K(20)$$

$$= \frac{1}{8}(24) = 3$$

Q.4 (a) Solution:

(i)



$$V_f = V_0 \times \frac{Z_2}{Z_1 + Z_2}$$

$$V_f = \left(1 + \frac{R_1}{R_2}\right)V_i \times \frac{1}{1 + Z_1 Y_2}$$

$$1 = \left(1 + \frac{R_1}{R_2}\right) \times \frac{1}{1 + \left(2R + \frac{1}{sC}\right)\left(\frac{1}{R} + 2sC\right)} \qquad (\therefore V_f = V_i)$$

$$1 = \left(1 + \frac{R_1}{R_2}\right) \times \frac{1}{1 + 2 + 4sCR + \frac{1}{sCR} + 2}$$

$$1 = \left(1+\frac{R_1}{R_2}\right)\times\frac{1}{5+j\left(4\omega CR-\frac{1}{\omega CR}\right)}$$

j terms should be zero,

$$4\,\omega_0 CR = \frac{1}{\omega_0 CR}$$

$$\omega_0^2 = \frac{1}{4R^2C^2}$$

$$\omega_0 = \frac{1}{2RC}$$

$$f_0 = \frac{1}{4\pi RC}$$

Given $R = 1.591\text{ k}\Omega$

and $C = 0.1\ \mu\text{F}$

$$f_0 = \frac{1}{4\pi\times 1.591\times 10^3\times 0.1\times 10^{-6}}$$

$$f_0 = 500\text{ Hz}$$

(ii) Magnitude analysis:

$$1 = \frac{\left(1+\frac{R_1}{R_2}\right)}{5}$$

$$5 = 1+\frac{R_1}{R_2}$$

$$R_1 = 4R_2$$

Given, $R_2 = 2\text{ k}\Omega$

$\therefore$ $R_1 = 8\text{ k}\Omega$

Q.4 (b) (i) Solution:

Given, A long narrow rod has an atomic density,

$$N = 5\times 10^{28}\text{ m}^{-3}$$

Atomic polarizability, $\alpha = 10^{-40}\text{ F-m}^2$

Applied axial field, $E = 1\text{ V/m}$

The Lorentz internal field is given by

$$E_i = \frac{E}{1-\frac{N\alpha}{3E_0}}$$

where $\in_0 = 8.854 \times 10^{-12}$ F/m

$$\therefore \quad E_i = \frac{1}{1-\frac{5\times10^{28}\times10^{-40}}{3\times8.854\times10^{-12}}} = \frac{1}{1-0.188}$$

$$E_i = \frac{1}{0.812} \approx 1.23 \text{ V/m}$$

Q.4 (b) (ii) Solution:

Ferromagnetic materials which have wide applications in electrical engineering has a disadvantage that they have low electrical resistivity. The laminations used for electrical machines have resistivity of about 14×10^{-4} ohm-m and the highest value obtainable in ferromagnetic alloys is less than 10^{-2} ohm-m. This disadvantage of the ferromagnetic materials limits their application in the high frequency alternating current applications, and results in high eddy current losses and poor magnetic utilization of materials occur in sheets even at low frequencies. Ferrites on the other hand, with useful magnetic properties have d.c. resistivity of many orders of magnitude higher than that of Iron and are used for frequencies upto microwave in transformer cores.

In ferrimagnetic substances, the magnetic moment of adjacent atoms are aligned in opposite, but the moments are not equal so that there is a net magnetic moment, i.e. if the net magnetization of magnetic sublattice is not zero, the material exhibits a net magnetic moment, however it is less than in ferromagnetic materials. This moment disappears above a curie temperature T_c, analogous to neel temperature at which thermal energy randomizes the individual magnetic moments and the materials becomes paramagnetic.

The general electric and magnetic characteristics of ferrites are:

- Very high resistivity, generally more than 10^5 ohm-cm.
- Microwave dielectric constant of the order of 10 - 12.
- Extremely low dielectric loss.
- High permeability.
- A saturation magnetization which is appreciable but noticeably smaller than that of ferromagnetic materials and low coercive field.
- A curie temperature which varies from 100° C to several hundred °C.

Q.4 (c) (i) Solution:

The flux linking with the moving coil is given by $N(A \cos \theta) \times B$

where $A = \frac{\pi}{4}d^2 =$ area of moving coil

$d =$ diameter of moving coil

$\theta =$ angle between axes of fixed and moving coils

$B =$ flux density

$N =$ number of turns of moving coil.

$\therefore$ Mutul inductance, $M = \frac{\left(\frac{\pi}{4}\right)d^2 NB\cos\theta}{I}$

where I is the current through the fixed or field coil.

$$\frac{dM}{d\theta} = \frac{\pi d^2 NB}{4I}\sin\theta$$

Deflection torque, $T_d = \frac{V}{R_p} I\cos\phi \frac{dM}{d\theta}$

$$= \frac{V}{R_p} I\cos\phi \frac{\pi d^2 NB}{4I}\sin\theta = \frac{\pi d^2 NBV}{4R_p}\cos\phi\sin\theta$$

$$= \frac{\pi(2.5\times10^{-2})^2\times500\times1.1\times10^{-3}\times100\times0.7}{4\times2000}\sin\theta$$

$\therefore$ $T_d = 9.45 \times 10^{-6} \sin\theta$ Nm

(i) Given, $\theta = 45°$

$\therefore$ $T_d = 9.45 \times 10^{-6} \times \sin 45° = 6.69 \times 10^{-6}$ Nm

(ii) Given, $\theta = 90°$

$\therefore$ $T_d = 9.45 \times 10^{-6} \times \sin 90°$

$= 9.45 \times 10^{-6}$ Nm

Q.4 (c) (ii) Solution:

Given, count displayed = 1133

Pulses of f_2 are counted during 100 cycles of f_1

$f_1 = 33$ kHz

Let T = time for 100 cycles of f_1

$$T = \frac{100}{f_1} = \frac{100}{33000} = \frac{1}{330}\text{sec}$$

The display shows the total number of pulses of frequency f_2 that occurred during this time T. Thus,

$$\text{Count} = f_2 \times T$$

$$1133 = f_2 \times \frac{1}{330}$$

$$\therefore \quad f_2 = 1133 \times 330$$

$$= 373890 \text{ Hz}$$

$$= 373.89 \text{ kHz}$$

Section-B

Q.5 (a) Solution:

$$\int_{-\infty}^{\infty} \frac{\sin x}{x^2 + 2x + 2} dx$$

$$I = \int_{-\infty}^{\infty} \frac{\sin z}{z^2 + 2z + 2} dz$$

$$\sin z = \text{imaginary part of } e^{iz}$$

$$= \int_{-\infty}^{\infty} \frac{\text{I.P of } e^{iz}}{z^2 + 2z + 2} dz$$

Poles are $z^2 + 2z + 2 = 0$

$$z = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$z = -1 - i$$

↓

Outside upper half

↓

Residue is 0

$$-1 + i$$

↓

inside upperhalf

$$\underset{z=-1+i}{\text{Res}}\, \phi(z) = \lim_{z \to -1+i} z - (-1+i) \frac{e^{iz}}{(z-(-1+i))(z-(-1-i))}$$

$$= \frac{e^{i(-1+i)}}{(-1+i)-(-1-i)} = \frac{e^{-i-1}}{-1+i+1+i} = \frac{e^{-i-1}}{2i}$$

$$I = \text{I.P. of } 2\pi i\left(\frac{e^{-i-1}}{2i}\right) = \text{I.P of } \pi\left(e^{-i}\cdot e^{-1}\right)$$

$$= \text{I.P of } \pi e^{-1}(\cos 1 - i\sin 1) = \frac{-\pi \sin 1}{e}$$

Q.5 (b) Solution:

The Fibonacci series is the sequence where the next number is determined as the sum of the previous two numbers of the sequence.

C Program:

```
#include<stdio.h>
int main ()
{
unsigned int fib[100]; //Use integer array to store the fibonacci numbers
fib[0] = 1;
fib[1] = 1;
for (int i = 2; i < 100; i++)
{
fib[i] = fib[i – 1] + fib[i – 2]; //Generate first 100 Fibonacci numbers.
}
for (int i = 0; i < 100; i++)
{
printf (“%d\n”, fib[i]); //Print the first 100 Fibonacci numbers
}
return 0;
}
```

Q.5 (c) Solution:

Wiedemann-Franz Lorenz law states that in metals the ratio of heat conductivity k to electrical conductivity σ at a constant temperature is same,

i.e. $$\frac{k}{\sigma} = \text{constant}$$

Where, k = heat conductivity at a temperature t °C.

σ = electrical conductivity at a temperature t °C.

Also, $$\frac{k}{\sigma} = LT \quad \text{...(i)}$$

Where T is absolute temperature and L is Lorenz number

At 273 K,

Diameter = 4 cm = 0.04 m

Thickness = 20 mm = 20×10^{-3} m

Resistivity, ρ = 70×10^{-9} Ω-m

Heat transported = P = 12 W

Using equation (i),

$$\frac{k}{\sigma} = \text{LT}$$

or $$k = \sigma\,\text{LT}$$

So, $$k = \frac{LT}{\rho} = \frac{2.23 \times 10^{-8} \times 273}{70 \times 10^{-9}} = 86.97 \text{ Wm}^{-1}\text{ K}^{-1}$$

by heat conduction formula,

$$\frac{\left(\frac{d\theta}{dt}\right)}{A} = k\left(\frac{\Delta T}{\Delta x}\right)$$

Power transmitted = thermal conductivity × Temperature gradient

$$\frac{P}{A} = k\left(\frac{\Delta T}{t}\right)$$

$$\Delta T = \frac{Pt}{Ak}$$

P = Heat transported,

t = length of disc

A = disk area,

k = thermal conductivity

$$\Delta T = \frac{12 \times 20 \times 10^{-3}}{\pi \times 4 \times 10^{-4} \times 86.97} = 2.196 \text{ K}$$

∴ Drop in temperature is 2.196 K

Q.5 (d) Solution:

Energy consumed in one minute with rated current and 0.8 p.f. lagging

$$= 240 \times 10 \times 0.8 \times \frac{1}{60} \times 10^{-3} = 0.032 \text{ kWh}$$

$\therefore$ Revolution made in one minute = 0.032 × 600 = 19.2

$\therefore$ Speed of disc = 19.2 rpm

When lag adjustment is altered: Steady speed N = KVI sin (Δ – ϕ)

If the lag adjustment is correctly done Δ = 90°

Under this condition, steady speed N = KVI sin (90° – ϕ) = KVI cos ϕ

$\therefore$ Error introduced because of incorrect lag adjustment

$$= \frac{KVI[\sin(\Delta - \phi) - \cos\phi]}{KVI\cos\phi} \times 100$$

$$= \frac{\sin(\Delta - \phi) - \cos\phi}{\cos\phi} \times 100 \text{ percent}$$

We have, $\Delta = 86°$

(i) At unity p.f. $\phi = 0$

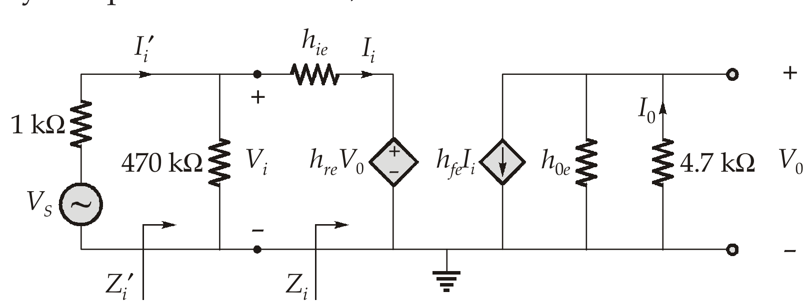
$$\therefore \quad \text{Error} = \frac{\sin(86° - 0°) - 1}{1} \times 100 = -0.24\%$$

(ii) At 0.5 p.f. lagging, $\phi = 60°$

$$\therefore \quad \text{Error} = \frac{\sin(86° - 60) - \cos 60°}{\cos 60°} \times 100 = -12.3\%$$

Q.5 (e) Solution:

AC equivalent hybrid parameter circuit,



(i) Current gain, $A_i = \dfrac{I_0}{I_i}$

$$I_0 = \frac{h_{fe} I_i \times \frac{1}{h_{0e}}}{\frac{1}{h_{0e}} + 4.7\text{ k}\Omega} = \frac{110 I_i \times \frac{1}{20 \times 10^{-6}}}{\frac{1}{20 \times 10^{-6}} + (4.7 \times 10^3)}$$

$$A_i = \frac{I_0}{I_i} = 100.548$$

(ii) Input impedance Z_i:

By applying KVL in the base emitter loop,

$$-V_i + h_{ie} I_i + h_{re} V_0 = 0$$

$$V_i = (1.6 \times 10^3) I_i + (2 \times 10^{-4})(-I_0 \times 4.7 \times 10^3)$$

$$= (1.6 \times 10^3) I_i - (2 \times 10^{-4})(A_i I_i \times 4.7 \times 10^3)$$

$$Z_i = \frac{V_i}{I_i} = 1.5\text{ k}\Omega$$

(iii) Voltage gain (A_V):

$$A_V = \frac{V_0}{V_i} = \frac{-I_0 \times 4.7 \times 10^3}{V_i} = \frac{-I_i A_i \times 4.7 \times 10^3}{V_i}$$

$$A_V = \frac{-A_i \times 4.7 \times 10^3}{Z_i} = \frac{-100.548 \times 4.7 \times 10^3}{1.5 \times 10^3} = -315$$

Q.6 (a) Solution:

Given, Y = 80 GPa and ρ = 2.65 g/cm^3 = 2650 kg/m^3.

So, the velocity of ultrasonic wave will be,

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{80 \times 10^9\text{ Pa}}{2650\text{ kg/m}^3}} = 5494\text{ m/s}$$

For fundamental mode, $n = 1$

For f = 1 kHz:

$$\lambda = \frac{v}{f} = \frac{5494}{1000}\text{m} = 5.494\text{ m}$$

The length of quartz crystal at 1 kHz for the fundamental mode is,

$$L = n\left(\frac{1}{2}\lambda\right) = \frac{\lambda}{2} \simeq 2.75\text{ m}$$

For f' = 1 MHz:

$$\lambda' = \frac{v}{f'} = \frac{5494}{10^6}\text{m} = 5.494 \text{ mm}$$

$$L' = n\left(\frac{1}{2}\lambda\right) = \frac{\lambda}{2} \simeq 2.75 \text{ mm}$$

From the values of L and L', it can be observed that,

- When f is small, a huge crystal is required which is impractical. As f increases, the crystal dimensions will be reduced to practically possible values.
- Increasing the value of n also increases the size of the quartz crystal.

Q.6 (b) (i) Solution:

Given,
$$\frac{dy}{dx} = x(y - x), \qquad x_0 = 2,$$
$$y_0 = 3, \qquad h = 0.2,$$
$$y(2.4) = \text{to be calculate}$$

Formulaa to be used for Ranga-Kutta 4th order:

$$y_{i+1} = y_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

Where,
$$K_1 = hf(x_i, y_i)$$
$$K_2 = hf\left[x_i + \frac{h}{2}, y_i + \frac{K_1}{2}\right]$$
$$K_3 = hf\left[x_i + \frac{h}{2}, y_i + \frac{K_2}{2}\right]$$
$$K_4 = hf(x_i + h, y_i + K_3)$$

First iteration:

$$h = 0.2, \quad x_0 = 2, \quad y_0 = 3$$
$$K_1 = 0.2[2 \times (3 - 2)] = 0.4$$
$$K_2 = 0.2[2.1 \times (3.2 - 2.1)] = 0.462$$
$$K_3 = 0.2[2.1 \times (3.231 - 2.1)] = 0.4750$$
$$K_4 = 0.2[2.2 \times (3.475 - 2.2)] = 0.5610$$

Now,
$$y(2.2) = 3 + \frac{1}{6}[0.4 + 2\times 0.462 + 2\times 0.4750 + 0.5610]$$
$$= 3.4725$$

Second iteration:

$$y(2.2) = 3.4750\,; \qquad x_1 = 2.2$$
$$K_1 = 0.2 \times [2.2 \times (3.4750 - 2.2)] = 0.56$$
$$K_2 = 0.2 \times [2.3 \times (3.755 - 2.3)] = 0.6693$$
$$K_3 = 0.2 \times [2.3 \times (3.8071 - 2.3)] = 0.6932$$
$$K_4 = 0.2 \times [2.4 \times (4.1657 - 2.4)] = 0.8475$$
$$y(2.4) = 3.4725 + \frac{1}{6}[0.56 + 2\times 0.6693 + 2\times 0.6932 + 0.8475]$$
$$= 4.1612$$

Q.6 (b) (ii) Solution:

Since,
$$\int_{-\infty}^{\infty} f(x)dx = 1,$$

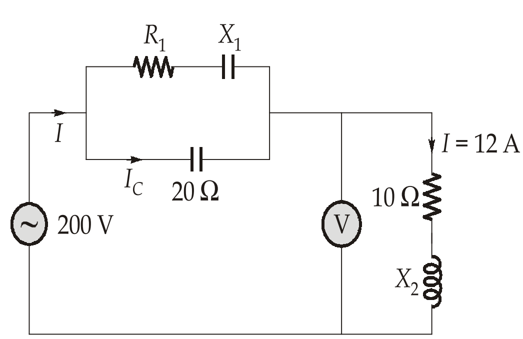
We have
$$a\int_{-\infty}^{\infty} e^{-2|x|}dx = 2a\int_{0}^{\infty} e^{-2x}dx = 2a\times\frac{1}{2} = a = 1$$

$$\text{Mean} = \int_{-\infty}^{\infty} x e^{-2|x|}dx = 0 \qquad \text{(Integrand is an odd function)}$$

Hence,
$$\text{Variance} = \int_{-\infty}^{\infty} x^2 e^{-2|x|}dx = 2\int_{-\infty}^{\infty} x^2 e^{-2x}dx$$
$$= 2\left[\frac{-x^2e^{-2x}}{2} - \frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4}\right]_0^{\infty} = \frac{1}{2}$$

Q.6 (c) Solution:

(i) Given,
$$I = 12\angle 0°\text{A}$$
$$P = 1800 \text{ W}$$



Since the voltmeter reads 200 V

Thus, $|Z_2| = \dfrac{200}{12} = 16.667\ \Omega$

where $|Z_2| = \sqrt{R_2^2 + X_2^2}$

$\Rightarrow$ $16.667 = \sqrt{(10)^2 + X_2^2}$

$\Rightarrow$ $(16.667)^2 = 100 + X_2^2$

$\Rightarrow$ $X_2 = 13.33375\ \Omega$

We have, $\overline{V_2} = IZ_2 = (12\angle 0°) \times (10 + j13.33375)$

$= (12\angle 0°) \times (16.667\angle 53.131°) = 200\angle 53.131°\ \text{V}$

Also, $P = VI \cos\phi$

$1800 = 200 \times 12 \times \cos\phi$

$\Rightarrow$ $\cos\phi = 0.75$

$\Rightarrow$ $\phi = 41.41°$

$\therefore$ Applied voltage, $\overline{V_s} = 200\angle 41.41°\text{V}$

Voltage across parallel branches

$= (200\angle 41.41°) - (200\angle 53.131°)$

$= 40.843\angle -42.73°\ \text{V}$

Current through capacitor,

$$I_C = \frac{40.843\angle -42.73°}{20\angle -90°} = 2.042\angle 47.27°\ \text{A}$$

Current through R_1 and X_1,

$= (12\angle 0° - 2.042\angle 47.27°) = 10.72\angle -8.043°\ \text{A}$

$$\therefore \quad Z_1 = \frac{40.843\angle -42.73°}{10.72\angle -8.043°}$$

$= 3.81\angle -34.687°\ \Omega = (3.133 - j2.1682)\ \Omega$

$\Rightarrow$ $R_1 = 3.133\ \Omega$

$X_1 = 2.1682\ \Omega$

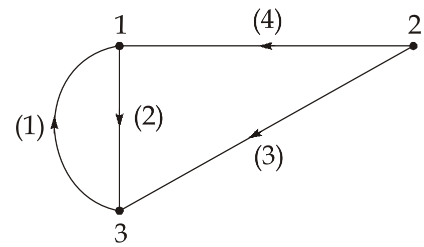
(ii) For drawing the oriented graph,

1. replace all resistors, inductors and capacitors by line segments.

2. replace voltage source by short circuit and current source by an open circuit,

3. assume directions of branch currents arbitrarily, and

4. number the nodes and branches.



Complete Incidence Matrix (A_a):

Nodes ↓	Branches → 1	2	3	4
1	–1	1	0	–1
2	0	0	1	1
3	1	–1	–1	0

$$A_a = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$$

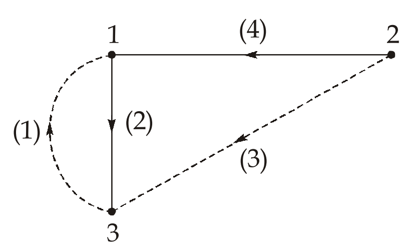
Eliminating the third row from the matrix A_a, we get the reduced incidence matrix A.

$$A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

To write the Tieset and f-cutset matrix, assume the tree with branches 2 and 4 as twigs and branches 1 and 3 as links.

Tieset Matrix (B):

A tie-set is a closed path in a graph containing one link and remaining branches are twigs. The number of tie-sets is equal to the number of links in the graph.

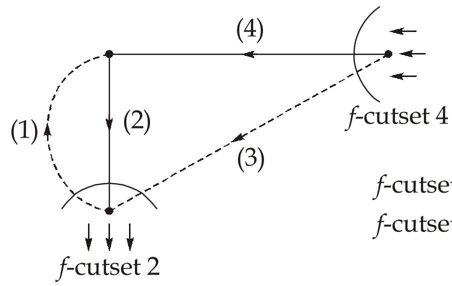


Twigs : {2, 4}
Links : {1, 3}
Tieset 1 : {1, 2}
Tieset 3 : {3, 2, 4}

$$B = \begin{array}{c} \\ 1 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

***f*-cutset matrix (Q):**

A cut-set is the smallest set of branches in a connected graph that, when removed, separates the graph into two sub-graphs. A fundamental cut-set of a graph with respect to a tree is a cut-set formed by one and only one twig and a set of links.



f-cutset 2: {2, 1, 3}

f-cutset 4: {4, 3}

$$Q = \begin{array}{c} \\ 2 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Q.7 (a) Solution:

For $t < 0$, given circuit is source-free RLC circuit.

$$\Rightarrow \quad i(0^-) = 0 \text{ A}$$

$$v(0^-) = 0 \text{ V}$$

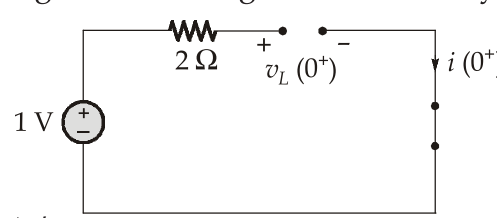
At $t = 0^+$: Since the inductor current and capacitor voltage cannot change instantaneously,

$$i(0^+) = i(0^-) = 0 \text{ A}$$

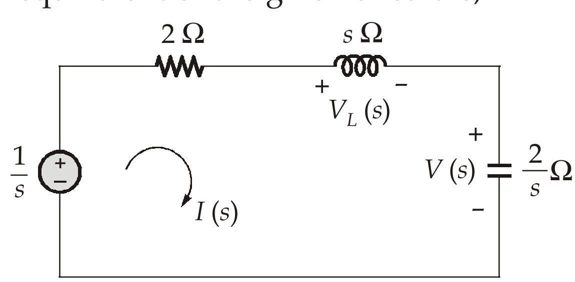
$$v(0^+) = v(0^-) = 0 \text{ V}$$

We have, $$v_L(0^+) = 1 \text{ V}$$

$$\Rightarrow \quad \frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{1}{1} = 1 \text{ A/s}$$



For $t \geq 0$: The s-domain equivalent of the given circuit is,



Apply KVL around the loop,

$$-\frac{1}{s} + \left(2 + s + \frac{2}{s}\right) I(s) = 0$$

$$I(s) = \frac{\left(\frac{1}{s}\right)}{\left(2 + s + \frac{2}{s}\right)} = \frac{\left(\frac{1}{s}\right)}{\left(\frac{s^2 + 2s + 2}{s}\right)}$$

$$\Rightarrow \qquad I(s) = \frac{1}{s^2+2s+2}$$

$$\therefore \qquad V(s) = \frac{2}{s}\times I(s) = \frac{2}{s}\times\frac{1}{s^2+2s+2} = \frac{A}{s}+\frac{Bs+C}{s^2+2s+2}$$

$$\Rightarrow \qquad 2 = A(s^2 + 2s + 2) + (Bs + C)s$$

On comparing the coefficients of 's^2'

$$0 = A + B \qquad \text{...(i)}$$

On comparing the coefficients of 's^1'

$$0 = 2A + C \qquad \text{...(ii)}$$

On comparing the coefficients of 's^0'

$$2 = 2A$$

$$\Rightarrow \qquad A = 1$$

From equation (i), $B = -1$

From equation (ii), $C = -2$

$$\therefore \qquad V(s) = \frac{1}{s}-\frac{(s+2)}{s^2+2s+2} = \frac{1}{s}-\frac{(s+1)}{(s+1)^2+1}-\frac{1}{(s+1)^2+1}$$

Take inverse Laplace transform on both sides,

$$v(t) = u(t) - e^{-t}\cos t\, u(t) - e^{-t}\sin t\, u\, lt)$$

$$\Rightarrow \qquad v(t) = 1 - e^{-t}\cos t - e^{-t}\sin t;\ t \ge 0$$

$$\frac{dv(t)}{dt} = -[e^{-t}(-\sin t) - e^{-t}\cos t] - [e^{-t}(\cos t) - e^{-t}\sin t]$$

$$= e^{-t}\sin t + e^{-t}\cos t - e^{-t}\cos t + e^{-t}\sin t$$

$$= 2\, e^{-t}\sin t$$

$$\frac{d^2v(t)}{dt^2} = 2[e^{-t}\cos t - e^{-t}\sin t]$$

At $t = 0^+$:

$$\frac{d^2v(0^+)}{dt^2} = 2[1 - 0] = 2\ \text{V/s}^2$$

For the given RLC-circuit, $R = 2\ \Omega$, $L = 1$ H, $C = \frac{1}{2}$F. The roots of the characteristic equation are given by

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $\alpha = \dfrac{R}{2L} = \dfrac{2}{2\times 1} = 1$

and $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{1\times\frac{1}{2}}} = \sqrt{2} = 1.414$

As $\alpha < \omega_0$, poles are complex conjugates with negative real parts. Thus, the circuit is underdamped.

Q.7 (b) (i) Solution:

The characteristic equation of A is

$$\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

By Cayley-Hamilton theorem, A must satisfy its characteristic equation, so that

$$A^2 - 4A - 5I = 0 \qquad \text{...(ii)}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 16 \\ 8 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 0$$

This verifies the theorem,

Multiplying (ii) by A^{-1},

We get $A - 4I - 5A^{-1} = 0$

$$A^{-1} = \frac{1}{5}(A - 4I)$$

$$= \frac{1}{5}\left\{\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

$$= \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

Now dividing the polynomial

$\lambda^5 - 4\lambda^4 - 7\lambda^3 + 11\lambda^2 - \lambda - 10I$ by the polynomial $\lambda^2 - 4\lambda - 5$, we obtain

$$\lambda^5 - 4\lambda^4 - 7\lambda^3 - \lambda - 10I = (\lambda^2 - 4\lambda - 5)(\lambda^2 - 2\lambda + 3) + \lambda + 5$$

Hence,

$$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = A + 5I$$

Which is linear polynomial in A.

Q.7 (b) (ii) Solution:

The characteristic equation of A is given by

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & 1 & -1 \\ -2 & 1-\lambda & 2 \\ 0 & 1 & 2-\lambda \end{vmatrix}$$

$$= \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\lambda = 1, 2, 3$$

Since the matrix A has three distinct eigen values, it has three linearly independent eigen vectors and hence it is diagonalizable.

The eigen vector corresponding to the eigen value $\lambda = 1$ is the solution of the system

$$(A - I)x = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution, $$X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

The eigen vector corresponding to the eigen value $\lambda = 2$ is the solution of the system

$$(A - 2I)X = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution, $$X_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

The eigen vector corresponding to the eigen value $\lambda = 3$ is the solution of the system,

$$(A - 3I)X = \begin{bmatrix} 0 & 1 & -1 \\ -2 & -2 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution, $X_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Hence, the modal matrix is given by

$$P = [X_1 \quad X_2 \quad X_3]$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and $P^{-1} = \begin{bmatrix} -1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

It can be verified that,

$$P^{-1}AP = \text{diag}(1, 2, 3)$$

We have, $D = \text{diag}\,(1, 2, 3)$

$$D^2 = \text{diag}\,(1, 4, 9)$$

$$A^2 + 5A + 3I = P(D^2 + 5D + 3I)P^{-1}$$

Now, $D^2 + 5D + 3I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 15 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 27 \end{bmatrix}$

$$A^2 + 5A + 3I = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 27 \end{bmatrix}\begin{bmatrix} -1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 25 & 8 & -8 \\ -18 & 9 & 18 \\ -2 & 8 & 19 \end{bmatrix}$$

Q.7 (c) Solution:

(i) Digital Voltmeter (DVM):

- A digital voltmeter (DVM) displays the value of AC or DC voltage being measured directly as discrete numerals in the decimal number system.
- Numerical readout is advantageous in many applications because it reduces human reading and interpolations because errors and eliminates parallax errors.
- The use of digital voltmeters increase the speed with which reading can be taken, also the output of digital voltmeter can be fed to memory devices for storage and future computations.

- A digital voltmeter is a versatile and accurate voltmeter which has many laboratory applications.

Types of DVMs:

(i) Ramp type DVM. (ii) Integrating type DVM. (iii) Potentiometric type DVM. (iv) Successive approximation type DVM. (v) Continuous type DVM.

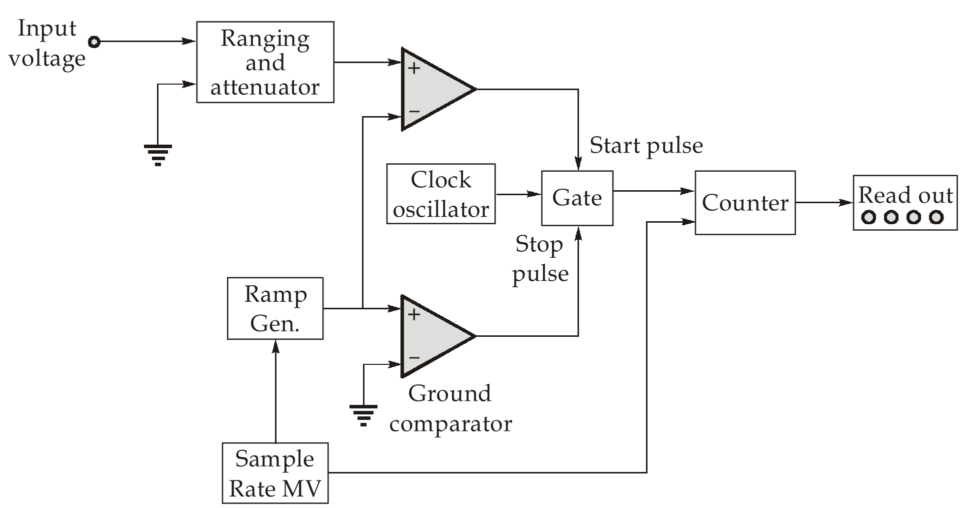
Basic Function:

In every case the basic function that is performed is an analog to digital (A/D) conversion.

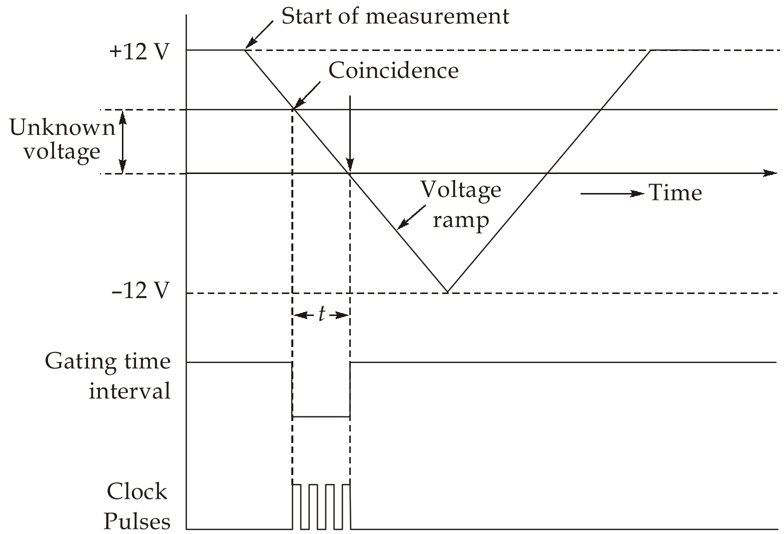
For example a voltage value may be changed to a proportional time interval, which starts and stops a clock oscillator. In turn the oscillator output is applied to an electronic counter which is provide with a read out in terms of voltage values.

Ramp Type Digital Voltmeter:

- When an analog voltage of ramp type is applied to the ramp type digital voltmeter it measure the time interval with an electronic time interval counter and count is displayed as a number of digits on electronic indicating tubes of the output read out of the voltmeter.
- Block diagram of ramp type DVM is shown on figure below:



- The conversion of a voltage value of a time interval is as shown in figure below:



(Timing diagram showing voltage to time conversion)

- The decimal number as indicated by the read out is a measure of the value of input voltage.
- The sample rate multivibrator determines the rate at which the measurement cycles are indicated.
- The sample rate circuit provides an indicating pulse for the ramp generator to start its next ramp voltage.
- At the same time it sends a pulse to the counter which sets all of them to 0. This momentarily removes the digital display of the readout.

(ii) Controlling torque at full scale deflection

$$T_c = 240 \times 10^{-6} \text{ N-m}$$

Deflecting torque at full scale deflection

$$T_d = N\,Bld\,I$$
$$= 100 \times 1 \times 40 \times 10^{-3} \times 30 \times 10^{-3}\, I$$
$$= 120 \times 10^{-3}\, I \text{ N-m}$$

At final steady state position,

$$T_d = T_c$$

or $$120 \times 10^{-3}\, I = 240 \times 10^{-6}$$

∴ Current at full scale deflection, I

$$= 2 \times 10^{-3} \text{ A} = 2 \text{ mA}$$

Let the resistance of the voltmeter circuit be R

∴ Voltage across the instrument = $2 \times 10^{-3}\, R$

This produces a deflection of 100 division

$\therefore$ Volts per division $= 2 \times 10^{-3} R/100$

This value should be equal to 1 in order to get 1 volt per division

$\therefore$ 2×10^{-3} R/100 $= 1$

or $R = 50000\ \Omega = 50\ \text{k}\Omega$

Q.8 (a) Solution:

Given, $y(1) = 1$

and $y(2) = 2$

The given differential equation is Cauchy's homogenous linear equation so,

Putting, $x = e^t$

i.e. $t = \log x$ ...(i)

the given equation becomes,

$$[D(D-1) + D + 1]y = t^2 \sin t$$

$$\Rightarrow \quad [D^2 + 1]y = t^2 \sin t$$

Now complete solution (CS) of this differential equation,

$$CS = CF + PI \quad \text{...(ii)}$$

Where complementary function CF is solved using auxiliary equation,

i.e. $D^2 + 1 = 0$

AE $m^2 + 1 = 0$

$m = \pm i$

$\Rightarrow$ $CF = C_1 \cos t + C_2 \sin t$ ...(iii)

And P.I.

$$\text{P.I.} = \frac{1}{D^2+1} t^2 \sin t = \frac{1}{D^2+1} t^2 (\text{I.P. of } e^{it})$$

Where, I.P. stands for imaginary part,

$$\Rightarrow \quad \text{P.I.} = \text{I.P. of } e^{it} \cdot \frac{1}{(D+i)^2+1} t^2$$

$$\text{P.I.} = \text{I.P. of } e^{it} \cdot \frac{1}{D^2+2iD} t^2$$

$$\text{P.I.} = \text{I.P. of } e^{it} \cdot \frac{1}{2iD\left(1+\frac{D}{2i}\right)} t^2$$

$$\text{P.I.} = \text{I.P. of } \frac{1}{2i} \cdot e^{it} \cdot \frac{1}{D} \cdot \left[1 - \frac{iD}{2}\right]^{-1} t^2$$

$$\text{P.I.} = \text{I.P. of } \frac{1}{2i}\cdot e^{it}\cdot\frac{1}{D}\cdot\left[1+\frac{iD}{2}+\left(\frac{iD}{2}\right)^2+....\right]t^2$$

$$\text{P.I.} = \text{I.P. of } \frac{1}{2i}\cdot e^{it}\cdot\frac{1}{D}\left[t^2+\frac{i}{2}\cdot 2t-\left(\frac{1}{4}\right)^2\right]$$

$$\text{P.I.} = \text{I.P. of } \frac{1}{2i}\cdot e^{it}\cdot\frac{1}{D}\left[t^2+it-\frac{1}{2}\right]$$

$$\text{P.I.} = \text{I.P. of } \frac{1}{2i}\cdot e^{it}\int\left(t^2+it-\frac{1}{2}\right)dt$$

$$\text{P.I.} = \text{I.P. of } \frac{1}{2i}\cdot e^{it}\left[\frac{t^3}{3}+\frac{it^2}{2}-\frac{t}{2}\right]$$

$$\text{P.I.} = \text{IP of } e^{it}\left[\frac{-it^3}{6}+\frac{t^2}{4}+\frac{it}{4}\right]$$

$$\text{P.I.} = \text{I.P. of } (\cos t+i\sin t)\left(\frac{-it^3}{6}+\frac{t^3}{4}+\frac{it}{4}\right)$$

$$\Rightarrow \quad \text{P.I.} = \left(\frac{-t^3}{6}+\frac{t}{4}\right)\cos t+\frac{t^2}{4}\sin t \quad ...(iv)$$

Using (ii), (iii) and (iv),

$$\text{C.S.} = C_1\cos t+C_2\sin t+\left(\frac{t}{4}-\frac{t^3}{6}\right)\cos t+\frac{t^2}{4}\sin t$$

Now using equation (i),

$$y = C_1\cos(\log x)+C_2\sin(\log x)+\left(\frac{\log x}{4}-\frac{\log^3 x}{6}\right)\cos(\log x)+\frac{\log^2 x}{4}\sin(\log x)$$

Now, $y(1) = C_1 \cos (0) + C_2 \sin (0) + 0 . \cos (0) + 0 . \sin (0) = 1$

$\Rightarrow \quad C_1 = 1$ ...(v)

and $y(2) = 1\cdot\cos(\log 2)+C_2\sin(\log 2)+\left(\frac{\log 2}{4}-\frac{\log^3 2}{6}\right)\cos(\log 2)$

$\Rightarrow \quad y(2) = 1.029 + C_2\cdot 0.2965 = 2$

$\Rightarrow \quad 0.2965\, C_2 = 0.971$

$\Rightarrow \quad C_2 = 3.27847 \approx 3.275$

$\Rightarrow$ Complete solution,

$$y = \cos\log x + 3.275\sin(\log x) + \left(\frac{\log x}{4} - \frac{\log^3 x}{6}\right)\cos(\log x) + \frac{\log^2 x}{4}\sin(\log x)$$

Q.8 (b) (i) Solution:

Consider the projection of S on the x-y plane

The projection is the circular region, $x^2 + y^2 \le 16$, $z = 0$ and the bounding curve C is circle $z = 0$, $x^2 + y^2 = 16$

We have,

$$\oint_C V \cdot dr = \oint_C (3x - y)dx - 2yz^2 dy - 2y^2 z dz$$

$$= \oint_C (3x - y)\, dx$$

Since, $z = 0$,

Setting, $x = 4\cos\theta$, $y = 4\sin\theta$, we obtain

$$\oint_C (3x - y)\, dx = \int_0^{2\pi} 4(3\cos\theta - \sin\theta)(-4\sin\theta)d\theta$$

$$= -16\int_0^{2\pi}\left[\frac{3}{2}\sin 2\theta - \frac{1}{2}(1 - \cos 2\theta)\right]d\theta$$

$$= 16\left(\frac{1}{2}\right)2\pi = 16\pi$$

Now,

$$\nabla \times V = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x - y & -2yz^2 & -2y^2 z \end{vmatrix}$$

$$= (-4yz + 4yz)\hat{a}_x - 0\hat{a}_y + 1\hat{a}_z = 1\hat{a}_z$$

$$n = \frac{2(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)}{2\sqrt{x^2 + y^2 + z^2}} = \frac{1}{4}(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)$$

$$(\nabla \times V)\cdot n = \frac{z}{4}$$

Therefore,

$$\iint_S (\nabla \times V)\cdot n\, dA = \iint_S \frac{z}{4} dA = \iint_R \frac{z}{4}\frac{dxdy}{n \cdot K} = \iint_R \frac{z}{4}\frac{dxdy}{(z/4)}$$

$$= \iint_R dx\,dy = 16\pi$$

Which is the area of the circular region in the x-y plane. Hence, Stoke's theorem is proved.

Q.8 (b) (ii) Solution:

The formula for the Modified Euler-Cauchy method is given by

$$y_{n+1} = y_n + K_2$$
$$K_1 = h\,f(x_n, y_n),$$
$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

We have the following results,

$$n = 0, \quad x_0 = 1, \quad y_0 = 2, \quad h = 0.2$$
$$K_1 = h_f(x_0, y_0) = 0.2f(1, 2) = 1$$
$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}K_1\right) = 0.2f(1.1, 2.5) = 1.492$$
$$y(1.2) = y_1 = y_0 + K_2 = 3.492$$
$$n = 1,$$
$$x = 1.2, \quad y_1 = 3.492$$
$$K_1 = hf(x_1, y_1) = 0.2f(1.2, 3.492) = 2.7268$$
$$K_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right) = 0.2f(1.3, 4.8554) = 5.0530$$
$$y(1.4) \approx y_2$$
$$= y_1 + K_2 = 3.492 + 5.0530 = 8.545$$

Q.8 (c) (i) Solution:

Given:

$$\beta_1 = 99; \beta_2 = 49$$
$$I_{E_{s1}} = 10I_{E_{s2}} = 5 \times 10^{-16}\text{ A}$$
$$V_{\text{out, max}} = 0.2\text{ V}$$
$$V_T = 25\text{ mV}$$

For the maximum output voltage,

$$I_{C_2(\max)} = \frac{V_{out,\max}}{R_C} = \frac{0.2}{0.5K} = 0.4\text{ mA}$$

$$I_{E2(\max)} = I_{C2(\max)}\left(\frac{\beta_2+1}{\beta_2}\right) = 0.4\left(\frac{50}{49}\right)$$

$$I_{E2(\max)} = 0.408 \text{ mA}$$

To calculate V_{BE_2}

For pnp transistor, $I_E = I_{E_s} e^{\frac{V_{EB}}{V_T}}$. We get,

$$V_{EB2(\max)} = V_T \ln\left(\frac{I_{E_2}}{I_{E_{s2}}}\right)$$

$$V_{EB2(\max)} = 0.025 \ln\left(\frac{0.408\times10^{-3}}{0.5\times10^{-16}}\right) = 0.743 \text{ V}$$

From the circuit, it is clear that $I_{E_2} = I_{E_1}$. For the npn transistor, $I_E = I_{E_s} e^{\frac{V_{BE}}{V_T}}$. Thus,

$$V_{BE1(\max)} = V_T \ln\left(\frac{I_{E_1}}{I_{E_{S1}}}\right)$$

$$= 0.025 \ln\left(\frac{0.408\times10^{-3}}{5\times10^{-16}}\right) = 0.685 \text{ V}$$

Thus, the maximum input voltage that can be applied to the circuit is given by

$$V_{\text{in}(\max)} = V_{BE_1(\max)} + V_{EB_2(\max)}$$

$$= 0.512 + 0.685$$

$$= 1.428 \text{ V}$$

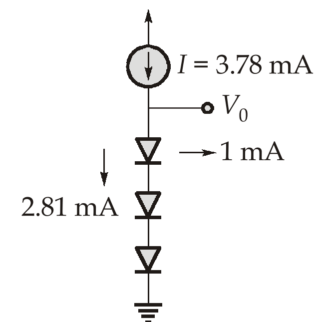
Q.8 (c) (ii) Solution:

As each diode is identical. So, the voltage across each diode is $V_D = \frac{V_0}{3}$.

Diode current, $$I = I_s e^{\frac{V_D}{\eta V_T}}$$

For $V_0 = 2$ V, $$I = I_S e\left(\frac{V_0/3}{V_T}\right) = 10^{-14} e\left(\frac{2/3}{0.025}\right) = 3.81 \text{ mA}$$

If a current of 1 mA is drawn away from the output terminals by a load, let's say now the changed voltage across each diode is $\Delta V = V_2 - V_1$



We can write,
$$\frac{I_2}{I_1} = e^{\frac{(V_2 - V_1)/3}{0.025}}$$

$$\frac{2.81}{3.81} = e^{\frac{(V_2 - 2)/3}{0.025}}$$

$$V_2 = 2 + 0.075 \ln(0.73)$$

On solving, we get
$$V_2 = 1.976 \text{ V}$$

Thus,
$$\Delta V = V_2 - 2 = 1.976 - 2$$
$$= -0.024 = -24 \text{ mV}$$

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