MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Civil Engineering Test No : 14

Section - A

1. (a) (i) Solution:

Super plasticizers: These are hydrodynamic lubricants which impart high workability to concrete by reducing friction between the grains or by reducing the amount of water to be added. They are improved version of plasticizers and interact both physically and chemically with cement particles. The mechanism of action of super plasticizers is same as that of plasticizers. Super plasticizers are anionic in nature and impart a negative charge to the cement particles, causing them to repel each other. Once the super plasticizers are added, the concrete should not be agitated as it will lead to an early loss of workability. The workability of the super plasticized concrete decreases more with time than that of normal concrete. The strength, water-cement ratio and creep of concrete with the admixture are same as that of concrete before the addition of plasticizer while shrinkage and surface absorption are reduced slightly and resistance to thawing and freezing gets improved.

By addition of superplasticizers, it is possible to obtain same strength with reduced cement content. However, the durability is reduced. The water-cement ratio of the superplasticized concrete may be reduced upto 30 percent to obtain the initial equality of workability.

A dose of 1 to 6 litres per cubic metre of concrete may be used.

Examples: Sulphonated melanin formaldehyde, naphthalene sulphonated formaldehyde, modified lingo-sulphate.

1. (a) (ii) Solution:

1. Batching is the process of measuring the different ingredients of concrete—cement, aggregates (fine and coarse), water, and admixtures—before mixing them to produce a fresh concrete mix. Precise batching is critical because the strength, durability, and workability of the concrete depend entirely on the correct proportions of these materials.

   There are two primary methods of batching used in construction: Volume Batching and Weigh Batching.

   (a) Volume Batching: In this method, the materials are measured by their loose volume using standardized measurement boxes, often called gauge boxes or frames.

   - **Process:** Measurement boxes are made to a specific volume (typically 35 liters, which is the volume of one bag of cement weighing 50 kg).
   - **Suitability:** This method is generally used for small-scale works or non-structural concrete where high precision is not mandatory.
   - **The "Bulking of Sand" Issue:** Volume batching is less accurate because the volume of fine aggregate (sand) varies significantly based on its moisture content. This phenomenon, known as bulking, can lead to an incorrect water-cement ratio if not adjusted.

   (b) Weigh Batching: In this method, all ingredients (except sometimes water and liquid admixtures) are measured by their weight using manual or electronic scales.

   - **Process:** Modern construction uses "Batching Plants" where digital sensors (load cells) weigh each material precisely before they enter the mixer.
   - **Suitability:** This is the standard for all major structural works, Ready-Mix Concrete (RMC), and high-strength concrete.
   - **Accuracy:** It eliminates the errors caused by the bulking of sand and ensures a highly consistent mix.

2. As-*Cu* treatment, commonly known as Copper-Chrome-Arsenic (*CCA*) treatment, is a highly effective water-borne preservative method used to protect timber from decay, fungi, and wood-boring insects. It is one of the most widely used industrial chemical treatments for outdoor and structural timber.

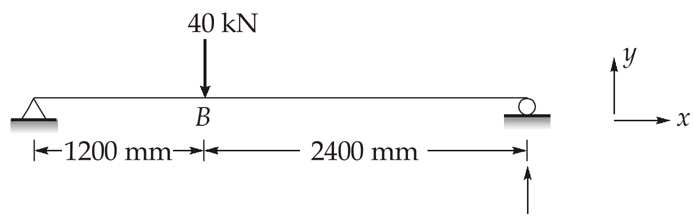
   The name *ASCU* is derived from its three primary chemical components: Arsenic, Copper, and Chromium.

Chemical Composition

The treatment involves a specific ratio of these three active ingredients, usually dissolved in water:

- **Arsenic** ($As_2O_5 \cdot 2H_2O$): Acts as a powerful insecticide and protects the wood against termites and other wood-boring pests.
- **Copper** ($CuSO_4 \cdot 5H_2O$): Functions as a fungicide to prevent rot, mold, and decay-causing fungi.
- **Chromium** ($K_2Cr_2O_7$): Acts as a "fixing agent." It chemically bonds the copper and arsenic to the wood fibers, ensuring the preservative does not leach out when the timber is exposed to rain or soil moisture.

1. (b) Solution:



Bending Moment under the point load,

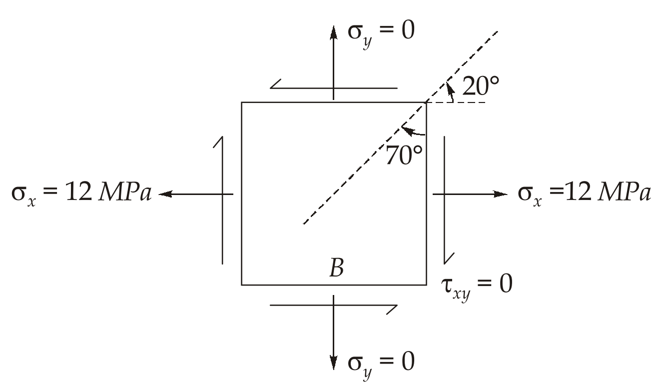
$$M_B = \frac{Pab}{l} = \frac{40 \times 1.2 \times 2.4}{3.6}$$

$\Rightarrow$ $$M_B = 32 \text{ kN-m}$$

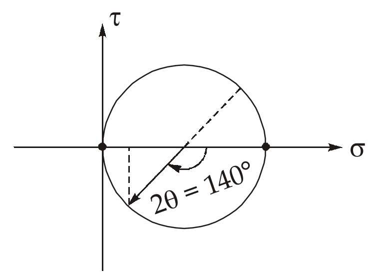
Bending tensile stress at B,

$$\sigma = \frac{M_B}{I} y_{max} = \frac{32 \times 10^6}{\dfrac{100 \times 400^2}{6}} = 12 \text{ MPa (tensile)}$$

State of stress at Point B,



here, $\sigma_x = 12$ MPa, $\sigma_y = 0$, $\tau_{xy} = 0$



Shear stress at point B along the grain,

$$\tau_{x'y'} = \left(\frac{12-0}{2}\right) \sin(180 - 140) = 3.856 \text{ MPa}$$

1. (c) Solution:

Anchorage length of bar at simply supported end of beam can be determined by,

$$\text{Development length, } L_d \le \frac{1.30\, M_u}{V_u} + l_0$$

$$\text{Anchorage length, l0} \ge L_d - \frac{1.3 M_u}{V_u}$$

where, $$L_d = \frac{0.87 \times f_y \times \phi}{4 \times \tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 1.92} = 940 \text{ mm}$$

$$M_u = 0.87 f_y A_{st}\left(d - 0.42 x_u\right)$$

Assume $x_u \le x_{u,\lim}$ (under-reinforced section)

∴ $$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

where $$A_{st} = 3 \times \frac{\pi}{4}(20)^2 = 942 \text{ mm}^2$$

∴ $$x_u = \frac{0.87 \times 415 \times 942}{0.36 \times 20 \times 250} = 188.95 \text{ mm}$$

and $$x_{u,\lim} = 0.48d = 0.48 \times 450 = 216 \text{ mm}$$

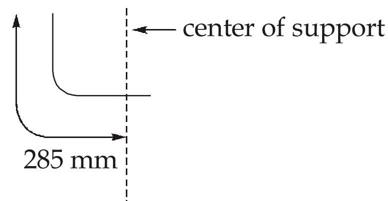
∵ $x_u < x_{u,\lim}$, therefore section is under-reinforced and computed value of x_u is correct.

⇒ $$M_u = 0.87 \times 415 \times 942 \times (450 - 0.42 \times 188.95) \text{ N-mm}$$

⇒ $$M_u = 126.06 \text{ kNm}$$

Hence, $$l_0 \geq 940 - \frac{1.3 \times 126.06 \times 10^6}{250 \times 10^3} \geq 284.50 \text{ mm}$$

∴ Therefore minimum anchorage length of 285 mm is required.



1. (d) (i) Solution:

Internal Torques in each segment:

$$T_1 = 55 \text{ Nm} = 55000 \text{ Nmm}$$
$$T_2 = 55 - 880 = -825 \text{ Nm} = -825000 \text{ Nmm}$$
$$T_3 = -825 + 275 = -550 \text{ Nm} = -550000 \text{ Nmm}$$
$$T_4 = -550 + 660 = 110 \text{ Nm} = 110000 \text{ Nmm}$$

Polar moment of inertia:

$$J = \frac{\pi d^4}{32}$$

$$J_1 = \frac{\pi \times 25^4}{32},\ J_2 = \frac{\pi \times 100^4}{32},\ J_3 = J_4 = \frac{\pi \times 75^4}{32}$$

Angle of twist:

$$\theta = \sum \left(\frac{TL}{GJ} \right)$$

⇒ $$\theta = \frac{T_1 L_1}{GJ_1} + \frac{T_2 L_2}{GJ_2} + \frac{T_3 L_3}{GJ_3} + \frac{T_4 L_4}{GJ_4}$$

$$\theta = \frac{55000 \times 1500}{G \times \frac{\pi (25)^4}{32}} + \frac{(-825000) \times 1000}{G \times \frac{\pi (100)^4}{32}} + \frac{(-550000) \times 1000}{G \times \frac{\pi (75)^4}{32}} + \frac{110000 \times 2000}{G \times \frac{\pi (75)^4}{32}}$$

$$\theta = \frac{2036.67}{G} \text{ rad}$$

1. (d) (ii) Solution:

Equivalent stiffness of fixed-fixed beam:

$$k_b = \frac{192EI}{L^3}$$

$$\Rightarrow \quad k_b = \frac{192 \times (3 \times 10^{10}) \times 0.0003}{6^3}$$

$$\Rightarrow \quad k_b = 8{,}000{,}000 \text{ N/m}$$

Total equivalent stiffness:

$$\frac{1}{k_{eq}} = \frac{1}{k_b} + \frac{1}{k_{spring}}$$

$$\Rightarrow \quad \frac{1}{k_{eq}} = \frac{1}{8000000} + \frac{1}{50000}$$

$$\Rightarrow \quad k_{eq} = 49689.441 \text{ N/m}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{49689.441}{4000}}$$

$$\Rightarrow \quad \omega_n = 3.525 \text{ rad/s}$$

Natural time period:

$$T_n = \frac{2\pi}{\omega_n}$$

$$\Rightarrow \quad T_n = \frac{2\pi}{3.525}$$

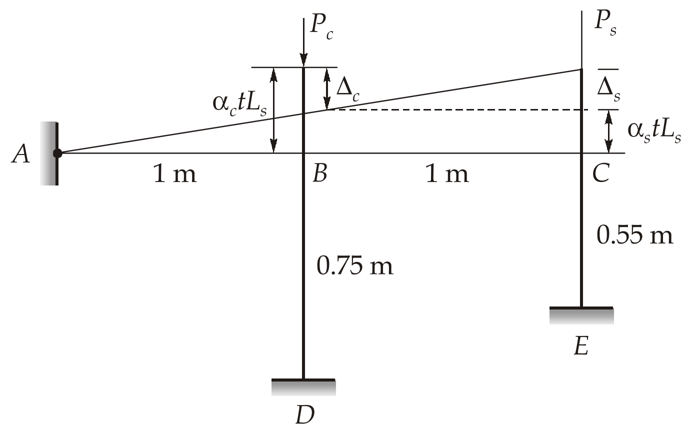
$$\Rightarrow \quad T_n = 1.783 \text{ scc}$$

1. (e) Solution:

Given
$\alpha_c = 18 \times 10^{-6}/°C$ $\alpha_s = 12 \times 10^{-6}/°C$
$E_c = 1 \times 10^5\ N/mm^2$ $E_s = 2 \times 10^5\ N/mm^2$
$A_c = 620\ mm^2$ $A_s = 450\ mm^2$

Free expansions–

Copper bar = $0.75 \times 18 \times 10^{-6} \times 32 = 4.32 \times 10^{-4}$ m
Steel bar = $0.55 \times 12 \times 10^{-6} \times 32 = 2.112 \times 10^{-4}$ m



Taking moment about A

$$2P_S = P_C \quad ...(i)$$

From similar triangle

$$\alpha_C tL_C - \Delta_C = (\alpha_S tL_S + \Delta_S)\frac{1}{2}$$

$$\Rightarrow \quad \alpha_S tL_S + \Delta_S = 2(\alpha_C tL_C - \Delta_C)$$

But
$$\Delta_s = \frac{P_S \times L_S}{A_S E_S} \quad \text{and} \quad \Delta_C = \frac{P_C \times L_C}{A_C E_C}$$

$$\therefore \quad (2\alpha_C tL_C - \alpha_S L_S t) = 2\Delta_C + \Delta_S$$

$$\Rightarrow \quad 2 \times 18 \times 10^{-6} \times 32 \times 750 - 12 \times 10^{-6} \times 550 \times 32 = \frac{4 \times P_S \times 750}{620 \times 10^5} + \frac{P_S \times 550}{450 \times 2 \times 10^5}$$

$$\Rightarrow \quad P_S = 11978.375\ N$$

From equation (i), $P_C = 23956.75$ N

$$\text{Stress in Steel Rod} = \frac{P_S}{A_S} = \frac{11978.375}{450} = 26.619\ N/mm^2\ \text{(Tensile)}$$

$$\text{Stress in copper Rod} = \frac{P_C}{A_C} = \frac{23956.75}{620} = 38.640\ N/mm^2\ \text{(Compressive)}$$

2. (a) Solution:

Given, Axial load = 1500 kN, SBC = 105 kN/m^2

Assuming 10% of column load as weight of footing and soil, approximate area of footing required:

$$= \frac{1.1 \times 1500}{105} = 15.71 \text{ m}^2$$

Provide 4.0 m × 4.0 m square footing giving total area = 16 m^2

Upward soil pressure on base slab

$$w_o = \frac{1500 \times 1.1}{16} = 103.125 \text{ kN/m}^2 < 105 \text{ kN/m}^2 \quad \text{(OK)}$$

Net upward pressure for design of base slab,

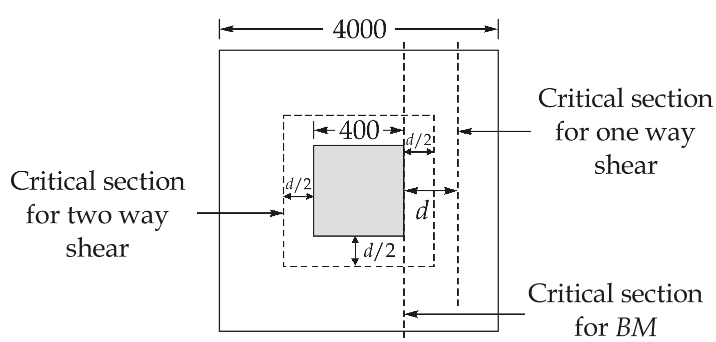
$$w_{\text{net}} = \frac{P}{A_{Provided}} = \frac{1500}{16} = 93.75 \text{ kN/m}^2$$

Factored net upward pressure

$$w_u = 140.625 \text{ kN/m}^2$$

Design for bending moment,

$$\therefore \quad M_u = \left[140.625 \times \left(\frac{4-0.4}{2}\right)^2 \times \frac{1}{2}\right] = 227.8 \approx 228 \text{ kNm}$$



∴ Effective depth required is given by,

$$BM = 0.138 f_{ck} bd^2$$

$$\Rightarrow \quad d = \sqrt{\frac{228 \times 10^6}{0.138 \times 25 \times 1000}} = 257.07 \text{ mm}$$

Adopt 550 mm effective depth and 600 mm overall depth of footing. Increased depth is taken due to shear consideration.

Area of tension steel is given by,

$$A_{st} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]bd = \frac{25}{2\times 415}\times\left[1-\sqrt{1-\frac{4.6\times 228\times 10^6}{25\times 1000\times 550^2}}\right]\times 1000\times 550$$

$\Rightarrow$ $$A_{st} = 1191.6 \text{ mm}^2$$

Now, $$A_{st,\,min} = 0.12\% \text{ of } bD$$

$$= \frac{0.12}{100}\times 1000\times 600 = 720 \text{ mm}^2 < 1191.6 \text{ mm}^2 \quad \text{(OK)}$$

$\therefore$ Number of 16 mm bars $$= \frac{1191.6}{\frac{\pi}{4}\times(16)^2} = 5.9 \simeq 6$$

$\therefore$ Centre to centre spacing between bars $$= \frac{1000}{6} = 166 \text{ mm}$$

$$= 150 \text{ mm}$$

$$p = \frac{6\times\frac{\pi}{4}\times(16)^2\times 100}{1000\times 550} = 0.219\%$$

One-way shear

The critical section is taken at distance '*d*' away from the face of column.

Shear force, $$V_u = 140.625\times\left[\left(\frac{4-0.4}{2}\right)-0.55\right] = 175.78 \text{ kN}$$

Nominal shear stress, $$\tau_v = \frac{V_u}{bd} = \frac{175.78\times 10^3}{1000\times 550} = 0.319 \text{ N/mm}^2$$

Shear strength of M25 concrete with 0.2% steel from given table

$$\tau_c = 0.338 \text{ N/mm}^2 > \tau_v \quad \text{...(OK)}$$

Two-way shear

The critical section is taken at a distance '0.5*d*' away from the face of column,

Shear force, $$V_u = 140.625\,[16 - (0.4 + 0.55)^2] = 2123.086 \text{ kN}$$

Nominal shear stress, $$\tau_v = \frac{V_u}{bd} = \frac{2123.086\times 1000}{4(400+550)\times 550} = 1.016 \text{ N/mm}^2$$

Shear strength of M25 concrete,

$$\tau_c' = k_s\times 0.25\sqrt{f_{ck}}$$

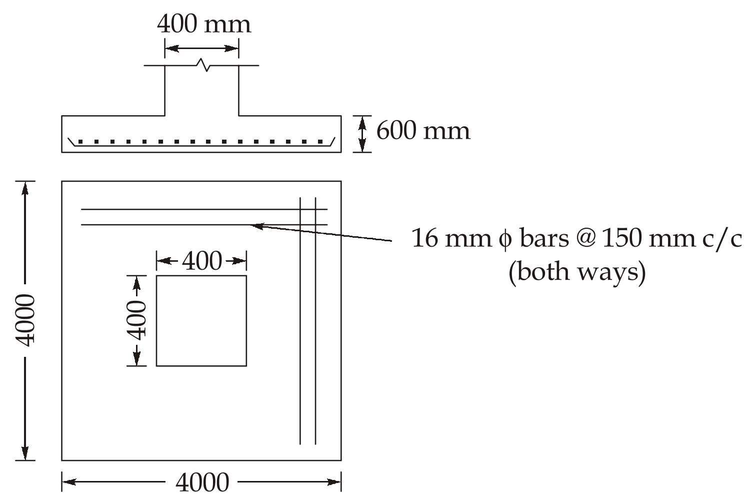
where, $k_s = 0.5+\beta_c \not> 1$ and $\beta_c = 1$ for square column

$\therefore \quad k_s = 0.5 + 1 \not> 1.0, k_s = 1.0$

$\therefore \quad = \tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25}$

$= 1.25 \text{ N/mm}^2 > 1.016 \text{ N/mm}^2$

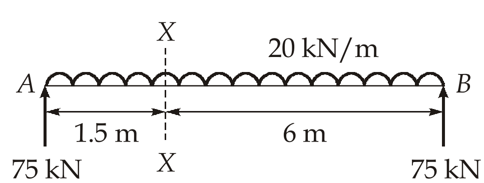
This shows that a footing having an effective depth of 550 mm will be safe in shear.



Reinforcement Details

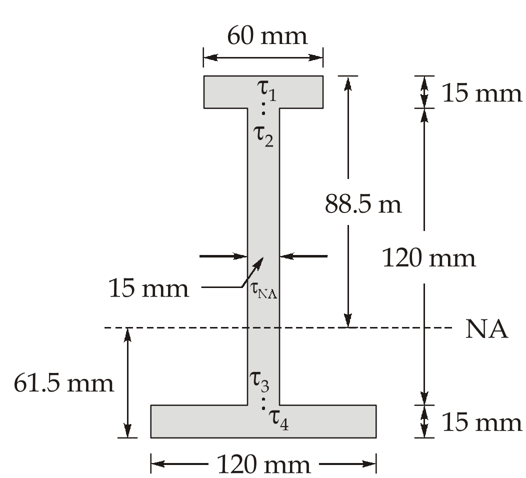
2. (b) Solution:

Given that: Unsymmetrical I-section



Shear force at section X-X = 75 – (20 × 1.5)

$V_x = 45 \text{ kN}$



Calculation for distance of *NA*

Distance of *NA* (centroid) of the section from the top fibre be y_1.

Then,
$$y_1 = \frac{(60\times15\times7.5)+(120\times15)\left(15+\frac{120}{2}\right)+(15\times120\times142.5)}{[(60\times15)+(120\times15)+(120\times15)]}$$

$$y_1 = 88.5 \text{ mm (from top)}$$

and,
$$I = \frac{1}{12}\times60\times(15)^3+60\times15\times(88.5-7.5)^2$$
$$+\frac{1}{12}\times15\times120^3+15\times120\times(88.5-75)^2$$
$$+\frac{1}{12}\times15^3\times120+15\times120\times(142.5-88.5)^2$$
$$I = 13692375 \text{ mm}^4$$

Shear stress at the bottom of the top flange

$$\tau_1 = \frac{V_x}{bI}(a\bar{y}) = \frac{(45\times10^3)}{(60)\times13692375}\times(60\times15)\times(88.5-7.5)$$

$\Rightarrow$ $$\tau_1 = 3.993 \text{ N/mm}^2$$

Shear stress at the same level, but in web

$$\tau_2 = 3.993\times\frac{60}{15} = 15.972 \text{ N/mm}^2$$

Shear stress at *NA*

$$a\bar{y} = a\bar{y} \text{ of top flange } + a\bar{y} \text{ of web above } NA$$

$$\tau_{NA} = \frac{45\times10^3}{15\times13692375}\times\left[60\times15\times(88.5-7.5)+15\times\frac{(88.5-15)^2}{2}\right]$$

$\Rightarrow$ $$\tau_{NA} = 24.850 \text{ N/mm}^2$$

Shear stress at the junction of web and lower flange

Considering the lower side of the section for finding $a\bar{y}$, we get

$$a\bar{y} = (120\times15)\times(61.5-7.5)$$
$$= 97200 \text{ mm}^3$$

$\therefore$ Shear stress at the junction of web and lower flange

$$\tau_4 = \frac{45\times10^3}{120\times13692375}\times97200$$

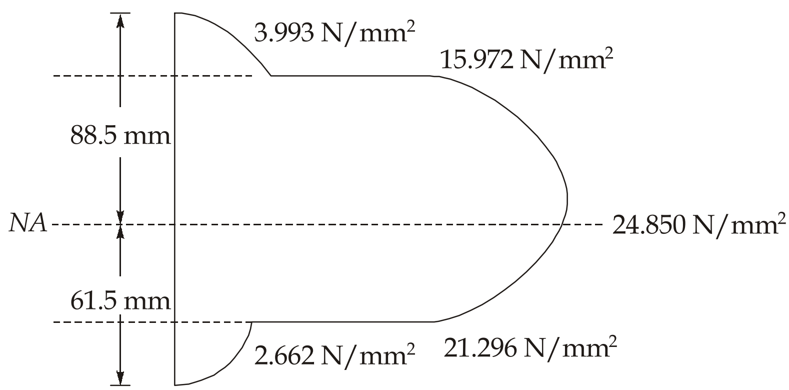
$= 2.662 \text{ N/mm}^2$

But at the above level, but in web

$$\text{Shear stress } \tau_3 = 2.662 \times \frac{120}{15} = 21.296 \text{ N/mm}^2$$

And at the extreme fibres, shear stress is zero.

Hence, the variation of shear stress across the depth of the section is as shown below.



Variation of shear stress across the depth

2. (c) Solution:

Given: M20 bolts of grade 4.6

Diameter of bolt, $d = 20$ mm

Diameter of bolt hole, $d_0 = 22$ mm

$f_{ub} = 400 \text{ N/m}^2$; $f_{yb} = 240 \text{ N/mm}^2$

Steel of grade E250 (Fe 410)

$\therefore$ $f_y = 250 \text{ N/mm}^2$; $f_u = 410 \text{ N/mm}^2$

Here, bolts are in single shear

Design strength of bolt:

Shear strength of bolt, $$V_{db} = V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} 1 \times A_{nb}$$

$$= \frac{400}{\sqrt{3} \times 1.25} \times 1 \times \frac{\pi}{4} \times (20)^2 \times 0.78 \times 10^{-3} \text{ kN}$$

$= 45.272$ kN

Also, Here $L_j = (80 \times 4 = 320 \text{ mm}) > (15d = 15 \times 20 = 300 \text{ mm})$

$\therefore$ $V_{db} = 45.272 \times \beta_{lj}$

where $\beta_{ij} = 1.075 - \frac{L_j}{200d}$

$$= 1.075 - \frac{320}{200 \times 20} = 0.995$$

$\therefore$ $V_{db} = 45.272 \times 0.995 = 45.05$ kN

Design tensile strength of bolt,

$$T_{db} = 0.9 \frac{f_{ub}}{\gamma_{mb}} \times A_{nb}$$

$\Rightarrow$ $T_{db} = 0.9 \times \frac{400}{1.25} \times 0.78 \times \frac{\pi}{4}(20)^2 \times 10^{-3}$ kN

$$\leq \frac{f_{yb}}{\gamma_{m0}} \times \frac{\pi}{4} d^2 = \frac{240}{1.1} \times \frac{\pi}{4}(20)^2 \times 10^{-3} \text{ kN}$$

$\Rightarrow$ $T_{db} = 70.57$ kN ≤ 68.54 kN

$\therefore$ $T_{db} = 68.54$ kN

Total depth of bracket plate

$$= 80 \times 4 + 2 \times 40 = 400 \text{ mm.}$$

h = Height of the upper most bolts from the lower edge of the bracket

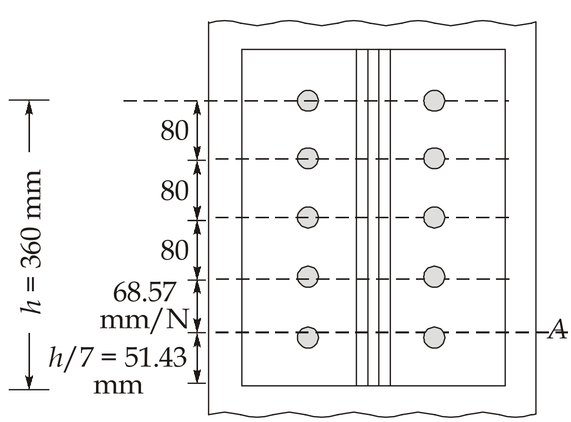
$h = 4 \times 80 + 40 = 360$ mm

The neutral axis is assumed to lie at $h/7$ from the bottom of the bracket i.e.

$$\frac{360}{7} = 51.43 \text{ mm}$$

$$\Sigma y_i = 2 \times [68.57 + 148.57 + 228.57 + 308.57]$$

$$= 1508.56 \text{ mm}$$



$$\Sigma y_i^2 = 2 \times [68.57^2 + 148.57^2 + 228.57^2 + 308.57^2]$$
$$= 348469.16 \text{ mm}^2$$

Now,
$$M' = \frac{M}{1 + \frac{2h}{21} \cdot \left(\frac{\Sigma y_i}{\Sigma y_i^2}\right)} = \frac{180 \times 0.25}{1 + \frac{2 \times 360}{21}\left(\frac{1508.56}{348469.16}\right)}$$

$\Rightarrow$ $M' = 39.184$ kNm

Tensile force in the critical bolt

$$T_b = \frac{M'}{\Sigma y_i^2} \times y_n = \frac{39.184 \times 10^3}{348469.16} \times 308.57 = 34.7 \text{ kN}$$

Shear force in the critical bolt,

$$V_{sb} = \frac{P}{\text{Number of Bolt}} = \frac{180}{2 \times 5} = 18 \text{ kN}$$

For safety, $\left(\frac{V_{sb}}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \le 1.0$

L.H.S $\left(\frac{18}{45.05}\right)^2 + \left(\frac{34.7}{68.54}\right)^2 = 0.416 < 1.0$

$\therefore$ The design is safe.

3. (a) Solution:

Given data

Width of sign, $w = 2.5$ m

Height of sign, $h = 1.5$ m

Outer diameter, $d_o = 0.25$ m

Inner diameter, $d_i = 0.20$ m

Wind pressure, $p = 1.8$ kPa = 1800 N/m^2

Offset, $e = 0.4$ m

Height to lower edge, $H_{low} = 5.0$ m

The total wind force acting on the sign is:

$$F = p \times (w \times h)$$
$$F = 1800 \times (2.5 \times 1.5) = 6750 \text{ N}$$

The vertical distance of the force from the base is:

$$H = h_{\text{low}} + \frac{h}{2}$$

$$\Rightarrow \quad H = 5.0 + \frac{1.5}{2} = 5.75 \text{ m}$$

The horizontal distance from the pole centreline is:

$$L = e + \frac{w}{2}$$

$$\Rightarrow \quad L = 0.4 + \frac{2.5}{2} = 1.65 \text{ m}$$

The bending moment at the base is:

$$M_{\text{Base}} = F \times H$$

$$\Rightarrow \quad M_{\text{Base}} = 6750 \times 5.75 = 38812.5 \text{ Nm}$$

The torque at the base is:

$$T = F \times L$$

$$T = 6750 \times 1.65 = 11137.5 \text{ Nm}$$

The shear force is:

$$V = F = 6750 \text{ N}$$

1. Stress analysis at point *A*:

Bending stress:

$$\sigma_A = \frac{M \times \frac{d_o}{2}}{I}$$

$$\sigma_A = \frac{38812.5 \times 0.125}{\frac{\pi}{64}\left(0.25^4 - 0.20^4\right)} = 42.855 \text{ MPa (tensile)}$$

Torsional shear stress:

$$\tau = \frac{T \times \frac{d_o}{2}}{J} = \frac{11137.5 \times 0.125}{\frac{\pi}{32}\left(0.25^4 - 0.20^4\right)} = 6.149 \text{ MPa}$$

Principal stresses:

$$\sigma_{1,\,2} = \frac{\sigma_A}{2} \pm \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,\,2} = \frac{42.855}{2} \pm \sqrt{\left(\frac{42.855}{2}\right)^2 + 6.149^2}$$

$$\sigma_1 = 43.72 \text{ MPa}$$
$$\sigma_2 = -0.865 \text{ MPa}$$

Maximum shear stress:

$$\tau_{max} = \sqrt{\left(\frac{42.855}{2}\right)^2 + (6.149)^2} = 22.292 \text{ MPa}$$

2. Stress analysis at point *B*:

First moment of area: $$Q = \frac{\pi}{8}d_o^2 \times \frac{2d_o}{3\pi} - \frac{\pi}{8}d_i^2 \times \frac{2d_i}{3\pi}$$

$\Rightarrow$ $$Q = \frac{1}{12}(d_o^3 - d_i^3) = \frac{1}{12}(0.25^3 - 0.20^3) = 6.3542 \times 10^{-4} \text{ m}^3$$

Transverse shear stress:

$$\tau_{trans} = \frac{V \times Q}{I \times (d_o - d_i)}$$

$$\tau_{trans} = \frac{6750 \times 6.3542 \times 10^{-4}}{\frac{\pi}{64}\left(0.25^4 - 0.20^4\right) \times 0.05} = 0.758 \text{ MPa}$$

Total shear stress: $$\tau_B = \tau + \tau_{trans}$$

$\Rightarrow$ $$\tau_B = 6.149 + 0.758 = 6.907 \text{ MPa}$$

Principal stresses: (Pure Shear Condition)

$$\sigma_1 = \sigma_1 = 6.907 \text{ MPa}, \sigma_2 = -6.907 \text{ MPa}$$

Maximum shear stress: $\tau_{max} = 6.907$ MPa

3. (b) (i) Solution:

Total cross-sectional area, $$A_c = 3900 \times 2 + 270 \times 8 = 9960 \text{ mm}^2$$

$\bar{y}$ from top of plate, $$\bar{y} = \frac{270 \times 8 \times 4 + 2 \times 3900 \times (125 + 8)}{9960} = 105.024 \text{ mm}$$

Now,

$$I_{xx} = 2 \times 3880 \times 10^4 + 2 \times 3900 \times \left(\frac{250}{2} - 105.024 + 8\right)^2 + \frac{270 \times 8^3}{12} + 270 \times 8 \times (105.024 - 4)^2$$

$\Rightarrow$ $$I_{xx} = 105.760 \times 10^6 \text{ mm}^4$$

Radius of gyration, $$r_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{105.760 \times 10^6}{9960}} = 103.046 \text{ mm}$$

Now, $I_{yy} = 2 \times 211 \times 10^4 + 2 \times 3900 \times (23 + 55)^2 + \frac{8 \times 270^3}{12}$

$\Rightarrow$ $I_{yy} = 64.797 \times 10^6 \text{ mm}^4$

Radius of gyration, $r_{yy} = r_{min} = \sqrt{\frac{64.797 \times 10^6}{9960}} = 80.658 \text{ mm}$

Effective length $= 4.8$ m

Slenderness ratio, $\lambda = \frac{4800}{80.658} = 59.51$

From table $f_{cd} = 183 - \frac{9.51}{10}(183 - 168)$

$\Rightarrow$ $f_{cd} = 168.735 \text{ N/mm}^2$

Load carrying capacity, $P_u = 168.735 \times 9960 \times 10^{-3} = 1680.60 \text{ kN}$

Service load $= \frac{P_u}{1.5} = \frac{1680.60}{1.5} = 1120.40 \text{ kN}$

3. (b) (ii) Solution:

Approximate solution considering gross section:

Area of a single wire, $A_w = \frac{\pi}{4} \times 8^2 = 50.265 \text{ mm}^2$

Total area of prestressing steel $= 9 \times 50.265 = 452.389 \text{ mm}^2$

Area of beam section, $A = 400 \times 350 = 140 \times 10^3 \text{ mm}^2$

Moment of inertia of beam section-$I = \frac{400 \times 350^3}{12} = 14.29 \times 10^8 \text{ mm}^4$

Distance of centroid of steel area from the soffit of beam,

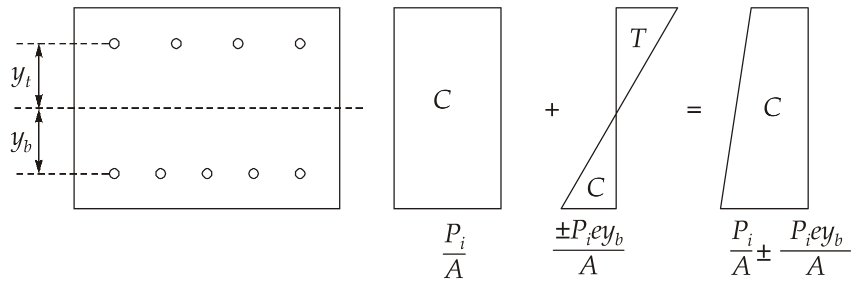
$$\bar{y} = \frac{4 \times (350 - 50) + 5 \times 50}{9} = 161.11 \text{ mm}$$

Prestressing force, $P_i = 0.8 \times 1470 \times 452.389\ N = 532.00 \text{ kN}$

Eccentricity of prestressing force,

$$e = \left(\frac{350}{2}\right) - 161.11 = 13.89 \text{ mm}$$

Stress diagrams due to P_i is shown below.



Since the wires are distributed above and below the *CG*, the losses are calculated for the top and bottom wires separately.

Stress at level of top wires ($y = y_c$ = 175 – 50 = 125 mm

$$(f_c)_t = \frac{P_i}{A} - \frac{P_i e}{I} y_t = \frac{532 \times 10^3}{140 \times 10^3} - \frac{532 \times 10^3 \times 13.89}{14.29 \times 10^8} \times 125$$

$$= 3.154 \text{ N/mm}^2$$

$$\text{Stress at bottom, } (f_c)_b = \frac{P_i}{A} + \frac{P_i e y_b}{I}$$

$$= \frac{532 \times 10^3}{140 \times 10^3} + \frac{532 \times 10^3 \times 13.89}{14.29 \times 10^8} \times 125$$

$$= 4.446 \text{ N/mm}^2$$

Loss of prestress in top wires = 6 × 3.154 × 4 × 50.265 = 3804.86 N

Loss of prestress in bottom wires = 6 × 4.446 × 5 × 50.265 = 6704.35 N

Total loss of prestress = 3.805 + 6.704 = 10.509 kN

$$\therefore \quad \text{Percentage loss} = \frac{10.504}{532} \times 100 = 1.97\%$$

3. (c) (i) Solution:

Strength of weld per mm length,

= 0.7 × 6 × 102.5 = 430.5 N

Total effective length of weld required = *L*

$$= \frac{250000}{430.5}$$

$$\therefore \quad L = 580.72 \text{ mm}$$

Let 'L_1' and 'L_2' be the weld lengths on two sides of the angle

$$\therefore \quad 2(L_1 + L_2) = L = 580.72$$

$$\Rightarrow \quad L_1 + L_2 = 290.36 \text{ mm}$$

The lengths 'L_1' and 'L_2' are such that the resultant strength of the weld passes through the C.G. of the section.

$$\therefore \quad \frac{L_1}{90 - 28.7} = \frac{L_1 + L_2}{90}$$

$$\Rightarrow \quad L_1 = (90 - 28.70) \times \frac{290.36}{90} = 197.8 \text{ mm}$$

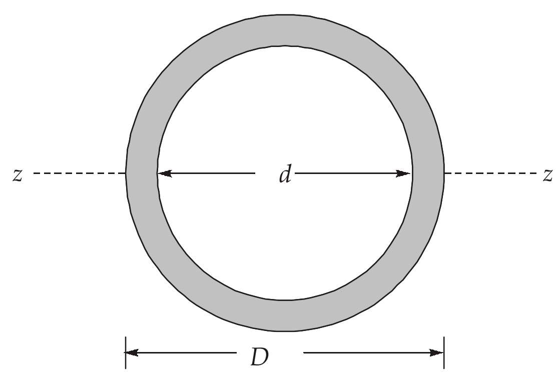
$$\therefore \quad L_2 = 290.36 - 197.8 = 92.56 \text{ mm}$$

$\therefore$ Adopt $L_1 = 200$ mm

$L_2 = 100$ mm

3. (c) (ii) Solution:

Moment of inertia about zz-axis,



$$I_z = \frac{\pi}{64}\left(D^4 - d^4\right)$$

Elastic section modulus, $$Z_{ez} = \frac{\pi}{64} \times \frac{\left(D^4 - d^4\right)}{D/2} = \frac{\pi}{32} \times \frac{D^4 - d^4}{D}$$

Plastic section modulus, $$Z_{pz} = \frac{A}{2}\left(\bar{y}_1 + \bar{y}_2\right)$$

$$A = \frac{\pi}{4}\left(D^2 - d^2\right)$$

$$\bar{y}_1 = \bar{y}_2 = \frac{\frac{1}{2} \times \left[\frac{\pi D^2}{4}\left(\frac{2}{3} \times \frac{D}{\pi}\right) - \frac{\pi d^2}{4}\left(\frac{2}{3} \times \frac{d}{\pi}\right)\right]}{\frac{1}{2} \times \left[\frac{\pi}{4}\left(D^2 - d^2\right)\right]}$$

$$= \frac{2}{3\pi} \times \frac{\left(D^3 - d^3\right)}{\left(D^2 - d^2\right)}$$

$$\therefore \quad Z_{pz} = \frac{1}{2}\times\frac{\pi}{4}(D^2-d^2)\left[2\times\frac{2}{3\pi}\times\frac{(D^3-d^3)}{(D^2-d^2)}\right] = \frac{1}{6}(D^3-d^3)$$

$$\text{Shape factor} = \frac{Z_{pz}}{Z_{ez}} = \frac{\frac{1}{6}(D^3-d^3)}{\frac{\pi}{32}\left(\frac{D^4-d^4}{D}\right)}$$

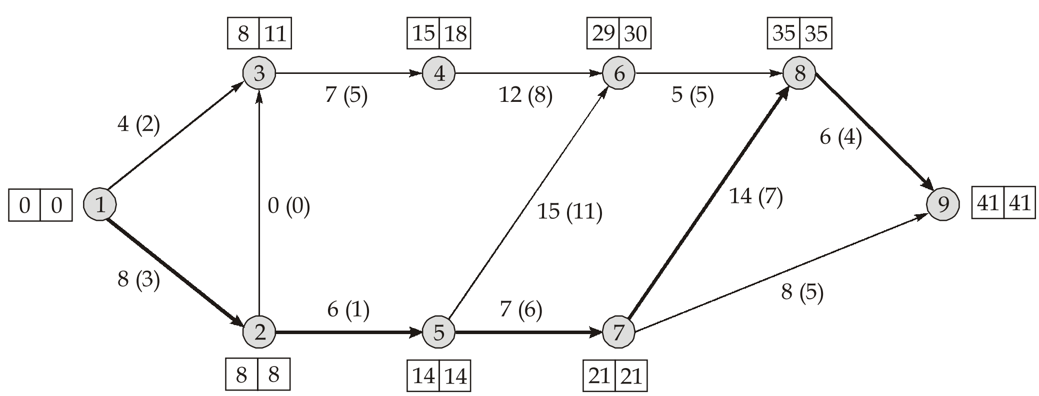
$$= \frac{32}{6\pi}\times\frac{D^3\left(1-\frac{d^3}{D^3}\right)}{D^3\left(1-\frac{d^4}{D^4}\right)} = \frac{32}{6\pi}\times\frac{\left(1-\frac{d^3}{D^3}\right)}{\left(1-\frac{d^4}{D^4}\right)}$$

Put, $k = \frac{d}{D}$

$$\text{Shape factor} = \frac{32}{6\pi}\times\frac{(1-k^3)}{(1-k^4)} = 1.7\frac{(1-k^3)}{(1-k^4)}$$

4. (a) Solution:

For the given activities, network diagram is shown below:



Activity	Duration in days		Direct cost (Rs.)		Cost slope $C_s = \frac{C_s - C_n}{t_n - t_s}$
	Normal	Crash	Normal	Crash	
1-2	8	3	700	1000	60
1-3	4	2	600	800	100
2-3	0	0	0	0	0
2-5	6	1	900	1150	50
3-4	7	5	250	450	100
4-6	12	8	1000	1600	150
5-6	15	11	1200	1600	100
5-7	7	6	1200	1400	200
6-8	5	5	1000	1000	-
7-8	14	7	600	740	20
7-9	8	5	600	1200	200
8-9	6	4	600	780	90

For normal duration

Project duration = 41 days

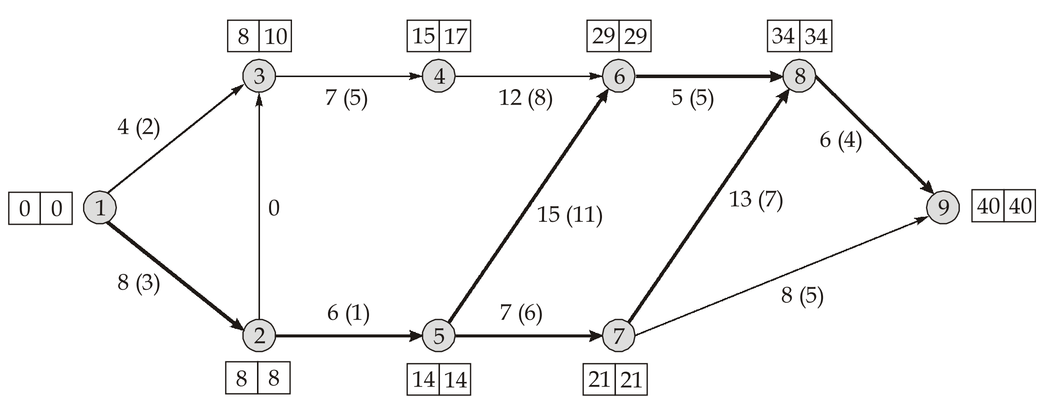
Total direct cost = 700 + 600 + 900 + 250 + 1000 + 1200 + 1200 + 1000 + 600 + 600 + 600

= Rs. 8650

Total indirect cost = 41 × 100 = Rs. 4100

Total project cost = 8650 + 4100 = Rs. 12750

Ist stage of crashing: Crashing activity 7-8 by 1 day



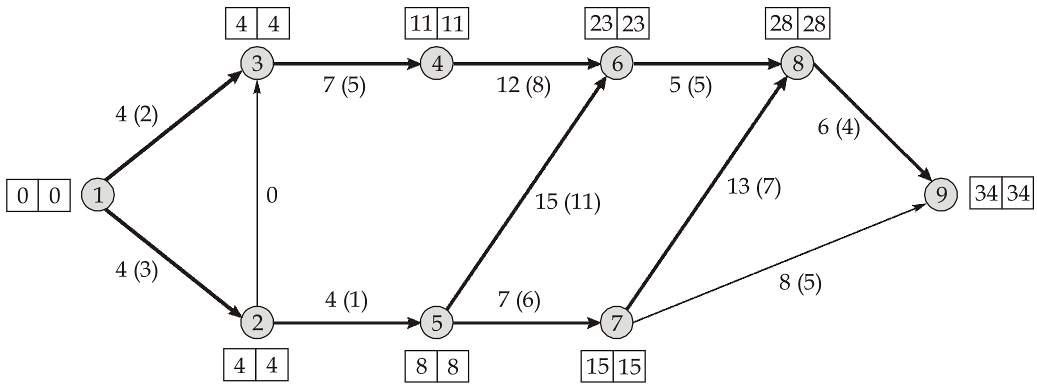
Time duration = 40 days

Total direct cost = 8650 + 20 = Rs. 8670

Total indirect cost = 40 × 100 = Rs. 4000

Total project cost = 8670 + 4000 = Rs. 12670

II[nd] stage of crashing: Crashing activity 1-2 by 4 days and 2-5 by 2 days.



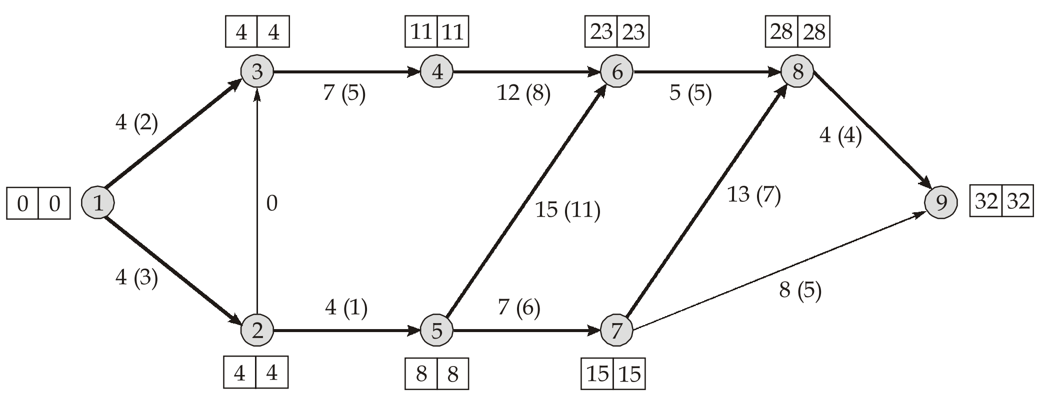
Time duration = 34 days

Total direct cost = 8670 + 4 × 60 + 2 × 50 = Rs. 9010

Total indirect cost = 34 × 100 = Rs. 3400

Total project cost = Rs. 12410

III[rd] stage of crashing: Crashing activity 8-9 by 2 days.



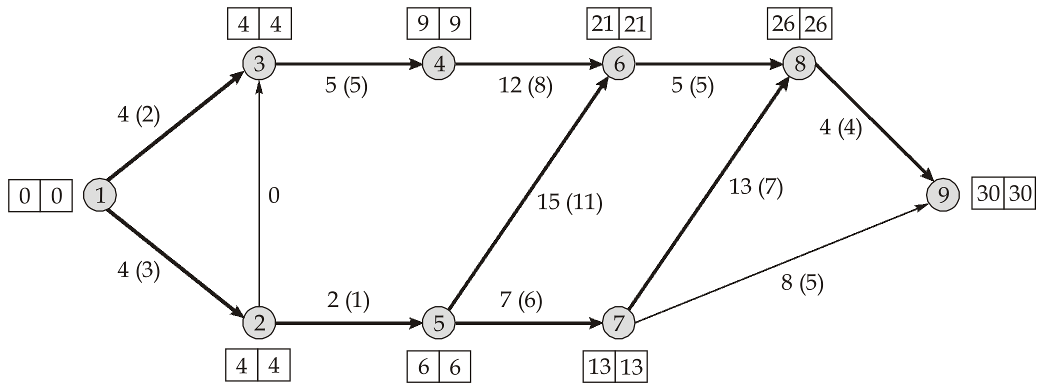
Time duration = 32 days

Total direct cost = 9010 + 2 × 90 = Rs. 9190

Total indirect cost = 32 × 100 = Rs. 3200

Total project cost = Rs. 12390

IVth stage of crashing: Crashing activity 2-5 and 3-4 by 2 days



$$\text{Time duration} = 30 \text{ days}$$

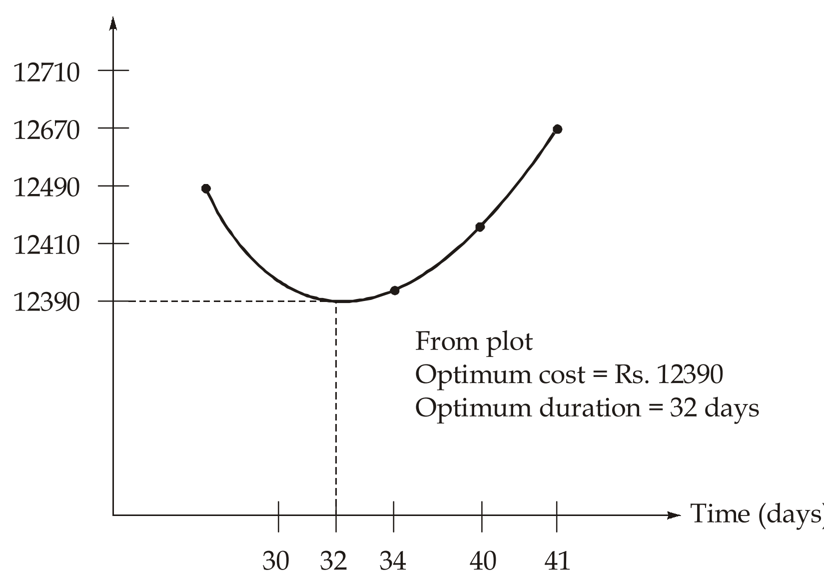
$$\text{Total direct cost} = 9190 + 2 \times (50 + 100) = \text{Rs. } 9490$$

$$\text{Total indirect cost} = 30 \times 100 = \text{Rs. } 3000$$

$$\text{Total project cost} = 9490 + 3000 = \text{Rs. } 12490$$

Further crashing will not be beneficial as crash slope will be more than indirect cost.

Plotting total cost vs time cost (Rs.)



4. (b) Solution:

Given data

$$\text{Diameter of column, } D = 450 \text{ mm}$$

$$\text{Grade of concrete, } f_{ck} = 20 \text{ N/mm}^2$$

$$\text{Grade of steel, } f_y = 250 \text{ N/mm}^2$$
$$\text{Factored axial load, } P_u = 1200 \times 10^3 \text{ N}$$
$$\text{Factored moment, } M_u = 100 \times 10^6 \text{ N-mm}$$
$$\text{Unsupported length, } L = 3000 \text{ mm}$$

Effective length for the given end conditions is

$$l_{eff} = 0.65 \times L$$
$$l_{eff} = 0.65 \times 3000 = 1950 \text{ mm}$$

Slenderness ratio is

$$\lambda = \frac{l_{eff}}{D} = \frac{1950}{450} = 4.333$$

Since $\lambda < 12$, the column is a short column.

For helical columns, the strength is taken as 1.05 times that of tied columns.

Adjusted axial load parameter is

$$\frac{P_u}{1.05 \times f_{ck} \times D^2} = \frac{1200 \times 1000}{1.05 \times 20 \times 450^2} = 0.282$$

Adjusted moment parameter is

$$\frac{M_u}{1.05 \times f_{ck} \times D^3} = \frac{100 \times 10^6}{1.05 \times 20 \times 450^3} = 0.052$$

From design charts for $f_y = 250 \text{ N/mm}^2$ and $d'/D = 0.1$,

$$\frac{p}{f_{ck}} = 0.065$$

Percentage of steel is

$$p = 0.065 \times 20 = 1.3\%$$

Area of longitudinal steel is,

$$A_{sc} = \frac{1.3 \times \pi \times 450 \times 450}{100 \times 4} = 2067.56 \text{ mm}^2$$

Using 20 mm diameter bars,

$$\text{Number of bars} = \frac{2067.56}{314.159} = 6.581$$

Provide 8 bars of 20 mm diameter

$$A_{sc,\ provided} = 8 \times \frac{\pi}{4}(20)^2\ 2513.274 \text{ mm}^2$$

clear cover = 45 mm

Core diameter is $D_c = 450 - 2 \times 45 = 360$ mm

Area of core is $A_c = \dfrac{\pi \times 360^2}{4} = 101787.602 \text{ mm}^2$

Gross area is $A_g = \dfrac{\pi \times 450^2}{4} = 159043.128 \text{ mm}^2$

Using 8 mm diameter helical bar,

Area of helix bar $a_\phi = 50.265 \text{ mm}^2$

Volume ratio condition is

$$\frac{V_h}{V_c} \geq 0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y}$$

$$\frac{V_h}{V_c} \geq 0.36\left(\frac{159043.128}{101787.602} - 1\right) \times \frac{20}{250} = 0.0162$$

Pitch of helix is $s = \dfrac{4 \times a_\phi \times (D_c - \phi_h)}{D_c^2 \times 0.014}$

$$\Rightarrow \quad \frac{\pi(D_c - \phi_h) \times \frac{\pi}{4}(8)^2}{\frac{\pi}{4} D_c^2 \times Pitch} \geq 0.0162$$

$$\Rightarrow \quad \frac{\pi(360 - 8) \times 64}{360^2 \times p} \geq 0.0162$$

$$\Rightarrow \quad p \leq 33.709 \text{ mm}$$

$$s = \frac{4 \times 50.265 \times (360 - 8)}{360^2 \times 0.0162} = 33.709 \text{ mm}$$

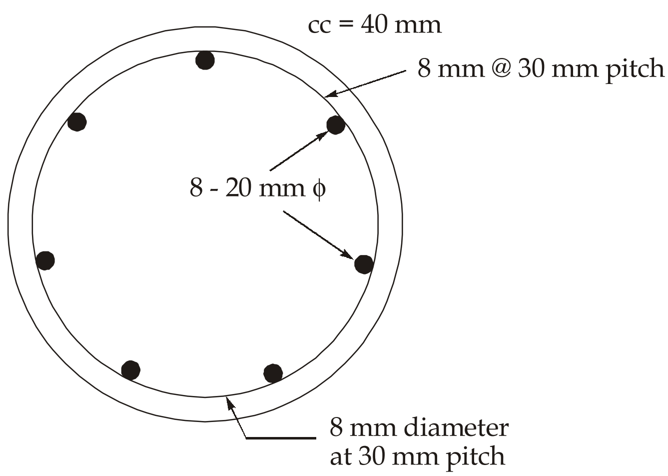
Pitch limits are $s \leq 75$ mm

$$s \leq \frac{D_c}{6} = 61.667 \text{ mm}$$

$$s \geq 25 \text{ mm}$$

$$s \geq 3 \times \phi_h = 24 \text{ mm}$$

Provide 8 mm diameter helical reinforcement at 30 mm pitch.



4. (c) Solution:

Given data

Cross-sectional area, $A = 2000 \text{ mm}^2$

Modulus of elasticity, $E = 2 \times 10^5 \text{ N/mm}^2$

Temperature rise, $\Delta T = 25°\text{C}$

Coefficient of linear expansion, $\alpha = 1.1 \times 10^{-5}/°\text{C}$

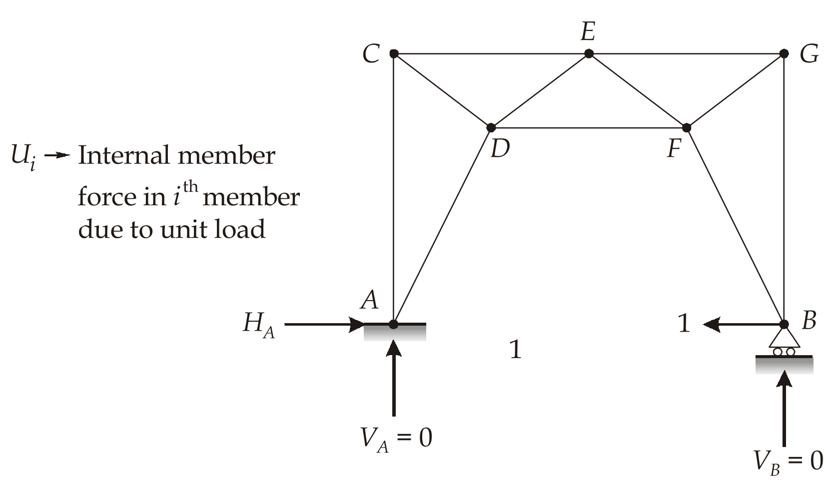
Lack of fit in DF, $\delta_L = -2 \text{ mm} = -2 \times 10^{-3} \text{ m}$

The horizontal thrust is given by:

$$H = \frac{\sum U_i(L_i \alpha \Delta T) + \sum U_i \delta_{L,i}}{\sum \frac{U_i^2 L_i}{AE}}$$

Apply horizontal unit load at B

Calculation of U_i :



From vertical equilibrium, $V_A + V_B = 0$

$\Sigma F_x = 0 \quad \Rightarrow \quad H_A = 1\,(\rightarrow)$

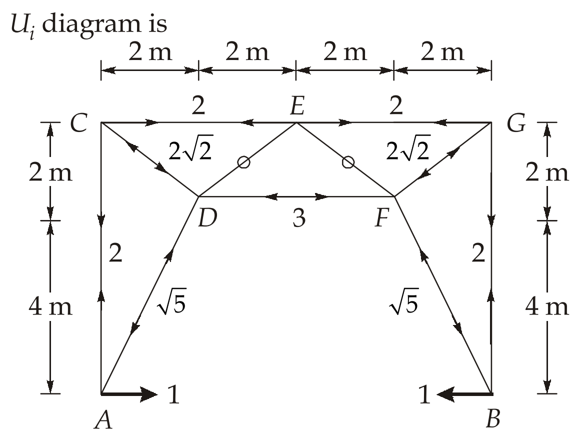
Taking moment about A, $\Sigma M_A = 0$

$$V_B \times 8 = 0$$

$$V_B = 0$$

$\therefore \quad V_A = 0$

U_i diagram is



Calculations are tabulated below:

Member	$l_i\,(m)$	U_i	$L_i\alpha\Delta T\,(\text{m})$	$U_i^2 L_i\,(\text{m})$	$U_i(L_i\alpha\Delta T)(\text{m})$	$U_i\delta_{L,i}\,(\text{m})$
AC	6	2	16.5×10^{-4}	24	33×10^{-4}	0
AD	$2\sqrt{5}$	$-\sqrt{5}$	12.3×10^{-4}	22.36	-27.503×10^{-4}	0
BF	$2\sqrt{5}$	$-\sqrt{5}$	12.3×10^{-4}	22.36	-27.503×10^{-4}	0
BG	6	2	16.5×10^{-4}	24	33×10^{-4}	0
DF	4	-3	11×10^{-4}	36	-33×10^{-4}	60×10^{-4}
ED	$2\sqrt{2}$	0	7.78×10^{-4}	0	0	0
EF	$2\sqrt{2}$	0	7.78×10^{-4}	0	0	0
CD	$2\sqrt{2}$	$-2\sqrt{2}$	7.78×10^{-4}	22.62	-22×10^{-4}	0
FG	$2\sqrt{2}$	$-2\sqrt{2}$	7.78×10^{-4}	22.62	-22×10^{-4}	0
CE	4	2	11×10^{-4}	16	22×10^{-4}	0
EG	4	2	11×10^{-4}	16	22×10^{-4}	0

$$\sum U_i^2 L_i = 205.96 \text{ m}$$

Here, $\sum U_i(L_i \alpha \Delta T) = -22.006 \times 10^{-4}$ m

$\sum U_i \delta_{L,i} = 60 \times 10^{-4}$ m

Horizontal thrust, $$H = \frac{(-22.006\times10^{-4})+(60\times10^{-4})}{\frac{205.96}{2000\times200000}}$$

$\Rightarrow$ $H = 7378.908$ N

$\Rightarrow$ $H = 7.379$ kN

The horizontal thrust developed at A and B is 7.379 kN.

Section B

5. (a) (i) Solution:

Seasoning of timber is the process of reducing the moisture content (sap) in freshly felled trees to a specific limit. Since "green" timber contains a high percentage of water, it is prone to shrinking, warping, and decay. The primary objective of seasoning is to make the timber stable, increase its strength, and make it resistant to fungal attacks.

Preservation of timber refers to the treatment of timber with specific chemicals (preservatives) to increase its life and durability. While seasoning removes moisture, preservation provides a protective barrier against external destructive agents like termites, wood borers, and fungi.

Methods of Applying Preservatives

- **Brushing:** Applying the preservative to the surface using a brush.
- **Spraying:** Using a spray gun to coat the timber surface.
- **Dipping and Steeping:** Immersing the timber in a tank filled with preservative for a few hours or days.
- **Charring:** Burning the exterior surface of the timber slightly to create a protective charcoal layer (often used for fence posts).
- **Hot and Cold Process:** Submerging timber in a hot preservative bath followed by a cold bath to create a vacuum that pulls the chemical deeper.
- **Pressure Process:** Using an airtight cylinder to force preservatives into the wood fibers under high pressure (most effective method).

5. (a) (ii) Solution:

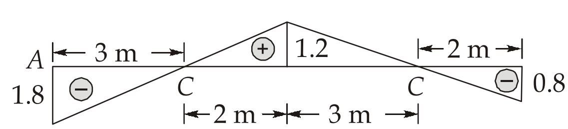
Defects in Concrete

Defects in concrete can occur due to poor mix design, faulty construction practices, or environmental factors. The most common defects include:

- **Cracking:** This is the most common defect. It can occur due to plastic shrinkage (water evaporating too fast), thermal expansion and contraction, or structural overloading.
- **Segregation:** This occurs when the heavy coarse aggregates separate from the fine materials (sand and cement paste). It usually happens when concrete is dropped from too great a height or over-vibrated, resulting in a non-homogeneous mix.
- **Bleeding:** This is a form of segregation where water rises to the surface of freshly placed concrete. It creates a weak, porous layer on the top (known as laitance) and reduces the bond between the concrete and reinforcement.
- **Honeycombing:** These are hollow spaces or voids left on the concrete surface or inside the mass, resembling a bee's honeycomb. It occurs when the mortar fails to fill the spaces between coarse aggregates, often due to insufficient vibration or a very dry mix.
- **Efflorescence:** A white crystalline deposit on the surface of the concrete. It is caused by soluble salts in the concrete ingredients reacting with water and migrating to the surface as the water evaporates.
- **Spalling:** The breaking or flaking of the concrete surface, often exposing the steel reinforcement. This is typically caused by the corrosion of internal steel bars, which expand and push the concrete outward.

5. (b) Solution:

ILD for bending moment at E



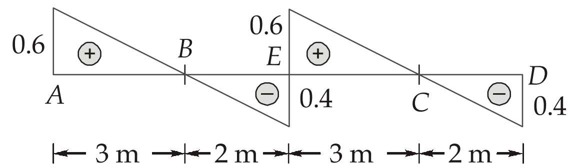
Maximum sagging moment at E,

$$(BM)_{max} = 15\left[\frac{1}{2}\times 1.2\times 5\right] = 45 \text{ kNm}$$

Maximum hogging moment at E,

$$(BM)_{min} = -15\left[\frac{1}{2}\times 2\times 0.8+\frac{1}{2}\times 3\times 1.8\right] = -52.5 \text{ kNm}$$

ILD for shear force at *E*



Maximum positive *SF* at *E*,

$$(SF)_{max} = 15\left[\frac{1}{2}\times 0.6\times 3+\frac{1}{2}\times 0.6\times 3\right] = 27 \text{ kN}$$

Maximum negative *SF* at *E*,

$$(BM)_{min} = -15\left[\frac{1}{2}\times 0.4\times 2+\frac{1}{2}\times 2\times 0.4\right] = -12 \text{ kN}$$

5. (c) Solution:

Flange width of ISHB 300 = 250 mm

Let thickness of weld = *t* mm

Upper weld will be in tension

Lower weld will be in compression

$$\text{Strength of weld } (f_{d,w}) = \frac{410}{\sqrt{3}\times 1.5} = 157.8 \text{ N/mm}^2$$

$$\text{shear stress in weld } (f_v) = \frac{80\times 10^3}{250\times 2\times t} = \frac{160}{t} \text{ N/mm}^2$$

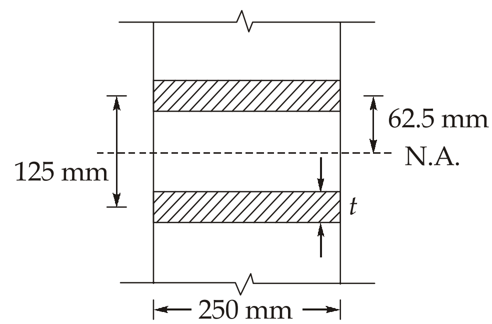
Bending stress due to bending moment, $(f_b) = \dfrac{My}{I}$

$$I = \left[\frac{250\times t^3}{12}+250\times t\times\left(\frac{125}{2}\right)^2\right]\times 2$$

$\Rightarrow$ $I = 1953125\, t \text{ mm}^4$ (By neglecting higher order of *t*)

then $$f_b = \left(\frac{80\times 1000\times 40\times 62.5}{953125t}\right) = \frac{102.40}{t}\text{N/mm}^2$$

then from IS 800:2007 $$f_r = \sqrt{f_b^2+3f_v^2} \le f_{dw}$$



$$\Rightarrow \qquad \sqrt{\left(\frac{102.40}{t}\right)^2 + 3\times\left(\frac{160}{t}\right)^2} \le 157.8$$

$$\Rightarrow \qquad \frac{295.442}{t} \le 157.8$$

$$\Rightarrow \qquad t \ge 1.872 \text{ mm}$$

$$\text{Size of weld, } s = \frac{1.872}{0.7} = 2.675 \text{ mm}$$

Providing 3 mm size of weld.

5. (d) Solution:

Unit load at Coordinate 1 ($P_1 = 1$)

Applying a unit downward vertical load at the free end:

- Vertical displacement at 1: $f_{11} = \dfrac{L^3}{3EI}$
- Rotation at 2: A downward load causes a clockwise rotation. Since coordinate 2 is defined as anticlockwise, the value is positive: $f_{21} = \dfrac{L^2}{2EI}$
- Axial displacement at 3: $f_{31} = 0$

Unit load at Coordinate 2 ($M_2 = 1$)

Applying a unit clockwise moment at the free end:

- Vertical displacement at 1: A clockwise moment causes a downward displacement. Since coordinate 1 is downward, the value is positive: $f_{12} = \dfrac{L^2}{2EI}$
- Rotation at 2: $f_{22} = \dfrac{L}{EI}$
- Axial displacement at 3: $f_{32} = 0$

Unit load at Coordinate 3 ($P_3 = 1$)

Applying a unit axial horizontal load (to the right) at the free end:

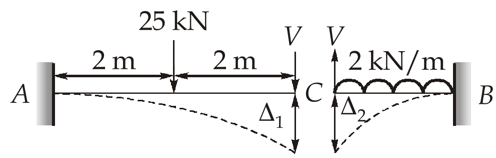
- Vertical displacement at 1: $f_{13} = 0$
- Rotation at 2: $f_{23} = 0$
- Axial displacement at 3: $f_{33} = \dfrac{L}{AE}$

The flexibility matrix is assembled as follows:

$$[f] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$\Rightarrow \qquad [f] = \begin{bmatrix} \dfrac{L^3}{3EI} & \dfrac{L^2}{2EI} & 0 \\ \dfrac{L^2}{2EI} & \dfrac{L}{EI} & 0 \\ 0 & 0 & \dfrac{L}{AE} \end{bmatrix}$$

5. (e) Solution:

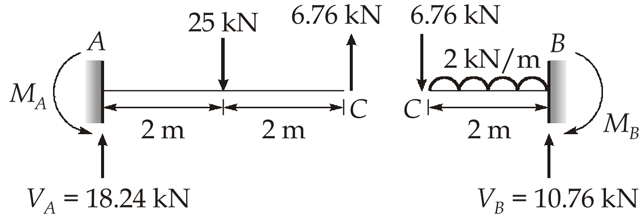


Using compatibility equation

$$\Delta_1 = \Delta_2$$

$$\Rightarrow \quad \frac{25\times 2^3}{3EI} + \frac{25\times 2^2}{2EI}\times 2 + \frac{V\times 4^3}{3EI} = \frac{2\times 2^4}{8EI} - \frac{V\times 2^3}{3EI}$$

$$V = -6.76 \text{ kN}$$

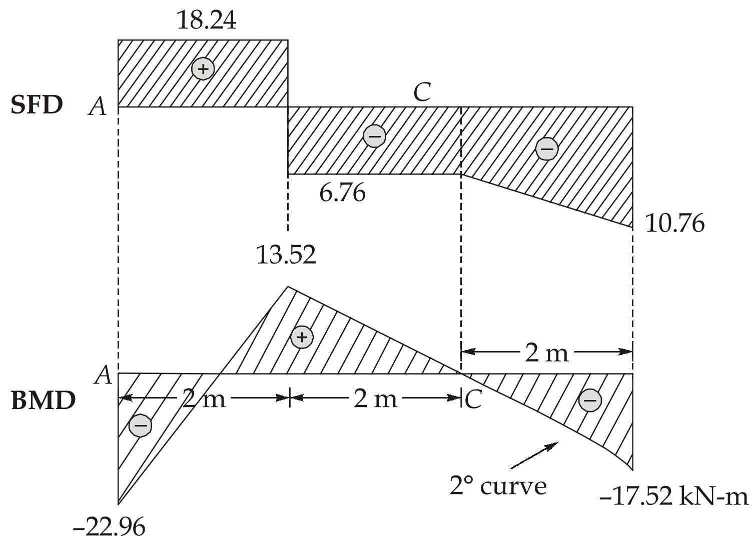


$$V_A = 25 - 6.76 = 18.24 \text{ kN } (\uparrow)$$

$$M_A = 25 \times 2 - 6.76 \times 4 = 22.96 \text{ kN-m (Hogging)}$$

Now,

$$V_B = 6.76 + 4 \times 4 = 10.76 \text{ kN } (\uparrow)$$
$$M_B = 6.76 \times 2 + 2 \times 2 \times 1 = 17.52 \text{ kN-m (Hogging)}$$



6. (a) Solution:

Given data

Allowable tensile stress, σ_t = 45 MPa

Allowable compressive stress, σ_c = 140 MPa

Flange width, b_f = 200 mm

Flange thickness, t_f = 50 mm

Web width, b_w = 50 mm

Web height, h_w = 125 mm

Total depth, H = 175 mm

Position of neutral axis from bottom,

$$\bar{y} = \frac{(10000 \times 25) + (6250 \times 112.5)}{10000 + 6250}$$

$$\bar{y} = 58.654 \text{ mm}$$

Distance of extreme fibres,

$$y_{bottom} = 58.654 \text{ mm}, \; y_{top} = 175 - 58.654 = 116.346 \text{ mm}$$

Moment of inertia about neutral axis,

$$I = \left[\frac{200\times 50^3}{12} + 10000(58.654-25)^2\right] + \left[\frac{50\times 125^3}{12} + 6250(112.5-58.654)^2\right]$$

$$I = 39668469.55 \text{ mm}^4$$

Reactions due to symmetry, $R_B = R_D = 0.5\,P$

Bending moment at support B,

$$M_B = -\left(\frac{P}{4}\times 1\right) = -0.25P \text{ N-m} = -250P \text{ N-mm}$$

Bending moment at center C,

$$M_C = \left(-\frac{P}{4}\times 2.5\right) + (0.5P\times 1.5) = 0.125P \text{ N-m} = 125P \text{ N-mm}$$

At support B, tension at top fibre,

$$45 = \frac{250P\times 116.346}{39668469.55}$$

$$P = 61371.465 \text{ N}$$

At support B, compression at bottom fibre,

$$140 = \frac{250P\times 58.654}{39668469.55}$$

$$P = 378735.345 \text{ N}$$

At center C, tension at bottom fibre,

$$45 = \frac{125P\times 58.654}{39668469.55}$$

$$P = 243472.722 \text{ N}$$

At center C, compression at top fibre,

$$140 = \frac{125P\times 116.346}{39668469.55}$$

$$P = 381866.896 \text{ N}$$

The allowable load is governed by the minimum value,

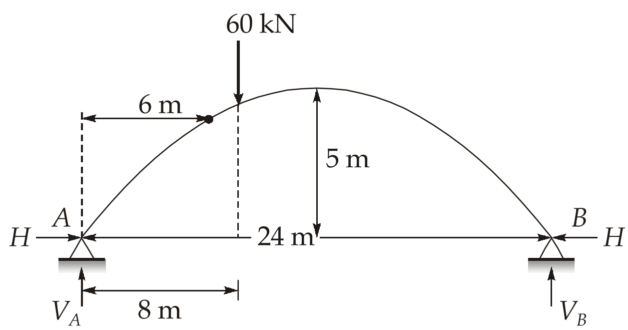
$$P = 61371.465 \text{ N} = 61.371 \text{ kN}$$

The maximum allowable value of P is 61.371 kN.

6. **(b) Solution:**

Given data

$$\text{Span}, L = 24 \text{ m}$$
$$\text{Rise}, h = 5 \text{ m}$$
$$\text{Point load}, W = 60 \text{ kN}$$



Taking moments about right support B,

$\Sigma M_B = 0 \quad \Rightarrow \quad V_A \times 24 - 60 \times (24 - 8) = 0$

$$V_A = 40 \text{ kN } (\uparrow)$$
$$V_B = 60 - 40 = 20 \text{ kN } (\uparrow)$$

For a two-hinged arch, horizontal thrust is given by

$$H = \frac{\int M_b y \, ds / EI}{\int y^2 \, ds / EI}$$

Since $I = I_0 \sec\theta$ and $ds = dx \sec\theta$,

$$\frac{ds}{I} = \frac{dx}{I_0}$$

Equation of parabola is

$$y = \frac{4hx(L-x)}{L^2} = \frac{4 \times 5 \times x(24-x)}{24^2} = \frac{5(24x - x^2)}{144}$$

Beam moment is

For $0 < x < 8$, $M_b = 40x$

For $8 < x < 24$, $M_b = 480 - 20x$

Numerator is

$$\int_0^{24} M_b y \, dx = \int_0^8 (40x)\left[\frac{5}{144}(24x - x^2)\right]dx + \int_8^{24} (480 - 20x)\left[\frac{5}{144}(24x - x^2)\right]dx$$

$$= 4266.667 + 11377.778 = 15644.44$$

Denominator is

$$\int_0^{24} y^2\,dx = \int_0^{24}\left[\frac{5}{144}(24x - x^2)\right]^2 dx = 320$$

Horizontal thrust is

$$H = \frac{15644.44}{320} = 48.889 \text{ kN}$$

Slope at section is

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$$

$\Rightarrow$ $$\tan\theta = \frac{4\times 5}{24^2}(24 - 2\times 6) = 0.417$$

$\Rightarrow$ $$\theta = 22.64^\circ \text{ (↻ from horizontal)}$$

Vertical shear at section is

$$V_x = V_A = 40 \text{ kN}$$

Normal thrust is

$$N = V_x \sin\theta + H\cos\theta$$

$$N = 40\sin(22.64^\circ) + 48.889\cos(22.64^\circ) = 60.519 \text{ kN}$$

Radial shear is

$$S = V_x\cos\theta - H\sin\theta$$

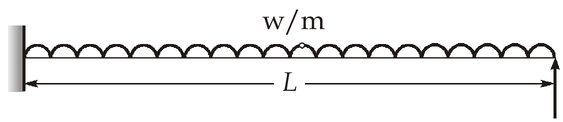
$$S = 40\cos(22.64^\circ) - 48.889\sin(22.64^\circ) = 18.098 \text{ kN}$$

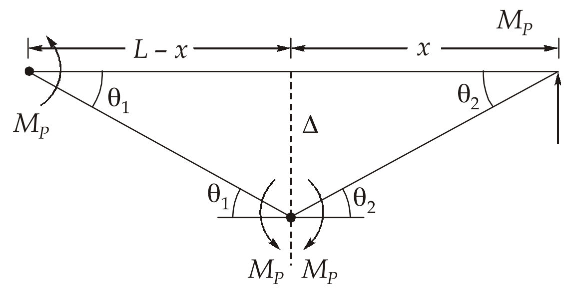
6. (c) (i) Solution:

A propped cantilever is statically indeterminate to degree one, hence number of plastic hinges required for collapse is two.

One plastic hinge forms at the fixed support where the bending moment is maximum. The second hinge forms in the span at a distance x from the propped end where the positive bending moment is maximum.

Let rotation at fixed end be θ_1 and rotation at propped end be θ_2.





From compatibility of deformation,

$$\delta = x\theta_2 = (L - x)\theta_1$$

$$\Rightarrow \qquad \theta_2 = \frac{(L-x)\theta_1}{x}$$

External work done by uniformly distributed load is equal to load multiplied by area under displacement diagram,

$$W_e = w \times \frac{1}{2} \times L \times \delta$$

$$\Rightarrow \qquad W_e = \frac{1}{2} wL(L-x)\theta_1$$

Internal work done is sum of work at plastic hinges,

$$W_i = M_p\theta_1 + M_p(\theta_1 + \theta_2)$$

$$\Rightarrow \qquad W_i = M_p\theta_1 + M_p\left(\theta_1 + \frac{(L-x)\theta_1}{x}\right)$$

$$\Rightarrow \qquad W_i = M_p\theta_1\left(2 + \frac{L-x}{x}\right)$$

$$\Rightarrow \qquad W_i = M_p\theta_1\left(\frac{x+L}{x}\right)$$

Equating external and internal work,

$$\frac{1}{2} wL(L-x)\theta_1 = M_p\theta_1\left(\frac{x+L}{x}\right)$$

$$\Rightarrow \qquad w = \frac{2M_p(L+x)}{Lx(L-x)}$$

For collapse load, differentiate with respect to x and equate to zero,

$$\frac{d}{dx}\left(\frac{L+x}{Lx-x^2}\right) = 0$$

$$\Rightarrow \quad (Lx - x^2)(1) - (L + x)(L - 2x) = 0$$

$$\Rightarrow \quad Lx - x^2 - (L^2 + 2Lx + Lx - 2x^2) = 0$$

$$\Rightarrow \quad x^2 + 2Lx - L^2 = 0$$

Solving,

$$x = L(\sqrt{2}-1) = 0.414L$$

Thus, plastic hinges form at fixed support and at 0.414 L from the propped end.

Substituting x = 0.414 L in load equation,

$$w_c = \frac{2M_p(L+0.414L)}{L(0.414L)(L-0.414L)}$$

$$\Rightarrow \quad w_c = \frac{11.657M_p}{L^2}$$

6. (c) (ii) Solution:

The design of a brick masonry wall is governed by several critical factors that determine its load-bearing capacity and long-term durability. These factors include:

- **Material Properties:** The compressive strength and overall quality of both the masonry units (bricks) and the mortar type used.
- **Execution Quality:** The standard of workmanship during construction, which ensures proper alignment and joint filling.
- **Bonding Patterns:** The specific arrangement of bricks (e.g., English bond, Flemish bond) which affects the monolithic action of the wall.
- **Geometric Constraints:** The unsupported height and length of the wall, which directly impact its susceptibility to buckling.
- **Loading Conditions:** The magnitude and combination of external loads (dead, live, wind, or seismic loads).
- The eccentricity of applied loads, which introduces bending stresses.
- **Architectural Features:** The placement, frequency, and dimensions of openings (windows and doors) that create weak points in the masonry.
- **Lateral Support:** The presence and location of intersecting cross-walls, floors, and roof diaphragms that provide lateral stability.

7. (a) Solution:

Given: Cantilever projection,

$$L = 2.4 \text{ m}$$

Materials used: M20, Fe415

$$f_{ck} = 20 \text{ N/mm}^2, \quad f_y = 415 \text{ N/mm}^2.$$

Depth of slab:

$$\frac{\text{Span}}{\text{Overall depth}} = 10 \qquad \text{(For cantilever slab)}$$

$$\Rightarrow \quad \text{Overall depth} = \frac{2.4 \times 1000}{10} = 240 \text{ mm}$$

$$\text{Nominal cover} = 20 \text{ mm}$$

$$\text{Diameter of bar used} = 10 \text{ mm}$$

$$\therefore \quad \text{Effective depth,} \quad d = 240 - 20 - \frac{10}{2} = 215 \text{ mm}$$

Let us provide maximum depth of slab as 240 mm at support and gradually reduce the depth to 120 mm at free end.

Load calculation:

$$\text{Self weight of slab} = 0.5\,(0.24 + 0.12) \times 25 = 4.5 \text{ kN/m}^2$$

$$\text{Live load} = 2 \text{ kN/m}^2$$

$$\text{Load due to finishes} = 1.5 \text{ kN/m}^2$$

$$\text{Total working load} = 8 \text{ kN/m}^2$$

$$\therefore \quad \text{Factored load } (w_u) = 1.5 \times 8 = 12 \text{ kN/m}^2$$

Check for depth: $$BM_u = \frac{w_u \times L^2}{2} = \frac{12 \times (2.4)^2}{2} = 34.56 \text{ kN-m/m}$$

For Fe 415 steel, $$BM_{u,\,\text{lim}} = 0.138 f_{ck} bd^2$$

$$\therefore \quad BM_u = BM_{u,\,\text{lim}}$$

$$\Rightarrow \quad 34.56 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$\Rightarrow \quad d = 111.9 \text{ mm} < 215 \text{ mm}.$$

Hence, the effective depth provided is sufficient to resist the design moment.

Also, provided depth is more than that required for balanced section and hence section is under-reinforced.

Reinforcement calculation:

$$A_{st} = \frac{0.5\, f_{ck}\, bd}{f_y}\left(1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck}\, bd^2}}\right)$$

$$= \frac{0.5 \times 20 \times 1000 \times 215}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 34.56 \times 10^6}{20 \times 1000 \times 215^2}}\right)$$

$$= 466.43 \text{ mm}^2$$

$\therefore$ Spacing of 10 mm ϕ bars $= \dfrac{1000 \times \frac{\pi}{4}(10)^2}{466.43} = 168.385 \text{ mm c/c}$

$\therefore$ Provide 10 mm ϕ bars @ 160 mm c/c

$$(A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4}(10)^2}{160} = 490.87 \text{ mm}^2 > (A_{st})_{\text{req.}}$$

Distribution reinforcement: $A_{st} = 0.12\%$ of A_g (For Fe 415 steel)

$$= \frac{0.12}{100} \times 1000 \times 240 = 288 \text{ mm}^2$$

Let us provide 10 mm ϕ bars as distribution bars

$\therefore$ Spacing $= \dfrac{1000 \times \frac{\pi}{4}(10)^2}{288} = 272.7 \text{ mm c/c}$

$\therefore$ Provide 10 mm ϕ bars @ 270 mm c/c.

$$(A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4}(10)^2}{270} = 290.89 \text{ mm}^2$$

Anchorage length: $L_d = \dfrac{(0.87 \cdot f_y)\phi}{4\,\tau_{bd}} = \dfrac{0.87 \times 415 \times 10}{4 \times 1.2 \times 1.6} = 470 \text{ mm}$

Main tension bars are extended into the support to a minimum length of 470 mm including anchorage value of hooks and 90° bends.

Check for deflection control:

$$\left(\frac{L}{d}\right)_{\max} = \left(\frac{L}{d}\right)_{\text{Basic}} \times k_t \times k_c \times k_f$$

$$P_t(\%) = \frac{A_{st}}{bd} \times 100 = \frac{490.87}{1000 \times 215} \times 100 = 0.23\%$$

$\therefore$ $f_s = 0.58 \times 415 \times \dfrac{466.43}{490.87} = 228.72 \text{ N/mm}^2$

From graph given modification factor for tension reinforcement)

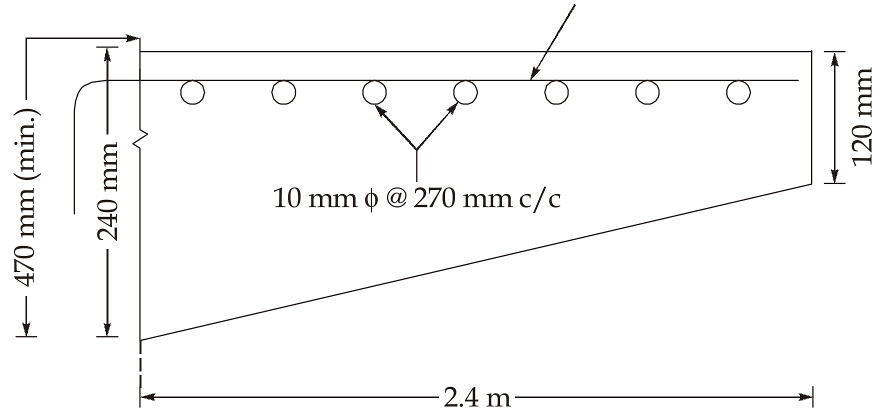
$$k_t = 1.8$$

$$\therefore \quad \left(\frac{L}{d}\right)_{max} = 7 \times 1.8 \times 1 \times 1 = 12.6$$

$$\left(\frac{L}{d}\right)_{provided} = \frac{2400}{215} = 11.16 < 12.6$$

Hence, the cantilever slab satisfies the deflection limit.

Reinforcement details:



7. **(b) Solution:**

Distribution Factors.

Joint	Member	Stiffness	Total Stiffness	Distribution factors
B	BA	$\frac{4EI}{4} = EI$	2EI	$\frac{1}{2}$
	BC	$\frac{4EI}{4} = EI$		$\frac{1}{2}$

Fixed End Moments:

$$\overline{M}_{ab} = \frac{20 \times 1 \times 3^2}{4^2} = -11.25 \text{ kNm}$$

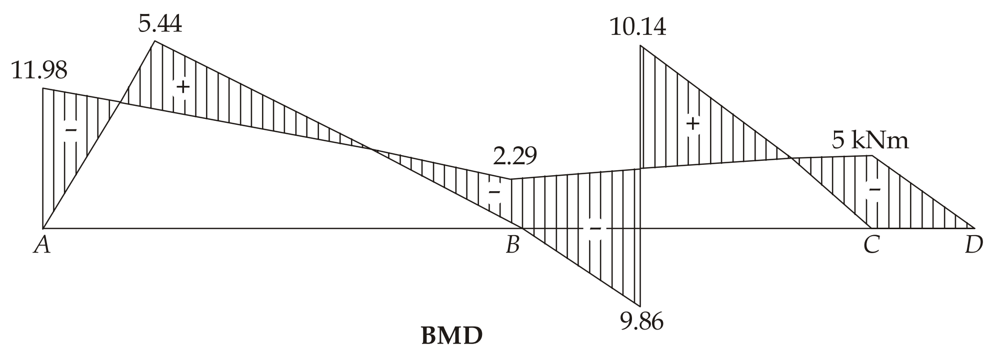
$$\overline{M}_{ba} = +\frac{20 \times 1^2 \times 3}{4^2} = +3.75 \text{ kNm}$$

$$\overline{M}_{bc} = +\frac{20 \times 2}{3^2}(3 \times 1 - 3) = 0$$

$$\overline{M}_{cb} = +\frac{20 \times 1}{3^2}(3 \times 2 - 3) = +6.67 \text{ kNm}$$

$$\overline{M}_{cd} = -5 \text{ kNm}$$

A		B (1/2)	B (1/2)	C	D
– 11.25	+ 3.75	+ 2.50 0 – 3.34	+ 5.00 + 6.57 – 6.67	– 5.00	
– 11.25	+ 3.75 – 1.46	– 0.84 – 1.45	+ 5.00	– 5.00	
– 0.73					
– 11.98	+ 2.29	– 2.29	– 5.00	– 5.00	



BMD

Calculation of support reactions

$$\text{B.M. at } B = V_a \times 4 - 20 \times 3 - 11.98 = -2.29$$

$$\Rightarrow \quad V_a = 17.42 \text{ kN}$$

$$\text{B.M. at } B = V_c \times 3 - 50 \times 4 - 20 = -2.29$$

$$V_c = 12.57 \text{ kN}$$

$$V_b = [20 + 5] - [17.42 + 12.57] = -4.99 \text{ kN} \downarrow$$

7. (c) (i) Solution:

$$\text{Mix proportion} = 1: 1.5: 3$$

$$\text{Water cement ratio} = 0.5$$

$$\text{Entrained air} = 1.5\% = 0.015 \text{ m}^3$$

$$\text{Volume of concrete, } V = 1 \text{ m}^3$$

$$\text{Unit weight of water, } \gamma_w = 9.81 \text{ kN/m}^3$$

Let volume of cement be V_c.

$$\text{Volume of sand} = 1.5V_c$$

Volume of coarse aggregate $= 3V_c$

Weight of cement $W_c = 14.5V_c$

Weight of sand $W_s = 16 \times 1.5V_c = 24V_c$

Weight of coarse aggregate $W_a = 15.5 \times 3V_c = 46.5V_c$

Weight of water $W_w = 0.5 \times 14.5V_c = 7.25V_c$

Using absolute volume relation,

$$1 = \frac{14.5V_c}{3.15 \times 9.81} + \frac{24V_c}{2.65 \times 9.81} + \frac{46.5V_c}{2.6 \times 9.81} + \frac{7.25V_c}{9.81} + 0.015$$

$$V_c = 0.249 \text{ m}^3$$

Weight of cement $W_c = 14.5 \times 0.249 = 3.612$ kN

Weight of sand $W_s = 24 \times 0.249 = 5.978$ kN

Weight of coarse aggregate $W_a = 46.5 \times 0.249 = 11.582$ kN

Weight of water $W_w = 7.25 \times 0.249 = 1.806$ kN

Converting into kg,

$$\text{Cement} = \frac{3.6116 \times 1000}{9.81} = 368.159 \text{ kg}$$

$$\text{Sand} = \frac{5.9779 \times 1000}{9.81} = 609.366 \text{ kg}$$

$$\text{Coarse aggregate} = \frac{11.5822 \times 1000}{9.81} = 1180.647 \text{ kg}$$

$$\text{Water} = \frac{1.8058 \times 1000}{9.81} = 184.079 \text{ kg}$$

7. (c) (ii) Solution:

The design of lacing systems for built-up compression members is governed by Clause 7.6 of IS 800 : 2007. The key recommendations are as follows:

General Requirements

- Lacing bars should be inclined at an angle θ between 40° and 70° to the axis of the built-up member.
- The lacing system should be uniform throughout the length of the column.
- Single lacing systems on opposite faces should preferably be mirrors of each other to avoid torsion.

Slenderness Ratio Limits

- The maximum slenderness ratio $\frac{l_e}{r}$ of the lacing bars shall not exceed 145.
- The effective length l_e for single lacing is the length between inner end rivets/welds. For double lacing or welded lacing, it is reduced to 0.7 times this length.
- To ensure the individual components do not fail before the whole member, the slenderness ratio of the individual main component between lacing connections should not exceed 50 or 0.7 times the maximum slenderness ratio of the member as a whole.

Design Forces

- **Transverse Shear:** The lacing system must be designed to resist a transverse shear force V equal to 2.5% of the maximum axial load on the column.
- If the member is subjected to external bending moments or lateral loads, the shear resulting from these must be added to the 2.5% value.
- This shear force V is divided equally among the parallel planes of the lacing system.

Thickness and Width

- **Minimum Thickness:** For single lacing, the thickness t should not be less than 1/40 of the effective length. For double lacing, it should not be less than 1/60 of the effective length.
- **Minimum Width:** In riveted/bolted construction, the minimum width of lacing bars should be approximately 3 times the nominal diameter of the end fastener.

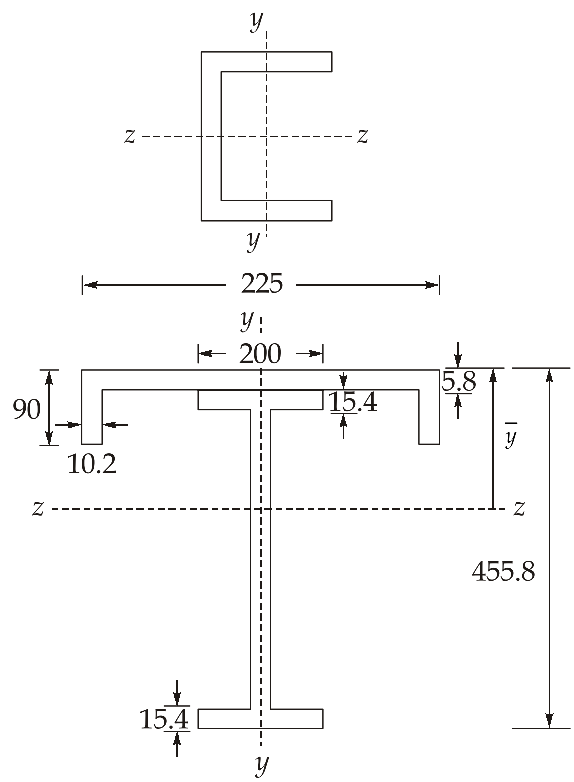
End Connections

- Lacing bars shall be connected to the main members by welding or by using at least one fastener at each end.
- **Tie Plates:** Lacing systems must be provided with tie plates (also known as batten plates) at the ends of the compression member and at points where the lacing system is interrupted.

8. (a) Solution:

For ISLC 225

$$I_{zz} = 2547.9 \times 10^4 \text{ mm}^4$$
$$I_{yy} = 209.5 \times 10^4 \text{ mm}^4$$



All dimension (in mm)

Depth of centroidal axis from the upper edge:

$$\bar{y} = \frac{3053 \times 24.6 + 10115(5.8 + 225)}{3053 + 10115} \simeq 183 \text{ mm.}$$

Moment of inertia of the composite section about the neutral axis

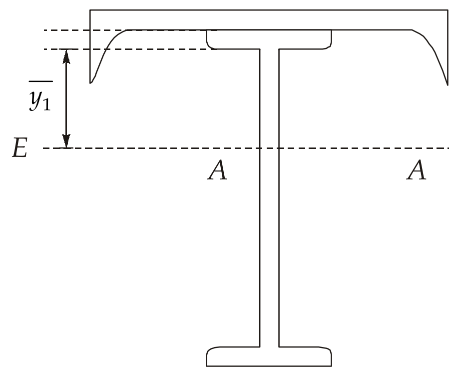
$$I_{NA} = \Sigma I_{self} + \Sigma A_i \left(\bar{y_i} - \bar{y}\right)^2$$
$$= 209.5 \times 10^4 + 3053\,(24.6 - 183)^2$$
$$+ 35057.6 \times 10^4 + 10115\,(230.8 - 183)^2$$
$$= 45.24 \times 10^7 \text{ mm}^4.$$

$\therefore$ Section modulus,

$$Z_{ez} = \frac{I_{NA}}{y_{max}} = \frac{45.24 \times 10^7}{(450 + 5.8 - 183)} = 1658.36 \times 10^3 \text{ mm}^3$$

Plastic modulus of section:

Equal area axis:



$$3053 + 200 \times 15.4 + \bar{y}_1 \times 9.2 = 200 \times 15.4 + (450 - 2 \times 15.4 - \bar{y}_1) \times 9.2$$

$$\Rightarrow \quad \bar{y}_1 = 43.68 \text{ mm}$$

(From lower surface of top flange of I-section)

Plastic modulus of the section about equal area axis,

$$Z_{pz_1} = 225 \times 5.8\left(43.68 + 15.4 + \frac{5.8}{2}\right)$$

$$+ \left[2 \times (90 - 5.8) \times 10.2 \times \left(43.68 + 15.4 - \frac{90 - 5.8}{2}\right)\right]$$

$$+ \ 200 \times 15.4\left(43.68 + \frac{15.4}{2}\right) + 43.68 \times 9.2 \times \frac{43.68}{2}$$

$$= 80883.9 + 29166.2 + 55490.4 + 8776.54$$

$$= 174.317 \times 10^3 \text{ mm}^3.$$

Plastic modulus of the section below equal area axis

$$Z_{Pz_2} = 200 \times 15.4 \times \left(450 - 15.4 - 43.68 - \frac{15.4}{2}\right)$$

$$+ \frac{(450 - 2 \times 15.4 - 43.68)^2 \times 9.2}{2}$$

$$= 1828.987 \times 10^3 \text{ mm}^3$$

$$\therefore \quad Z_{pz} = 174.317 \times 10^3 + 1828.987 \times 10^3$$

$$= 2003.304 \times 10^3 \text{ mm}^3.$$

Plastic modulus of the compression flange about *yy*-axis.

$$Z_{pz,y} = \frac{200 \times 15.4 \times 200}{4} + \frac{2 \times (225 - 2 \times 10.2)^2 \times 5.8}{8} + 2 \times \left(10.2 \times 90 \times \frac{225 - 10.2}{2}\right)$$

$$= 411.885 \times 10^3 \text{ mm}^3.$$

Classification of section:

Outstand of flange of *I*-section

$$b = \frac{b_f}{2} = \frac{200}{2} = 100 \text{ mm}$$

$$\frac{b}{t_f} = \frac{100}{15.4} = 6.5 < 8.4\,\varepsilon \quad (\varepsilon = 1)$$

Outstand of flange of channel section

$$b = b_f - t_w = 90 - 5.8 = 84.2 \text{ mm.}$$

$$\frac{b}{t_f} = \frac{84.2}{10.2} = 8.25 < (8.4\,\varepsilon)$$

$$\frac{d}{t_w} \text{ of } I\text{-section} = \frac{h - 2t_f}{t_w} = \frac{450 - 2 \times 15.4}{9.2} = 45.565 < 84\,\varepsilon.$$

Hence, the entire section is plastic.

∴ Local moment capacity

$$M_{dz} = \beta_b Z_{pz} \frac{f_y}{\gamma_{m0}} \leq 1.2 Z_{ez} \frac{f_y}{\gamma_{m0}}$$

$$= 1.0 \times 2003.304 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}$$

$$\leq 1.2 \times 1658.36 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}$$

$$= 455.3 \text{ kNm} \leq 452.28 \text{ kNm}$$

∴ $$M_{dz} = 452.28 \text{ kNm} > 253.8 \text{ kNm}$$ (OK)

∴ $$Z_{ey}.\ (\text{Compression flange}) = \frac{3574.6 \times 10^4}{112.5} = 317.742 \times 10^3 \text{ mm}^3.$$

∴ $$M_{dy.f} = 1.0 \times 411.885 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}$$

$$\leq 1.2 \times 317.742 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}$$

$$= 93.61 \text{ kNm} \leq 86.65 \text{ kNm}$$

$$\therefore \quad M_{dy.f} = 86.65 \text{ kNm}$$

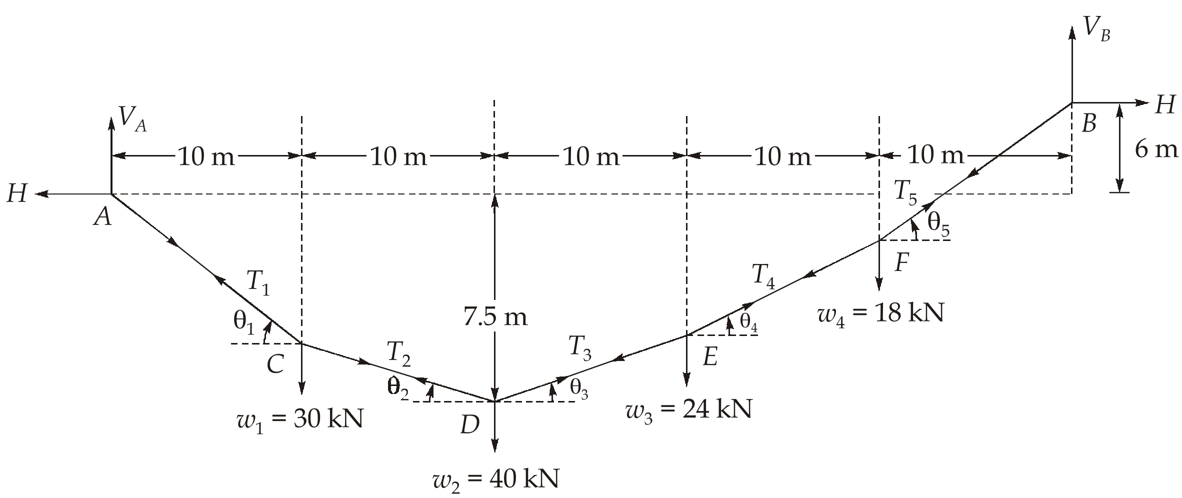
Check for biaxial bending

$$\frac{M_z}{M_{dz,f}} + \frac{M_y}{M_{dy,f}} = \frac{253.8}{452.28} + \frac{7.4}{86.65} = 0.64 < 1.0. \quad \text{(OK)}$$

8. (b) Solution:

Let tension in segments of cable are T_1, T_2, T_3, T_4 and T_5 and inclination are θ_1, θ_2, θ_3, θ_4 and θ_5 from horizontal as shown in free body diagram.

FBD of cable



$$\Sigma f_y = 0$$

$$\Rightarrow \quad V_A + V_B = 30 + 40 + 24 + 18$$

$$\Rightarrow \quad V_A + V_B = 112 \text{ kN} \quad \text{...(i)}$$

Taking moment about point D (Consider left side)

$$\Sigma M_D = 0$$

$$\Rightarrow \quad V_A \times 20 - H \times 7.5 - 30 \times 10 = 0$$

$$\Rightarrow \quad V_A = \left(\frac{7.5H}{20} + 15\right) \quad \text{...(ii)}$$

Taking moment about point D (Consider right side)

$$\Sigma M_D = 0$$

$$\Rightarrow \quad V_B \times 30 - H \times (7.5 + 6) - 24 \times 10 - 18 \times 20 = 0$$

$$\Rightarrow \quad V_B = \left(\frac{13.6H}{30} + 20\right) \quad \text{...(iii)}$$

On putting the value of V_A ann V_B in equation (i), we get

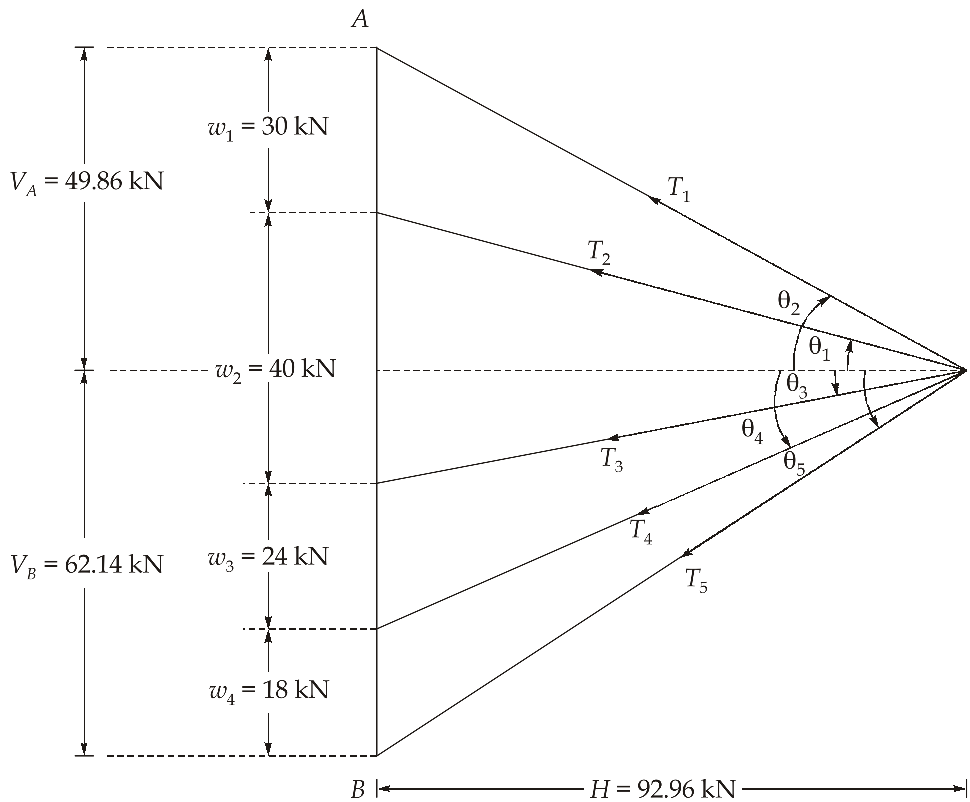
$$\left(\frac{7.5H}{20}+15\right)+\left(\frac{13.6H}{30}+20\right)=112$$

$$H = 92.96 \text{ kN}$$

Now:

From equation (ii), $V_A = \frac{7.5\times 92.96}{20}+15 = 49.86 \text{ kN}$

From equaiton (iii), $V_B = \frac{13.6\times 92.96}{30}+20 = 62.14 \text{ kN}$



(Funicular polygon)

From function diagram

Tension in each segment of cable

$$T_1 = \sqrt{V_A^2+H^2} = \sqrt{49.86^2+92.96^2} = 105.49 \text{ kN}$$

$$T_2 = \sqrt{(V_A-w_1)^2+H^2} = \sqrt{(49.86-30)^2+92.96^2} = 95.06 \text{ kN}$$

$$T_3 = \sqrt{(V_B-w_3-w_4)^2+H^2} = \sqrt{(62.14-24-18)^2+92.96^2} = 95.12 \text{ kN}$$

$$T_4 = \sqrt{(V_B - w_4)^2 + H^2} = \sqrt{(62.14 - 18)^2 + 92.96^2} = 102.91 \text{ kN}$$

$$T_5 = \sqrt{V_B^2 + H^2} = \sqrt{62.14^2 + 92.96^2} = 111.82 \text{ kN}$$

Inclination of each segment of cable from horizontal.

$$\tan\theta_1 = \frac{V_A}{H} \Rightarrow \theta_1 = \tan^{-1}\left(\frac{49.86}{92.96}\right) \Rightarrow \theta_1 = 28.207°$$

$$\tan\theta_2 = \frac{(V_A - w_1)}{H} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{19.86}{92.96}\right) \Rightarrow \theta_2 = 12.059°$$

$$\tan\theta_3 = \frac{(V_B - w_3 - w_4)}{H} \Rightarrow \theta_3 = \tan^{-1}\left(\frac{20.14}{92.96}\right) \Rightarrow \theta_3 = 12.224°$$

$$\tan\theta_4 = \frac{(V_B - w_4)}{H} \Rightarrow \theta_4 = \tan^{-1}\left(\frac{44.14}{92.96}\right) \Rightarrow \theta_4 = 25.40°$$

$$\tan\theta_5 = \frac{V_B}{H} \Rightarrow \theta_5 = \tan^{-1}\left(\frac{62.14}{92.96}\right) \Rightarrow \theta_5 = 33.761°$$

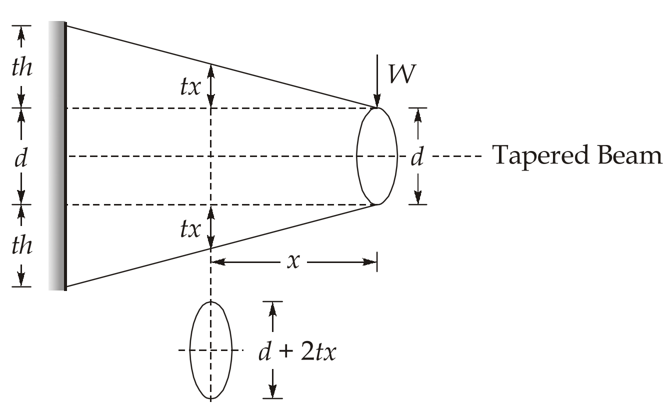
Final length of cable

$$\Rightarrow \quad L_{\text{final}} = \frac{10}{\cos\theta_1} + \frac{10}{\cos\theta_2} + \frac{10}{\cos\theta_3} + \frac{10}{\cos\theta_4} + \frac{10}{\cos\theta_5}$$

$$\Rightarrow \quad L_{\text{final}} = \frac{10}{\cos(28.207°)} + \frac{10}{\cos(12.059°)} + \frac{10}{\cos(12.224°)} + \frac{10}{\cos(25.40°)} + \frac{10}{\cos(33.761°)}$$

$$L_{\text{final}} = 54.903 \text{ m}$$

8. (c) (i) Solution:



Circular Cross-section

Diameter at a section at distance x from the free end is

$$d_x = d + 2tx$$

Bending moment at the section is

$$M_x = Wx$$

Section modulus for circular section is

$$Z_x = \frac{\pi d_x^3}{32} = \frac{\pi(d+2tx)^3}{32}$$

Bending stress is

$$\sigma_x = \frac{M_x}{Z_x} = \frac{Wx}{\frac{\pi(d+2tx)^3}{32}}$$

$$\sigma_x = \frac{32Wx}{\pi(d+2tx)^3}$$

Let $\sigma_x = C\frac{x}{(d+2tx)^3}$, where $C = \frac{32W}{\pi}$

For maximum bending stress,

$$\frac{d\sigma_x}{dx} = 0$$

Differentiating using quotient rule,

$$\frac{d}{dx}\left[\frac{x}{(d+2tx)^3}\right] = 0$$

$$\Rightarrow (d + 2tx)^3 - x \times 3(d + 2tx)^2 \times 2t = 0$$

$$\Rightarrow (d + 2tx)^3 - 6tx(d + 2tx)^2 = 0$$

Dividing by $(d + 2tx)^2$,

$$\Rightarrow (d + 2tx) - 6tx = 0$$

$$\Rightarrow d + 2tx - 6tx = 0$$

$$\Rightarrow d - 4tx = 0$$

$$\Rightarrow x = \frac{d}{4t}$$

Thus, the distance from the free end at which the bending stress is maximum is $x = \frac{d}{4t}$.

Substituting $x = \frac{d}{4t}$,

$$\sigma_{max} = \frac{32W\left(\frac{d}{4t}\right)}{\pi\left(d + 2t \times \frac{d}{4t}\right)^3}$$

$$\Rightarrow \quad \sigma_{max} = \frac{\frac{32Wd}{4t}}{\pi\left(\frac{3d}{2}\right)^3}$$

$$\Rightarrow \quad \sigma_{max} = \frac{\frac{8Wd}{t}}{\pi \times \frac{27d^3}{8}}$$

$$\Rightarrow \quad \sigma_{max} = \frac{8Wd}{t} \times \frac{8}{27\pi d^3}$$

$$\Rightarrow \quad \sigma_{max} = \frac{64W}{27\pi td^2}$$

Thus, the maximum bending stress is

$$\sigma_{max} = \frac{64W}{27\pi td^2} \text{ at } x = \frac{d}{4t} \text{ (from free end)}$$

8. (c) (ii) Solution:

Main Components of Brick Earth

1. Silica

- High-quality bricks usually contain 50–60% silica, existing in both free and combined states.
- It helps maintain the brick's shape by reducing cracking, shrinkage, and distortion.
- Proper amounts improve uniformity and give a smoother texture.
- Too much silica makes the brick fragile.
- It enhances the heat resistance of clays that have low alumina content.

2. Alumina

- Alumina is a key ingredient in clay, responsible for its plastic nature when mixed with water, allowing easy molding.
- In excessive amounts, it leads to high shrinkage and deformation during heating, reducing the brick's strength.
- Ideally, bricks contain about 20–30% alumina.

- It also contributes to the ability of bricks to withstand high temperatures.

3. **Lime (around 5%)**
   - Lime functions as a binding agent during burning.
   - It reduces shrinkage during the drying stage and helps combine silica and alumina effectively.
4. **Iron Oxide**
   - This compound determines the color of bricks, producing shades from yellow to deep red depending on its concentration.
   - When present in higher percentages (around 8–10%), it can create darker shades like blue or purple, and with manganese, nearly black.
   - It also enhances strength, durability, and resistance to moisture.
5. **Magnesia**
   - Found in small quantities (generally below 1%), it can add a yellowish tint along with iron oxide.
   - It helps reduce warping by slowing the softening process during firing.
   - Excessive magnesia can weaken the brick and cause deterioration.