MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 13

Full Syllabus Test (Paper-II)

Section A

Q.1 (a) Solution:

Given:

$$\text{Refractive index of core, } n_1 = 1.46,$$

$$2a = 8\ \mu\text{m} \quad \Rightarrow \quad a = 4\ \mu\text{m},$$

Now, Numerical aperture,
$$\text{NA} = n_1(2\Delta)^{1/2}$$
$$= 1.46[2 \times 0.003]^{1/2}$$
$$\text{NA} = 0.113$$

Acceptance angle is given as,

$$\sin\theta_a = \text{NA} = 0.113$$
$$\theta_a = 6.49° \cong 6.5°$$

The condition for single mode propagation inside the fiber is normalized frequency, $V \leq 2.405$, $V = 2.405$ corresponds to a minimum wavelength λ_c given as,

$$V = \frac{2\pi a\, NA}{\lambda_c} = 2.405$$

$$\lambda_c = \frac{[2\pi a\, NA]}{(2.405)} = \frac{2\pi \times 4 \times 10^{-6} \times 0.113}{2.405}$$

$$\lambda_c = 1.18\ \mu\text{m}$$

Illumination wavelengths shorter than 1.18 μm will result in a multimode operation. Thus, the single mode cut-off wavelength λ_c of the fiber is 1.18 μm.

Q.1 (b) Solution:

We can write the frequency components in the message signal $m(t)$ as,

$$f_1 = 100 \text{ Hz}$$
$$f_2 = 200 \text{ Hz}$$
$$f_3 = 400 \text{ Hz}$$
$$\text{Carrier frequency, } f_4 = 400 \text{ kHz}$$
$$\text{Local Oscillator frequency, } f_{LO} = 100.02 \text{ kHz}$$

(i) SSB-SC signal with upper sideband is given by,

$$\phi_{\text{USB}} = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t,$$

where $m_h(t)$ is the hilbert transform of $m(t)$

and $$m(t) = A_m(\cos\omega_1 t + \cos\omega_2 t + \cos\omega_3 t)$$

We know that, $$H[\cos\omega t] = \sin\omega t$$

Thus, $$\phi_{\text{USB}}(t) = A_m(\cos\omega_1 t + \cos\omega_2 t + \cos\omega_3 t)\cos\omega_c t - A_m(\sin\omega_1 t + \sin\omega_2 t + \sin\omega_3 t)\sin\omega_c t$$

Using the identity $\cos(A + B) = \cos A.\cos B + \sin A.\sin B$, we get

$$\phi_{\text{USB}}(t) = A_m[\cos(\omega_c + \omega_1)t + \cos(\omega_c + \omega_2)t + \cos(\omega_c + \omega_3)t]$$

Now, it is passed through a detector.

Detector output is $\phi_{\text{USB}}\sin\omega_{LO}t$

So, the frequency components at output are,

$$(f_c + f_1) \pm f_{LO}, (f_c + f_2) \pm f_{LO}, (f_c + f_3) \pm f_{LO}$$
$$= (400 + 0.1 \pm 100.02)\text{kHz}, (400 + 0.2 \pm 100.02)\text{ kHz}, (400 + 0.4 \pm 100.02)\text{ kHz}$$
$$= 500.12 \text{ kHz}, 300.08 \text{ kHz}, 500.22 \text{ kHz}, 300.18 \text{ kHz}, 500.42 \text{ kHz}, 300.38 \text{ kHz}$$

The frequency components of detector output is,

300.08 kHz, 300.18 kHz, 300.38 kHz, 500.12 kHz, 500.22 kHz, 500.42 kHz

(ii) If only LSB is transmitted,

$$\phi_{\text{LSB}}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

On solving it, we get

$$\phi_{\text{LSB}}(t) = A_m[\cos(\omega_c - \omega_1)t + \cos(\omega_c - \omega_2)t + \cos(\omega_c - \omega_3)t]$$

Now, when passed through detector, frequency components at output is,

$(f_c - f_1) \pm f_{LO}, (f_c - f_2) \pm f_{LO}, (f_c - f_3) \pm f_{LO}$

$= (400 - 0.1 \pm 100.02)\text{kHz}, (400 - 0.2 \pm 100.02)\text{ kHz}, (400 - 0.4 \pm 100.02)\text{ kHz}$

$= 499.92\text{ kHz}, 299.88\text{ kHz}, 499.82\text{ kHz}, 299.78\text{ kHz}, 499.62\text{ kHz}, 299.58\text{ kHz}$

The frequency components of detector output are,

299.58 kHz, 299.78 kHz, 299.88 kHz, 459.62 kHz, 499.82 kHz, 499.92 kHz

Q.1 (c) Solution:

FCFS: In First-Come, First-Serve (FCFS) scheduling algorithm, the process that arrives first in the ready queue gets executed first. The Gantt chart is as below:

A	B	C	D	E

0 3 7 10 15 22

Process Name	Arrival Time	Burst Time	Completion Time	TAT Time	Waiting Time
A	0	3	3	3	0
B	3	4	7	4	0
C	7	3	10	3	0
D	9	5	15	6	1
E	12	7	22	10	3

Average Waiting Time = 4/5 = 0.8 ms

RR (T_Q = 3): Round Robin scheduling algorithm treats all processes equally and allocates a fixed time slice, known as a quantum (T_Q), to each process. The Gantt chart can thus be obtained as below,

A	B	B	C	D	E	D	E

0 3 6 7 10 13 16 18 22

Process Name	Arrival Time	Burst Time	Completion Time	TAT Time	Waiting Time
A	0	3	3	3	0
B	3	4	7	4	0
C	7	3	10	3	0
D	9	5	18	9	4
E	10	7	22	12	5

Average Waiting Time = 9/5 = 1.8 ms

SRTF: In Shortest Remaining Time First (SRTF), the process with the least time left to finish is selected for execution. The Gantt chart can thus be obtained as below,

A	B	C	C	D	D	E

0 3 7 9 10 12 15 22

Process Name	Arrival Time	Burst Time	Completion Time	TAT Time	Waiting Time
A	0	3	3	3	0
B	3	4	7	4	0
C	7	3	10	3	0
D	9	5	15	6	1
E	10	7	22	12	3

Average Waiting Time = 4/5 = 0.8 ms

Waiting Time: FCFS = SRTF < RR

For the given process set, FCFS and SRTF scheduling algorithm provide the optimal average waiting time, while Round Robin scheduling algorithm is the least effective due to higher average waiting time.

Q.1 (d) Solution:

For the given control system:

$$G(s)H(s) = \frac{K}{(s^2+4s+4)(s+3)} = \frac{K}{s^3+7s^2+16s+12}$$

Substituting $s = j\omega$, we get

$$G(j\omega)H(j\omega) = \frac{K}{-j\omega^3 - 7\omega^2 + j16\omega + 12}$$

$$= \frac{K}{j(16\omega - \omega^3) + (12 - 7\omega^2)}$$

$$G(j\omega)\,H(j\omega) = \frac{K(12-7\omega^2) - j(16\omega - \omega^3)}{(12-7\omega^2)^2 + (16\omega - \omega^3)^2} \quad ...(i)$$

For imaginary part to be zero,

$$(16\omega - \omega^3) = 0$$

$$\omega(16 - \omega^2) = 0$$

$$\omega = 4 \text{ rad/sec}$$

At ω = 4 rad/sec, phase cross-over frequency, (ω_{pc}) occurs.

Now,

$$G(j\omega)H(j\omega)\big|_{\omega=\omega_{pc}=4\,\text{rad/sec}} = \frac{K}{12-7(4)^2} = \frac{-K}{100}$$

$\therefore$ Gain margin $= \dfrac{1}{\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}}} = \dfrac{1}{\frac{K}{100}} = \dfrac{100}{K}$ ...(ii)

Required value of gain margin,

$$GM \geq 4$$

Comparing with equation (ii), we get

$$\frac{100}{K} \geq 4$$

$$K \leq 25 \quad \text{...(iii)}$$

For position error constant :

$$K_p > 2$$

We know, $K_p = \lim\limits_{s\to 0} G(s)H(s)$

$$K_p = \lim_{s\to 0}\frac{K}{(s+2)^2(s+3)} = \frac{K}{12}$$

$\therefore$ $\dfrac{K}{12} > 2;\ K > 24$ ...(iv)

From (iii) and (iv), allowable range of K :

$$24 < K \leq 25$$

Q.1 (e) Solution:

Given,

Part 1: $P_{\text{avg}} = \dfrac{2\sin^2\theta\cos\phi}{r^2}\,\hat{a}_r\ \text{W/m}^2$

$\therefore$ Radiation intensity $U(\theta,\phi) = r^2 P_{\text{avg}}$

$$U(\theta,\phi) = 2\sin^2\theta\cos\phi\,\hat{a}_r$$

Directive gain $G_d = \dfrac{U(\theta,\phi)}{U_{avg}}$

$$G_d = \frac{4\pi U(\theta,\phi)}{P_{rad}} \quad \text{...(1)}$$

We have,
$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi/2} U(\theta,\phi)\sin\theta\, d\theta\, d\phi$$
$$= \int_0^{\pi}\int_0^{\pi/2} 2\sin^2\theta\cos\phi\cdot\sin\theta\, d\theta\, d\phi$$
$$= \int_0^{\pi} 2\sin^3\theta\, d\theta \int_0^{\pi/2}\cos\phi\, d\phi = \left(\frac{8}{3}\right)\times 1$$
$$\left\{\because \int_0^{\pi}\sin^3\theta d\theta = 2\int_0^{\pi/2}\sin^3\theta d\theta = 2\times\frac{2}{3} = \frac{4}{3}\right\}$$
$$\therefore \quad P_{rad} = \frac{8}{3}\text{W}$$

$\therefore$ From equation (1),

$$\text{Directive gain } G_d(\theta,\phi) = \frac{4\pi\times 2\sin^2\theta\cdot\cos\phi}{8/3} = 3\pi\sin^2\theta\cos\phi$$

Part 2: Directivity $D = G_d(\theta, \phi)|_{max} = 3\pi = 9.425$

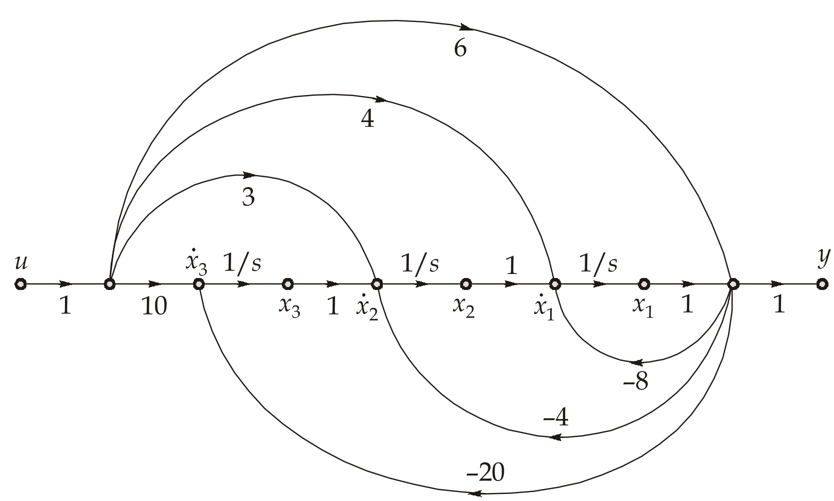
Q.2 (a) Solution:

$$\frac{Y(s)}{U(s)} = \frac{6+4/s+3/s^2+10/s^3}{1-(-8/s-4/s^2-20/s^3)} \quad ...(1)$$

The signal flow graph is shown in figure below. Comparing the above with the Mason's gain formula, the following information is obtained.

$$\frac{Y(s)}{U(s)} = \frac{\Sigma P_k\Delta_k}{\Delta} = \frac{\Sigma P_k\Delta_k}{1-\Sigma L_i}$$

where P_k are the forward path gains and L_i are the feedback loop gains.



1. There are four forward paths having gains 6, $4/s$, $3/s^2$ and $10/s^3$.
2. There are three feedback paths having gain $-8/s$, $-4/s^2$ and $-20/s^3$.

With the above information, the signal flow graph is obtained as shown. The state space equations can be written as,

$$y = x_1 + 6u$$
$$\dot{x}_1 = x_2 - 8y + 4u$$
$$\dot{x}_1 = x_2 - 48u - 8x_1 + 4u = -8x_1 + x_2 - 44u$$
$$\dot{x}_2 = x_3 - 4y + 3u = -4x_1 + x_3 - 21u$$
$$\dot{x}_3 = -20y + 10u = -20x_1 - 120u + 10u = -20x_1 - 110u$$

Phase variable representation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -8 & 1 & 0 \\ -4 & 0 & 1 \\ -20 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -44 \\ -21 \\ -110 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 6u$$

Q.2 (b) Solution:

(i) Transmitted power, P_t = 50 W

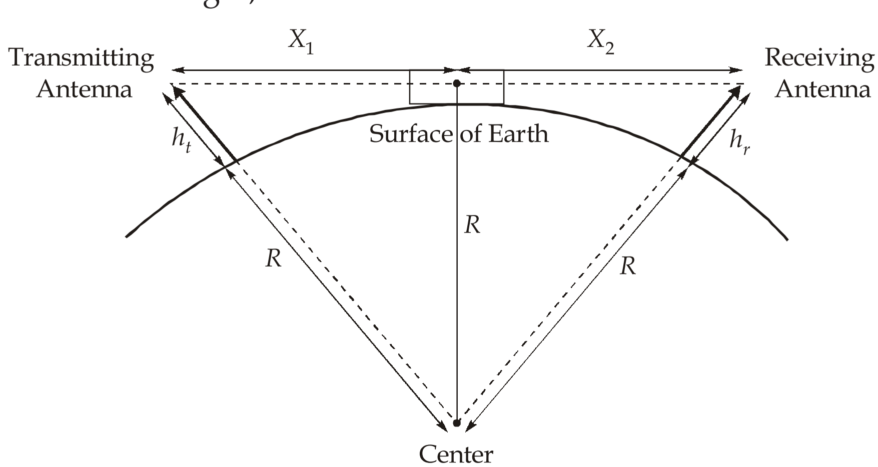
frequency, f = 120 MHz

Distance, d = ?

Transmitter height, h_t = 35 m

Receiver height, h_r = 30 m

field strength, E = ?



From the above figure,

$$(R + h_t)^2 = x_1^2 + R^2$$

$$R^2 + h_t^2 + 2h_tR = x_1^2 + R^2$$

Since $h_t^2 \ll 2h_tR$, we get

$$x_1^2 = 2h_tR$$

$$x_1 = \sqrt{2h_tR}$$

Assuming radius of earth, R = 6370 km, we get

$$x_1 = \sqrt{2h_t \times 10^{-3} \times 6370} = 3.57\sqrt{h_t} \text{ Km}$$

Here, h_t is in meters. Because atmospheric refraction typically bends radio waves downward, the signal travels further than a strict optical line of sight. An equivalent "effective" Earth radius is used, modifying the calculation by a standard refraction multiplier of 4/3. Thus,

$$d_1 = 3.57\sqrt{h_t \times \frac{4}{3}} = 4.123\sqrt{h_t} \text{ Km}$$

Similarly, we obtain

$$d_2 = 4.123\sqrt{h_r} \text{ Km}$$

Thus, the maximum line of sight distance is given by

$$d = 4.123\left[\sqrt{h_t} + \sqrt{h_r}\right]$$

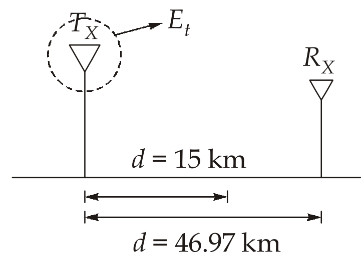
$$d = 4.123\left[\sqrt{35} + \sqrt{30}\right] = 46.97 \text{ km}$$

The received field strength at a distance 'd' is given by

$$E = \frac{88\sqrt{P_t}h_th_r}{\lambda d^2}, \qquad \text{where } \lambda = f/c$$

$$E = \frac{88\sqrt{50} \times 35 \times 30}{\left(\frac{3 \times 10^8}{120 \times 10^6}\right) \times (46.97 \times 10^3)^2} = 1.183 \times 10^{-4} \text{ V/m}$$

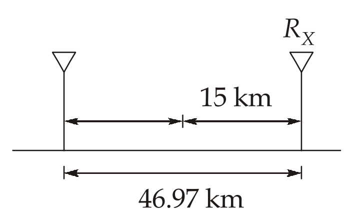
(ii) The field strength at 15 km from T_x



$$E = \frac{88\sqrt{P_t}\, h_t h_r}{\lambda d^2}$$

$$E = \frac{88\sqrt{50}\times 35\times 30}{\left(\dfrac{3\times 10^8}{120\times 10^6}\right)\times (15\times 10^3)^2} = 1.161\times 10^{-3}\ \frac{\text{V}}{\text{m}}$$

At 15 km from R_X:



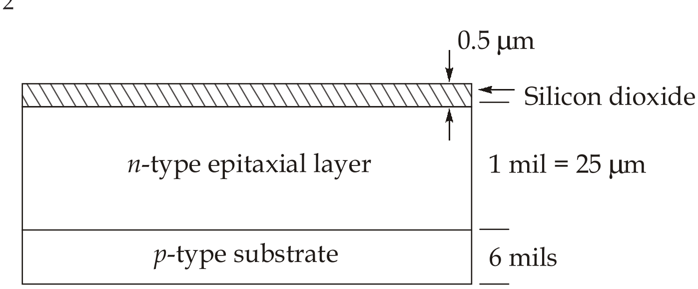
$$d = 46.97 - 15 = 31.97 \text{ km}$$

$$E = \frac{88\sqrt{P_t}\, h_t h_r}{\lambda d^2}$$

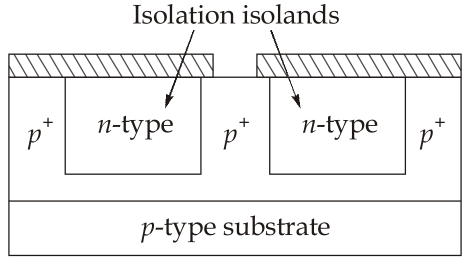
$$E = \frac{88\sqrt{50}\times 35\times 30}{\left(\dfrac{3\times 10^8}{120\times 10^6}\right)\times (31.97\times 10^3)^2} = 0.256\times 10^{-3}\ \frac{\text{V}}{\text{m}}$$

Q.2 (c) Solution:

Step-1 : Epitaxial growth: An n-type epitaxial layer typically 25 μm thick, is grown onto a p-type substrate which has a resistivity of above 10 Ω-cm. After polishing and cleaning, a thin layer of SiO_2 is formed over the entire wafer as shown in the figure below:

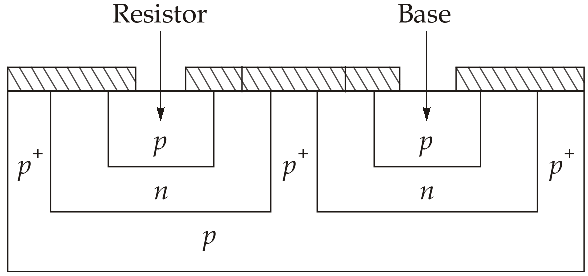


Step-2 : Isolation diffusion: In figure, the wafer is shown with the oxide removed in three different places. This removal is accompanied by means of a lithographic etching process. The remaining SiO_2 serves as a mask for the diffusion of acceptor impurities (like boron). The wafer is now subjected to the so called isolation diffusion, which takes place at the temperature and for the time interval required for the *p*-type impurities to penetrate the *n*-type epitaxial layer and reach the *p*-type substrate.



These sections are called isolation islands or isolated regions because they are separated by back to back *p*-*n* junction. Their purpose is to electrically isolate different circuit components.

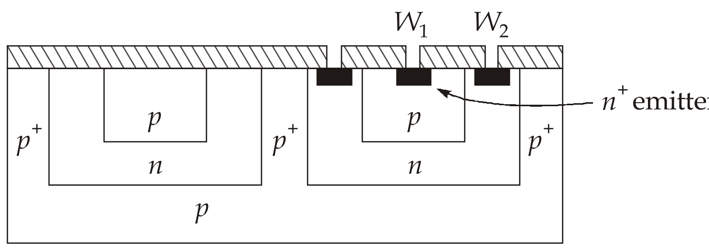
Step-3 : Base diffusion: In this process, a layer of oxide is formed over the wafer and the photolithographic process is again used to create the pattern of openings shown in figure. The *p*-type impurities (boron) are diffused through these openings into the islands of *n*-type epitaxial silicon.



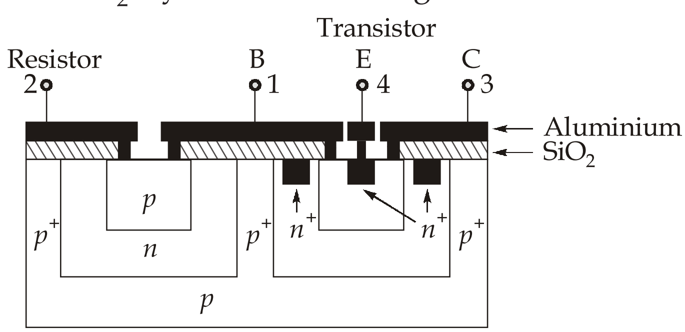
In this way, the transistor base regions as well as resistors are formed. It is important to control the depth of this diffusion so that it is shallow and doesn't penetrate to the substrate.

Step-4 : Emitter diffusion: A new layer of oxide is formed over entire surface and masking and etching processes are used again to open windows in *p*-type regions as shown in figure. Through those opening are diffused *n*-type impurities (phosphorus) for the formation of transistor emitter.

Additional windows W_1 and W_2 dare made into *n*-regions to which a lead is to be connected using aluminium as ohmic contact. During the diffusion of phosphorus, a heavy concentration (called n^+) is formed at the points where contacts with aluminium is to be made.



Step-5 : Aluminium metallization: All *p-n* junctions and resistor for circuit have been formed. It is necessary to interconnect the various components of the IC as dictated by the desired circuit. To make these interconnections, a fourth set of windows are opened into a newly formed SiO_2 layer as shown in figure.



The interconnection are first made using vacuum deposition of a thin even coating of aluminium over the entire wafer. The photoresist technique is then applied to etch away all undesired aluminium areas, leaving the desired pattern of interconnection between resistor and transistors.

Q.3 (a) Solution:

Let $h(n)$, $h_1(n)$ and $h_2(n)$ represent the unit sample responses corresponding to the system functions $H(z)$, $H_1(z)$ and $H_2(z)$, respectively. It follows that

$$h_1(n) = \left(\frac{1}{2}\right)^n u(n), \quad h_2(n) = \left(\frac{1}{4}\right)^n u(n)$$

$$H(z) = H_1(z) \cdot H_2(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{1}{1-\frac{1}{4}z^{-1}}$$

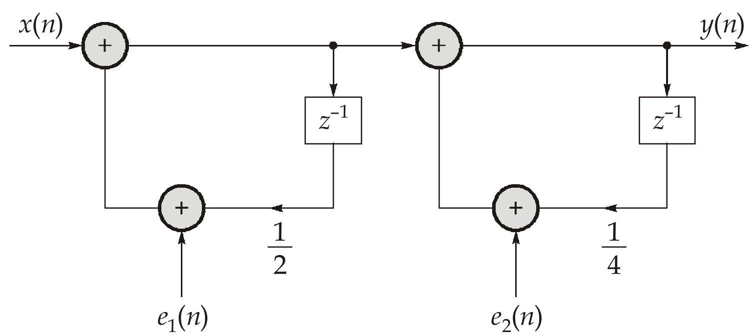
Using partial fraction expansion,

$$H(z) = \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{(-1)}{1-\frac{1}{4}z^{-1}}$$

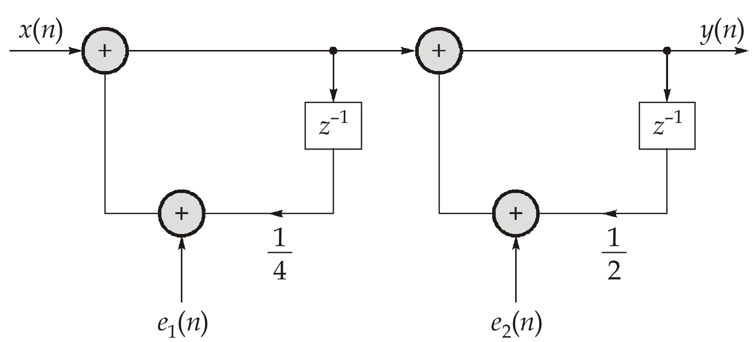
Taking inverse z-transform,

$$h(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right]u(n)$$

The two cascade realizations of the system are shown below:



(a) Cascade realization I



(b) Cascade realization II

In the first cascade realization,

Note: The first noise source $e_1(n)$ passes through the entire remaining cascade, while the second noise source $e_2(n)$ passes only through the second section. Thus, the variance of the round-off noise at the output is

$$\sigma_{q1}^2 = \sigma_e^2\left[\sum_{n=0}^{\infty} h^2(n) + \sum_{n=0}^{\infty} h_2^2(n)\right]$$

In the second cascade realization, the variance of the round-off noise at the output is

$$\sigma_{q2}^2 = \sigma_e^2\left[\sum_{n=0}^{\infty} h^2(n) + \sum_{n=0}^{\infty} h_1^2(n)\right]$$

Now, $$\sum_{n=0}^{\infty} h_1^2(n) = \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\sum_{n=0}^{\infty} h_2^2(n) = \sum_{n=0}^{\infty}\left(\frac{1}{16}\right)^n = \frac{1}{1-\frac{1}{16}} = \frac{16}{15}$$

$$\sum_{m=0}^{\infty} h^2(n) = \sum_{n=0}^{\infty}\left[4\left(\frac{1}{4}\right)^n - 4\left(\frac{1}{8}\right)^n + \left(\frac{1}{16}\right)^n\right]$$

$$= \frac{4}{1-\frac{1}{4}} - \frac{4}{1-\frac{1}{8}} + \frac{1}{1-\frac{1}{16}} = 1.83$$

Therefore,
$$\sigma_{q1}^2 = 2.90\sigma_e^2$$
$$\sigma_{q2}^2 = 3.16\sigma_e^2$$

and the ratio of noise variances is

$$\frac{\sigma_{q2}^2}{\sigma_{q1}^2} = 1.09$$

Hence, the noise power in the second cascade realization is 9% larger than in the first realization.

Q.3 (b) Solution:

$$\frac{Y(s)}{R(s)} = \frac{K(s+1)}{(s+1)(s^2+2s+2)+K} = \frac{K(s+1)}{s^3+3s^2+4s+(K+2)}$$

Characteristic equation :

$$s^3 + 3s^2 + 4s + (K + 2) = 0$$

Let
$$(K + 2) = K'$$

$$1+\frac{K'}{s^3+3s^2+4s} = 0 \qquad \text{...(i)}$$

Comparing eqn. (i) with $1 + G(s)H(s) = 0$;

$$G(s)H(s) = \frac{K'}{s^3+3s^2+4s} = \frac{K'}{s(s^2+3s+4)}$$

Substituting $s = j_{\omega}$, we get

$$G(j\omega)H(j\omega) = \frac{K'}{j\omega(4-\omega^2+j3\omega)}$$

$$|G(j\omega)H(j\omega)| = \frac{K'}{(\omega)\left(\sqrt{(4-\omega^2)^2+9\omega^2}\right)}$$

$$\angle G(j\omega)H(j\omega) = -90° - \tan^{-1}\left(\frac{3\omega}{4-\omega^2}\right)$$

At $\omega = \omega_{pc}$; $\quad \angle G(j\omega)H(j\omega) = -180°$

$$-180° = -\left(90° + \tan^{-1}\frac{3\omega_{pc}}{4-\omega_{pc}^2}\right) \Rightarrow \omega_{pc} = 2 \text{ rad/sec}$$

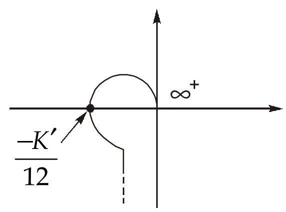
$$|G(j\omega)H(j\omega)|_{\omega_{pc}} = \frac{-K'}{12}$$

At $\omega = 0^+$, $G(j0^+)H(j0^+) = 8\angle{-90°}$

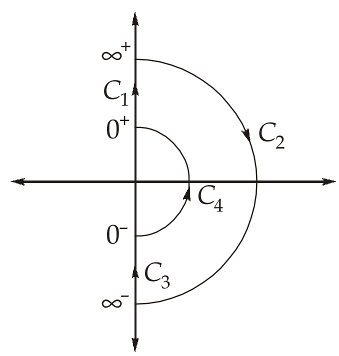
At $\omega = 8^+$, $G(j8^+)H(j8^+) = 0\angle{-270°}$

$\because$ $G(s)H(s)$ is a minimum phase system and an all pole system.

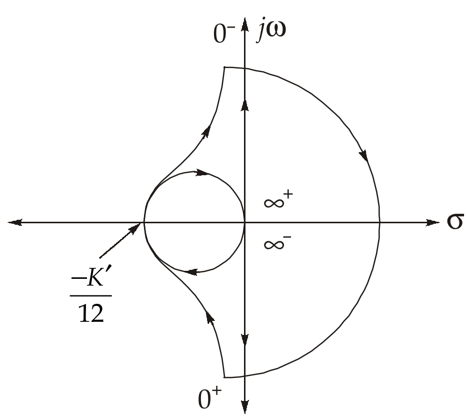
Its polar part is :



Nyquist Contour :



Nyquist Plot :



For the system to be stable, closed loop poles on RH side of s-plane i.e. $Z = 0$. Since no open loop poles lies in the right side of s-plane, hence $P = 0$. According to Nyquist stability criteria,

$$N = P - Z$$

$$N = 0 - 0 = 0$$

∴ For stability, we need zero encirclements of point $(-1 + j0)$.

$$\therefore \quad \frac{-K'}{12} > -1$$

$$\frac{K'}{12} < 1$$

$$K' < 12$$

$$K + 2 < 12$$

Thus, $K < 10$ for stability

Q.3 (c) Solution:

(i) The bandwidth of the signal is

$$B = 5 \text{ kHz}$$

$$\text{Nyquist rate} = 2B = 2 \times 5000 = 10{,}000 \text{ Hz}$$

Hence, $\text{sampling rate} = 2 \times \text{Nyquist rate} = 2 \times 10{,}000$

$$\therefore \quad \text{Sampling rate } (r) = 20{,}000 \text{ samples/second}$$

Since the samples are quantized into 256 equally likely levels there will be,

$$n = \log_2(256) = 8 \text{ bits/sample}$$

The information rate (R) = $n \times r = 8 \times 20{,}000$

$$R = 160 \text{ kbps}$$

(ii) To check for error-free transmission on the AWGN channel with

$$B = 10 \text{ kHz and } \frac{S}{N} = 40 \text{ dB}$$

In decibels, $$\left(\frac{S}{N}\right)_{\text{dB}} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$40 = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\frac{S}{N} = 10^4$$

The capacity of AWGN channel is given by,

$$C = B\log_2\left[1 + \frac{S}{N}\right]$$

$$C = 10000 \log_2[1 + 10^4] = 132.88 \text{ kbps}$$

For error-free transmission, $R \le C$. In this case, $C = 132.88$ kbps and $R = 160$ kbps.

∴ The information rate is more than the capacity of the channel. Hence, error free transmission is not possible in this case.

(iii) $\frac{S}{N}$ ratio for error-free transmission, with B = 10 kHz.

$$R \le C = B\log_2\left[1+\frac{S}{N}\right]$$

$$160 \text{ K} \le 10\text{K}\log_2\left[1+\frac{S}{N}\right]$$

$$\left(\frac{S}{N}\right) \ge 65535$$

In decibels,
$$\left(\frac{S}{N}\right)_{dB} \ge 10 \log_{10}(65535)$$

$$\left(\frac{S}{N}\right)_{dB} \ge 48.16 \text{ dB}$$

(iv) Given $\left(\frac{S}{N}\right)$ ratio is 40 dB

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$40 = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\frac{S}{N} = 10^4$$

For error-free transmission over AWGN channel,

$$R \le C = B\log_2\left[1+\frac{S}{N}\right]$$

$$160 \times 10^3 \le B \log_2[1 + 10^4]$$

$$\frac{160\times10^3}{\log_2\left[1+10^4\right]} \le B$$

$$\therefore \quad B \ge 12.04 \text{ kHz}$$

Q.4 (a) Solution:

(i) We know that

Profile index for graded index fiber is given as

$$n(r) = n_1\left[1-2\Delta\left(\frac{r}{a}\right)^{\alpha}\right]^{1/2} \qquad \text{for } 0 \le r \le a$$

Where, n_1 = core refractive index when, $r = 0$

Given, $$n(0) = n_1 = 1.53$$

For triangular profile, $\alpha = 1$. Thus, at $r = 2\ \mu m$,

$$n(2) = 1.53\left[1-2\Delta\left(\frac{2}{3}\right)^1\right]^{1/2} = 1.497$$

$$1-2\Delta\left(\frac{2}{3}\right) = \left(\frac{1.497}{1.53}\right)^2$$

$$\left[\frac{1-0.957}{2}\right]\frac{3}{2} = \Delta$$

$$\Delta = 0.032$$

And $$\Delta = \frac{n_1^2-n_2^2}{2n_1^2} = 0.032$$

$$\frac{1.53^2-n_2^2}{2(1.53)^2} = 0.032$$

On solving, we get $$n_2 = 1.48$$

Thus, we get relative refractive index, $\Delta = 0.032$ and cladding refractive index, $n_2 = 1.46$.

(ii) At $\lambda = 1.55\ \mu m$

Normalised frequency, $$V = \frac{2\pi}{\lambda}aNA$$

$$V = \frac{2\pi}{1.55}\times 3\times n_1\sqrt{2\Delta}$$

$$V = \frac{2\pi}{1.55}\times 3\times 1.53\sqrt{2\times 0.032} = 4.707$$

Number of modes, $$M_n = \left[\frac{\alpha}{\alpha+2}\right]\frac{V^2}{2}$$

Since $\alpha = 1$ for triangular profile, we get

$$M_n = \frac{V^2}{6} = 3.69$$

Number of complete modes = 3

Approximate number of modes = 4

(iii) For single mode operation in the fiber, the normalized cut-off frequency,

$$V \leq 2.405\left[\frac{\alpha+2}{\alpha}\right]^{1/2}$$

Since $\alpha = 1$, thus

$$V \leq 2.405\left[\frac{3}{1}\right]^{1/2} = 4.166$$

(iv) For a step index fiber ($\alpha = \infty$) to operate in single mode,

$$V \leq 2.405$$

We have,

$$V = \frac{2\pi}{\lambda} aNA$$

$$2.405 \geq \frac{2\pi}{\lambda} \cdot 3\times10^{-6} \times 1.53\sqrt{2\times0.032}$$

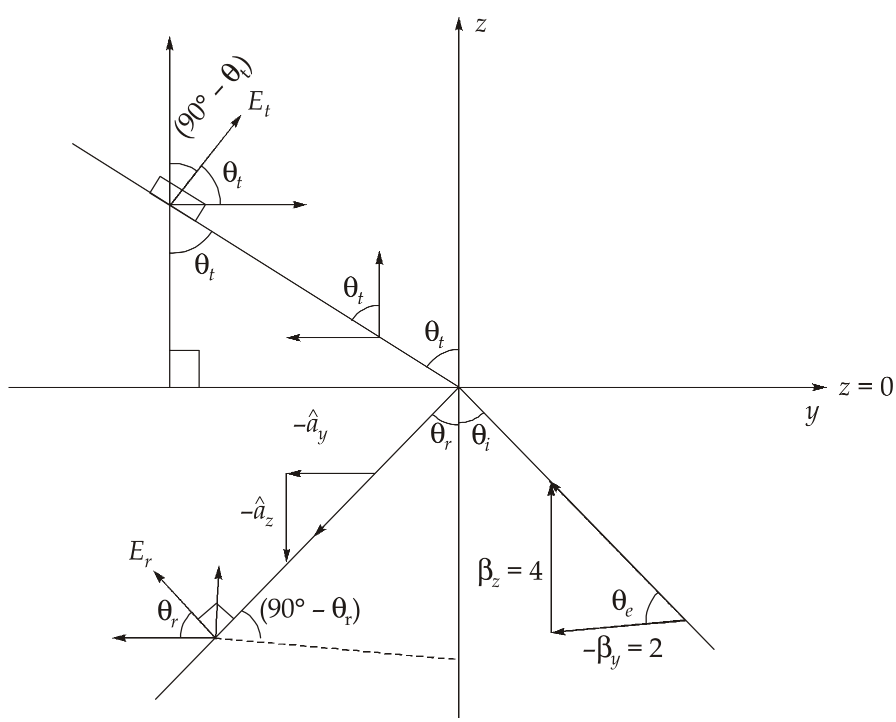
$$\lambda \geq 3.03\ \mu m$$

Q.4 (b) Solution:

We have,

Incidence electric field as,

$$\vec{E_i} = (10\hat{y}+5\hat{z})\cos(\omega t+2y-4z)\ \text{V/m}$$



(i) Incidence region:

Direction of wave: $-\hat{a}_y + \hat{a}_z$ $\quad\left(\vec{\beta}_1 = -2\hat{a}_y + 4\hat{a}_z\right)$

Electric field direction, E_i: $\hat{a}_y + \hat{a}_z$

Direction of incidence magnetic field, H_i: $-\hat{a}_x$ $\quad[\because\ \hat{a}_E \times \hat{a}_H = \hat{a}_k]$

Intrinsic impedance, $\eta_1 = 120\pi\ \Omega$

In free space,
$$c = \frac{\omega}{\beta}$$
$$\omega = 3\times 10^8 \times \sqrt{4+16} \qquad \left\{\beta = \sqrt{\beta_x^2 + \beta_z^2}\right\}$$
$$\omega = (13.42 \times 10^8)\ \text{rad/sec}$$

From the figure,
$$\tan\theta_e = \frac{P}{b};\quad \text{and}\quad \theta_i = 90° - \theta_e$$
$$\tan\theta_e = \frac{4}{2} \qquad \theta_i = 90° - 63.43°$$
$$\theta_e = 63.43° \qquad \theta_i = 26.56°$$

The incidence magnetic field is given as,
$$\vec{H}_i = H_{i0}\cos(\omega t + 2y - 4z)(-\hat{a}_x)\ \text{A/m}$$

where
$$H_{i0} = \frac{|E_{i0}|}{\eta}$$
$$H_{i0} = \frac{\sqrt{100+25}}{120\pi} = 0.03$$
$$\vec{H}_i = 0.03\cos(13.42\times 10^8 t + 2y - 4z)(-\hat{a}_x)\ \text{A/m}$$

(ii) Reflected wave:

Direction of reflected wave: $-\hat{a}_y - \hat{a}_z$. Thus, $\vec{\beta}_r = -2\hat{a}_y - 4\hat{a}_z$

Angle of reflection, $\phi_r = \theta_i = 26.56°$

Using Snell's law,
$$n_1 \sin\theta_i = n_2 \sin\theta_t$$
$$\sin 26.56° = \sqrt{4}\sin\theta_t$$
$$\theta_t = 12.92°$$

Since magnetic field is perpendicular to the plane of incidence, thus the wave is parallely polarized. For parallel polarization,

$$\text{Reflection coefficient, } \Gamma_p = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}$$

$$\Gamma_p = \frac{\tan(12.92° - 26.56°)}{\tan(12.92° + 26.56°)}$$

$$\Gamma_p = -0.294$$

Negative sign of Γ_p indicates that the component of E_i parallel to the interface changes phase and the direction of magnetic field parallel to the interface remain unchanged. Thus,

Direction of reflected electric wave $\hat{E}_r : -\hat{a}_y + \hat{a}_z$

Direction of reflected magnetic wave $\hat{H}_r : -\hat{a}_x$

As,
$$|\Gamma_p| = \frac{|E_{r0}|}{|E_{i0}|}$$

$$|\Gamma_p| = \frac{|E_{r0}|}{|E_{i0}|} = \frac{|E_{r0}|}{\sqrt{100+25}}$$

$$|E_{r0}| = 0.294\sqrt{125}$$

$$|E_{r0}| = 3.29$$

The reflected electric field is given as,

$$\vec{E}_r = (-|E_{r0}|\cos\theta_r \hat{a}_y + |E_{r0}|\sin\theta_r \hat{a}_z)\cos(\omega t - \vec{\beta}_r.(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)) \text{ V/m}$$

$$\vec{E}_r = (-3.29\cos 26.56°\hat{a}_y + 3.29\sin 26.56°\hat{a}_z)\cos(13.42\times10^8 t + 2y + 4z) \text{ V/m}$$

$$\vec{E}_r = (-2.94\hat{a}_y + 1.47\hat{a}_z)\cos(\omega t + 2y + 4z)\text{V/m}$$

The reflected magnetic field is given as,

$$\vec{H}_r = H_{r0}\cos(\omega t + 2y + 4z)(-\hat{a}_x) \text{ A/m}$$

where
$$H_{r0} = \frac{|E_{r0}|}{120\pi}$$

$$H_{r0} = \frac{3.29}{120\pi} = 8.73\times10^{-3}$$

$$\vec{H}_r = -8.73\cos(13.42\times10^8 t + 2y + 4z)\hat{a}_x \text{ mA/m}$$

(iii) Transmitted wave:

Direction of transmitted electric field and magnetic field are same as incidence field.

In dielectric medium, $\eta_2 = \frac{120}{\sqrt{\epsilon_{r2}}} = \frac{120\pi}{\sqrt{4}} = 60\pi\,\Omega$

$$v_p = \frac{\omega}{|\beta_2|} = \frac{3\times10^8}{\sqrt{\epsilon_{r2}}}$$

$$|\beta_2| = \frac{13.42\times10^8\times\sqrt{4}}{3\times10^8}$$

$$|\beta_2| = 8.95 \text{ m}^{-1}$$

We have, $|\beta_z| = |\beta_2| \cos\theta_t = 8.95 \cos 12.92°$

$$|\beta_z| = 8.72 \text{ m}^{-1}$$

$$|\beta_y| = |\beta_2| \sin\theta_t = 8.95 \sin 12.92°$$

$$|\beta_y| = 2 \text{ m}^{-1}$$

Thus, $\beta_t = (-2\hat{a}_y + 8.72\hat{a}_z)\text{m}^{-1}$

For parallel polarization, transmission coefficient, τ_p

$$\tau_p = (1+\Gamma)\frac{\cos\theta_i}{\cos\theta_t}$$

$$\tau_p = (1-0.294)\frac{\cos 26.56°}{\cos 12.92°}$$

$$\tau_p = 0.65$$

We have, $|\tau_p| = \frac{|E_{t0}|}{|E_{i0}|}$

$$0.65 = \frac{|E_{t0}|}{\sqrt{125}}$$

$$|E_{t0}| = 7.27$$

We have, $E_y = |E_{t0}|\cos\theta_t$ and $E_z = |E_{t0}| \sin\theta_t$

$E_y = 7.27 \cos 12.92°$ $\quad$ $E_t = 7.27 \sin 12.92°$

$E_y = 7.09$ $\quad$ $E_z = 1.63$

Thus, $\vec{E}_t = (7.09\hat{y} + 1.63\hat{z})\cos(13.42\times10^8 t + 2y - 8.72z)$ V/m

We have, $$H_{t0} = \frac{|E_{t0}|}{\eta}$$

$$H_{t0} = \frac{7.27}{60\pi}$$

$$H_{t0} = 0.039$$

Thus, $$\vec{H}_t = -0.039\cos(13.42\times10^8 t + 2y - 8.72z)\hat{a}_x \text{ A/m}$$

(iv) Brewster angle:

The Brewster angle is defined as a specific angle of incidence at which incident wave with *p*-polarization is not reflected and is given as:

$$\tan\theta_B = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

$$\theta_B = \tan^{-1}\left(\sqrt{\frac{4\epsilon_0}{\epsilon_0}}\right)$$

$$\theta_B = 63.43°$$

Q.4 (c) Solution:

(i) Direct addressing mode: In direct addressing mode, memory address is directly specified in the instruction.

```
MOV A, #55H
MOV 40H, A
MOV 41H, A
MOV 42H, A
MOV 43H, A
MOV 44H, A
MOV 45H, A
```

(ii) Register indirect addressing mode without loop: In register indirect addressing mode, register R0 is used to hold the target memory address, denoted as @R0. Here, we manually increment the pointer without using a loop control instruction.

```
MOV A, #55H
MOV R0, #40 H
MOV @R0, A
INC R0
MOV @R0, A
INC R0
```

```
         MOV @R0, A
         INC R0
         MOV @R0, A
         INC R0
         MOV @R0, A
         INC R0
         MOV @R0, A
```

(iii) Register indirect addressing mode with loop: This approach utilizes a counter register along with the DJNZ (Decrement and Jump if Not Zero) instruction to loop through the memory block. The memory address is incremented for each loop iteration using the INC instruction.

```
         MOV A, #55H
         MOV R0, #40 H
         MOV R2, #06 H
AGAIN:   MOV @R0, A
         INC R0
         DJNZ R2, AGAIN
```

Section B

Q.5 (a) Solution:

Given, $Z_0 = 60\ \Omega$, $\alpha = 20$ m Np/m, $v = 0.6c$, $f = 100$ MHz

For a distortionless line,

$$RC = LG$$

$$G = \frac{RC}{L} \quad ...(1)$$

and

$$Z_0 = \sqrt{\frac{L}{C}} \quad ...(2)$$

$$\alpha = \sqrt{RG} = \sqrt{R \cdot \frac{RC}{L}} \quad \text{(from equation 1)}$$

$$\alpha = R\sqrt{\frac{C}{L}} \quad ...(3)$$

From equation (2) and (3)

$$\alpha = \frac{R}{Z_0}$$

$$R = \alpha Z_0 \quad ...(4)$$

Also, $$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \quad ...(5)$$

From equation (2) and (5),

$$L = \frac{Z_0}{v} \quad ...(6)$$

Also, $$G = \frac{\alpha^2}{R} \quad ...(7)$$

From equation (2) and (5),

$$vZ_0 = \frac{1}{C}$$

$$C = \frac{1}{vZ_0} \quad ...(8)$$

Now, substituting the given values in respective equation.

From equation (4), $$R = \alpha Z_0 = 20 \times 10^{-3} \times 60$$

$$R = 1.2\ \Omega/\text{m}$$

From equation (6), $$L = \frac{Z_0}{v}$$

$$= \frac{60}{0.6 \times 3 \times 10^8}$$

$$L = 333.33\ \text{nH/m}$$

From equation (7), $$G = \frac{\alpha^2}{R}$$

$$= \frac{(20 \times 10^{-3})^2}{1.2}$$

$$G = 333.33\ \mu\text{S/m}$$

From equation (8), $$C = \frac{1}{vZ_0}$$

$$= \frac{1}{0.6 \times 10^8 \times 3 \times 60}$$

$$C = 92.59\ \text{pF/m}$$

and wavelength, $$\lambda = \frac{v}{f} = \frac{0.6 \times 3 \times 10^8}{100 \times 10^6}$$

$$\lambda = 1.8\ \text{m}$$

Q.5 (b) Solution:

Given:

For fresh water: $f = 1$ MHz, $\in_r = 80$, $\sigma = 5 \times 10^{-3}$ S/m

For 'good' earth: $f = 1$ GHz, $\in_r = 15$, $\sigma = 10 \times 10^{-3}$ S/m

$$\eta_1 = 377\ \Omega \text{ (for air)}$$

For fresh water:

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\in}}$$

$$= \sqrt{\frac{j \times 2\pi \times 10^6 \times 4\pi \times 10^{-7}}{5 \times 10^{-3} + j2\pi \times 10^6 \times 8.854 \times 10^{-12} \times 80}}$$

$$= \sqrt{1179.55\angle 48.328^\circ}$$

$$\eta_2 = 34.344\angle 24.164^\circ\ \Omega$$

$$\text{Reflection coefficient, } \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\Gamma = \frac{34.344\angle 24.164^\circ - 377}{34.344\angle 24.164^\circ + 377} = \frac{31.334 + j14.059 - 377}{31.334 + j14.059 + 377}$$

$$\Gamma = 0.847\angle 175.7^\circ$$

For 'good' earth:

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\in}}$$

$$= \sqrt{\frac{j \times 2\pi \times 10^9 \times 4\pi \times 10^{-7}}{10 \times 10^{-3} + j2\pi \times 10^9 \times 8.854 \times 10^{-12} \times 15}}$$

$$= \sqrt{9461.24\angle 0.6866^\circ} = 97.27\angle 0.3433^\circ\ \Omega$$

Reflection coefficient,

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{97.27\angle 0.3433^\circ - 377}{97.27\angle 0.3433^\circ + 377}$$

$$= \frac{97.268 + j0.583 - 377}{97.268 + j0.583 + 377}$$

$$\Gamma = 0.59\angle 179.8^\circ \simeq -0.6$$

Q.5 (c) Solution:

We have; $y[n] = 0.2x[n] + 0.2x[n-2] + 0.4x[n-3]$

Taking Z-transform

$$Y(z) = 0.2X(z) + 0.2z^{-2}X(z) + 0.4z^{-3}X(z)$$

$$\frac{Y(z)}{X(z)} = H(z) = 0.2 + 0.2z^{-2} + 0.4z^{-3}$$

Taking inverse z-transform,

$$h[n] = 0.2 + 0.2\delta[n-2] + 0.4\delta[n-3]$$

i.e., $h[n] = \{0.2, 0, 0.2, 0.4\}$

Given, input $x[n] = \{-1, 1, 0, -1\}$

We know, $y[n] = x[n] * h[n]$

Using tabular method for convolution,

	–1	1	0	–1
0.2	–0.2	0.2	0	–0.2
0	0	0	0	0
0.2	–0.2	0.2	0	–0.2
0.4	–0.4	0.4	0	–0.4

$$y[n] = \{-0.2, 0.2, -0.2, -0.4, 0.4, -0.2, -0.4\}$$

The summation of output is given as:

$$\Sigma y[n] = -0.2 + 0.2 - 0.2 - 0.4 + 0.4 - 0.2 - 0.4$$

$$= -0.8$$

Q.5 (d) Solution:

We have; $G(s) = \frac{25}{s(s+6.25)}$ and $H(s) = 1$

Now, the characteristic equation is

$$1 + G(s)H(s) = 0$$

$$\therefore \quad 1 + \frac{25}{s(s+6.25)} = 0$$

$$s^2 + 6.25s + 25 = 0$$

We know,

For standard 2[nd] order system, the characteristic equation is given as;

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

On comparing, we get,
$$\omega_n = 5 \text{ rad/sec}$$
$$2\xi\omega_n = 6.25 \quad \Rightarrow \quad \xi = 0.625$$

(i) Resonance peak (M_r):

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times 0.625\sqrt{1-(0.625)^2}} = 1.024$$

(ii) Resonant frequency (ω_r):

$$\omega_r = \omega_n\sqrt{1-2\xi^2} = 5\sqrt{1-2(0.625)^2}$$
$$= 2.338 \text{ rad/s}$$

(iii) Bandwidth (B.W)

$$\text{B.W} = \omega_n\sqrt{\sqrt{4\xi^4 - 4\xi^2 + 2} + (1-2\xi^2)}$$
$$= 5.573 \text{ rad/s}$$

Q.5 (e) Solution:

The silicon-on-sapphire or spinel technology refers to the technology wherein different parts of the device circuit are built on small, separate islands of silicon that are fabricated over insulating substrates, in effect providing a certain degree of isolation between circuits on different islands and reduces parasitic capacitances in the device. This leads to higher speed and reduced power consumption.

Other good features:

1. Completely depleted type MOSFET transistor.
2. Complete separation of elements is possible.
3. Junction capacitance is extremely small, which benefits high frequency performance and low power consumption characteristics.
4. Realisation of high performance passive element (eg. inductor).
5. Extremely high radiation resistance.
6. Low cross talk.

Drawbacks:

1. Because of the lattice parameter mismatch between the grown silicon layer and the sapphire substrate, misfit dislocations, edge dislocations and stacking faults are

frequently encountered in SOS devices, with the defect density varying inversely with the distance from the substrate.

2. The difference between the coefficients of thermal expansion of silicon and sapphire also results in a residual stress within the silicon layer, which tends to reduce hole mobility. This, coupled with the lower hole and electron mobilities caused by defects, ultimately results in SOS wafers yielding MOS devices with poorer performance in comparison to those fabricated on bulk silicon.

Q.6 (a) Solution:

NBFM signal,

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin 2\pi f_m t$$

For the signal $A \cos 2\pi f_c t + B \sin 2\pi f_c t$, the envelope is given as $\sqrt{A^2 + B^2}$.

$\therefore$ Envelope of the NBFM wave,

$$e(t) = \sqrt{A_c^2 + A_c^2 \beta^2 \sin^2(2\pi f_m t)}$$

$$\Rightarrow \quad e(t) = A_c \sqrt{1 + \beta^2 \sin^2(2\pi f_m t)}$$

The maximum value of envelope $e(t)$ occurs when $\sin^2(2\pi f_m t) = 1$.

Hence, $$e(t)_{\max} = A_c \sqrt{1+\beta^2}$$

The minimum value of envelope $e(t)$ occurs when $\sin^2(2\pi f_m t) = 0$.

Hence, $$e(t)_{\min} = A_c$$

The ratio of maximum to the minimum amplitude is given by

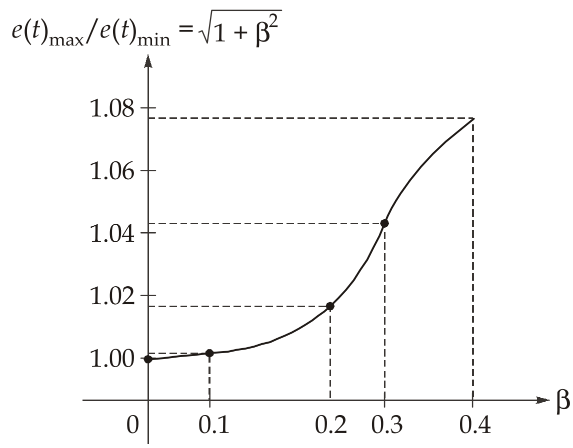
$$\frac{e(t)_{\max}}{e(t)_{\min}} = \frac{A_c\sqrt{1+\beta^2}}{A_c}$$

$$\frac{e(t)_{\max}}{e(t)_{\min}} = \sqrt{1+\beta^2}$$

Over the interval $0 \le \beta = 0.4$, we can obtain the following values:

β	$\sqrt{1+\beta^2}$
0	1
0.1	1.005
0.2	1.0198
0.3	1.044
0.4	1.077

Plot of $\frac{e(t)_{\max}}{e(t)_{\min}}$ versus "β" :



Given NBFM signal is

$$s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t)$$

$$\Rightarrow \quad s(t) = A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}[\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_c - f_m)t]$$

$$= A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\cos[2\pi(f_c + f_m)t] - \frac{\beta A_c}{2}\cos[2\pi(f_c - f_m)t]$$

∴ Total average power of NBFM signal is (assuming 'R' as the antenna resistance)

$$P_t = \frac{1}{R}\left[\frac{A_c^2}{2} + \left(\frac{\beta A_c}{2\sqrt{2}}\right)^2 + \left(\frac{-\beta A_c}{2\sqrt{2}}\right)^2\right]$$

$$= \frac{1}{R}\left[\frac{A_c^2}{2} + \frac{\beta^2 A_c^2}{8} + \frac{\beta^2 A_c^2}{8}\right] = \frac{A_c^2}{2R}\left[1 + \frac{\beta^2}{4} \times 2\right]$$

$$P_t = P_c\left[1 + \frac{\beta^2}{2}\right]$$

where, $P_c = \frac{A_c^2}{2R}$ is the average power of unmodulated carrier signal

$$\Rightarrow \quad \frac{P_t}{P_c} = \left(1 + \frac{\beta^2}{2}\right)$$

In terms of percentage,

$$\frac{P_t}{P_c} = \left(1 + \frac{\beta^2}{2}\right) \times 100$$

Q.6 (b) Solution:

(i) With error rate control, open loop transfer function of the system is,

$$G(s) = \frac{10(1+sk_e)}{s(s+2)}$$

Therefore, the closed-loop transfer function is

$$\frac{\theta_C(s)}{\theta_R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{10(1+sk_e)}{s(s+2)}}{1+\frac{10(1+sk_e)}{s(s+2)}} = \frac{10+10sk_e}{s^2+s(2+10k_e)+10}$$

Comparing the characteristic equation $s^2 + s(2 + 10k_e) + 10 = 0$ with the standard form of the characteristic equation of a second-order system,

$$s^2+2\xi\omega_n s+\omega_n^2 = 0, \text{ we get}$$

$$\omega_n^2 = 10, \quad \omega_n = \sqrt{10} = 3.16 \text{ rad/sec}$$

and $\quad 2\xi\omega_n = 2 + 10k_e$

$\therefore$ For $\xi = 0.6$, $\quad k_e = \frac{2\xi\omega_n - 2}{10} = \frac{2\times0.6\times3.16-2}{10} = 0.18$

The settling time, $\quad t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.6\times3.16} = 2.11 \text{ sec}$

The peak overshoot, $\quad M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{\frac{-\pi\times0.6}{\sqrt{1-0.6^2}}} = 0.0949$

The percentage peak overshoot

$$\%M_p = 0.0949 \times 100\% = 9.49\%$$

Therefore, the steady-state error for unit ramp input,

$$e_{ss} = \frac{1}{k_v} = \frac{1}{\lim_{s\to0} sG(s)} = \frac{1}{\lim_{s\to0} s\frac{10(1+sk_e)}{s(s+2)}}$$

$$= \frac{1}{5} = 0.2 \text{ rad}$$

(ii) Without error rate control i.e. for $k_e = 0$, the open loop transfer function of the system is given by

$$G(s) = \frac{10}{s(s+2)}$$

Therefore, the closed-loop transfer function is

$$\frac{\theta_C(s)}{\theta_R(s)} = \frac{\dfrac{10}{s(s+2)}}{1+\dfrac{10}{s(s+2)}} = \frac{10}{s^2+2s+10}$$

Comparing this transfer function with the standard form of the transfer function of a second-order system $\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$, we get

$$\omega_n^2 = 10$$

$$\therefore \qquad \omega_n = \sqrt{10} = 3.16 \text{ rad/sec}$$

$$2\xi\omega_n = 2$$

$$\therefore \qquad \xi = \frac{2}{2\omega_n} = \frac{2}{2\times 3.16} = 0.32$$

The settling time, $$t_s = \frac{4}{\xi\omega_n} = \frac{4}{1} = 4 \text{ sec}$$

The peak overshoot, $$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{\frac{-\pi\times 0.32}{\sqrt{1-0.32^2}}} = 0.351 = 35.1\%$$

For unit ramp input, steady-state error,

$$e_{ss} = \frac{1}{k_v} = \frac{1}{\lim\limits_{s\to 0} sG(s)} = \frac{1}{\lim\limits_{s\to 0} s\dfrac{10}{s(s+2)}} = \frac{1}{5} = 0.2 \text{ rad}$$

Q.6 (c) Solution:

We have, Bit Rate, R_b = 10 Gbps

Attenuation (α) = $0.25\dfrac{\text{dB}}{\text{km}}$

Laser Diode power = 2 dB = P_t

APD level required to maintain BER = –24 dB = $P_{r(\min)}$

BER = 10^{-11}

Power margin = ?

WDM MUX loss = –3 dB

OADM loss = –4 dB

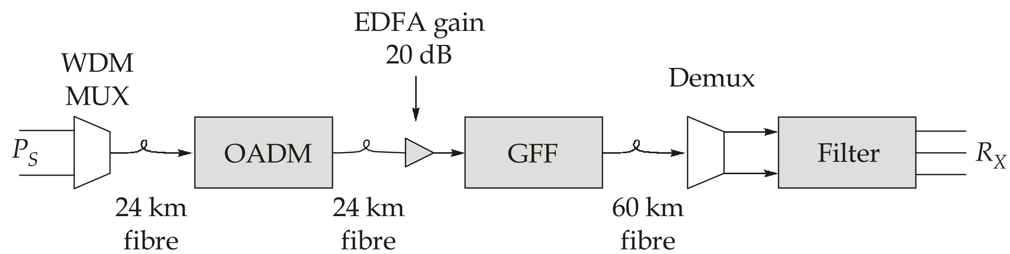
EDFA coupling loss = –1 dB

EPFA gain = 20 dB

Gain filtering loss = –3 dB

WDM Demux loss = –3 dB

Output filter loss = –3 dB



Length of the fiber = 24 + 24 + 60 = 108 km

To achieve the required BER,

$$P_{r(\min)} = P_t - P_L \text{ (dB)},$$

where P_L are the maximum allowed losses

$$\therefore \quad P_L = P_t - P_{r(\min)}$$

Allowed losses:

$$P_L = 2 \text{ dB} - (-24 \text{ dB})$$

$$P_L = 26 \text{ dB}$$

Actual losses:

Actual loss = $L_{\text{WDM MUX}} + L_{\text{OADM}} + L_{\text{EDFA}} - G_{\text{EDFA}} + L_{\text{GFF}} + L_{\text{WDM(Demux)}} + L_{\text{filter}} + (\alpha \times L_{\text{fiber}})$

$$\text{Actual Loss} = 3 + 4 + 1 - 20 + 3 + 3 + 3 + 0.25 \times 108$$

$$\text{Actual Loss} = 24 \text{ dB}$$

The power margin is thus given by

$$\text{PM} = \text{Allowed Losses - Actual Losses}$$

$$\text{PM} = 26 \text{ dB} - 24 \text{ dB} = 2 \text{ dB}$$

Q.7 (a) Solution:

The envelope detector input is

$$v(t) = s(t) - s(t - T)$$

$$= A_c \cos[2\pi f_c t + \phi(t)] - A_c \cos[2\pi f_c (t - T) + \phi(t - T)]$$

$$= -2A_c \sin\left[\frac{2\pi f_c (2t - T) + \phi(t) + \phi(t - T)}{2}\right] \sin\left[\frac{2\pi f_c T + \phi(t) - \phi(t - T)}{2}\right] \quad ...(1)$$

where, $\phi(t) = \beta \sin(2\pi f_m t)$

The phase difference $\phi(t) - \phi(t-T)$ is

$$\phi(t) - \phi(t-T) = \beta \sin(2\pi f_m t) - \beta \sin[2\pi f_m (t-T)]$$
$$= \beta[\sin(2\pi f_m t) - \sin(2\pi f_m t)\cos(2\pi f_m T) + \cos(2\pi f_m t)\sin(2\pi f_m T)]$$
$$= \beta[\sin(2\pi f_m t) - \sin(2\pi f_m t) + 2\pi f_m T \cos(2\pi f_m t)]$$
$$[\because \cos(2\pi f_m T) \approx 1 \text{ and } \sin(2\pi f_m T) \approx 2\pi f_m T]$$
$$= 2\pi \Delta f T \cos(2\pi f_m t)$$

where $\Delta f = \beta f_m$

Given that delay line produces a phase shift of $\pi/2$ radians at the carrier frequency f_c.

Thus, $2\pi f_c T = \dfrac{\pi}{2}$, we may write

$$\sin\left[\frac{2\pi f_c T + \phi(t) - \phi(t-T)}{2}\right] = \sin[\pi f_c T + \pi \Delta f T \cos(2\pi f_m t)]$$
$$= \sin\left[\frac{\pi}{4} + \pi\Delta f T \cos(2\pi f_m t)\right]$$
$$= \sqrt{2}\cos[\pi\Delta f T\cos(2\pi f_m t)] + \sqrt{2}\sin[\pi\Delta f T \cos(2\pi f_m t)]$$
$$= \sqrt{2} + \sqrt{2}\pi\Delta f T\cos(2\pi f_m t)$$

where we have made use of the fact that $\pi\Delta f T = \pi\beta f_m T << 1$. Thus, $\cos[\pi\Delta f T \cos(2\pi f_m t)] \approx 1$ and $\sin[\pi\Delta f T\cos(2\pi f_m t)] \approx \pi\Delta f T\cos(2\pi f_m t)]$. We may therefore rewirte eqn. (1) as:

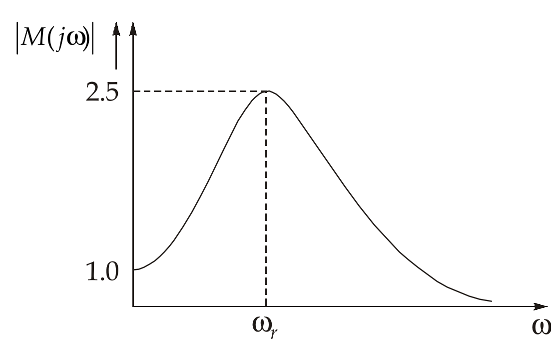
$$v(t) = -2\sqrt{2}A_c\left[[1 + \pi\Delta f T\cos(2\pi f_m t)]\sin\left(\pi f_c(2t - T) + \frac{\phi(t) + \phi(t-T)}{2}\right)\right]$$

Accordingly, the envelope detector output is

$$a(t) = 2\sqrt{2}A_c[1 + \pi\Delta f T\cos(2\pi f_m t)]$$

which, except for a bias term, is proportional to the modulating wave. Thus, the given circuit works as a FM demodulator.

Q.7 (b) Solution:



For an underdamped second-order system with $0 < \xi < 0.707$, the resonant peak is given by

$$M_r = \frac{K}{2\xi\sqrt{1-\xi^2}} = 2.5$$

$$5\xi\sqrt{1-\xi^2} = K$$

$$25\xi^2(1-\xi^2) = K^2$$

$$\xi^2(1-\xi^2) = 0.04K^2$$

$$\xi^4 - \xi^2 + 0.04K^2 = 0$$

$$\xi^2 = \frac{1 \pm \sqrt{1-0.16K^2}}{2}$$

$$\xi = \frac{\pm\sqrt{1-\sqrt{1-0.16K^2}}}{\sqrt{2}} \text{ or } \frac{\pm\sqrt{1+\sqrt{1-0.16K^2}}}{\sqrt{2}}$$

$$= \pm 0.707\sqrt{1-\sqrt{1-0.16K^2}} \text{ or } \pm 0.707\sqrt{1+\sqrt{1-0.16K^2}}$$

$\because$ ξ is a positive real number and $\xi < \frac{1}{\sqrt{2}}$ for resonant peak to occur.

$\therefore$ $$\xi = +0.707\sqrt{1-\sqrt{1-0.16K^2}}$$

[Because $+0.707\sqrt{1+\sqrt{1-0.16K^2}}$ will be >0.707 as $\sqrt{1+x} > 1$ always whether x is greater than 1 or not]

$\because$ $$|M(j\omega)|_{\omega=0} = 1.0 = \frac{K\omega_n^2}{0+0+\omega_n^2}$$

$\therefore$ $$K = 1.0$$

$\therefore$ $$\xi = +0.707\sqrt{1-\sqrt{1-0.16}}$$

$$\xi = 0.2043$$

Q.7 (c) Solution:

(i) Given AM signal,

$$u(t) = [20 + 2\cos 3000\pi t + 10\cos 6000\,\pi t]\cos 2\pi f_c t$$

$$u(t) = 20\cos 2\pi f_c t + 2\cos 3000\pi t \cdot \cos 2\pi f_c t + 10\cos 6000\pi t \cdot \cos 2\pi f_c t$$

$$u(t) = 20\cos 2\pi f_c t + \cos[2\pi(f_c + 1500)t] + \cos[2\pi(f_c - 1500)t] + 5\cos[2\pi(f_c + 3000)t] + 5\cos[2\pi(f_c - 3000)t]$$

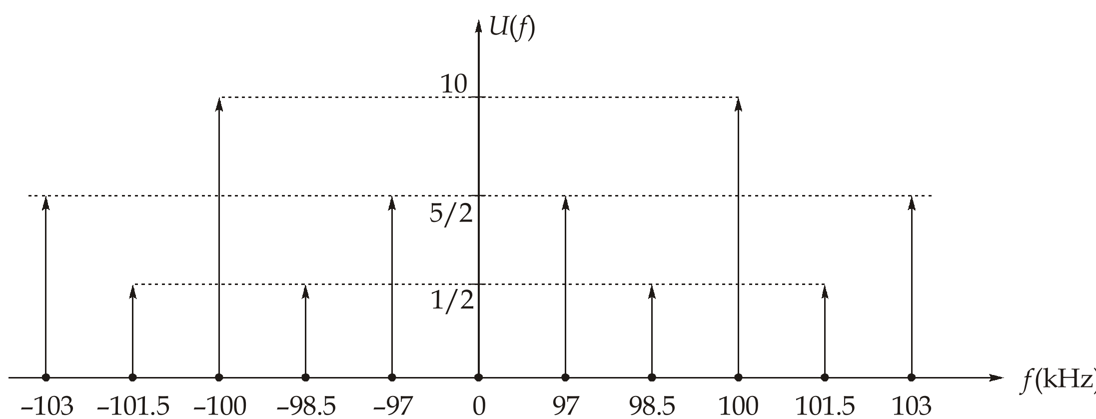
We know that,

$$\cos 2\pi f_c t \longleftrightarrow \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

Spectrum of $u(t)$ is thus obtained as

$$U(f) = 10[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{2}\left[\delta[f - (f_c + 1500)] + \delta[f + (f_c + 1500)]\right]$$
$$+ \frac{1}{2}\left[\delta\left[f - \left(f_c - 1500\right)\right] + \delta\left[f + \left(f_c - 1500\right)\right]\right]$$
$$+ \frac{5}{2}\left[\delta\left[f - \left(f_c + 3000\right)\right] + \delta\left[f + \left(f_c + 3000\right)\right]\right]$$
$$+ \frac{5}{2}\left[\delta\left[f - \left(f_c - 3000\right)\right] + \delta\left[f + \left(f_c - 3000\right)\right]\right]$$

Given f_c = 100 kHz, thus the spectrum plot is given as,



(ii) From the figure,

$\Rightarrow$ Power content of carrier frequency, $f_c = \dfrac{(20)^2}{2} = 200$ W

$\Rightarrow$ Power content of the frequency f_c + 1500 is same as the power content of the frequency f_c –1500, and is equal to,

$$P = \frac{(1)^2}{2} = 0.5 \text{ W}$$

⇒ Power content of the frequency f_c + 3000 is same as the power content of the frequency f_c – 3000, and is equal to,

$$P = \frac{(5)^2}{2} = \frac{25}{2}\text{W} = 12.5 \text{ W}$$

(iii) $u(t) = 20\left[1+\frac{1}{20}(2\cos 3000\pi t + 10\cos 6000\pi t)\right]\cos 2\pi f_c t$

Comparing with standard multi-tone AM signal,

$s(t) = A_c[1+ \mu_1 \cos 2\pi f_{m1}t + \mu_2 \cos 2\pi f_{m2}t]\cos 2\pi f_c t$

$$A_c = 20 \text{ V}, \mu_1 = \frac{1}{10} = 0.1,\ \mu_2 = 0.5$$

$$\therefore \quad \text{Modulation index, } \mu = \sqrt{\mu_1^2+\mu_2^2}$$

$$= \sqrt{(0.1)^2+(0.5)^2} = \frac{\sqrt{26}}{10}$$

$$\mu = 0.5099$$

(iv) $$\text{Total power } P_t = P_c\left(1+\frac{\mu^2}{2}\right)$$

where $$P_c = \frac{A_c^2}{2} = \frac{20\times 20}{2} = 200 \text{ W}, \mu = 0.5099$$

$$\therefore \quad P_t = 200\left[1+\frac{(0.5099)^2}{2}\right] = 226 \text{ W}$$

$$\text{Sideband power, } P_{SB} = P_t - P_c = \frac{P_c\mu^2}{2} = \frac{200\times(0.5099)^2}{2} = 26 \text{ W}$$

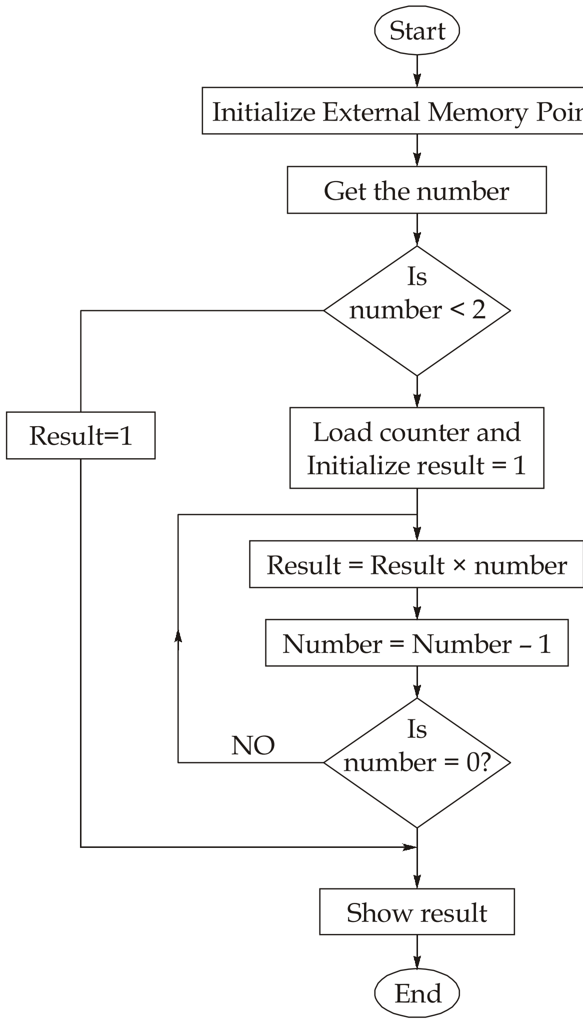
Ratio of sideband power to total power,

$$\eta = \frac{P_{SB}}{P_t} = \frac{26}{226} = 0.11504$$

$$\%\eta = 11.504\%$$

Q.8 (a) Solution:

(i) The flowchart for finding the factorial of a number is as given below:



(ii)

MOV	DPTR, #9000H	; Pointer to external memory location 9000H
MOV	A, @ DPTR	; Read number from external memory into accumulator
MOV	R3, #01 H	; Initialize Low byte of result
MOV	R4, #00 H	; Initialize High byte of result
SUBB	A, #02 H	; Compare whether number < 2
JC	LAST	; If carry generated, jump to LAST (factorial = 1)
MOVX	A, @DPTR	; Reload number from memory
MOV	R3, A	; Store initial value of number in R3 (low byte result)
DEC	A	

```
         MOV    R6, A        ; R6 used as multiplication counter
LOOP:    MOV    R5, R6       ; Copy multiplier into R5 for repeated addition
         MOV    R0, #00 H    ; Clear low byte of temporary product
         MOV    R1, #00 H    ; Clear high byte of temporary product
LOOP 1:  MOV    A, R0        ; LOOP 1 performs multiplication
         ADD    A, R3        ; Add low byte of current result
         MOV    R0, A
         MOV    A, R1
         ADDC   A, R4        ; Add high byte of current result
         MOV    R1, A
         DJNZ   R5, LOOP 1   ; Repeat addition R5 times
         MOV    R3, R0       ; Update factorial low byte
         MOV    R4, R1       ; Update factorial high byte
         DJNZ   R6, LOOP     ; Decrement R6 and continue multiplication
                               until it becomes zero
LAST:    INC    DPTR
         MOV    A, R3        ; Load low byte of factorial result
         MOVX   @DPTR, A     ; Store low byte of factorial result
         INC    DPTR
         MOV    A, R4        ; Load high byte of factorial result
         MOVX   @DPTR, A     ; Store high byte of factorial result
         END
```

Q.8 (b) Solution:

In a multiprocessor-based telecommunication switch, deadlocks can lead to catastrophic network failures, such as dropped calls or lost data packets. Circular wait is one of the four necessary conditions (alongside mutual exclusion, hold-and-read, and no preemption) required for a deadlock to occur.

1. **Analysis of Circular Wait in Telecommunication Switches:**

   Circular Wait occurs when a closed chain of processors exists such that each processor holds at least one resource needed by the next processor in the chain.

   Contribution to Deadlocks

   - **Resource Dependency:** In a switch, resources such as memory buffers, line cards, and routing table are finite. If Processor P_1 holds Buffer A and requests

Buffer B, while P_2 holds Buffer B and requests Buffer A, a circular dependency is formed.

- **Packet Processing Stalls:** Telecommunication switches often use "store-and-forward" or "cut-through" switching. If a circular wait occurs in buffer allocation, the flow of packets stops entirely, leading to a buffer overflow at the ingress ports.
- **Priority Inversion:** In real-time systems, a high-priority process might be waiting for a resource held by a low-priority process involved in a circular wait, causing the entire high-priority task to miss its deadline.

2. Deadlock Prevention Techniques:

To prevent deadlocks without violating the strict real-time constraints of a telecommunication switch, we must eliminate at least one of the four Coffman conditions. Preventing Circular Wait is often the most viable strategy.

A. Hierarchical Resource Allocation (Resource Ordering)

This is the most common technique to break circular wait. All resources are assigned a unique integer rank.

- **Rule:** A processor can only request resources in strictly increasing (or decreasing) order. For example, if a processor holds a resource R_1, it can only request resource R_2 or higher.
- **Real-Time Advantage:** It is a static prevention method. It does not require complex runtime checks like the Banker's Algorithm, thus maintaining low latency for packet switching.
- **Utilization Trade-off:** While it prevents circular wait, it may lead to lower utilization if a processor must grab a high-rank resource early, even if it doesn't need it immediately.

B. Time-out Based Preemption

In telecommunication, "No Preemption" is often a requirement to avoid data corruption. However, controlled preemption via timers can be used.

- **Mechanism:** If a processor cannot acquire a resource within a defined "Real-Time Window," it must release all its currently held resources and retry.
- **Real-Time Advantage:** Ensures that no single process can hang the switch indefinitely. It is crucial for maintaining the Quality of Service (QoS).

C. Priority Ceiling Protocol (PCP)

Designed specifically for real-time multiprocessor systems to prevent deadlocks and minimize priority inversion.

- **Mechanism:** Each resource is assigned a "priority ceiling," which is the priority of the highest-priority task that may lock it. A task can only lock a resource if its priority is strictly higher than the priority ceilings of all resources currently locked by other tasks.
- **Benefit:** PCP inherently prevents circular wait and ensures that a high-priority task is never blocked by a lower-priority task for more than one critical section.

3. Balancing Utilization and Constraints

To maintain high resource utilization while ensuring real-time performance, switches often employ a Hybrid Approach:

1. **Static Partitioning:** Dividing resources (like buffers) into pools for different traffic classes (e.g., Voice vs. Data). This limits the scope of a potential circular wait to a specific pool.
2. **Wait-Die/Wound-Wait Schemes:** Using timestamps to decide whether a process waits or is rolled back when competing for a resource. This is more dynamic than simple ordering and allows for better utilization of line-card buffers.

Q.8 (c) Solution:

(i) 1. Using FIFO: In this algorithm, the page that has been present for the longest time is replaced.

No. of main memory frames = 4

	7	0	1	2	0	3	0	4	2	3	0	3	2	1	2	0	1	7	0	1
0	7	7	7	7	7	3	3	3	3	3	3	3	3	3	2	2	2	2	2	2
1		0	0	0	0	0	0	4	4	4	4	4	4	4	4	4	4	7	7	7
2			1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
3				2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1
					*		*		*	*		*	*			*	*		*	*

No. of page hits = 10

No. of page faults = 20 – 10 = 10

$$\text{Hit ratio} = \frac{\text{Number of page hits}}{\text{Total pages}} = \frac{10}{20} = 0.5$$

2. Using LRU: In this algorithm, page which is least recently used will be replaced.

	7	0	1	2	0	3	0	4	2	3	0	3	2	1	2	0	1	7	0	1
0	7	7	7	7	7	3	3	3	3	(3)	3	(3)	3	3	3	3	3	7	7	7
1		0	0	0	(0)	0	(0)	0	0	0	(0)	0	0	0	0	(0)	0	0	(0)	0
2			1	1	1	1	1	4	4	4	4	4	4	1	1	1	(1)	1	1	(1)
3				2	2	2	2	2	(2)	2	2	2	(2)	2	(2)	2	2	2	2	2
					*		*		*	*	*	*	*		*	*	*		*	*

No. of page hits = 12

No. of page faults = 20 – 12 = 8

$$\therefore \quad \text{Hit Ratio} = \frac{\text{Number of page hits}}{\text{Total pages}} = \frac{12}{20} = 0.6$$

3. Using Optimal Page Replacement: In this algorithm, pages are replaced which would not be used for the longest duration of time in the future.

	7	0	1	2	0	3	0	4	2	3	0	3	2	1	2	0	1	7	0	1
0	7	7	7	7	7	3	3	3	3	3	3	3	3	3	3	3	3	7	7	7
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2			1	1	1	1	1	4	4	4	4	4	4	1	1	1	1	1	1	1
3				2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
					*		*		*	*	*	*	*		*	*	*		*	*

No. of page hits = 12

No. of page faults = 20 – 12 = 8

$$\therefore \quad \text{Hit Ratio} = \frac{\text{Number of page hits}}{\text{Total pages}} = \frac{12}{20} = 0.6$$

(ii) 1. The criteria used in deciding which strategy is best utilized for a particular file is given below:

Contiguous: If file is usually accessed sequentially or if file is relatively small, then use contiguous strategy. Also preferred when fast direct access is required because file blocks are stored continuously.

Linked: If file is large and usually accessed sequentially, then use linked strategy. Useful when file size is dynamic or unpredictable, since blocks can be allocated anywhere in memory.

Indexed: If file is large and usually accessed randomly, then use indexed strategy. Since an index block stores pointers to all file blocks, any block can be accessed directly.

2. Caches allow components of differing speeds to communicate more efficiently by storing data from the slower device, temporarily, in a faster device (the cache). Caches store frequently used data and instructions and are closer to the CPU, thereby reducing the average access time, which improves overall system performance. Caches are, more expensive than the device they are caching for, so increasing the number or size of cache would increase system cost.

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