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Detailed Solutions

ESE-2026 Mains Test Series

Electrical Engineering Test No : 13

Section-A

Q.1 (a) Solution:

(i) When there is no speed control

The system is open loop, then the loop gain $\frac{1}{R} = 0.0$

(a) AFRC $\beta = D + \frac{1}{R}$.

Here $\frac{1}{R} = 0$

$$\Rightarrow \quad \beta = D$$

$$\beta = D = 0.1 \text{ PU MW/Hz}$$

(b) $\Delta f_{ss} = \frac{-M}{\beta}$

$$M = 2\% \Rightarrow \frac{2}{100}$$

$$= \frac{-2}{100} \times \frac{1}{0.1} = -0.2 \text{ Hz}$$

(c) Operating frequency $f' = f + \Delta f_{ss} = 50 + (-0.2) = 48.8$ Hz

(ii) With $R = 4$

(a) AFRC $\beta = D + \frac{1}{R} = 0.1 + \frac{1}{4} = 0.35 \text{ MW/Hz}$

(b) $\Delta_{fss} = \frac{-M}{\beta} = \frac{-2}{100} \times \frac{1}{0.35} = -0.0571 \text{ Hz}$

(c) The operating frequency

$$f' = 50 + (-0.0571)$$
$$= 49.9429 \text{ Hz}$$

Q.1 (b) Solution:

(i) For maximum field current,

$$\alpha_f = 0°$$

Now,
$$V_f = \frac{3V_{ml}}{\pi} = \frac{3 \times 208\sqrt{2}}{\pi} = 280.90 \text{ Volts}$$

Field current,
$$I_f = \frac{V_f}{R_f} = \frac{280.90}{145} = 1.9372 \text{ A}$$

Now, we know that

$$\omega = \frac{2\pi N}{60}$$

$\Rightarrow$
$$\omega = \frac{2\pi \times 900}{60}$$

$\Rightarrow$
$$\omega = 94.24 \text{ rad/s}$$

and Armature current,
$$I_a = \frac{T_a}{K_v I_f} = \frac{116}{1.2 \times 1.9372} = 49.90 \text{ A}$$

Now,
$$E_b = K_v I_f . w = 1.2 \times 1.9372 \times 94.24 = 219.09 \text{ V}$$

and
$$V_a = E_b + I_a R_a = 219.09 + 49.90 \times 0.25 = 231.568 \text{ V}$$

$$V_a = 231.568 = \frac{3V_{ml}}{\pi} \cos\alpha_a$$

$$231.568 = \frac{3 \times 208\sqrt{2}}{\pi} \cos\alpha_a$$

$\Rightarrow$
$$\alpha_a = 34.474°$$

(ii) $\alpha_a = 0$ and

$$V_a = \frac{3 \times 208\sqrt{2}}{\pi} = 280.90 \text{ V}$$

$$E_b = 280.90 - 49.90 \times 0.25 = 268.423 \text{ V}$$

And the speed,

$$\omega = \frac{E_g}{K_v I_f} = \frac{268.423}{1.2 \times 1.9372} = 115.47 \text{ rad/s or } 1102.657 \text{ rpm}$$

(iii)
$$\omega = \frac{1800\pi}{30} = 188.5 \text{ rad/sec}$$

$$E_b = 268.423 \text{ V} = 1.2 \times 188.5 \times I_f$$

$\Rightarrow$
$$I_f = \frac{268.423}{1.2 \times 188.50} = 1.1867 \text{ A}$$

$$V_f = I_f R_f = 172.066 \text{ Volts}$$

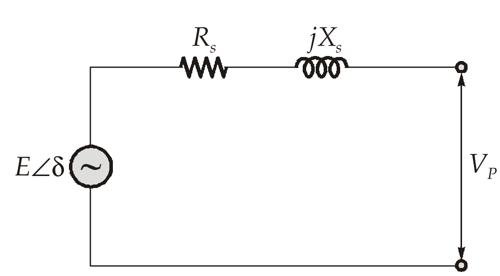
Now,
$$V_f = \frac{3V_{ml}}{\pi}\cos\alpha$$

$$172.066 = \frac{3 \times \sqrt{2} \times 208}{\pi}\cos\alpha_f$$

$$\alpha_f = \cos^{-1}\left(\frac{171.50\pi}{3 \times 208\sqrt{2}}\right)$$

$$\alpha_f = 52.225°$$

Q.1 (c) Solution:



Given: $R_s = 1.5\ \Omega$, $jX_s = j30\ \Omega$

$$V_P = \frac{13.5}{\sqrt{3}} = 7.794 \text{ kV}$$

$$\phi = \cos^{-1}0.8 = 36.87° \text{ (lagging)}$$

Now,
$$3V_P|I|\cos\phi = 1280 \times 10^3$$

$$\text{Current, } |I| = \frac{1280 \times 10^3}{3 \times 7.794 \times 10^3 \times 0.8}$$

$$\therefore \quad I = 68.428 \angle -36.87° \text{ A}$$

Now,
$$E\angle\delta = (R_s + jX_s)I + V_P$$
$$= (1.5 + j30)\,(68.428\angle -36.87°) + V_P$$
$$= (1.5 + j30)\,(68.428\angle -36.87°) + 7794\angle 0°$$
$$= 9244\angle 9.845° \text{ V}$$

Now,
$$\text{Voltage regulation} = \frac{|E| - |V_P|}{|V_P|} = \frac{9244 - 7794}{7794} = 18.6\%$$

Q.1 (d) Solution:

Given signals $y_1(t) = x(2t)$

$$y_2(t) = x\left(\frac{t}{2}\right)$$

(i) Let $y_1(t)$ be periodic with period T_0. Hence,

$$y_1(t) = y_1(t + T_0) \quad \text{...(i)}$$

Now consider,
$$y_1(t) = x(2t) \quad \text{...(ii)}$$

i.e.,
$$x(2t) = y_1(t)$$

$$x\left(2\left[t + \frac{T}{2}\right]\right) = y_1\left(t + \frac{T}{2}\right)$$

$$x(2t + T) = y_1\left(t + \frac{T}{2}\right)$$

Let
$$\frac{T}{2} = T_0$$

$$\Rightarrow \quad T = 2T_0$$

$$\therefore \quad x(2t + T) = y_1(t + T_0)$$

Now from equation (i) and (ii),

$$x(2t + T) = y_1(t) = x(2t)$$

Hence, $x(t)$ is periodic signal with period T, where $T = 2T_0$.

(ii) Let $x(t)$ be periodic with period T, hence

$$x(t) = x(t + T)$$

Now consider
$$x\left(\frac{t}{2}\right) = x\left(\frac{t}{2} + T\right) \quad \text{...(iii)}$$

and $$y_2(t) = x\left(\frac{t}{2}\right) \quad \text{...(iv)}$$

$$y_2(t+T_0) = x[(t+T_0)/2]$$

$$= x\left[\frac{t}{2}+\frac{T_0}{2}\right]$$

Let $\frac{T_0}{2} = T \Rightarrow T_0 = 2T$

$$\therefore \quad y_2(t+T_0) = x\left[\frac{t}{2}+T\right]$$

From equation (iii) and (iv),

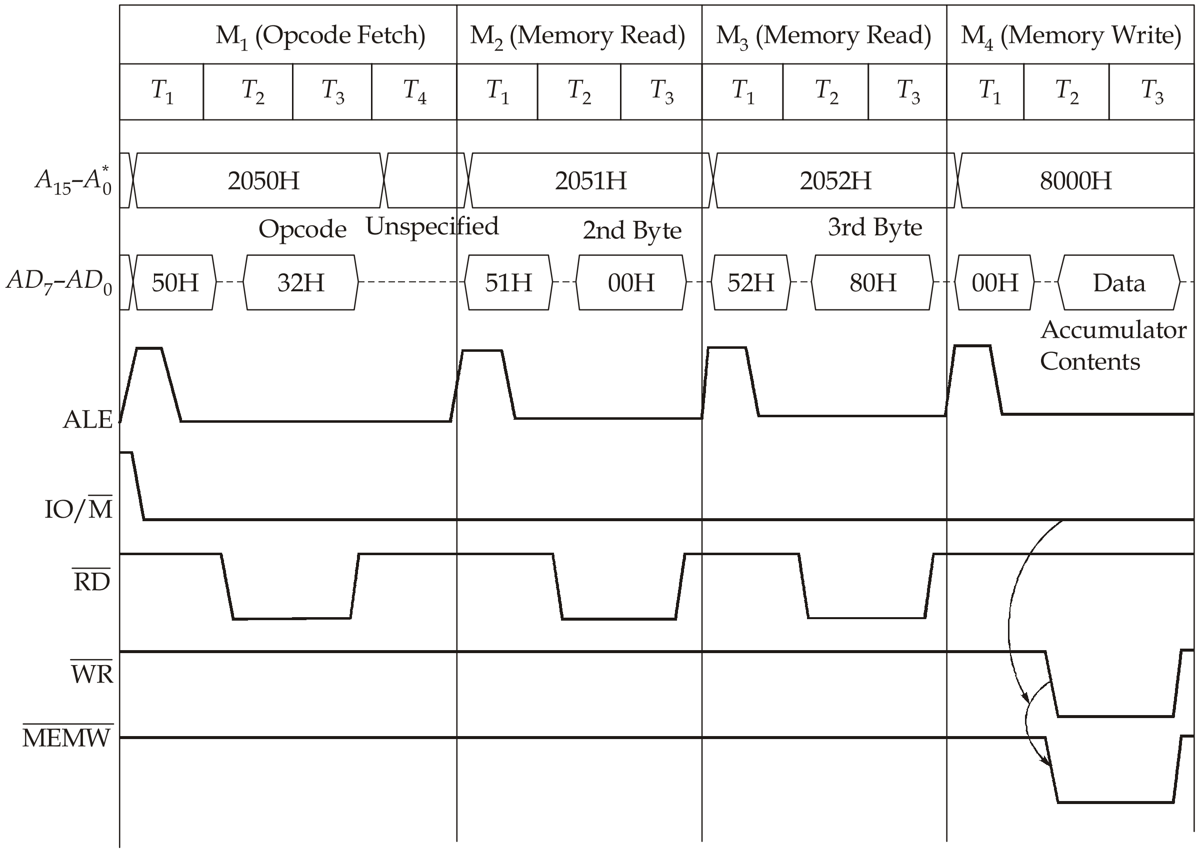
$$y_2(t+T_0) = x\left(\frac{t}{2}\right) = y_2(t)$$

Hence, $y_2(t)$ is periodic signal with period T_0, where $T_0 = 2T$.

Q.1 (e) Solution:

In 8085 instruction set,

- STA stands for Store Accumulator content in memory
- In this instruction, Accumulator 8-bit content will be stored to a memory location whose 16-bit address is indicated in the instruction.
- This instruction is of 3-bytes and takes 4 machine cycle and 13 T-States.
- The first byte will contain the opcode hex 32 H.
- The next byte in memory will hold 00H and after that 80 H will be kept in the last third byte.
- Opcode Fetch transfers the opcode (32H) from the memory location 2050H to the instruction register.
- The 2-byte address is then transferred, 1-byte at a time, from the memory locations 2051H and 2052H to the temporary register.
- During the opcode fetch and memory read machine cycles, the microprocessor places the memory addresses to be read on the address bus and activates ALE in the first T-state (T_1) in order to latch lower order address and uses the lower order address bus as data bus along with the control signals $\overline{\text{MEMR}}$ to read the data from the memory in the remaining T-states.
- The microprocessor places memory address 8000H in the fourth machine cycle on entire address bus.
- The accumulator contents are sent on the data bus, followed by the control signal $\overline{\text{MEMW}}$ to store the accumulator contents at memory location 8000H.



Q.2 (a) Solution:

(i) The battery terminal voltage V_0 is

$$V_0 = \frac{3V_{mL}}{\pi}\cos\alpha = E + I_0R$$

$$V_0 = \frac{3\times\sqrt{2}\times 230}{\pi}\cos\alpha = 200 + (20\times 0.5)$$

But $$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha = 210 \text{ V}$$

Firing angle delay, $$\alpha = \cos^{-1}\frac{210\times\pi}{3\sqrt{2}\times 230} = 47.46^\circ$$

As given load current is constant, the supply current i_A is quasi square wave of aplitude 20 A. i_A flows for 120° (or $2\pi/3$ radians) over every half cycle of 180° or π radians.

∴ Rms value of the supply current I_s over π radians is

$$I_s = \left[\frac{1}{\pi}(20)^2\frac{2\pi}{3}\right]^{1/2} = 20\sqrt{\frac{2}{3}} = 16.33 \text{ A}$$

Rms value of output current,

$$I_{or} = 20 \text{ A}$$

$$\text{Power delivered to load} = EI_0 + I^2_{or}.R$$
$$= [200 \times 20 + (20)^2 \times 0.5] = 4200 \text{ W}$$

Now $\sqrt{3}V_s I_s \cos\phi = 4200$ W

$\therefore$ $$\text{Input supply } pf = \frac{4200}{\sqrt{3} \times 230 \times 16.33} = 0.645 \text{ lag}$$

(ii) When battery is delivering power, then

$$V_0 = -200 + 20 \times 0.5 = -190 \text{ V}$$

When power flows from dc source to ac load, the 3-phase full-converter then works as a 3-phase line commutated inverter

$\therefore$ $$\frac{3V_{ml}}{\pi}\cos\alpha = -190 \text{ V}$$

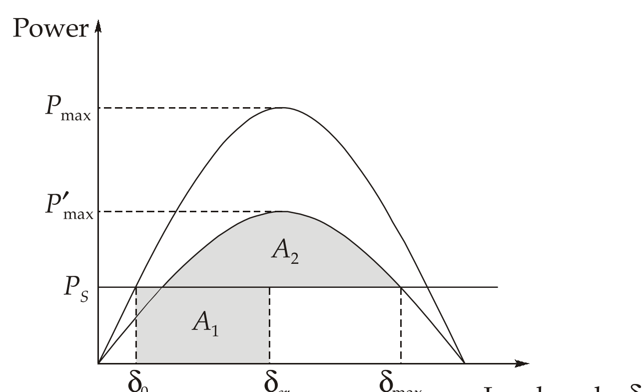
Firing angle delay, $$\alpha = \cos^{-1}\left[\frac{-190 \times \pi}{3\sqrt{2} \times 230}\right] = 127.71°$$

Q.2 (b) Solution:

The power angle curves are shown in figure below:

For curve A, i.e. when both lines are in operation,

Transfer reactance, $$X = 0.25 + \frac{0.5 \times 0.4}{0.5 + 0.4} + 0.05$$
$$= 0.522 \text{ pu}$$



Steady state power limit, $$P_{max} = \frac{EV}{X} = \frac{1.2 \times 1}{0.522} = 2.298 \text{ pu}$$

For initial condition, $P_0 = 1.0$ pu

$$\text{Initial load angle, } \delta_0 = \sin^{-1}\left(\frac{P_0}{P_{max}}\right)$$

$$= \sin^{-1}\left(\frac{1}{2.298}\right) = 0.45 \text{ rad(elec) or } 25.797°$$

For curve B, when one line is in operation

$$\text{Transfer reactance, } X' = 0.25 + 0.5 + 0.05 = 0.8 \text{ pu}$$

$$\text{Steady state power limit, } P'_{max} = \frac{EV}{X'} = \frac{1.2 \times 1.0}{0.8} = 1.5 \text{ pu}$$

Electrical power develoed, $P_E' = P'_{max}\sin\delta = 1.5\sin\delta$ and load angle,

$$\delta_m = 180° - \sin^{-1}\left(\frac{1}{1.5}\right) = 180° - 41.81°$$

$$= 138.19° \text{ or } 2.41 \text{ radian (elec)}$$

Applying equal area criterion for clearing angle δ_c.

$$A_1 = P_0(\delta_c - \delta_0) = 1.0\,(\delta_c - 0.45)$$

$$= \delta_c - 0.45.$$

$$A_2 = \int_{\delta_c}^{\delta_m}\left(P_E' - P_0\right)d\delta = \int_{\delta_c}^{138.19}(1.5\sin\delta - 1)\,d\delta$$

$$= [-1.5\cos\delta - \delta]_{\delta_c}^{138.19°}$$

$$= -1.5(\cos 138.19° - \cos\delta_c) - (2.41 - \delta_c)$$

$$= 1.5\cos\delta_c + \delta_c - 1.292$$

$A_1 = A_2$, we have,

$$\delta_c - 0.45 = 1.5\cos\delta_c + \delta_c - 1.292$$

or

$$\cos\delta_c = \frac{1.292 - 0.45}{1.5} = 0.561$$

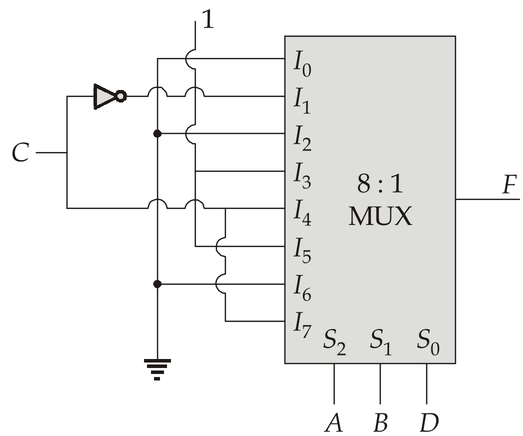
$$\text{Critical clearing angle, } \delta_c = \cos^{-1}(0.5613) = 55.85° \text{ (electrical)}$$

Q.2 (c) (i) Solution:

$$F(A, B, C, D) = \Sigma m(1, 5, 7, 9, 10, 11, 15)$$

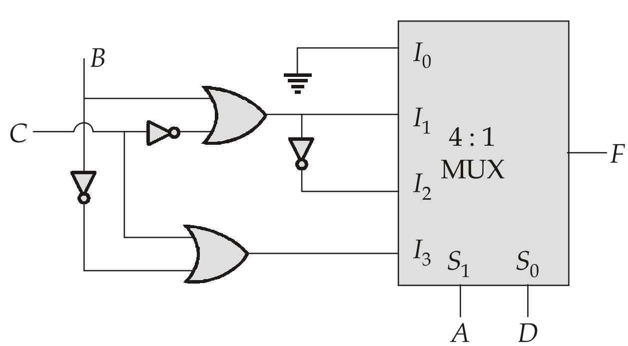
1. 8 × 1 Multiplexer with A, B and D as select lines

	I_0 $\bar{A}\bar{B}\bar{D}$	I_1 $\bar{A}\bar{B}D$	I_2 $\bar{A}B\bar{D}$	I_3 $\bar{A}BD$	I_4 $A\bar{B}\bar{D}$	I_5 $A\bar{B}D$	I_6 $AB\bar{D}$	I_7 ABD
$\bar{C}$	0	(1)	4	(5)	8	(9)	12	13
C	2	3	6	(7)	(10)	(11)	14	(15)
	0	$\bar{C}$	0	1	C	1	0	C

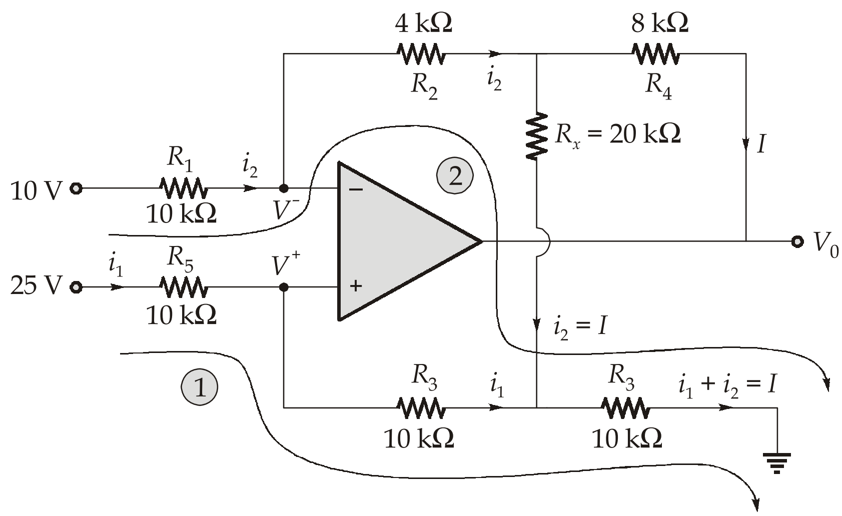


2. 4 : 1 Multiplexer with A and D as select lines.

	I_0 $\bar{A}\bar{D}$	I_1 $\bar{A}D$	I_2 $A\bar{D}$	I_3 AD
$\bar{B}\bar{C}$	0	(1)	8	(9)
$\bar{B}C$	2	3	(10)	(11)
$B\bar{C}$	4	(5)	12	13
BC	6	(7)	14	(15)
	$I_0 = 0$	$I_1 = (B+\bar{C})$	$I_2 = \bar{B}C$	$I_3 = (\bar{B}+C)$
			$I_2 = \overline{(B+\bar{C})}$	



Q.2 (c) (ii) Solution:



By KVL in input loop:

$$-10 + 10i_2 - 10i_1 + 25 = 0$$

$$\Rightarrow \quad i_1 - i_2 = 1.5 \quad \text{...(i)}$$

By KVL in loop-1,

$$-25 + 10i_1 + 10i_1 + 10(i_1 + i_2 - I) = 0$$

$$30i_1 + 10i_2 - 10I = 25 \quad \text{...(ii)}$$

By KVL in loop-2,

$$-10 + 10i_2 + 4i_2 + 20(i_2 - I) + 10(i_1 + i_2 - I) = 0$$

$$10i_1 + 44i_2 - 30I = 10 \quad \text{...(iii)}$$

On solving equation (i),(ii) and (iii), we get

$$i_1 = \frac{2}{3}\text{mA}$$

$$i_2 = \frac{-5}{6}\text{mA}$$

$$I = \frac{-4}{3}\text{mA}$$

Now by KVL,

$$-10 + 10i_2 + 4i_2 + 8I + V_0 = 0$$

$$V_0 = 10 - 14i_2 - 8I$$

$$= 10 - 14\times\left(\frac{-5}{6}\right) - 8\left(\frac{-4}{3}\right)$$

$$V_0 = 32.33 \text{ Volts}$$

Q.3 (a) Solution:

(i) This problem, we will proceed using the eigen function property of LSI system.

If the input to an LSI system is $x(n) = \cos\omega_o n$, then the response will be

$$y(n) = \left|H(e^{j\omega_o})\right|\cos(n\omega_o + \phi_n(\omega_o))$$

Therefore, we need to find the frequency response of the system we know the unit sample response of ideal low pass filter,

$$H_1(e^{j\omega}) = \begin{cases} 1; & |\omega| \le \omega_c \\ 0; & \omega_c < |\omega| \le \pi \end{cases}$$

Therefore, $$h_1(n) = \frac{\sin n\omega_c}{\pi n}$$

Because, $h(n) = 2h_1(n-1)$ with $\omega_c = \dfrac{\pi}{2}$, an expression may be derived for $H(e^{j\omega})$ in terms of $H_1(e^{j\omega})$ as follows :

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-jn\omega} = \sum_{n=-\infty}^{\infty} h_1(n-1)\cdot e^{-j\omega n}$$

$$= 2\sum_{n=-\infty}^{\infty} h_1(n)e^{-j(n+1)\omega} = 2e^{-j\omega}\sum_{n=-\infty}^{\infty} h_1(n)e^{-j\omega n}$$

$$= 2e^{-j\omega}H_1(e^{j\omega})$$

Therefore, $$H(e^{j\omega}) = \begin{cases} 2e^{-j\omega}, & |\omega| \le \dfrac{\pi}{2} \\ 0, & \dfrac{\pi}{2} < |\omega| \le \pi \end{cases}$$

Because $|H(e^{j\omega})| = 0$ at $\omega = \dfrac{3\pi}{4}$, the sinusoid in x(n) is filtered out and the output is simply.

$$y(n) = 2\left|H(e^{j\pi/4})\right|\cos\left(\frac{n\pi}{4} + \phi_n\left(\frac{\pi}{4}\right)\right)$$

$$y(n) = 4\cos\left(\frac{n\pi}{4} - \frac{\pi}{4}\right) = 4\cos\left((n-1)\frac{\pi}{4}\right)$$

(ii) First we take the one-sided z-transform of each term in the difference equation.

$$Y(z) = z^{-1}Y(z) + Y(-1) - [z^{-2}Y(z) + z^{-1}Y(-1) + Y(-2)] + \frac{1}{2}X(z) + \frac{1}{2}z^{-1}X(z)$$

Substituting the given values for the initial condition, we have

$$Y(z) = z^{-1}Y(z) + \frac{3}{4} - z^{-2}Y(z) - \frac{3}{4}z^{-1} - \frac{1}{4} + \frac{1}{2}X(z) + \frac{1}{2}z^{-1}X(z)$$

Because $$x(n) = \left(\frac{1}{2}\right)^n u(n) \Rightarrow X(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)}; \quad |z| > \frac{1}{2}$$

which gives $$Y(z) = \frac{\frac{1}{2} - \frac{3}{4}z^{-1}}{1 - z^{-1} + z^{-2}} + \frac{\frac{1}{2} + \frac{1}{2}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - z^{-1} + z^{-2})}$$

Expanding the second term using partial fraction expansion, we have

$$Y(z) = \frac{\frac{1}{2} - \frac{3}{4}z^{-1}}{1 - z^{-1} + z^{-2}} + \frac{\frac{1}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{z^{-1}}{1 - z^{-1} + z^{-2}}$$

$$Y(z) = \frac{\frac{1}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{1}{2} + \frac{1}{4}z^{-1}}{1 - z^{-1} + z^{-2}}$$

Therefore, $$y(n) = \left(\frac{1}{2}\right)^{n+1} u(n) + \left[\frac{\sqrt{3}}{6}\sin\left(\frac{4\pi}{3}\right) + \frac{\sqrt{3}}{3}\sin(n-1)\frac{\pi}{3}\right]u(n)$$

Q.3 (b) Solution:

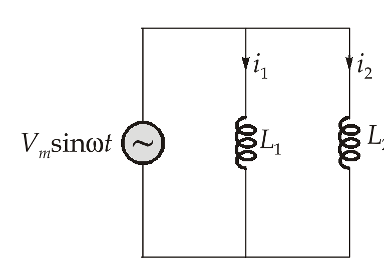
(i) $$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

$$= \frac{1}{2}\left(\frac{\partial L_{11}}{\partial \theta}\right)i_1^2 + \left(\frac{\partial L_{12}}{\partial \theta}\right)i_1 i_2 + \frac{1}{2}\left(\frac{\partial L_{22}}{\partial \theta}\right)i_2^2$$

Substituting the value of inductance.

$$T_f = -(\sin\theta)\, i_1 i_2$$

Using KVL in given circuit



$$V_m \sin\omega t = 2\frac{di_1}{dt} + \cos\theta\frac{di_2}{dt}$$

and $$V_m \sin\omega t = (\cos\theta)\frac{di_1}{dt} + 2\frac{di_2}{dt}$$

Solving these we get,

$$\frac{di_1}{dt} = \frac{di_2}{dt} = \frac{V_m \sin\omega t}{(2+\cos\theta)}$$

Integrating, $$i_1 = i_2 = -\frac{V_m \cos\omega t}{\omega(2+\cos\theta)}$$

Substituting in T_f, $$T_f = -\frac{V_m^2 \cos^2 \omega t}{\omega^2 (2+\cos\theta)^2}\sin\theta$$

$$T_f(av) = -\frac{V_m^2 \sin\theta}{2\omega^2 (2+\cos\theta)^2}$$

Given: $\theta = 30°$, $$V = 100 \sin 314t$$

$\therefore$ $$T_f(av) = -\frac{(100)^2 \sin 30°}{2(2+\cos 30°)^2 \times (314)^2} = -3.086886 \times 10^{-3}\text{ Nm}$$

(ii) If coil 2 is shorted, $$0 = \cos\theta\frac{di_1}{dt} + 2\frac{di_2}{dt}$$

$$\frac{di_2}{dt} = -\frac{1}{2}(\cos\theta)\frac{di_1}{dt}$$

or $$i_2 = -\frac{1}{2}(\cos\theta)i_1$$

Given: $$i_1 = I_m \sin\omega t$$

$\therefore$ $$i_2 = -\frac{1}{2}I_m(\cos\theta)\sin\omega t$$

Substituting in T_f, $$T_f = (\sin\theta)\times\frac{1}{2}I_m^2 \sin^2 \omega t \cos\theta$$

$$= \frac{1}{2}I_m^2(\sin\theta)\times(\cos\theta)\sin^2\omega t$$

$$T_f(av) = \frac{1}{8}I_m^2(\sin 2\theta) \quad [\because 2\sin\theta\cos\theta = \sin 2\theta]$$

Given: $\theta = 45°$, $$I_m = \sqrt{2}$$

$\therefore$ $$T_f(av) = \frac{1}{8}\times 2\sin 90° = 0.25\text{ Nm}$$

Q.3 (c) Solution:

Assume $$(\text{kVA})_{base} = 30\text{ MVA}$$

$$(\text{kV})_{base} = 11\text{ kV at generator terminal}$$

For generator,

$$X_1 = X_2 = j0.1 \times \frac{30}{20} = j0.15 \text{ pu}$$

and $$X_0 = j0.15 \times \frac{30}{20} = j0.225 \text{ pu}$$

For transformer (1)

$$X_1 = X_2 = X_0 = 0.12 \text{ pu}$$

For transformer (2)

$$X_0 = X_1 = X_2 = j0.05 \times \frac{30}{20} = j0.075$$

For transmission line,

$$Z_{base} = \frac{22^2}{30} = 16.133\ \Omega$$

Hence, pu impedance of transmission line is

$$Z_{TL(pu)} = \frac{1 + j5}{16.133} = (0.062 + j0.31) \text{ pu}$$

Since load is unity power factor, so it can be represented by equivalent resistance R.

$$R_{actual} = \frac{V^2}{P} = \frac{(1.1)^2}{10} = 0.121$$

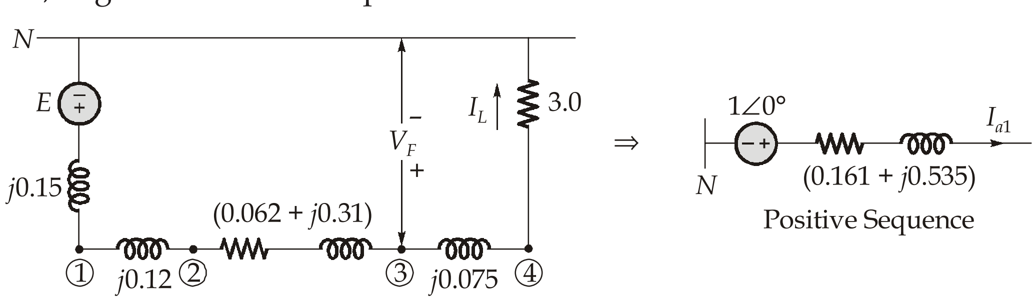
$$R_{pu} = \frac{R_{actual}}{(kV)_b^2} \times S_{base} = \frac{0.121}{(1.1)^2} \times 30 = 3 \text{ pu}$$

Per unit value of fault resistance,

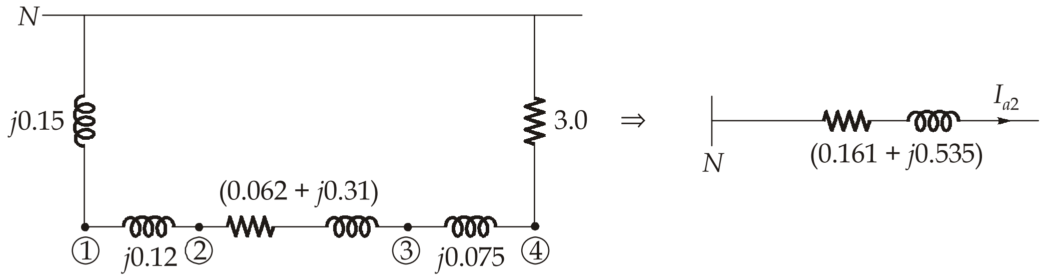
$$R_{f(pu)} = \frac{6.6 \times 30}{(22)^2} = 0.409$$

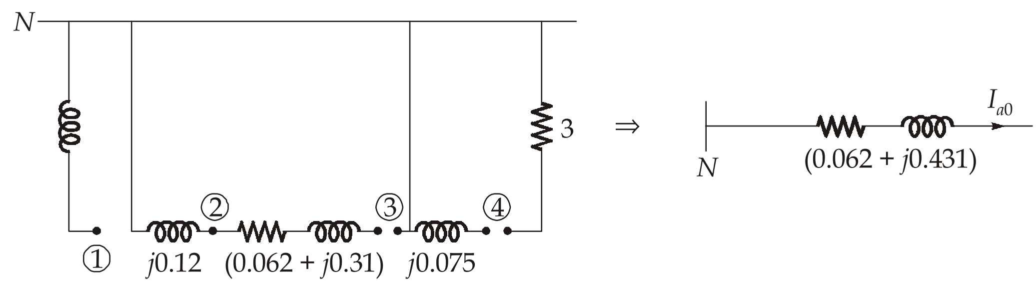
Load current, $$I_{L(pu)} = \frac{P_{pu}}{V_{pu}} = \frac{\left(\frac{10}{30}\right)}{1.0} = 0.33\angle 0° \text{ pu}$$

Positive, negative and zero-sequence network are shown as



Positive Sequence





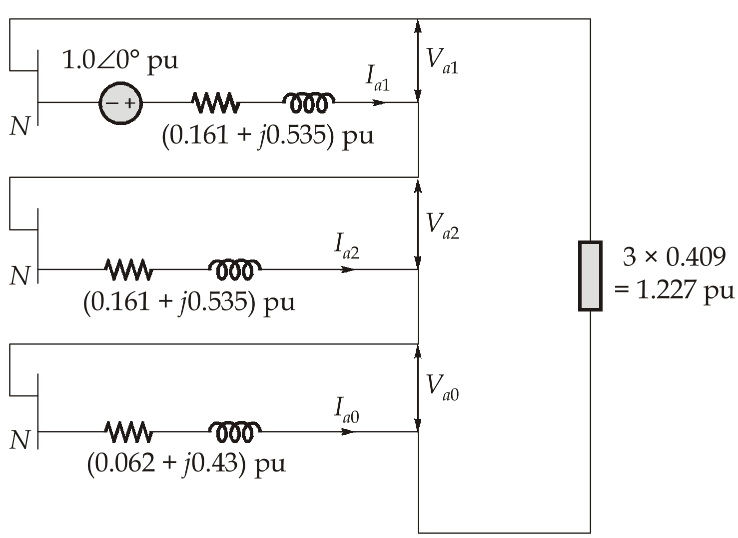
Assuming a prefault voltage of 1.0 pu at node (4), Thevenins equivalent voltage at node (3),

$$V_F = 1.0 + 0.33 \times j0.075 = 1.0\angle 1.46° \text{ pu}$$

Assuming that V_F is the reference phasor at the fault point, i.e., node (3).

$$V_F = 1.0\angle 0° \text{ pu}$$

The interconnected network for an SLG at node (3) is shown



From the interconnected network

$$I_{a0} = \frac{V_F}{Z_1 + Z_2 + Z_0 + 3R_f}$$

$$= \frac{1}{0.384 + j1.50 + 1.227}$$

$$I_{a0} = \frac{1}{1.611 + j1.501} = 0.459\angle{-43°} \text{ pu}$$

Fault current, $I_f = 3I_{a0} = 1.3624\angle{-43°}$ pu

Actual fault current, $I_f = 1.3624 \times \dfrac{30 \times 10^3}{22\sqrt{3}} = 1072.66\angle{-43°}$ Amp

Q.4 (a) Solution:

As this voltage waveform has quarter wave symmetry,

$$a_n = 0,\ a_o = 0$$

$$b_n = \frac{4V_s}{\pi}\left[\int_0^{\alpha_1} \sin(n\omega t)d(\omega t) - \int_{\alpha_1}^{\alpha_2} \sin(n\omega t)d(\omega t) + \int_{\alpha_2}^{\pi/2} \sin(n\omega t)d(\omega t)\right]$$

$$= \frac{4V_s}{\pi}\left[\frac{(1-\cos n\alpha_1)}{n} - \frac{(\cos n\alpha_1 - \cos n\alpha_2)}{n} + \frac{\cos n\alpha_2}{n}\right]$$

$$b_n = \frac{4V_s}{n\pi}[1 - 2\cos n\alpha_1 + 2\cos n\alpha_2]$$

Therefore, $b_1 = \dfrac{4 \times 200}{\pi}[1 - 2\cos(23.62°) + 2\cos(33.30°)]$

$$b_1 = 213.692 \text{ V}$$

$$b_3 = 0;\quad b_5 = 0$$

$$b_7 = \frac{4 \times 200}{7\pi}[1 - 2\cos(7 \times 23.62°) + 2\cos(7 \times 33.30°)]$$

$$b_7 = 63.08 \text{ V}$$

$$b_9 = \frac{4 \times 200}{9\pi}[1 - 2\cos(9 \times 23.62°) + 2\cos(9 \times 33.30°)]$$

$$b_9 = 104.015 \text{ Volts}$$

$$b_{11} = \frac{4 \times 200}{11\pi}[1 - 2\cos(11 \times 23.62°) + 2\cos(11 \times 33.30°)]$$

$$b_{11} = 77.35 \text{ Volts}$$

Therefore, output voltage expression upto 11th harmonic is

$$v_o(t) = 213.70 \sin(\omega_o t) + 63.08 \sin(7\omega_o t) + 104.015 \sin(9\omega_o t) + 77.35 \sin(11\omega_o t) \text{ Volts}$$

Output current expression can be given as

$$\vec{Z}_1 = R + j\omega_o L = 10 + j314.15 \times 20 \times 10^{-3} = 11.81\angle 32.14° \ \Omega$$

$$\vec{Z}_7 = 10 + j14\pi = 45.10\angle 77.20° \ \Omega$$

$$\vec{Z}_9 = 10 + j18\pi = 57.42\angle 80° \ \Omega$$

$$\vec{Z}_{11} = 10 + j22\pi = 69.83\angle 81.76° \ \Omega$$

$$i_o(t) = \frac{213.70}{11.81}\sin(\omega_o t - 32.14°) + \frac{63.08}{45.10}\sin(7\omega_o t - 77.80°) + \frac{104.015}{57.42}\sin(9\omega_o t - 80°) + \frac{77.35}{69.83}\sin(11\omega_o t - 81.76°)$$

$$i_o(t) = 18.09 \sin(\omega_o t - 32.14°) + 1.40 \sin(7\omega_o t) + 1.81 \sin(9\omega_o t - 80°) + 1.107 \sin(11\omega_o t - 81.76°) \text{ A}$$

$$I_{o(\text{rms})} = \frac{\sqrt{18.09^2 + 1.40^2 + 1.81^2 + 1.107^2}}{\sqrt{2}}$$

$$I_{o(\text{rms})} = 12.917 \text{ A}$$

Power delivered to load, $P_L = I_{o(\text{rms})}^2 \times R$

$$P_L = (12.917)^2 \times 10$$

$$P_L = 1668.489 \text{ kW}$$

$\Rightarrow$ Distortion factor, $g = \dfrac{I_{o1}}{I_{or}} = \dfrac{12.791}{12.917} = 0.99$

As $I_{o1} = \dfrac{18.09}{\sqrt{2}}$

Total harmonic distortion in output current is,

$$\text{T.H.D.}_\text{I} = \sqrt{\frac{1}{g^2} - 1} = \sqrt{\left(\frac{1}{0.99}\right)^2 - 1}$$

$$\text{T.H.D.}_\text{I} = 0.1424 \text{ or } 14.24\%$$

Q.4 (b) Solution:

(i) Given, open-loop transfer function,

$$G(s)H(s) = \frac{K(s+20)}{(s+2)(s+4)(s+10)}$$

The open-loop system has three finite poles i.e., p = 3 and one finite zero. i.e., z = 1.

The number of asymptotes,

$$p - z = 3 - 1 = 2$$

The angle of asymptotes,

$$\theta_A = \frac{(2m+1)180^\circ}{p-z}$$

where, m = 0, 1

for $m = 0;\ \theta_A = \frac{180^\circ}{2} = 90^\circ$

for $m = 1;\ \theta_A = \frac{3\times 180^\circ}{2} = 270^\circ$

There are two asymptotes at 90° and 270° respectively.

(ii) The asymptote's real axis intercept is called centroid (σ),

$$\sigma = \frac{\Sigma(\text{sum of open-loop poles}) - \Sigma(\text{sum of open-loop zeros})}{p-z}$$

$$= \frac{-10-4-2-(-20)}{3-1}$$

$$\therefore \quad \sigma = 2$$

(iii) The root locus exist on real axis to the left of an odd number of poles and zeros of open-loop transfer function $G(s)H(s)$. Hence, root locus exists on the real-axis for $-4 \le s \le -2$ and $-20 \le s \le -10$.

Break-away points: The characteristic equation is

$$1 + G(s)\cdot H(s) = 0$$

$$\Rightarrow \quad 1+\frac{K(s+20)}{(s+2)(s+4)(s+10)} = 0$$

$$\Rightarrow \quad K = \frac{-(s+2)(s+4)(s+10)}{(s+20)} = \frac{(-s+2)(s^2+14s+40)}{(s+20)}$$

$$\Rightarrow \quad K = \frac{-(s^3+16s^2+68s+80)}{(s+20)}$$

For break-away points,

$$\frac{dK}{ds} = 0$$

$\Rightarrow$ $-(3s^2 + 32s + 68)(s + 20) + (s^3 + 16s^2 + 68s + 80) = 0$

$\Rightarrow$ $-3s^3 - 92s^2 - 708s - 1360 + s^3 + 16s^2 + 68s + 80 = 0$

$\Rightarrow$ $2s^3 + 76s + 640s + 1280 = 0$

$\Rightarrow$ $s = -27.04, -8, -2.96$

Since, root locus exist on real axis for $-4 \le s \le -2$. Hence, $s = -2.96$ is the valid break-point.

Intersection with imaginary axis.

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \quad 1+\frac{K(s+20)}{(s+2)(s+4)(s+10)} = 0$$

$\Rightarrow$ $s^3 + 16s^2 + 68s + 80 + Ks + 20K = 0$

$\Rightarrow$ $s^3 + 16s^2 + (68 + K)s + (80 + 20K) = 0$

Applying Routh-Hurwitz criterion,

$$\begin{array}{c|cc} s^3 & 1 & 68+K \\ s^2 & 16 & 80+20K \\ s^1 & \dfrac{16(68+K)-(80+20K)}{16} & \\ s^0 & 80+20K & \end{array}$$

For roots to lie on imaginary axis, odd row must be zero.

Hence, $16(68 + K) - (80 + 20K) = 0$

$\Rightarrow$ $1088 + 16K - 80 - 20K = 0$

$\Rightarrow$ $4K = 1008$

$\Rightarrow$ $K = 252$

The location of poles at imaginary axis are given by

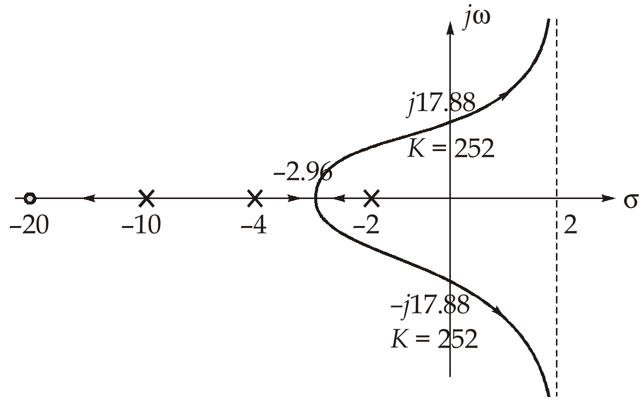
$$16s^2 + (80 + 20K) = 0$$

$\Rightarrow$ $16s^2 = -(80 + 20 \times 252) = -5120$

$\Rightarrow$ $s^2 = -320$

$\Rightarrow$ $s = \pm j17.88$

The root locus of the given system,



(iv) Since the root locus crosses over to the right half of $j\omega$ plane, the system will become unstable for a sufficiently high value of the feedback gain (K). In addition, large gain in the stable region leads to increased overshoot and longer settling time; both of these qualities are generally undesirable.

Q.4 (c) Solution:

(i) For rotor circuit, $\frac{R_2}{s} >> X_2 \quad \therefore$ Neglect X_2

$$R_2 = \frac{61}{2} = 30.5 \text{ m}\Omega$$

$$\omega_s = \frac{120f}{P} \times \frac{2\pi}{60} = \frac{4\pi f}{P} = \frac{4\pi \times 50}{12} = 52.36 \text{ rad/s}$$

$$\omega = \omega_s(1 - s) = 52.36 \times (1 - 0.045) = 50 \text{ rad/s}$$

At full load $\quad 200 \times 10^3 = \omega T_{fan} = 50\, T_{fan}$

$$T_{fan} = 4000 \text{ Nm}$$

Torque developed in motor,

$$T_d = \frac{3}{\omega_s} \cdot \frac{I_2^2 R_2}{s}$$

$$4000 = \frac{3}{52.36} \cdot \frac{I_2^2 \times 30.5 \times 10^{-3}}{(0.045)}$$

$\Rightarrow \quad I_2 = 320.94 \text{ A}$

Rotor circuit voltage (stand still),

$$E_2 = I_2\left(\frac{R_2}{s}\right) = 320.94 \times \frac{30.5 \times 10^{-3}}{0.045} = 217.53 \text{ V}$$

The fan speed is to be reduced to

$$n = 450 \text{ rpm}$$

$$= 450 \times \frac{2\pi}{60} \text{rad/s} = 47.12 \text{ rad/s}$$

$$\text{Slip, } s = \frac{500 - 450}{500} = 0.1$$

$$I_{2\,(\text{new})} = \frac{217.53}{R_t / s} = \frac{21.753}{R_t}, \quad R_t = \text{total rotor resistance}$$

$$T_{(\text{new})} = 4000 \times \left(\frac{47.12}{50}\right)^2 = 3552.47 \text{ Nm}$$

$$T_{(\text{new})} = \frac{3}{\omega_s} \cdot \frac{I_{2(\text{new})}^2 \cdot R_t}{s}$$

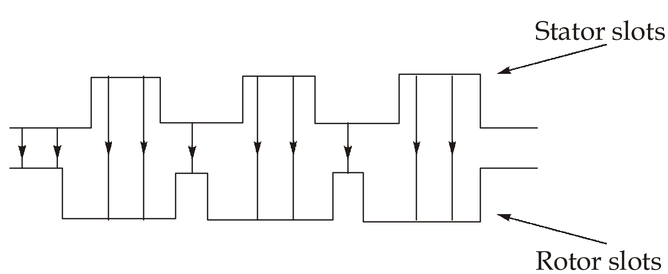
$$3552.47 = \frac{3}{\omega_s} \cdot \left(\frac{21.753}{R_t}\right) \times \frac{R_t}{0.1}$$

$$R_t = 76.32 \text{ m}\Omega$$

$$R_{\text{ext}} = 76.32 - 30.5 = 45.82 \text{ m}\Omega$$

(ii) Cogging or Magnetic locking - with the number of stator slots S_1 equal to or an integral multiple of rotor slots S_2, the variation of reluctance as a function of space will be quite pronounced resulting in strong alignment forces at the instant of starting. These forces may create an aligning torque stronger than the accelerating torque with consequent failure of the motor to start.

This phenomenon is known as 'cogging'.



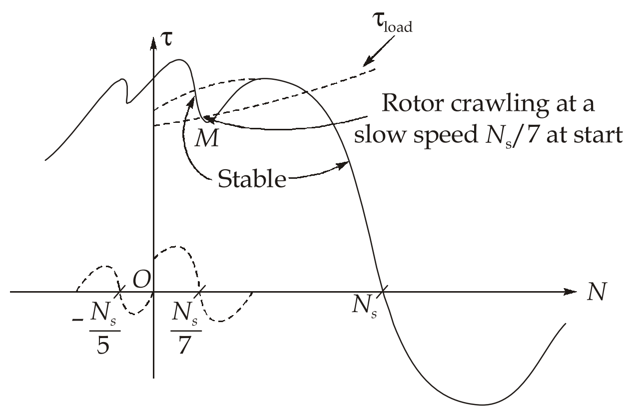
Cogging can be avoided by using skewed rotor.

Crawling : Occurs due to space harmonics of the *mmf* wave e.g. fifth and seventh harmonics which correspond to poles five and seven times that of the fundamental.

The space-phase difference are

ϕ	A	B	C	
Fundamental ϕ_1	$\phi_m\angle 0°$	$\phi_m\angle -120°$	$\phi_m\angle +120°$	
ϕ_3 (9^{th}, 15^{th}, 21^{th}....)	0°	–360°	+360°	$\rightarrow$ Constitute zero sequence
ϕ_5 (11^{th}, 17^{th}, 23^{rd}....)	0°	+120°	–120°	$\rightarrow$ Constitute negative sequence $\Rightarrow \phi_5$ rotates in opposite direction
ϕ_7 (13^{th}, 19^{th}, 25^{th}....)	0°	–120°	+120°	$\rightarrow$ Constitute positive sequence

The 5^{th} and 7^{th} harmonic produce their own asynchronous (induction) torques of the same generator torque-slip shape as that of fundamental. A marked 'saddle' effect is observed with the **stable** region of operation (negative torque-slip slope) around $\frac{1}{7}$th normal speed $\left(s = \frac{6}{7}\right)$.



Note: Crawling and cogging both are particularly related to squirrel cage induction motor.

Section-B

Q.5 (a) Solution:

(i) Surge tank: The load on a generator keeps on fluctuating. Therefore the water intake to the turbine has to be regulated according to the load. A reduction in load on the alternator causes the governor to close the turbine gates. Sudden closure of turbine gates creates an increased pressure known as water hammer, in the penstock, when the governor opens the turbine gates suddenly to admit more water, there is a tendency to cause a vaccum in the penstock. The function of the surge tank is to absorb these sudden changes in water requirements so as to prevent water hammer and vaccum.

The surge tank helps in stablising the velocity and pressure in the penstock and reduces water hammer and negative pressure.

(ii) **Penstock:** A penstock carries water from the water storage system to the tubine. It may be a low pressure type or high pressure type. A low pressure penstock may consists of flume or a steel pipe. The high pressure penstock consists of thick steel pipes. The diameter may be upto a few meters for large units. Each turbine has a separate penstock. Small size plants have penstocks of concrete.

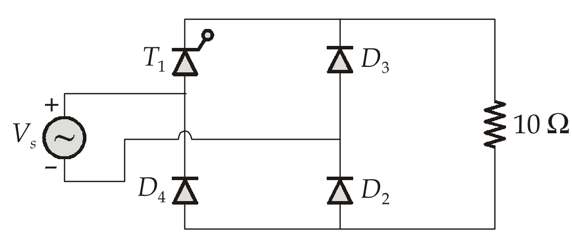
A penstock may be carried below earth surface or exposed.

(iii) **Spillway:** Every dam is provided with an arrangement to discharge excess. water during floods. The arrangement may be a spillway or a by-pass tunnel or conduit. The spillway should be so designed as to discharge the major flood waters without damage to the dam but at the same time maintain a predetermined head. Spillways are classified as over flow spillway, chute spillway, side channel spillway, shaft spillway and siphone spillway.

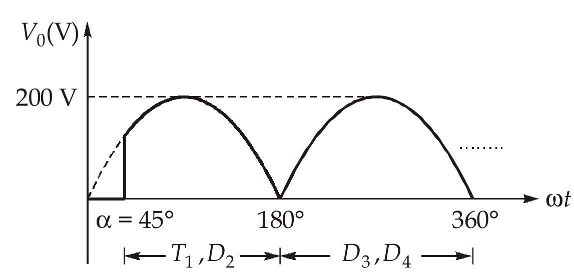
The type of spillway for a particular location depends on type of dam and topographical, hydrological and geological factors.

(iv) **Tailrace:** A tailrace is required to discharge the water, leaving the turbine into the river. It is necessary that the draft tube must maintain water sealed all the time. Impulse turbines do not need a draft tube and discharges water directly. The design and size of the tailrace should be such that water has a free exit and the jet of water, after it leaves turbine, has unimpeded passage.

Q.5 (b) Solution:



The output voltage (V_0) waveform is shown below,



The rms voltage,
$$V_{0r} = \sqrt{\frac{1}{2\pi}\left(\int_{\alpha}^{\pi}(200\sin\omega t)^2 d(\omega t) + \int_{\pi}^{2\pi}(-200\sin\omega t)^2 d(\omega t)\right)}$$

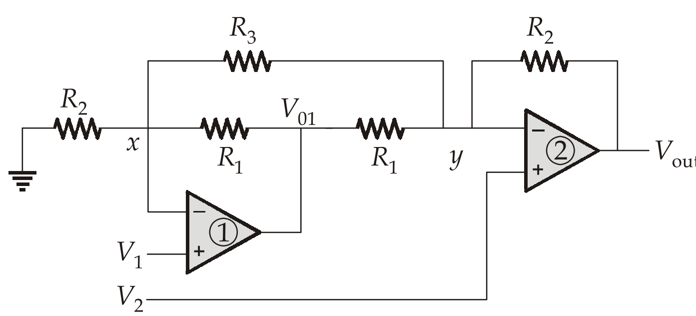
$$V_{0r} = \frac{200}{\sqrt{2\pi}}\left[\sqrt{\int_{\alpha}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t + \int_{\pi}^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t}\right]$$

$$V_{0r} = \frac{200}{\sqrt{2\pi}}\left[\sqrt{\frac{1}{2}\left(\left(\pi - \frac{\pi}{4}\right) + \frac{1}{2}\sin 2\times\frac{\pi}{4}\right) + \left(\frac{\pi}{2} - 0\right)}\right]$$

$$V_{0r} = 138.17 \text{ Volt}$$

$$P_0 = \frac{V_{0\,\text{rms}}^2}{R} = \frac{138.17^2}{10} = 1909.115 \text{ Watts}$$

Q.5 (c) Solution:



Let the voltage at node x is V_x and at node y is V_y.

Now due to virtual short circuit,

$$V_x = V_1 \text{ and } V_y = V_2$$

For op-amp (1)

Applying KCL at inverting terminal,

$$\frac{0 - V_1}{R_2} = \frac{V_1 - V_2}{R_3} + \frac{V_1 - V_{01}}{R_1}$$

$$\frac{V_{01}}{R_1} = V_1\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{V_2}{R_3} \quad \text{...(i)}$$

For op-amp (2)

Applying KCL at inverting terminal,

$$\frac{V_2 - V_{01}}{R_1} + \frac{V_2 - V_1}{R_3} + \frac{V_2 - V_{out}}{R_2} = 0$$

$$V_2\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{V_1}{R_3}-\frac{V_{out}}{R_2} = \frac{V_{01}}{R_1} \quad ...(ii)$$

From equation (i) and (ii),

$$V_1\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{V_2}{R_3} = V_2\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{V_1}{R_3}-\frac{V_{out}}{R_2}$$

$$\frac{V_{out}}{R_2} = [V_2-V_1]\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]+\frac{(V_2-V_1)}{R_3}$$

$$\frac{V_{out}}{(V_2-V_1)} = \left[1+\frac{R_2}{R_1}+\frac{R_2}{R_3}\right]+\frac{R_2}{R_3}$$

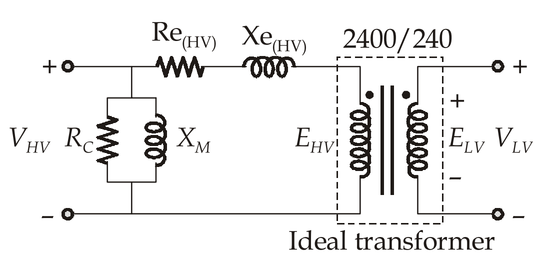
$$\frac{V_{out}}{(V_2-V_1)} = 1+\frac{R_2}{R_1}+\frac{2R_2}{R_3}$$

when $R_1 = R_2 = R_3$

then
$$\frac{V_{out}}{(V_2-V_1)} = 2+1+1$$

$$\frac{V_{out}}{(V_2-V_1)} = 4$$

Q.5 (d) Solution:



Equivalent circuit

(i) From open circuit test, $R_C' = \frac{V_{OC}^2}{P_{OC}} = \frac{240^2}{186} = 309.67\ \Omega$

$$X_C' = \frac{R_C'}{\tan\left(\cos^{-1}\left(\frac{P_{OC}}{V_{OC}I_{OC}}\right)\right)} = 44.91\ \Omega$$

$$R_C = a^2R_C' = 10^2 \times 309.677 = 30.97\ \text{k}\Omega$$

$$X_C = a^2X_C' = 10^2 \times 44.91 = 4.49\ \text{k}\Omega$$

From short circuit test, $R_{e(HV)} = \dfrac{P_{SC}}{I_{SC}^2} = \dfrac{620}{20.8^2} = 1.433\ \Omega$

$$X_{e(HV)} = \text{Img}\left[\frac{V_{SC}}{I_{SC}}\angle\cos^{-1}\left(\frac{P_{SC}}{V_{SC}I_{SC}}\right)\right] = 1.808\ \Omega$$

(ii) Voltage regulation, $\text{VR} = Ze_{(pu)} \cos(\theta_e - \phi)$

$$= \frac{|(1.433 + j1.808)|}{\dfrac{(2400)^2}{50\times 10^3}}\cos\left[\tan^{-1}\left(\frac{1.808}{1.433}\right) - \cos^{-1}0.8\right]$$

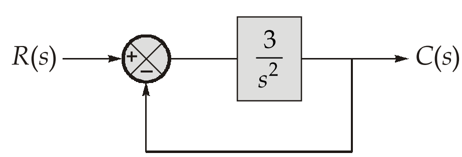
$$= 0.01937 \text{ or } 1.937\%$$

$$\text{Efficiency} = \frac{P_o}{P_o + P_{loss}} = \frac{50\times10^3\times0.8}{50\times10^3\times0.8 + \left(\dfrac{50\times10^3}{2400}\right)^2\times1.433 + 186}$$

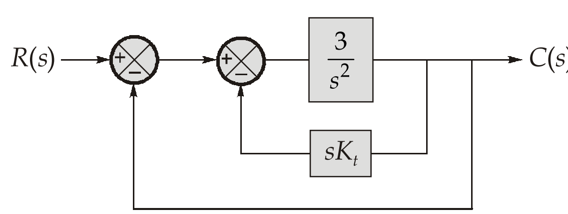
$$= 0.9802 \text{ or } 98.02\%$$

Q.5 (e) Solution:

Given uncompensated system,



With tachometer feedback, the block diagram of the compensated system is



The overall transfer function

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\dfrac{3}{s^2}}{1+\dfrac{3}{s^2}(sK_t)}}{1+\dfrac{\dfrac{3}{s^2}}{1+\dfrac{3}{s^2}(sK_t)}} = \frac{\dfrac{3}{s^2+3sK_t}}{1+\dfrac{3}{s^2+3sK_t}} = \frac{3}{s^2+3sK_t+3}$$

The characteristic equation,

$$q(s) = s^2 + 3sK_t + 3$$

Comparing it with standard second order characteristic equation, we get

$$\omega_n = \sqrt{3} \text{ rad/sec}$$

$$2\xi\omega_n = 3K_t \quad \text{...(i)}$$

Now given,
$$\%M_p = 30\%$$

$$0.3 = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$-\frac{\xi}{\sqrt{1-\xi^2}} = \ln(0.3)$$

$$\therefore \quad \xi = 0.357$$

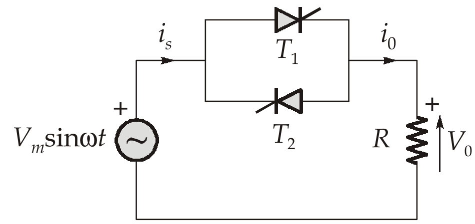
Substituting this value in equation (i),

$$2 \times 0.357 \times \sqrt{3} = 3K_t$$

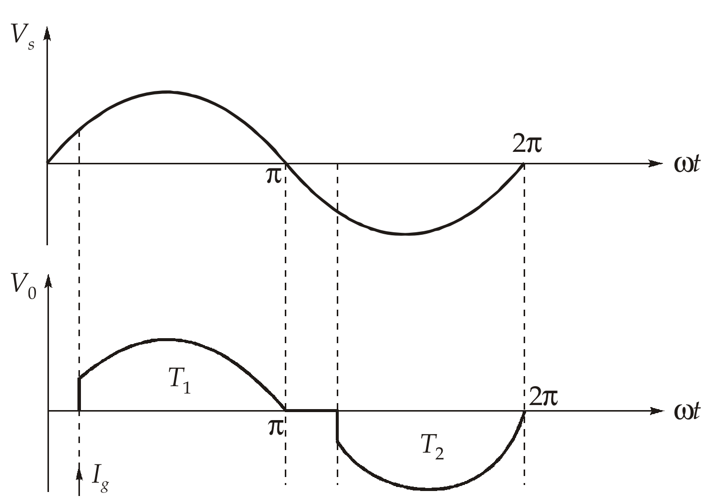
$$\therefore \quad K_t = 0.412$$

Q.6 (a) (i) Solution:

Circuit diagram of single phase full wave ac voltage controller with resistive load is



Wave forms :



From output voltage waveform, rms voltage can be written as,

$$V_{or} = \left\{\frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d(\omega t)\right\}^{1/2} = \frac{V_m}{\sqrt{\pi}}\left(\int_{\alpha}^{\pi} \frac{1-\cos 2\omega t}{2} \cdot d(\omega t)\right)^{1/2}$$

$$V_{or} = \frac{V_m}{\sqrt{2\pi}}\left(\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}\right)^{1/2}$$

$$V_{or} = \frac{V_m}{\sqrt{2\pi}}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]^{1/2}$$

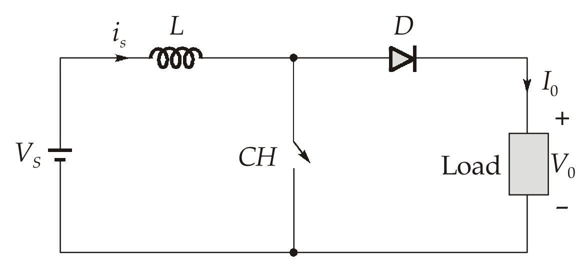
where α = firing angle

Putting the values given in question,

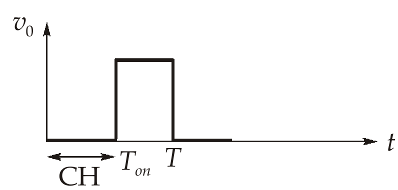
$$V_{or} = \frac{230\sqrt{2}}{\sqrt{2}\sqrt{\pi}}\left[\pi - \frac{\pi}{6} + \frac{\sin 60°}{2}\right]^{1/2} \qquad = 226.66 \text{ V}$$

Q.6 (a) (ii) Solution:

Step up chopper:



Output voltage waveform



Output voltage for step up chopper is given by,

$$V_0 = \frac{V_s}{1-\alpha}$$

$$1 - \alpha = \frac{200}{600} = \frac{1}{3}$$

$$\alpha = \frac{2}{3}$$

where, α = duty ratio = $\frac{T_{on}}{T}$

pulse width of output voltage is equal to T_{off}

$$T_{off} = T - T_{on}$$

$$\Rightarrow \quad \frac{T_{off}}{T} = \frac{T - T_{on}}{T} = 1 - \alpha$$

$$T_{off} = \frac{(1-\alpha)}{f} = \frac{1 - \frac{2}{3}}{25 \times 10^3} = 13.33\ \mu s$$

Now pulse width of output is halved,

$$\alpha' = \frac{T'_{on}}{T} = \frac{T - T'_{off}}{T} = 1 - \frac{T'_{off}}{T}$$

$$\alpha' = 1 - \frac{1}{50} \times 25 = \frac{5}{6}$$

New output voltage, $$V_0 = \frac{V_s}{1-\alpha} = \frac{200}{1 - \frac{5}{6}} = 1200\ \text{V}$$

Q.6 (b) Solution:

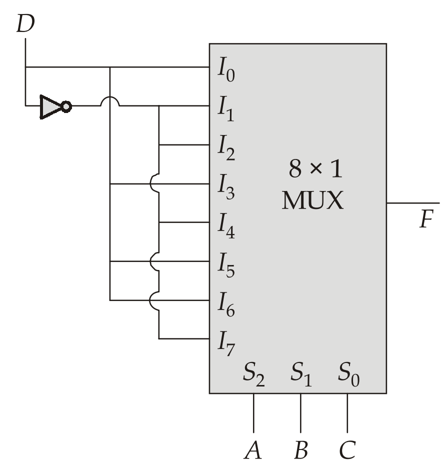
Truth table for 4-bit parity checker.

Inputs				Output
A	*B*	*C*	*D*	*F*
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$F = \Sigma m(1, 2, 4, 7, 8, 11, 13, 14)$$

(i) 8 × 1 MUX with A, B, C as select lines S_2, S_1, S_0

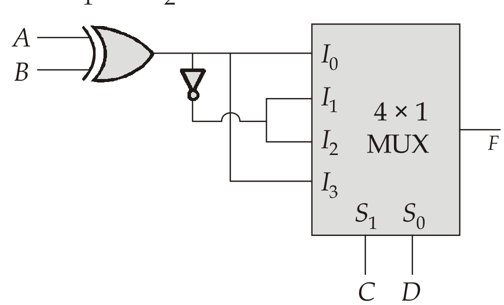
	I_0 $\bar{A}\bar{B}\bar{C}$	I_1 $\bar{A}\bar{B}C$	I_2 $\bar{A}B\bar{C}$	I_3 $\bar{A}BC$	I_4 $A\bar{B}\bar{C}$	I_5 $A\bar{B}C$	I_6 $AB\bar{C}$	I_7 ABC
$\bar{D}$	0	(2)	(4)	6	(8)	10	12	(14)
D	(1)	3	5	(7)	9	(11)	(13)	15
	D	$\bar{D}$	$\bar{D}$	D	$\bar{D}$	D	D	$\bar{D}$



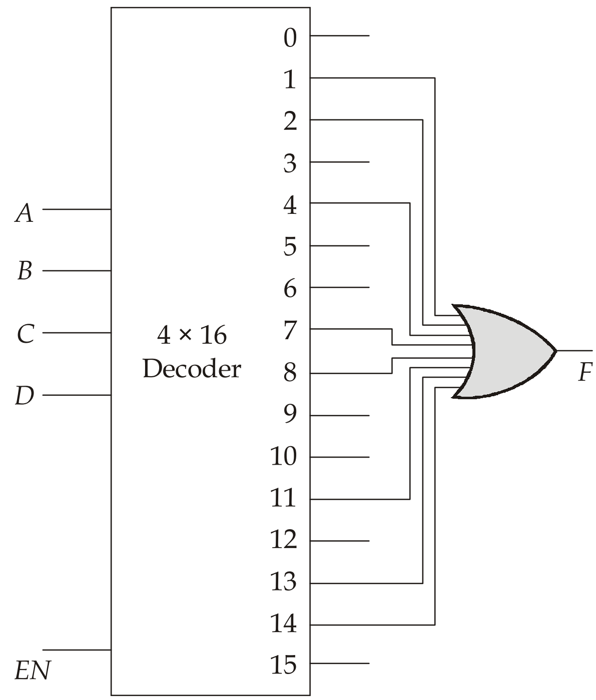
(ii) 4 × 1 MUX with AB as input and C, D as select lines.

	I_0 $\bar{C}\bar{D}$	I_1 $\bar{C}D$	I_2 $C\bar{D}$	I_3 CD
$\bar{A}\bar{B}$	0	(1)	(2)	3
$\bar{A}B$	(4)	5	6	(7)
$A\bar{B}$	(8)	9	10	(11)
AB	12	(13)	(14)	15
	$\bar{A}B+A\bar{B}$	$\bar{A}\bar{B}+AB$	$\bar{A}\bar{B}+AB$	$\bar{A}B+A\bar{B}$

$$I_0 = I_3 = A \oplus B$$
$$I_1 = I_2 = A \odot B$$

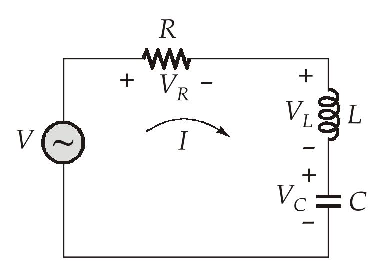


(iii) 4 × 16 Decoder



Q.6 (c) Solution:

Series R-L-C circuit



$$V = V_R + j(V_L - V_C) \quad \text{...(i)}$$

$$Z = R + j(X_L - X_C) \quad \text{...(ii)}$$

Condition of resonance

Net reactive voltage drop $(V_L - V_C) = 0$

Imaginary part of impedance,

$$Im[Z] = 0$$

$$V_L - V_C = 0$$

$$V_L = V_C$$

$$IX_L = IX_C$$

$$X_L = X_C$$

and hence, $$Z = R$$

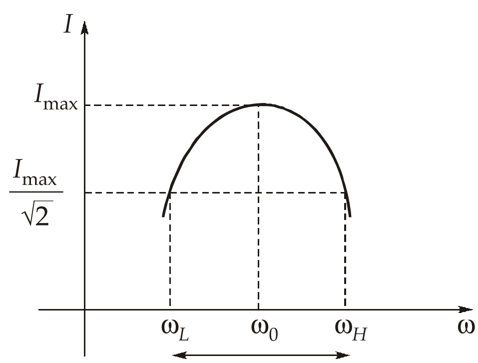
$$\therefore \quad \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$\text{Resonant frequency, } f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

The current in the circuit, $$I = \frac{V}{Z} = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$



ω_L = lower 3-dB frequency or lower half-power frequency

ω_H = higher 3-dB frequency or upper half-power frequency

$$I_{max} = \frac{V}{R} \text{ (at resonant frequency)}$$

$$I = \frac{V}{Z} = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$|I| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At 3-dB frequency,

$$|I| = \frac{I_{max}}{\sqrt{2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

$$\frac{1}{2R^2} = \frac{1}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$2R^2 = R^2+\left(\omega L-\frac{1}{\omega C}\right)^2$$

$$R^2 = \left(\omega L-\frac{1}{\omega C}\right)^2$$

$$\left(\omega L-\frac{1}{\omega C}\right) = \pm R$$

At upper 3-dB frequency, $\omega = \omega_H$

$$\omega_H L-\frac{1}{\omega_H C} = +R \qquad ...(1)$$

At lower 3-dB frequency, $\omega = \omega_L$

$$\omega_L L-\frac{1}{\omega_L C} = -R \qquad ...(2)$$

From equation (1), $\omega_H^2 LC-1 = \omega_H RC$

$$\omega_H^2-\frac{R}{L}\omega_H-\frac{1}{LC} = 0$$

$$\omega_H = \frac{\frac{R}{L}\pm\sqrt{\left(\frac{R}{L}\right)^2+\frac{4}{LC}}}{2}$$

$$\omega_H = \left[\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}\right]\text{rad/sec}$$

At Lower 3-dB frequency, $\omega = \omega_L$, using equation (i)

$$\omega_L^2 LC-1 = -R\omega_L C$$

$$\omega_L^2+\frac{R}{L}\omega_L-\frac{1}{LC} = 0$$

$$\omega_L = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2}$$

$$\omega_L = \left(\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right) \text{rad/sec}$$

$$\text{Bandwidth} = \omega_H - \omega_L$$

$$= \frac{R}{2L} - \left(\frac{-R}{2L}\right) = \frac{R}{L} \text{rad/sec}$$

Q.7 (a) Solution:

(i) The total line parameters are :

$$R = 0.125 \times 400 = 50.0\ \Omega$$

$$X = 0.4 \times 400 = 160.0\ \Omega$$

$$Y = 2.8 \times 10^{-6} \times 400\angle 90°$$

$$= 1.12 \times 10^{-3}\angle 90° \text{ Mho}$$

$$Z = R + jX = (50.0 + j160.0) = 168.0\angle 72.6°\ \Omega$$

$$YZ = 1.12 \times 10^{-3}\angle 90° \times 168\angle 72.6°$$

$$= 0.188\angle 162.6°$$

At no-load,

$$V_S = AV_R\ ; \qquad I_S = CV_R$$

A and C are computed as follows:

$$A = 1 + \frac{1}{2}YZ$$

$$= 1 + \frac{1}{2} \times 0.188\angle 162.6° = 0.91 + j0.028$$

$$|A| = 0.91$$

$$C = Y\left(1 + \frac{YZ}{6}\right) = 1.12 \times 10^{-3}\angle 90°\left(1 + \frac{0.188}{6}\angle 162.6°\right)$$

$$= 1.09 \times 10^{-3}\angle 90.55°$$

Now,
$$|V_R|_{\text{line}} = \frac{220}{|A|} = \frac{220}{0.911} = 241.53 \text{ kV}$$

$$|I_s| = |C||V_R| = 1.09 \times 10^{-3} \times \frac{241.53}{\sqrt{3}} \times 10^3 = 151.52 \text{ A}$$

(ii) Maximum permissible no-load receiving end voltage = 235 kV

$$|A| = \left|\frac{V_S}{V_R}\right| = \frac{220}{235} = 0.936$$

Now,
$$A = 1+\frac{1}{2}YZ$$
$$= 1+\frac{1}{2}l^2 \times j2.8\times 10^{-6} \times (0.125 + j0.4)$$
$$= (1 - 0.56 \times 10^{-6}l^2) + j0.175 \times 10^{-6}l^2$$

Since the imaginary part will be less than $\frac{1}{10}$th of the real part, $|A|$ can be approximated as

$$|A| = 1 - 0.56 \times 10^{-6}l^2 = 0.936$$

$\therefore$
$$l^2 = \frac{1-0.936}{0.56\times 10^{-6}}$$
$$l = 338 \text{ km}$$

Q.7 (b) Solution:

(i) The input power to the motor at rated conditions

$$P_{IN} = \frac{P_{OUT}}{\eta} = \frac{100\times 746}{0.96}\text{W/hp} = 77.7 \text{ kW}$$

(ii) Line current at rated values,

$$I_L = \frac{P}{\sqrt{3}V_L\cos\phi} = \frac{77.7 \text{ k}\Omega}{\sqrt{3}\times 440\times 0.85} = 119.94 \text{ A}$$

So phase current at rated conditions,

$$I_\phi = \frac{I_L}{\sqrt{3}} = \frac{119.94}{\sqrt{3}} = 69.25 \text{ A}$$

(iii) Reactive power supplied by the motor to power system at rated condition

$$Q_{\text{rated}} = 3\, V_\phi\, I_A \sin\theta = 3 \times 440 \times (69.25) \times \sin(31.788°)$$
$$= 48.152 \text{ kVAR}$$

The stator copper loss at rated condition

$$P_{cu} = 3I_A{}^2R_A = 3 \times (69.25)^2 \times 0.3$$
$$= 4.316 \text{ kW}$$

(iv) The internal generated voltage at rated condition,

$$E_A = V_\phi - R_A I_A - jX_s I_A$$
$$= 440\angle 0° - (0.3\,\Omega) \times (69.25\angle 31.788°) - j(4)(69.25\angle 31.788°)$$
$$= 619.377\angle -23.44°\ \text{V}$$

(v) Converted power, $P_{conv} = P_{IN} - P_{cu} = 77.7 - 4.316$ W

$$= 73.384\ \text{kW}$$

(vi) If E_A is decreased by 10%

The new value of $E'_A = 0.9 \times 619.377 = 557.439$ V

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{619.377}{557.439}\sin(-23.44)\right)$$
$$= \sin^{-1}(1.11 \times -0.3977)$$
$$= \sin^{-1}(-0.441) = -26.16°$$

So,
$$I_A' = \frac{V_\phi - E'_A}{jX_s} = \frac{440\angle 0° - 557.439\angle -26.16°}{j4}$$
$$= 63.332\angle 13.74°\ \text{A}$$

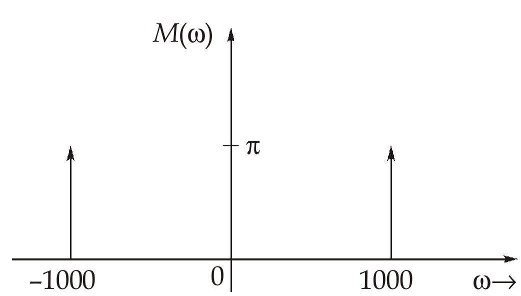
So, reactive power supplied by the motor to the power system,

$$Q' = 3V_\phi I_A' \sin\theta'$$
$$= 3 \times 440 \times 63.332 \times \sin 13.74°$$
$$= 19.855\ \text{kVAR}$$

Q.7 (c) Solution:

(i) Let message signal $m(t)$ spectra = $M(\omega)$

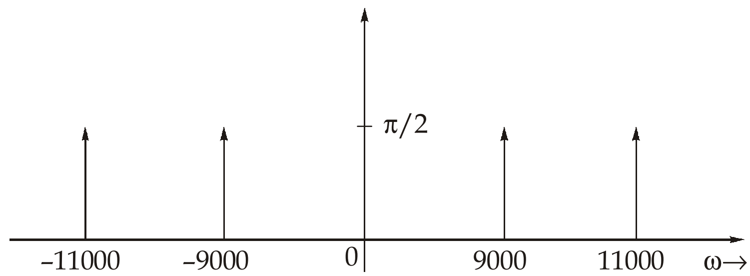
$$m(t) = \cos(1000t)$$



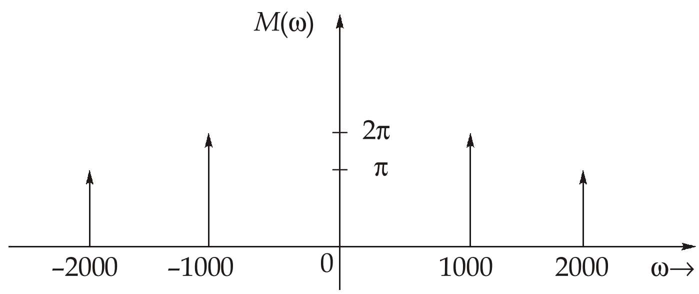
$$\psi_{DSB\text{-}SC}(t) = m(t)\cos(10{,}000t)$$
$$= \cos(1000t)\cos(10000t)$$

$$= \frac{1}{2}\left[\underbrace{\cos(9000t)}_{LSB} + \underbrace{\cos(11000t)}_{USB}\right]$$

Modulated signal spectrum,



(ii) For $m(t) = 2\cos(1000t) + \cos(2000t)$

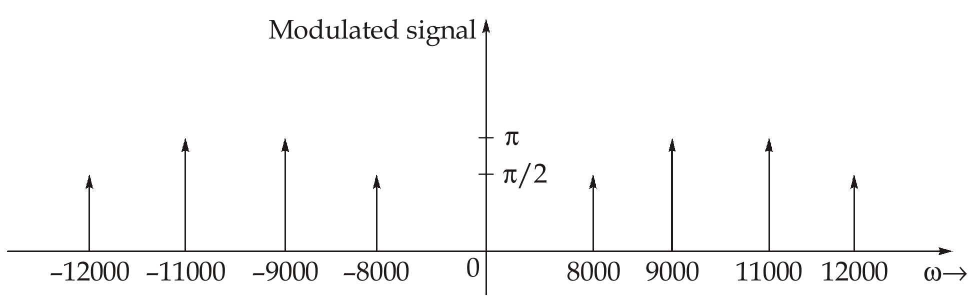


$$\psi_{DSB\text{-}SC}(t) = m(t)\cos(10000t)$$

$$= [2\cos 1000t + \cos 2000t]\cos 10000t$$

$$= \cos 9000t + \cos 11000t + \frac{1}{2}[\cos 8000t + \cos 12000t]$$

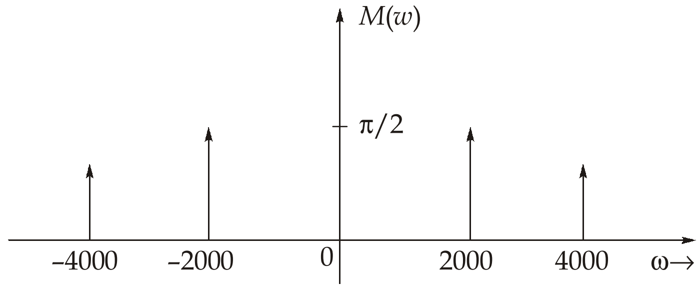
$$= \underbrace{\cos 9000t + \frac{1}{2}\cos 8000t}_{LSB} + \underbrace{\cos 11000t + \frac{1}{2}\cos 12000t}_{USB}$$



(iii) For $m(t) = \cos(1000t)\cos(3000t)$

$$= \frac{1}{2}[\cos 2000t + \cos 4000t]$$

$$\therefore \left(\cos A \cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]\right)$$



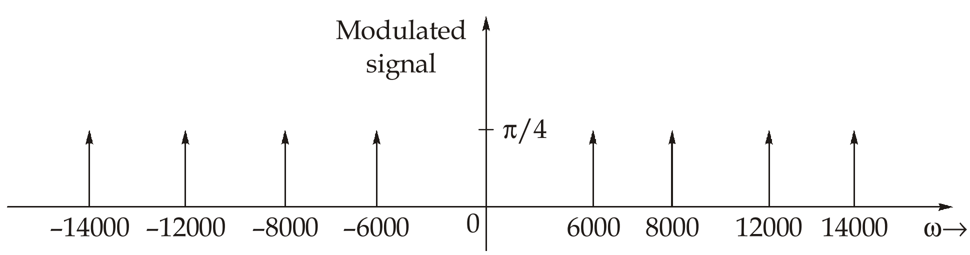
$$\psi_{DSB\text{-}SC}(t) = m(t)\cos(10000t)$$

$$= \frac{1}{2}[\cos 2000t + \cos 4000t]\cos(10000t)$$

$$= \frac{1}{2}[\cos(2000t)\cos(10000t) + \cos(4000t)\cos(10000t)]$$

$$= \frac{1}{4}[\cos(8000t) + \cos(12000t) + \cos(14000t) + \cos(6000t)]$$

$$= \frac{1}{4}\left[\underbrace{\cos(6000t)+\cos(8000t)}_{LSB} + \underbrace{\cos(12000t)+\cos(14000t)}_{USB}\right]$$



Q.8 (a) Solution:

(i) Consider the difference equation relating $w(n)$ and $x(n)$ for system S_1.

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

Taking z-transform,

$$W(z) = \frac{1}{2}z^{-1}W(z) + X(z)$$

$$W(z) = \frac{X(z)}{\left(1-\frac{1}{2}z^{-1}\right)}$$

$$\frac{W(z)}{X(z)} = H_1(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)} \quad ...(1)$$

Let $H_1(z)$ is the transfer function of system S_1.

Now, consider the difference equation relating $y(n)$ and $w(n)$ for system S_2.

$$y(n) = \alpha y(n-1) + \beta w(n)$$

Taking z-transform

$$Y(z) = \alpha z^{-1}Y(z) + \beta W(z)$$

$$Y(z)[1 - \alpha z^{-1}] = \beta W(z)$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{\beta}{(1-\alpha z^{-1})} \quad ...(2)$$

where, $H_2(z)$ is the transfer function of system S_2.

Now, consider $H(z)$ as the transfer function of whole system. Hence,

$$H(z) = \frac{Y(z)}{X(z)} = H_1(z)\cdot H_2(z) \quad ...(3)$$

$$H(z) = H_1(z)\cdot H_2(z)$$

$$\therefore \quad H(z) = \frac{\beta}{(1-\alpha z^{-1})\left(1-\frac{1}{2}z^{-1}\right)} \quad \text{(from eqn (1) and (2))}$$

$$H(z) = \frac{\beta}{1+\frac{\alpha z^{-2}}{2}-\frac{1}{2}z^{-1}-\alpha z^{-1}}$$

$$H(z) = \frac{\beta}{1-\left(\alpha+\frac{1}{2}\right)z^{-1}+\frac{\alpha}{2}z^{-2}} \quad ...(4)$$

Now consider the difference equation relating $x(n)$ and $y(n)$,

$$y(n) = \frac{-1}{8}y(n-2)+\frac{3}{4}y(n-1)+x(n)$$

Taking z-transform,

$$Y(z) = \frac{-1}{8}z^{-2}Y(z)+\frac{3}{4}z^{-1}Y(z)+X(z)$$

$$Y(z)\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right] = X(z)$$

$\therefore$ $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right]} \quad ...(5)$$

Now comparing equation (4) and (5), we get

$$\alpha + \frac{1}{2} = \frac{3}{4} \Rightarrow \alpha = \frac{1}{4}$$

and $$\beta = 1$$

(ii) Given, $$X(z) = \frac{1}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

We can write, $$X(z) = X_1(z)\cdot X_2(z)$$

where, $$X_1(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)}$$

and $$X_2(z) = \frac{1}{\left(1+\frac{1}{4}z^{-1}\right)}$$

Now consider, $X_1(z)$,

Taking inverse z-transform, we get

$$x_1(n) = IZT\left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]$$

$$x_1(n) = \left(\frac{1}{2}\right)^n u(n) \quad ...(1)$$

Consider $X_2(z)$,

Taking inverse z-transform, we get

$$x_2(n) = IZT\left[\frac{1}{1+\frac{1}{4}z^{-1}}\right]$$

$$x_2(n) = \left(\frac{-1}{4}\right)^n u(n) \quad ...(2)$$

Using convolution property of z-transform, $x(n)$ is the convolution of $x_1(n)$ and $x_2(n)$

$\therefore$ $$x(n) = x_1(n) * x_2(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x_1(n-k)\cdot x_2(k)$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{(n-k)} \cdot \left(\frac{-1}{4}\right)^{k} u(k)u(n-k)$$

$$= \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-k} \cdot \left(\frac{-1}{4}\right)^{k}$$

$$= \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} \left[\frac{(-1/4)}{(1/2)}\right]^{k}$$

$$= \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} \left(\frac{-1}{2}\right)^{k}$$

$$= \left(\frac{1}{2}\right)^{n} \left[\frac{1-\left(\frac{-1}{2}\right)^{n+1}}{1-\left(-\frac{1}{2}\right)}\right]$$

$$= \left(\frac{1}{2}\right)^{n} \left[\frac{1-\left(\frac{-1}{2}\right)^{n+1}}{\frac{3}{2}}\right]$$

$$= \left(\frac{1}{2}\right)^{n} \cdot \frac{2}{3} \cdot \left[1-\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)^{n}\right]$$

$$= \frac{2}{3}\left(\frac{1}{2}\right)^{n} + \frac{1}{2} \times \frac{2}{3}\left(\frac{1}{2}\right)^{n} \cdot \left(\frac{-1}{2}\right)^{n}$$

$$x(n) = \left[\frac{2}{3}\left(\frac{1}{2}\right)^{n} + \frac{1}{3}\left(\frac{-1}{4}\right)^{n}\right] u(n)$$

Q.8 (b) Solution:

(i) slip, $s = \dfrac{-n}{m}\cos\alpha = -a\cos\alpha_m$

For 25% speed range, S_m = 0.25 thus below the synchronous speed

$$0.25 = -a\cos 165°$$

$$a = \frac{-0.25}{\cos 165°} = 0.2588 \approx 0.259$$

$$\frac{n}{m} = a \text{ or } \frac{2}{m} = 0.259 \text{ or } m = 7.722$$

(ii) For a speed of 780 rpm

$$\text{slip, } s = \frac{1000 - 780}{1000} = 0.22$$

$$V_{d1} = \frac{3\sqrt{6}}{\pi} s \frac{V}{n} = \frac{3\sqrt{6}}{\pi} \times \frac{0.22 \times \frac{440}{\sqrt{3}}}{2} = 65.363 \text{ V}$$

$$V_{d2} = \frac{3\sqrt{6}}{\pi} \frac{V}{m} \cos\alpha$$

$$= \frac{3\sqrt{6}}{\pi} \frac{440}{\sqrt{3} \times 7.722} \times \cos 140° = -58.95 \text{ V}$$

$$R'_S = 0.1 \times (0.5)^2 = 0.025,$$

$$R_r = 0.08 \times 0.5 \times 0.5 = 0.02$$

$$I_d = \frac{V_{d1} + V_{d2}}{2(SR'_s + R_r) + R_d} = \frac{65.363 - 58.95}{2(0.22 \times 0.025 + 0.02) + 0.01}$$

$$= 105.11 \text{ A}$$

For 800 rpm $\quad \text{slip, } s = \frac{1000 - 800}{1000} = 0.2$

$$\text{Torque, } T = \frac{|V_{d2}| I_d}{s\,\omega_{ms}} = \frac{58.95 \times 105.11}{0.22 \times 104.72} \approx 269 \text{ N-m}$$

(iii) $\quad \text{Rated slip} = \frac{1000 - 970}{1000} = 0.03$

$$\text{Rated torque} = \frac{\frac{3}{104.72} \times \left(\frac{440}{\sqrt{3}}\right)^2 \times \frac{0.08}{0.03}}{\left((0.1)^2 + \frac{0.08}{0.03}\right)^2 + (0.7)^2} = 605.32 \text{ N-m}$$

$$\text{Half rated torque} = 302.66 \text{ N-m}$$

For 800 rpm, $\quad \text{slip, } s = \frac{1000 - 800}{1000} = 0.2$

$$V_{d1} = \frac{3\sqrt{6}}{\pi} \times \frac{0.2 \times \frac{440}{\sqrt{3}}}{2} = 59.42 \text{ V}$$

$$V_{d2} = \frac{3\sqrt{6}}{\pi} \times \frac{\frac{440}{\sqrt{3}}}{7.722} \cos\alpha = 76.95 \cos\alpha$$

$$I_d = \frac{59.42 + 76.95\cos\alpha}{2(0.2 \times 0.025 + 0.02) + 0.01} = 990.33 + 1282.5 \cos\alpha$$

$$\text{Torque, } T = \frac{|V_{d2}| I_d}{s\omega_{ms}} = \frac{76.95|\cos\alpha| \times (990.33 + 1282.5\cos\alpha)}{0.2 \times 104.72}$$

$$T = (3.673\ |\cos\alpha|)\ (990.33 + 1282.5 \cos\alpha)$$

Let, $\cos\alpha = -X,$

$$\text{Torque, } T = (3.673X)\ (990.33 - 1282.5X)$$

This should be equal to half rated torque 302.66 N-m

$$(3.673X)\ (990.33 - 1282.5X) = 302.66$$

$$X^2 - 0.772\ X + 0.06425 = 0$$

$$X = 0.677 \text{ and } 0.0949$$

$$\alpha = 132.6° \text{ and } 95.45°$$

Later value of α corresponds to operation in unstable part of characteristics.

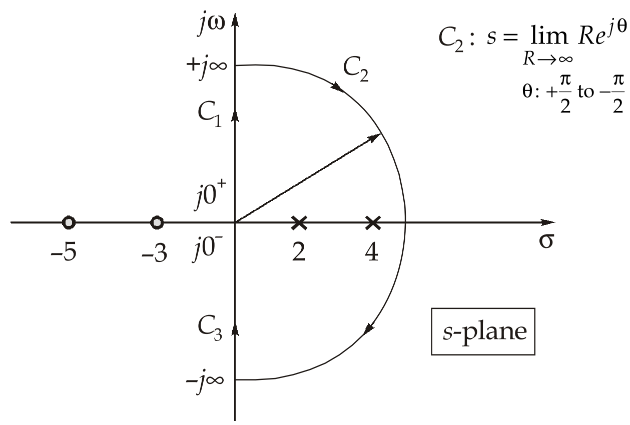
Q.8 (c) Solution:

The open-loop transfer function of the given system is,

$$L(s) = G(s)H(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)} \quad \text{...(i)}$$

Analysis using Nyquist stability criterion:

- The number of open-loop poles in the right-half of s-plane is, $P = 2$ (at $s = 2$ and $s = 4$).
- The Nyquist plot of the given system can be obtained by mapping the contour which encloses the entire right-half of s-plane on to the $L(s) = G(s)H(s)$ plane. This contour which encloses the right-half of s-plane is called as the Nyquist contour, which is shown below.



Segment C_1 of the Nyquist contour can be mapped on to the $L(s)$ plane as follows:

- For this segment, $s = j\omega$ and ω is varying from 0^+ to ∞.
- For the purpose of mapping C_1, the open-loop transfer function of the given system can be modified by replacing "s" with "$j\omega$".

$$L(j\omega) = \frac{K(3+j\omega)(5+j\omega)}{(-2+j\omega)(-4+j\omega)}$$

At $\omega = 0^+$, $$L(j\omega) = \frac{15K}{8}\angle 0°$$

At $\omega = \infty$, $$L(j\omega) = K\angle 0°$$

At $\omega = 1$, $$L(j\omega) = 1.75K\angle 70.35°$$

- The intersection points of Nyquist plot with 180° line can be investigated as follows:

$$\angle L(j\omega) = \tan^{-1}\left(\frac{\omega}{3}\right) + \tan^{-1}\left(\frac{\omega}{5}\right) - 180° + \tan^{-1}\left(\frac{\omega}{2}\right) - 180° + \tan^{-1}\left(\frac{\omega}{4}\right) = -180°$$

$$\tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) + \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right) = 180°$$

$$8\omega(8 - \omega^2) + 6\omega(15 - \omega^2) = 0$$

$$\omega(64 - 8\omega^2 + 90 - 6\omega^2) = 0$$

$$14\omega_{pc}^2 - 154 = 0$$

$$\omega_{pc}^2 = 11$$

$$\omega_{pc} = \sqrt{11} \text{ rad/sec} = 3.316 \text{ rad/sec}$$

$$|L(j\omega)|\Big|_{\omega = \omega_{pc} = \sqrt{11}} = \frac{4K}{3} = 1.33K$$

- The intersection points of Nyquist plot with 90° line can be investigated as follows:

$$\tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) + \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right) = 90°$$

$$\tan^{-1}\left[\frac{8\omega(8-\omega^2) + 6\omega(15-\omega^2)}{(15-\omega^2)(8-\omega^2) - 48\omega^2}\right] = 90°$$

$$120 - 71\omega_{90}^2 + \omega_{90}^4 = 0$$

$$\omega_{90}^2 = \frac{71 \pm \sqrt{(71)^2 - 480}}{2} = 1.73,\ 69.3$$

Valid value ω_{90} will be less than ω_{pc}.

So, $\omega_{90} = \sqrt{1.73} = 1.316$ rad/sec

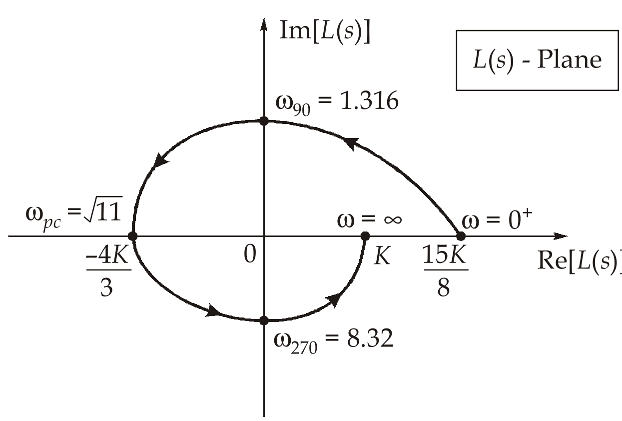
- The intersection points of the Nyquist plot with 270° line can be investigated as follows:

$$\tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) + \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right) = 270°$$

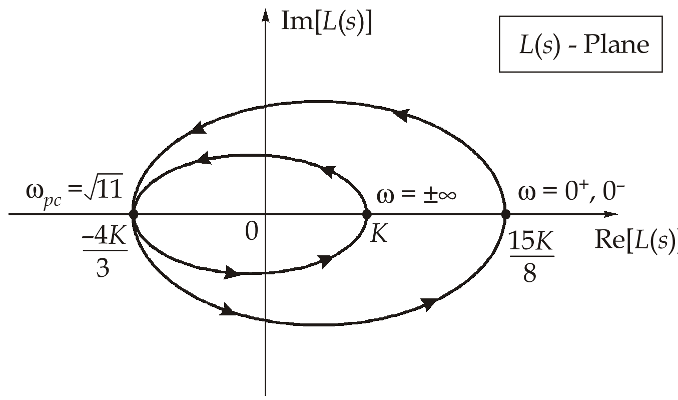
$$120 - 71\omega_{270}^2 + \omega_{270}^4 = 0$$

$$\omega_{270} = \sqrt{69.3} = 8.32 \text{ rad/sec} \qquad (\because \omega_{270} > \omega_{pc})$$

- Nyquist plot corresponding to the segment C_1 of the Nyquist contour is as follows:



Segment C_3 of the Nyquist contour can be mapped by taking the mirror image (about real axis) of the Nyquist plot corresponding to segment C_1 as shown below:

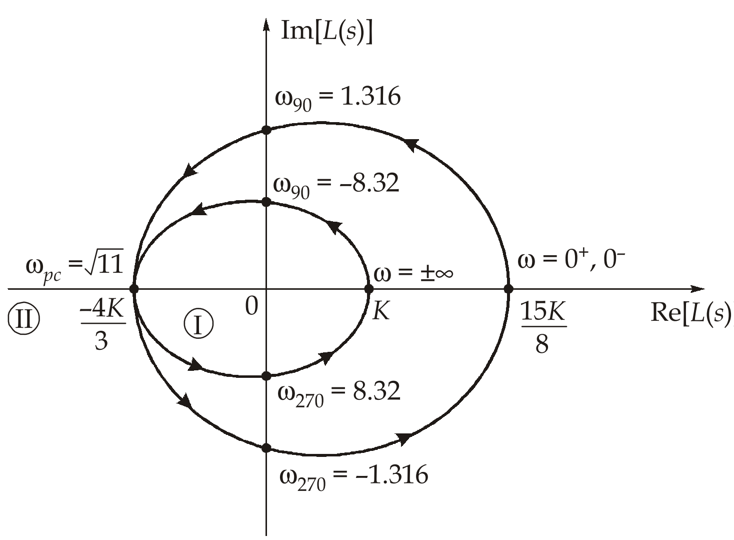


Segment C_2 of the Nyquist contour can be mapped on to the $L(s)$-plane as follows:

- Along the segment C_2, "s" can be written as $\lim_{R\to\infty} Re^{j\theta}$ where θ varies from $+\frac{\pi}{2}$ to $-\frac{\pi}{2}$.

$$L(s) = \lim_{R\to\infty} \frac{K(3+Re^{j\theta})(5+Re^{j\theta})}{(-2+Re^{j\theta})(-4+Re^{j\theta})} = K\angle 0°$$

- So, the segment C_2 of the Nyquist contour can be mapped on to the $L(s)$-plane as a point $K\angle 0°$.
- The overall Nyquist plot of the given system can be drawn as follows:



Nyquist stability criterion:

- To determine the stability of the closed-loop system, the number of encirclements of the point $(-1 + j0)$ by the Nyquist plot should be determined, which will be equal to,

$$N_{(-1+j0)} = P - Z$$

where, P = Number of open-loop poles in the right-half of s-plane

Z = Number of closed-loop poles in the right-half of s-plane

- The closed-loop system will be stable, when $Z = 0$.
- When $Z = 0$, $N_{(-1 + j0)} = P - 0 = P = 2$
- When the point $(-1 + j0)$ lies in the region-I of the Nyquist plot,

  $N_{(-1 + j0)} = 2 \Rightarrow$ system will be stable

  When the point $(-1 + j0)$ lies in the region-II of the Nyquist plot,

  $N_{(-1 + j0)} = 0 \Rightarrow$ system will be unstable as $Z = P - N = 2$.
- So, to make the closed-loop system stable, the Nyquist plot should encircle the point $(-1 + j0)$, which is possible when $\frac{4K}{3} > 1$.
- Hence, the system will be stable for,

$$\frac{4K}{3} > 1$$

$$K > \frac{3}{4} = 0.75$$

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