MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Mechanical Engineering Test No : 13

Section : A

1. (a) Solution:

The deflection due to w and the reaction will be equal and opposite in nature.

Deflection due to reaction only = $\dfrac{RL^3}{3EI}$

Deflection due to w only

$$EI\frac{d^2y}{dx^2} = M(x)$$

$$\Rightarrow \quad EI\frac{d^3y}{dx^3} = \frac{dM}{dx} = V(x)$$

$$\Rightarrow \quad EI\frac{d^4y}{dx^4} = \frac{dV(x)}{dx} = -w(x) = -w_0\left(\frac{x}{L}\right)^2$$

$$\Rightarrow \quad EI\frac{d^3y}{dx^3} = \frac{-w_0x^3}{3L^2} + C_1 \qquad ...(1)$$

as shear force at $x = 0$ is zero

So, $\quad 0 = 0 + C_1$

$\Rightarrow \quad C_1 = 0$

from eq. (1)

$$EI\frac{d^2y}{dx^2} = \frac{-w_0x^4}{12L^2}+C_2 \qquad ...(2)$$

as $M(x) = 0$ at $x = 0$

⇒ $C_2 = 0$

So from eq. (2)

$$EI\frac{dy}{dx} = \frac{-w_0x^5}{60L^2}+C_3$$

at $x = L, \frac{dy}{dx}=0$

So, $C_3 = \frac{w_0L^3}{60}$

$$\Rightarrow \quad EI\frac{dy}{dx} = \frac{w_0}{60L^2}(L^5-x^5)$$

$$\Rightarrow \quad EIy = \frac{w_0}{60L^2}\left(L^5x-\frac{x^6}{6}\right)+C_4$$

at $x = L$, $y = 0$

$$\Rightarrow \quad 0 = \frac{w_0}{60L^2}\left(L^6-\frac{L^6}{6}\right)+C_4$$

$$\Rightarrow \quad C_4 = -\frac{5w_0L^4}{360}$$

$$\Rightarrow \quad EIy = \frac{w_0}{60L^2}\left(L^5x-\frac{x^6}{6}\right)-\frac{5w_0L^4}{360}$$

at $x = 0$,

$$y = -\frac{w_0L^4}{72EI}$$

This will be equal to deflection due to reaction alone.

$$\frac{RL^3}{3EI} = \frac{w_0L^4}{72EI}$$

$$\Rightarrow \quad R = \frac{w_0L}{24} = \frac{6000\times 2}{24} = 500 \text{ N}$$

1. (b) Solution:

Given data : Speed ratio = 12; $m_1 = m_2 = 3.6$ mm; $m_3 = m_4 = 2.5$ mm; $T_{min} = 20$

Distance between centres, $C = 180$ mm

$$r_1 + r_2 = C$$

$$\frac{m_1}{2}[T_1 + T_2] = 180$$

$$\frac{3.6}{2}[T_1 + T_2] = 180$$

$$T_1 + T_2 = 100 \quad \text{...(i)}$$

Also,
$$r_3 + r_4 = C$$

$$\frac{m_3}{2}[T_3 + T_4] = C$$

$$\frac{2.5}{2}[T_3 + T_4] = 180$$

$$T_3 + T_4 = 144 \quad \text{...(ii)}$$

$\because$ Speed ratio = 12

$$\frac{T_2 \times T_4}{T_1 \times T_3} = 12$$

$$\Rightarrow \quad \left(\frac{T_2}{T_1}\right) \times \left(\frac{T_4}{T_3}\right) = 12 \quad \text{...(ii)}$$

Let, $\frac{T_2}{T_1} = 2$ and $\frac{T_4}{T_3} = 6$

From equation (i), $T_1 = \frac{100}{3} = 33.33 \times$ Not possible

Let, $\frac{T_2}{T_1} = 4$ and $\frac{T_4}{T_3} = 3$

$$T_1 = \frac{100}{5} \text{ and } T_3 = \frac{144}{4} = 36$$

$$T_2 = 20 \times 4 = 80; \quad T_4 = 108$$

So, $T_2 = 80; T_1 = 20; T_4 = 108; T_3 = 36$ **Ans.**

1. (c) Solution:

Given: $P = 10$ kN, $\tau_{per} = 90$ N/mm^2

Primary shear stress:

Let t is the throat of each weld. There are two welded W_1 and W_2 and their throat area is given by,

$$A = 2(60t) = 120t \text{ mm}^2$$

The primary shear stress is given as,

$$\tau_1 = \frac{P}{A} = \frac{10 \times 10^3}{(120t)} = \left(\frac{250}{3t}\right) \text{N/mm}^2$$

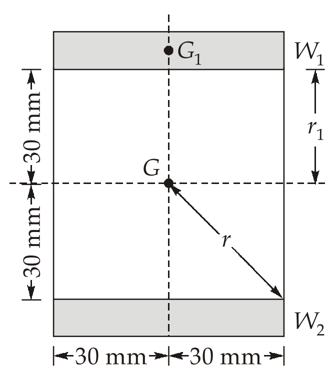
Secondary shear stress:

The two welds are symmetrical and G is the centre of gravity of the two welds,]

$$e = \frac{60}{2} + 120 = 150 \text{ mm}$$

$$M = P \times e = (10 \times 10^3)(150) = 1.5 \times 10^6 \text{ N-mm}$$

The distance r of the farthest point in the weld from the centre of gravity is given by:



$$r = \sqrt{30^2 + 30^2} = 42.43 \text{ mm}$$

The polar moment of inertia of the two welds is given as

$$J = J_1 + J_2 = 60t\left[\frac{60^2}{12} + 30^2\right] + 60t\left[\frac{60^2}{12} + 30^2\right] \quad \because J_1 = J_2$$

$$= (144000t) \text{ mm}^4$$

The secondary shear stress is given as:

$$\tau_2 = \frac{Mr}{J} = \frac{(1.5 \times 10^6)(42.43)}{(144000t)} = \left(\frac{441.98}{t}\right) \text{N/mm}^2$$

Resultant shear stress:

$$\tau_R = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos 45°}$$

$$\tau_R = \frac{1}{t}\sqrt{441.98^2 + \left(\frac{250}{3}\right)^2 + 2 \times 441.98 \times \frac{250}{3} \times \cos 45°}$$

$$\tau_R = \frac{504.36}{t}$$

Since the permissible shear stress for the weld material is 90 N/mm^2.

$$\Rightarrow \quad \frac{504.36}{t} = 90$$

$$\Rightarrow \quad t = 5.604 \simeq 5.6 \text{ mm}$$

1. (d) Solution:

Given : m_1 = 140 kg ; m_2 = 70 kg; M = 85 kg; k = 110 mm = 0.11 m; r_1 = 240 mm = 0.24 m; r_2 = 120 mm = 0.12 m

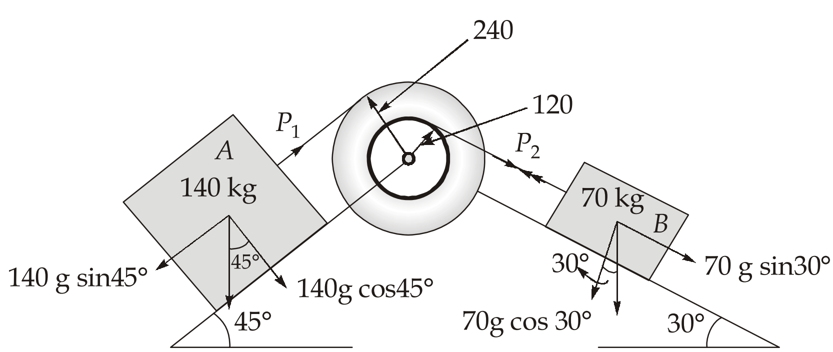
Let P_1 = Pull in the string carrying 140 kg mass, and

P_2 = Pull in the string carrying 70 kg mass.

a_1 = Acceleration of the 140 kg mass,

a_2 =Acceleration of the 70 kg mass, and

α = Angular acceleration of the pulley.



We know that mass moment of inertia of the pulley,

$$I = mk^2 = 85(0.11)^2 = 1.0285 \text{ kg-m}^2$$

First of all, consider the motion of 140 kg mass, which is coming down. We know that the forces acting on it, the plane, $m_1 g \sin \theta_1$ = 140 × 9.81 sin 45° = 140 × 9.81 × 0.707 = 971.14 N (downwards), and P_1 newtons (upwards). As the mass is moving downwards, therefore resultant force.

$$= 971.14 - P_1 \text{ newtons} \quad \text{...(i)}$$

Since the mass is moving downwards with an acceleration (a_1), therefore force acting on the body

$$= 140a_1 \text{ newtons} \quad \text{...(ii)}$$

Equating equations (i) and (ii),

$$971.14 - P_1 = 140a_1 \quad \text{...(iii)}$$

Now consider the motion of 70 kg mass, which is going up-wards. We know that the forces acting on it, along the plane, are $m_2 g \sin\theta_2 = 70 \times 9.8 \sin 30° = 70 \times 9.8 \times 0.5 =$ 343.35 N (downwards) and P_2 newtons upwards. As the mass is moving upwards, therefore resultant force

$$= P_2 - 343.35 \text{ newtons} \quad \text{...(iv)}$$

Since the mass is moving upwards with an acceleration (a_2), therefore force acting on the body

$$= 70a_2 \text{ newtons} \quad \text{...(v)}$$

Equating equations (iv) and (v),

$$P_2 - 343.35 = 70a_2 \quad \text{...(vi)}$$

Now consider the motion of the pulley which is rotating about its axis due to downward motion of the 140 kg mass tied to the string. We know that linear acceleration of the 140 kg mass is equal to the angular acceleration of the pulley.

$\therefore \quad a_1 = r_1 \cdot \alpha = 0.24\alpha$

Similarly, $\quad a_2 = r_2 \cdot \alpha = 0.12\alpha$

and $\quad$ Torque, $T = P_1 \cdot r_1 - P_2 \cdot r_2 = P_1 \times 0.24 - P_2 \times 0.12 \quad$ ...(vii)

We also known that torque on the pulley,

$$T = I \cdot \alpha = 1.0285\,\alpha \quad \text{...(viii)}$$

Equating equations (vii) and (viii),

$$= 0.24P_1 - 0.12P_2 = 1.0285\alpha$$

or $\quad P_1 - 0.5P_2 = 4.285\alpha$

$\therefore \quad P_1 = 0.5P_2 + 4.285\alpha \quad$ ...(ix)

Substituting the value of P_1 in equation (iii),

$$971.14 - (0.5P_2 + 4.285\alpha) = 140a_1 = 140 \times 0.24\alpha$$

$$971.14 - 0.5P_2 - 4.285\alpha = 33.6\alpha$$

$\therefore \quad 971.14 - 0.5P_2 = 37.5\alpha + 4.285\alpha = 37.88\alpha$

Multiplying both sides by 2

$$1942.28 - P_2 = 75.77\alpha \quad \text{...(x)}$$

From equation (vi), we find that

$$P_2 - 343.35 = 70a_2 = 70 \times 0.12\alpha = 8.4\alpha \quad \text{...(xi)}$$

Adding equation (x) and (xi),

$$1598.93 = 84.17\alpha$$

$$\therefore \quad \alpha = 18.99 \text{ rad/s}^2$$

Now substituting the value of α in equations (x),

$$1942.28 - P_2 = 75.77\alpha = 75.77 \times 18.99 = 1438.87$$

$$\therefore \quad P_2 = 503.4 \text{ N}$$ **Ans.**

Again substituting the value of α and P_2 in equation (ix),

$$P_1 = (0.5 \times 503.4) + (4.285 \times 18.99) = 333.07 \text{ N}$$ **Ans.**

Acceleration of the masses A and B

We know that the acceleration of mass A (i.e. 150 kg),

$$a_1 = r_1\alpha = 0.24 \times 18.99 = 4.56 \text{ m/s}^2$$ **Ans.**

Similarly, $$a_2 = r_2\alpha = 0.12 \times 18.97 = 2.28 \text{ m/s}^2$$ **Ans.**

1. (e) Solution:

Given : $m = 120$ kg; $l = 2b = 90$ mm; $W_{rec} = 2.5$ kg; $N = 750$ rpm;

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 750}{60} = 78.539 \text{ rad/s}$$

(i) Damping is neglected

$$c = 0$$

$$\xi = 0$$

$$\text{Transmissibility, } \in_f = \frac{1}{22}$$

$$\frac{\sqrt{1 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}} = \frac{1}{22}$$

$$\pm\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{1}{22}$$

$$\frac{1}{-1+\left(\frac{78.539}{\omega_n}\right)^2} = \frac{1}{22}$$

$$23 = \left(\frac{78.539}{\omega_n}\right)^2$$

$$\omega_n = 16.376 \text{ rad/s}$$

$$\sqrt{\frac{k_{eq}}{m}} = 16.376$$

$$\sqrt{\frac{k_{eq}}{120}} = 16.376$$

$$k_{eq} = 32.182 \times 10^3 \text{ N/m}$$

$$k_{eq} = 32.182 \text{ N/mm}$$

$$\omega_n = 16.376 \text{ rad/s}$$

$$\sqrt{\frac{k_{eq}}{m}} = 16.376$$

$$\sqrt{\frac{k_{eq}}{120}} = 16.376$$

$$k_{eq} = 32.182 \times 10^3 \text{ N/m}$$

$$k_{eq} = 32.182 \text{ N/mm}$$

(ii) Damping is present

$$x_1 = 0.75x_0$$

$\Rightarrow$
$$\frac{x_0}{x_1} = \frac{4}{3}$$

Taking ln both sides,

$$\ln\left(\frac{x_0}{x_1}\right) = \ln\left(\frac{4}{3}\right)$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.28768$$

$$\xi = 0.04573$$

$$F_0 = m_{rec} \cdot r \cdot \omega^2$$

$$= 2.5 \times \left(\frac{0.090}{2}\right) \times (78.539)^2 = 693.94 \text{ N}$$

$$\text{Transmissibility, } \in_f = \frac{\sqrt{1+\left(2\times 0.04573 \times \frac{78.539}{16.376}\right)^2}}{\sqrt{\left[1-\left(\frac{78.539}{16.376}\right)^2\right]^2 + \left(2\times 0.04573\times\frac{78.539}{16.376}\right)^2}}$$

$$\in_f = 0.04962$$

$$\frac{F_T}{F_0} = 0.04962$$

$$F_T = 0.04962 \times 693.94 = 34.43 \text{ N}$$

At resonance $\omega = \omega_n$

$$\Rightarrow \quad \frac{\omega}{\omega_n}$$

$$(\in_f)_{\text{reso}} = \frac{\sqrt{1+(2\xi)^2}}{2\xi}$$

$$= \frac{\sqrt{1+(2\times 004573)^2}}{2\times 0.04573} = 10.979$$

$$(\in_f)_{\text{reso}} = m_{rec} \cdot r \cdot \omega_n^2$$

$$= 2.5 \times \left(\frac{0.09}{2}\right) \times (16.376)^2 = 30.169 \text{ N}$$

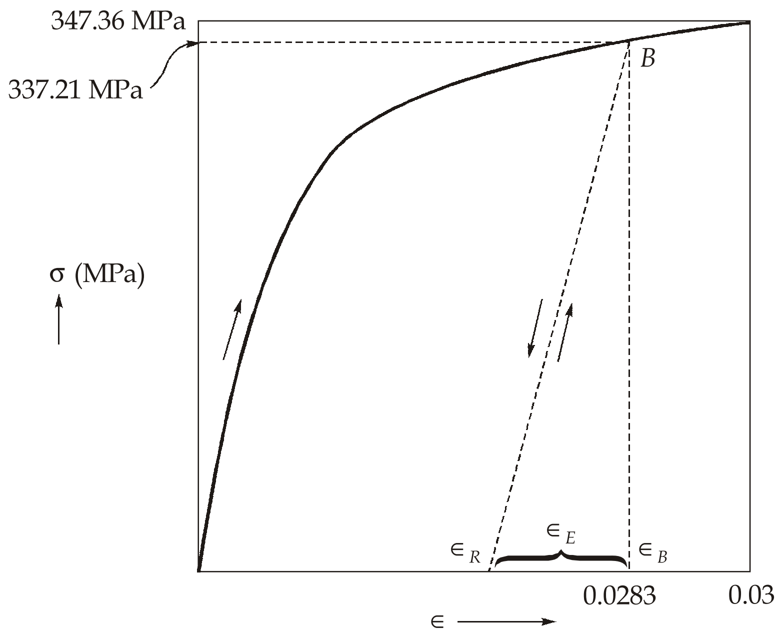
$$f_t = (\in_t)_{reso} \times (f_o)_{reso}$$

$$= 10.979 \times 30.169 = 331.23 \text{ N}$$

2. (a) Solution:

(i) Given : $L = 1.219$ m; $d = 3.175$ mm $= 3175 \times 10^{-3}$ m; $P = 2.67$ kN

$$\sigma = \frac{22{,}000 \in}{1 + 30 \in}$$



Initial slope of stress strain curve

$$\frac{d\sigma}{d \in} = \frac{(1 + 30 \in)(22{,}000) - (22{,}000)(30 \in)}{(1 + 30 \in)^2}$$

$$= \frac{22{,}000}{(1 + 30 \in)^2}$$

At $\in = 0$, $$\frac{d\sigma}{d \in} = 22000 \text{ MPa}$$

Initial slope = 22000 MPa

(ii) Elongation δ of the wire

$$\sigma = \frac{P}{A} = \frac{2.67 \times 10^3}{\frac{\pi}{4} \times (3.175)^2}$$

$$\sigma = 337.23 \text{ MPa}$$

So, $337.23 = \dfrac{22000\,\epsilon}{1 + 30\,\epsilon}$

$\epsilon = 0.02837$

Elongation, $\delta = \epsilon L = 0.0283 \times 1.219 = 0.03459$ m

Stress and strain at point B,

$\sigma = 337.21$ MPa

$\epsilon = 0.02837$

Elastic recovery $\epsilon_E = \dfrac{\sigma_B}{\text{slope}} = \dfrac{337.23}{22000} = 0.0153$

Residual strain, $\epsilon_R = \epsilon_B - \epsilon_E$

$= 0.0283 - 0.0153$

$= 0.013$

(iii) Permanent set $= \epsilon_R \times L$

$= 0.013 \times 1.219$

$= 0.0158$ m

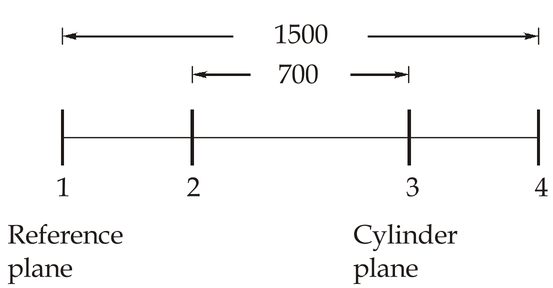
(iv) Proportional limit when reloaded $= \sigma_B$

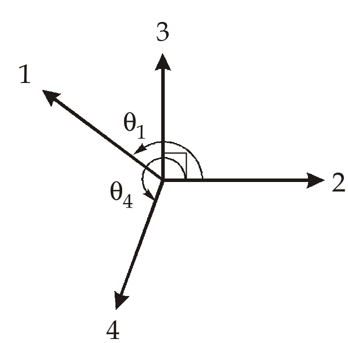
$\sigma_B = 337.21$ MPa **Ans.**

2. (b) Solution:

Total mass to be balanced = Rotating mass + C × Reciprocating mass

$= 290 + \dfrac{2}{3} \times 330 = 510$ kg





Side view of crank
$\theta \rightarrow$ Measured from cylinder position

Plane	Mass (m) (in kg)	Radius (r) (in m)	mr	l (in m)	mrl	Angle from reference plane, θ (degree)
1	m_1	0.650	$0.65m_1$	0	0	θ_1
2	510	0.300	153	0.4	61.2	0
3	510	0.300	153	1.1	168.3	90°
4	m_4	0.650	$0.65m_4$	1.5	$0.975m_4$	θ_4

Moment balance,

$$\Sigma m_i r_i l_i \sin\theta_i = 0$$

$$0 \times \sin\theta_1 + 61.2 \times \sin 0° + 168.3 \sin 90° + 0.975\, m_4 \sin\theta_4 = 0$$

$$m_4 \sin\theta_4 = -172.6 \quad \text{...(i)}$$

$$\Sigma m_i r_i l_i \cos\theta_i = 0$$

$$0 \times \cos\theta_1 + 61.2 \times \cos 0 + 168.3 \cos 90° + 0.975\, m_4 \cos\theta_4 = 0$$

$$m_4 \cos\theta_4 = -62.769 \quad \text{...(ii)}$$

Square and add equation (i) and (ii)

$$m_4 = \sqrt{(172.6)^2 + (62.769)^2}$$

$$m_4 = 183.66 \text{ kg}$$

From equation (i), $\sin\theta_4 = -0.93978$

$$\theta_4 = 250.01°$$

Also, $\Sigma m_i r_i \sin\theta_i = 0$

$$0.65m_1 \sin\theta_1 + 153 \sin 0° + 153 \sin 90 + 0.65 \times 183.66 \times \sin 250.01 = 0$$

$$m_1 \sin\theta_1 = -62.799 \quad \text{...(iii)}$$

$$\Sigma m_i r_i \cos\theta_i = 0$$

$$0.65m_1 \cos\theta_1 + 153 \cos 0° + 153 \cos 90° + 0.65 \times 183.66 \times \cos 250.01 = 0$$

$$m_1 \cos\theta_1 = -172.6 \quad \text{...(iv)}$$

Square and add (iii) and (iv),

$$m_1 = \sqrt{(62.799)^2 + (172.6)^2}$$

$$m_1 = 183.66 \text{ kg}$$

From equation (iii)

$$\sin\theta_1 = \frac{-62.799}{183.66} = -0.3419$$

$$\theta_1 = 199.99°$$

$$\theta_1 \simeq 200°$$

(ii)
$$\omega = \frac{V}{r} = \frac{60 \times \frac{5}{18}}{\left(\frac{1.9}{2}\right)} = 17.543 \text{ rad/s}$$

$$\text{Swaying couple} = \pm\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$$

$$= \pm\frac{1}{\sqrt{2}}\left(1-\frac{2}{3}\right)\times 330 \times 0.3 \times 17.543^2 \times 0.7$$

$$= 5026.95 \text{ Nm}$$

(iii)
$$\text{Variation in tractive force} = \pm\sqrt{2}(1-c)mr\omega^2$$

$$= \pm\sqrt{2}\left(1-\frac{2}{3}\right)\times 330 \times 0.3 \times 17.543^2$$

$$= 14362.71 \text{ N}$$

(iv) Balance mass for reciprocating parts only

$$= 183.66 \times \frac{\frac{2}{3}\times 330}{510} = 79.225 \text{ kg}$$

$$\text{Hammer blow} = mr\omega^2$$

$$= 79.225 \times 0.65 \times 17.543^2 = 15848.5 \text{ N}$$

$$\text{Dead load} = 3.8 \times 1000 \times 9.81 = 37278 \text{ N}$$

$$\text{Maximum pressure on rails} = 37278 + 15848.5 = 53126.5 \text{ N}$$

$$\text{Minimum pressure on rails} = 37278 - 15848.5 = 21429.5 \text{ N}$$

(v) Maximum speed of locomotive without lifting wheels from the rails will be when the dead loads becomes equal to hammer blow.

i.e.
$$79.225 \times 0.65 \times \omega^2 = 37278$$

$$\omega = 26.905 \text{ rad/s}$$

$$\text{Velocity of wheels} = r\,\omega$$

$$= 26.905 \times \frac{1.9}{2} \text{ m/s} = 25.56 \text{ m/s} = 92.016 \text{ km/h}$$

2. (c) Solution:

$$\text{Tensile load, } F_T = 18 \text{ kN} = 18 \times 10^3 \text{ N}$$

$$\text{Shear load, } F_s = 12 \text{ kN} = 12 \times 10^3 \text{ N}$$

$$\text{Yield stress, } \sigma_{ys} = 328.6 \text{ MPa, FOS} = 2.5$$

$$\therefore \quad \text{Allowable stress, } \sigma_e = \frac{\sigma_{ys}}{FOS} = \frac{328.6}{2.5} = 131.44 \text{ MPa}$$

$$\text{Tensile stress, } \sigma_x = \frac{F_T}{A} = \frac{18 \times 10^3}{A}$$

$$\text{Shear stress, } \tau_{xy} = \frac{F_s}{A} = \frac{12 \times 10^3}{A} \qquad (\sigma_y = 0, \text{ not given})$$

(i) According to Rankine's theory of failure,

$$\sigma_e = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$131.44 = \frac{1}{2}\left[\frac{18 \times 10^3}{A} + \sqrt{\left(\frac{18 \times 10^3}{A}\right)^2 + 4\left(\frac{12 \times 10^3}{A}\right)^2}\right]$$

$$A = 182.59 = \frac{\pi d_c^2}{4}$$

$$\therefore \quad \text{Core dia. } d_c = 15.25 \text{ mm}$$

(ii) According to maximum shear stress theory,

$$\sigma_e = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$131.44 = \sqrt{\left(\frac{18 \times 10^3}{A}\right)^2 + 4\left(\frac{12 \times 10^3}{A}\right)^2}$$

$$\therefore \quad A = 228.24 = \frac{\pi d_c^2}{4}$$

$$\therefore \quad \text{Core diameter, } d_c = 17.05 \text{ mm}$$

(iii) According to Von-Mises theory of failure,

$$\sigma_e = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$131.44 = \sqrt{\left(\frac{18 \times 10^3}{A}\right)^2 + 3\left(\frac{12 \times 10^3}{A}\right)^2}$$

$\therefore \quad A = 209.19 = \frac{\pi d_c^2}{4}$

$\therefore$ Core diameter, d_c = 16.32 mm

(iv) According to Saint Venant's theory of failure,

$$\sigma_e = \frac{1}{2}\left[(1-\mu)(\sigma_x)+(1+\mu)\sqrt{\sigma_x^2+4\tau_{xy}^2}\right]$$

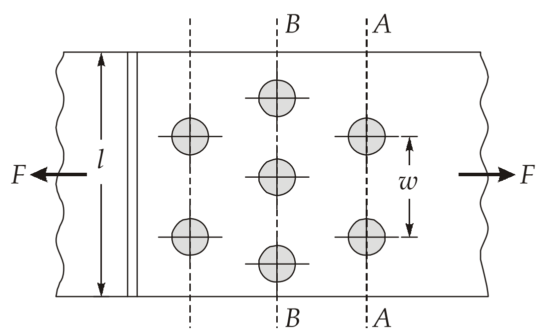
$$131.44 = \frac{1}{2}\left[(1-0.298)\frac{18\times10^3}{A}+(1+0.298)\sqrt{\left(\frac{18\times10^3}{A}\right)^2+4\left(\frac{12\times10^3}{A}\right)^2}\right]$$

$$A = 196.196 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core diameter, d_c = 15.81 mm

3. (a) Solution:

Given : $l \times t$ = 180 mm × 12 mm; d = 18 mm; w = 50 mm; σ_{yt} = 200 MPa; t = 150 MPa; σ_p = 300 MPa; F = 200 kN



Load carrying capacity according to shear strength of rivets.

$$F_1 = n\frac{\pi}{4}d^2\tau = 7\times\frac{\pi}{4}\times(18)^2\times(150)$$

$$= 267.19 \text{ kN}$$

Load carrying according to crushing strength,

$$F_2 = n \times d \times t(\sigma_p)$$

$$= 7 \times 18 \times 12 \times 300$$

$$F_2 = 453.6 \text{ kN}$$

Load carrying capacity according to tensile strength, there are two cases.

Case I : If the plate fails in tension at cross-section A-A

$$(F_{tension})_{A\text{-}A} = (l - 2d)t(\sigma_{yt})$$
$$= (180 - 2 \times 18) \times 12 \times 200$$
$$= 345.6 \text{ kN}$$

Case II : If the plate fails in tension at cross-section B-B and rivets on cross-section A-A must either fail in shear or in crushing.

$$(F_{tension})_{B\text{-}B} = (l - 3d)t(\sigma_{yt})$$
$$= (180 - 3 \times 18) \times 12 \times 200$$
$$= 302.4 \text{ kN}$$

Force causes the shear failure of the rivet at cross-section A-A

$$(F_{shear})_{A\text{-}A} = 2 \times \frac{\pi}{4} d^2 \times \tau$$
$$= 2 \times \frac{\pi}{4}(18)^2 \times 150 = 76.34 \text{ kN}$$

Force causes the crushing failure of the rivet at cross-section A-A,

$$(F_{crushing})_{A\text{-}A} = 2 \times \text{d} \times t \times \sigma_p$$
$$= 2 \times 18 \times 12 \times 300$$
$$= 129.6 \text{ kN}$$

The force required for tensile failure at B-B and simultaneous shear (A-A) is

$$F_3 = (F_{tension})_{B\text{-}B} + (F_{shear})_{A\text{-}A}$$
$$= 302.4 + 76.34$$
$$= 378.74 \text{ kN}$$

The force required for tensile failure at B-B and simultaneous crushing of rivet at A-A is

$$F_4 = (F_{tension})_{B\text{-}B} + (F_{crushing})_{A\text{-}A}$$
$$= 302.4 + 129.6$$
$$= 432 \text{ kN}$$

Therefore maximum allowable load for joint is

minimum $(F_1, F_2, F_3, F_4) = 267.19$ kN

and applied load is 200 kN. It means strength of joint is sufficient. No need to improve design.

3. (b) Solution:

Given : $L = 40$ m; $A = 45$ mm^2; $E = 170$ GPa; d = difference in length = 110 mm ;

$\sigma_Y = 500$ MPa

Initial stretching of cable 1.

Initially, cable 1 supports all of the load.

Let, W_1 = load required to stretch cable 1 to the same length as cable 2

$$W_1 = \frac{EA}{L}d = \frac{170 \times 45}{40 \times 10^3} \times 110 \text{ kN}$$

$$W_1 = 21.0375 \text{ kN}$$

$$\delta_1 = 110 \text{ mm (elongation of cable 1)}$$

$$\sigma_1 = \frac{W_1}{A} = \frac{Ed}{L} = \frac{21.0375 \times 10^3}{45} = 467.5 \text{ MPa}$$

$$\sigma_1 < \sigma_Y$$

(i) Yield load W_Y

Cable 1 yields first,

$$F_1 = \sigma_Y A$$

$$= (500 \times 45) \times 10^{-3} \text{ kN}$$

$$= 22.5 \text{ kN}$$

$$\delta_{1Y} = \text{Total elongation of cable 1}$$

$$\delta_{1Y} = \frac{F_1 L}{EA} = \frac{\sigma_Y L}{E} = \frac{500 \times 40}{170 \times 10^3} = 0.1176 \text{ m}$$

$$\delta_Y = \delta_{1Y} = 117.6 \text{ mm}$$

$$\delta_{2Y} = \text{elongation of cable 2}$$

$$= \delta_{1Y} - \text{d}$$

$$= 117.64 - 110 = 7.64 \text{ mm}$$

$$F_2 = \frac{EA}{L}\delta_{2Y} = \frac{170 \times 45 \times 7.64}{40 \times 10^3} = 1.453 \text{ kN}$$

$$W_y = F_1 + F_2$$

$$W_y = 22.5 + 1.453 = 23.95 \text{ kN}$$

(ii) Plastic load W_P

$$F_1 = \sigma_Y A, \quad F_2 = \sigma_Y A$$

$$W_P = 2\sigma_Y A = 2 \times 500 \times 45 \times 10^{-3} \text{ kN}$$

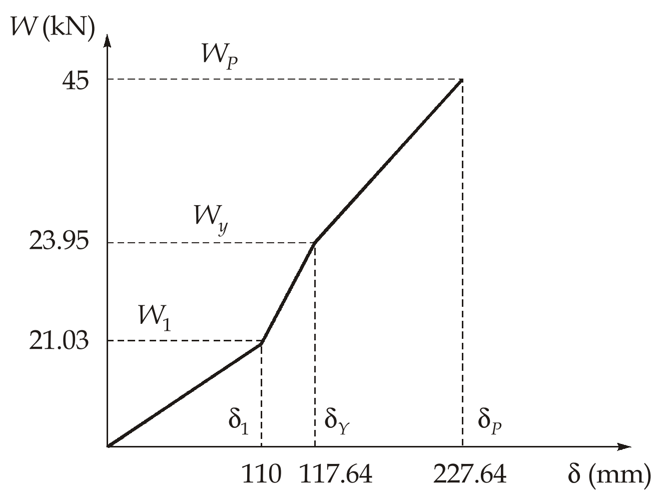
$$= 45 \text{ kN}$$

Elongation of cable 2, $\delta_{2P} = \dfrac{F_2 L}{EA} = \dfrac{\sigma_Y L}{E} = \dfrac{500 \times 40 \times 10^3}{170 \times 10^3} = 117.64$ mm

$$\delta_{1P} = \delta_{2P} + d$$

$$= 117.64 + 110 = 227.64 \text{ mm}$$

(iii) Load displacement diagram



$$\frac{W_Y}{W_1} = \frac{23.95}{21.0375} = 1.1389$$

$$\frac{\delta_Y}{\delta_1} = \frac{117.64}{110} = 1.0694$$

$$\frac{W_P}{W_Y} = \frac{45}{23.95} = 1.878$$

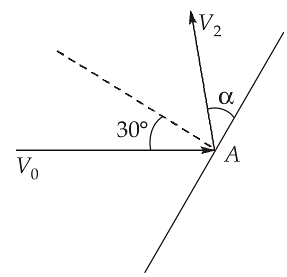
$$\frac{\delta_P}{\delta_Y} = \frac{227.64}{117.6} = 1.935$$

$$0 < W < W_1 : \text{slope} = \frac{21.0375 \times 10^3}{0.11} = 191250 \text{ N/m}$$

$$W_1 < W < W_Y : \text{slope} = \frac{(23.95 - 21.0375) \times 10^3}{0.1176 - 0.110} = 382657 \text{ N/m}$$

$$W_Y < W < W_P : \text{slope} = \frac{(45 - 23.95) \times 10^3}{(0.22764 - 0.11764)} = 191250 \text{ N/m}$$

3. (c) (i) Solution:



Considering wall to be frictionless, momentum will be conserved along the wall

$$MV_o \sin 30° = MV_2 \cos\alpha \quad \text{...(i)}$$

$$\Rightarrow \quad V_2 \cos\alpha = 4$$

In perpendicular direction of the wall

$$e = \frac{-(\text{Velocity of separation})}{(\text{Velocity of approach})}$$

$$0.9 = \frac{0 - (-V_2 \sin\alpha)}{V_o \cos 30° - 0} \quad \text{...(1)}$$

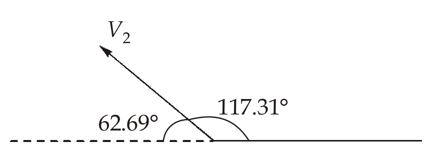
$$V_2 \sin\alpha = 6.235 \text{ m/s}$$

From (1) and (2)

$$V_2 = 7.408 \text{ m/s}$$

$$\alpha = 57.31°$$

So, ball will rebound to (60 + 57.31)° from horizontal at $V_2 = 7.408$ m/s



$$V_{2y} = V_2 \sin(62.69°) = 6.582 \text{ m/s}$$

$$V_{2x} = V_2 \cos(62.69°) = 3.400 \text{ m/s}$$

Using equation of motion for uniform acceleration in vertical direction

$$s = ut + \frac{1}{2}at^2$$

$$-1 = 6.582t - \frac{9.81}{2}t^2$$

$\Rightarrow$ $4.905t^2 - 6.582t - 1 = 0$

$\Rightarrow$ $t = 1.48$ or -0.13 sec

So, $t = 1.48$ sec

$$BD = V_{2x}t$$
$$= 3.4 \times 1.48 = 5.032 \text{ m}$$
$$CD = \frac{1}{\tan 60°} = 0.577 \text{ m}$$
$$d = BC = BD - CD = 4.455 \text{ m}$$

Given : $m_1 = 140$ kg ; $m_2 = 70$ kg; $M = 85$ kg; $k = 110$ mm $= 0.11$ m; $r_1 = 240$ mm $= 0.24$ m; $r_2 = 120$ mm $= 0.12$ m

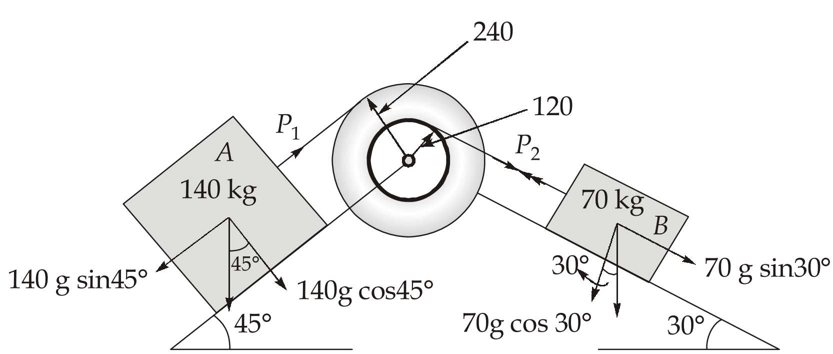
Let P_1 = Pull in the string carrying 140 kg mass, and

P_2 = Pull in the string carrying 70 kg mass.

a_1 = Acceleration of the 140 kg mass,

a_2 =Acceleration of the 70 kg mass, and

α = Angular acceleration of the pulley.



We know that mass moment of inertia of the pulley,

$$I = mk^2 = 85(0.11)^2 = 1.0285 \text{ kg-m}^2$$

First of all, consider the motion of 140 kg mass, which is coming down. We know that the forces acting on it, the plane, $m_1 g \sin\theta_1 = 140 \times 9.81 \sin 45° = 140 \times 9.81 \times 0.707 =$

971.14 N (downwards), and P_1 newtons (upwards). As the mass is moving downwards, therefore resultant force.

$$= 971.14 - P_1 \text{ newtons} \quad ...(i)$$

Since the mass is moving downwards with an acceleration (a_1), therefore force acting on the body

$$= 140a_1 \text{ newtons} \quad ...(ii)$$

Equating equations (i) and (ii),

$$971.14 - P_1 = 140a_1 \quad ...(iii)$$

Now consider the motion of 70 kg mass, which is going up-wards. We know that the forces acting on it, along the plane, are $m_2 g \sin\theta_2 = 70 \times 9.8 \sin 30° = 70 \times 9.8 \times 0.5 =$ 343.35 N (downwards) and P_2 newtons upwards. As the mass is moving upwards, therefore resultant force

$$= P_2 - 343.35 \text{ newtons} \quad ...(iv)$$

Since the mass is moving upwards with an acceleration (a_2), therefore force acting on the body

$$= 70a_2 \text{ newtons} \quad ...(v)$$

Equating equations (iv) and (v),

$$P_2 - 343.35 = 70a_2 \quad ...(vi)$$

Now consider the motion of the pulley which is rotating about its axis due to downward motion of the 140 kg mass tied to the string. We know that linear acceleration of the 140 kg mass is equal to the angular acceleration of the pulley.

$\therefore$ $$a_1 = r_1 \cdot \alpha = 0.24\alpha$$

Similarly, $$a_2 = r_2 \cdot \alpha = 0.12\alpha$$

and $$\text{Torque, } T = P_1 \cdot r_1 - P_2 \cdot r_2 = P_1 \times 0.24 - P_2 \times 0.12 \quad ...(vii)$$

We also known that torque on the pulley,

$$T = I \cdot \alpha = 1.0285\,\alpha \quad ...(viii)$$

Equating equations (vii) and (viii),

$$= 0.24P_1 - 0.12P_2 = 1.0285\alpha$$

or $$P_1 - 0.5P_2 = 4.285\alpha$$

$\therefore$ $$P_1 = 0.5P_2 + 4.285\alpha \quad ...(ix)$$

Substituting the value of P_1 in equation (iii),

$$971.14 - (0.5P_2 + 4.285\alpha) = 140a_1 = 140 \times 0.24\alpha$$

$$971.14 - 0.5P_2 - 4.285\alpha = 33.6\alpha$$

$\therefore \quad 971.14 - 0.5P_2 = 37.5\alpha + 4.285\alpha = 37.88\alpha$

Multiplying both sides by 2

$$1942.28 - P_2 = 75.77\alpha \quad ...(x)$$

From equation (vi), we find that

$$P_2 - 343.35 = 70a_2 = 70 \times 0.12\alpha = 8.4\alpha \quad ...(xi)$$

Adding equation (x) and (xi),

$$1598.93 = 84.17\alpha$$

$\therefore \quad \alpha = 18.99 \text{ rad/s}^2$

Now substituting the value of α in equations (*x*),

$$1942.28 - P_2 = 75.77\alpha = 75.77 \times 18.99 = 1438.87$$

$\therefore \quad P_2 = 503.4 \text{ N}$ **Ans.**

Again substituting the value of α and P_2 in equation (ix),

$P_1 = (0.5 \times 503.4) + (4.285 \times 18.99) = 333.07 \text{ N}$ **Ans.**

Acceleration of the masses A and B

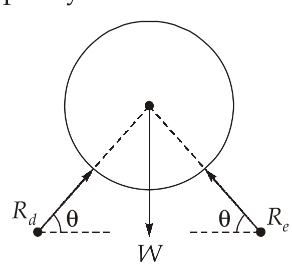
We know that the acceleration of mass A (i.e. 150 kg),

$a_1 = r_1\alpha = 0.24 \times 18.99 = 4.56 \text{ m/s}^2$ **Ans.**

Similarly, $a_2 = r_2\alpha = 0.12 \times 18.97 = 2.28 \text{ m/s}^2$ **Ans.**

3. (c) (ii) Solution:

The free body diagram of the upper cylinder is shown below:

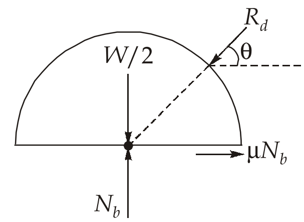


This cylinder is in equilibrium under the action of three coplanar forces, R_d, R_e and W.

From the consideration of equilibrium

$$R_d = R_e = \frac{W}{2\sin\theta} \quad \text{... (i)[Symmetrical configuration]}$$

The free body diagram of the lower left semicircular cylinder is shown below.



Again, from equilibrium consideration,

$$\Sigma F_H = 0 \Rightarrow -R_d \cos\theta + \mu N_b = 0 \quad \text{... (ii)}$$

$$\Sigma F_V = 0 \Rightarrow N_b - \frac{W}{2} - R_d \sin\theta = 0$$

$$N_b - \frac{W}{2} - \frac{W}{2\sin\theta} \times \sin\theta = 0 \text{ [From equation (i)]}$$

$$N_b = W$$

On putting, $N_b = W$, $\mu = 0.5$ and $R_d = \dfrac{W}{2\sin\theta}$ in equation (ii)

We get, $-\dfrac{W}{2\sin\theta} \times \cos\theta + 0.5 \times W = 0$

$$0.5W = \frac{W}{2} \times \cot\theta$$

$$\cot\theta = 1 \quad \text{... (iii)}$$

Also, from the geometry of the figure.

$$\cot\theta = \frac{b/2}{\sqrt{4r^2 - \frac{b^2}{4}}}$$

$$\frac{b/2}{\sqrt{4r^2 - \frac{b^2}{4}}} = 1 \text{[From equation (iii)]}$$

$$\frac{b}{2} = \sqrt{4r^2 - \frac{b^2}{4}}$$

On squaring both sides

$$\frac{b^2}{4} = 4r^2 - \frac{b^2}{4}$$

$$\frac{2b^2}{4} = 4r^2$$

$$\frac{b^2}{2} = 4r^2$$

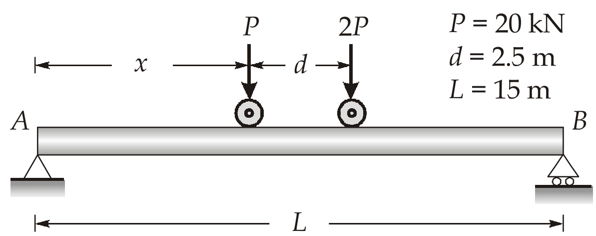
$$b^2 = 8r^2$$

$$b = 2\sqrt{2}r$$

Hence, the maximum distance is $b = 2\sqrt{2}r$, between the centers B and C for the equilibrium to be possible.

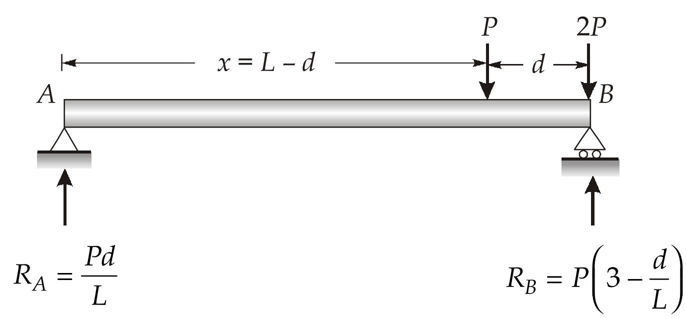
4. (a) Solution:

Moving loads on a beam



(i) Maximum shear force

By inspection, the maximum shear force occurs at support B when the larger load is placed close to, but not directly over, that support.



$$\Sigma M_A = 0$$

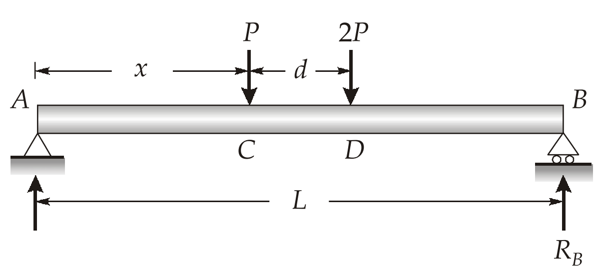
$$R_B \times L - 2P \times L - P(L - d) = 0$$

$$V_{max} = R_B = P\left(3 - \frac{d}{L}\right) = 56.66 \text{ kN}$$

$$x = L - d = 15 - 2.5 = 12.5 \text{ m}$$

(ii) Maximum bending moment

By inspection, the maximum bending moment occurs at point D, under the larger load $2P$.



Reaction at support B :

$$R_B = \frac{P}{L}x + \frac{2P}{L}(x+d) = \frac{P}{L}(2d+3x) \quad \text{...(i)}$$

Bending moment at D :

$$M_D = R_B(L-x-d) = \frac{P}{L}(2d+3x)(L-x-d)$$

$$= \frac{P}{L}[-3x^2 + (3L-5d)x + 2d(L-d)] \quad \text{...(ii)}$$

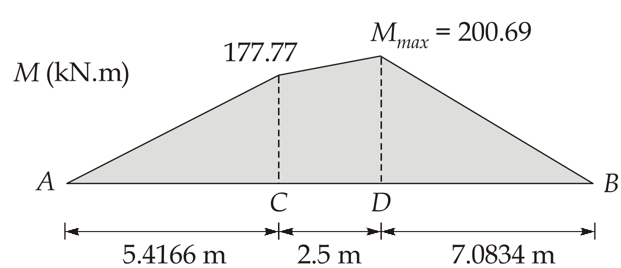
$$\frac{dM_D}{dx} = \frac{P}{L}(-6x+3L-5d) = 0$$

Solve for x : $$x = \frac{L}{6}\left(3 - \frac{5d}{L}\right) = \frac{15}{6}\left(3 - \frac{5\times 2.5}{15}\right) = 5.4166 \text{ m}$$

Substitute x into equation (ii)

$$M_{max} = \frac{P}{L}\left[-3\left(\frac{L}{6}\right)^2\left(3-\frac{5d}{L}\right)^2 + (3L-5d)\times\left(\frac{L}{6}\right)\left(3-\frac{5d}{L}\right) + 2d(L-d)\right]$$

$$= \frac{PL}{12}\left(3-\frac{d}{L}\right)^2 = 200.69 \text{ N m}$$



From equation (i)

$$R_B = \frac{20}{15}\times(2\times 2.5 + 3\times 5.4166) = 28.33 \text{ kN}$$

$$R_A = 3 \times 20 - 28.33 = 31.67 \text{ kN}$$

$$M_C = R_A \times x = 31.67 \times 5.4166$$

$$= 171.54 \text{ kNm}$$

$$M_D = M_{max} = 200.69 \text{ N-m}$$

4. (b) Solution:

Given: ω_2 = 12 rad/sec, Number of links (n) = 6

$$\text{Number of I-centres} = \frac{n(n-1)}{2} = \frac{6\times 5}{2} = 15$$

I-centres: $I_{12}, I_{13}, I_{14}, I_{15}, I_{16}$

$I_{23}, I_{24}, I_{25}, I_{26}$

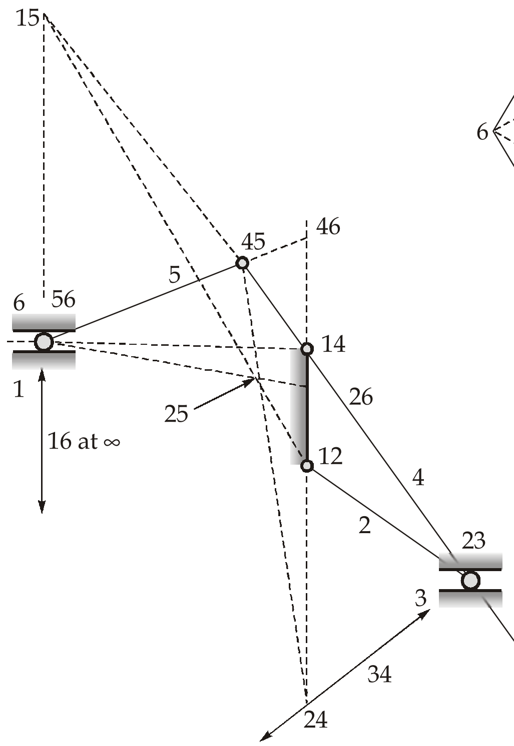
I_{34}, I_{35}, I_{36}

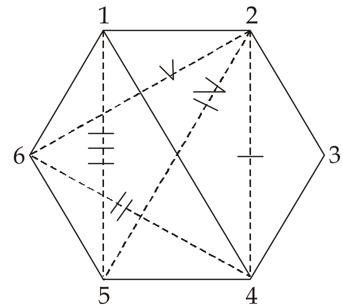
I_{45}, I_{46}

I_{56}

I-centre 26 is needed to be located as the velocity of the link 2 is known and that of 6 is to be found.

- Locate the fixed I-centres I_{12}, I_{23}, I_{34}, I_{45}, I_{56}, I_{16} and I_{14}.
- Locate I-centre I_{24} at the intersection of lines joining I-centre I_{23}, I_{34} and I_{12}, I_{14}.
- Locate I-centre I_{46} at the intersection of lines joining I-centre I_{14}, I_{16} and I_{45}, I_{56}.
- I-centre I_{16} is perpendicular to AS and lies at infinity. Joining I_{12} and I_{16} means a line passing through OA.
- Now, while locating I-centre I_{26} at the inter-section of lines joining I-centre I_{12}, I_{16} and I_{24}, I_{46} it is observed the they lie on the same vertical line OA. Thus, I-centre I_{26} cannot be located by using this path.
- Locate I-centre I_{15} and I_{25} using Kennedy's theorem.
- Now, locate I-centre I_{26} which lies at the intersection of lines joining I-centres I_{12}, I_{16} and I_{25}, I_{56}.





Now, as the velocity of the I-centre I_{26} is the same whether it is considered to lie on the link 2 or 6.

$$\therefore \quad V_{26} = \omega_2 I_{12} I_{26} = V_s$$

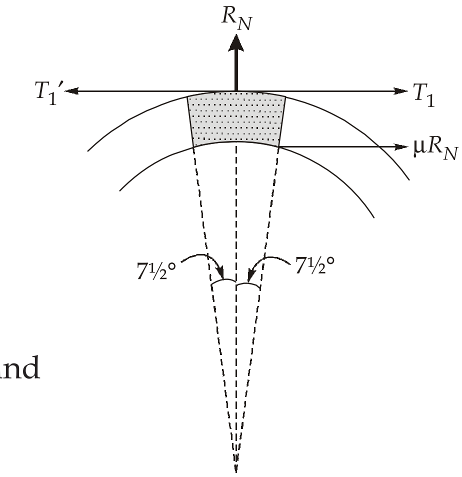
$$V_s = \omega_2 (I_{12} I_{26}) = 12 \times 0.137$$

$$V_s = 1.644 \text{ m/s}$$

4. (c) Solution:

Since $OA > OB$, therefore the force at C must act downward. Also, the drum rotates clockwise, therefore the end of the band attached to A will be slack with tension T_2 (least tension) and the end of the band attached to B will be tight with tension T_1 (greatest tension).

Consider one of the blocks (say first block) as shown is figure.



This is in equilibrium under the action of the following four forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T'_1) or the tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction (μR_N).

Resolving the forces radially, we have

$$(T_1 + T'_1)\sin 7.5° = R_N$$

Resolving the forces tangentially, we have

$$(T_1 - T'_1)\cos 7.5° = \mu R_N$$

Dividing equation (ii) by (i), we have

$$\frac{(T_1 - T_1')\cos 7.5°}{(T_1 + T_1')\sin 7.5°} = \mu$$

or $$\frac{(T_1 - T_1')}{(T_1 + T_1')} = \mu \tan 7.5°$$

$\therefore$ $$\frac{T_1}{T_1'} = \frac{1 + \mu \tan 7.5°}{1 - \mu \tan 7.5°}$$

Similarly, for the other blocks, the ratio of tensions $\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'}$ etc., remains constant.

Therefore for 12 blocks having greatest tension T_1 and least tension T_2 is

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7.5°}{1 - \mu \tan 7.5°}\right)^{12}$$

Least force required at C

Let P = Least force required at C

Now, diameter of band, $D = d + 2t = 0.85 + 2 \times 0.75$

$D = 1$ m

and power absorbed $= \frac{(T_1 - T_2) \times \pi D N}{60}$

$$\therefore \quad T_1 - T_2 = \frac{250 \times 10^3 \times 60}{\pi \times 1 \times 320} = 14920.77 \text{ N}$$

We have proved that,

$$\frac{T_1}{T_2} = \left(\frac{1+\mu\tan 7.5°}{1-\mu\tan 7.5°}\right)^{12}$$

$$\frac{T_1}{T_2} = \left(\frac{1+0.4\tan 7.5°}{1-0.4\tan 7.5°}\right)^{12} = 3.54$$

$$\therefore \quad T_1 = 3.54T_2$$

Also, $3.54T_2 - T_2 = 14920.77$

$$\therefore \quad T_2 = 5874.32 \text{ N}$$

and $T_1 = 20795.09$ N

Now taking moments about O, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 30$$

or $P \times 500 = 5874.32 \times 150 - 20795.09 \times 30$

$$\therefore \quad P = 514.59 \text{ N}$$ **Ans.**

Section : B

5. (a) Solution:

Given data : V_c = 200 m/min; α = 8°; w = 2 mm; t_1 = 0.2 mm; μ = 0.5; τ_s = 400 MPa;

$$\beta = \tan^{-1}(\mu) = 26.565°$$

$$\beta = 26.565°$$

According to Lee-shafer's theory

$$\phi + \beta - \alpha = 45°$$

$$\phi + 26.565 - 8 = 45°$$

$$\phi = 26.435°$$

$$\text{Shear force; } F_s = \frac{\tau_s \cdot w \cdot t_1}{\sin\phi} = \frac{400 \times 2 \times 0.2}{\sin(26.435°)} = 359.40 \text{ N}$$

$$F_s = R \cdot \cos(\phi + \beta - \alpha)$$

$$R = \frac{359.40}{\cos(26.435° + 26.565 - 8)}$$

$$R = 508.27 \text{ N}$$

$$F_c = R\cos(\beta - \alpha) = 481.82 \text{ N} \quad \textbf{Ans.}$$

$$F_T = R\sin(\beta - \alpha) = 161.82 \text{ N} \quad \textbf{Ans.}$$

5. (b) Solution:

(i) Flame hardening : This is the simplest form of heat treatment process. The workpiece is heated by means of a gas torch (oxy-acetylene flame) followed by a water spray on the heated parts. The heat from the torch penetrates only to a small depth up to 3 mm on the surface and consequently the steel in the outer layers gets quenched to martensite and bainite. This process is suitable for any complex shape of the component such as crank shaft, large gear, cams etc. with carbon percentage ranging from 0.3 to 0.6%.

The characteristic of the process are :

- Hard and highly wear resistance surface (deep case depths).
- Good bending fatigue strength.
- Fair resistance to seizure.
- Fair dimensional control possible.
- Fair freedom from quench cracks
- Medium capital investment

(ii) Induction hardening : This is similar to flame hardening process where the heating of the component surface is achieved by the electromagnetic induction. The workpiece such as crank shaft is enclosed in the magnetic field of an alternating (10 kHz to 2 MHz) current conductor to obtain case depth of the order of 0.25 to 1.5 mm. This causes induction heating of the workpiece. The heated workpiece is then quenched by water spray. The induction heat penetrates only the outer surface of the workpiece and as a result only the skin gets hardened by the quenching process. The whole process is very fast (5 seconds 4 minutes) and results in hard outer surface which is wear resistant).

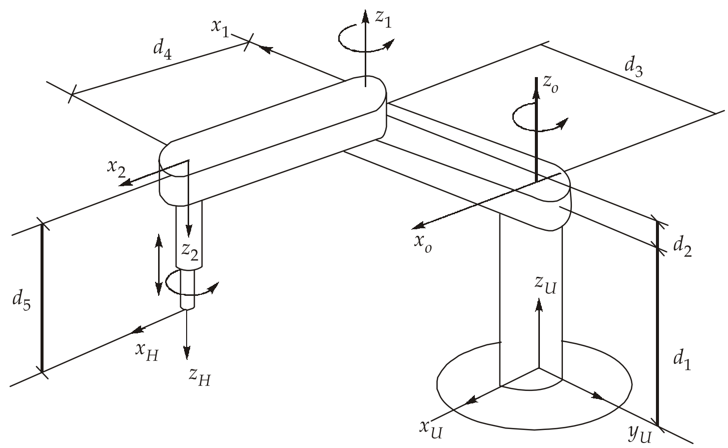
(iii) Precipitation hardening :

- It is one of the principal mechanism by which the non-ferrous alloys are heat treated.
- In the precipitation hardening, the alloy is heated and soaked to form a single phase solid solution. This is then cooled quickly (quenched in water) so that the single phase structure is retained even at the room temperature.
- The hardness obtained is a function of precipitation temperature since the size of the precipitating particles and their distribution control the movement of dislocations.

- If the precipitation process occurs at room temperature, it is called age hardening.

5. (c) (i) Solution:

1. As per given configuration diagram of Robot assigning the coordinate from based on D-H representation.



2. D-H parameter table :

Parameter / Position	θ	d	a	α
0–1	θ_1	d_2	d_3	0
1–2	θ_2	0	d_4	180°
2–H	θ_3	d_5	0	0

5. (c) (ii) Solution:

Polar (Spherical) Configuration : The polar configuration is illustrated in figure (a). It consists of a telescopic link (prismatic joint) that can be raised or lowered about a horizontal revolute joint. These two links are mounted on a rotating base. This arrangement of joints, known as RRP configuration, gives the capability of moving the arm end-point within a partial spherical shell space as work volume, as shown in figure (a).

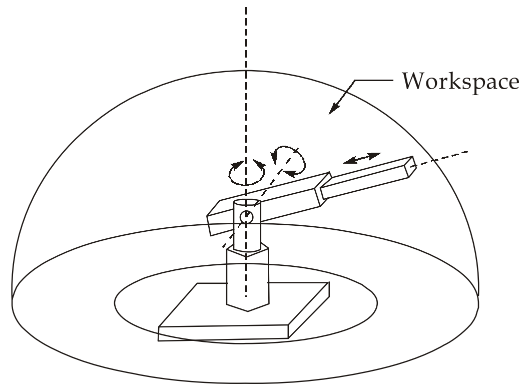


Fig. (a) A 3-DOF polar arm configuration and its workspace

This configuration allows manipulation of objects on the floor because its shoulder joint allows its end-effector to go below the base. Its mechanical stiffness is lower than Cartesian and cylindrical configurations and the wrist positioning accuracy decreases with the increasing radial stroke. The construction is more complex. Polar arms are mainly employed for industrial applications such as machining, spray painting and so on. Alternate polar configuration can be obtained with other joint arrangements such as RPR, but PRR will not give a spherical work volume.

Articulated (Revolute or Jointed-arm) Configuration : The articulated arm is the type that best simulates a human arm and a manipulator with this type of an arm is often referred as an anthropomorphic manipulator. It consists of two straight links, corresponding to the human "forearm" and "upper arm" with two rotary joints corresponding to the "elbow" and "shoulder" joints. These two links are mounted on a vertical rotary table corresponding to the human waist joint. Figure (b) illustrates the joint-link arrangement for the articulated arm. This configuration (RRR) is also called revolute because three revolute joints are employed. The work volume of this configuration is spherical shaped, and with proper sizing of links and design of joints, the arm endpoint can sweep a full spherical space. The arm endpoint can reach the base point and below the base, as shown in figure (b) This anthropomorphic structure is the most dexterous one, because all the joints are revolute, and the positioning accuracy varies with arm endpoint location in the workspace. The range of industrial applications of this arm is wide.

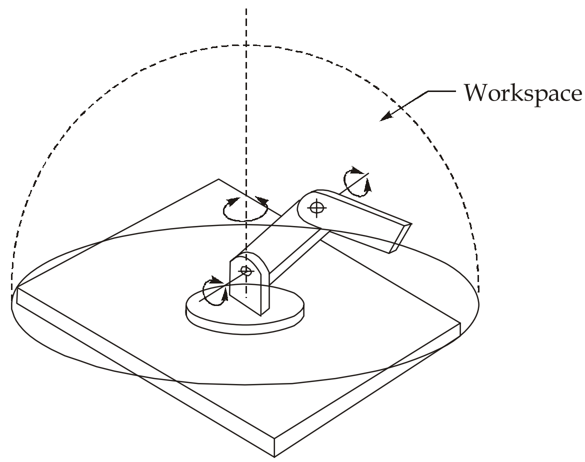


Fig. (b) A 3-DOF articulated arm configuration and its workspace

5. (d) Solution:

Given : R = 18000 units per year; k = 3000 units per month = 36000 units per year;

c_o = Rs.500 per unit per setup; c_c = 0.15 per month = Rs.1.8 per unit per year;

c_s = Rs. 15 per unit per year

(i)

$$\text{EOQ } (Q_0) = \sqrt{\left(\frac{2Rc_0}{c_c}\right)\left(\frac{k}{k-r}\right)\left(\frac{c_s+c_c}{c_s}\right)}$$

$$= \left[\frac{2\times 18000\times 500}{1.8}\times\frac{36000}{18000}\times\frac{16.8}{15}\right]^{1/2}$$

$$= 4732.86 \text{ units} = 4733 \text{ units}$$ **Ans.**

(ii) Optimum number of shortages,

$$Q_2 = \left(\frac{c_c}{c_c+c_s}\right)\left(1-\frac{R}{k}\right)Q_0$$

$$Q_2 = \frac{1.8}{1.8+15}\left(1-\frac{18000}{36000}\right)\times 4733$$

$$= 253.55 \simeq 254 \text{ units}$$

(iii) Maximum inventoy level $= Q_0\times\left(\frac{k-R}{k}\right)-Q_2$

$$= 4733\times\left(\frac{36000-18000}{36000}\right) - 254 = 2112.5 \simeq 2113 \text{ units}$$

(iv) Production time, $\frac{Q_0}{k} = \frac{4733}{36000} = 0.1315$ years or 1.58 months

(v) Time between setups, $\frac{Q_0}{R} = \frac{4733}{18000} = 0.2629$ years or 3.15 months

5. (e) Solution:

A PID controller is a widely used feedback controller that combines proportional integral and derivative action to improve the performance of a control system in terms of stability, transient response and steady state accuracy.

Output of a PID controller is given by:

$$V_0(t) = k_p e(t) + k_I \int e(t)dt + k_D \frac{de(t)}{dt}$$

$$\text{Transfer function, } G(s) = \frac{V_o(s)}{E(s)} = k_p + \frac{k_I}{s} + k_D s$$

$$G(s) = \frac{k_D s^2 + k_P s + k_I}{s}$$

PID controller introduces two zeros and one pole at origin, with this although system behaviour remains almost unchanged, its steady-state error will go to zero.

Effect of proportional controller (k_P)

- Provides control action proportional to error.
- Improves speed of response.
- it reduces steady state error but cannot eliminate it completely.

Effect of Integral control (k_I)

- Eliminates steady state error by integrating error over time.
- Adss a pole at origin increasing system type.
- May reduce stability and slow down response.

Effect of derivative control (k_D)

- Acts on rate of change of error
- Improves stability and damping
- Reduces overshoot and improves transient response

A PID controller combines the advantage of all three control actions:

- Fast response (Proportional)
- Zero steady - state error (Integral)
- Improved stability and damping (Derivative)

6. (a) Solution:

Given, Initial dimensions of billet 24 × 25 × 160 mm^3.

Final dimensions of billet 6 × 100 × 160 mm^3 as width never changes after forging of rectangular billet and all the calculations of forging operation are done on the final dimensions of billet.

So, the dimensions of billet are

Thickness of billet (h) = 6 mm

Length of billet ($2L$) = 100 mm

Width of billet (w) = 160 mm

Coefficient of friction (μ) = 0.24

Step 1: Checking for the sticking

As we know sticking length in case of rectangular billet is given by

$$\text{Sticking length } (x_s) = L - \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right)$$

$$x_s = 50 - \frac{6}{2\times 0.24}\ln\left(\frac{1}{2\times 0.24}\right)$$

$$x_s = 40.82 \text{ mm}$$

+ve sign shows sticking will occur during the forging operation.

Step II :

As we know that forging stress during sticking zone is given by

$$P_{x2} = \frac{K'}{\mu} + \frac{2K'}{h}(x_s - x)$$

as forging of rectangular billet is a case of plane strain,

So, $$2K' = \frac{2\sigma_0}{\sqrt{3}} \quad \text{where, } \sigma_0 = \text{mean flow stress}$$

$$\sigma_0 = 8 \text{ N/mm}^2 \text{ (given)}$$

$$\Rightarrow \quad P_{x2} = \frac{\sigma_0}{\sqrt{3}\mu} + \frac{2\sigma_0}{\sqrt{3}h}(x_s - x)$$

$$P_{x2} = \left(\frac{8}{\sqrt{3}\times 0.24}\right) + \left(\frac{2\times 8}{\sqrt{3}\times 6}\right)(40.82 - x)$$

$$P_{x2} = 19.24 + 62.846 - 1.539x$$

$$P_{x_2} = 82.09 - 1.539x$$

So, forging load during sticking $= 2 \times \int_0^{x_s} P_{x_2} \cdot w dx$

$$= (2 \times 160) \times \int_0^{40.82} (82.09 - 1.539x) dx$$

$$= 320 \times \left| 82.09x - \frac{1.539x^2}{2} \right|_0^{40.82}$$

$$= 320 \times \left| (82.09 \times 40.82) - \frac{1.539}{2} \times (40.82)^2 \right|$$

$$= 661.99 \text{ kN}$$

Step III :

Forging stress during Non-sticking zone. As we know forging stress during non-sticking zone is given by

$$P_{x_1} = 2K' e^{\frac{2\mu}{h}(L-x)}$$

$$P_{x_1} = \left(\frac{2 \times \sigma_0}{\sqrt{3}}\right) \cdot e^{\frac{2 \times 0.24}{6} \times (50-x)}$$

$$P_{x_1} = 9.237 \cdot e^{\frac{(50-x)}{12.5}}$$

So, Forging load $= 2 \times \int_{x_s}^{L} P_{x_1} \cdot w.dx$

$$= (2 \times w) \int_{40.82}^{50} 9.237 \cdot e^{\left(\frac{50-x}{12.5}\right)} dx$$

$$= 2 \times 160 \times 125.16$$

$$= 40051.63 \text{ or } 40.05 \text{ kN}$$

So, Total forging load = Load (1) + Load (2) = 661.99 + 40.05

Total forging load = 702.04 kN

6. (b) Solution:

Period (t)	Demand (dt)	t^2	$t \times dt$
0	102	0	0
1	106	1	106
2	109	4	218
3	113	9	339
4	117	16	468
5	118	25	590
$\sum t = 15$	$\sum dt = 665$	$\sum t^2 = 55$	$\sum tdt = 1721$

$$\sum t = 15$$

$$d_t = a + bt \quad \text{...(i)}$$

$$\sum d_t = \sum a + b\sum t$$

$$\sum d_t = na + b\sum t \quad \text{...(ii)}$$

Multiplying t in equation (i), both sides

$$td_t = ta + tbt^2$$

$$\sum td_t = \sum(ta) + \sum(bt^2)$$

$$\sum td_t = a\sum t + b\sum t^2 \quad \text{...(iii)}$$

Putting values in equation (ii) and (iii),

$$665 = 6a + 15b$$

$$1721 = 15a + 55b$$

On solving, $a = 102.47$

$$b = 3.342$$

$$d_t = 102.47 + 3.342t$$

Forecast for the 11th period would be when $t = 10$

$$d_t = 102.47 + 3.342 \times 10$$

$$= 135.89$$

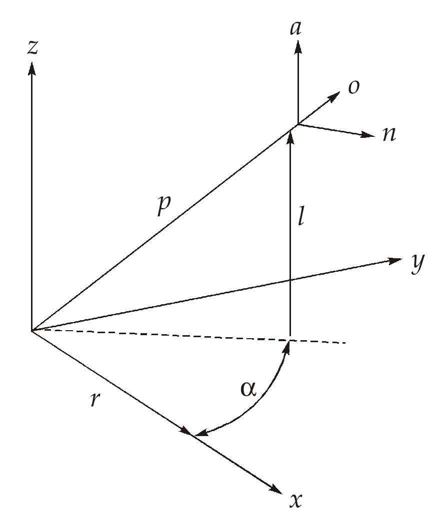
t	d_t	$F_t = 102.47 + 3.342t$	$(d_t - F_t)^2$
0	102	102.47	0.2209
1	106	105.812	0.0353
2	109	109.154	0.0237
3	113	112.496	0.2540
4	117	115.838	1.3502
5	118	119.18	1.3924
			$\Sigma(d_f - f_t)^2 = 3.2765$

Standard error of the estimate,

$$S_{tdf} = \sqrt{\frac{(d_t - F_t)^2}{n-2}} = \sqrt{\frac{3.2765}{6-2}} = 0.9050$$ **Ans.**

6. (c) Solution:

A cylindrical coordinate system includes two linear translations and one rotation. The sequence is a translation of r along the x-axis, a rotation of α about the z-axis, and a translation of l along the z-axis, as shown in figure below. Since these transformations are all relative to the universe frame. The total transformation caused by these three transformations is found by pre-multiplying by each matrix as follows:



Cylindrical coordinates

$$^{R}T_P = T_{cyl}(r,\alpha,l) = Trans(0,0,l)Rot(z,\alpha)Trans(r,0,0)$$

$$^{R}T_P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c\alpha & -s\alpha & 0 & 0 \\ s\alpha & c\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{R}T_P = T_{cyl}(r,\alpha,l) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First three columns represent the orientation of the frame after this series of transformations. In this case, we are only interested in the position of the origin of the frame, or the last column.

The original orientation of the frame can be restored by rotating the n, o, a frame about the a-axis an angle of $-\alpha$, which is equivalent to post multiplying the cylindrical coordinate matrix by a rotation matrix of Rot $(a, -\alpha)$.

$$T_{cyl} \times \text{Rot}\,(a, -\alpha) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c(-\alpha) & -s(-\alpha) & 0 & 0 \\ s(-\alpha) & c(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & rc\alpha \\ 0 & 1 & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore,

$$T = \begin{bmatrix} 1 & 0 & 0 & -2.394 \\ 0 & 1 & 0 & 6.578 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & rc\alpha \\ 0 & 1 & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By comparing it with given matrix:

$$l = 9$$

$$rc\alpha = -2.394 = r\cos\alpha$$

$$rs\alpha = 6.578 = r\sin\alpha$$

$$\tan\alpha = -\frac{6.578}{2.394} = -2.7477$$

$$\alpha = -70° \text{ or } 110°$$

As $s\alpha$ and $c\alpha$ are positive and negative, respectively. Therefore α is in the second quadrant.

$$\alpha = 110°$$

$$r\sin\alpha = 6.578$$

$$\Rightarrow \quad r = 7$$

As derived above, the original orientation of the robot will be given as

$$^{R}T_P = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the value of r, α and l, we get

$$^{R}T_P = \begin{bmatrix} -0.342 & -0.9397 & 0 & -2.394 \\ 0.9397 & -0.342 & 0 & 6.578 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. (a) (i) Solution:

On the basis of mechanical characteristics upon heating:

Thermoplastics	Thermosetting polymers
Soften on heating and eventually liquefy.	Do not soften on heating once formed.
Harden again on cooling (reversible process).	Become permanent hard during formation.
Secondary bonding forces decrease with temperature, allowing chain movement under stress.	Covalent crosslinks restrict chain motion at high temperature.
Can be reheated and reshaped repeatedly, and become permanently degraded if heated excessively.	Heating to very high temperature causes degradation instead of softening.
Generally softer compared to thermosets.	Generally hard, stronger, and dimensionally more stable

According to possible molecular structures:

Thermoplastics :

- Mostly linear polymers.
- May have some branched structures.
- Chains are held together mainly be secondary bonding forces.

Thermosetting polymers :

- Network polymers.

- Adjacent chains are connected by covalent crosslinks.
- Crosslinking is extensive (about 10 - 50% of repeat units crosslinked).

7. (a) (ii) Solution:

Residual thermal stresses are introduced in a glass piece during cooling due to difference in cooling rate and thermal contraction between the surface and interior regions.

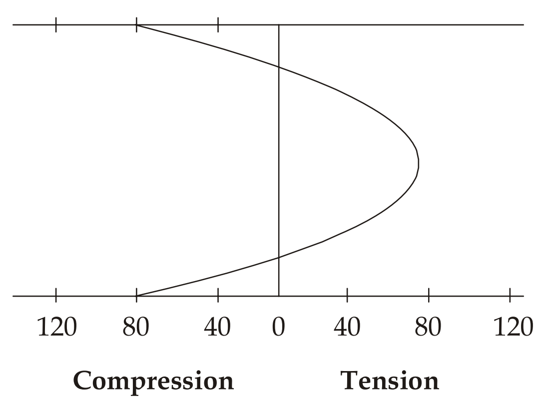
When glass is cooled from an elevated temperature :

- The surface cools faster than the interior.
- Faster cooling causes the surface to contract earlier.
- The interior, still at higher temperature cools and contracts later.
- Because both regions are rigidly connected, this unequal contraction generates internal thermal stresses.

In normal cooling, these stresses may weaken the glass and can even cause thermal shock.

In thermal tempering :

- The surface is cooled rapidly and becomes rigid first (below strain point).
- The interior remains hot and plastic for some time.
- As the interior cools further, it attempts to contract more than the rigid surface allows.
- This results in compressive stresses at the surface and tensile stresses in the interior after cooling to room temperature.

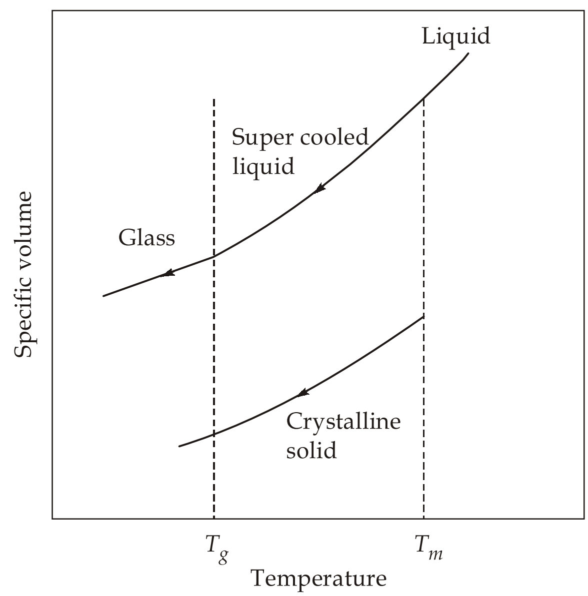


Room temperature residual stress distribution over the cross-section of a tempered glass plate

Difference between glass transition temperature and glass melting temperature:

Glass transition temperature : Glass transition temperature is characteristics of glassy (noncrystalline) materials, which do not solidify at a definite temperature like crystalline materials. Upon cooling, the viscosity increases continuously and the specific volume decreases gradually. At glass transition temperature, there is only a slight decrease in the slope of the specific volume-temperature curve. Below this temperature, the material is considered as a glass; about it, the material is first a supercooled liquid and then a liquid. The glass transition temperature lies above the strain point.

Melting temperature (T_m) : Melting temperature is characteristic of crystalline materials. At this temperature, there is definite transformation from solid to liquid accompanied by a discontinuous decrease in specific volume. In the case of glasses (for processing purposes), the melting point corresponds to the temperature at which the viscosity is about 10 Pa.s at which the glass is fluid enough to be considered a liquid.



7. (b) (i) Solution:

1. Arc characteristics is given by

$$I_a = 24\,(V - 20)$$

Volt-amp characteristics of power source is given by

$$I_t^2 = -700(V - 65)$$

as current-voltage variation is parabolic, so the power source is constant current source.

Hence, for the stability of arc;

$$I_t = I_a$$

(i.e. current across transformer is equal to current across arc)

or $$I_t^2 = I_a^2$$

$$-700(V-65) = [24(V-20)]^2$$

$$-700V + 45500 = 576V^2 + 230400 - 23040V$$

$$576V^2 - 22340V + 184900 = 0$$

Solve this equation, we get roots of this equation as; V = 26.81 volts, 11.97 volts

$$V = 11.97 \text{ volts}; I_a = 24(V-20)$$
$$= 24(11.97 - 20) = -\text{ve}$$

So neglect, V = 11.97 volts

Hence, Voltage (V) = 26.81 volts

Current of arc at this voltage = $I = 24(V - 20)$

$\Rightarrow$ $I = 24(26.81 - 20) = 163.4$ Amps

So, Power of arc = $V \times I$

$= 26.81 \times 163.4 = 4381.8$ W

or Power = 4.381 kW **Ans.**

2. Arc length voltage is given by

$$V = 32 + 5.6l$$

Volt-amp characteristic of power source is given by

$$I_t^2 = -700(V-65)$$

On putting value of V in the above equation, we get

$$I_t^2 = -700(32 + 5.6l - 65)$$

$$= -700(5.6l - 33) = \{-700(5.6l - 33)\}^{1/2}$$

As power (P) is given by, $P = V \times I$

$$= (32 + 5.6l) \times \{-700(5.6l - 33)\}^{1/2}$$

For maximum power, $\frac{dp}{dl} = 0$

$$\frac{dp}{dl} = \left\{(32 + 5.6l) \times \frac{1}{2}\{-700(5.6l - 33)\}^{-\frac{1}{2}} \times (-700 \times 5.6)\right\} + \left\{\{-700(5.6l - 33)\}^{\frac{1}{2}}\right\} \times 5.6$$

equals $\frac{dp}{dl} = 0$

$$\Rightarrow \quad \frac{(32+5.6l)\{-700(5.6l-33)\}^{-\frac{1}{2}}(-700\times 5.6)}{2} = \left[\{-700(5.6l-33)\}^{\frac{1}{2}}\times 5.6\right]\times -1$$

$$\Rightarrow \quad \frac{(32+5.6l)\times(-700\times 5.6)}{2} = +700(5.6l-33)\times 5.6$$

$$\Rightarrow \quad \frac{(32+5.6l)}{2} = (33-5.6l)$$

$$\Rightarrow \quad (32+5.6l) = 66-11.2l$$

$$\Rightarrow \quad 16.8l = 34$$

$$l = 2.024 \text{ mm}$$

Hence, optimum arc length for maximum power

$$l_{opt} = 2.024 \text{ mm}$$

7. (b) (ii) Solution:

In Plasma Arc Welding (PAW), a concentrated plasma arc is produced and directed towards the weld area. The arc is stable and reaches temperature as high as 33,275 K. Plasma is ionized hot gas, composed of nearly equal numbers of electrons and ions. The plasma is initiated between the tungsten electrode and orifice, using a low-current Pitot arc. Unlike the arc in other processes, the plasma arc is concentrated, because it is forced through a relatively small orifice. Operating current are usually below 100A, but they can be higher for special applications. When a filler metal is used, it is fed into the arc, as in GTAW. Arc and weld zone shielding is supplied through an outer shielding ring, using gases such as argon, helium, or mixture of these gases.

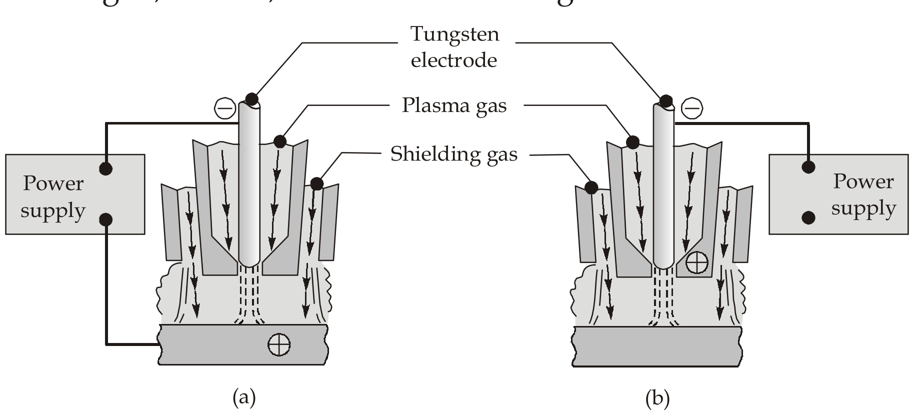


Fig. *Two type of plasma arc welding processes (a) transferred and (b) non-transferred*

There are two methods of plasma arc welding. In the transferred arc method, the part being welded is a part of the electrical circuit. The arc thus transfers from the electrode to the workpiece, hence the terms transferred. In the non-transferred method, the arc is between the electrode and the nozzle, and the heat is carried to the workpiece by the plasma gas; this technique is also used for thermal spraying.

7. **(c) Solution:**

Monitoring Methods	Direction of Problem by	Determination of Nature of Problem by Analysis of Measurement
Visual Monitoring	Overall Appearance	Colouring/Shape/Texture
Performance Monitoring	Rate of Output	Uniform Quality Level/Rate of Output/Uniformity
Vibration/Noise Level Monitoring	Overall Noise/Vibration Level	Frequency of Noise Level/Signal Waveform/Signal Statistics
Wear Monitoring	Amount of Debris/Friction Colour	Shape/Size/Size Distribution of Debris/Chemical Composition
Corrosion Monitoring	Colour/Chemical Analysis	Variation in Coating Thickness/Chemical Composition

(a) **Visual Monitoring:** It is the monitoring technique, which involves inspection and recording of surface appearance. Normally, monitoring is done by naked eyes, but it can require the use of telescopes, microscopes, and various other optical equipments. Sometimes, it may even involve the use of borescopes to see inaccessible places, or the use of photography or surface imprinting for record purpose.

(b) **Vibration Monitoring:** The noise and vibration are the most important parameters to monitor a machine, particularly in the moving parts such as shafts, rotors, cutting tools, gears, etc. The vibration level is recorded by attaching a transducer like velocity probe, accelerometer, or proximity probe to the machine. Special equipment is also available for using the output from the sensor to indicate the nature of vibration problem and even its precise cause. In some cases, it may become necessary to use the principles of Sonics and acoustics.

(c) **Wear Debris Monitoring:** This works on the principle that the working surfaces of a machine are washed by the lubricating oil, and any damage to them should be detectable from particles of wear debris in the oil. If the debris consists of relatively large ferrous lumps such as those generated by the fatigue of rolling element bearings and gears or the pitting of cams and taproots, these can be picked up by removable magnetic plugs inserted in the oil return lines. For small debris particle, spectrographic analysis or microscopic examination of oil samples after magnetic separation are commonly used techniques. Another popular technique is SOAP analysis for debris monitoring.

(d) **Performance and Behaviour Monitoring:** Here we check the performance of a machine or component to see whether it is behaving correctly. This may, for example, involve monitoring the performance of a bearing by measuring its temperature to see whether it is carrying out its function of transmitting load between moving surfaces with minimum friction.

(e) **Corrosion Monitoring:** This is usually applied to fixed plant containing aggressive materials and is intended to monitor the rates of internal corrosion of the walls of the plant. This may be done by drilling sentinel holes part way, through the wall, which can be plugged when they leak or by inserting readily removable coupons of material of which the corrosion rate is assumed to relate to that of plant.

8. (a) (i) Solution:

$$p_c = \text{Specific cutting power}$$

$$p_c = \frac{F_c}{bt_1} \quad \text{...(i)}$$

$$\frac{F_s}{\left(\dfrac{bt_1}{\sin\phi}\right)} = \tau_s \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{\tau_s}{p_c} = \frac{F_s}{\left(\dfrac{bt_1}{\sin\phi}\right)} \times \frac{bt_1}{F_c} = \frac{F_s}{F_c}\sin\phi \quad \text{...(iii)}$$

From Merchant's circle, we know

$$\frac{F_s}{F_c} = \frac{\cos(\phi+\beta-\alpha)}{\cos(\beta-\alpha)}$$

Given: $\alpha = 0°$,

So from equation (iii):

$$\frac{\tau_s}{p_c} = \frac{\cos(\phi+\beta)}{\cos\beta}\sin\phi$$

$$\frac{\tau_s}{p_c} = \frac{(\cos\phi\,\cos\beta - \sin\phi\sin\beta)}{\cos\beta}\sin\phi$$

$$\frac{\tau_s}{p_c} = (\cos\phi - \sin\phi\,\tan\beta)\,\sin\phi$$

$$= (\cos\phi - \mu\sin\phi)\sin\phi \quad (\because \mu = \tan\beta)$$

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$

$$\tan\phi = r \qquad (\because \alpha = 0°)$$

$$\frac{\sin\phi}{\cos\phi} = r \qquad ...(iv)$$

Also,
$$\tan^2\phi + 1 = \sec^2\phi = \frac{1}{\cos^2\phi}$$

$$r^2 + 1 = \frac{1}{\cos^2\phi} \Rightarrow \cos\phi = \frac{1}{\sqrt{1+r^2}}$$

From eq. (iv)
$$\sin\phi = \frac{r}{\sqrt{1+r^2}}$$

Now,
$$\frac{\tau_s}{p_c} = \left(\frac{1}{\sqrt{1+r^2}} - \frac{r}{\sqrt{1+r^2}}\mu\right)\frac{r}{\sqrt{1+r^2}}$$

$$= r\left(\frac{1}{1+r^2} - \frac{\mu r}{1+r^2}\right)$$

$$\frac{\tau_s}{p_c} = \frac{r(1-\mu r)}{(1+r^2)}$$

8. **(a) (ii) Solution:**

Corrosion can be defined as gradual destruction of material by chemical or electrochemical reaction with their environment. Corrosion is a natural process, it converts a refined material (metal) in its more chemically stable form such as oxide, hydroxide etc. Corrosion engineering is the field of engineering which is used for controlling and stopping corrosion.

Formation of iron oxides is known as rusting, it is one of the most popular example of electrochemical corrosion. It generally produces oxide(s) or salt(s) of original metal and results in distinctive orange colouration of the metal.

Corrosion may also occur in polymer or ceramics. For polymer and ceramics most common term used is degradation. Because of corrosion, properties of material changes such as strength, appearance and permeability to liquid and gases.

Although this definition is applicable to any type of material, but it is usually reserved for metallic alloys. Of the 105 known chemical elements, approximately eighty are metals, and about half of these can be alloyed with other metals, giving rise to more than 40,000 different alloys. Each of the alloys will have different physical, chemical, and mechanical properties, but all of them can corrode to some extent, and in different ways.

When newly made steel is first exposed to air, its originally shiny surface will be covered with rust in a few hours. The tendency of metals to corrode is related to the low stability of the metallic state. Metals occurs either in the pure metallic state, the zero oxidation state, or in the form of compounds with other elements (they acquire positive states of oxidation). In the natural world, the most metals are found as compound with other elements, indicating the greater stability of their oxidized forms. For this reason, to obtain the pure metal form its compound, it is necessary to put in energy. The reverse is true when a metal is exposed to its environment, it tends to release this stored energy through the processes of corrosion. This is rather analogous to what happens when an object is suspended at a point above the ground (equivalent to the metallic state). When allowed to fall or reach to stable state, it returns to a position of minimum energy on the ground (equivalent to the metal's oxidized state).

The chemical reactions that take place in corrosion processes are reduction-oxidation (redox) reactions. Such reactions require a species of material that is oxidized (the metal), and another that is reduced (the oxidizing agent). Thus the complete reaction can be divided into two partial reactions: one oxidation and the other reduction. In oxidation, the metal loses electrons. The zone in which this happens is known as the anode. In the reduction reaction, the oxidizing agent gains the electrons that have been shared by the metal, and the zone in which this happens is the cathode.

Uniform corrosion : Uniform corrosion is defined as a type of corrosion attack that is more or less uniformly distributed over the entire exposed surface of a metal. It is also referred as the corrosion that proceeds at approximately the same rate over the exposed metal surface. Cast irons and steels corrode uniformly when exposed to open atmosphere, soils and natural water, leading to the rusty appearance.

Mechanism: The anodic reaction in the corrosion process is always the oxidation reaction

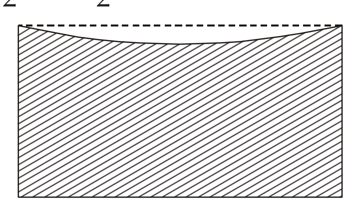
$$M = M^+ + e^-$$

In acidic environments, the cathodic process is mainly the reduction of hydrogen ions.

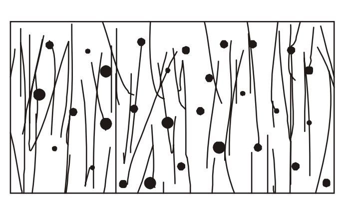
$$2H^+ + 2e^- = H_2$$

In alkaline or neutral environment, reduction of dissolved oxygen is the predominant cathodic process that causes uniform corrosion.

$$O_2 + 2H_2O + 4e^- = 4\,OH^-$$



Uniform Corrosion



Pitting Corrosion

Pitting: It is a form very localized corrosion attack in which small pits or holes form. They ordinarily penetrate from the top of a horizontal surface downward in a nearly vertical direction. It is an extremely insidious type of corrosion, often going undetected and with very little material loss until failure occurs.

The mechanism for pitting is that the oxidation occurs within the pit itself, with complementary reduction at the surface. It is supposed that gravity causes the pits to grow downward, the solution at the pit tip becoming more concentrated and dense as pit growth progresses. A pit may be initiated by a localized surface defect such as a scratch or a slight variation in composition.

8. **(b) Solution:**

As per given information

$$L_{12} + d_3 - L_4 = r_{34}$$

$$d_3 = r_{34} + L_4 - L_{12}$$

$$L_2C_{12} + L_{11}C_1 = r_{14} \quad \text{...(1)}$$

$$L_2S_{12} + L_{11}S_1 = r_{24} \quad \text{...(2)}$$

Squaring eqn. (1) and (2), then adding

$$(r_{14})^2 + (r_{24})^2 = L_{11}^2 + L_2^2 + 2L_{11}L_2C_2$$

$$C_2 = \frac{r_{14}^2 + r_{24}^2 - L_{11}^2 - L_2^2}{2L_{11}L_2} \quad \text{...(3)}$$

$$S_2 = \pm\sqrt{1 - C_2^2} \quad \text{..(4)}$$

The solution for θ_2 is obtained from equation (3) and (4)

$$\theta_2 = A \tan 2(S_2, C_2)$$

Now, θ_2 is used to compute θ_1

$$L_2(C_1C_2 - S_1S_2) + L_{11}C_1 = r_{14} \text{ (expanding } C_{12}\text{, i.e., } \cos(\theta_1 + \theta_2)]$$

$$L_2(S_1C_2 + C_1S_2) + L_{11}S_1 = r_{24} \text{ (expanding } S_{12}\text{, i.e., } \sin(\theta_1 + \theta_2)]$$

$$(L_{11} + L_2C_2)C_1 - (L_2S_2)S_1 = r_{14} \quad \text{...(5)}$$

$$(L_{11} + L_2C_2)S_1 + (L_2S_2)C_1 = r_{24} \quad \text{...(6)}$$

Let

$$L_{11} + L_2C_2 = r\cos\phi$$

$$L_2S_2 = r\sin\phi$$

$$r = \sqrt{(L_{11} + L_2C_2)^2 + (L_2S_2)^2}$$

$$\phi = A\tan 2\left(\frac{L_2S_2}{r}, \frac{L_{11} + L_2C_2}{r}\right)$$

From equation (5) and (6)

$$r\cos(\theta_1 + \phi) = r_{14} \quad \text{...(7)}$$

$$r \sin(\theta_1 + \phi) = r_{24} \qquad ...(8)$$

From (7) and (8)

$$\theta_1 = A\tan 2\left(\frac{r_{24}}{r}, \frac{r_{14}}{r}\right) - \phi$$

Now, θ_1, θ_2 and d_3 are known, θ_4 can be computed.

$$C_{124} = r_{11}$$
$$S_{124} = r_{21}$$
$$\theta_1 + \theta_2 - \theta_4 = A\tan 2(r_{21}, r_{11})$$
$$\theta_4 = \theta_2 + \theta_1 - A\tan 2(r_{21}, r_{11})$$

Now all variables θ_1, θ_2, θ_4, d_3 are known.

8. (c) (i) Solution:

Total requirement = 32

Total capacity = 32

Finding initial feasible solution using Vogel's approximation method

	A	B	C	D		
1	20 X	12 11	10 X	15 X	11/11	[2]
2	10	22 X	10	20	8	[0]
3	15	20 X	12	8	13	[4]
	5	11/11	8	8		
	[5]	[8]	[0]	[7]		

	A	C	D		
2	10	10	20 X	8	[0]
3	15	12	8 8	13/8	[4]
	5	8	8/8		
	[5]	[2]	[12]		

	A	C		
2	10 5	10	8/5	[0]
3	15 X	12	5	[3]
	5	8		
	[5]	[2]		

10 3	3
12 5	5
8	

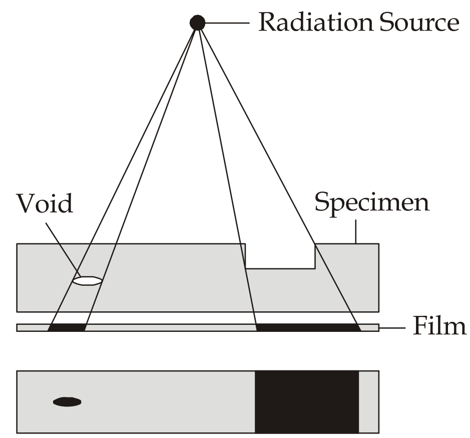
Initial feasible solution is

	A	B	C	D
1	20	12 (11)	10	15
2	10 (5)	22	10 (3)	20
3	15	20	12 (5)	8 (8)

$$\text{Cost} = 10 \times 5 + 12 \times 11 + 10 \times 3 + 12 \times 5 + 8 \times 8$$

$$= \text{Rs. } 336 \text{ thousands}$$

8. (c) (ii) Solution:



Plan View of the Film Darker areas (after processing)

Fig. Schematic of Radiographic Testing

Radiographic Testing is a nondestructive testing method of impacting materials for hidden flaws by using the ability of short wavelength electromagnetic radiation (high energy photons) to penetrate various materials. In this method the part is placed between the radiation source and a piece of film. A pattern will be generated on the film depending on the thickness of the inspected part. Where ever there is a defect, the amount of radiation absorbed will be different and corresponding pattern will be generated. Detection of subsurface defects is the major advantages of this test but proper safety measures have to be taken to prevent exposure to these radiations. Figure schematically shows a typical radiographic testing set-up.