MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Civil Engineering Test No : 13

Section - A

1. (a) Solution:

Given data

Discharge diverted from canal (Q_C) = 160 litres/sec

Discharge delivered to field (Q_f) = 125 litres/sec

Area of the field (A) = 2.5 hectares = 25000 m^2

Irrigation time (t) = 6 hours = 6 × 3600 sec

Runoff loss (R_f) = 450 m^3

Root zone depth (D) = 1.5 m

Depth of penetration at head (d_1) = 1.6 m

Depth of penetration at tail (d_2) = 1.0 m

Available water holding capacity = 20 cm/m = 0.2 m/m

Moisture depletion level = 50%

Water conveyance efficiency is the ratio of water delivered to the field to the water diverted from the canal.

$$\eta_c = \frac{Q_f}{Q_c} \times 100 = \frac{125}{160} \times 100$$

$$\Rightarrow \qquad \eta_c = 78.125\%$$

Now, total volume of water delivered to the field.

$$V_f = Q_f \times t$$

$\Rightarrow$ $$V_f = 125 \times 10^{-3} \times 6 \times 3600 = 2700 \text{ m}^3$$

Volume of water stored in the root zone is

$$V_s = V_f - R_f$$

$\Rightarrow$ $$V_s = 2700 - 450 = 2250 \text{ m}^3$$

Water application efficiency is

$$\eta_a = \frac{V_s}{V_f} \times 100 = \frac{2250}{2700} \times 100$$

$\Rightarrow$ $$\eta_a = 83.333\%$$

Now, water needed in the root zone before irrigation.

Available moisture capacity is

$$= D \times 0.2 = 1.5 \times 0.2 = 0.3 \text{ m}$$

Moisture deficit to be filled at 50% depletion is

$$= 0.3 \times 0.5 = 0.15 \text{ m}$$

$$V_n = A \times \text{moisture deficit}$$

$\Rightarrow$ $$V_n = 25000 \times 0.15 = 3750 \text{ m}^3$$

Water storage efficiency is

$$\eta_s = \frac{V_s}{V_n} \times 100 = \frac{2250}{3750} \times 100 = 60\%$$

Average depth of penetration is

$$d_{avg} = \frac{d_1 + d_2}{2} = \frac{1.6 + 1.0}{2} = 1.3 \text{ m}$$

Numerical deviation from mean at head is

$$|1.6 - 1.3| = 0.3 \text{ m}$$

Numerical deviation from mean at tail is

$$|1.0 - 1.3| = 0.3 \text{ m}$$

Average numerical deviation is

$$y = \frac{0.3 + 0.3}{2}$$

$\Rightarrow$ $$y = 0.3 \text{ m}$$

Water distribution efficiency is

$$\eta_d = \left(1 - \frac{y}{d_{avg}}\right) \times 100$$

$$\Rightarrow \quad \eta_d = \left(1 - \frac{0.3}{1.3}\right) \times 100 = 76.923\%$$

1. (b) Solution:

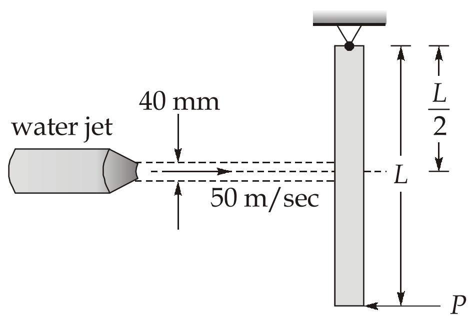
(i) Given data

Weight of plate (W) = 10 kN = 10000 N

Diameter of jet (d) = 40 mm = 0.04 m

Velocity of jet (v) = 50 m/s

Density of water (ρ) = 1000 kg/m^3



Force exerted by the water jet is given by

$$F_{jet} = \rho A v^2$$

$$\Rightarrow \quad F_{jet} = 1000 \times \frac{\pi}{4}(0.04)^2 \times 50^2$$

$$\Rightarrow \quad F_{jet} = 3141.593 \text{ N}$$

For horizontal force at the bottom edge, taking moments about the hinge at the top edge,

$$P \times L = F_{jet} \times \frac{L}{2}$$

$$\Rightarrow \quad P = \frac{3141.593}{2} = 1570.797 \text{ N}$$

(ii) Given data

Relative density of oil (R.D.) = 0.88

Density of oil (ρ) = 0.88 × 1000 = 880 kg/m^3

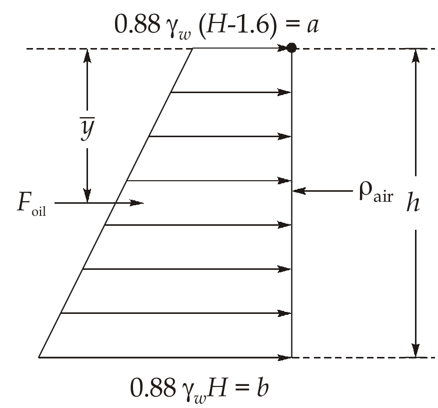
Air pressure (P_{air}) = 38 kPa = 38000 N/m^2

Gate height (h) = 1.6 m

Gate width (w) = 0.5 m

Acceleration due to gravity (g) = 9.81 m/s^2

Moment due to air pressure about the hinge is



$$M_{air} = F_{air} \times \frac{h}{2}$$

$$\Rightarrow \quad M_{air} = P_{air} \times w \times h \times \frac{h}{2}$$

$$\Rightarrow \quad M_{air} = 38 \times 0.5 \times 1.6 \times \frac{1.6}{2}$$

$$\Rightarrow \quad M_{air} = 24.320 \text{ kN-m}$$

Moment due to oil pressure about the hinge is

$$M_{oil} = F_{oil} \times \bar{y} = \left[\frac{1}{2}(a+b)hw\right] \times \left(\frac{a+2b}{a+b}\right)\frac{h}{3}$$

$$\Rightarrow \quad M_{oil} = 0.5 \times 1.6 \times 0.5 \times 0.88 \times 1000 \times 9.81[H - 1.6 + 2H] \times \frac{1.6}{3}$$

$$\Rightarrow \quad M_{oil} = (5524.99\,H - 2946.66)$$

For the gate to just begin rotating counterclockwise, moments must be equal.

$$M_{oil} = M_{air}$$

$$\Rightarrow \quad -2946.66 + 5524.992H = 24.320 \times 10^3$$

$$\Rightarrow \quad H = 4.935 \text{ m}$$

1. (c) Solution:

Given data

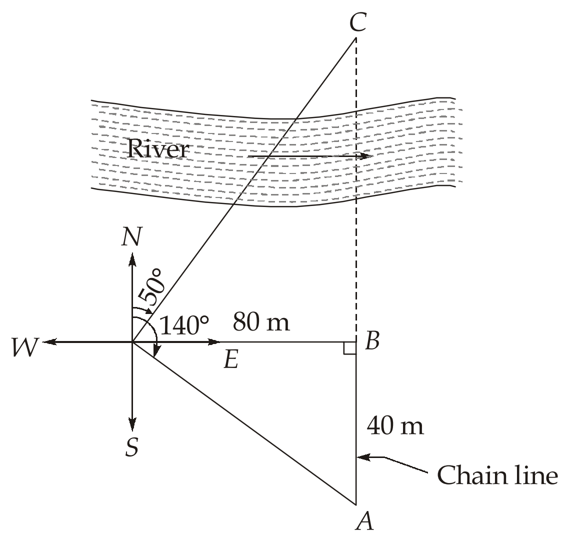
Length of perpendicular BE = 80 m

Length of AB = 40 m

Bearing of EC = 50°

Bearing of EA = 140°

Chainage of B = 350.25 m



In the geometry of the survey, since BE is perpendicular to the chain line ABC, there are two right-angled triangles $\Delta\, ABE$ and $\Delta\, CBE$ with common side BE.

The internal angle is

$$\angle AEC = 140° - 50°$$

$$\Rightarrow \quad \angle AEC = 90°$$

Since $\angle AEC = 90°$, triangle AEC is right-angled and BE acts as altitude to hypotenuse AC.

$$BE^2 = AB \times BC$$

$$\Rightarrow \quad BC = \frac{BE^2}{AB}$$

$$\Rightarrow \quad BC = \frac{80 \times 80}{40}$$

$$\Rightarrow \quad BC = 160 \text{ m}$$

Chainage of point C is

Chainage of C = Chainage of B + BC

$\Rightarrow$ Chainage of C = 350.25 + 160

$\Rightarrow$ Chainage of C = 510.25 m

1. (d) Solution:

Total head at entry (bottom of Soil-A, Point 1): H_1 = 120 + 80 + 60 = 260 cm

Total head at exit (top of Soil-B): H_{exit} = 120 + 80 = 200 cm

Total head loss (h_L) = 260 – 200 = 60 cm

Unit weight of water (γ_w) = 9.81 kN/m^3

1. Quantity of water flowing per minute per unit area (*q*/*A*)

First, we calculate the equivalent hydraulic conductivity (K_{eq}) for flow perpendicular to the layers:

$$K_{eq} = \frac{L_A + L_B}{\frac{L_A}{K_A} + \frac{L_B}{K_B}}$$

$$\Rightarrow \quad K_{eq} = \frac{120 + 80}{\frac{120}{2 \times 10^{-4}} + \frac{80}{5 \times 10^{-4}}}$$

$$\Rightarrow \quad K_{eq} = 2.632 \times 10^{-4} \text{ cm/s}$$

Now, using Darcy's Law for velocity ($v = K_{eq} \times i$):

$$v = K_{eq} \times \left(\frac{h_L}{L_A + L_B}\right)$$

$$\Rightarrow \quad v = 2.632 \times 10^{-4} \times \frac{60}{200}$$

$$\Rightarrow \quad v = 7.895 \times 10^{-5} \text{ cm/s}$$

Discharge per minute per unit area (q/A in cm^3/min/cm^2):

$$q/A = v \times 1 \times 1 = 7.895 \times 10^{-5} \text{ cm}^3\text{/min/cm}^2$$

$$\Rightarrow \quad q/A = 7.895 \times 10^{-5} \times 60$$

$$\Rightarrow \quad q/A = 4.737 \times 10^{-3} \text{ cm}^3\text{/min/cm}^2$$

2. Piezometric height above *X*-*X*

The total head at any point is the sum of the pressure head (h_p) and datum head (z). The piezometric height above datum is $h_p + z$. Since the flow is upward, head loss occurs in the direction of flow (from Point 1 to Point 3).

At Point 1 (Bottom of Soil-*A*, z = 0):

The water is supplied from the reservoir directly to the bottom.

Piezometric height = 260 cm

At Point 2 (Interface, z = 120 cm):

$$\text{Head loss in Soil-A } (h_{LA}) = i_A \times L_A = \frac{v}{K_A} \times L_A$$

$$\Rightarrow \quad (h_{LA}) = \frac{7.895 \times 10^{-5}}{2 \times 10^{-4}} \times 120$$

$$\Rightarrow \quad (h_{LA}) = 47.37 \text{ cm}$$

$$\text{Total head at Point 2 } (H_2) = H_1 - h_{LA} = 260 - 47.37 = 212.63 \text{ cm}$$

Piezometric height at point 2 = 212.63 cm

At Point 3, $z = 120 + 40 = 160$ cm

$$\text{Head loss from Point 2 to Point 3 } (h_{L2-3}) = \frac{v}{K_B} \times 40$$

$$h_{L2-3} = \frac{7.895 \times 10^{-5}}{5 \times 10^{-4}} \times 40$$

$$\Rightarrow \quad h_{L2-3} = 6.316 \text{ cm}$$

$$\text{Total head at Point 3 } (H_3) = H_2 - h_{L2-3} = 212.63 - 6.316 = 206.314 \text{ cm}$$

Piezometric height at point 3 = 206.314 cm

3. Effective stress at mid-height of Soil-*A*

Mid-height of Soil-*A* is at $z = 60$ cm from datum.

Depth from top surface (D) = 80 + 60 = 140 cm = 1.4 m.

Total Stress (σ):

$$\sigma = (\gamma_{sat,B} \times L_B) + (\gamma_{sat,A} \times 60 \text{ cm})$$

$$\Rightarrow \quad \sigma = (19 \times 0.8) + (21 \times 0.6) = 27.8 \text{ kN/m}^2$$

Pore Water Pressure (u):

Total head at mid-height (H_m) = H_1 – (head loss in bottom half of Soil-*A*)

$$H_m = 260 - \left(\frac{h_{LA}}{2}\right) = 260 - 23.685 = 236.315 \text{ cm}$$

$$\text{Pressure head } (h_p) = H_m - z = 236.315 - 60 = 176.315 \text{ cm} = 1.763 \text{ m}$$

Poue water pressure, $u = h_p \times \gamma_w = 1.763 \times 9.81$

$$\Rightarrow \quad u = 17.295 \text{ kN/m}^2$$

Effective Stress (σ'):

$$\sigma' = \sigma - u$$

$$\Rightarrow \quad \sigma' = 27.8 - 17.295$$

$$\Rightarrow \quad \sigma' = 10.505 \text{ kN/m}^2$$

1. (e) **Solution:**

River training works refer to various engineering measures and structures constructed on or along a river to control its flow, protect its banks, and maintain a specific channel alignment. The primary goal is to manage the river's natural tendency to erode, meander, or overflow, ensuring the safety of nearby infrastructure and improving navigability.

Types of River Training Works

River training is generally classified based on the specific objective of the project. There are three main types:

1. **High Water Training (Training for Discharge)**

   The main objective is flood control. These works are designed to provide sufficient bridge or channel capacity to pass the maximum flood discharge safely without overtopping the banks.

   - **Example: Embankments or Levees.** These are earthen banks constructed parallel to the river course to confine the floodwater within a specific boundary, preventing it from spreading into the surrounding plains.

2. **Low Water Training (Training for Depth)**

   This is primarily done to maintain a sufficient depth of water in the channel during the dry season, usually for navigation or to ensure water intake for irrigation and power plants.

   - **Example: Bandalling.** A temporary structure made of bamboo or wooden poles and mats used in shallow rivers to restrict the flow to a narrow channel, which increases the velocity and scours the bed to maintain depth.

3. **Mean Water Training (Training for Sediment)**

   The goal is to provide an efficient channel alignment that can transport the sediment load effectively, preventing both excessive erosion and heavy siltation. It aims to keep the river in a stable "mean" condition.

   - **Example: Groynes (Spurs).** These are structures built transverse to the river flow, extending from the bank into the river. They deflect the current away from the bank to prevent erosion and encourage silting between the spurs.

2. (a) Solution:

(i) Given Data:

Weight of plane (W) = 34335 N

Wing area (A) = 25 m^2

$$\text{Velocity } (V) = 300 \text{ km/hr} = \frac{300 \times 1000}{3600} = 83.333 \text{ m/s}$$

Engine Power (P_{total}) = 8500 kW = 8500 × 10^{3}W

Power used for drag (P_D)= 70% of P_{total} = 0.70

Density of air (ρ) = 1.21 kg/m^3

1. Coefficient of Lift (C_L):

For steady level flight, Lift (L) = Weight (W).

$$L = \frac{1}{2} \times \rho \times A \times V^2 \times C_L$$

$$\Rightarrow \quad 34335 = \frac{1}{2} \times 1.21 \times 25 \times 83.333^2 \times C_L$$

$$\Rightarrow \quad C_L = 0.327$$

2. Coefficient of Drag (C_D):

Power used to overcome drag (P_{drag}):

$$P_{drag} = 0.70 \times 8500 \times 10^3 = 5950000 \text{ W}$$

Now, Drag force (F_D) using the power formula $P = F_D \times V$:

$$F_D = \frac{P_{drag}}{V} = \frac{5950000}{83.333} = 71400.285 \text{ N}$$

Using the drag force equation:

$$F_D = \frac{1}{2} \times \rho \times A \times V^2 \times C_D$$

$$\Rightarrow \quad 71400.285 = \frac{1}{2} \times 1.21 \times 25 \times 83.333^2 \times C_D$$

$$\Rightarrow \quad C_D = 0.679$$

(ii) Given Data:

Width of channel at section 1, B_1 = 4.0 m

Depth of flow at section 1, y_1 = 1.8 m

Width of channel at section 2, B_2 = 2.5 m

Height of hump, Δz = 0.2 m

Drop in water surface, $d = 0.1$ m

Head loss, $h_L = 0.05$ m

Acceleration due to gravity, $g = 9.81$ m/s^2

The depth of flow at the contracted section (y_2) is calculated based on the water surface drop:

$$y_2 = y_1 - \Delta z - d$$

$$\Rightarrow \quad y_2 = 1.8 - 0.2 - 0.1$$

$$\Rightarrow \quad y_2 = 1.5 \text{ m}$$

Applying the Energy Equation between section 1 and section 2 including head loss:

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta z + h_L$$

Substitute $V = \frac{Q}{A} = \frac{Q}{By}$

$$y_1 + \frac{Q^2}{2g(B_1 y_1)^2} = y_2 + \frac{Q^2}{2g(B_2 y_2)^2} + \Delta z + h_L$$

$$\Rightarrow 1.8 + \frac{Q^2}{2 \times 9.81 \times (4.0 \times 1.8)^2} = 1.5 + \frac{Q^2}{2 \times 9.81 \times (2.5 \times 1.5)^2} + 0.2 + 0.05$$

$$\Rightarrow \quad Q = 4.351 \text{ m}^3/\text{s}$$

$$\Rightarrow \quad \text{Discharge, } Q = 4.351 \text{ m}^3/\text{s}$$

2. (b) (i) Solution:

The triaxial shear test is a fundamental laboratory method used to determine the shear strength parameters of soil. During this test, particularly in undrained conditions, water within the soil voids experiences pressure changes. Understanding the significance of this pore water pressure and how it relates to applied stresses through Skempton's parameters is essential for predicting soil behavior under field loading.

Significance of Pore Water Pressure

In a triaxial test, the total stress applied to a soil sample is resisted by both the soil skeleton (effective stress) and the water in the pores (pore water pressure). The significance of monitoring pore water pressure (u) includes:

1. **Effective Stress Principle:** According to Terzaghi's principle, the shear strength of soil depends on effective stress (σ'), not total stress (σ). By measuring u, we can calculate.

$$\sigma' = \sigma - u$$

2. **Volume Change Characteristics:** In undrained tests (CU or UU), pore pressure measurements substitute for volume change data, indicating whether a soil tends to dilate (negative u) or contract (positive u).

3. **Stability Analysis:** It allows for the determination of "undrained" strength parameters which are critical for "short-term" stability analysis of structures like embankments or foundations.

Skempton's Pore Water Pressure Parameters

A.W. Skempton (1954) proposed that the change in pore water pressure (Δu) in a soil mass subjected to changes in principal stresses can be expressed using two empirical parameters, A and B.

1. **Parameter *B* (Confining Pressure Stage)**

When a soil sample is subjected to an all-round (cell) pressure $\Delta\sigma_3$ under undrained conditions, the change in pore pressure is Δu_c. The ratio is defined as:

$$B = \frac{\Delta u_c}{\Delta \sigma_3}$$

For fully saturated soils, $B = 1$.

For dry soils, $B = 0$.

For partially saturated soils, $0 < B < 1$.

2. **Parameter A (Shearing/Deviatoric Stage)**

When the deviator stress $\Delta\sigma_d$ (where $\Delta\sigma_d = \Delta\sigma_1 - \Delta\sigma_3$) is applied during the shearing stage, it causes a further change in pore pressure $\Delta\sigma_d$. This is expressed via the parameter A, though it is often represented as the product AB:

$$\Delta u_d = A \times B \times \Delta\sigma_d$$

$$\Rightarrow \quad \bar{A} = AB$$

The value of A at failure (A_f) provides insight into the soil type:

Highly sensitive clays: 0.75 to 1.5

Normally consolidated clays: 0.5 to 1

Over-consolidated clays: –0.5 to 0

General Equation

Combining both stages, the total change in pore water pressure is given by Skempton's general equation:

$$\Delta u = B \times [\Delta\sigma_3 + A \times (\Delta\sigma_1 - \Delta\sigma_3)]$$

2. (b) (ii) Solution:

Given $G = 2.7$ and for $\beta = 34°$; stability No. (S_n) are:

ϕ	S_n
6°	0.122
7°	0.116
14°	0.074

Height of cutting, $H = 7.5$ m

Cohesion, $c = 22$ kN/m^2

Void ratio, $e = 0.65$

Angle of internal friction, $\phi = 14°$

Specific gravity, $G = 2.7$

Side slope = 1.5 H : 1V

$\Rightarrow \tan\beta = 1/1.5$

$\Rightarrow \beta = 33.7° \approx 34°$

$\beta = 34°$

Preliminary Calculations:

Saturated unit weight, γ_{sat}:

$$\gamma_{sat} = \frac{(G+e)\cdot\gamma_w}{1+e}$$

$$\Rightarrow \gamma_{sat} = \frac{(2.7+0.65)9.81}{1+0.65}$$

$$\Rightarrow \gamma_{sat} = 19.917 \text{ kN/m}^3$$

Submerged unit weight, γ':

$$\gamma' = \gamma_{sat} - \gamma_w$$

$$\Rightarrow \gamma' = 19.917 - 9.81$$

$$\Rightarrow \gamma' = 10.107 \text{ kN/m}^3$$

1. Water level rises to full height (Submerged Case)

   For submerged soil, the effective stress is reduced by buoyancy. The stability number for

   $\phi = 14°$ is taken from the table as:

   $S_n = 0.074$

Factor of safety with respect to cohesion–

$$F_c = \frac{c}{S_n \gamma' H}$$

$$\Rightarrow \qquad F_c = \frac{22}{0.074 \times 10.107 \times 7.5}$$

$$\Rightarrow \qquad F_c = 3.921$$

2. Sudden drawdown of water level (Sudden Drawdown Case)

For sudden drawdown, the effective friction angle is reduced as:

$$\phi_w = \phi \frac{\gamma'}{\gamma_{sat}}$$

$$\Rightarrow \qquad \phi_w = 14 \times \frac{10.107}{19.917}$$

$$\Rightarrow \qquad \phi_w = 7.104°$$

Using linear interpolation for S_n at $\phi_w = 7.104°$ between $\phi = 7°$ ($S_n = 0.116$) and $\phi = 14°$ ($S_n = 0.074$):

$$S_n = 0.116 + \frac{0.074 - 0.116}{(14-7)}(7.104 - 7)$$

$$\Rightarrow \qquad S_n = 0.115$$

Factor of safety with respect to cohesion using γ_{sat}:

$$F_c = \frac{c}{S_n \gamma_{sat} H}$$

$$\Rightarrow \qquad F_c = \frac{22}{0.115 \times 19.917 \times 7.5}$$

$$\Rightarrow \qquad F_c = 1.281$$

2. (c) (i) Solution:

Rotating Biological Contactor (*RBC*) is a fixed-film aerobic secondary treatment system for wastewater. Microorganisms grow on the surface of rotating discs partially submerged in wastewater. The discs rotate slowly, alternately exposing the attached biomass to wastewater and atmospheric oxygen.

The system consists of four primary components:

1. **Shaft:** A central horizontal axis that supports and rotates the media.
2. **Media:** Large-diameter, closely spaced circular plastic discs, typically made of HDPE, on which microorganisms attach.

3. **Drive Mechanism:** An electric motor or air-drive rotates the shaft at 1–2 revolutions per minute.
4. **Tank (Trough):** Holds the wastewater and maintains approximately 40% submergence of disc surface area.

Operational Principle:

The *RBC* process relies on the development of a biomass (zoogleal slime) on the disc surfaces.

Submerged Phase: Microorganisms absorb organic nutrients (BOD) from the wastewater.

Aerobic Phase: As the discs rotate out of the liquid, a thin film of wastewater remains on the biomass surface, absorbing oxygen from the air. Aerobic bacteria metabolize the organic matter during this phase.

Excess biomass sloughs off due to shearing forces from rotation and the weight of the growth. This sloughed material is carried to a secondary clarifier for removal.

Key Equations:

The total surface area of media required is calculated from the hydraulic loading and organic loading rate:

$$A = \frac{QS_i}{L_{org}}$$

Where A is the total surface area of the media, Q is the influent flow rate, S_i is the influent *BOD* concentration, and L_{org} is the organic loading rate.

Advantages:

1. Low Energy Consumption: RBCs require less power than activated sludge systems since no heavy aeration is needed.
2. Small Footprint: The high surface area of discs allows efficient use of space.
3. Resilience: Fixed-film design provides resistance to shock loads or toxic spikes in waste water

2. (c) (ii) Solution:

Given data

Population, P = 60,000

Dry solid concentration, S_d = 80 g/capita/day = 0.08 kg/capita/day

Dry solid loading rate, L = 120 kg/m^2/year

Size of one bed, a = 20 m × 10 m

Number of times a bed can be used in a year, n_t = 12

Solid content of digested sludge, s = 8% = 0.08

Specific gravity of digested sludge, G = 1.03

Total dry solids produced per year, W:

$$W = \text{Population} \times \text{Dry solid concentration} \times 365$$

$$\Rightarrow \quad W = 60000 \times 0.08 \times 365$$

$$\Rightarrow \quad W = 1{,}752{,}000 \text{ kg/year}$$

Total area of drying beds required, A:

$$A = \frac{W}{L}$$

$$\Rightarrow \quad A = \frac{1{,}752{,}000}{120}$$

$$\Rightarrow \quad A = 14{,}600 \text{ m}^2$$

$$\text{Area of one drying bed} = 20 \times 10 = 200 \text{ m}^2$$

Number of operational beds, n:

$$n = \frac{A}{\text{Area of one drying bed}}$$

$$\Rightarrow \quad n = \frac{14{,}600}{200}$$

$$\Rightarrow \quad n = 73$$

Total number of beds including two standby beds, N:

$$N = n + 2$$

$$\Rightarrow \quad N = 73 + 2$$

$$\Rightarrow \quad N = 75$$

Mass of dry solids per bed per cycle, **w**:

$$w = \frac{W}{nn_t}$$

$$\Rightarrow \quad w = \frac{1{,}752{,}000}{73 \times 12}$$

$$\Rightarrow \quad w = 2000 \text{ kg}$$

Volume of wet sludge applied per cycle, V:

$$V = \frac{w}{sG1000}$$

$$\Rightarrow \quad V = \frac{2,000}{0.08 \times 1.03 \times 1000}$$

$$\Rightarrow \quad V = 24.272 \text{ m}^3$$

Depth of sludge application, d:

$$d = \frac{V}{\text{Area of one drying bed}} = \frac{24.272}{200} = 0.121 \text{ m}$$

3. (a) Solution:

Given data

Sum of Northings: $\sum N = 400.25 + 150.75 = 551$

Sum of Southings: $\sum S = 400.50 + 150.25 = 550.75$

Sum of Eastings: $\sum E = 300.5 + 50.75 = 351.25$

Sum of Westings: $\sum W = 100.25 + 250.75 = 351$

Arithmetic sum of Latitudes, $L = 400.25 + 150.75 + 400.5 + 150.25 = 1101.75$

Arithmetic sum of Departures, $D = 300.5 + 100.25 + 250.75 + 50.75 = 702.25$

(i) Magnitude and direction of closing error

Error in Latitude, e_L:

$$e_L = \sum N - \sum S$$

$$\Rightarrow \quad e_L = 551 - 550.75 = 0.25$$

Error in Departure, e_D:

$$e_D = \sum E - \sum W$$

$$\Rightarrow \quad e_D = 351.25 - 351 = 0.25$$

Magnitude of closing error, e:

$$e = \sqrt{e_L^2 + e_D^2} = \sqrt{0.25^2 + 0.25^2} = 0.354 \text{ unit}$$

Direction of closing error, θ:

$$\tan\theta = \frac{e_D}{e_L} = \frac{0.25}{0.25} = 1$$

$$\Rightarrow \quad \theta = \tan^{-1}(1) = 45^\circ$$

Since both e_L and e_D are positive, the closing error lies in the North-East (N 45°E) quadrant.

(ii) Corrected consecutive coordinates using Transit Rule

Transit Rule:

$$\text{Correction to Latitude} = -e_L \frac{|\text{Latitude of line}|}{L}$$

$$\text{Correction to Departure} = -e_D \frac{|\text{Departure of line}|}{D}$$

Line *AB*:

$$c_{L_{AB}} = -0.25 \times \frac{400.25}{1101.75} = -0.091$$

$$c_{D_{AB}} = -0.25 \times \frac{300.5}{702.25} = -0.107$$

$$\text{Corrected Latitude} = 400.25 - 0.091 = 400.159$$

$$\text{Corrected Departure} = 300.5 - 0.107 = 300.393$$

Line *BC*:

$$c_{L_{BC}} = -0.25 \times \frac{150.75}{1101.75} = -0.034$$

$$c_{D_{BC}} = -0.25 \times \frac{100.25}{702.25} = -0.036$$

$$\text{Corrected Latitude} = 150.75 - 0.034 = 150.716$$

$$\text{Corrected Departure} = -100.25 - 0.036 = -100.286$$

Line *CD*:

$$c_{L_{CD}} = -0.25 \times \frac{400.5}{1101.75} = -0.091$$

$$c_{D_{CD}} = -0.25 \times \frac{250.75}{702.25} = -0.089$$

$$\text{Corrected Latitude} = -400.5 - 0.091 = -400.591$$

$$\text{Corrected Departure} = -250.75 - 0.089 = -250.839$$

Line *DA*:

$$c_{L_{DA}} = -0.25 \times \frac{150.25}{1101.75} = -0.034$$

$$c_{D_{DA}} = -0.25 \times \frac{50.75}{702.25} = -0.018$$

$$\text{Corrected Latitude} = -150.25 - 0.034 = -150.284$$

$$\text{Corrected Departure} = 50.75 - 0.018 = 50.732$$

(iii) Independent coordinates of all stations

Given coordinates of A: (100, 100)

Station B:

$$N_B = 100 + 400.159 = 500.159$$
$$E_B = 100 + 300.393 = 400.393$$

Station C:

$$N_C = 500.159 + 150.716 = 650.875$$
$$E_C = 400.393 - 100.286 = 300.107$$

Station D:

$$N_D = 650.875 - 400.591 = 250.284$$
$$N_D = 300.107 - 250.839 = 49.268$$

Independent coordinates of stations:

Line/Station	Corrected Latitude	Corrected Departure	Station	Independent Northing (N)	Independent Easting (E)
A	–	–	A	100	100
AB	400.159	300.393	B	500.159	400.393
BC	150.716	–100.286	C	650.875	300.107
CD	–400.591	–250.839	D	250.284	49.268
DA	–150.284	50.732	A	100	100

3. (b) (i) Solution:

Height of the dam, H

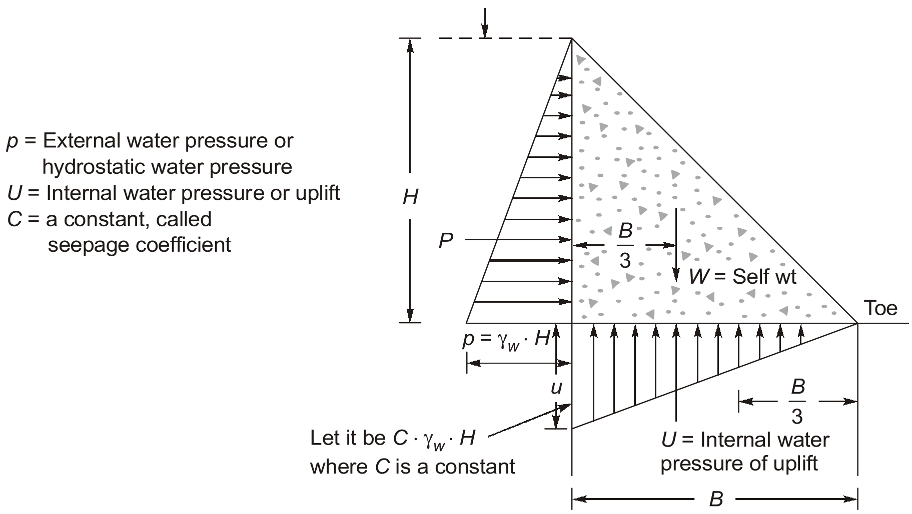
Base width of the dam, B

Specific gravity of dam material, G

Unit weight of water, w

Uplift pressure intensity factor, C (where CwH is the pressure at the heel)

Coefficient of friction, μ



An elementary profile of a gravity dam is a theoretical triangular section having zero width at the water level and maximum width at the base. This profile is derived by considering only the primary forces: the self-weight of the dam and the external water pressure.

1. Base width for No Tension

For no tension at the heel, the resultant must pass through the outer middle third (toe).

The forces involved are:

Weight of dam, $W = \frac{1}{2}BHGw$, acting at $B/3$ from the heel.

Horizontal water pressure, $P = \frac{1}{2}wH^2$, acting at $H/3$ from the base.

Uplift force, $U = \frac{1}{2}CwHB$, acting at $B/3$ from the heel.

Taking moments about the toe:

$$\Sigma M_{toe} = 0$$

$$\Rightarrow \qquad W\cdot\frac{2B}{3} - U\cdot\frac{2B}{3} - P\cdot\frac{H}{3} = 0$$

Substituting values:

$$\left(\frac{1}{2}BHGw\right)\frac{2B}{3} - \left(\frac{1}{2}CwHB\right)\frac{2B}{3} = \left(\frac{1}{2}wH^2\right)\frac{H}{3}$$

$$\Rightarrow \qquad \frac{B^2Hw(G-C)}{3} = \frac{wH^3}{6}$$

$$\Rightarrow \qquad B^2 = \frac{H^2}{2(G-C)}$$

Base Width: $B = \dfrac{H}{\sqrt{2(G-C)}}$

If uplift is considered full ($C = 1$), then $B = \dfrac{H}{\sqrt{2(G-1)}}$

2. Base width for Safety against Sliding

For safety against sliding:

$$\mu(W-U) \geq P$$

Substituting values:

$$\mu\left[\frac{1}{2}BHGw - \frac{1}{2}CwHB\right] = \frac{1}{2}wH^2$$

$$\Rightarrow \qquad \mu\frac{1}{2}BHw(G-C) = \frac{1}{2}wH^2$$

$$\Rightarrow \qquad B = \frac{H}{\mu(G-C)}$$

Base Width:

$$B = \frac{H}{\mu(G-C)}$$

If uplift is considered full ($C = 1$), then

$$B = \frac{H}{\mu(G-1)}$$

3. (b) (ii) Solution:

Given h = 22 cm

wheel load stress due to corner loading = 31.4 kg/cm^2,

wheel load stress due to edge loading = 33.6 kg/cm^2,

warping stress at corner region during summer = 9.2 kg/cm^2,

warping stress at corner region during winter = 7.4 kg/cm^2,

warping stress at edge region during summer = 8.5 kg/cm^2,

warping stress at edge region during winter= 6.3 kg/cm^2,

frictional stress during summer = 5.2 kg/cm^2,

and frictional stress during winter is 4.1 kg/cm^2.

Analysis of Edge Region:

The critical stress at the edge occurs during summer when wheel load and warping stresses are additive and frictional stress acts in compression, so it is subtracted.

Case 1: During Summer

$$\text{Critical Stress at Edge} = S_e + S_{we,s} - S_{f,s}$$

$$\text{Critical Stress at Edge} = 33.6 + 8.5 - 5.2$$

$$\text{Critical Stress at Edge} = 36.9 \text{ kg/cm}^2$$

Case 2: During Winter

$$\text{Critical Stress at Edge} = S_e + S_{we,w} + S_{f,w}$$

$$\text{Critical Stress at Edge} = 33.6 + 6.3 + 4.1$$

$$\text{Critical Stress at Edge} = 44 \text{ kg/cm}^2$$

Analysis of Corner Region:

At the corner, frictional stress is negligible because corners are free to move. The critical stress is the sum of wheel load and warping stresses.

Case 3: During Summer

$$\text{Critical Stress at Corner} = S_c + S_{wc,s}$$

$$\text{Critical Stress at Corner} = 31.4 + 9.2$$

$$\text{Critical Stress at Corner} = 40.6 \text{ kg/cm}^2$$

Case 4: During Winter

$$\text{Critical Stress at Corner} = S_c + S_{wc,w}$$

$$\text{Critical Stress at Corner} = 31.4 + 7.4$$

$$\text{Critical Stress at Corner} = 38.8 \text{ kg/cm}^2$$

Conclusion:

By comparing all the calculated values, the maximum stress occurs at the edge during winter.

$$\text{Most Critical Stress Value} = 44 \text{ kg/cm}^2$$

3. (c) (i) Solution:

1. General Energy Equation

The total energy H at a section is the sum of the datum head z, pressure head y, and velocity head $v^2/2g$:

$$H = z + y + \frac{v^2}{2g}$$

Differentiating with respect to the distance x along the channel bed:

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right)$$

2. Defining the Slopes

Energy slope (S_f) represents the rate of energy loss:

$$-\frac{dH}{dx} = S_f$$

Bed slope (S_o) represents the rate of change of bed elevation:

$$-\frac{dz}{dx} = S_o$$

Substituting into the differential equation:

$$-S_f = -S_o + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right)$$

$$\frac{dy}{dx}\left(1 + \frac{d}{dy}\left(\frac{v^2}{2g}\right)\right) = S_o - S_f$$

3. Kinetic Energy Term (Fr^2)

For a wide rectangular channel, discharge per unit width is $q = vy$, so $v = q/y$.

$$\frac{d}{dy}\left(\frac{q^2}{2gy^2}\right) = -\frac{q^2}{gy^3}$$

Since the Froude number squared is $F_r^2 = \frac{v^2}{gy} = \frac{q^2}{gy^3}$, the term becomes $-F_r^2$.

Thus, the dynamic equation simplifies to:

$$\frac{dy}{dx}(1 - F_r^2) = S_o - S_f \Rightarrow \frac{dy}{dx} = \frac{S_o - S_f}{1 - F_r^2}$$

4. Incorporating Manning's Equation

For a wide channel, hydraulic radius $R \approx y$. Manning's equation for flow at any depth y is:

$$Q = \frac{1}{n} A R^{2/3} S_f^{1/2} \Rightarrow S_f = \frac{n^2 Q^2}{A^2 y^{4/3}} \qquad \{\because A = By\}$$

For normal flow ($y = y_o$), the energy slope equals the bed slope ($S_f = S_o$):

$$S_o = \frac{n^2 Q^2}{A^2 y_o^{4/3}} \qquad \{\because A = By_o\}$$

Taking the ratio of S_f to S_o:

$$\frac{S_f}{S_o} = \frac{y_o^{10/3}}{y^{10/3}} = \left(\frac{y_o}{y}\right)^{10/3}$$

5. Relationship with Critical Depth

Critical depth y_c occurs when $F_r = 1$, so:

$$\Rightarrow \qquad 1 = \frac{q^2}{g y_c^3} \Rightarrow q^2 = g y_c^3$$

Substituting into the Froude number expression for any depth y:

$$F_r^2 = \frac{g y_c^3}{g y^3} = \left(\frac{y_c}{y}\right)^3$$

6. Final Dynamic Equation

Substituting the expressions for S_f / S_o and Fr^2 into the GVF equation:

$$\frac{dy}{dx} = S_o \frac{\left[1 - \left(\frac{y_o}{y}\right)^{10/3}\right]}{\left[1 - \left(\frac{y_c}{y}\right)^3\right]}$$

This equation explicitly shows how the water surface profile depends on the ratios of the actual depth y to the normal depth y_o and the critical depth y_c.

3. (c) (ii) Solution:

Given data

Width of channel, $b = 1.5$ m

Discharge, $Q = 1.2\ m^3/s$

Initial depth, $y = 0.45$ m

Chezy's coefficient, $C = 60\ m^{1/2}/sec$

Discharge per unit width:

$$q = \frac{Q}{b} = \frac{1.2}{1.5} = 0.8\ m^2/s$$

Calculate Critical Depth (y_c)

Critical depth for a rectangular channel:

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{0.8^2}{9.81}} = 0.402\ m$$

Since $y = 0.45\ m > y_c = 0.402\ m$, the flow is subcritical.

Analysis for Case (i) – Slope $S = 1/800$

Normal depth (y_o) is obtained from Chezy's equation:

$$Q = AC\sqrt{RS}, \quad A = by_o$$

$$\Rightarrow \quad 1.2 = (1.5y_o)60\sqrt{\frac{1.5y_o}{1.5+2y_o} \times \frac{1}{800}}$$

Solving,

$$y_0 = 0.641\ m$$

Examination:

$$y_o = 0.641\ m, \quad y_c = 0.402\ m$$

Since $y_o > y_c$, the slope is Mild (M).

The initial depth $y = 0.45$ m lies between y_c and y_o ($0.402 < 0.45 < 0.641$), representing an M2 profile.

Result: Depth will decrease downstream

Analysis for Case (ii) – Slope $S = 1/300$

Using Chezy's equation for the steeper slope:

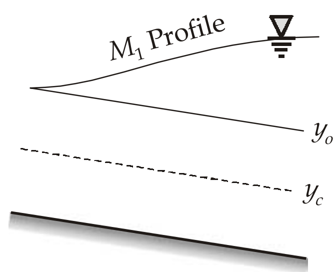
$$1.2 = (1.5 \times y_o) \times 60\sqrt{\frac{1.5y_o}{1.5+2y_o} \cdot \frac{1}{300}}$$

Solving, $y_o = 0.439$ m

Examination: $y_o = 0.439$ m, $y_c = 0.402$ m

$\Rightarrow$ $y_c < y_o < y$

Slope is mild and pfofile is M_1



Slope is mild (M)

Result: Depth will increase downstream.

4. (a) (i) Solution:

Given,

$$\text{Discharge, } Q = 1700\ l/\text{sec}\} = 1.7\ \text{m}^3/\text{sec}$$

$$\text{Head, } H_m = 21\ \text{m}$$

$$N = 315\ \text{rpm}$$

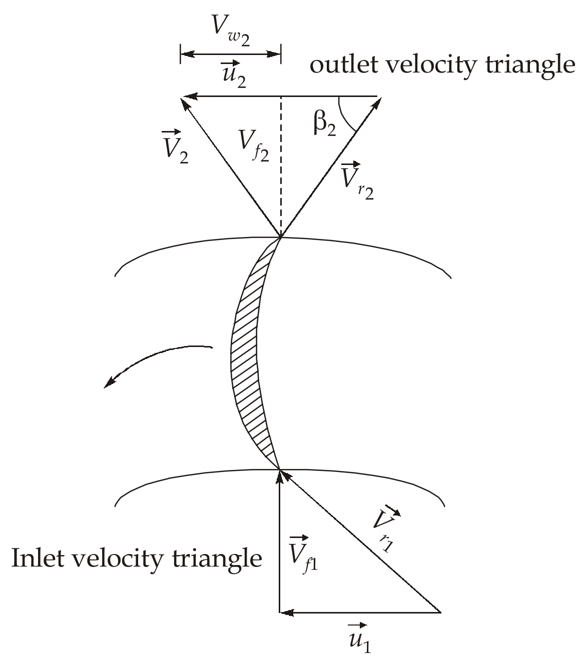
Diameter of impeller at outlet (D_2) = 1.5 m

Flow velocity at outlet (V_{f2}) = 2.45 m/sec

Vane angle at outlet (β_2) = 30°

$$D_1 = \frac{D_2}{2}$$

$\because$ $\beta_2 < 90°$, this is a backward curved vane.



1. Manometric efficiency (η_{mano})

$$(\eta_{mano}) = \frac{gH_m}{V_{w_2} u_2} \quad \text{...(i)}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.5 \times 315}{60} = 24.74 \text{ m/sec}$$

From outlet velocity triangle,

$$\tan\beta_2 = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\Rightarrow \quad V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\beta_2}$$

$$\Rightarrow \quad V_{w_2} = 24.74 - \frac{2.45}{\tan 30^\circ} = 20.496 \text{ m/sec}$$

Substituting values of H_m, V_{w2} and u_2 in equation (i)

$$\eta_{mano} = \frac{9.81 \times 21}{20.496 \times 24.74} = 0.4063$$

$$\therefore \quad \eta_{mano} = 40.63\%$$

2. Power required by the pump (i.e. shaft power)

We know that, $\eta_{overall} = \frac{\text{WHP}}{\text{SP}} = \frac{\gamma Q H_m}{\text{SP}}$

$$\Rightarrow \quad SP = \frac{\gamma Q H_m}{\eta_o}$$

$$\because \quad \eta_0 = 40\%$$

$$\therefore \quad SP = \frac{\gamma Q H_m}{0.4} = \frac{9810 \times 1.7 \times 21}{0.4} = 875{,}542.50 \text{ watt}$$

$\therefore$ Power required by the pump = 875.54 kW

3. Minimum starting speed

The flow will commence only if centrifugal force is greater than manometric head

i.e. $$\frac{u_2^2 - u_1^2}{2g} \geq H_m$$

$$\Rightarrow \quad \frac{\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2}{2g} \geq H_m$$

$$\Rightarrow \quad \left(\frac{\pi N}{60}\right)^2 (D_2^2 - D_1^2) > 2g\, H_m$$

$$\Rightarrow \quad N \geq \sqrt{\frac{(2gH_m)(60)^2}{\pi^2 (D_2^2 - D_1^2)}}$$

$$\Rightarrow \quad N_{min} = \sqrt{\frac{2 \times 9.81 \times 21 \times 60^2}{\pi^2 (1.5^2 - 0.75^2)}} \qquad \left[\because D_1 = \frac{D_2}{2}\right]$$

$$\Rightarrow \quad N_{min} = 298.427 \text{ r.p.m.}$$

4. (a) (ii) Solution:

$$\text{Actual discharge, } Q_{act} = 0.022 \text{ m}^3/\text{s}$$
$$\text{Pump speed, } N = 75 \text{ r.p.m}$$
$$\text{Stroke length, } L = 60 \text{ cm} = 0.6 \text{ m}$$
$$\text{Piston diameter, } D = 25 \text{ cm} = 0.25 \text{ m}$$
$$\text{Total lift, } H = 18 \text{ m}$$

Piston area

$$A = \frac{\pi}{4}D^2$$

$$\Rightarrow \quad A = \frac{\pi}{4} \times 0.25^2 = 0.0491 \text{ m}^2$$

Theoretical discharge of a single-acting reciprocating pump,

$$Q_{th} = \frac{A \times L \times N}{60}$$

$$\Rightarrow \quad Q_{th} = \frac{0.0491 \times 0.6 \times 75}{60}$$

$$\Rightarrow \quad Q_{th} = 0.0368 \text{ m}^3/\text{s}$$

Slip of the pump, $S = Q_{th} - Q_{act}$

$$\Rightarrow \quad S = 0.0368 - 0.022$$

$$\Rightarrow \quad S = 0.0148 \text{ m}^3/\text{s}$$

$$\because \quad \text{Percentage slip} = \frac{S}{Q_{th}} \times 100$$

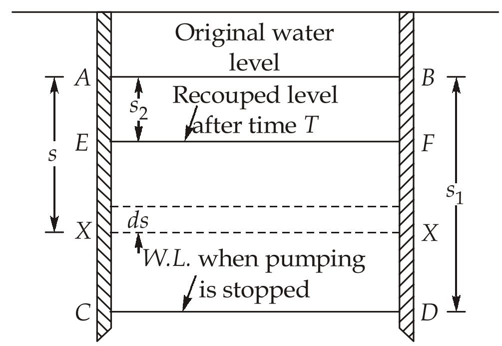
$$\Rightarrow \quad \text{Percentage slip} = \frac{0.0148}{0.0368} \times 100$$

$$\Rightarrow \quad \text{Percentage slip} = 40.2\%$$

4. (b) (i) Solution:

Step 1: Concept of Recuperating Test

A recuperating test is a method used to determine the specific capacity or yield of an open well. In this test, water is first rapidly pumped from the well to depress the water level to a certain depth. Pumping is then stopped, and the rise of the water level (recuperation) is observed over time. The rate of rise indicates the well's ability to yield water, particularly useful for wells with low discharge.



Step 2: Relationship between discharge and depression head

The rate of seepage into the well is proportional to the depression head:

$$Q = C' \times A \times s$$

For a small time interval dt, the water level rises by ds. The volume entering the well is:

$$dV = -A\,ds$$

Equating inflow and volume change:

$$C' A\,s\,dt = -A\,ds$$

$$\Rightarrow \quad C'\,dt = -\frac{ds}{s}$$

Step 3: Integration to determine specific capacity

Integrating from $t = 0(s = s_1)$ to $t = T(s = s_2)$:

$$\int_0^T C'\,dt = -\int_{s_1}^{s_2} \frac{1}{s}\,ds$$

$$\Rightarrow \quad C'T = -[\ln(s)]_{s_1}^{s_2} = \ln\left(\frac{s_1}{s_2}\right)$$

$$\Rightarrow \quad C' = \frac{1}{T}\ln\left(\frac{s_1}{s_2}\right) = \frac{2.303}{T}\log_{10}\left(\frac{s_1}{s_2}\right)$$

Step 4: Yield under constant depression head

Once the specific capacity C' is known, the yield Q under a constant depression head H is:

$$Q = C' \cdot A \cdot H$$

This gives the discharge of the well corresponding to a given head, derived from the recuperation behaviour.

4. (b) (ii) Solution:

Given data

Initial depression head, $s_1 = 4$ m

Recuperation depth, $\Delta s = 2.5$ m

Final depression head, $s_2 = s_1 - \Delta s = 4 - 2.5 = 1.5$ m

Time for recuperation, $T = 90$ min

Diameter of well, $d = 3$ m

Working depression head, $H = 4$ m

Cross-sectional area of the well,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 3^2 = 7.069 \text{ m}^2$$

Specific capacity per unit area,

$$C' = \frac{2.303}{T} \log_{10}\left(\frac{s_1}{s_2}\right)$$

$$\Rightarrow \quad C' = \frac{2.303}{90} \log_{10}\left(\frac{4}{1.5}\right)$$

$$\Rightarrow \quad C' = 0.011 \text{ min}^{-1}$$

Yield from the well,

$$Q = C' \times A \times H$$

$$\Rightarrow \quad Q = 0.011 \times 7.069 \times 4$$

$$\Rightarrow \quad Q = 0.308 \text{ m}^3/\text{min}$$

To express the yield in cubic meters per hour,

$$Q = 0.308 \times 60 = 18.491 \text{ m}^3/\text{hr}$$

4. (c) Solution:

Given data

$$\text{Wheel load, } P = 4100 \text{ kg}$$

$$\text{Tyre pressure, } p = 7 \text{ kg/cm}^2$$

Dial calibration: 100 divisions = 210 kg $\Rightarrow$ 1division = 2.1 kg

Standard load at 2.5 mm penetration = 1370 kg

Standard load at 5.0 mm penetration = 2055 kg

CBR of sandy soil, $CBR_{\text{sandy}} = 8\%$

CBR of poorly graded gravel, $CBR_{\text{poor}} = 25\%$

CBR of well-graded gravel, $CBR_{\text{well}} = 92\%$

Minimum surface thickness, $t_{\text{surface}} = 6 \text{ cm}$

CBR calculation for Specimen No. 1

Load at 2.5 mm penetration,

$$\text{Load}_{2.5} = 38 \times 2.1 = 79.8 \text{ kg}$$

$$CBR_{2.5} = \frac{79.8}{1370} \times 100 = 5.825\%$$

Load at 5.0 mm penetration,

$$\text{Load}_{5.0} = 52 \times 2.1 = 109.2 \text{ kg}$$

$$CBR_{5.0} = \frac{109.2}{2055} \times 100 = 5.314\%$$

Higher value is taken,

$$CBR_1 = 5.825\%$$

CBR calculation for Specimen No. 2

Load at 2.5 mm penetration,

$$\text{Load}_{2.5} = 40 \times 2.1 = 84 \text{ kg}$$

$$CBR_{2.5} = \frac{84}{1370} \times 100 = 6.131\%$$

Load at 5.0 mm penetration,

$$\text{Load}_{5.0} = 55 \times 2.1 = 115.5 \text{ kg}$$

$$CBR_{5.0} = \frac{115.5}{2055} \times 100 = 5.62\%$$

Higher value is taken, $CBR_2 = 6.131\%$

Average subgrade *CBR*,

$$CBR_{subgrade} = \frac{5.825 + 6.131}{2} = 5.978\%$$

Pavement thickness calculations

The formula for thickness *T* above a layer with *CBR* value *C* is,

$$T = \sqrt{\frac{1.75 \times P}{C} - \frac{P}{p \times \pi}}$$

Total thickness above subgrade,

$$T_{subgrade} = \sqrt{\frac{1.75 \times 4100}{5.978} - \frac{4100}{7 \times \pi}} = 31.84 \text{ cm}$$

Thickness above sandy soil,

$$T_8 = \sqrt{\frac{1.75 \times 4100}{8} - \frac{4100}{7 \times \pi}} = 26.654 \text{ cm}$$

Thickness above poorly graded gravel,

$$T_{25} = \sqrt{\frac{1.75 \times 4100}{25} - \frac{4100}{7 \times \pi}} = 10.028 \text{ cm}$$

Design thickness of individual layers

Thickness of compacted sandy soil,

$$T_{sandy} = 31.84 - 26.654 = 5.186 \text{ cm}$$

Thickness of poorly graded gravel,

$$T_{poor} = 26.654 - 10.028 = 16.626 \text{ cm}$$

Thickness of well-graded gravel,

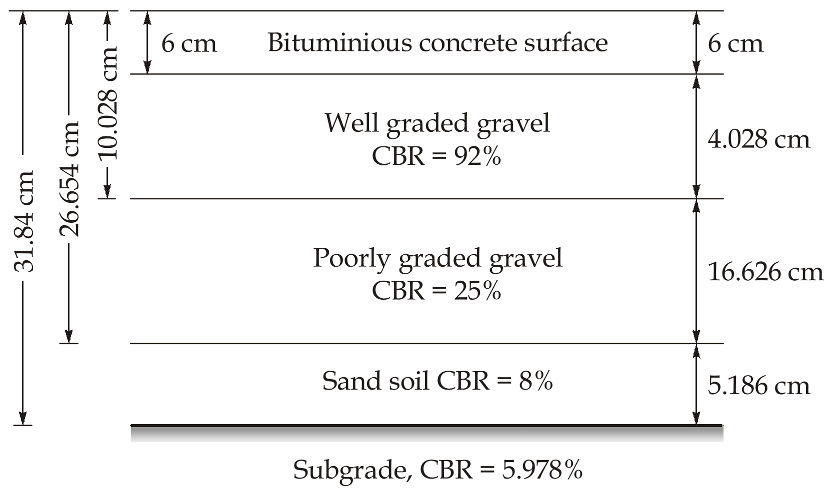
$$t_{well} = 10.028 - 6 = 4.028 \text{ cm}$$

Thickness of bituminous concrete surface,

$$t_{surface} = 6 \text{ cm}$$

Total pavement thickness,

$$T_{total} = 31.84 \text{ cm}$$



Section B

5. (a) Solution:

A. Building blocks of clay minerals:

There are two types of building blocks of clay mineral viz:

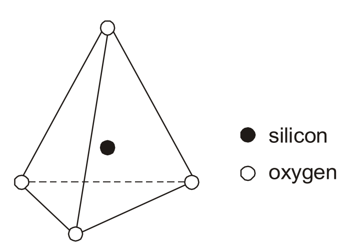
1. **Silica tetrahedral unit.**
2. **Octahedral unit.**

1. Silica tetrahedral unit:

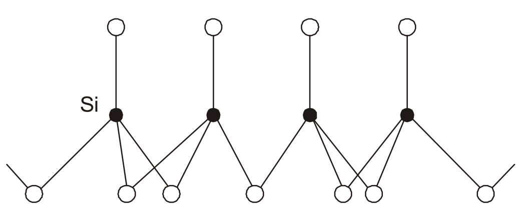
In this unit, 4 oxygen atoms enclose silica at center of tetrahedron with all the oxygen at base of tetrahedron lying in same common plane and is shared by 2 tetrahedron units.

Net charge over tetrahedron unit is -1.

It is represented as a trapezoid.



Single tetrahedral unit

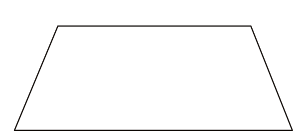


$4 \times (-2) = -8$

$4 \times (+4) = +16$

$6 \times (-2) = -12$

Net = –4



2. **Octahedral unit:**

In this unit, 6 hydroxyl (OH^-) molecules enclose Al, Mg or Fe at centre of octahedral geometry. Each hydroxyl molecule is shared between 3 units.

If Al^{+3} is at center, it is termed as gibbsite unit, similarly for Mg and Fe it is known as brucite and ferrite unit respectively and is represented as rectangular symbol.

Net charge present over octahedral unit is +1.

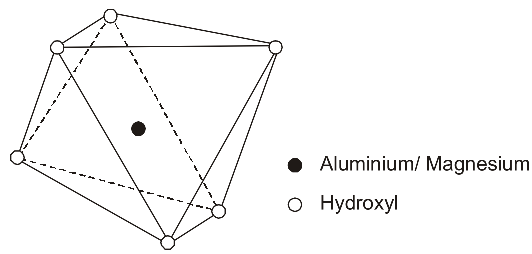


Fig. Single octahedral unit

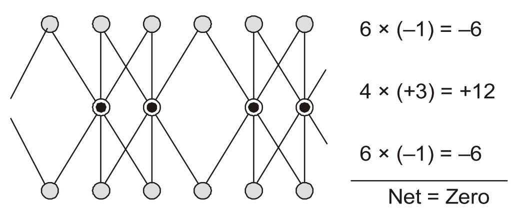


Fig. Octahedral sheet

B. Effect of water content on compaction:

As the water content is increased, the soil particles get lubricated. The soil mass becomes more workable and the particles have closer packing. The dry density of the soil increases with an increase in the water content till the optimum water content is reached. At that stage, the air voids attain approximately a constant volume. With further increase in water content, the air voids do not decrease, but the total voids (air plus water) increase and the dry density decreases. Thus the higher dry density is achieved upto the optimum water content due to forcing air out from the soil voids. After the optimum water content is reached, it becomes more difficult to force air out to further reduce the air voids.

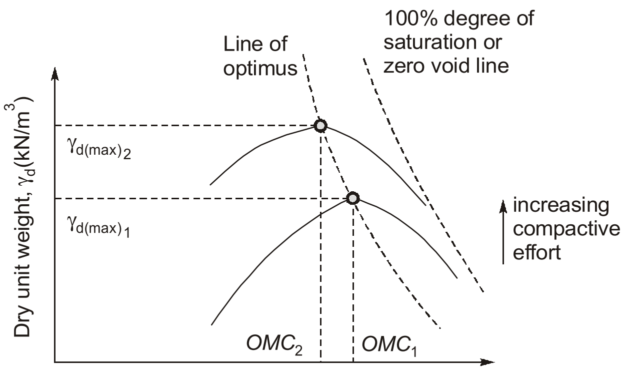


Fig. Compaction Curve

5. (b) Solution:

Target overall kill efficiency = 95%

$$\text{Disinfection model, } N_t = \frac{N_0}{1+kt}$$

At $t = 10$ min, 50% killed $\Rightarrow$ $N_t = 0.5\,N_0$

At 75% killed $\Rightarrow$ $N_t = 0.25\,N_0$

New rate after 75% kill, $k' = 3k$

Finding initial kill rate k

At $t = 10$ min,

$$0.5N_0 = \frac{N_0}{1+10k}$$

$$\Rightarrow \quad 0.5 = \frac{1}{1+10k}$$

$$\Rightarrow \quad 1 + 10k = 2$$

$$\Rightarrow \quad k = 0.1 \text{ min}^{-1}$$

Finding time to reach 75% kill

$$0.25\,N_0 = \frac{N_0}{1+0.1t_{75}}$$

$$\Rightarrow \quad 0.25 = \frac{1}{1+0.1t_{75}}$$

$$\Rightarrow \quad 1 + 0.1t_{75} = 4$$

$$\Rightarrow \quad t_{75} = 30 \text{ min}$$

Finding time required for last 20% kill

New rate after 75% kill,

$$k' = 3 \times 0.1 = 0.3 \text{ min}^{-1}$$

At start of final phase,

$$N'_{\text{initial}} = 0.25N_0$$

At 95% kill,

$$N'_{\text{final}} = 0.05N_0$$

Using the model,

$$N'_{\text{final}} = \frac{N'_{\text{initial}}}{1+k'\Delta t}$$

$$\Rightarrow \quad 0.05N_0 = \frac{0.25N_0}{1+0.3\Delta t}$$

$$\Rightarrow \quad 0.05 = \frac{0.25}{1+0.3\Delta t}$$

$$\Rightarrow \quad 1+0.3\Delta t = 5$$

$$\Rightarrow \quad 0.3\Delta t = 4$$

$$\Rightarrow \quad \Delta t = 13.333 \text{ min}$$

5. (c) Solution:

Given data

Thickness of clay layer, $H_1 = 4$ m

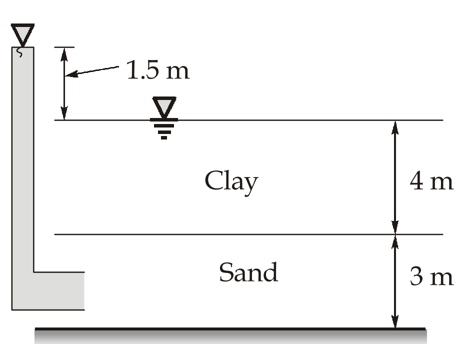
Saturated unit weight of clay, $\gamma_{sat1} = 20$ kN/m^3

Thickness of sand layer, $H_2 = 3$ m

Saturated unit weight of sand, $\gamma_{sat2} = 18.5$ kN/m^3

Artesian head above ground level, $h = 1.5$ m

Unit weight of water, $\gamma_w = 9.81$ kN/m^3



Stress calculations at ground level (z = 0 m)

$$\sigma = 0 \text{ kPa}, u = 0 \text{ kPa}, \sigma' = 0 \text{ kPa}$$

Stress calculations at bottom of clay layer (z = 4 m)

$$\sigma = 4 \times 20 = 80 \text{ kPa}, u = 4 \times 9.81 = 39.24 \text{ kPa}$$

$$\sigma' = 80 - 39.24 = 40.76 \text{ kPa}$$

Pore water pressure considering artesian head,

$$u = (4 + 1.5) \times 9.81 = 53.955 \text{ kPa}$$

$$\sigma' = \sigma - u = 80 - 53.955 = 26.045 \text{ kPa}$$

Stress calculations at top of rock (z = 7 m)

$$\sigma = 80 + (3 \times 18.5) = 135.5 \text{ kPa}$$

$$u = (7 + 1.5) \times 9.81 = 83.385 \text{ kPa}$$

$$\sigma' = 135.5 - 83.385 = 52.115 \text{ kPa}$$

Increase in effective stress at top of rock

Initial effective stress,

$$\sigma' = 52.115 \text{ kPa}$$

Reduction in artesian head,

$$\Delta h = 0.8 \text{ m}$$

New pore water pressure,

$$u_2 = (8.5 - 0.8) \times 9.81 = 75.537 \text{ kPa}$$

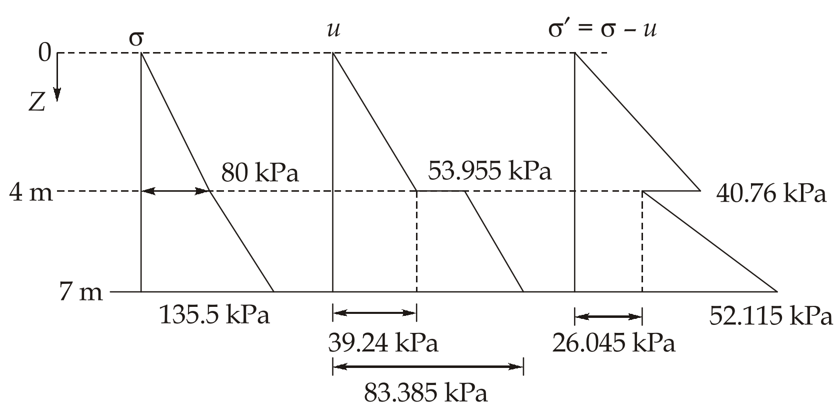
New effective stress,

$$\sigma_2' = 135.5 - 75.537 = 59.963 \text{ kPa}$$

Increase in effective stress,

$$\Delta\sigma' = \sigma_2' - \sigma_1' = 59.963 - 52.115 = 7.848 \text{ kPa}$$

$$\Delta\sigma' = 0.8 \times 981 = 7.848 \text{ kPa}$$



5. (d) Solution:

Initial pressure, P_1 = 1 atm

Initial temperature, T_1 = 273 K

Initial molar volume, V_1 = 22.4 L/mol

Final pressure, P_2 = 1.5 atm

Final temperature, P_2 = 25°C = 298 K

Concentration, 150 ppb = 0.15 ppm

Molecular weight of SO_2, M = 64 g/mol

Using gas law relation,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \quad V_2 = \left(\frac{P_1V_1}{T_1}\right)\left(\frac{T_2}{P_2}\right)$$

$$\Rightarrow \quad V_2 = \left(\frac{1\times 22.4}{273}\right)\left(\frac{298}{1.5}\right) = 16.299 \text{ L/mol}$$

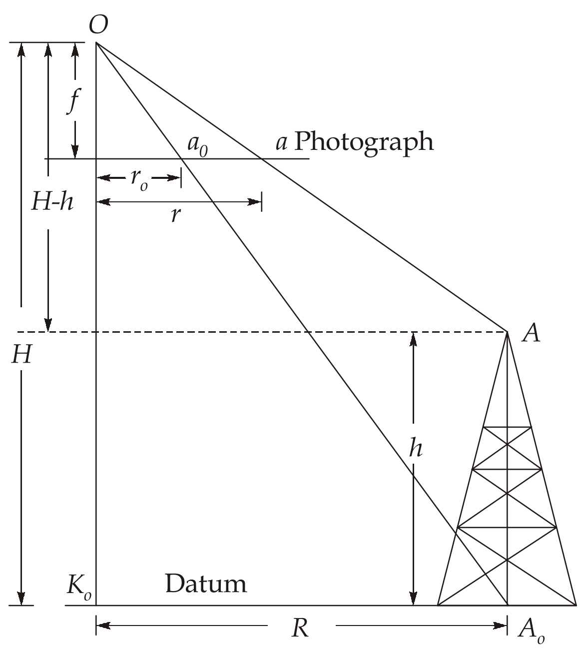
Concentration in $\mu g/m^3$,

$$C = \frac{0.15\times 64}{16.299}\times 10^3$$

$$\Rightarrow \quad C = 589.008\ \mu g/m^3$$

5. **(e) Solution:**

When the ground is not horizontal, the scale of the photograph varies from point to point and is not constant. Since the photograph is the perspective view, the ground relief is shown in perspective on the photograph. Every point on the photograph is therefore, displaced from their true orthographic position. This displacement is called relief displacement.



As is clear from the figure, the point a is displaced outward from its datum photograph position, the displacement being along the corresponding radial lines from the principal point. The radial distance aa_0 is the relief displacement of A. The point k has not been displaced since it coincides with the principal point of the photograph. To calculate the amount of relief displacement,

Let r = Radial distance of a from k.

r_0 = Radial distance of a_0 from k

R = Distance horizontal of A or A_0 on ground from K_0

Then, from similar triangles,

$$\frac{f}{H-h} = \frac{r}{R} \Rightarrow r = \frac{Rf}{H-h} \quad ...(i)$$

Also,
$$\frac{f}{H} = \frac{r_0}{R} \Rightarrow r_0 = \frac{Rf}{H} \quad ...(ii)$$

Hence the relief displacement (d) is given by

$$\therefore \qquad d = r - r_0 = \frac{Rf}{H-h} - \frac{Rf}{H}$$

$$\Rightarrow \qquad \boldsymbol{d} = \frac{Rfh}{H(H-h)} \qquad \text{...(iii)}$$

From equation (i) and (ii)

$$R = \frac{r(H-h)}{f} = \frac{r_0 H}{f}$$

Substituting the values of R in (3), we get,

$$d = \frac{r(H-h)}{f}\frac{fh}{H(H-h)} = \frac{rh}{H}$$

Also,

$$d = \frac{r_0 H}{f}\frac{fh}{H(H-h)} = \frac{r_0 h}{H-h}$$

6. (a) Solution:

Given data

Volume of mould, V = 1000 ml

Mass of mould, M_m = 1000 g

Specific gravity of solids, G_s = 2.65

Density of water, ρ_w = 1 g/ml

1. Calculation of bulk density and dry density

Mass of wet soil, M = (Mass of mould + wet soil) – 1000

$$\text{Bulk density, } \rho_b = \frac{M}{1000}$$

$$\text{Dry density,} \rho_d = \frac{\rho_b}{1+\frac{w}{100}}$$

w(%)	M(g)	ρ_b(g/ml)	ρ_d(g/ml)
8.5	1850	1.85	1.705
10.5	2010	2.01	1.819
12.5	2120	2.12	1.884
14.5	2080	2.08	1.817
16.5	2020	2.02	1.734

From the above values, maximum dry density occurs at

$$MDD = 1.884 \text{ g/ml}, \; OMC = 12.5\%$$

2. Zero air voids line

$$\rho_{zav} = \frac{G_s \times \rho_w}{1 + \frac{w \times G_s}{100}}$$

At $w = 8.5\%$, $$\rho_{zav} = \frac{2.65 \times 1}{1 + \frac{8.5 \times 2.65}{100}} = 2.163 \text{ g/ml}$$

At $w = 12.5\%$, $$\rho_{zav} = \frac{2.65 \times 1}{1 + \frac{12.5 \times 2.65}{100}} = 1.991 \text{ g/ml}$$

At $w = 16.5\%$, $$\rho_{zav} = \frac{2.65 \times 1}{1 + \frac{16.5 \times 2.65}{100}} = 1.844 \text{ g/ml}$$

3. Degree of saturation at maximum dry density

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

$$\Rightarrow \quad 1.884 = \frac{2.65 \times 1}{1 + e}$$

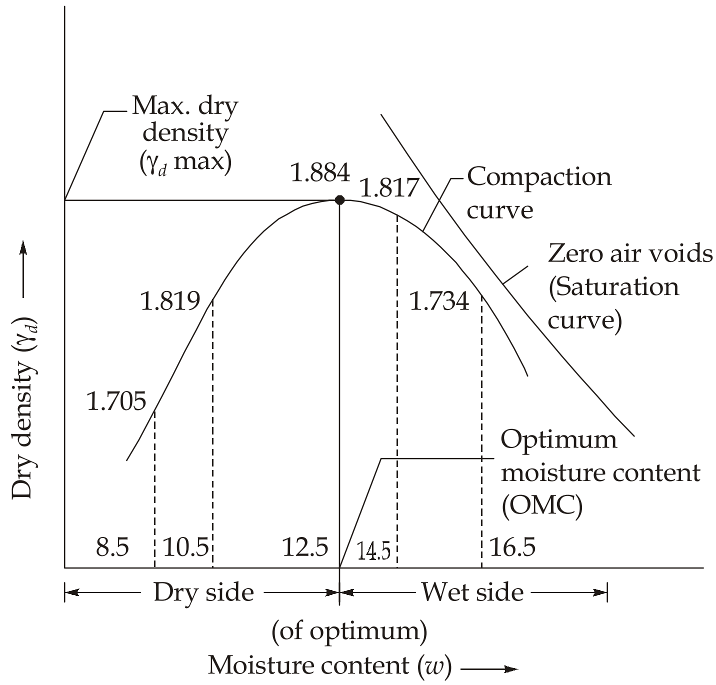
$$\Rightarrow \quad e = 0.407$$

Using relation,

$$S \times e = w \times G_s$$

$$\Rightarrow \quad S = \frac{0.125 \times 2.65}{0.407}$$

$$\Rightarrow \quad S = 0.814 = 81.4\%$$



6. (b) (i) Solution:

The magnitude of the rise in pressure is deduced by considering the energy changes of the system.

$$\text{Kinetic energy of the moving fluid} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(\rho Al)v^2$$

Strain energy stored in water (after closure of valve)

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times P \times \frac{P}{K} \times (Al) = \frac{P^2}{2K}(Al)$$

Where K is the bulk modulus of the liquid. When the whole column of fluid has been instantaneously stopped (quick closure of the valve), kinetic energy will be stored uniformly throughout the length of fluid as strain energy if the pipe is assumed to be perfectly rigid.

$$\therefore \qquad \frac{1}{2}(\rho Al)v^2 = \frac{P^2}{2K}(Al)$$

$$\Rightarrow \quad P = v\sqrt{K\rho} = v\sqrt{\frac{K}{\rho}\rho^2}$$

$$\Rightarrow \quad \frac{P}{\rho g} = \frac{v}{g}\sqrt{\frac{K}{\rho}}$$

6. (b) (ii) Solution:

For sudden closure of the valve provided in an elastic pipe, the pressure rise is given by:

$$\frac{P}{\rho g} = \frac{v}{g}\sqrt{\frac{K}{\rho\left(1+\frac{DK}{Et}\right)}} \quad \text{or} \quad \frac{P}{\rho g} = \frac{v}{g}\sqrt{\frac{k}{\rho\left(1+\frac{Dk}{Et}\right)}}$$

Flow velocity,

$$\Rightarrow \quad v = \frac{Q}{A} = \frac{2.15}{\frac{\pi}{4}(1)^2}$$

$$\Rightarrow \quad v = 2.737 \text{ m/s}$$

$$\therefore \quad \frac{P}{\rho g} = \frac{v}{g}\sqrt{\frac{k}{\rho\left(1+\frac{Dk}{Et}\right)}}$$

$$= \frac{2.737}{9.81}\sqrt{\frac{2\times 10^9}{10^3\times\left(1+\frac{1\times 2\times 10^9}{2\times 10^{11}\times 0.01}\right)}}$$

$$\Rightarrow \quad \frac{P}{\rho g} = 279$$

$$\Rightarrow \quad P = 279 \times 9.81 = 2737 \text{ kPa}$$

$$\Rightarrow \quad \sigma_L = \frac{2737\times 1}{4\times 0.01} = 68.425 \text{ MPa}$$

$$\Rightarrow \quad \sigma_C = 2 \times \sigma_L = 136.85 \text{ MPa}$$

6. (c) Solution:

Given data

Rainfall intensities, i_1 = 1.2 cm/h, i_2 = 2.8 cm/h, i_3 = 1.6 cm/h

Duration of each period, t = 4 h

ϕ-index, ϕ = 3 mm/h = 0.3 cm/h

Base flow = 15 m^3/s

Calculation of effective rainfall

$$R = (i - \phi) \times t$$

$$R_1 = (1.2 - 0.3) \times 4 = 3.6 \text{ cm}$$

$$R_2 = (2.8 - 0.3) \times 4 = 10 \text{ cm}$$

$$R_3 = (1.6 - 0.3) \times 4 = 5.2 \text{ cm}$$

Time (h)	4-h U.H. (m^3/s)	DRH_1 (3.6 × U.H.)	DRH_2 (10 × U.H. lagged 4h	DRH_3 (5.2 × U.H. lagged 8h)	Total DRH (m^3/s)	Base Flow (m^3/s)	Total Runoff (m^3/s)
0	0	0	-	-	0	15	15
2	12.52	45.072	-	-	45.072	15	60.072
4	21.32	76.752	0	-	76.752	15	91.752
6	23.54	84.744	125.2	-	209.944	15	224.944
8	17.84	64.224	213.2	0	277.424	15	292.424
10	14.79	53.244	235.4	65.104	353.748	15	368.748
12	12.18	43.848	178.4	110.864	333.112	15	348.112
14	10.04	36.144	147.9	122.408	306.452	15	321.452
16	8.26	29.736	121.8	92.768	244.304	15	259.304
18	6.51	23.436	100.4	76.908	200.744	15	215.744
20	4.98	17.928	82.6	63.336	163.864	15	178.864
22	3.95	14.22	65.1	52.208	131.528	15	146.528
24	3.05	10.98	49.8	42.952	103.732	15	118.732
26	2.26	8.136	39.5	33.852	81.488	15	96.488
28	1.60	5.76	30.5	25.896	62.156	15	77.156
30	1.07	3.852	22.6	20.54	46.992	15	61.992
32	0.53	1.908	16	15.86	33.768	15	48.768
34	-	-	10.7	11.752	22.452	15	37.452
36	-	-	5.3	8.32	13.62	15	28.62
38	-	-	-	5.564	5.564	15	20.564
40	-	-	-	2.756	2.756	15	17.756

7. (a) (i) Solution:

Given,
$$K_x = 0.002 \text{ cm/sec}$$
$$K_z = 0.0025 \text{ cm/sec}$$
$$\therefore \quad K' = \sqrt{K_x K_z} = \sqrt{0.002 \times 0.0025} = 2.236 \times 10^{-3} \text{ cm/sec}$$
$$= 2.236 \times 10^{-3} \times 10^{-2} \times 86400$$
$$= 1.932 \text{ m/day}$$
$$H = \text{Head available} = 3 - 1.5 = 1.5 \text{ m}$$
$$N_f = \text{No. of flow channels} = 6$$
$$N_d = \text{No. of equipotential drops} = 12$$

1. Quantity of seepage loss per unit width (q)
$$= K'H\frac{N_f}{N_d} = 1.932 \times 1.5 \times \frac{6}{12} = 1.45 \text{ m}^3\text{/day/m}$$

2. Head drop between two equipotential lines (Δh)
$$= \frac{H}{N_d} = \frac{1.5}{12} = 0.125 \text{ m}$$

Seepage pressure at point A $= (H - 3\Delta h)\gamma_w$
$$= (1.5 - 3 \times 0.125) \times 9.81$$
$$= 11.036 \text{ kN/m}^2$$

Seepage pressure at point B $= (H - 5\Delta h)\gamma_w$
$$= (1.5 - 5 \times 0.125) \times 9.81$$
$$= 8.58 \text{ kN/m}^2$$

Seepage pressure at point C $= (H - 7\Delta h)\gamma_w$
$$= (1.5 - 7 \times 0.125) \times 9.81$$
$$= 6.131 \text{ kN/m}^2$$

For point E,

Average number of head drop at E $= \left(\frac{8+9}{2}\right) = 8.5$

$\therefore$ Seepage pressure at point E $= (1.5 - 8.5 \times 0.125) \times 9.81$
$$= 4.29 \text{ kN/m}^2$$

Seepage pressure at point D $= (H - 10\Delta h)\gamma_w$
$$= (1.5 - 10 \times 0.125) \times 9.81$$

$= 2.45 \text{ kN/m}^2$

3. Pore water pressure at point B $= (H - n\Delta h - D.H.)\gamma_w$

$= [1.5 - 5 \times 0.125 - (-10)] \times 9.81$

$= 106.68 \text{ kN/m}^2$

Pore water pressure at point D $= (H - n\Delta h - D.H.)\gamma_w$

$= [1.5 - 10 \times 0.125 - (-6)] \times 9.81$

$= 61.3125 \text{ kN/m}^2$

4. Since, $l = 1.2$ m

$\therefore$ Exit gradient $= i_{exit} = \dfrac{\Delta h}{l} = \dfrac{0.125}{1.2} = 0.10416$

5. $\because$ $\eta = 0.35$

$\therefore$ Void ratio $(e) = \dfrac{n}{1-n} = \dfrac{0.35}{1-0.35} = 0.538$

Critical gradient $(i_c) = \dfrac{G-1}{1+e} = \dfrac{2.67-1}{1+0.538} = 1.085$

Factor of safety against piping $= \dfrac{i_c}{i_{exit}} = \dfrac{1.085}{0.10416} = 10.417$

7. (a) (ii) Solution:

Terzaghi's design of protective filter is primarily based on the grain size distribution of the grain size distribution of the filter material and the protected material, which are as follows:

- $$\frac{D_{15(\text{filter})}}{D_{85(\text{Protected material})}} < 5$$

  This ensures that no significant invasion of particles from the protected soil to the filter shall takes placed.

- $$4 < \frac{D_{15(\text{filter})}}{D_{15(\text{Protected material})}} < 20$$

  First part will ensure that sufficient head is lost in flow through the filters without a build up of seepage pressure.

- $$\frac{D_{50(\text{filter})}}{D_{50(\text{Protected material})}} < 25$$

  Second part of 2nd specification and third specification are additional guidelines for the selection of filter material.

7. (b) (i) Solution:

Index properties are those properties of soil that are used to identify, classify, and indicate the engineering behavior of the soil. These properties do not necessarily measure the engineering strength (like shear strength or permeability) directly, but they provide a reliable "index" of how the soil will behave under different loading or environmental conditions. They are primarily used to distinguish one soil type from another in a laboratory setting.

Categories:

Index properties are generally divided into two main categories based on whether they describe the soil mass as a whole or the individual particles that compose it.

1. **Soil Aggregate Properties**

   These properties depend on the soil's collective structure, its history, and its current physical state (such as moisture content and density). They are crucial for undisturbed soil mass analysis.

   - **Water Content:** The ratio of the weight of water to the weight of solids in a given soil mass.
   - **Specific Gravity:** The ratio of the unit weight of soil solids to the unit weight of distilled water at 4°C.
   - **In-situ Density:** The density of the soil in its natural state in the ground.
   - **Relative Density (Density Index):** Used specifically for cohesionless soils to indicate the degree of packing (looseness or denseness).
   - **Consistency Limits (Atterberg Limits):** These define the boundary water contents at which a fine-grained soil passes from one state to another (Liquid Limit, Plastic Limit, and Shrinkage Limit).
   - **Thixotropy and Sensitivity:** Properties describing the loss and regain of strength in fine-grained soils when disturbed.

2. **Soil Particle Properties**

These properties depend solely on the nature of the individual grains, regardless of how they are arranged in a soil mass. These are typically determined using disturbed or remolded soil samples.

- **Grain Size Distribution:** The percentage of various grain sizes present in the soil, determined through sieve analysis (for coarse-grained soil) or sedimentation analysis (for fine-grained soil).
- **Particle Shape:** The physical form of the grains (e.g., angular, sub-angular, rounded, or flaky), which significantly affects the internal friction and packing.

7. (b) (ii) Solution:

Leachate is a highly concentrated, toxic liquid formed when rainwater or moisture percolates through decomposing waste in a landfill. As the water moves downward, it undergoes physical, chemical, and biological reactions, picking up dissolved or suspended components from the waste.

Composition and Factors

The characteristics of leachate vary significantly based on the age of the landfill and the type of waste deposited. Its composition is generally influenced by:

- **Organic Matter:** High levels of Chemical Oxygen Demand (COD) and Biochemical Oxygen Demand (BOD).
- **Inorganic Salts:** High concentrations of chlorides, sulfates, and bicarbonates.
- **Heavy Metals:** Presence of lead, copper, mercury, and cadmium from industrial or electronic waste.
- **Pathogens:** Bacteria and viruses from organic decomposition.

The Environmental Problem

The primary concern with leachate is its potential to escape the landfill boundaries:

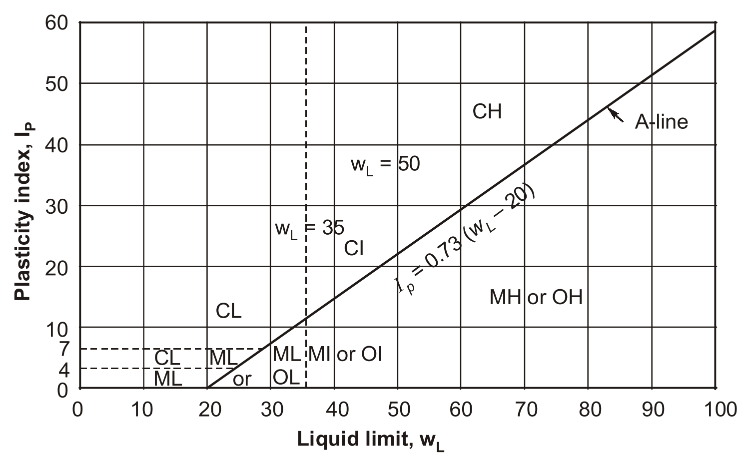
- **Groundwater Contamination:** If the landfill liner fails or is non-existent, leachate can seep into the soil and reach underlying aquifers, rendering local water sources undrinkable.
- **Surface Water Pollution:** Runoff can carry leachate into nearby streams and ponds, leading to eutrophication and the death of aquatic life.
- **Soil Toxicity:** Persistent pollutants can alter the soil chemistry, affecting plant growth and entering the food chain.

Management and Control

Modern engineering practices aim to mitigate the leachate problem through multiple barriers:

- **Bottom Liners:** Using impermeable layers like High-Density Polyethylene (HDPE) or compacted clay.
- **Leachate Collection Systems:** A network of perforated pipes and gravel layers at the base to channel the liquid to a sump.
- **Treatment Plants:** Once collected, leachate must be treated using biological (aerobic/anaerobic) or chemical (flocculation/oxidation) methods before safe disposal.

7. (b) (iii) Solution:



7. (c) Solution:

Compute the volume of raw sludge, having 5% solids content. We can compute it using the following equation:

$$V_1 = \frac{W_s}{\rho_w \times S_1 \times P_{s1}}$$

Density of water, $\rho_w = 1000\ \text{kg/m}^3$

$$V_1 = \frac{450}{1000 \times 1.02 \times 0.05} = 8.824\ \text{m}^3$$

= volume of sludge before digestion

Compute the total solids in digested sludge

Fixed solids FS in raw sludge = 25% of TS (as VS in raw sludge is 75%)

= 0.25 × 450 kg

= 112.5 kg

Also, as 65% of *VS* is destroyed in digested sludge, 35% *VS* will remain in the sludge after digestion.

Therefore,

VS remaining in digested sludge = 0.35 × (0.75 × 450) = 118.125 kg

So the total mass of solids in digested sludge is *FS* of raw sludge + *VS* remaining after digestion

$$= 112.5 \text{ kg} + 118.125 \text{ kg}$$

$$= 230.625 \text{ kg}$$

Compute the volume of digested sludge having 11% solids content

$$\text{Volume of digested sludge}, V_2 = \frac{230.625}{1000 \times 1.04 \times 0.11}$$

$$= 2.016 \text{ m}^3$$

So, the volume of sludge after digestion is 2.016 m^3

Compute the reduction in sludge volume

$$\textbf{Reduction in sludge volume} = \frac{(\text{volume of raw sludge}) - (\text{volume of digested sludge})}{(\text{volume of raw sludge})} \times 100$$

$$= \frac{8.824 - 2.016}{8.824} \times 100$$

$$= 77.153\%$$

So, the reduction in sludge volume is 77.153%

Compute the specific gravity of solids in digested sludge

We can compute it, using the equation given below derived

$$S_{s(d)} = \frac{P_s \times S}{1 - S(1 - P_s)}$$

where

$S_{s(d)}$ = Specific gravity of solids in digested sludge

P_s = Percentage of solids in digested sludge

$1 - P_s$ = Percentage of water in digested sludge

S = Specific gravity of digested sludge

Now, substituting the values in equation, we have

$$S_{s(d)} = \frac{0.11 \times 1.04}{1 - 1.04(1 - 0.11)} = 1.538$$

Compute the specific gravity of dewatered sludge, S_d

We can compute it, using the following equation

$$\frac{1}{S_d} = \frac{P_s}{S_{s(d)}} + \frac{(1-P_s)}{1}$$

where, W_d = weight of dewatered sludge

S_d = specific gravity of dewatered sludge

S = SP gravity of solid

Substituting the values, we get

$$\frac{1}{S_d} = \frac{0.22}{1.538} + \frac{(1-0.22)}{1}$$

(specific gravity of water is 1)

$$= 0.923$$

Therefore, $S_d = 1.083$

Compute the volume of dewatered sludge (V_d)

$$V_d = \frac{W_s}{\rho_w \times S_d \times P_s}$$

$$\Rightarrow \quad V_d = \frac{230.625}{1000 \times 1.083 \times 0.22} = 0.968 \text{ m}^3$$

So, the volume of dewatered sludge after digestion = 0.968 m^3

8. (a) **Solution:**

Given Data:

- Population = 2,00,000
- Rate of solid waste production = 800 g/capita-day = 0.8 kg/capita-day
- Design period = 25 years
- Landfill height = 12 m

Component	Mass fraction (w_i)	Density (ρ_i)	w_i/ρ_i
Food waste	0.0943	288	0.000327
Paper	0.4317	81.7	0.005284
Plastics	0.0181	64	0.000283
Cardboard	0.0650	99.3	0.000655
Textiles	0.0020	64	0.000031
Rubber	0.0088	128	0.000069
Leather	0.0150	160	0.000094
Garden Trimming	0.1432	104	0.001377
Wood	0.0350	240	0.000146
Glass	0.0749	194	0.000386
Tin cans	0.0520	88.1	0.000590
Non-ferrous metal	0.0100	160	0.000063
Ferrous metal	0.0400	320	0.000125
Dirt, ashes	0.0100	480	0.000021
Total	**1.0000**		**0.009451**

Step 1: Average density of solid waste

The average density is calculated using the weighted harmonic mean of individual component densities:

$$\frac{1}{\rho_{avg}} = \sum \frac{w_i}{\rho_i}$$

From the table, summing w_i/ρ_i for all components:

$$\sum \frac{w_i}{\rho_i} = 0.009451$$

$$\rho_{avg} = \frac{1}{0.009451} = 105.81 \text{ kg/m}^3$$

Step 2: Total mass of waste produced in 25 years

Total mass = Population × Waste generation rate × Days per year × Design period

Total mass = 200,000 × 0.8 × 365 × 25

Total mass = 1,460,000,000 kg

Step 3: Volume of uncompacted waste

$$V_{\text{uncompacted}} = \frac{\text{Total mass}}{\rho_{\text{avg}}}$$

$$V_{\text{uncompacted}} = \frac{1,460,000,000}{105.81} = 13,798,448.15 \text{ m}^3$$

Step 4: Volume of compacted solid waste

Average compaction ratio of the waste mixture:

$$\text{Compaction ratio} = \Sigma w_i \times \text{Compaction}_i$$

$$\text{Compaction ratio} = 4.148$$

$$V_{\text{compacted waste}} = \frac{V_{\text{uncompacted}}}{\text{Compaction ratio}}$$

$$\Rightarrow \quad V_{\text{compacted waste}} = \frac{13,798,448.15}{4.148} = 3,326,530.41 \text{ m}^3$$

Step 5: Total landfill volume

Including the ratio of fill to compacted waste:

$$V_{\text{landfill}} = V_{\text{compacted waste}} \times \text{Fill ratio}$$

$$V_{\text{landfill}} = 3,326,530.41 \times 1.65 = 5,488,775.18 \text{ m}^3$$

Step 6: Area required for landfill

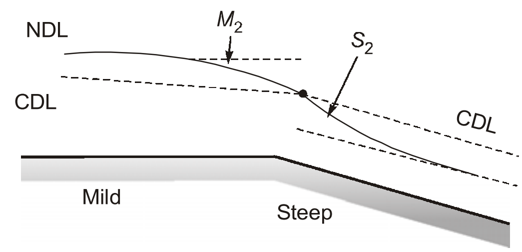
$$\text{Area} = \frac{V_{\text{landfill}}}{\text{Landfill height}}$$

$$\Rightarrow \quad \text{Area} = \frac{5,488,775.18}{12} = 457,397.93 \text{ m}^2$$

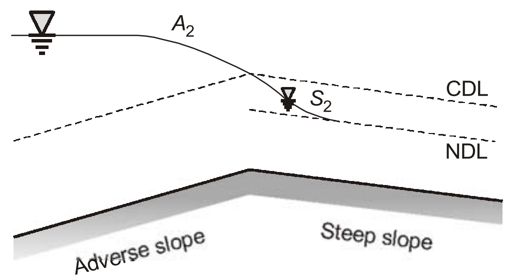
$$\text{Area in hectares} = \frac{457,397.93}{10,000} = 45.74 \text{ ha}$$

8. (b) (i) Solution:

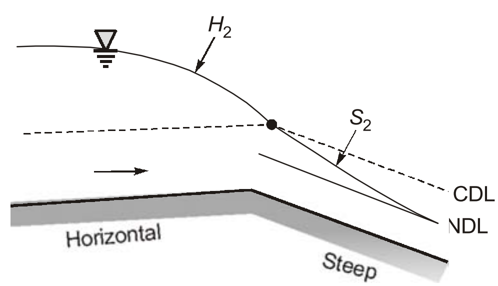
1.



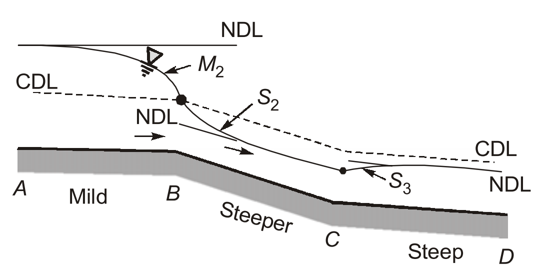
2.



3.



4.



8. (b) (ii) Solution:

Trapezoidal Rule

The formula for volume is:

$$V = \frac{d}{2}\times[(A_1 + A_7) + 2\times(A_2 + A_3 + A_4 + A_5 + A_6)]$$

$$\Rightarrow \quad V = \frac{40}{2}\times[(250+750)+2\times(380+460+520+610+680)]$$

$$\Rightarrow \quad V = 126000 \text{ m}^3$$

Simpson's 1/3rd Rule

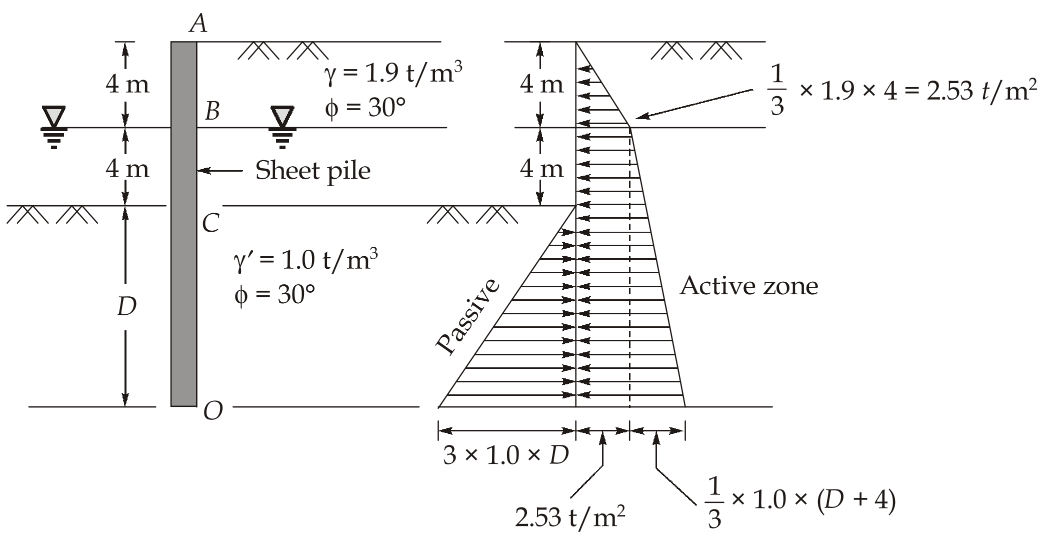
The formula for volume is:

$$V = \frac{d}{3} \times [(A_1 + A_7) + 4 \times (A_2 + A_4 + A_6) + 2 \times (A_3 + A_5)]$$

$$\Rightarrow \quad V = \frac{40}{3} \times [(250 + 750) + 4 \times (380 + 520 + 680) + 2 \times (460 + 610)]$$

$$\Rightarrow \quad V = 126133.333 \text{ m}^3$$

8. (c) (i) Solution:



Passive and active earth pressure coefficient,

$$K_p = \tan^2(45° + \phi/2) = \tan^2(60°) = 3$$

$$K_a = \frac{1}{K_p} = \frac{1}{3}$$

Taking moments of all forces about the point O

$$\frac{1}{2} \times 2.53 \times 4 \times \left(D + 8 - \frac{2}{3} \times 4\right) + 2.53 \times (D + 4) \times \frac{(D+4)}{2}$$

$$+\frac{1}{2} \times \left[\frac{1}{3} \times 1.0 \times (D+4)\right] \times (D+4) \times \frac{1}{3} \times (D+4) = \frac{1}{2} \times 3D \times D \times \frac{D}{3}$$

on simplification,

$$\Rightarrow \quad D = 9.7 \text{ m}$$

hence, the depth of embedment provided

$$= 1.3 \times 9.7$$

$$= 12.61 \text{ m}$$

8. (c) (ii) Solution:

$N_1 = 200$ rpm, $N_2 = 280$ rpm, $H_1 = 22$ m, $\eta = 0.85$, $P_1 = 4500$ kW

(a) $N_s = \dfrac{N_1\sqrt{P_1}}{H_1^{5/4}} = \dfrac{200\sqrt{4500}}{22^{5/4}} = 281.584$

(b) $P_1 = \eta\gamma Q_1 H_1 = 0.85 \times 9.81 \times Q_1 \times 22 = 4500$

$\Rightarrow \quad Q_1 = 24.530 \text{ m}^3/\text{s}$

Since diameter is same: $\dfrac{N_1}{H_1^{1/2}} = \dfrac{N_2}{H_2^{1/2}}$

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \quad \Rightarrow H_2 = 22 \times \left(\frac{280}{200}\right)^2 = 43.12 \text{ m}$$

By similarity unit relationship for discharge:

$$Q_2 = Q_1\left(\frac{H_2}{H_1}\right)^{1/2} = 24.530 \times \sqrt{\frac{43.12}{22}} = 34.342 \text{ m}^3/\text{s}$$

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