Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 12

Full Syllabus Test (Paper-I)

Section A

Q.1 (a) Solution:

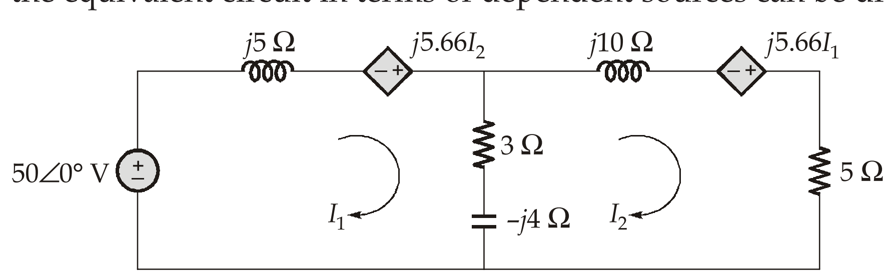
For a magnetically coupled circuit, the mutual inductance M is given by

$$M = K\sqrt{5(10)}$$

$$X_m = K\sqrt{X_{L_1}X_{L_2}}$$

$$= 0.8\sqrt{5(10)} = 5.66\ \Omega$$

Based on the dot convention, the current I_1 enters the dotted terminal of L_1 and I_2 leaves the dotted terminal of L_2. Thus, the mutually induced emf opposes the self-induced emf. Hence, the equivalent circuit in terms of dependent sources can be drawn as



Applying KVL to Mesh 1,

$$50\angle 0° - j5I_1 - 3(I_1 - I_2) + j4(I_1 - I_2) + j5.66I_2 = 0$$

$$50\angle 0° = (3 + j1)I_1 - (3 + j1.66)I_2 \quad ...(1)$$

$$(3 + j1)I_1 + (-3 - j1.66)I_2 = 50\angle 0°$$

Applying KVL to Mesh 2,

$$j4(I_2 - I_1) - 3(I_2 - I_1) - j10I_2 + j5.66I_1 - 5I_2 = 0$$

$$j4\, I_2 - j4\, I_1 - 3I_2 + 3I_1 - j10I_2 + j5.66I_1 - 5I_2 = 0$$

$$-j4I_2 + j4I_1 + 3I_2 - 3I_1 + j10I_2 - j5.66I_1 + 5I_2 = 0$$

$$(-3 - j1.66)I_1 + (8 + j6)I_2 = 0 \quad ...(2)$$

By Cramer's rule,

$$I_2 = \frac{\begin{vmatrix} 3+j1 & 50\angle 0° \\ -3-j1.66 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j1 & -3-j1.66 \\ -3-j1.66 & 8+j6 \end{vmatrix}}$$

$$= \frac{150+j83}{(18+j26)-(6.244+j9.96)} = \frac{150+j83}{11.756+j16.04}$$

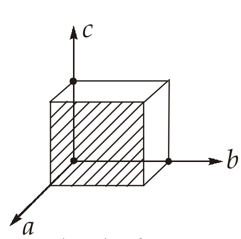
$$= 8.62\ \angle{-24.79°}\text{ A}$$

Thus, $V = 5I_2 = 5(8.62\ \angle{-24.79°}) = 43.1\ \angle{-24.79°}\text{ V}$

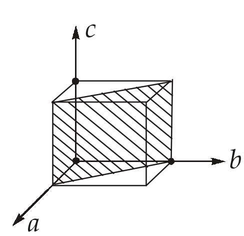
Q.1 (b) Solution:

The Miller indices (*hkl*) are formally defined as the reciprocals of the fractional intercepts that a plane makes with the crystallographic axes (x, y, z). Thus,

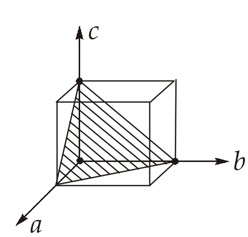
- The (100) plane intersects the x-axis at 1 and is parallel to the y and z axes, effectively forming a face of the unit cell.
- The (110) plane intersects the x and y axis at 1 and is parallel to the z-axis, forming a diagonal plane through the unit cell.
- The (111) plane intersects the x, y, and z axes all at 1, forming a triangular cross-section as shown below.



(100) Plane



(110) Plane



(111) Plane

Since (100) plane represents a face of the unit cell, there are six equivalent (100) planes in a cubic crystal: $(100), (\bar{1}00), (010), (0\bar{1}0), (001), (00\bar{1})$

In a face – centerd cubic structure.

$$a\sqrt{2} = 4r$$

where a = side of unit cell

r = ionic radius

Given : $r = 1.06$ Å

$$a = \frac{4r}{\sqrt{2}} = \frac{4\times 1.06}{\sqrt{2}} = 2.998 \text{ Å}$$

The interplanar spacing for a cubic lattice with given Miller indices (hkl) is given by,

$$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

Thus, Inter planar separation for (111) planes is

$$d = \frac{a}{\sqrt{3}} = \frac{2.998}{\sqrt{3}} = 1.731 \text{ Å}$$

Q.1 (c) Solution:

When two coils are connected in series aiding configuration,

$$L_{additive} = L_1 + L_2 + 2M = 0.60 \text{ H} \quad ...(i)$$

$$L_{subtractive} = L_1 + L_2 - 2M = 0.40 \text{ H} \quad ...(ii)$$

When two coils are connected in series opposing configuration,

By adding equations (i) and (ii)

$$2(L_1 + L_2) = 1.0$$

$$L_1 + L_2 = 0.5$$

Given, $L_1 = 0.15$ H

∴ $L_2 = 0.5 - 0.15 = 0.35$ H

(i) Substituting the values of L_1 and L_2 in equation (i)

$$2M = 0.60 - 0.50$$

$$M = \frac{0.10}{2} = 0.05 \text{ H}$$

(ii) The coefficient of coupling, $K = \dfrac{M}{\sqrt{L_1 \cdot L_2}} = \dfrac{0.05}{\sqrt{0.15\times 0.35}}$

$$K = 0.22$$

Q.1 (d) Solution:

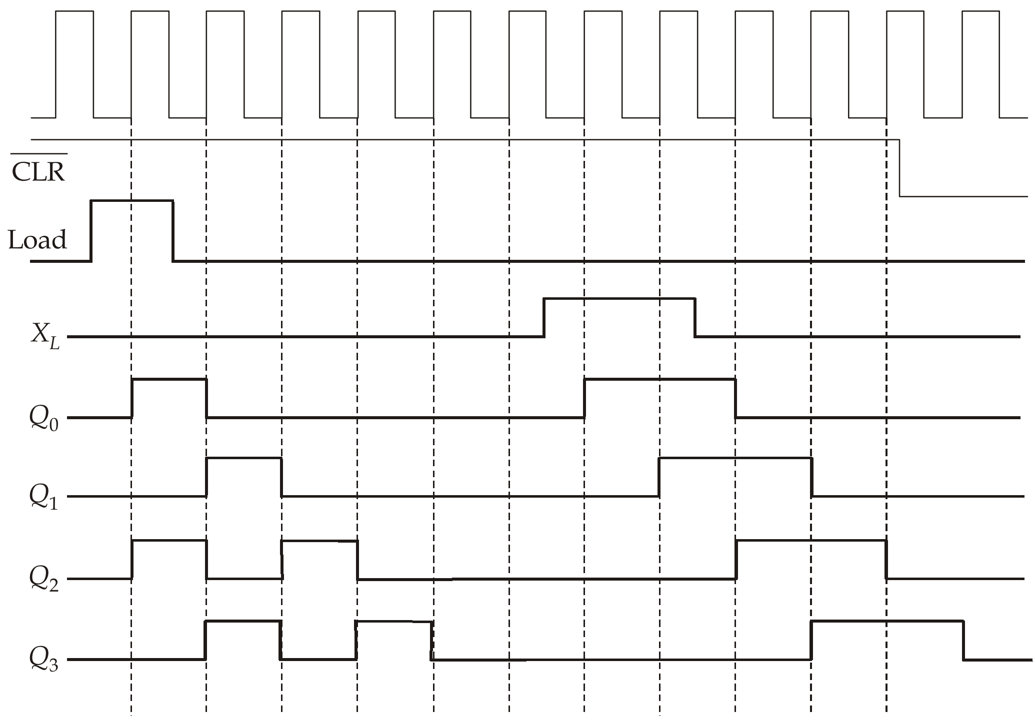
The circuit is a 4-bit shift register with a parallel load feature controlled by 2:1 MUX.

–When **Load = 1**, the MUX selects the external inputs X_i. The next state becomes $Q_i = X_i$.

–When **Load = 0**, it works as a right-shift register and the serial input to Flip-flop 1 is X_L.

–When $\overline{\textbf{CLR}} = \textbf{0}$, the flip-flop outputs are set to 0.

Inputs			Next State			
CK	$\overline{CLR}$	Load	Q_3	Q_2	Q_1	Q_0
X	0	X	0	0	0	0
↑	1	0	0	0	0	0
↑	1	1	0	1	0	1



Q.1 (e) Solution:

(i) Each transistor is biased at an emitter current of 0.5 mA. Thus,

$$r_{e1} = r_{e2} = \frac{V_T}{I_E} = \frac{25\text{ mV}}{0.5\text{ mA}} = 50\ \Omega$$

In differential analysis, as the circuit is symmetric and $V_1 = -V_2$, the node joining two emitter resistances can be assumed as AC ground. Thus, R_{EE} would be dead resistor. The input differential resistance can now be found as

$$R_{id} = 2(\beta + 1)(r_e + R_E)$$
$$= 2 \times 101 \times (50 + 150) \cong 40\text{ k}\Omega$$

(ii) From the given circuit,

$$\frac{v_{id}}{v_{sig}} = \frac{R_{id}}{R_{sig} + R_{sig} + R_{id}}$$

$$= \frac{40}{5+5+40} = 0.8 \text{ V/V}$$

The voltage gain from the bases to the output is

$$\frac{v_0}{v_{id}} \cong \frac{\text{Total resistance in the collectors}}{\text{Total resistance in the emitters}}$$

$$= \frac{2R_C}{2(r_e + R_E)} = \frac{2\times 10}{2(50+150)\times 10^{-3}} = 50 \text{ V/V}$$

The overall differential voltage gain can now be found as

$$A_d = \frac{v_0}{v_{sig}} = \frac{v_{id}}{v_{sig}}\frac{v_0}{v_{id}} = 0.8\times 50 = 40 \text{ V/V}$$

(iii) We have,
$$A_{cm} = \frac{V_{02}}{V_{cm}} - \frac{V_{01}}{V_{cm}} = -\frac{R_{C2}}{2R_{EE}} + \frac{R_{C1}}{2R_{EE}}$$

Common mode gain in worst case is equal to maximum possible common mode gain. A_{CM} is maximum if $R_{C1} = R_1 + \Delta R_C$ and $R_{C2} = R - \Delta R_C$. Thus,

$$A_{cm} = \frac{2\Delta R_C}{2R_{EE}} = \frac{R_C}{R_{EE}}\frac{\Delta R_C}{R_C}$$

Given $\Delta R_C / R_C = 0.01$. Thus,

$$A_{cm} = \frac{10}{200}\times 0.01 = 5\times 10^{-4} \text{ V/V}$$

(iv)
$$\text{CMRR} = 20\log\frac{A_d}{A_{cm}}$$

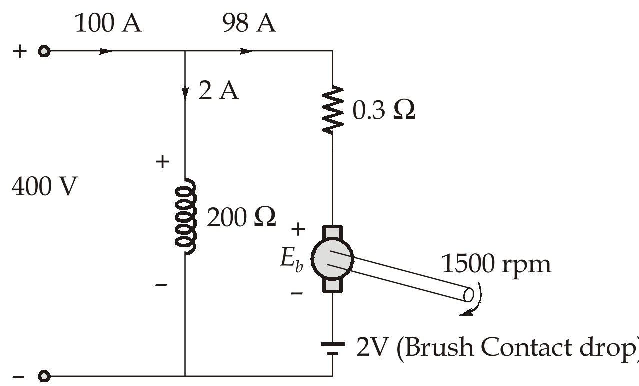
$$= 20\log\frac{40}{5\times 10^{-4}} = 98 \text{ dB}$$

(v) For transistor,
$$r_0 = \frac{V_A}{I/2} = \frac{100}{0.5\times 10^{-3}} = 200 \text{ k}\Omega$$

Using eq.,
$$R_{icm} = (\beta+1)\left(R_{EE} \parallel \frac{r_0}{2}\right)$$

$$= 101(200 \text{ k}\Omega \parallel 100 \text{ k}\Omega) = 6.7 \text{ M}\Omega$$

Q.2 (a) Solution:



$$\text{Field current} = I_f = \frac{400}{200} = 2 \text{ A}$$

At full load, armature current,

$$I_{a_1} = I_L - I_f = 98 \text{ A}$$

By applying KVL:

$$-400 + (98 \times 0.3) + E_b + 2 = 0$$

$$E_{b1} = 400 - (98 \times 0.3) - 2 = 368.6 \text{ V}$$

For DC motor, $E_b \propto N\phi$. Thus,

$$E_b \propto N \quad (\text{as } \phi \text{ is constant})$$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} \quad \text{...(i)}$$

(i) At half load, $I_L = 50$ A

$\therefore$ $I_{a_2} = I_L - I_f = 50 - 2 = 48$ A

$$E_{b2} = 400 - (48 \times 0.3) - 2 = 383.6 \text{ V}$$

Substituting all values in equation (i), we get

$$\frac{368.6}{383.6} = \frac{1500}{N_2}$$

$$N_2 = \frac{1500 \times 383.6}{368.6} = 1561 \text{ rpm}$$

(ii) At 150% of full load, $I_L = 150$ A

$$I_{a_3} = 150 - 2 = 148 \text{ A}$$

$$E_{b3} = 400 - (148 \times 0.3) - 2 = 353.6 \text{ V}$$

Substituting all values in equation (i)

$$\frac{368.6}{353.6} = \frac{1500}{N_3}$$

$$N_3 = \frac{1500 \times 353.6}{368.6} = 1439 \text{ rpm}$$

Q.2 (b) Solution:

We have; $W = 2L = 5\ \mu m$

i.e., $L = 2.5\ \mu m$

$\therefore \quad \frac{W}{L} = 2$

DC Analysis: For DC, the capacitors act as open circuit. Since, no current flows through gate, hence current through $R_G = 10\ M\Omega$ is zero. We have, $V_{DS} = V_{GS}$, hence the transistor is in saturation. Therefore,

$$I_D \cong \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

$\because \quad V_{GS} = V_{DS} = V_D = V_0$

$$I_D \cong \frac{1}{2} \times 0.125 \times 2 \times (V_D - 1.5)^2$$

but $V_D = 15 - I_D \cdot R_D = 15 - 10I_D$

$\therefore \quad 8I_D = [15 - 10I_D - 1.5]^2$

$$100I_D^2 - 278I_D + 182.25 = 0$$

On solving, we get $I_D = 1.06$ mA, 1.72 mA

$\therefore \quad I_D = 1.06$ mA ..... for MOS to be in saturation region.

[For $I_D = 1.72$ mA, $V_{GS} = V_D = 15 - 10\ I_D < V_T$]

Thus, $V_D = 15 - (10 \times 1.06) = 4.4$ V

The small signal parameters of the MOSFET can be obtained as below,

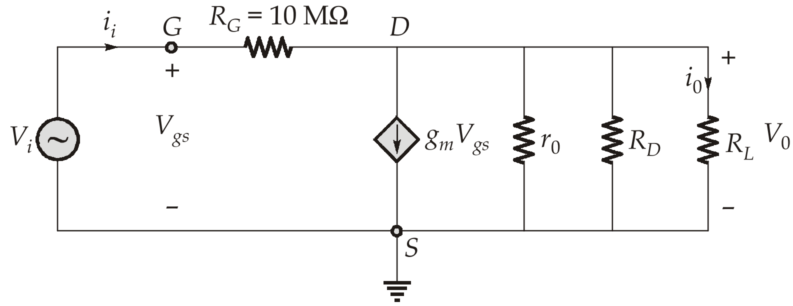
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)$$

$$= 0.25 \times (4.4 - 1.5)$$

$$= 0.725 \text{ mA/V}$$

and $$r_0 = \frac{V_A + V_D}{I_D} = \frac{50 + 4.4}{1.06} = 51.32\ k\Omega \quad \left(V_A = \frac{1}{\lambda} = 50 \text{ V}\right)$$

Small signal model of amplifier is given as:



(i) KCL at node 'D':

$$\frac{V_i - V_0}{R_G} = g_m V_{gs} + \frac{V_o}{R_x}$$

where $R_x = r_o \,||\, R_D \,||\, R_L = 51.32 \,||\, 10 \,||\, 10 = 4.55\ \text{k}\Omega$

$\because$ $V_i = V_{gs}$

we get, $$V_i\left(\frac{1}{R_G} - g_m\right) = V_0\left(\frac{1}{R_G} + \frac{1}{R_x}\right)$$

$$A_V = \frac{V_0}{V_i} = \frac{\frac{1}{R_G} - g_m}{\frac{1}{R_G} + \frac{1}{R_x}} = \frac{\frac{1}{10\times 10^3} - 0.725}{\frac{1}{10\times 10^3} + \frac{1}{4.55}}$$

$$= -\frac{0.725}{0.22} = -3.3$$

(ii) To get input resistance:

$$i_i = \frac{V_i - V_0}{R_G} = \frac{V_i - V_i(A_V)}{R_G}$$

$$i_i = \frac{V_i(1 - A_V)}{R_G}$$

$$R_i = \frac{V_i}{i_i} = \frac{R_G}{1 - A_V} = \frac{10}{1 + 3.3} = 2.32\ \text{M}\Omega$$

(iii) Current gain:

Apply KCL at drain:

$$i_i = g_m V_{gs} + \frac{V_0}{R_x}$$

$\because$ $V_{gs} = V_i = i_i R_G + V_0$

$$i_i = g_m(i_i R_G + V_0) + \frac{V_0}{R_x}$$

But $$V_0 = i_0 \cdot R_L$$

Hence, $$\frac{i_0}{i_i} = \frac{1 - g_m R_G}{g_m R_L + \frac{R_L}{R_x}} = \frac{1 - 0.725 \times 10^4}{0.725 \times 10 + \frac{10}{4.55}}$$

∴ Current gain (A_I) = –767.27

Q.2 (c) Solution:

There are three types of cubic crystal structures:

- **Simple Cubic Crystal structure (SCC):**

  In this structure, there is one Lattice point at each of the eight corners of the unit cell. It has a coordination number of six.

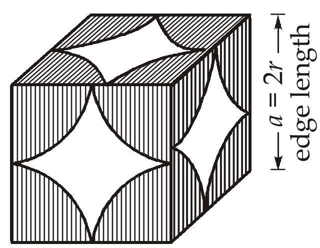
  Each corner atom contributes (1/8) atom to the unit cell. Hence,

  $$\text{Number of atoms/unit cell} = \frac{1}{8} \times 8 = 1$$

  For simple cubic crystal,

  $$a = 2r$$

  where a is the unit cell radius and r is the radius of atom.



  Atomic Packing Factor (APF) of the crystal structure is defined as the ratio of total volume of the atoms per unit cell to the volume of the unit cell. It is also known as packing efficiency (η)

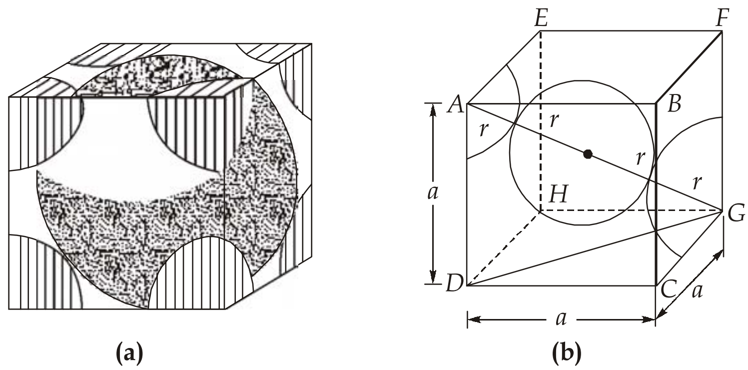
  Here, $$\text{APF} = \frac{\text{Number of atoms/unit cell} \times \text{Volume of atom}}{\text{Volume of unit cell}}$$

  $$= \frac{1 \times \frac{4}{3}\pi r^3}{a^3} = \frac{1 \times (\pi/6) a^3}{a^3} = \frac{\pi}{6} = 0.524$$

∴ % APF = 52.4% filled

- **Body Centered Cubic structure (BCC):**

  In this structure, in an unit cell, there are eight atoms at corners and another atom is at the body center. It has a coordination number of eight.



Body centered cubic structure

From the figure, $GD = \sqrt{a^2 + a^2} = \sqrt{2}a$

Along the body diagonal,

$$AG^2 = AD^2 + GD^2 \Rightarrow (4r)^2 = a^2 + 2a^2$$

$$r = \frac{a\sqrt{3}}{4}$$

Number of atoms/unit cell = $\frac{1}{8} \times 8 + 1 = 2$

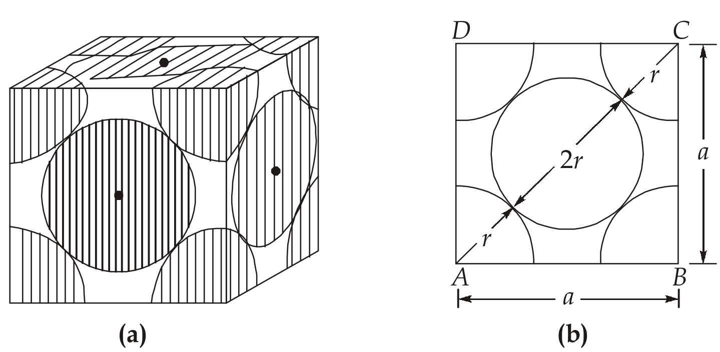
Now, $$(\text{APF})_{BCC} = \frac{N \times \text{Volume of each atom}}{\text{Total volume of unit cell}}$$

$$= \frac{2 \times \frac{4}{3}\pi\left(\frac{a\sqrt{3}}{4}\right)^3}{a^3} = \frac{\frac{2 \times \pi\sqrt{3}}{16}a^3}{a^3} = \frac{\pi\sqrt{3}}{8} = 0.68$$

$\therefore$ % APF = 68% filled

- **Face Centered Cubic structure (FCC):**

  In this structure, one atom lies at each corner of the cube in addition to one atom at the center of each face. The coordination number of FCC structure is (4 + 4 + 4 = 12)



Face centered cubic structure

From the figure, $a^2 + a^2 = (4r)^2$

Radius of atom, $= r = \frac{a\sqrt{2}}{4}$

and for FCC,

Number of atoms/unit cell = $\frac{1}{8}\times 8 + \frac{1}{2}\times 6 = 4$

Now, $(\text{APF})_{\text{FCC}} = \frac{N \times \text{Volume of each sphere}}{\text{Total volume of each cell}}$

$$= \frac{4\times\frac{4}{3}\pi\times\left(\frac{a\sqrt{2}}{4}\right)^3}{a^3} = \frac{4\times\frac{\sqrt{2}\pi a^3}{24}}{a^3} = \frac{\sqrt{2}\pi}{6} = \frac{\pi}{3\sqrt{2}} = 0.74$$

∴ % APF = 74 % filled

Q.3 (a) Solution:

(i) Given:

Total current, $I_{\text{total}} = 60$ mA

Dark current, $I_d = 1.5$ mA

Light intensity, $E = 10$ W/m^2

Circular window diameter, $d = 10$ mm $= 0.01$ m

Radius $r = \frac{d}{2} = 5\text{ mm} = 5\times 10^{-3}\text{ m}$

1. Photocurrent,

$$I_{ph} = I_{\text{total}} - I_d$$

$$I_{ph} = 60 - 1.5 = 58.5 \text{ mA}$$

2. Power of the incident light

Area(A) of photodiode window given as circular,

$$A = \pi r^2$$

$$A = \pi(5 \times 10^{-3})^2$$

$$A = \pi \times 25 \times 10^{-6} = 7.854 \times 10^{-5} \text{ m}^2$$

Power incident on the photodiode:

$$P = E \times A$$

$$P = 10 \times 7.854 \times 10^{-5}$$

$$P = 7.854 \times 10^{-4} \text{ W}$$

$$P = 0.785 \text{ mW}$$

3. Responsivity,

$$R = \frac{I_{ph}}{P}$$

Converting current into ampere,

$$I_{ph} = 58.5 \text{ mA} = 0.0585 \text{ A}$$

$$\therefore \quad R = \frac{0.0585}{7.854\times10^{-4}}$$

$$R \approx 74.5 \text{ A/W}$$

(ii) From part (i) , the results are:

$$\text{Incident optical power, } P = 7.854 \times 10^{-4} \text{ W}$$

$$\text{Photocurrent, } I_{ph} = 58.5 \text{ mA} = 0.0585 \text{ A}$$

Given, $\text{Wavelength, } \lambda = 510 \text{ nm} = 510 \times 10^{-9} \text{ m}$

1. Number of photons falling per second,

$$N = \frac{P}{E_{ph}}$$

where E_{ph}, Energy of one photon,

$$E_{ph} = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{510\times10^{-9}}$$

$$E_{ph} = 3.90 \times 10^{-19} \text{ J}$$

Therefore, $$N = \frac{7.854\times10^{-4}}{3.90\times10^{-19}}$$

$$N \approx 2.01 \times 10^{15}$$

2. Quantum efficiency

Number of electrons per second

$$n_e = \frac{I_{ph}}{e} = \frac{0.0585}{1.6\times10^{-19}}$$

$$n_e = 3.66 \times 10^{17}$$

$$\text{Quantum efficiency, } \eta = \frac{\text{Electrons emitted per second}}{\text{Photons incident per second}} = \frac{n_e}{N}$$

$$\eta = \frac{3.66\times10^{17}}{2.01\times10^{15}} = 182$$

$$\text{Quantum efficiency} \approx 182 \ (\approx 1.82 \times 10^{2} \text{ or } 18200\%)$$

Note: A Quantum Efficiency (QE) greater than 1 indicates that each incident photon is producing multiple electrons.

Q.3 (b) Solution:

For the two-winding transformer, rated current for 11500 V winding

$$= \frac{100 \times 1000}{11500} = 8.69 \text{ A}$$

and rated current for 2300 V winding

$$= \frac{100 \times 1000}{2300} = 43.48 \text{ A}$$

It is to be noted that if the windings of the 2-winding transformer are connected in series to form an autotransformer, the rated currents are not exceeded.

In series aiding connection, the voltages of the two windings add together to produce a higher output voltage. There are two possible configurations for autotransformers:

(i) First Configuration (High-voltge winding as common winding)

The winding AB is for 2300 V and winding BC for 11500 V as shown in figure (a).

Here, $V_{AB} = 2300 \text{ V}, V_{BC} = 11500 \text{ V}$

$\therefore$ $V_H = V_{AB} + V_{BC} = 2300 + 11500 = 13800 \text{ V}$

$V_L = V_{BC} = 11500 \text{ V}$

Therefore, the voltage ratio for the autotransformer of figure (a) is

$$a_H = \frac{V_H}{V_L} = \frac{13800}{11500}$$

By KCL at point B,

$$I_L = I_{AB} + I_{CB} = 43.48 + 8.69 = 52.17 \text{ A}$$

The current distribution is shown in figure (a)

kVA of the autotransformer of 13800/11500 ratio:

$$= \frac{V_L I_L}{1000} = \frac{11500 \times 52.17}{1000} = 600 \text{ kVA}$$

or $$= \frac{V_H I_H}{1000} = \frac{13800 \times 43.48}{1000} = 600 \text{ kVA}$$

The weight of copper in a transformer is proportional to the volt-ampere rating of the windings. In an autotransformer, the saving in conductor material compared to a two-winding transformer is defined by the reciprocal of the transformation ratio. Thus, saving in conductor material

$$= \frac{1}{a_H} = \frac{11500}{13800} = 0.833 \text{ pu}$$

$$= 83.3 \text{ percent}$$

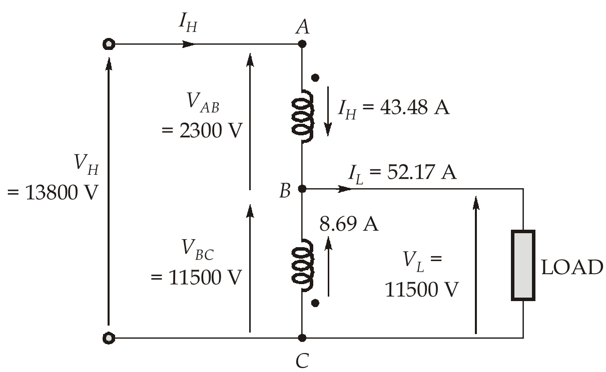


Fig. (a)

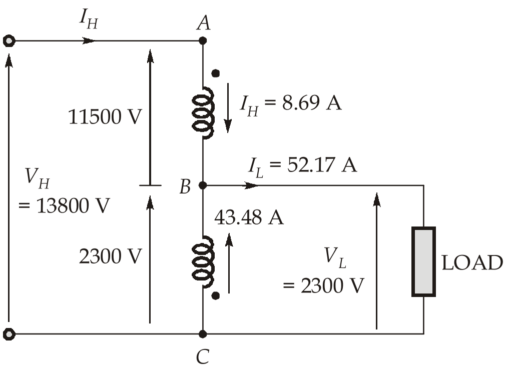


Fig. (b)

(ii) Second configuration (Low-voltage winding as common winding)

Here the winding *AB* is for 11500 V and *BC* for 2300 V, as shown in fig. (b). Therefore, V_{AB} = 11500 V and V_{BC} = 2300 V. Thus,

$$V_H = V_{AB} + V_{BC} = 11500 + 2300 = 13800 \text{ V}$$

$$V_L = V_{BC} = 2300 \text{ V}$$

$$a_L = \frac{V_H}{V_L} = \frac{13800}{2300}$$

By KCL at point *B*,

$$I_L = I_{AB} + I_{CB} = 8.69 + 43.48 = 52.17 \text{ A}$$

The current distribution is shown in figure. (b)

kVA of the autotransformer of 13800/2300 V ratio

$$= \frac{V_L I_L}{1000} = \frac{2300 \times 52.17}{1000} = 120 \text{ kVA}$$

$$= \frac{V_H I_H}{1000} = \frac{13800 \times 8.69}{1000} = 120 \text{ kVA}$$

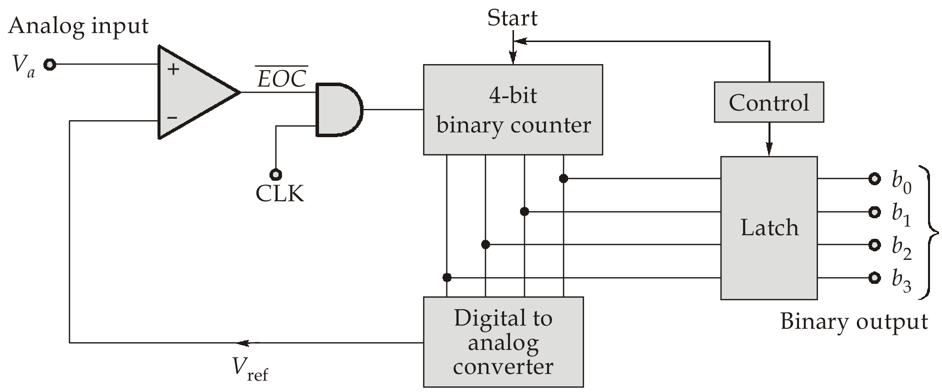
Saving in conductor material $= \frac{1}{a_L} = \frac{2300}{13800} = 0.166 \text{ pu} = 16.6 \text{ per cent}$

Q.3 (c) Solution:

(i) Single Slope Type ADC/Counter Type ADC:

- It is the simplest type of ADC which employs a binary counter, an analog comparator, a control circuit (an AND gate) and a DAC as shown in below figure.

- The counter type A/D converter is also known as a digital ramp ADC, because the waveform at the output of the DAC is a staircase waveform (step-by-step ramp).



Logic diagram of a counter type ADC

Operation:

- In a counter type ADC, the analog voltage (V_a) which is to be converted is applied to the non-inverting terminal of the comparator. The output from the DAC (V_{ref}) is applied to the inverting terminal of the Op-amp.
- The counter is used to count the number of clock pulses applied. A start pulse is applied to reset the counter to zero. Initially, V_{ref} = 0 V.
- As long as the analog input signal is greater than the reference voltage provided by DAC, the output of the comparator is HIGH (logic 1), the AND gate is enabled and, so, the clock pulses are transmitted to the counter and thus, V_{ref} is increased.
- When the output of DAC V_{ref} becomes greater than V_a, then the comparator output become LOW (logic 0) and the counter will stop counting. At this time, the output of the counter will provide the digital output proportional to the analog input. The control logic loads the binary count into the latches and resets the counter. Thus, beginning another count sequence to sample the input value.

Conversion Time:

The conversion time is the time interval between the starting of the conversion and the time the comparator output is LOW (stopping the counter). The maximum conversion time of a counter-type ADC is determined by the maximum count of the counter, which is (2^N – 1) for an n-bit counter. The maximum number of clock pulses required for N-bit conversion is (2^N – 1). Thus,

$$t_{c(max)} = (2^N - 1) \times t_{CLK}$$

$$\text{Average conversion time} = \frac{t_{c(max)}}{2}$$

where, N = number of bits in the counter.
Note that the conversion time depends upon input analog voltage V_a.

Disadvantages:

- It has large conversion time (essentially doubles for each bit that is added to the counter).
- The resolution can be improved only at the cost of a longer 't_c'.
- It is used for low speed application.

(ii) From circuit: $R_{eq} = R_1 + [R_2 \| 5]$

From voltage division rule:

$$V_0 = V_s \times \frac{R_2 \| 5}{1 + R_1 + R_2 \| 5}$$

$$\frac{V_0}{V_s} = \frac{R_2 \| 5}{1 + R_{eq}}$$

$$0.05 = \frac{\frac{R_2 \cdot 5}{R_2 + 5}}{1 + 39}$$

$$2 = \frac{5R_2}{R_2 + 5}$$

$$2R_2 + 10 = 5R_2$$

$$R_2 = \frac{10}{3} = 3.33\text{ k}\Omega$$

$\because$ $$R_{eq} = R_1 + [R_2 \| 5]$$

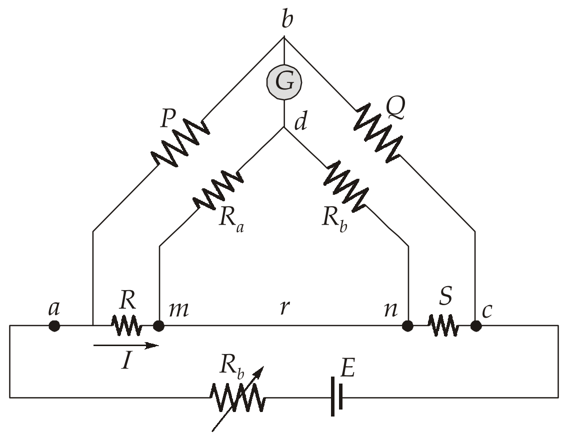
$$39 = R_1 + 2$$

$$R_1 = 37\text{ k}\Omega$$

Q.4 (a) Solution:

Kelvin's Double Bridge:

Resistances having a value under 1 Ω are known as low resistances. The resistance of leads and contacts, though small, are appreciable in comparison in the case of low resistances. Kelvin double bridge is a modification of the wheatstone bridge and overcomes these difficulties by eliminating the effects of lead and contact resistances.



The Kelvin's Double bridge incorporates the idea of a second set of ratio arms – hence the name double bridge and the use of four terminal resistors for the low resistance arms. The first of ratio arms is P and Q. The second set of ratio arms R_a and R_b is used to connect the galvanometer to a point d at the appropriate potential between points m and n to eliminate the effect of connecting lead of resistance r between the known resistance R, and the standard resistance S.

The ratio R_a/R_b is made equal to P/Q. Under balance conditions, there is no current through the galvanometer, which means that the voltage drop between a and b, E_{ab} is equal to the voltage drop E_{amd} between a and d.

Now, $$E_{ab} = \frac{P}{P+Q} \cdot E_{ac}$$

and $$E_{ac} = I\left[R + S + \frac{(R_a+R_b)r}{R_a+R_b+r}\right] \quad \text{... (i)}$$

and $$E_{amd} = I\left[R + \frac{R_a}{R_a+R_b}\left\{\frac{(R_a+R_b)r}{R_a+R_b+r}\right\}\right] = I\left[R + \frac{R_a r}{R_a+R_b+r}\right] \quad \text{... (ii)}$$

For zero galvanometer deflection, $E_{ab} = E_{amd}$ i.e.

$$\frac{P}{P+Q} \cdot I\left[R + S + \frac{(R_a+R_b)r}{R_a+R_b+r}\right] = I\left[R + \frac{R_a r}{R_a+R_b+r}\right]$$

or $$R = \frac{P}{Q} \cdot S + \frac{R_b r}{R_a+R_b+r}\left[\frac{P}{Q} - \frac{R_a}{R_b}\right] \quad \text{... (iii)}$$

Now if $\frac{P}{Q} = \frac{R_a}{R_b}$, equation (iii) becomes

$$R = \frac{P}{Q} \cdot S$$

As the expression of R does not involve r, hence the effect of r (contact resistance of leads) is eliminated, provided the two set of ratio arms have equal ratios.

For the given Kelvin Bridge, the two set of ratio arms should have equal ratios i.e.

$$\frac{R_4}{R_2} = \frac{R_b}{R_a}$$

Therefore,
$$\frac{R_4}{R_2} = \frac{R_b}{R_a} = \frac{1}{1000} \quad \text{(given)}$$

Given,
$$R_1 = 0.5\,R_2 \text{ and } R_1 = 5\ \Omega$$

Thus,
$$R_2 = \frac{5}{0.5} = 10\ \Omega$$

Therefore,
$$\frac{R_4}{10} = \frac{1}{1000}$$

$$R_4 = 10 \times \frac{1}{1000} = 0.01\ \Omega$$

Q.4 (b) Solution:

Observations:

- Input applied to gate (via C_{C1})
- Output taken from drain (via C_{C2})
- Source is AC grounded (through C_S)

Thus, the given circuit is a Common Source (CS) amplifier

DC Analysis (Q-point)

Given:

$$V_{TP} = -1.5 \text{ V}, K_p = 2 \text{ mA/V}^2$$

The gate is connected to ground through R_G. Therefore, $V_G = 0$

Thus,
$$V_{SG} = V_S - V_G = V_S$$

We have,

$$V_D = -9 + I_D R_D = -9 + 2I_D \quad \text{...(i)}$$

For pMOS to be in saturation,

$$V_{SD} \geq V_{SG} + V_{TP}$$
$$V_S - V_D \geq V_S - 0 + V_{TP}$$
$$V_D \geq -V_{TP} \Rightarrow V_D \leq 1.5 \text{ V}$$

Thus, the highest voltage that keeps the transistor in saturation region is,

$$V_{D\max} = 1.5 \text{ V}$$

If $V_D > 1.5$ V, the transistor enters the triode region. The lower limit of the drain voltage occurs when the transistor cuts off completely ($I_D = 0$). From equation (i),

$$V_{D\min} = -9 \text{ V}$$

Therefore the midpoint of the saturation region is:

$$V_{DQ} = \frac{V_{D\max} + V_{D\min}}{2} = \frac{1.5 - 9}{2} = -3.75 \text{ V}$$

We have,

$$V_{DQ} = -9 + 2I_{DQ} \quad \Rightarrow \quad -3.75 = -9 + 2I_{DQ}$$

$$I_{DQ} = 2.625 \text{ mA}$$

In saturation region,

$$I_D = \frac{K_P}{2}(V_{SG} - |V_{TP}|)^2 = (V_S - 1.5)^2 \text{ mA}$$

For $I_{DQ} = 2.625$ mA,

$$2.625 = (V_{SQ} - 1.5)^2 \quad \Rightarrow \quad \pm 1.62 = V_{SQ} - 1.5$$

Since $V_{SG} > |V_{TP}|$, thus

$$V_{SQ} = 3.12 \text{ V}$$

Small-Signal Parameters:

Transconductance (g_m)

$$g_m = \sqrt{2K_p I_{DQ}} = \sqrt{2 \times 2 \text{ mA/V}^2 \times 2.625 \text{ mA}} = 3.24 \text{ mS}$$

Output Resistance (r_o)

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{0.01 \times 2.625 \times 10^{-3}} = 38.1 \text{ k}\Omega$$

The voltage gain of the Common source amplifier is given by

$$A_v = -g_m (r_o \parallel R_D \parallel R_L)$$

Case 1: When R_L is Disconnected ($R_L = \infty$)

$$A_{v1} = -g_m (r_o \parallel R_D) = -3.24(38.1 \parallel 2) = -6.14$$

Case 2: When R_L is Connected ($R_L = 20$ kΩ)

$$A_{v2} = -g_m (r_o \parallel R_D \parallel R_L) = -3.24(38.1 \parallel 2 \parallel 20) = -5.62$$

Thus, $$\% \text{ change in gain} = \frac{|A_{v2}| - |A_{v1}|}{|A_{v1}|} \times 100$$

$$\% \text{ change in gain} = \frac{5.62 - 6.14}{6.41} \times 100 = 8.47\% \text{ decrease}$$

Q.4 (c) Solution:

(i) $R = 20\ \Omega$; $L = 200\ \mu H$; $f = 10^6$ Hz; $V = 230$ V; $R_S = 8000\ \Omega$

At $f = 10^6$ Hz, $X_L = 2\pi fL = 2 \times \pi \times 10^6 \times 200 \times 10^{-6} = 1256.6\ \Omega$

For a tank circuit, resonant frequency,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

To achieve resonance at 10^6 Hz,

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{200 \times 10^{-6} \times C} - \frac{(20)^2}{(200 \times 10^{-6})^2}}$$

$$C = 126.65 \times 10^{-12} \text{ F} = 126.65 \text{ pF}$$

Quality Factor $$Q_0 = \frac{2\pi f_0 L}{R}$$

$$= \frac{2\pi \times 10^6 \times 200 \times 10^{-6}}{20} = 62.83$$

Dynamic Impedance, $$Z = \frac{L}{CR}$$

$$= \frac{200 \times 10^{-6}}{126.65 \times 10^{-12} \times 20} = 78958\ \Omega$$

Total equivalent impedance of the circuit at resonance

$$Z_{eq} = 78958 + 8000 = 86958\ \Omega$$

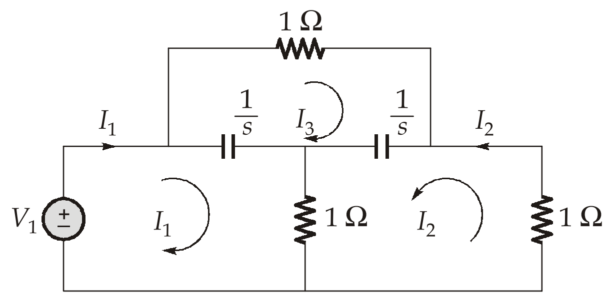
Total circuit current at resonance

$$\frac{V}{Z_{eq}} = \frac{230}{86958}$$

$$= 2.645 \times 10^{-3} \text{ A}$$

$$= 2.65 \text{ mA}$$

(ii) The transformed network in s-domain is shown in figure below:



Applying KVL to Mesh 1,

$$V_1 = \left(\frac{1}{s}+1\right)I_1 + I_2 - \frac{1}{s}I_3 \qquad \text{...(i)}$$

Applying KVL to Mesh 2,

$$0 = I_1 + \left(2+\frac{1}{s}\right)I_2 + \frac{1}{s}I_3 \qquad \text{...(ii)}$$

Applying KVL to Mesh 3,

$$0 = -\frac{1}{s}I_1 + \frac{1}{s}I_2 + \left(\frac{2}{s}+1\right)I_3$$

Writing these equations in matrix form,

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s}+1 & 1 & \frac{-1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ \frac{-1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Using Cramer's rule, $I_1 = \dfrac{\Delta_1}{\Delta}$

where $\Delta = \begin{vmatrix} \frac{1}{s}+1 & 1 & \frac{-1}{s} \\ 1 & 2+\frac{1}{s} & \frac{1}{s} \\ \frac{-1}{s} & \frac{1}{s} & \frac{2}{s}+1 \end{vmatrix}$

$$= \left(1+\frac{1}{s}\right)\left[\left(2+\frac{1}{s}\right)\left(1+\frac{2}{s}\right)-\frac{1}{s^2}\right]-1\left[(1)\left(1+\frac{2}{s}\right)+\frac{1}{s^2}\right]$$

$$-\frac{1}{s}\left[(1)\left(\frac{1}{s}\right)+\left(\frac{1}{s}\right)\left(2+\frac{1}{s}\right)\right]$$

$$= \frac{s^2+5s+2}{s^2}$$

and

$$\Delta_1 = \begin{vmatrix} V_1 & 1 & \frac{-1}{s} \\ 0 & 2+\frac{1}{s} & \frac{1}{s} \\ 0 & \frac{1}{s} & \frac{2}{s}+1 \end{vmatrix}$$

$$= V_1\left[\left(2+\frac{1}{s}\right)\left(1+\frac{2}{s}\right)-\frac{1}{s^2}\right] = V_1\left(\frac{2s^2+5s+1}{s^2}\right)$$

$$I_1 = V_1\left(\frac{2s^2+5s+1}{s^2+5s+2}\right)$$

Driving-point admittance

$$Y_{11}(s) = \frac{I_1}{V_1} = \frac{2s^2+5s+1}{s^2+5s+2}$$

We have,

$$I_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{s}+1 & V_1 & \frac{-1}{s} \\ 1 & 0 & \frac{1}{s} \\ \frac{-1}{s} & 0 & \frac{2}{s}+1 \end{vmatrix}$$

$$= -V_1\left[\frac{2}{s}+1+\frac{1}{s^2}\right] = -V_1\left(\frac{s^2+2s+1}{s^2}\right)$$

$$I_2 = -V_1\left(\frac{s^2+2s+1}{s^2+5s+2}\right)$$

Transfer admittance, $Y_{12}(s) = \dfrac{I_2}{V_1} = -\dfrac{s^2+2s+1}{s^2+5s+2}$

Section B

Q.5 (a) Solution

Given data:

Anode voltage, $E_a = 2000$ V

Length of deflecting plates, $l_d = 1.5$ cm $= 1.5 \times 10^{-2}$ m

Distance between deflecting plates, $d = 5$ mm $= 5 \times 10^{-3}$ m

Distance between screen and the center of the deflecting plates,

$$L = 50 \text{ cm} = 0.5 \text{ m}$$

(i) When electrons are accelerated through the anode voltage V_a, their potential energy is converted entirely into kinetic energy. Thus,

$$eV_a = \frac{1}{2}mv_{ox}^2$$

Velocity of beam, $$v_{ox} = \sqrt{\frac{2eE_a}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2000}{9.1\times10^{-31}}} = 26.5 \times 10^6 \text{ m/sec}$$

(ii) Deflection sensitivity, $$S = \frac{Ll_d}{2dE_a} = \frac{0.5\times1.5\times10^{-2}}{2\times5\times10^{-3}\times2000} = 0.375 \text{ mm/V}$$

(iii) Deflection factor, $$G = \frac{1}{S} = \frac{1}{0.375} = 2.66 \text{ V/mm}$$

Q.5 (b) Solution:

Absorption coefficient (α) of a semiconductor is the measure of how far into a material light of a particular wavelength can penetrate before it is absorbed.

It is defined as the fractional decrease in light intensity per unit distance inside the material given by Beer-Lambert's Law,

i.e., $$I(x) = I_0 e^{-\alpha x}$$

where, I_0 = incident light intensity

$I(x)$ = intensity after travelling distance 'x'

α = absorption coefficient (cm^{-1})

Given data:

Thickness, $t = 0.46$ μm $= 0.46 \times 10^{-4}$ cm

$h\nu = 3$ eV

$\alpha = 6 \times 10^4$ cm^{-1}

Incident power, $P_0 = 11$ mW $= 0.011$ W

(i) Total energy absorbed per second,

$$P_{abs} = P_0(1 - e^{-\alpha t})$$

where $\alpha t = 6 \times 10^4 \times 0.46 \times 10^{-4} = 2.76$

$\therefore$ $P_{abs} = 0.011[1 - e^{-2.76}] = 0.0103$ W

(or) $P_{abs} \simeq 10.3$ mW

So, energy absorbed per second = 10.3 mJ/s.

(ii) Each absorbed photon has energy,

$$h\nu = 3 \text{ eV}$$

For GaAs, $E_g = 1.43$ eV

Excess thermal energy per electron,

$$E_{\text{excess}} = h\nu - E_g$$
$$= 3 - 1.43$$
$$E_{\text{excess}} = 1.57 \text{ eV}$$

Fraction converted to heat,

$$\frac{E_{\text{excess}}}{h\nu} = \frac{1.57}{3} = 0.523$$

$\therefore$ Thermal power released,

$$P_{\text{thermal}} = 0.523 \times 10.3 \text{ mW} \simeq 5.4 \text{ mW}$$

Q.5 (c) Solution:

(i) From the figure, $I_4 = 40$ A ...(i)

Meshes 2 and 3 form a supermesh. Writing current equation for supermesh,

$$I_3 - I_2 = 5V_x$$

But $V_x = \frac{1}{5}(I_2 - I_1)$

$$I_3 = 2I_2 - I_1 \quad \text{...(ii)}$$

Applying KVL to supermesh,

$$-\frac{1}{5}(I_2 - I_1) - \frac{1}{20}I_2 - \frac{1}{15}I_3 - \frac{1}{2}(I_3 - I_4) = 0 \Rightarrow 12I_1 - 15I_2 - 34I_3 = -1200 \quad \text{...(iii)}$$

Applying KVL to Mesh 1,

$$-6 - \frac{1}{10}I_1 - \frac{1}{5}(I_1 - I_2) - \frac{1}{6}(I_1 - I_4) = 0 \Rightarrow 14I_1 - 6I_2 = 20 \quad \text{...(iv)}$$

Solving Eqs (i), (ii), (iii) and (iv), we get

$$I_1 = 10 \text{ A}$$

$$I_2 = 20 \text{ A}$$
$$I_3 = 2(20) - 10 = 30 \text{ A}$$
$$I_4 = 40 \text{ A}$$

(ii) Applying KVL to Mesh 1,

$$-96I_1 - (100 + 4 + j200)(I_1 - I_2) + 10\angle 0° = 0$$

$$(200 + j200)I_1 - (104 + j200)I_2 = 10\angle 0° \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-(1 - j50 + 100)I_2 - (100 + 4 + j200)(I_2 - I_1) = 0$$

$$-(104 + j200)I_1 + (205 + j150)I_2 = 0 \quad \text{...(ii)}$$

Writing equations in matrix form,

$$\begin{bmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$I_1 = \frac{\begin{vmatrix} 10\angle 0° & -(104+j200) \\ 0 & 205+j150 \end{vmatrix}}{\begin{vmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{vmatrix}}$$

$$I_1 = \frac{10(205+j150)}{(11000+j71000)-(-29184+j41600)}$$

$$= \frac{10(205+j150)}{40184+j29400}$$

$$I_1 \approx 5.102 \times 10^{-2}\angle 0° \text{A}$$

$$I_2 = \frac{\begin{vmatrix} 200+j200 & 10\angle 0° \\ -(104+j200) & 0 \end{vmatrix}}{\begin{vmatrix} 200+j200 & -(104+j200) \\ -(104+j200) & 205+j150 \end{vmatrix}} = \frac{10(104+j200)}{40184+j29400}$$

$$= 4.53 \times 10^{-2}\angle 26.33° \text{ A}$$

We have

$$V_{AB} = 100I_2 - (4 + j200)(I_1 - I_2)$$
$$= 4.53\angle 26.33° - (4 + j200) \times 10^{-2} \times (5.102 - 4.06 - j2)$$
$$= 4.53\angle 26.33° - 200.04\angle 88.85° \times (1.042 - j2)$$
$$= 4.53\angle 26.33° - 2\angle 88.85° \times 2.255\angle -62.48°$$
$$= 0$$

Q.5 (d) Solution:

In the matched transistor, the reverse saturation currents are equal. Applying KVL considering $v_1 = 0$ (virtual short).

$$v_2 = v_{BE2} - v_{BE1} \quad \text{...(i)}$$

A grounded BJT can also be utilized, since its emitter current and base-to-emitter voltage are related by

$$I_E = I_S e^{v_{BE}/V_T} \quad \text{...(ii)}$$

Taking the logarithm of both sides of Eq. (ii), we get

$$v_{BE} = V_T \ln \frac{I_E}{I_S} \quad \text{...(iii)}$$

Using equations (i) and (iii), with $I_C \approx I_E$,

$$v_2 = V_T \ln \frac{I_{E2}}{I_S} - V_T \ln \frac{I_{E1}}{I_S} = -V_T \ln \frac{I_{C1}}{I_{C2}} \quad \text{...(iv)}$$

According to Eq (i), v_2 is the difference between two small voltages. If V_R is several volts in magnitude, then $v_s << V_R$.

$$I_{C2} \approx I_{E2} = \frac{V_R - v_2}{R_2} \approx \frac{V_R}{R_2} \quad \text{...(v)}$$

$$I_{C1} \approx I_{E1} = \frac{v_s - v_1}{R_1} \approx \frac{v_s}{R_1} \quad \text{...(vi)}$$

From equation (iv), we get

$$v_2 = -V_T \ln\left(\frac{v_s R_2}{V_R R_1}\right) = V_T\left(\ln\left(\frac{R_2}{V_R R_1}\right) - \ln v_s\right)$$

The second op-amp is a non-inverting amplifier with,

$$v_0 = \left(1 + \frac{R_4}{R_3}\right) v_2$$

$$v_0 = \left(1 + \frac{R_4}{R_3}\right) V_T \left(\ln\left(\frac{R_2}{V_R R_1}\right) - \ln v_s\right)$$

Since R_1, R_2, R_3, R_4, V_R and V_T are constant, we can write

$$v_0 = K_1 - K_2 \ln v_s$$

Hence, the output voltage v_0 is proportional to $\ln v_s$.

Q.5 (e) Solution

The magnetization for a paramagnetic spin system with N spins is given by,

$$M = \frac{Np_B^2\mu_0 H}{kT}$$

Magnetization per spin in Bohr magnetons becomes

$$M' = \frac{M}{Np_B} = \frac{p_B \mu_0 H}{kT}$$

Where, p_B = Bohr magnetron

$= 9.27 \times 10^{-24}$ Am^2

$\because$ $\mu_0 = 4\pi \times 10^{-7}$ H/m

$H = 10^6$ A/m

k – Boltzmann's constant = 1.38×10^{-23} J/K

T – Temperature = 300 K (given)

So,

$$M' = \frac{9.27\times10^{-24}\times4\pi\times10^{-7}\times10^{6}}{1.38\times10^{-23}\times300}$$

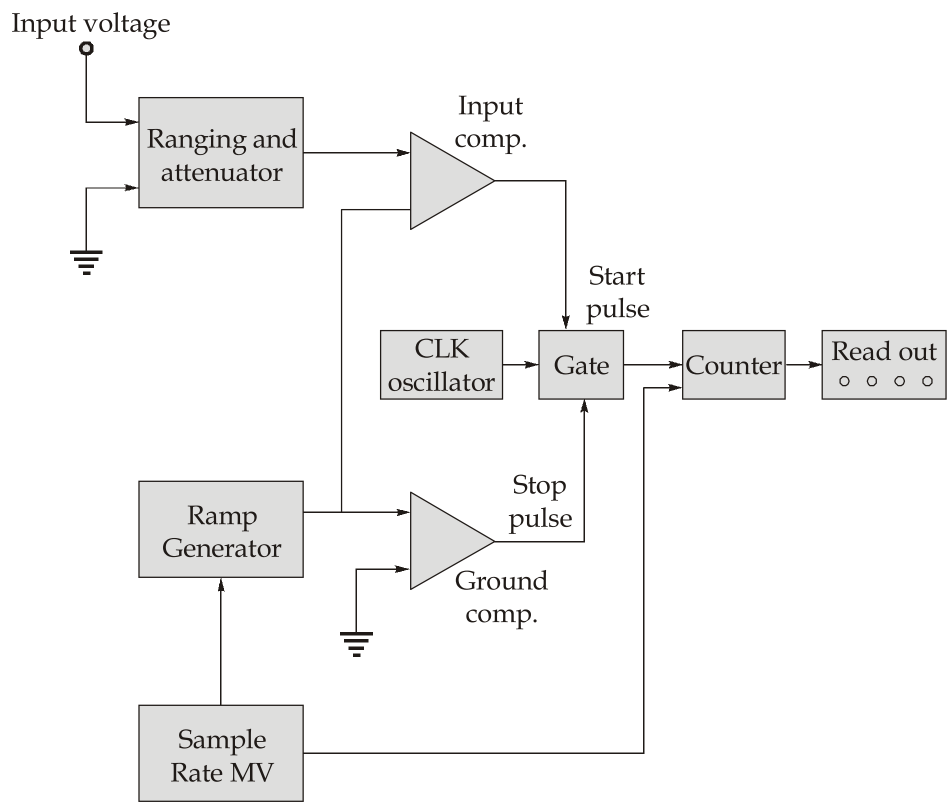
$= 2.81 \times 10^{-3}$ Bohr magneton

Q.6 (a) Solution:

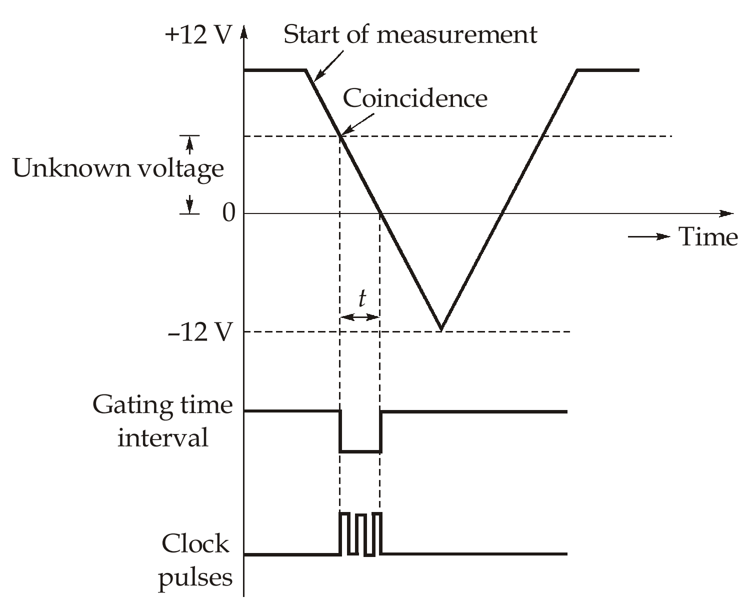
Ramp type DVM

The operating principle of a ramp type digital voltmeter is to measure the time that a linear ramp voltage takes to change from level of input voltage to zero voltage (or vice-versa). This time interval is measured with an electronic time interval counter and the count is displayed as a number of digits on electronic indicating tubes of the output readout of the voltmeter. At the start of the measurement, a ramp voltage is initiated (counter is reset to 0 and sample rate multivibrator gives a pulse which initiates the ramp generator). The ramp voltage value is continuously compared with the voltage being measured (unknown voltage).

At the instant the value of ramp voltage is equal to that of unknown voltage, a coincidence circuit, called an input comparator, generates a pulse which opens a gate. The ramp voltage continues to decrease till it reaches zero voltage. At this instant, another comparator called ground comparator generates a pulse and close the gate. The time elapsed between opening and closing of the gate is 't' as indicated in figure. During this time interval, pulses from a clock pulse generator pass through the gate and are counted and displayed. The count is a measure the magnitude of the input voltage, which is displayed by the readout. Therefore, the voltage is converted into time and the time count represents the magnitude of the voltage. The sample rate multivibrator determines the rate of cycle of measurement. The sample rate circuit provides an initiating pulse for the ramp generator to start its next ramp voltage. At the same time a reset pulse is generated, which resets the counter to the zero state.



Block diagram of a ramp DVM



Timing diagram showing voltage to time conversion

Q.6 (b) Solution:

(i) Oxide thickness $t_{ox} = 30 \text{ nm} = 3 \times 10^{-6} \text{ cm}$

Substrate doping $N_A = 10^{16} \text{ cm}^{-3}$

Flat-band voltage $V_{FB} = -2 \text{ V}$

Gate work function (Al)

$$\phi_m = 4.1 \text{ V}$$

Oxide capacitance $C_{ox} = \dfrac{\varepsilon_{ox}}{t_{ox}} = \dfrac{3.45\times10^{-13}}{3\times10^{-6}}$

$$C_{ox} = 1.15 \times 10^{-7} \text{ F/cm}^2 = 115 \text{ nF/cm}^2$$

Fermi potential $\phi_F = V_T \ln\left(\dfrac{N_A}{n_i}\right) = 0.026 \ln\left(\dfrac{10^{16}}{10^{10}}\right)$

$$= 0.026 \ln(10^6) = 0.026 \times 13.82$$

$$\phi_F \approx 0.36 \text{ V}$$

Threshold voltage $V_T = V_{FB} + 2\phi_F + \dfrac{\sqrt{2q\varepsilon_s N_A (2\phi_F)}}{C_{ox}}$

$$= -2 + 0.72 + \frac{\sqrt{2(1.6\times10^{-19})(1.04\times10^{-12})(10^{16})(0.72)}}{1.15\times10^{-7}}$$

$$V_T = -2 + 0.72 + 0.425$$

$\therefore$ $V_T \approx -0.86 \text{ V}$

Construct C-V curve (High-frequency)

Regions

1. Accumulation region ($V_G < V_{FB}$): Capacitance is maximum

$$C \approx C_{ox}$$

2. Flat-band voltage:

$$V_G = V_{FB} = -2 \text{ V}$$

3. Depletion region: Capacitance decreases with increasing gate voltage.
4. Threshold voltage, $V_T = -0.86$ V
5. Inversion Region: At low frequency, beyond the threshold voltage V_T, the MOS capacitance increases back toward the maximum oxide capacitance C_{ox} because minority carriers respond to the AC signal. At high frequency, the capacitance remains at the minimum value C_{min}. and does not increase.The low frequency and high frequency C-V characteristics curves of a MOS capacitor are shown.

To find the minimum capacitance $C_{\min}$ for the high-frequency MOS C-V curve, use:

$$C_{\min} = \frac{C_{ox}C_{d(\max)}}{C_{ox}+C_{d(\max)}}$$

where $C_{d(\max)}$ is the maximum depletion capacitance.

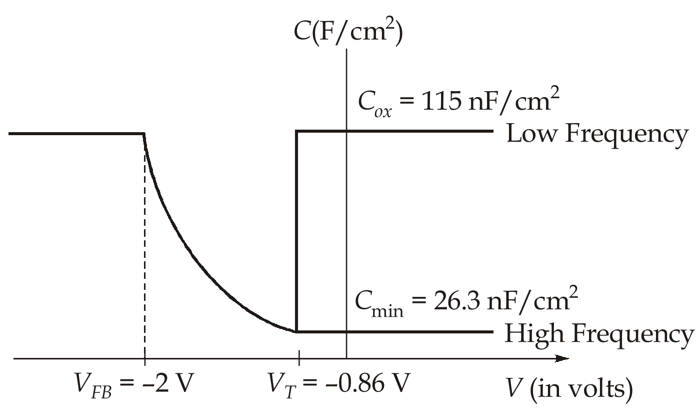
The maximum depletion width:

$$W_{d(\max)} = \sqrt{\frac{4\varepsilon_s\phi_F}{qN_A}} = \sqrt{\frac{4(1.04\times10^{-12})(0.36)}{(1.6\times10^{-19})(10^{16})}} = 3.06\times10^{-5}\text{cm}$$

Thus, $$C_{d(\max)} = \frac{\varepsilon_s}{W_{d(\max)}} = \frac{1.04\times10^{-12}}{3.06\times10^{-5}} = 3.40\times10^{-8}\text{ F/cm}^2$$

Hence, $$C_{\min} = \frac{(1.15\times10^{-7})(3.40\times10^{-8})}{1.15\times10^{-7}+3.40\times10^{-8}}$$

$$\approx 2.63\times10^{-8}\text{ F/cm}^2 = 26.3\text{ nF/cm}^2$$



(ii) We know that,

Flatband voltage,

$$V_{FB} = \phi_{ms} - \frac{Q_{ox}}{C_{ox}} \quad ...(i)$$

For Si:

$$\phi_s = \chi + \frac{E_g}{2} + \phi_F = 4.05+0.56+0.36 = 4.97\text{ V}$$

Work function difference,

$$\phi_{ms} = \phi_m - \phi_s = 4.1 - 4.97 = -0.87\text{ eV}$$

From equation (i), $$V_{FB} = -2\text{ V}$$

$$-2 = -0.87 - \frac{Q_{ox}}{C_{ox}}$$

$$\frac{Q_{ox}}{C_{ox}} = 1.13$$

$$Q_{ox} = 1.13 \times 1.15 \times 10^{-7}$$

$$Q_{ox} = 1.3 \times 10^{-7}\ \text{C/cm}^2$$

Q.6 (c) Solution:

(i) First, writing the complete incidence matrix A_a such that the sum of all the entries in each column of A_a is zero, we have,

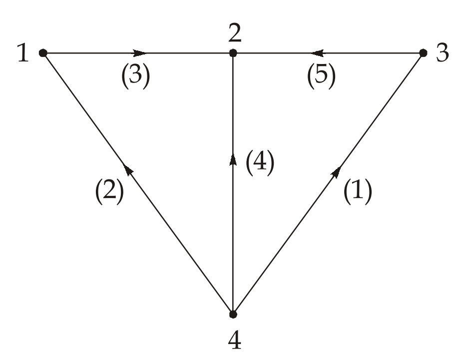
Nodes ↓, Branches ←

$$A_a = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 0 & 0 & -1 & -1 & -1 \\ 3 & -1 & 0 & 0 & 0 & 1 \\ 4 & 1 & 1 & 0 & 1 & 0 \end{array}$$

For the incidence matrix,

$$a_{ij} = \begin{cases} 1, \text{ if the branch } j \text{ is incident at node } i \text{ and is oriented away from node } i \\ -1, \text{ if the branch } j \text{ is incident at node } i \text{ and is oriented towards the node } i \\ 0, \text{ if the branch } j \text{ is not incident at node } i \end{cases}$$

The oriented graph can be drawn with the matrix A_a as,



The number of possible trees = $|AA^T|$

A tree is a connected graph containing no cycles. Consider the tree with branches {3, 4, 5}.

$$AA^T = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

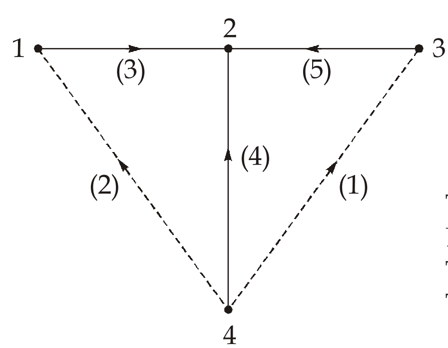
$$|AA^T| = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 2(6-1)+1(-2) = 8$$

The number of possible trees = 8.

Tieset Matrix (B)

A tieset is formed by adding a link to the tree to create a fundamental loop. The reference direction of the tie-set (loop current) is established by the direction of the link.

$$B = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 & 0 \end{bmatrix} \end{matrix}$$

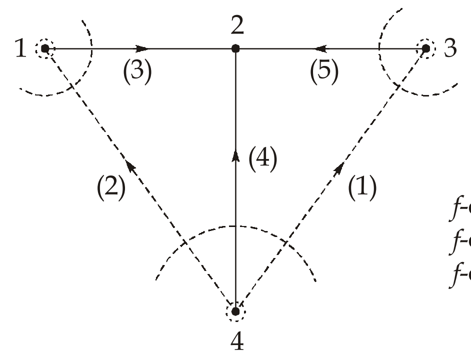


Twigs : [3, 4, 5]
Links: [1, 2]
Tieset 1: [1, 4, 5]
Tieset 2: [2, 3, 4]

f-cutset Matrix (Q)

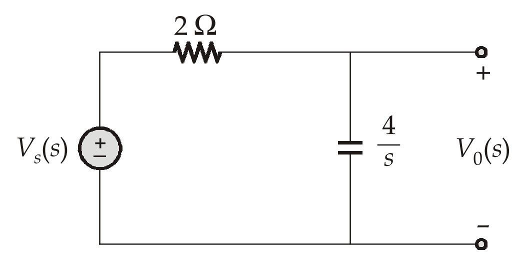
A cutset is formed by removing one tree branch and some links such that the removal of these branches causes the graph to be cut into exactly two parts. The reference direction of the cut-set follows the direction of the tree branch.

$$Q = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$



f-cutset 3: [3, 2]
f-cutset 4: [4, 2, 1]
f-cutset 5: [5, 1]

(ii) The transformed network is shown in figure below:



$$V_s(s) = \frac{1}{2}\frac{s}{s^2+1}$$

By voltage-division formula,

$$V_0(s) = V_s(s)\times\frac{\frac{4}{s}}{2+\frac{4}{s}} = \frac{2V_s(s)}{s+2} = \frac{s}{(s^2+1)(s+2)}$$

By partial-fraction expansion,

$$V_0(s) = \frac{As+B}{s^2+1}+\frac{C}{s+2}$$

$$s = (As + B)(s + 2) + C(s^2 + 1)$$

$$s = s^2(A + C) + s(2A + B) + (2B + C)$$

Comparing coefficients of s^2, s and s^0, we have

$$A + C = 0$$

$$2A + B = 1$$

$$2B + C = 0$$

Solving the equations, we get

$$A = 0.4$$
$$B = 0.2$$
$$C = -0.4$$

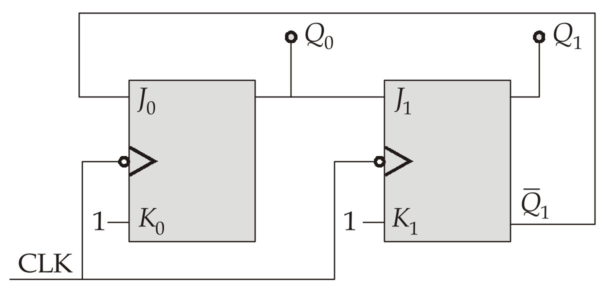
$$V_0(s) = \frac{0.4s + 0.2}{s^2+1} - \frac{0.4}{s+2} = \frac{0.4s}{s^2+1} + \frac{0.2}{s^2+1} - \frac{0.4}{s+2}$$

Taking the inverse Laplace transform,

$$v_0(t) = 0.4 \cos t + 0.2 \sin t - 0.4e^{-2t} \quad \text{for } t > 0$$

Q.7 (a) Solution:

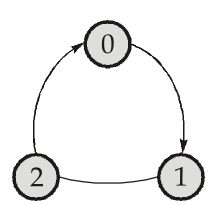
(i) Using the given information, the circuit can be obtained as below:



2-Stage synchronous counter

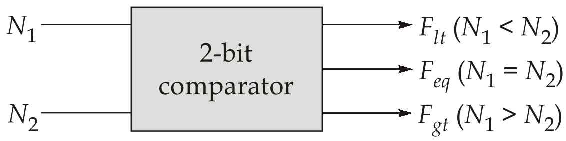
Present state		FF_1 inputs		FF_0 inputs		Next State	
Q_0	Q_0	J_1	K_1	J_0	K_0	Q_1	Q_0
0	0	0	1	1	1	0	1
0	1	1	1	1	1	1	0
1	0	0	1	0	1	0	0

So, state diagram of the counter can be drawn as below:



Since the counter cycles through 3 unique states, the given synchronous counter is 3 : 1 counter (MOD 3 counter).

(ii) Consider the below 2-bit comparator circuit,



N_1 and N_2 are two inputs for which the output of the comparator is specified by three binary variables that indicate whether,

$$N_1 > N_2,\ N_1 = N_2 \text{ or } N_1 < N_2$$

denoted by F_{gt}, F_{eq} and F_{gt} respectively. The truth table of the circuit is as shown below. From the truth table, we obtain

$$F_{gt} = \sum(m_4, m_8, m_9, m_{12}, m_{13}, m_{14})$$

$F_{gt} = 1$ if the MSB is strictly greater, or if the MSBs are equal and the LSB of N_1 is greater. Thus, we can write

$$F_{gt} = A_1\bar{B}_1 + (A_1 \odot B_1)\, A_0\bar{B}_0$$

Similarly,

$$F_{lt} = \sum(m_1, m_2, m_3, m_6, m_7, m_{11})$$

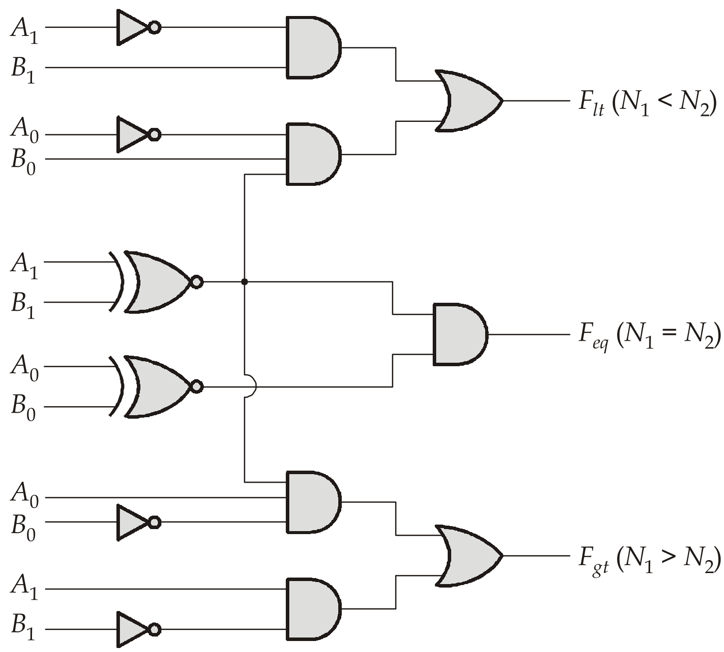
$$= \bar{A}_1 B_1 + (A_1 \odot B_1)\, \bar{A}_0 B_0$$

$$F_{eq} = \sum(m_0, m_5, m_{10}, m_{15})$$

$$= (A_1 \odot B_1)\,(A_0 \odot B_0)$$

Minterm	A_1	A_0	B_1	B_0	F_{gt}	F_{eq}	F_{lt}
m_0	0	0	0	0	0	1	0
m_1	0	0	0	1	0	0	1
m_2	0	0	1	0	0	0	1
m_3	0	0	1	1	0	0	1
m_4	0	1	0	0	1	0	0
m_5	0	1	0	1	0	1	0
m_6	0	1	1	0	0	0	1
m_7	0	1	1	1	0	0	1
m_8	1	0	0	0	1	0	0
m_9	1	0	0	1	1	0	0
m_{10}	1	0	1	0	0	1	0
m_{11}	1	0	1	1	0	0	1
m_{12}	1	1	0	0	1	0	0
m_{13}	1	1	0	1	1	0	0
m_{14}	1	1	1	0	1	0	0
m_{15}	1	1	1	1	0	1	0

Logic circuit:

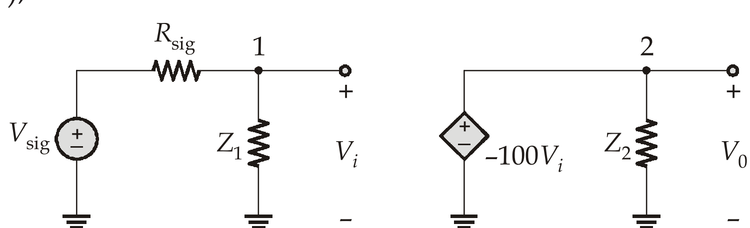


Q.7 (b) Solution:

(i) **1.** Given a circuit where $V_2 = KV_1$ and Z is connected between nodes V_1 and V_2, then using Miller's Theorem Z can be replaced by grounded impedances Z_1 and Z_2 at node V_1 and V_2 respectively where

$$Z_1 = \frac{Z}{1-K} \quad \text{and} \quad Z_2 = \frac{Z}{1-1/K}$$

For Z = 1 MΩ, employing Miller's theorem results in the equivalent circuit in Fig. (a), where



(a)

$$Z_1 = \frac{Z}{1-K} = \frac{1000\text{ k}\Omega}{1+100} = 9.9\text{ k}\Omega$$

$$Z_2 = \frac{Z}{1-\frac{1}{K}} = \frac{1\text{ M}\Omega}{1+\frac{1}{100}} = 0.99\text{ M}\Omega$$

The voltage gain can be found as follows:

$$\frac{V_0}{V_{sig}} = \frac{V_0}{V_i}\frac{V_i}{V_{sig}} = -100\times\frac{Z_1}{Z_1+R_{sig}}$$

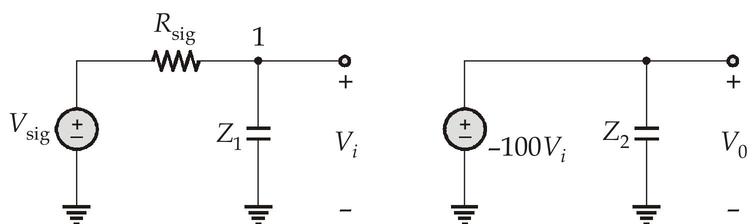
$$= -100\times\frac{9.9}{9.9+10} = -49.7\text{ V/V}$$

2. For Z as a 1-pF capacitance—that is, $Z = 1/sC$ — applying Miller's theorem allows us to replace Z by Z_1 and Z_2, where

$$Z_1 = \frac{Z}{1-K} = \frac{1/sC}{1+100} = \frac{1}{s(101C)}$$

$$Z_2 = \frac{Z}{1-\frac{1}{K}} = \frac{1}{1.01}\frac{1}{sC} = \frac{1}{s(1.01C)}$$

It follows that Z_1 is a capacitance with $C_1 = 101C = 101$ pF and that Z_2 is a capacitance with $C_2 = 1.01\,C = 1.01$ pF. The resulting equivalent circuit is shown in figure (b), from which the voltage gain can be found as follows:



(b)

$$\frac{V_0}{V_{sig}} = \frac{V_0}{V_i}\frac{V_i}{V_{sig}} = -100\frac{1/sC_1}{1/(sC_1)+R_{sig}}$$

$$= \frac{-100}{1+sC_1R_{sig}}$$

$$= \frac{-100}{1+s\times101\times1\times10^{-12}\times10\times10^{3}}$$

$$= \frac{-100}{1+s\times1.01\times10^{-6}}$$

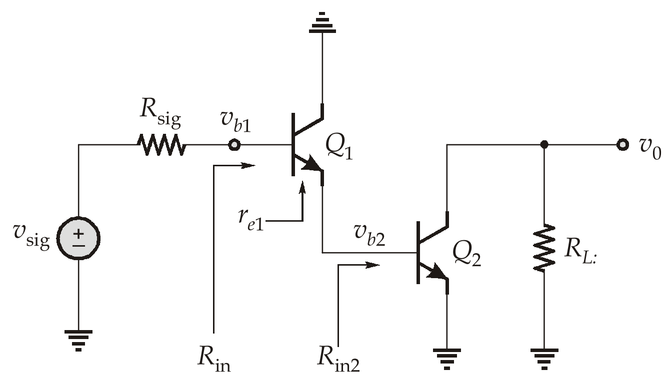
This is the transfer function of a first-order low-pass network with a dc gain of –100 and a 3-dB frequency f_{3dB} of

$$f_{3dB} = \frac{1}{2\pi\times1.01\times10^{-6}} = 157.6\text{ kHz}$$

(ii) At an emitter current of 1 mA, Q_1 and Q_2 have

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{25\text{ mV}} = 40\text{ mA/V}$$

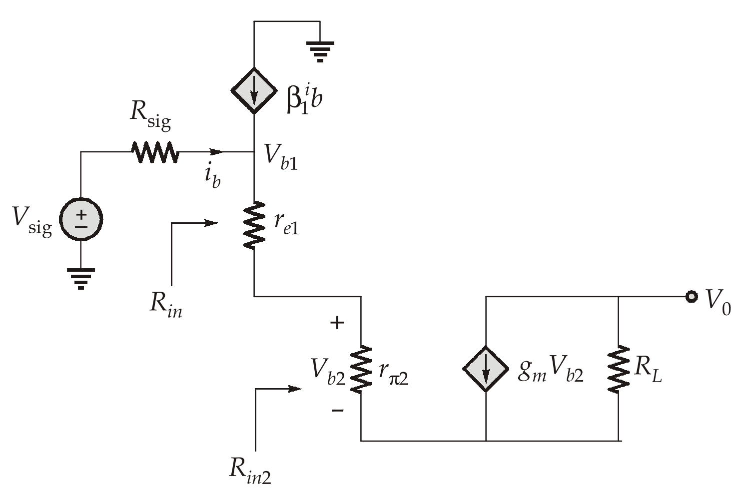
Considering the input signal source v_{sig} having resistance R_{sig} = 4 kΩ and the load resistance R_L = 4 kΩ, the ac equivalent circuit can be obtained as below:



We have,
$$r_e = \frac{1}{g_m} = \frac{1}{40\times10^{-3}} = 25\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5\text{ k}\Omega$$

Using the small signal model of BJT as shown below,



$$R_{in2} = r_{\pi2} = 2.5\text{ k}\Omega$$
$$R_{in} = (\beta_1 + 1)(r_{e1} + R_{in2})$$
$$= 101(0.025 + 2.5) \approx 255\text{ k}\Omega$$

Using voltage division rule,

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in}+R_{sig}} = \frac{255}{255+4} = 0.98 \text{ V/V}$$

and $$\frac{v_{b2}}{v_{b1}} = \frac{R_{in2}}{R_{in2}+r_{e1}} = \frac{2.5}{2.5+0.025} = 0.99 \text{ V/V}$$

Also, $$\frac{v_0}{v_{b2}} = -g_{m2}R_L = -40 \times 4 = -160 \text{ V/V}$$

Thus, $$G_v = \frac{v_0}{v_{sig}} = \frac{v_0}{v_{b2}} \times \frac{v_{b2}}{v_{b1}} \times \frac{v_{b1}}{v_{sig}}$$

$$= -160 \times 0.99 \times 0.98 \approx -155 \text{ V/V}$$

For comparison, a CE amplifier operating under the same conditions will have

$$R_{in} = r_\pi = 2.5 \text{ k}\Omega$$

$$G_v = \frac{R_{in}}{R_{in}+R_{sig}}(-g_m R_L)$$

$$= \frac{2.5}{2.5+4}(-40\times4)$$

$$= -61.5 \text{ V/V}$$

Q.7 (c) Solution:

Open-circuit test: The open-circuit test results are used to find the core loss components: magnetizing resistance (R_0) and reactance (X_0).

$$V_1 = 250 \text{ V}, I_0 = 1 \text{ A}, P_i = 80 \text{ W}$$

$$P_i = V_1 I_0 \cos\phi_0$$

$$\cos\phi_0 = \frac{P_i}{V_1 I_0} = \frac{80}{250\times1} = 0.32$$

$$I_W = I_0 \cos\phi_0 = 1 \times 0.32 = 0.32 \text{ A}$$

$$I_\mu = I_0 \sin\phi_0 = \sqrt{I_0^2 - I_W^2} - \sqrt{1^2-(0.32)^2} = 0.947 \text{ A}$$

$$R_0 = \frac{V_1}{I_W} = \frac{250}{0.32} = 781.25\ \Omega$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{250}{0.947} = 264\ \Omega$$

Short-circuit test

The SC test is used to determine the equivalent resistance R_e and leakage reactance X_e of the transformer. The results of the short-circuit test are given in terms of the h.v. side, while the results of the open-circuit test are in terms of low-voltage side. The results obtained in short-circuit test are, therefore, converted in terms of the l.v. side.

$$V_{2sc} = 20 \text{ V}, P_{cfl} = 100 \text{ W}, I_{2sc} = 12 \text{ A}$$

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} = \frac{250}{500} = 0.5$$

Short-circuit voltage referred to the l.v. side

$$V_{1sc} = V_{2sc}\frac{T_1}{T_2} = 20 \times \frac{250}{500} = 10 \text{ V}$$

Short-circuit current referred to the l.v. side

$$I_{1sc} = I_{2sc}\frac{T_2}{T_1} = 12 \times \frac{500}{250} = 24 \text{ A}$$

We have,

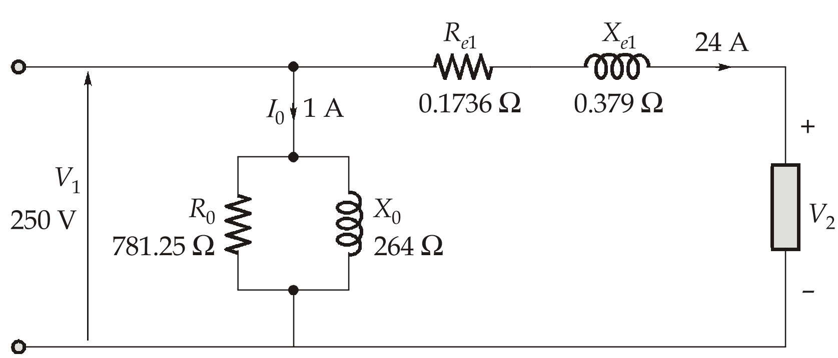
$$P_{cfl} = I_{1sc}^2 R_{e_1}$$

$$R_{e1} = \frac{P_{cfl}}{I_{1sc}^2} = \frac{100}{(24)^2} = 0.1736\ \Omega$$

$$Z_{e_1} = \frac{V_{1sc}}{I_{1sc}} = \frac{10}{24} = 0.417\ \Omega$$

$$X_{e_1} = \sqrt{Z_{e_1}^2 - R_{e_1}^2} = \sqrt{(0.417)^2 - (0.1736)^2} = 0.379\ \Omega$$

The equivalent circuit is shown in figure below:



Q.8 (a) Solution:

Given: $$V_s(t) = 50\cos 2\sqrt{2}t + 10\cos\frac{1}{\sqrt{2}}t \text{ V}$$

Since circuit is linear and has two source frequencies, we can use superposition theorem.

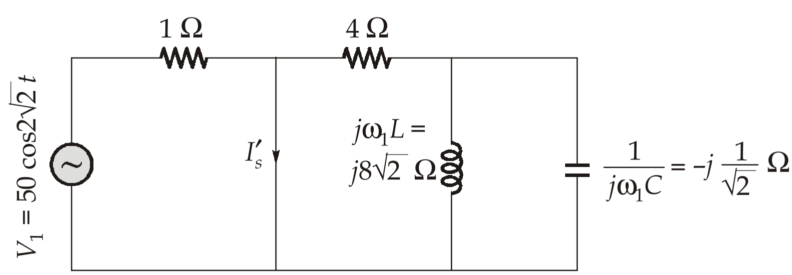
Taking $V_1 = 50\cos 2\sqrt{2}t$ Volt; we have $\omega_1 = 2\sqrt{2}$ rad/sec

Series L-C branch of circuit where $C = 0.25$ F and $L = 0.5$ H is having resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25\times0.5}} = 2\sqrt{2} \text{ rad/sec}$$

that is equal to source frequency.

So, LC branch will act as short circuit at frequency ω_1.



Due to short-circuit, No current flows through 4 Ω resistor.

$$I'_{4\,\Omega} = 0 \quad \text{...(1)}$$

$$\text{Source current, } I'_s = \frac{50}{\sqrt{2}\times1} = \frac{50}{\sqrt{2}} \text{ A}$$

$$\text{Source Power, } P_{s1} = (V_1)_{rms}\cdot I'_s\cdot\cos\phi_s$$

[$\cos\phi_s = 1$ as the circuit becomes resistive]

$$= \frac{50}{\sqrt{2}}\times\frac{50}{\sqrt{2}} = 1250 \text{ W}$$

Now, taking source voltage,

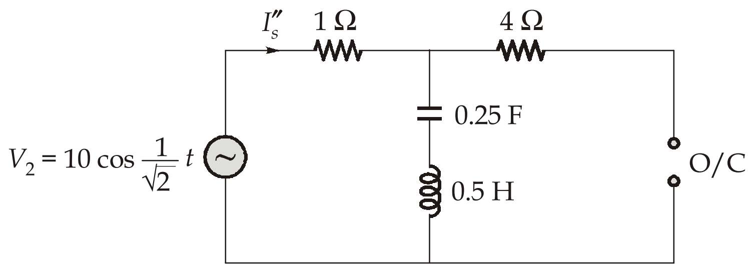
$$V_2 = 10\cos\frac{1}{\sqrt{2}}t \text{ Volt}$$

thus, $$\omega_2 = \frac{1}{\sqrt{2}} \text{ rad/sec}$$

Here, parallel branch of L-C in the circuit, having $L = 4$ H and $C = 0.5$ F is having resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times0.5}} = \frac{1}{\sqrt{2}} \text{ rad/sec}$$

that is equal to source frequency. So, parallel LC branch acts as open circuit. The equivalent circuit is thus obtained as below,



No current flows through 4 Ω resistor

$$I''_{4\,\Omega} = 0$$

$$\text{Source current, } I_s'' = \frac{10}{\sqrt{2}(Z_{eq})}$$

where

$$Z_{eq} = R + j(X_L - X_C)$$

$$= 1 + j\left(\omega_2 L - \frac{1}{\omega_2 C}\right)$$

$$= 1 + j\left[\frac{1}{\sqrt{2}} \times 0.5 - \frac{1}{\frac{1}{\sqrt{2}} \times 0.25}\right]$$

$$= 1 + j\left[\frac{1}{2\sqrt{2}} - 4\sqrt{2}\right]$$

$$= (1 - j5.303)\ \Omega$$

$$I_s'' = \frac{10}{\sqrt{2}[1 - j5.303]}$$

$$= 1.310\angle 79.31°\ \text{A}$$

Power delivered by source,

$$P_s'' = (V_2)_{rms} \cdot I_s'' \cos\phi_s$$

$$= \frac{10}{\sqrt{2}} \times 1.31 \cos 79.31°$$

$$= 1.718\ \text{W}$$

So, using superposition theorem, current through 4 Ω resistor

$$I_{4\,\Omega} = I'_{4\Omega} + I''_{4\Omega} = 0\text{ A}$$

Power delivered by source

$$P_s = P'_s + P''_s$$
$$= 1250 + 1.718$$
$$= 1251.718\text{ W}$$

Q.8 (b) Solution:

Given: $R_1 = 100\ \Omega$ and $R_2 = 50\ \Omega$. The uncertainties in resistance values:

$$w_{R_1} = 0.1\text{ W and } \omega_{R_2} = 0.03\ \Omega$$

When the two resistances are connected in series, the resultant resistance is

$$R_s = R_1 + R_2$$
$$\frac{\partial R_s}{\partial R_1} = 1\ ;\ \frac{\partial R_s}{\partial R_2} = 1$$

Hence uncertainty in the total resistance

$$w_R = \pm\sqrt{\left(\frac{\partial R_s}{\partial R_1}\right)^2 w_{R_1}^2 + \left(\frac{\partial R_s}{\partial R_2}\right)^2 w_{R_2}^2}$$
$$= \pm\sqrt{(1)^2\,(0.1)^2 + (1)^2\,(0.03)^2} = \pm 0.1044\ \Omega$$

Thus, the combined resistance in series can be expressed as,

$$R_s = 150 \pm 0.1044\ \Omega$$

When the two resistances are connected in parallel, the resultant resistance is

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{100\times 50}{150} = 33.33\ \Omega$$

Now, $$R_p = (R_1R_2)\,(R_1 + R_2)^{-1}$$

$$\frac{\partial R_p}{\partial R_1} = (R_2)\,(R_1 + R_2)^{-1} - R_1R_2(R_1 + R_2)^{-2}$$
$$= \frac{R_2}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2} = \frac{50}{150} - \frac{100\times 50}{(150)^2}$$
$$\frac{\partial R_p}{\partial R_1} = 0.111$$
$$\frac{\partial R_p}{\partial R_2} = \frac{R_1}{R_1 + R_2} - \frac{R_1 R_2}{(R_1 + R_2)^2} = \frac{100}{150} - \frac{100\times 50}{(150)^2}$$
$$\frac{\partial R_p}{\partial R_2} = 0.444$$

Hence, uncertainty in total resistance is,

$$w_R = \pm\sqrt{\left(\frac{\partial R_p}{\partial R_1}\right)^2 w_{R_1}^2 + \left(\frac{\partial R_p}{\partial R_2}\right)^2 w_{R_2}^2}$$

$$= \pm\sqrt{(0.111)^2\,(0.1)^2 + (0.444)^2\,(0.03)^2}$$

$$w_R = \pm\,0.01734\ \Omega$$

Thus, the combined resistance in parallel can be expressed as,

$$R_p = 33.33 \pm 0.01734\ \Omega$$

Q.8 (c) Solution:

Given,

$$\text{Mobility, } \mu = 700\ \text{cm}^2/\text{Vs}$$

$$\text{Gate length, } L = 0.3\ \mu\text{m} = 3 \times 10^{-5}\ \text{cm}$$

$$\text{Gate width, } W = 1.0\ \mu\text{m} = 1 \times 10^{-4}\ \text{cm}$$

Threshold voltage is to be shifted from, –0.20 → +0.30 V

$$\therefore \quad \Delta V_T = +0.50\ \text{V}$$

(i) Implant dose required

Threshold shift due to oxide charge:

$$\Delta V_T = \frac{Q_{impl}}{C_{ox}},$$

where Q_{impl} is the implant (Boron) charge density

$$Q_{impl} = C_{ox} \times \Delta V_T$$

Given, $t_{ox} = 5\ \text{nm} = 5 \times 10^{-7}\ \text{cm}$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-13}}{5\times10^{-7}} = 6.9\times10^{-7}\ \text{F/cm}^2$$

$$\therefore \quad Q_{impl} = 6.9 \times 10^{-7} \times 0.5 = 3.45 \times 10^{-7}\ \text{C/cm}^2$$

Thus, required Boron Dose, $= \dfrac{Q}{q} = \dfrac{3.45\times10^{-7}}{1.6\times10^{-19}} = 2.16 \times 10^{12}\ \text{cm}^{-2}$

(ii) Drain current using square-law,

Let $k = \mu C_{ox}\dfrac{W}{L}$

Here, $\dfrac{W}{L} = \dfrac{1\times10^{-4}}{3\times10^{-5}} \approx 3.33$

Thus, $k = 700 \times 6.9 \times 10^{-7} \times 3.33$

$$k \approx 1.61 \times 10^{-3} \text{ A/V}^2$$

For $V_{GS} = 2$ V, Overdrive voltage,

$$V_{ov} = V_{GS} - V_T = 2.0 - 0.30 = 1.7 \text{ V}$$

1. Given, $V_{DS} = 1$ V

Since, $V_{DS} < V_{ov} \Rightarrow$ MOSFET is operating in linear region

$$I_D = k\left[(V_{ov})V_D - \frac{V_D^2}{2}\right]$$

$$= 1.61 \times 10^{-3}[1.7(1) - 0.5]$$

$$= 1.61 \times 10^{-3}(1.2)$$

$$I_D \approx 1.93 \text{ mA}$$

2. Given, $V_{DS} = 2$ V

Now, $V_{DS} > V_{ov} \Rightarrow$ MOSFET is operating in saturation region

$$I_D = \frac{1}{2}kV_{ov}^2$$

$$= \frac{1}{2}(1.61 \times 10^{-3})(1.7)^2$$

$$= 2.33 \times 10^{-3}$$

$$I_D \approx 2.33 \text{ mA}$$

3. Given, $V_{DS} = 3$ V: $V_{DS} > V_{ov}$

Thus, MOSFET is operating in saturation region.

$\therefore$ $I_D \approx 2.33$ mA

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