MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Electrical Engineering Test No : 12

Section-A

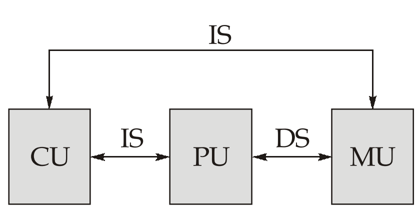
Q.1 (a) Solution:

Based on the number of instruction and data streams that can be processed simultaneously, computing systems are classified into four major categories as per Flynn's classification:

1. **Single Instruction Stream, Single Data Stream (SISD):**

   A computer with a single processor is called a Single Instruction stream, Single Data stream (SISD) computer. It represents the organization of a single computer containing a control unit, a processor unit and a memory unit. Instructions are executed sequentially and the system may or may not have internal parallel processing. Parallel processing may be achieved by means of a pipeline processing.

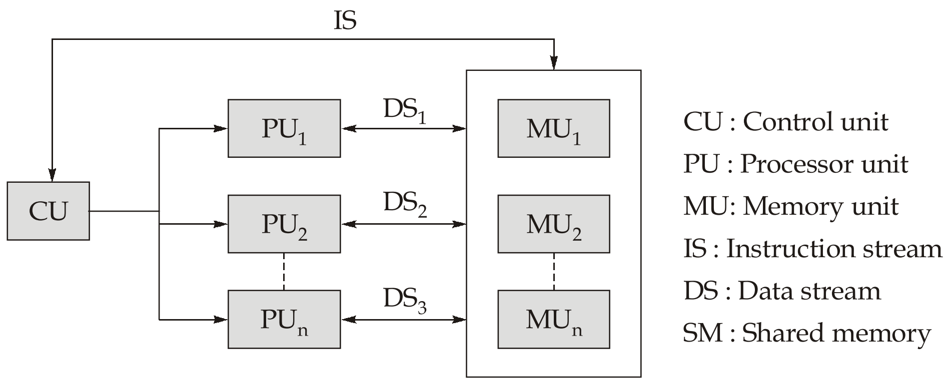
   In such a computer a single stream of instructions and a single stream of data are accessed by the processing elements from the main memory, processed and the results are stored back in the main memory. SISD computer organization is shown in figure below.



CU : Control unit
PU : Processor unit
MU: Memory unit
IS : Instruction stream
DS : Data stream

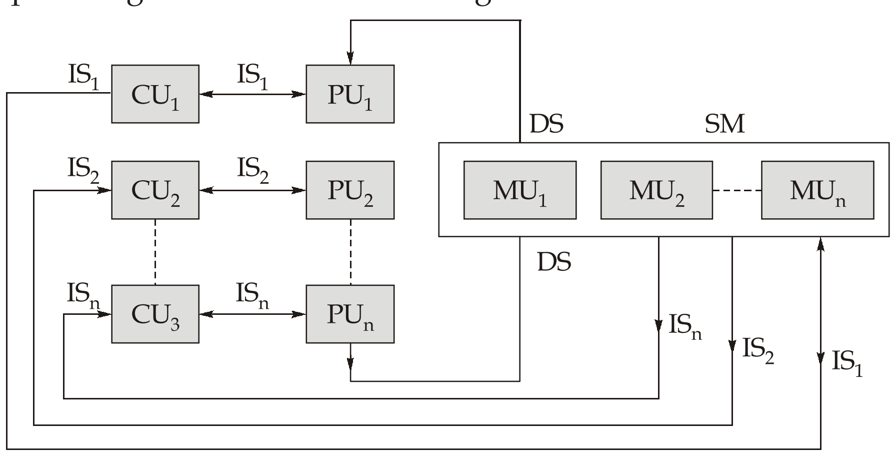
2. **Single Instruction Stream, Multiple Data Stream (SIMD):**

   It represents an organization of computer which has multiple processors under the supervision of a common control unit. All processors receive the same instruction from the control unit but operate on different items of the data. SIMD computers are used to solve many problems in science which require identical operations to be applied to different data sets synchronously. Examples are adding a set of matrices simultaneously, such as $\sum_i \sum_k (a_{ik} + b_{ik})$. Such computers are known as array processors. SIMD computer organization is shown in figure below:



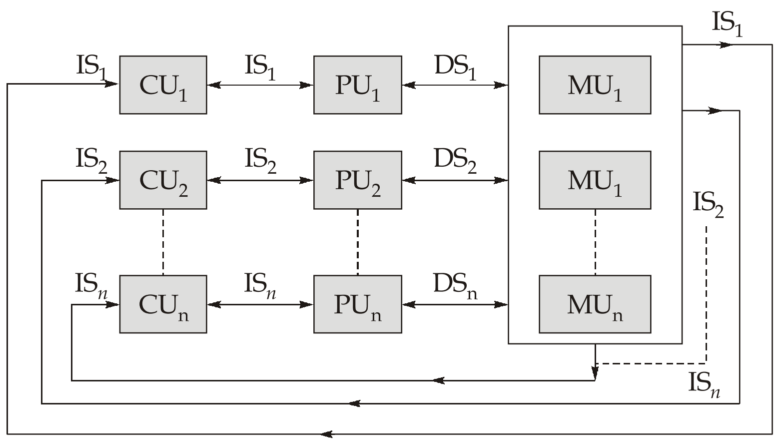
3. **Multiple Instruction Stream, Single Data Stream (MISD):**

It refers to the computer in which several instruction manipulate the same data stream concurrently. In this structure, different processing elements run different programs on the same data. This type of processor may be generalized using a 2-dimensional arrangement of processing elements. Such a structure is known as systolic processor. MISD computer organization is shown in figure below:



4. **Multiple Instruction Stream, Multiple Data Stream (MIMD):**

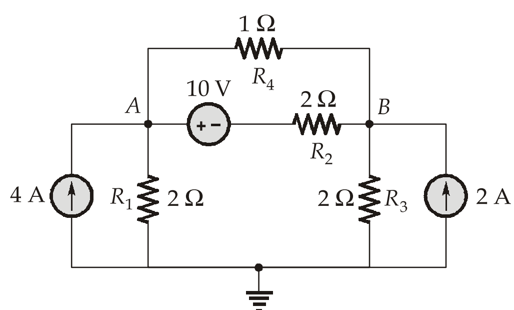
MIMD computers are the general purpose parallel computers. Its organization refers to a computer system capable of processing several programs at a same time i.e. it can execute several instructions, on different data sets, all at the same time. MIMD systems include all multiprocessing systems. MIMD computer organization is shown in figure below.



CU : Control unit
PU : Processor unit
MU: Memory unit
IS : Instruction stream
DS : Data stream

Q.1 (b) Solution:

There are two reference nodes in the given circuit,



Let us name them as A and B.

By applying the KCL at node A, we get,

$$\frac{V_A}{2\,\Omega} + \frac{V_A - V_B - 10\text{ V}}{2\,\Omega} + \frac{V_A - V_B}{1\,\Omega} = 4\text{ A}$$

$$V_A + V_A - V_B - 10\text{ V} + 2V_A - 2V_B = 8\text{ V}$$

$$4V_A - 3V_B = 18\text{ V} \qquad \text{...(i)}$$

By applying KCL at node B, we get,

$$\frac{V_A - V_B}{1\,\Omega} + \frac{V_A - V_B - 10\text{ V}}{2\,\Omega} + 2\text{ A} = \frac{V_B}{2\,\Omega}$$

$$2V_A - 2V_B + V_A - V_B - 10\text{ V} + 4\text{ V} - V_B = 0$$

$$3V_A - 4V_B = 6\text{ V} \qquad \text{...(ii)}$$

By solving equations (i) and (ii), we get,

$$V_A = \frac{54}{7}\text{ V} \quad \text{and} \quad V_B = \frac{30}{7}\text{ V}$$

The power dissipated by R_1, $P_1 = \frac{V_A^2}{R_1} = \frac{(54/7)^2}{2}$ W = 29.755 W

The power dissipated by R_2, $P_2 = \frac{(V_A - V_B - 10\text{ V})^2}{R_2} = \frac{\left(\frac{24}{7} - 10\right)^2}{2}$ W = 21.6 W

The power dissipated by R_3, $P_3 = \frac{V_B^2}{R_3} = \frac{(30/7)^2}{2}$ W = 9.184 W

The power dissipated by R_4, $P_4 = \frac{(V_A - V_B)^2}{R_4} = \frac{(24/7)^2}{1}$ W = 11.755 W

Q.1 (c) Solution:

The crystal structure of NaCl is FCC, i.e., No. of atoms per unit cell (N) = 4

Lattice parameter for NaCl is $a = 2(r_a + r_c)$

r_a = radius of anion ; r_c = radius of cation

We know,

$$\text{APF} = \frac{N \times \frac{4}{3}\pi\left[r_a^3 + r_c^3\right]}{a^3} = \frac{4 \times \frac{4}{3}\pi\left[r_a^3 + r_c^3\right]}{8(r_a + r_c)^3}$$

$$= \frac{\frac{2\pi}{3}\left[1 + \left(\frac{r_c}{r_a}\right)^3\right]}{\left(1 + \frac{r_c}{r_a}\right)^3}$$

We have,

$$\frac{r_c}{r_a} = 0.45$$

$$= \frac{\frac{2\pi}{3}\left[1 + (0.45)^3\right]}{(1 + 0.45)^3} = \frac{\frac{2\pi}{3}[1 + 0.091]}{(1.45)^3}$$

$$\text{APF} = 0.749 \approx 75\%$$

Q.1 (d) Solution:

The correlation coefficient r is measure of strength of relation between or among variables such that if,

$r = +1 \Rightarrow$ Perfect positive correlation

$1 > r > 0 \Rightarrow$ Positive relationship

$r = 0 \Rightarrow$ No relationship

$0 > r > -1 \Rightarrow$ Negative relationship

$r = -1 \Rightarrow$ Perfect negative relationship

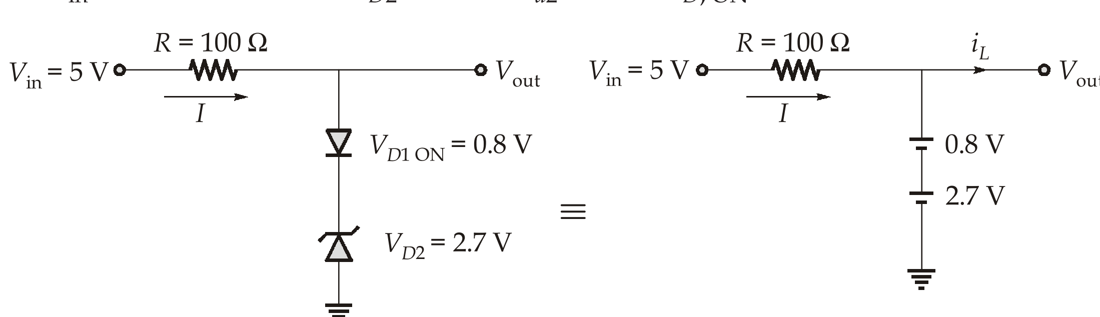
Observation	x	y	Deviation of $x(x-\bar{X})$	$(x-\bar{X})^2$	Deviation of $y(x-\bar{Y})$	$(y-\bar{Y})^2$	Product of Deviations $(x-\bar{X})(y-\bar{Y})$
1	12	50	–1.5	2.25	8.40	70.56	–12.60
2	13	54	–0.50	0.25	12.40	153.76	–6.20
3	10	48	–3.50	12.25	6.40	40.96	–22.40
4	9	47	–4.50	20.25	5.40	29.16	–24.30
5	20	70	6.50	42.25	28.40	806.56	184.60
6	7	20	–6.50	42.25	–21.60	466.56	140.40
7	4	15	–9.50	90.25	–26.60	707.56	252.70
8	22	40	8.50	72.25	–1.60	2.56	–13.60
9	15	35	1.50	2.25	–6.60	43.56	–9.90
10	23	37	9.50	90.25	–4.60	21.16	–43.70
	$\Sigma x = 135$ $\bar{X} = \frac{135}{10}$ $= 13.5$	$\Sigma y = 416$ $\bar{Y} = \frac{416}{10}$ $= 41.6$	$\Sigma(x-\bar{X})$ $= 0$	$\Sigma(x-\bar{X})^2$ $= 374.50$	$\Sigma(y-\bar{Y})$ $= 0$	$\Sigma(y-\bar{Y})^2$ $= 2342.40$	$\Sigma\begin{bmatrix}(x-\bar{X}) \\ -(y-\bar{Y}\end{bmatrix}$ $= 445.0$

Now, Correlation coefficient, $r = \dfrac{\Sigma\left[\left(x-\bar{X}\right)\cdot\left(y-\bar{Y}\right)\right]}{\sqrt{\Sigma\left(x-\bar{X}\right)^2}\cdot\sqrt{\Sigma\left(y-\bar{Y}\right)^2}}$

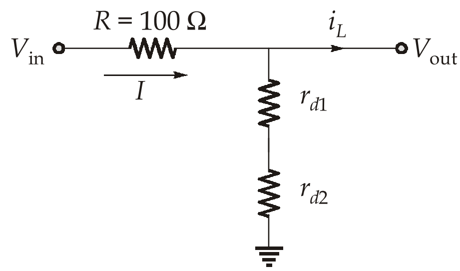
$$= \frac{445}{\sqrt{374.50}\cdot\sqrt{2342.40}} = 0.475$$

Q.1 (e) Solution:

Given: $V_{in} = 5$ V, $R = 100\ \Omega$, $V_{D2} = 2.7$ V, $r_{d2} = 5\ \Omega$, $V_{D,ON} = 0.8$ V



Assuming r_{d1} and r_{d2} as the small signal resistance of the diodes D_1 and D_2 respectively, the small signal model can be drawn as



$$I = \frac{5-0.8-2.7}{100}$$

$$I = 15\text{ mA}$$

$$r_{d1} = \frac{V_T}{I} = \frac{25.8}{15} = 1.72\ \Omega$$

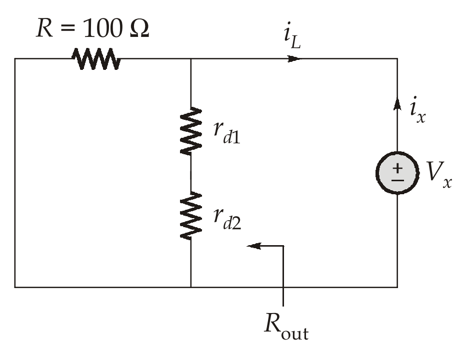
$$\text{Line regulation} = \frac{V_{out}}{V_{in}}$$

$$V_{out} = (r_{d1} + r_{d2})I \quad \text{...(i)}$$

$$V_{in} = (R + r_{d1} + r_{d2})I \quad \text{...(ii)}$$

$$\text{Line regulation} = \frac{V_{out}}{V_{in}} = \frac{r_{d1}+r_{d2}}{R+r_{d1}+r_{d2}} = \frac{5+1.72}{100+5+1.72} = 0.063$$

$$\text{Load regulation} = \left|\frac{V_{out}}{i_L}\right|$$



$$-R_{out} = \frac{V_{out}}{-i_x} = \frac{V_{out}}{i_L} = [R \parallel (r_{d1} + r_{d2})]$$

$$= -[100 \parallel (1.72 + 5)] = -6.297$$

$$\left|\frac{V_{out}}{i_L}\right| = 6.297$$

Thus, a 1 mA change in the load current results in a 6.297 mV change in the output voltage.

Q.2 (a) (i) Solution:

For $0 \le x < 1$

$$\frac{dy}{dx} + \frac{3y}{x} = 1$$

$$\text{I.F} = e^{\int \frac{3}{x} dx} = e^{3\log_e x} = x^3$$

$\therefore$ Solution is

$$yx^3 = \int x^3 dx = \frac{1}{4}x^4 + c$$

$$y = \frac{1}{4}x + \frac{c}{x^3}, 0 \le x < 1$$

using $y\left(\frac{1}{2}\right) = \frac{1}{8}$, we get $c = 0$

So,

$$y = \frac{x}{4}, 0 \le x < 1$$

For $x > 1$

$$\frac{dy}{dx} + \frac{3}{x}y = \frac{1}{x}$$

$$\text{I.F} = x^3$$

$\therefore$ Solution is,

$$yx^3 = \int x^2 dx = \frac{1}{3}x^3 + c$$

So,

$$y = \frac{1}{3} + \frac{c}{x^3}, x > 1$$

Thus,

$$y = \begin{cases} \frac{x}{4}, 0 \le x < 1 \\ \frac{1}{3} + \frac{c}{x^3}, x > 1 \end{cases}$$

Since $y(x)$ is continuous, we have

$$\lim_{x \to 1^-} \frac{x}{4} = \lim_{x \to 1^+} \left(\frac{1}{3} + \frac{c}{x^3}\right)$$

$$c = -\frac{1}{12}$$

So,

$$y = \begin{cases} \frac{x}{4} & , 0 \le x \le 1 \\ \frac{1}{3} - \frac{1}{12x^3}, & x > 1 \end{cases}$$

Q.2 (a) (ii) Solution:

Given equation in symbolic form is

$$(D^3 - 2D^2D')Z = 2e^{2x} + 3x^2 y$$

Its A.E. is $m^3 - 2m^2 = 0$, where $m = 0, 0, 2$

$$\therefore \quad \text{C.F.} = f_1(y) + x f_2(y) + f_3(y + 2x)$$

$$\text{P.I.} = \frac{1}{D^3 - 2D^2D'}(2e^{2x} + 3x^2y)$$

$$= 2\cdot\frac{1}{D^3 - 2D^2D'}e^{2x} + 3\cdot\frac{1}{D^3\left(1 - \frac{2D'}{D}\right)}x^2y$$

$$= 2\cdot\frac{1}{2^3 - 2\cdot 2^2(0)}e^{2x} + \frac{3}{D^3}\cdot\left(1 - \frac{2D'}{D}\right)^{-1} x^2y$$

$$= \frac{1}{4}e^{2x} + \frac{3}{D^3}\left(1 + \frac{2D'}{D} + \frac{4D'^2}{D^2} + ...\right)x^2y$$

$$= \frac{1}{4}e^{2x} + \frac{3}{D^3}\left(x^2y + \frac{2}{D}x^2\cdot 1\right)$$

$$= \frac{1}{4}e^{2x} + \frac{3}{D^3}\left(x^2y + \frac{2}{3}x^3\right)$$

$$= \frac{1}{4}e^{2x} + 3y\frac{x^5}{3\times 4\times 5} + 2\frac{x^6}{4\times 5\times 6}$$

$$\left[\because \frac{1}{D}f(x) = \int f(x)dx\right]$$

$$\left[\because \frac{1}{D^3}f(x) = \int\left[\int\left(\int f(x)\,dx\right)dx\right]dx\right]$$

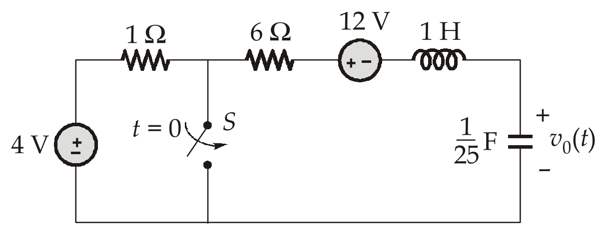
$$= \frac{e^{2x}}{4} + \frac{x^5y}{20} + \frac{x^6}{60}$$

Hence the complete solution is

$$Z = f_1(y) + x f_2(y) + f_3(y + 2x) + \frac{1}{60}\left(15e^{2x} + 3x^5y + x^6\right)$$

Q.2 (b) Solution:

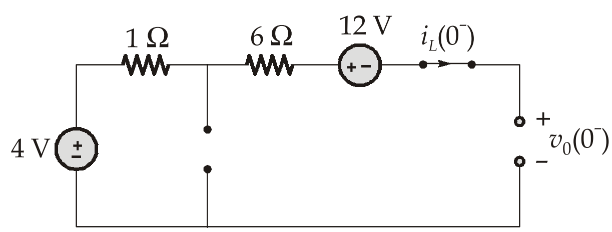
By applying the source transformation to the two current sources in the given circuit, it can be reduced to the following form.



At $t = 0^-$:

At $t = 0^-$, the circuit will be in steady state. In this state for DC excitation, an inductor acts as a short circuit and a capacitor acts as an open circuit.

So, the equivalent of the given circuit at $t = 0^-$ can be drawn as shown below.

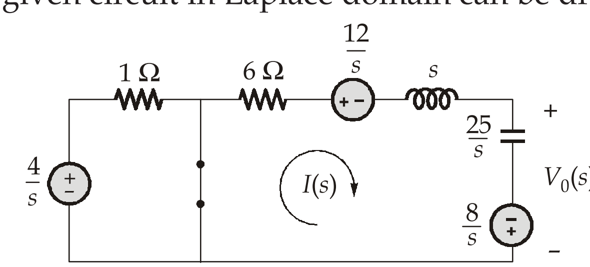


$$i_L(0^-) = 0 \text{ A}$$

$$v_0(0^-) = -12 \text{ V} + 4 \text{ V} = -8 \text{ V}$$

For $t > 0$:

The equivalent of the given circuit in Laplace domain can be drawn as shown below:



$$V_0(s) = I(s)\left(\frac{25}{s}\right) - \frac{8}{s}$$

$$I(s) = \frac{\frac{8}{s} - \frac{12}{s}}{6 + s + \frac{25}{s}} = -\frac{4}{s^2 + 6s + 25}$$

$$V_0(s) = -\frac{100}{s(s^2 + 6s + 25)} - \frac{8}{s}$$

By using partial fractions,

$$\frac{100}{s(s^2+6s+25)} = \frac{A}{s}+\frac{Bs+C}{s^2+6s+25}$$

$$A(s^2 + 6s + 25) + Bs^2 + Cs = 100$$

$$(A + B)s^2 + (6A + C)s + 25A = 100$$

$25A = 100 \Rightarrow$ $A = 4$

$A + B = 0 \Rightarrow$ $B = -A = -4$

$6A + C = 0 \Rightarrow$ $C = -6A = -24$

So,
$$V_0(s) = -\left(\frac{4}{s}-\frac{4s+24}{s^2+6s+25}\right)-\frac{8}{s} = -\frac{12}{s}+\frac{4s+24}{(s+3)^2+16}$$

$$= 4\left[-\frac{3}{s}+\frac{(s+3)}{(s+3)^2+16}+\frac{3}{(s+3)^2+16}\right]$$

By taking the inverse Laplace transform of $V_0(s)$, we get,

$$v_0(t) = \left[-12+4e^{-3t}\cos(4t)+3e^{-3t}\sin(4t)\right]u(t)\text{ V}$$

Q.2 (c) Solution:

Given : $\rho = 1.82 \times 10^{-8}\ \Omega\text{-m}$, $E_F = 5.5$ eV, $n = 6.2 \times 10^{28}/\text{m}^3$

(i) We know, conductivity,

$$\sigma = \frac{ne^2\tau}{m}$$

$$\tau = \frac{m\sigma}{ne^2} = \frac{m}{\rho ne^2}$$

$$= \frac{9.1\times10^{-31}}{1.82\times10^{-8}\times6.2\times10^{28}\times(1.6\times10^{-19})^2}$$

$$= 3.15 \times 10^{-14}\text{ sec}$$

But mobility,
$$\mu_n = \frac{e\tau}{m}$$

$$\mu_n = \frac{1.6\times10^{-19}\times3.15\times10^{-14}}{9.1\times10^{-31}}$$

$$\mu_n = 5.538 \times 10^{-3}\ \text{m}^2/\text{V-S}$$

(ii) Drift velocity,

$$v_d = \mu_n E$$

$$= 5.54 \times 10^{-3} \times \frac{2.5}{1 \times 10^{-2}} = 1.3846 \text{ m/s}$$

(iii) We know,

Fermi energy, $$E_F = \frac{1}{2} m V^2$$

$$V = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2 \times 5.5 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.391 \times 10^6 \text{ m/s}$$

(iv) The mean free path,

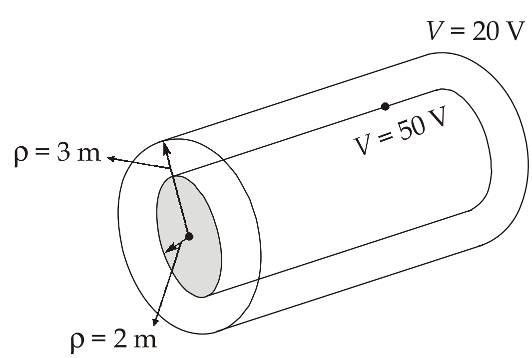
$$\lambda = V.\tau$$

$$= 1.3907 \times 10^6 \times 3.15 \times 10^{-14}$$

$$\lambda = 4.38 \times 10^{-8} \text{ m}$$

Q.3 (a) Solution:

(i) Two coaxial conducting cylinders



We know that from Laplace equation,

$$\nabla^2 \cdot V = 0$$

$$\nabla \cdot \nabla V = 0$$

$$\frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\rho\frac{\partial V}{\partial \rho} + \frac{\partial}{\partial \phi}\frac{1}{\rho}\frac{\partial V}{\partial \phi} + \frac{\partial}{\partial z}\rho\frac{\partial V}{\partial z}\right] = 0$$

$\therefore$ $$V = f(\rho),$$

Therefore, $$\frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right)\right] = 0$$

$$\frac{d}{d\rho}\left(\rho\frac{dV}{d\rho}\right) = 0$$

On integrating the above equation,

$$\rho\frac{dV}{d\rho} = A$$

$$\frac{dV}{d\rho} = \frac{A}{\rho}$$

Again intergrating,

$$V = A\ln\rho + B \quad \text{...(i)}$$

Now at $\rho = 2$ m, $V = 50$ V

$$50 = A\ln 2 + B \quad \text{...(ii)}$$

Also at $\rho = 3$ m, $V = 20$ V

$$20 = A\ln 3 + B \quad \text{...(iii)}$$

On solving equation (ii) and (iii), we get

$$A = -73.98$$

and $B = 101.28$

Therefore, $V = -79.98\ln\rho + 101.28$

Electric field intensity, $E = -\nabla V = \dfrac{73.98}{\rho}$

Point $P(3, 1, 2)_{\text{rectangular}} \rightarrow P(\sqrt{10}, 18.43°, 2)_{\text{cylindrical}}$

$$|E|_{(3,1,2)} = |E|_{(\sqrt{10},\,18.43°,\,2)} = \left|\frac{73.98}{\sqrt{10}}\right| = 23.40 \text{ V/m}$$

(ii) Two radial conducting plates,

$$V = 50 \text{ V at } \phi = 10°$$

and $V = 20$ V at $\phi = 30°$

$$V = f(\phi)$$

Therefore from Laplace equation,

$$\nabla^2 V = 0$$

$$\frac{1}{\rho}\left[\frac{\partial}{\partial\phi}\frac{1}{\rho}\frac{\partial V}{\partial\phi}\right] = 0$$

$$\frac{1}{\rho^2}\frac{d^2V}{d\phi^2} = 0$$

On integrating the above expression,

$$\frac{dV}{d\phi} = A$$

Again integration,

The solution of Laplace equation is

$$V = A\phi + B \quad ...(i)$$

Now at $\phi = 10°$, $V = 50$ V

$$50 = A \times 10 + B \quad ...(ii)$$

Also at $\phi = 30°$, $V = 20$ V

$$20 = 30A + B \quad ...(iii)$$

On solving equation (ii) and (iii), we get

$$A = -85.94\,;$$

$$B = 65$$

Now the general solution is

$$V = -85.94\phi + 65$$

Electric field,

$$E = -\nabla V = -\frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\rho$$

$$|\vec{E}| = \frac{85.94}{\rho}$$

At $\rho = \sqrt{10}\,\text{m}$,

$$|\vec{E}| = \frac{85.94}{\sqrt{10}}$$

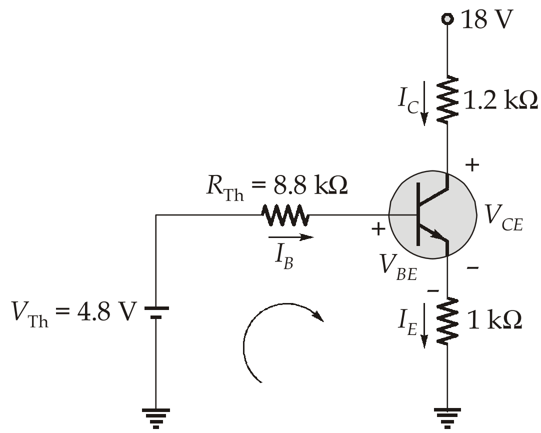
$$= 27.17 \text{ V/m}$$

Q.3 (b) Solution:

DC Analysis : Thevenins equivalent across base-terminals,

$$R_{th} = R_1 \parallel R_2 = \frac{33 \times 12}{33 + 12} = 8.8 \text{ k}\Omega$$

$$V_{th} = \frac{R_2}{R_1 + R_2}V_{cc} = \frac{12}{33 + 12} \times 18 = 4.8 \text{ V}$$



Apply KVL in base-emitter loop,

$$-4.8 + 8.8kI_B + V_{BE} + 1kI_E = 0$$

$$I_B = \frac{4.8 - 0.7}{8.8 + 81 \times 1} \text{ mA} = 0.04566 \text{ mA}$$

$$I_C = \beta I_B = 3.65 \text{ mA}$$

Apply KVL in C-E loop,

$$-18 + 1.2kI_C + V_{CE} + 1kI_E = 0$$

$$V_{CE} = 18 - (1.2 \times 3.65 + 1 \times 81 \times 0.04566)$$

$$V_{CE} = 9.92154 \text{ V}$$

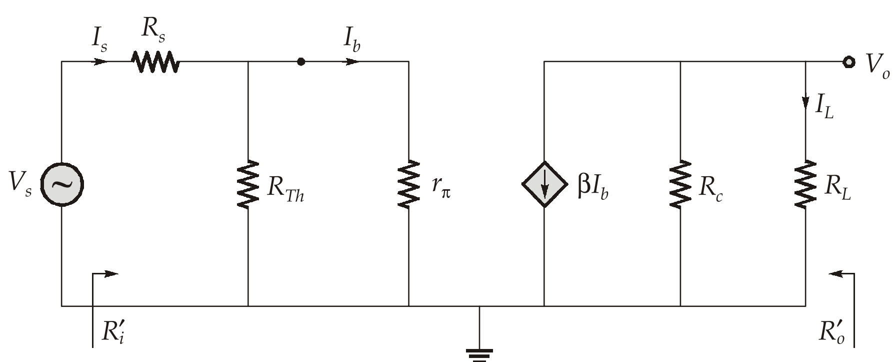
Operating point (9.92154 V, 3.65 mA)

AC Analysis :

$$r_\pi = \frac{V_T}{I_B} = \frac{26 \times 10^{-3}}{0.04566 \times 10^{-3}}$$

$$r_\pi = 569.426\ \Omega \simeq 0.569\ \text{k}\Omega$$

Small signal model,



(i) Current gain,
$$A_I = \frac{I_L}{I_s}$$

$$I_L = \frac{-\beta I_b R_c}{R_c + R_L} \quad ...(1)$$

Also,
$$I_b = \frac{R_{Th}}{r_\pi + R_{Th}} I_s \quad ...(2)$$

$$I_s = \left(\frac{r_\pi + R_{Th}}{R_{Th}}\right) I_b$$

$$A_I = \frac{I_L}{I_s} = \frac{-\left(\frac{R_c}{R_c + R_L}\right)\beta I_b}{\left(\frac{R_{Th} + R_\pi}{R_{Th}}\right) I_b}$$

$$A_I = \frac{-80 \times \left(\frac{1.2}{1.2+2}\right)}{\left(\frac{8.8+0.569}{8.8}\right)} = -28.178$$

(ii) Voltage gain,
$$A_V = \frac{V_o}{V_s}$$

$$V_o = I_L . R_L \quad ...(1)$$

$$V_s = I_s[R_s + (R_{Th} \parallel r_\pi)] \quad ...(2)$$

$$A_v = \frac{V_o}{V_s} = \frac{R_L I_L}{[R_s + (R_{Th} \parallel r_\pi)]I_s}$$

$$= \frac{R_L}{R_s + (R_{Th} \parallel r_\pi)} \cdot A_I$$

$$= \frac{2}{1 + \frac{(8.8 \times 0.569)}{8.8 + 0.569}} \times -28.178$$

$$A_v = -36.7273$$

(iii) Input resistance,
$$R'_i = R_s + (R_{Th} \parallel r_\pi)$$

$$R'_i = 1 + \frac{0.569 \times 8.8}{8.8 + 0.569} = 1.534 \text{ k}\Omega$$

(iv) Output resistance, $R'_o = R_C \| R_L$

$$R'_o = \frac{1.2 \times 2}{1.2 + 2} = 0.75 \text{ k}\Omega$$

$$R'_o = 750\ \Omega$$

Q.3 (c) Solution:

(i)

$$z_{11} = \frac{V_1}{I_1} \text{ with } I_2 = 0$$

$$= 10 \| 2.5 + 8 = 10\ \Omega$$

$$z_{21} = \frac{V_2}{I_1} \text{ with } I_2 = 0$$

$$= \frac{8I_1 + \dfrac{2.5 \times 5I_1}{12.5}}{I_1} = 9\ \Omega$$

$$z_{22} = \frac{V_2}{I_2} \text{ with } I_1 = 0$$

$$= 5 \| 7.5 + 8 = 11\ \Omega$$

$$z_{12} = \frac{V_1}{I_2} \text{ with } I_1 = 0$$

$$= \frac{8I_2 + \dfrac{5}{12.5} \times 2.5I_2}{I_2} = 9\ \Omega$$

In matrix form, $z_{oc} = \begin{bmatrix} 10 & 9 \\ 9 & 11 \end{bmatrix} \Omega$

(ii)

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = \frac{25 \times 10^{-3}}{25 \times 10^{-6}} = 1000\ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = \frac{1 \times 10^{-3}}{25 \times 10^{-6}} = 40$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = \frac{0.125 \times 10^{-6}}{5 \times 10^{-3}} = 25\ \mu\text{S}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = \frac{2.5 \times 10^{-6}}{5 \times 10^{-3}} = 5 \times 10^{-4}$$

Q.4 (a) Solution:

(i) Since, applied electric field is in z-direction, hence all the dipoles are aligned towards electric field and polarization induced will also be in z-direction. Hence,

$\vec{p} = \left(\frac{1}{10\pi}\right) a_z \times 10^{-27}$ C/m

We know; $\vec{P} = \chi_e \varepsilon_0 \vec{E}$;

$$N \cdot \vec{p} = \chi_e \varepsilon_0 \vec{E}$$

where, $\vec{P}$ = polarization; N = no. of molecules per unit volume;

$\vec{p}$ = Dipole moment; $\vec{E}$ = Applied field

$$\therefore \quad 3 \times 10^{18} \times \frac{1}{10\pi} \times 10^{-27} = \chi_e (8.85 \times 10^{-12})(5)$$

$$\chi_e = 2.15$$

Thus; $\varepsilon_r = 1 + \chi_e = 3.15$

Now; Internal field $(E_i) = E_{ext} + \frac{\gamma \cdot P}{\varepsilon_0}$

here; $\gamma = \frac{1}{3}$ for Lorentz field

$$P = N \cdot \vec{p} = 3 \times 10^{18} \times \frac{1}{10\pi} \times 10^{-27}$$

$$= 9.54 \times 10^{-11} \text{ C/m}^2$$

$$\therefore \quad E_i = 5 + \frac{9.54 \times 10^{-11}}{3 \times 8.854 \times 10^{-12}}$$

$$= 8.59 \text{ V/m}$$

$$\therefore \quad D = \varepsilon_0 \varepsilon_r \cdot E_i$$

$$= 8.854 \times 10^{-12} \times 3.15 \times 8.59$$

$$= 0.24 \text{ nC/m}^2$$

(ii) 1. Photoconductive effect:

- When Radiant energy falls on semiconductor device its resistance decreases, this effect is known as photoconductive effect. It can be categorised as:
- **Intrinsic Photoconductivity:** Due to radiant energy, covalent bonds break and electron hole pairs gets generated and due to this, overall amount of electron hole pairs increases in given semiconductor. Hence, conductivity improves. The minimum energy of a photon required for intrinsic excitation is the forbidden-gap energy (E_a) of the semiconductor material.

- **Extrinsic Photoconductivity:** A photon may excite a donor electron into the conduction band or a valence electron may go into an acceptor state. Hence, the ionization of impurities in semiconductors provides free electrons and holes which lead to increase in conductivity.

2. **Photovoltaic Effect:**
   - The photovoltaic effect is a process that generates voltage or electric current in a photovoltaic cell when it is exposed to sunlight.
   - It is this effect that make solar panels useful, as it is how the cells within the panel convert sunlight to electrical energy.
   - The photovoltaic effect occurs in solar cells. These solar cells are composed of two different types of semiconductors p-type and n-type; that are joined together to create a p-n junction.
   - When light of a suitable wavelength is incident on these cells, energy from the photon is transferred to an atom of the semiconducting material in the p-n junction. This causes the electrons to jump to a higher energy state known as the conduction band leaving behind a “hole” in the valence band. This movement of the electron as a result of added energy creates two charge carriers, an electron-hole pair. Because of the electric field that exists as a result of the p-n junction, electrons and holes move in the opposite direction. This motion of the electron creates an electric current in the cell.

Q.4 (b) (i) Solution:

Now, $R_1 = 20\ \Omega$; $L_1 = 0.22$ H; $R_4 = 750\ \Omega$; $R_3 = 40\ \Omega$; $L_3 = 0.1$ H

At balance,

$$(R_1 + j\omega L_1)R_4 = R_2(R_3 + j\omega L_3)$$

Thus the two balance equations are:

$$R_1 = \frac{R_2 R_3}{R_4}$$

and
$$L_1 = \frac{R_2 L_3}{R_4}$$

From above, we have

$$\frac{L_1}{L_3} = \frac{R_2}{R_4} = \frac{R_1}{R_3}$$

∴ Value of R_2 required for balance,

$$R_2 = R_4\frac{L_1}{L_3} = 750\times\frac{0.22}{0.1} = 1650\ \Omega$$

and
$$\frac{L_1}{L_3} = \frac{R_2}{R_4} = 2.2$$

Now examine the value of ratio $\frac{R_1}{R_3}$ for the existing circuit, we have

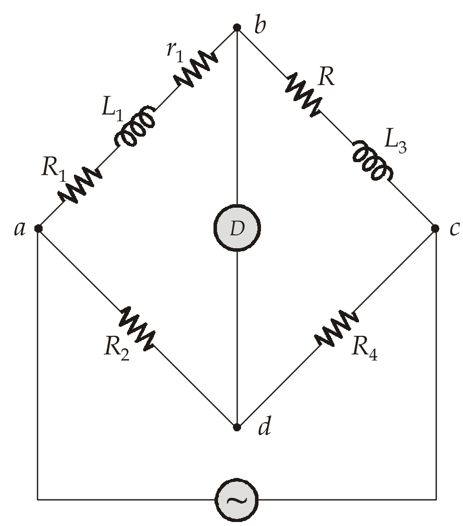
$$\frac{R_1}{R_3} = \frac{20}{40} = 0.5$$

The value of this ratio should be 2.2 for both resistive and inductive balance and therefore we must add a series resistance to arm *ab*. Let this series resistance be r_1. Therefore

$$\frac{R_1+r_1}{R_3} = 2.2$$

$$r_1 = 2.2\times 40 - 20 = 68\ \Omega$$

The modified circuit is shown below,



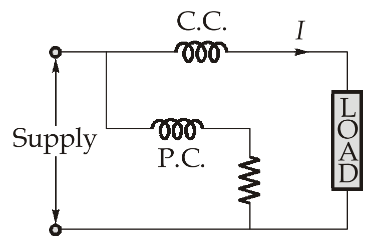
Q.4 (b) (ii) Solution:

Given :
$$R_{cc} = 0.06\ \Omega,\ X_{cc} = 0.02,\ R_{pc} = 6250\ \Omega$$
$$I_L = 10\text{ A at } 0.174\text{ pf (lagging)},\ V_L = 25\text{ V}$$

Power consumed by the load,

$$P_L = V_L I_L \cos\phi$$
$$P_L = 25\times 10\times 0.174 = 43.5\text{ W}$$

Case I : When the pressure coil is connected on the supply side.



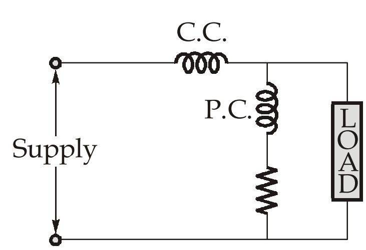
The current,
$$I_{cc} = 10\angle -\cos^{-1}(0.174) = 10\angle -79.98° \simeq 10\angle -80°$$
$$= (1.74 - j9.847)\text{ A}$$

Voltage drop across current coil
$$V_{cc} = I_{cc}(R_{cc} + jX_{cc})$$
$$= (1.74 - j9.847)(0.06 + j0.02)$$
$$V_{cc} = 0.632\angle -61.544°\text{ V}$$
$$= (0.301 - j0.556)\text{ V}$$

Voltage across pressure coil,
$$V_{pc} = V_L + V_{cc}$$
$$= 25 + (0.301 - j0.556)$$
$$= 25.301 - j0.556$$
$$= 25.307\angle -1.259°\text{ Volts}$$

Power indicated by wattmeter
$$= 25.307 \times 10 \times \cos(80° - 1.259)$$
$$= 49.41\text{ Watts}$$
$$\%\text{ Error} = \frac{49.415 - 43.5}{43.5} \times 100\% = 13.6\%$$

Case-II : When pressure coil is connected on the load side.



Power loss in pressure coil,
$$P_{L(pc)} = \frac{V_L^2}{R_{pc}} = \frac{(25)^2}{6250}$$

$$P_{L(pc)} = 0.1 \text{ W}$$

$$\text{Power indicated by wattmeter} = 43.5 + 0.1 = 43.6$$

$$\% \text{ Error} = \frac{43.6 - 43.5}{43.5} \times 100\% = 0.23\%$$

Q.4 (c) Solution:

DC calculations:

The dc or quiescent gate to source voltage,

$$V_{GS\,Q} = V_{DD} \times \frac{R_2}{R_1 + R_2}$$

$$= 10 \times \frac{29.1}{29.1 + 70.9} = 2.91 \text{ V}$$

The quiescent drain current,

$$I_{DQ} = K_n \left(V_{GSQ} - V_t\right)^2$$

$$= 0.5 \times 10^{-3} (2.91 - 1.5)^2$$

$$\approx 1 \text{ mA}$$

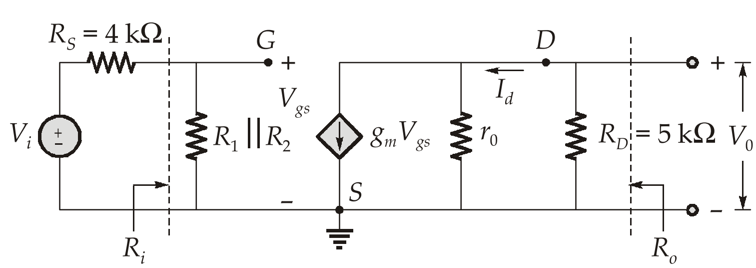
The quiescent drain to source voltage,

$$V_{DS\,Q} = V_{DD} - I_{DQ} R_D$$

$$= 10 - (5 \times 1) = 5 \text{ V}$$

$V_{DS\,Q} > V_{GS\,Q} - V_t$, the transistor is biased in saturation region.

Small signal equivalent circuit:



Small signal output resistance,

Amplifier input resistance,

$$R_i = \frac{R_1 \times R_2}{R_1 + R_2} = \left(\frac{70.9 \times 29.1}{70.9 + 29.1}\right) = 20.63 \text{ k}\Omega$$

Amplifier output resistance,

$$R_0 = r_0 \parallel R_D = \left(\frac{100 \times 5}{100 + 5}\right) K = 4.76 \text{ k}\Omega$$

Small signal trans-conductance,

$$g_m = 2K_n\left(V_{GSQ} - V_t\right)$$

$$= 2 \times 0.5 \times 10^{-3} (2.91 - 1.5)$$

$$g_m = 1.41 \text{ mA/V}$$

Small signal voltage gain,

$$A_V = \frac{V_0}{V_i}$$

$$V_0 = (-g_m V_{gs}) \times \frac{r_0 \times R_D}{r_0 + R_D}$$

$$V_{gs} = V_i \times \frac{R_i}{R_i + R_S}$$

$$V_i = V_{gs} \times \left(\frac{R_i + R_S}{R_i}\right)$$

$$A_V = \frac{V_0}{V_i} = -g_m\left(\frac{r_0 R_D}{r_0 + R_D}\right)\left(\frac{R_i}{R_i + R_S}\right)$$

$$= -1.41 \times 4.76 \times \left(\frac{20.63}{20.63 + 4}\right)$$

$$A_V = -5.62$$

Section-B

Q.5 (a) Solution:

Given quadratic form,

$$P(x) = 4x_1^2 + 4x_2^2 + x_3^2 - 2x_1x_3$$

The above equation can be written as

$$P(x) = 4x_1^2 + 4x_2^2 + x_3^2 - x_1x_2 - x_1x_2$$

We know that quadratic form,

$$P(x) = x^T A x$$

$$= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Where, $A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a symmetric matrix

Calculation of eigen values of A,

$$|A - \lambda I| = \begin{vmatrix} 4-\lambda & -1 & 0 \\ -1 & 4-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)[(\lambda-1)(\lambda-4)] + 1[\lambda - 1] + 0 = 0$$

$$(4 - \lambda)[\lambda^2 - 5\lambda + 4] + \lambda - 1 = 0$$

$$\lambda^3 - 9\lambda^2 + 23\lambda - 15 = 0$$

On solving, we get

$$\lambda = 1, 5, 3$$

For orthogonal transformation,

Let, $x = Py$

$$Q = x^T A x$$

$$= (Py)T_A Py$$

$$= y^T P^T A P y$$

$$Q = y^T (P^T A P) y$$

$[\because P^T AP = P^{-1}AP = D$(digonal matrix)$]$

$Q = y^T D y$ ... New orthogonal transformation

$$Q = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$Q = y_1^2 + 5y_2^2 + 3y_3^2$$

Therefore the new orthogonal set of axes $0(y_1, y_2, y_3)$ is

$$Q_{new} = y_1^2 + 5y_2^2 + 3y_3^2$$

or

$$= 5y_1^2 + y_2^2 + 3y_3^2$$

or

$$= 5y_1^2 + 3y_2^2 + y_3^2$$

or

$$= 3y_1^2 + y_2^2 + 5y_3^2$$

or

$$= 3y_1^2 + 5y_2^2 + y_3^2$$

Q.5 (b) Solution:

As the switch is initially opened for a long time, the voltage across the capacitor at $t = 0^-$ is zero and the current flowing through the inductor at $t = 0^-$ is also zero.

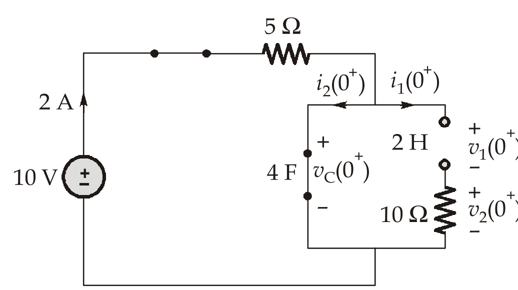
(i) To determine $\frac{dv_1(t)}{dt}$ and $\frac{dv_2(t)}{dt}$ at $t = 0^+$:

$$v_1(t) = L\frac{di_1(t)}{dt}$$

$$v_2(t) = (10\ \Omega)\ i_1(t)$$

$$v_C(t) = v_1(t) + v_2(t)$$

$$i_2(t) = C\frac{d\,v_C(t)}{dt}$$



At $t = 0^+$, the current through inductor is zero and the voltage across capacitor is zero. Hence, the inductor acts as open circuit and capacitor acts as short circuit.

At $t = 0^+$:

$$i_1(0^+) = 0\text{ A}$$

$$i_2(0^+) = 2\text{ A}$$

$$v_1(0^+) = v_2(0^+) = v_C(0^+) = 0\text{ V}$$

$$C\frac{dv_C(t)}{dt} = i_2(t)$$

So,

$$\frac{dv_C(0^+)}{dt} = \frac{1}{C}i_2(0^+) = \frac{2}{4}\text{ V/sec} = 0.50\text{ V/sec}$$

$$\frac{dv_2(t)}{dt} = (10\,\Omega)\frac{di_1(t)}{dt} = \frac{10\,\Omega}{2\,\text{H}}v_1(t)$$

So,
$$\frac{dv_2(0^+)}{dt} = 5v_1(0^+) = 0\text{ V/sec}$$

$$v_1(t) + v_2(t) = v_c(t)$$

$$\frac{dv_1(t)}{dt} = \frac{dv_C(t)}{dt} - \frac{dv_2(t)}{dt}$$

So,
$$\frac{dv_1(0^+)}{dt} = \frac{dv_C(0^+)}{dt} - \frac{dv_2(0^+)}{dt} = 0.50 - 0 = 0.50\text{ V/sec}$$

(ii) To determine $\frac{d^2v_2(t)}{dt^2}$ at $t = 0^+$:

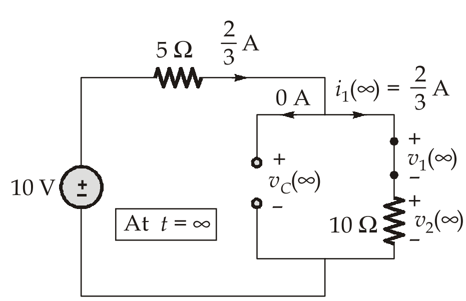
$$v_2(t) = (10\,\Omega)\,i_1(t)$$

$$\frac{d^2v_2(t)}{dt^2} = (10\,\Omega)\frac{d}{dt}\left[\frac{di_1(t)}{dt}\right] = \frac{10\,\Omega}{2\,\text{H}}\frac{d}{dt}[v_1(t)]$$

So,
$$\frac{d^2v_2(0^+)}{dt^2} = 5\frac{dv_1(0^+)}{dt} = 5\,(0.50) = 2.50\text{ V}^2/\text{sec}^2$$

(iii) To determine $v_1(t)$ and $v_2(t)$ at $t = \infty$:

- At $t = \infty$, the circuit will be in steady state. In this state, a capacitor acts as an open circuit and an inductor acts as a short circuit, for DC excitation.
- So, the equivalent of the circuit given in the question at $t = \infty$ can be drawn as,

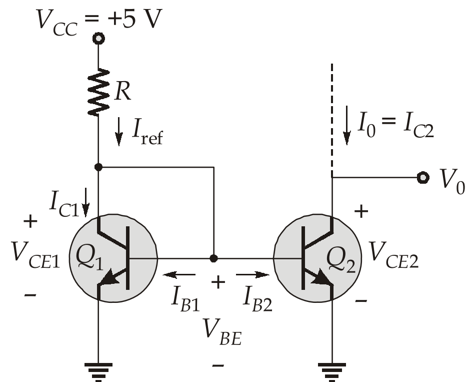


$$v_1(\infty) = 0\text{ V}$$

$$v_2(\infty) = (10\,\Omega)\,i_1(\infty)$$

$$= 10\times\frac{2}{3}\text{ V} = \frac{20}{3}\text{ V} = 6.67\text{ V}$$

Q.5 (c) Solution:



$$I_0 = \left(\frac{I_{ref}}{1+\frac{2}{\beta}}\right)\left(1+\frac{\Delta V_{CE}}{V_A}\right)$$

where,
$$\Delta V_{CE} = V_{CE2} - V_{CE1}$$
$$= V_0 - 0.7$$
$$= 2 - 0.7 = 1.3 \text{ V}$$

Therefore,
$$I_0 = \left(\frac{I_{ref}}{1+\frac{2}{100}}\right)\left(1+\frac{1.3}{50}\right)$$

Therefore,
$$I_{ref} \simeq I_0 = 0.5 \text{ mA}$$

but,
$$I_{ref} = \frac{V_{CC} - V_{BE}}{R} = \frac{4.3}{R} \quad \text{...(i)}$$
$$I_{ref} = I_{C1} + I_{B1} + I_{B2} \quad \text{...(ii)}$$

and $I_{B1} \approx I_{B2}$, as the effect of base width modulation is less reflected in I_B.

$$\frac{I_{C1}}{I_{C2}} = \frac{\left(1+\frac{V_{CE1}}{V_A}\right)}{\left(1+\frac{V_{CE2}}{V_A}\right)} = \frac{\left(1+\frac{0.7}{50}\right)}{\left(1+\frac{2}{50}\right)}$$

$\because$ $V_{CE1} = V_{BE1} = 0.7$ V; and $V_{CE2} = V_0 = 0.2$ V

$$\frac{I_{C1}}{I_{C2}} = 0.975$$

$\therefore$
$$I_{C1} = 0.975 I_0 \quad \text{.... } I_0 = I_{C2} = 0.5 \text{ mA}$$
$$= 0.4875 \text{ mA}$$

But, $$I_{C1} = \beta I_{B1}\left(1+\frac{V_{CE1}}{V_A}\right)$$

...when finite value of early voltage present

$$0.4875 = 100 I_{B1}\left(1+\frac{0.7}{50}\right)$$

$$I_{B1} \cong I_{B2} = 4.8\ \mu A$$

Now, from equation (ii) $$I_{ref} = I_{C1} + I_{B1} + I_{B2} = 0.497 \text{ mA}$$

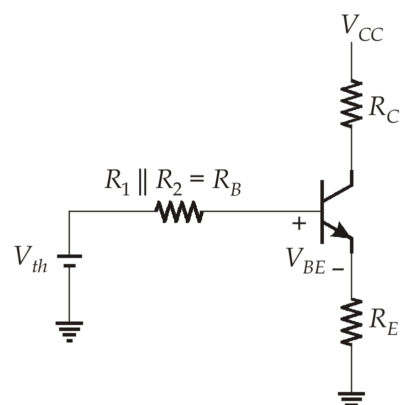
from equation (i),

$$\text{Resistance, } R = \frac{4.3}{I_{ref}} = \frac{4.3}{0.5} = 8.6\ \text{K}\Omega$$

Q.5 (d) Solution:

$$S' = \frac{\partial I_C}{\partial V_{BE}}$$

Using Thevenin equivalent, the circuit reduces to



KVL in base emitter loop,

$$-V_{th} + I_B R_B + V_{BE} + (I_B + I_C)R_E = 0$$

$$I_B[R_B + R_E] + I_C R_E + V_{BE} = V_{th}$$

$$I_B[R_B + R_E] + I_C R_E = V_{th} - V_{BE} \quad \text{...(i)}$$

Now, $$I_C = \beta I_B + (1+\beta) I_{CO}$$

$$I_B = \frac{I_C - (1+\beta) I_{CO}}{\beta}$$

Put in equation (i),

$$\left[\frac{I_C - (1+\beta) I_{CO}}{\beta}\right][R_B + R_E] + I_C R_E = V_{th} - V_{BE}$$

$$I_C(R_B + R_E) - (1 + \beta)I_{CO}(R_B + R_E) + \beta I_C R_E = \beta(V_{th} - V_{BE})$$

$$I_C[R_B + (1 + \beta)R_E] = \beta(V_{th} - V_{BE}) + (1 + \beta)I_{CO}(R_B + R_E)$$

$$I_C = \frac{\beta(V_{th} - V_{BE})}{[R_B + (1+\beta)R_E]} + \frac{(1+\beta)I_{CO}(R_B + R_E)}{[R_B + (1+\beta)R_E]}$$

Differentiating w.r.t V_{BE} and consider β and I_{CO} as constant

$$\frac{\partial I_C}{\partial V_{BE}} = \frac{-\beta}{[R_B + (1+\beta)R_E]} = S'$$

If $(1 + \beta)R_E >>> R_B$ then

$$\frac{\partial I_C}{\partial V_{BE}} = S' = \frac{-\beta}{(1+\beta)R_E}$$

$$S' \approx \frac{-1}{R_E} \quad \text{For self bias}$$

Q.5 (e) Solution:

The resultant circuit is shown in Fig. (a). The circuit can be analysed as below.

Input	Conducting states of D_1 and D_2	Output
$v_i < (V_{S2} - V_{\gamma 2}) = 1.7$ V	D_1 OFF and D_2 ON	$v_0 = 1.7$ V
$(V_{S2} - V_{\gamma 2}) < v_i < (V_{S1} + V_{\gamma 1})$ i.e. 1.7 V $< v_i <$ 3.65 V	D_1 OFF and D_2 OFF	$v_0 = v_i$
$v_i > (V_{S1} + V_{\gamma 1}) = 3.65$ V	D_1 ON and D_2 OFF	$v_0 = 3.65$ V

where we have used $V_{S_2} = 2$ V, $V_{\gamma 1} = 0.65$ V, $V_{\gamma 2} = 0.3$ V in the above calculations given earlier. The output waveform corresponding to the sinusoidal input is illustrated in

Fig. (*b*) where,

$$\alpha_1 = \sin^{-1}\left(\frac{3.65}{5}\right) = 0.818,$$

$$\alpha_2 = \pi - \alpha_1 = 2.323,$$

$$\alpha_3 = \sin^{-1}\left(\frac{1.7}{5}\right) = 0.347$$

and

$$\alpha_4 = \pi - \alpha_3 = 2.795.$$

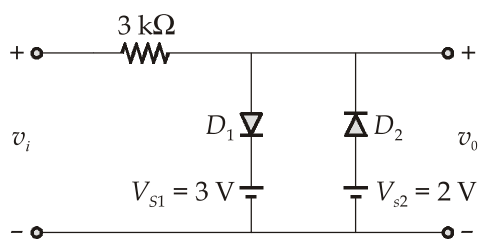


Figure (a)

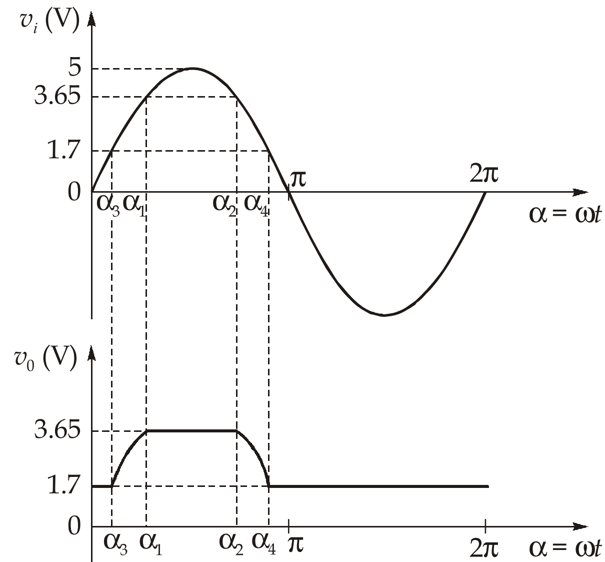


Figure (b)

Q.6 (a) Solution:

Let, $y = \sin x - \log_e x + e^x$

The value of y wrt is calculated in following table,

x :	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y :	3.0295 y_0	2.7975 y_1	2.8976 y_2	3.1660 y_3	3.5597 y_4	4.0698 y_5	4.7042 y_6

(i) Now using trapezoidal method,

$$\int_{0.2}^{1.4} y\,dx = \frac{h}{2}\{(y_0 + y_n) + 2(y_1 + y_2 + y_3 .....y_{n-1})\}$$

$$\Rightarrow \int_{0.2}^{1.4} (\sin x - \ln x + e^x)dx = \frac{0.2}{2}\begin{Bmatrix}(3.0295 + 4.7042) \\ +2(2.7975 + 2.8976 + 3.1660 + 3.5597 + 4.0698)\end{Bmatrix}$$

$$= 4.07149 \quad ...(i)$$

(ii) Simpson's $\frac{1}{3}$rd rule

Now, $$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + ...) + 2(y_2 + y_4 + y_6 + ...)]$$

$$\int_{0.2}^{1.4} (\sin x - \ln x + e^x)dx = \frac{0.2}{3}[(y_1 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{0.2}{3}[7.7337 + 40.1332 + 12.9146]$$

$$= \frac{0.2}{3}[(60.7815)] = 4.0521 \quad \text{...(ii)}$$

(iii) Simpson's rule $\frac{3}{8}$th

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\{(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + ...) + 2(y_3 + y_6 + y_9 + ...)\}$$

$$= \frac{3 \times 0.2}{8}[7.7337 + 2(3.1660) + 3(13.3246)]$$

$$= 4.05296 \quad \text{...(iii)}$$

Now using integral calculus,

$$\int_{0.2}^{1.4} (\sin x - \ln x + e^x)\,dx = \int_{0.2}^{1.4} \sin x\,dx - \int_{0.2}^{1.4} \ln x\,dx + \int_{0.2}^{1.4} e^x\,dx$$

$$= -\cos x\Big|_{0.2}^{1.4} - (x\ln x - x)\Big|_{0.2}^{1.4} + e^x\Big|_{0.2}^{1.4}$$

$$= \cos 0.2 - \cos 1.4 - [1.4 \ln 1.4 - 1.4 - 0.2 \ln 0.2 + 0.2] + e^{1.4} - e^{0.2}$$

$$\int_{0.2}^{1.4} (\sin x - \ln x + e^x)\,dx = 4.0508 \quad \text{...(iv)}$$

Now from equation (i), (ii), (iii) and (iv), we get

|error in trapezoidal method |

$$|e_T| = |4.07149 - 4.0508|$$

$$= 20.69 \times 10^{-3}$$

$$\left|\text{error in Simpson's } \frac{1}{3}\text{rd method}\right|$$

$$\left|e_{s\frac{1}{3}}\right| = |4.0521 - 4.0508| = 1.3 \times 10^{-3}$$

$$\left|\text{error in Simpson's } \frac{3}{8}\text{th method}\right|,$$

$$\left|e_{s\frac{3}{8}}\right| = |4.05296 - 4.0508| = 2.1625 \times 10^{-3}$$

So, $$|e_T| > \left|e_{s\frac{3}{8}}\right| > \left|e_{s\frac{1}{3}}\right|$$

So in this integration, Simpson's $\frac{1}{3}$rd is most accurate and trapezoidal method is least accurate.

Q.6 (b) Solution:

(i) FIFO:

In FIFO, replace the cache block which is having the longest time stamp.

1-miss	4-miss	8-miss	5-miss	20-miss	17-miss	19-miss	5-hit	6-miss
1	1	1	1	20	20	20	20	20
	4	4	4	4	17	17	17	17
		8	8	8	8	19	19	19
			5	5	5	5	5	6

9-miss	11-miss	4-miss	4-hit	3-miss	5-miss	6-miss	9-miss	17-miss
9	9	9	9	9	5	5	5	5
17	11	11	11	11	11	6	6	6
19	19	4	4	4	4	4	9	9
6	6	6	6	3	3	3	3	17

$$\text{Hit ratio} = \frac{2}{18} = 0.11$$

(ii) LRU:

In LRU, replace the cache block that has been in the cache longest without no reference to it.

1-miss	4-miss	8-miss	5-miss	20-miss	17-miss	19-miss	5-hit	6-miss
			5	20	17	19	5	6
		8	8	5	20	17	19	5
	4	4	4	8	5	20	17	19
1	1	1	1	4	8	5	20	17

9-miss	11-miss	4-miss	4-hit	3-miss	5-miss	6-miss	9-miss	17-miss
9	11	4	4	3	5	6	9	17
6	9	11	11	4	3	5	6	9
5	6	9	9	11	4	3	5	6
19	5	6	6	9	11	4	3	5

$$\text{Hit ratio} = \frac{2}{18} = \frac{1}{9} = 0.11$$

(iii) Direct mapped cache:

It maps each block of main memory into only one possible cache line given by k mod n.

1-miss;	1 mod 4 = 1
4-miss;	4 mod 4 = 0
8-miss;	8 mod 4 = 0
5-miss;	5 mod 4 = 1
20-miss;	20 mod 4 = 0
17-miss;	17 mod 4 = 1
19-miss;	19 mod 4 = 3
5-miss;	5 mod 4 = 1
6-miss;	6 mod 4 = 2
9-miss;	9 mod 4 = 1
11-miss;	11 mod 4 = 3
4-miss;	4 mod 4 = 0
4-hit;	4 mod 4 = 0
3-miss;	3 mod 4 = 3
5-miss;	5 mod 4 = 1
6-hit;	6 mod 4 = 2
9-miss;	9 mod 4 = 1
17-miss;	17 mod 4 = 1

0	~~4~~ ~~8~~ ~~20~~ ~~4~~
1	~~1~~ ~~5~~ ~~17~~ ~~5~~ ~~9~~ ~~5~~ ~~9~~ 17
2	6
3	~~19~~ ~~11~~ 3

k mod n = ?

$$\text{Hit Ratio} = \frac{2}{18} = \frac{1}{9} = 0.11$$

(iv) 2-way set associative with LRU:

$$s = \frac{n}{P\text{-way}} = \frac{4}{2} = 2$$

The cache lines are grouped into two sets. It maps each block of main memory into only one possible set given by k mod s. The mapping within a set is done using LRU i.e. replacing the block that has been in the cache longest without no reference to it.

1-miss	1 mode 2 = 1
4-miss	4 mode 2 = 0
8-miss	8 mode 2 = 0
5-miss	5 mode 2 = 1
20-miss	20 mode 2 = 0
17-miss	17 mode 2 = 1

2 way	set	
Set 0	~~4~~ ~~20~~ 4	~~8~~ 6
Set 1	~~1~~ ~~17~~ ~~5~~ ~~11~~ ~~5~~ 17	~~5~~ ~~19~~ ~~9~~ ~~3~~ 9

k mod s = i

19-miss	19 mode 2 = 1
5-miss	5 mode 2 = 1
6-miss	6 mode 2 = 0
9-miss	9 mode 2 = 1
11-miss	11 mode 2 = 1
4-miss	4 mode 2 = 0
4-hit	4 mode 2 = 0
3-miss	3 mode 2 = 1
5-miss	5 mode 2 = 1
6-hit	6 mode 2 = 0
9-miss	9 mode 2 = 1
17-miss	17 mode 2 = 1

$$\text{Hit ratio} = \frac{2}{18} = 0.11$$

Q.6 (c) Solution:

(i) Total resistance of the circuit when the meter is converted to a 250 V voltmeter.

$$R = \frac{250}{100 \times 10^{-3}} = 2500\ \Omega$$

Resistance of the series resistor:

$$R_s = 2500 - 320 = 2180\ \Omega$$

When a voltage of 250 V is applied continuously, the total circuit resistance becomes 2180 + 369 = 2549 Ω

As the current through the coil decreases,

$$\therefore \quad \text{Voltmeter reading} = \frac{2500}{2549} \times 250$$

$$= 245.2\ \text{V}$$

∴ Error due to self-heating

$$= \frac{245.2 - 250}{250} \times 100 = -1.92\%$$

i.e., meter will read 1.92% low.

(ii)
$$\text{Peak voltage } (V_m) = (4\ \text{div}) \times \frac{1}{1\ \frac{\text{div}}{\text{volt}}} = 4\ \text{volt}$$

∴ For sinusoidal wave:

$$V_{RMS} = \frac{V_m}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.82 \text{ volt}$$

$$T_{signal} = 10 \text{ div} \times \frac{4 \text{ ms}}{\text{div}}$$

$$= 40 \text{ ms per cycle.}$$

$$\therefore \quad f_{signal} = \frac{1}{T_{Signal}} = \frac{1}{40 \times 10^{-3}} = 25 \text{ Hz}$$

Q.7 (a) Solution:

(i) Given:

$$l = 100 \text{ cm} = 1 \text{ m}$$

$$\varepsilon_r' = 3.4$$

$$\tan\delta = 0.005$$

$$f = 100 \text{ MHz} = 10^8 \text{ Hz}$$

$$V = 230 \text{ Volts}$$

Calculations:

We know;

$$E = \frac{V}{l} = \frac{230}{1} = 230 \text{ Volt/m}$$

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

$\therefore$ loss factor $(\varepsilon_r'') = (\tan\delta)\cdot\varepsilon_r'$

$$= 0.005 \times 3.4 = 0.017$$

Dielectric loss $(W) = \frac{1}{2}\omega\varepsilon_0\varepsilon_r''\cdot E^2$

$$= \frac{1}{2} \times 2\pi \times f \times \varepsilon_0\, \varepsilon_r'' \cdot E^2$$

$$W = \pi \times (10^8) \times 8.854 \times 10^{-12} \times 0.017 \times (230)^2$$

$$W = 2.5 \text{ Watt}$$

(ii) We have; $D = 4$ cm, $dx = 20$ mm $= 20 \times 10^{-3}$ m

$$\rho = 60 \text{ n}\Omega\text{m} = 60 \times 10^{-9}\ \Omega\text{m}$$

Rate of heat transfer, $\frac{dQ}{dt} = 20$ W

Now, from Wiedemann-Franz law;

$$K = \sigma L T$$

K = Coefficient of thermal conductivity

L = Lorenz number = 2.45×10^{-8} WΩ/K^2

σ = Electrical conductivity

T = temperature = 300 K

$$\therefore \quad K = \frac{1}{\rho} \cdot LT = \frac{1}{60 \times 10^{-9}} \times 2.45 \times 10^{-8} \times 300$$

$$K = 122.5 \text{ W.m}^{-1} \text{ K}^{-1}$$

From thermal conductivity equation:

$$\frac{dQ}{dt} = KA\left(\frac{dT}{dx}\right)$$

$$\therefore \quad dT = \frac{Q \cdot (dx)}{K \cdot A} = \frac{20 \times 20 \times 10^{-3}}{122.5 \times \pi(2 \times 10^{-2})^2}$$

$$dT = 2.59 \text{ K}$$

$\therefore$ Temperature drop across the disk is equal to 2.59 Kelvin.

Q.7 (b) Solution:

The characteristic equation is

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

or $\quad \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

To verify the theorem, we must show that

$$A^3 - 6A^2 + 9A - 4I = 0 \qquad \text{...(i)}$$

Now,
$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

Similarly, $\quad A^3 = A^2 \times A$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$\therefore \; A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

This verifies the theorem.

To obtain A^{-1}, multiply both sides of equation (i) by A^{-1}, we get

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

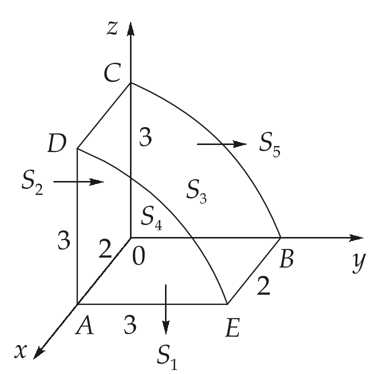
or $$4A^{-1} = A^2 - 6A + 9I$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Q.7 (c) Solution:

The given closed surface is combination of:



S_1 : Rectangular face *OAEB* in *xy*-plane

S_2 : Rectangular face *OADC* in *xz*-plane

S_3 : Circular quadrant *OBC* in *yz*-plane

S_4 : Circular quadrant *AED*

S_5 : Circular surface *BCDE* of cylinder in first octant.

$$\int_S F \cdot \hat{n}\, ds = \sum_{i=1}^{5} \int_{S_i} F \cdot N ds \qquad \text{...(i)}$$

Now, $\int_{S_1} F \cdot \hat{n}\, ds = \int_{S_i} \left(2x^2 y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}\right)\cdot\left(-\hat{k}\right) ds$

$$\int_{S_1} F \cdot \hat{n}\, ds = 4\int_{S_1} xz^2 ds$$

For xy-plane, $z = 0$

$\Rightarrow$ $\int_{S_1} F \cdot \hat{n}\, ds = 4\int_{S_1} xz^2 ds = 0$ ...(ii)

Now, $\int_{S_1} F \cdot \hat{n}\, ds = \int_{S_2} \left(2x^2 y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}\right)\cdot\left(-\hat{j}\right) dS$

$$= \int y^2 ds$$

In xz-plane, $y = 0$

$\Rightarrow$ $\int_{S_2} F \cdot \hat{n}\, ds = \int_{S_3} y^2 ds = 0$ ...(iii)

Now, $\int_{S_3} F \cdot \hat{n}\, ds = \int_{S_3} \left(2x^2 y\hat{i} - y^2 j + 4xz^2\hat{k}\right)\cdot\left(-\hat{i}\right) ds$

$$= \int_{S_3} 2x^2 y\, ds$$

In yz-plane, $x = 0$

$\Rightarrow$ $\int_{S_3} F \cdot \hat{n}\, ds = \int_{3} 2x^2 y\, ds = 0$ ...(iv)

Now, $\int_{S_4} F \cdot \hat{n}\, ds = \int_{S_4} \left(2x^2 y\hat{i} - y^2 j + 4xz^2\hat{k}\right)\cdot \hat{i}\, ds$

$$= \int_{S_4} 2x^2 y\, ds = \int_0^3 \int_0^{\sqrt{9-z^2}} 8y\, dy\, dz$$

$$= 4\int_0^3 \left(9 - z^2\right) dz$$

$\Rightarrow$ $\int_{S_4} F \cdot \hat{n}\, ds = 4\left(\left(9z - \frac{z^3}{3}\right)\Bigg|_0^3\right) = 4 \times 18 = 72$ ...(v)

To find N in S_5 we have,

$$\hat{n} = \frac{\nabla(y^2+z^2)}{|\nabla(y^2+z^2)|} = \frac{\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)(y^2+z^2)}{\left|\nabla\left(y^2+z^2\right)\right|}$$

$$= \frac{2y\,\hat{j}+2z\,\hat{k}}{\sqrt{(2y)^2+(2z)^2}} = \frac{y\,\hat{j}+z\hat{k}}{\sqrt{y^2+z^2}}$$

Also given, $y^2 + z^2 = 9$

$$\Rightarrow \quad \hat{n} = \frac{y\,\hat{j}+z\hat{k}}{\sqrt{9}} = \frac{y\,\hat{j}+z\hat{k}}{3}$$

$$\Rightarrow \quad |\hat{n}\cdot\hat{K}| = \frac{Z}{3}$$

So,

$$ds = \frac{dxdy}{\frac{z}{3}}$$

$$\Rightarrow \quad \int_{S_5} F\cdot\hat{n}\,ds = \int_0^2\int_0^3\left(\frac{-y^3+4xz^3}{3}\right)\frac{dydx}{\frac{z}{3}}$$

$$= \int_0^2\int_0^3\left(\frac{-y^3}{z}+4xz^2\right)dy\,dx$$

Let, $y = 3\sin\theta$

$\Rightarrow$ $dy = 3\cos\theta\, d\theta$

and $(3\sin\theta)^2 + z^2 = 9$

$\Rightarrow$ $z = 3\cos\theta$

$$= \int_0^2\int_0^{\pi/2}\left[\frac{-27\sin^3\theta}{3\cos\theta}+4x(9\cos^2\theta)\right]3\cos\theta\,d\theta\,dx$$

$$s = \int_0^2\left[-27\times\frac{2}{3}\times 108x\times\frac{2}{3}\right]dx = 108 \qquad \text{...(vi)}$$

Using equation (i), (ii), (iii), (iv), (v) and (vi),

$$\int_S F\cdot\hat{n}\,ds = 72 + 108 = 180$$

Q.8 (a) (i) Solution:

The auxiliary equations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Since the given P.D.E. is in the form

$$P_p + Q_q = R$$

Where,

$$P = (y - z)x^2$$

$$Q = y^2(z - x)$$

$$R = f(x, y, z) = Z^2(x - y)$$

Therefore,

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$$

Using $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}$ as multiplier, we get

$$\frac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{0} = \frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2} = 0 \quad \text{...(i)}$$

On intergrating equation (i),

We get, $$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = a \quad \text{...(ii)}$$

Again using $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ as multiplier, we get

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0 \quad \text{...(iii)}$$

On integration equation (iii),

$$\log x + \log y + \log z = \log b$$

$$xyz = b \quad \text{...(iv)}$$

From (ii) and (iv), the general solution is:

$$f\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$$

Q.8 (a) (ii) Solution:

Given A is a skew-Hermitian matrix

Therefore, $$A^\theta = -A$$

Let λ be the eigen value of λ and X is the corresponding eigen vector,

Then, $$AX = \lambda X \quad \text{...(i)}$$

$$(AX)^\theta = (\lambda X)^\theta$$

$$X^\theta \cdot A^\theta = \lambda^\theta \cdot X^\theta$$

$$-X^\theta \cdot A = \lambda^\theta X^\theta = \bar{\lambda} X^\theta \qquad \left[\because A^\theta = -A \text{ and } \lambda^\theta = \bar{\lambda}\right]$$

Post multiplying by X,

$$-X^\theta AX = \bar{\lambda} X^\theta X \quad \text{...(ii)}$$

Pre-multiplying equation (i) by X^θ

$$X^\theta AX = \lambda X^\theta X \quad \text{...(iii)}$$

From equation (ii) and (iii),

$$X^\theta AX = -\bar{\lambda} X^\theta X$$

$$\lambda X^\theta X = -\bar{\lambda} X^\theta X$$

$$(\lambda + \bar{\lambda}) X^\theta X = 0$$

$$\lambda = -\bar{\lambda}$$

This is possible either $\lambda = 0$ or λ = purely imaginary

Hence eigen values of a skew-Hermitian matrix are either zero or purely imaginary.

Q.8 (b) Solution:

Total resistance of instrument circuit $(R_T) = \dfrac{150}{0.05} = 3000\ \Omega$

Resistance of coil $(R) = 400\ \Omega$

$\therefore$ Series resistance $(R_s) = R_T - R = 3000 - 400 = 2600\ \Omega$.

(i) Change in resistance of coil/°C:

$$= 0.004 \times 400 \times 1 = 1.60\ \Omega$$

Change in swamping resistance /°C

$$= 0.00015 \times 2600 \times 1$$

$$= 0.39\ \Omega$$

$\therefore$ Total change in resistance of instrument circuit /°C:

$$= 1.60 + 0.39 = 1.99\ \Omega$$

Now, Resistance temperature co-efficient of instrument = $\dfrac{\text{Change in resistance}/^\circ\text{C}}{\text{Total resistance}}$

$$= \frac{1.99}{3000} = 0.00066/^\circ\text{C}.$$

(ii) Reactance of coil at 100 Hz:

$$= 2\pi \times 100 \times 0.75 = 471.2\ \Omega$$

Impedance of instrument at 100 Hz

$$= \sqrt{(3000)^2 + (471.2)^2} = 3037\ \Omega$$

Current drawn by instrument at 100 Hz,

$$= \frac{150}{3037} = 0.0494\ \text{Amp.}$$

∴ Reading of instrument at 100 Hz

$$= \frac{0.0494}{0.05} \times 150 = 148.2\ \text{V}$$

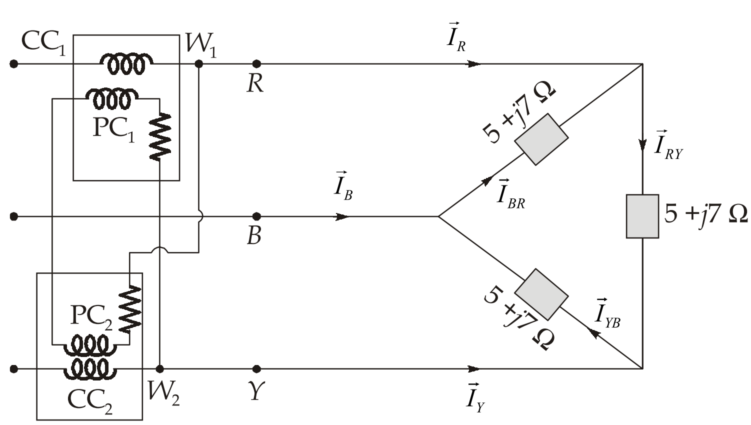
$$\text{Error} = \frac{148.2 - 150}{150} \times 100 = -1.2\%$$

(iii) In order that there is no frequency error, value of capacitance to be connected across R_s,

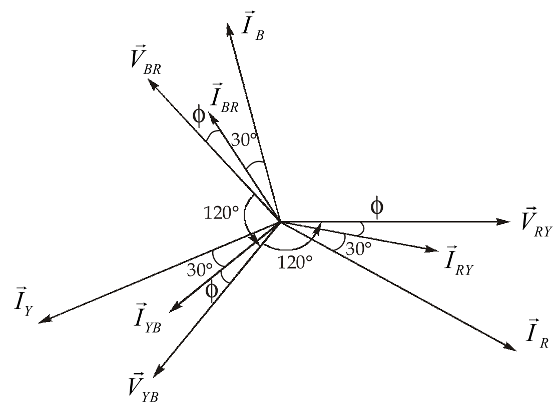
$$C = 0.41\frac{L}{R_s^2} = 0.41 \times \frac{0.75}{(2600)^2} = 0.0455\ \mu\text{F}$$

Q.8 (c) Solution:

Watthour meters W_1 and W_2 connection diagram:



$$Z_{RY} = Z_{YB} = Z_{BR}\ \text{(Balanced load)}$$



$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{7}{5}\right) = 54.46°$$

Watt hour meter reading of $W_1 = V_{YB}I_R \cos(90° - \phi) \times t$ hr

$= tV_{YB}I_R \sin\phi = tV_L I_L \sin\phi$ Wh

Watt hour meter reading of $W_2 = V_{BR}I_Y \cos(90° - \phi) \times t$ hr

$= tV_{BR}I_Y \sin\phi = tV_L I_L \sin\phi$ Wh

Reactive kVAh $Q_{3\phi} = \sqrt{3}V_L I_L \sin\phi \times t = \sqrt{3} \times$ Wh meter readings of W_1 or W_2

$$\text{Watt hour meter readings} = 1\text{ hr} \times 400 \times \left(\frac{\sqrt{3} \times 400}{\sqrt{5^2 + 7^2}}\right) \times \sin(54.46°) = 26.22 \text{ kWh}$$

Total reactive kVAh $= \sqrt{3} \times 26.22 = 45.4$ kVAh

For unbalanced load, $I_R \neq I_Y \neq I_B$

Phase load angles, $\phi_R \neq \phi_Y \neq \phi_B$

Therefore, this method won't work to calculate reactive VAh of the unbalanced load.

❍❍❍❍