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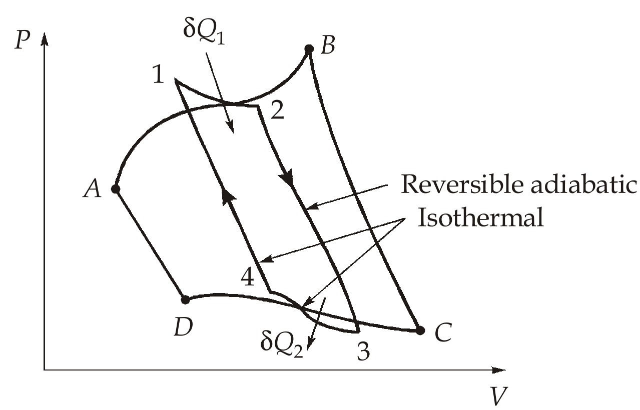
Detailed Solutions

ESE-2026 Mains Test Series

Mechanical Engineering Test No : 12

Section : A

1. (a) Solution:



Consider a general thermodynamic process *ABCD*.

Let this process be broken down into smaller cycle - example 1234 consisting of reversible adiabatic and isothermal process.

Efficiency of this reversible cycle, $\eta_{\text{Carnot}} = 1 - \dfrac{T_2}{T_1}$

or
$$\eta = 1 - \frac{\delta Q_2}{\delta Q_1} = 1 - \frac{T_2}{T_1}$$

$\Rightarrow$
$$\frac{\delta Q_1}{T_1} + \left(\frac{-\delta Q_2}{T_2}\right) = 0$$

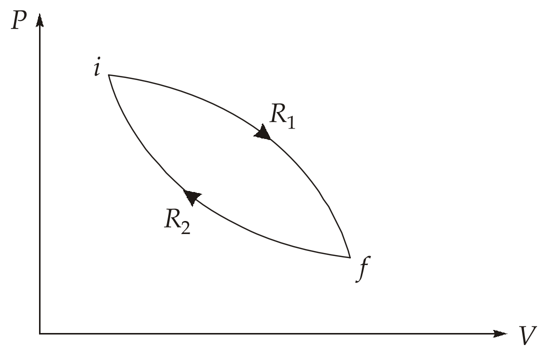
Since δQ_2 is heat rejected, it is taken as negative.

Similarly rest of the process *ABCD* can also be broken down into similar cycles, so heat-

$$\frac{\delta Q_1}{T_1} + \frac{\delta Q_2}{T_2} + \frac{\delta Q_3}{T_3} + \frac{\delta Q_4}{T_4} + \ldots\ldots\ldots\ldots\ldots = 0$$

$$\Rightarrow \qquad \sum \frac{\delta Q_i}{T_i} = 0 \quad \Rightarrow \quad \oint \frac{\delta Q}{T} = 0$$

Now, consider a cycle composed of two reversible process - R_1 and R_2 for cycle.



$$\oint \frac{\delta Q}{T} = 0$$

$$\Rightarrow \qquad \int_i^f \left(\frac{\delta Q}{T}\right)_{R_1} + \int_f^i \left(\frac{\delta Q}{T}\right)_{R_2} = 0$$

$$\Rightarrow \qquad \int_i^f \left(\frac{\delta Q}{T}\right)_{R_1} = \int_i^f \left(\frac{\delta Q}{T}\right)_{R_2}$$

Hence $\frac{\delta Q}{T}$ represents a property since it only depends on initial and final states. This property is known as entropy and is defined for a reversible process, heat exchanged δQ_{rev} at temperature T.

$$\therefore \qquad \text{Entropy change, } dS = \frac{\delta Q_{rev}}{T}$$

1. (b) Solution:

Given : $p_1 = 1$ bar; $T_1 = 50 + 273 = 323$ K; $Q = 2200$ kJ/kg

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times 10^5 \times 14^{1.4} = 40.23 \times 10^5 \text{ N/m}^2$$

$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 r^{(\gamma-1)} = 323 \times 14^{0.4} = 928.22 \text{ K}$$

For unit mass:

Consider the process 2-3,

$$Q_{2-3} = \frac{1}{2} \times 2200 = 1100 \text{ kJ/kg}$$

$$Q_{2-3} = m\int_2^3 (0.709 + 0.000028T)dT$$

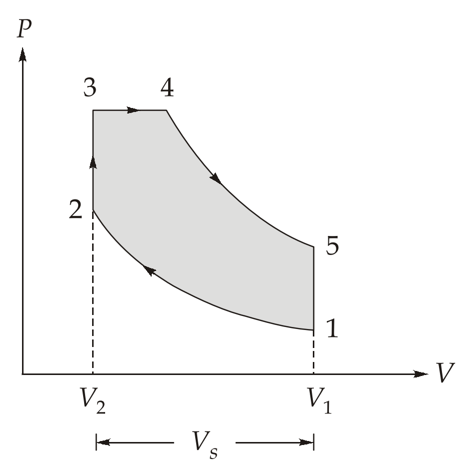
$$1100 = 0.709 \times (T_3 - 928.22) + \frac{0.000028}{2} \times (T_3^2 - 928.22^2)$$

$$T_3 = 2384.4 \text{ K}$$

$$p_3 = p_2\left(\frac{T_3}{T_2}\right) = 40.23 \times \left(\frac{2384.4}{928.22}\right) \times 10^5$$

$$= 103.34 \times 10^5 \text{ N/m}^2$$

Consider the process 3-4,



$$Q_{3-4} = \frac{1}{2} \times 2200 = 1100 \text{ kJ/kg}$$

$$C_p = C_v + R = 0.996 + 0.000028T$$

$$Q_{3-4} = m\int_3^4 C_p dT$$

$$1100 = \int_{T_3}^{T_4} (0.996 + 0.000028T)dT$$

$$1100 = 0.996 \times (T_4 - 2384.4) + \frac{0.000028}{2} \times (T_4^2 - 2384.4^2)$$

$$T_4 = 3405.72 \text{ K}$$

$$V_4 = V_3\left(\frac{T_4}{T_3}\right) = \frac{3405.72}{2384.4}V_3 = 1.428V_3$$

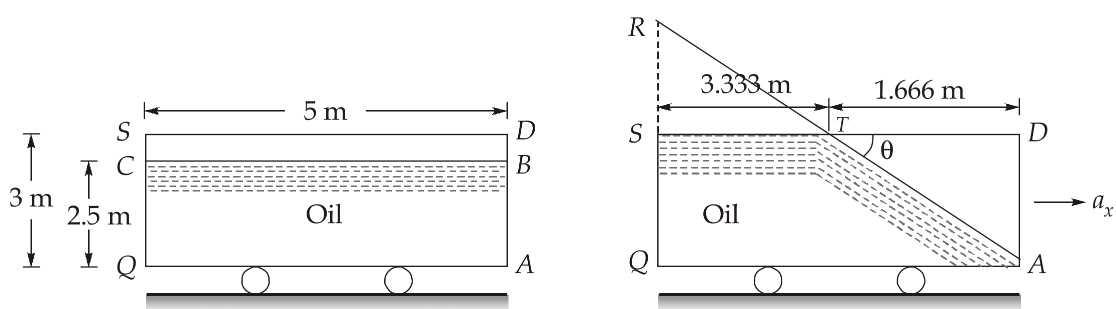
$$V_s = V_1 - V_3 = V_3(r-1) = 13V_3$$

$$\text{Cut-off\%} = \frac{V_4 - V_3}{V_s} \times 100 = \frac{V_4 - V_3}{13V_3} \times 100$$

$$= \frac{1.428 - 1}{13} \times 100 = 3.29\%$$ **Ans.**

1. (c) Solution:

Corner A of the oil tanker will get exposed on account of uniform acceleration of the tanker in the direction indicated in the figure shown below.



(i) Tanker stationary (ii) Tanker in motion

The oil surface which was initially horizontal (indicated by BC) now assumes a profile ATS. The angle of inclination θ of line AT (with the horizontal) will be determined from the condition that the quantity of oil inside the tanker remains the same.

Volume of oil inside the tanker $= 2.5 \times 3 \times 5 = 37.5 \text{m}^3$

Volume of empty space inside the tanker = 0.5 × 3 × 5 = 7.5 m^3

From figure (ii) $\tan\theta = \frac{AD}{TD} = \frac{3}{x}$

Volume of empty space $= \frac{1}{2} \times AD \times TD \times 3 = \frac{1}{2} \times 3 \times x \times 3 = 4.5x$

Equating volumes of empty space in the two cases, i.e. before and after the motion

$$4.5x = 7.5$$

$$x = 1.666 \text{ m}$$

Also, $$\tan\theta = \frac{a_x}{g}$$

$$\frac{a_x}{g} = \frac{3}{1.666} \Rightarrow a_x = \frac{3 \times 9.81}{1.666} = 17.66 \text{ m/s}^2$$

From Δs ARQ and RST,

$$\frac{QA}{QR} = \frac{ST}{SR}$$

Let SR = y

$$\frac{5}{3+y} = \frac{3.333}{y}$$

which result in $y = 6$ m

Hydrostatic force on the left side face QS

$$F_h = \frac{1}{2}\left[P_S + P_Q\right] \times A = \frac{1}{2}(9.81 \times 800)(6+9) \times 9$$

$$= 529.74 \text{ kN}$$

1. (d) Solution:

For lumped system, $Bi < 0.1$

Given data : $k = 50$ W/mK, $\alpha = 1.3 \times 10^{-5}$ m^2/s; $h = 300$ W/m^{2}K; $d = 38$ mm; $T_i = 600$°C; $T_\infty = 50$°C

$$Bi = \frac{hL_c}{k} = \frac{h\left(\frac{D}{6}\right)}{k} = \frac{300 \times \frac{0.038}{6}}{50} = 0.038$$

$$Bi < 0.1$$

⇒ Bearing can be treated as lumped system with minimal error in calculation.

$$F_0 = \frac{\alpha t}{L^2} = \frac{\alpha t}{\left(\frac{D}{6}\right)^2} = \frac{\left(1.3 \times 10^{-5}\right)t}{\left(\frac{0.038}{6}\right)^2} = \left(\frac{117}{361}\right)t$$

(i) Time required for ball bearings to reach 200°C

$$\frac{\theta}{\theta_i} = e^{-B_i F_o}$$

$$\frac{200-50}{600-50} = e^{-0.038\times\frac{117}{361}t}$$

$$t = 105.4973 \text{ sec}$$ **Ans.**

Total amount of heat removed from bearing during 105.4973 sec duration from start.

$$Q = hA(T-T_\infty) = \int_0^t hA(T_i - T_\infty)e^{-BiF_o}\cdot dt$$

$$= hA\theta_i \int_0^t e^{\left(\frac{-hAt}{\rho cV}\right)}\cdot dt = hA\theta_i\left[e^{\left(\frac{-hAt}{\rho cV}\right)}\right]_0^t \times \rho cV$$

$$= \rho cV\theta_i\left[e^{\left(\frac{-hAt}{\rho cV}\right)} - 1\right]$$

$$= \frac{50}{1.3\times10^{-5}}\times\frac{4}{3}\times\pi\times(0.019)^3\times(600-50)\times\left[e^{-0.038\times\frac{117}{361}\times105.497} - 1\right]$$

$$\left[\text{As } \alpha = \frac{k}{\rho c};\ \therefore \rho c = \frac{k}{\alpha}\right]$$

$$Q = -44.201 \text{ kJ}$$ **Ans.**

(ii) Instantaneous heat transfer rate at $t = 0$ or ($F_o = 0$)

$$Q = hA(T_i - T_\infty)$$

$$= 300 \times 4\pi(0.019)^2(600 - 50) = 748.516 \text{ W}$$ **Ans.**

Instantaneous heat transfer rate at $t = 105.4973$ seconds or $F_o = 34.19$

$$Q = hA(T_i - T_\infty)e^{-Bi\cdot Fo}$$

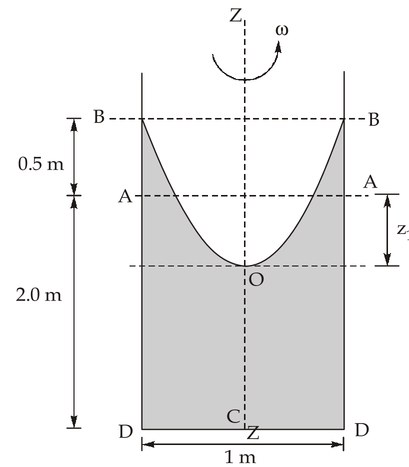
$$= 300 \times 4\pi(0.019)^2(600 - 50)e^{-0.038\times 34.19}$$

$$= 204.1535 \text{ W}$$ **Ans.**

Q.1 (e) Solution:

(i) As shown in the figure, if no water is spilled, the volume of the paraboloid of revolution BOB equals the volume above the original water level A-A. Volume of paraboloid of revolution = 1/2 (volume of circumscribing cylinder)

$$= \frac{1}{2}\left[\frac{\pi}{4}(1)^2(0.5+z_1)\right]$$



$$\frac{1}{2}\times\frac{\pi}{4}(1)^2(z_1+0.5) = \frac{\pi}{4}(1)^2\times 0.5$$

$$z_1 = 1 - 0.5$$

$$z_1 = 0.5 \text{ m}$$

Any point on the paraboloid of revolution is given as

$$z = \frac{\omega^2 r^2}{2g}$$

for point B the coordinates r and z with O as origin are $r = 0.5$ m,

$$z = 0.5 + 0.5 = 1 \text{ m.}$$

$$1.0 = \frac{\omega^2 \times 0.5^2}{2\times 9.81}$$

$$\omega^2 = \frac{2\times 9.81}{0.25} \quad \text{or} \quad \omega = \sqrt{78.48} = 8.86 \text{ rad/s}$$

The corresponding speed in RPM is

$$N = \frac{60\omega}{2\pi} = \frac{60\times 8.86}{2\pi} = 84.61 \text{ rpm}$$ **Ans. (i)**

(ii) For $\omega = 6$ rad/s

$$z = \frac{\omega^2 r^2}{2g} = \frac{(6)^2\times(0.5)^2}{19.62} = 0.46 \text{ m from O}$$

The origin O drops $\frac{1}{2}z$ = 0.23 m below the original water level A-A and hence point O is now (2 – 0.23) = 1.77 m above the bottom of the tank.

Pressure intensity at the centre of the bottom of the tank is

$$P_c = 9.81 \times 1.77 = 17.36 \text{ kPa}$$ **Ans.**

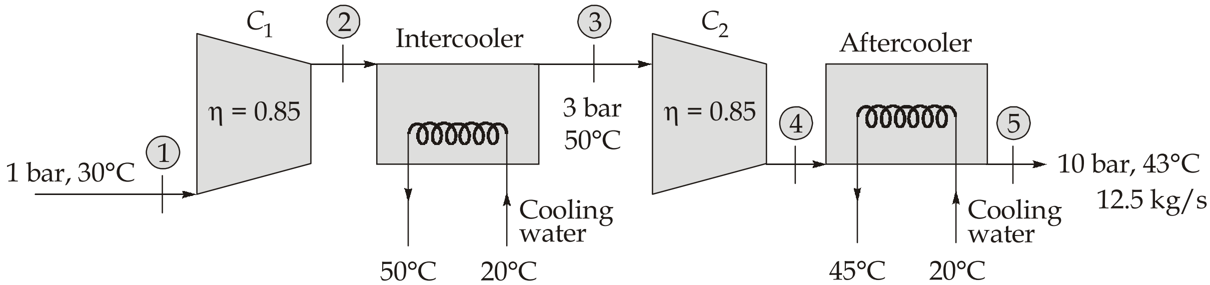
Similarly, at the walls the depth of water will be

$$2 + 0.23 = 2.23 \text{ m}$$

Pressure intensity at the bottom of the tank near the wall is

$$P_D = 9.81 \times 2.23 = 21.87 \text{ kPa}$$ **Ans.**

2. (a) Solution:



Assumptions :

1. Perfectly leakproof arrangement.
2. Steady flow process.
3. No change in bulk KE and PE of the system.
4. Air is treated as ideal gas with constant specific heats.
5. $C_{p,\,air}$ = 1.005 kJ/kgK, $C_{p,water}$ = 4.18 (kJ/kgK)
6. Perfectly insulated compressors.

(i)

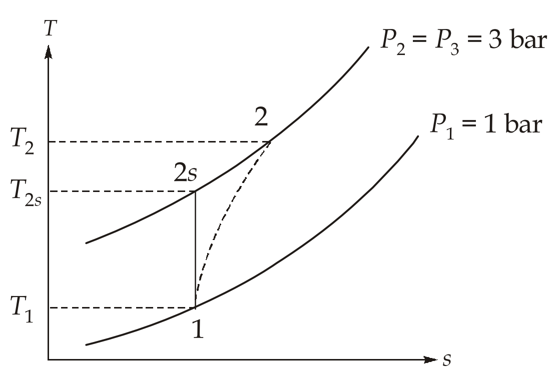
At section (1)

P_1 = 1 bar; T_1 = 30°C = 303 K

At section (2)

$$T_2 - T_1 = \left(\frac{T_{2s} - T_1}{\eta}\right); \quad \left\{W_{act} = \frac{W_{ideal}}{\eta_c}\right\}$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta}$$



$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (273+30)\left(\frac{3}{1}\right)^{\frac{0.4}{1.4}}; \quad \{\because\ P_2 = P_3 = 3\text{ bar}\}$$

$$T_{2s} = 414.7276 \text{ K}$$

$$T_2 = 303 + \frac{414.7276 - 303}{0.85}$$

$$T_2 = 434.444 \text{ K} = 161.44°\text{C}$$

At section (3)

$$P_3 = 3 \text{ bar and } T_3 = 50°\text{C} = 323 \text{ K}$$

$$\dot{Q}_{rejected\,by\,air} = \dot{Q}_{gained\,by\,water}$$

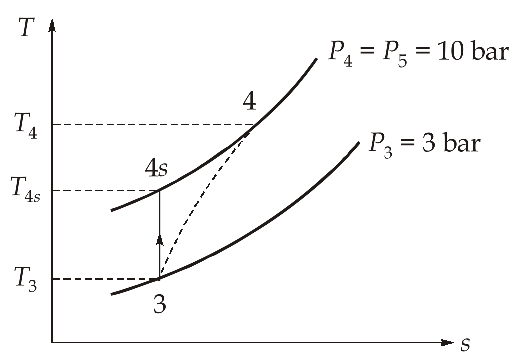
$$\Rightarrow \quad \dot{m}_a c_{pa} \Delta T_a = \dot{m}_w c_{pw} \Delta T_w$$

$$\Rightarrow \quad \dot{m}_w = \dot{m}_a \left(\frac{c_{pa}}{c_{pw}}\right)\left(\frac{\Delta T_a}{\Delta T_w}\right)$$

$$\Rightarrow \quad \dot{m}_w = \frac{(12.5)(1.005)(161.44 - 50)}{(4.18) \times (50 - 20)} = 11.164 \text{ kg/s}$$ **Ans.**

(Water flow rate in intercooler)

At section (4)



$$T_4 - T_3 = \left(\frac{T_{4s} - T_3}{\eta_c}\right)$$

$$T_4 = T_3 + \left(\frac{T_{4s} - T_3}{\eta_c}\right)$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = (273 + 50)\left(\frac{10}{3}\right)^{\frac{0.4}{1.4}} = 455.613 \text{ K}$$

$$T_4 = 323 + \frac{455.613 - 323}{0.85}$$

$$T_4 = 479.0157 \text{ K} = 206.0157°\text{C}$$

At section (5)

$$\dot{Q}_{rejected\,by\,air} = \dot{Q}_{gained\,by\,water}$$

$$\Rightarrow \quad \dot{m}_a c_{p_a} \Delta T_a = \dot{m}_w c_{p_w} \Delta T_w$$

$$\Rightarrow \quad \dot{m}_w = \dot{m}_a \left(\frac{c_{p_a}}{c_{p_w}}\right)\left(\frac{\Delta T_a}{\Delta T_w}\right)$$

$$\Rightarrow \quad \dot{m}_w = 12.5\left(\frac{1.005}{4.18}\right)\left(\frac{206.0157 - 43}{45 - 20}\right)$$

$$= 19.597 \text{ kg/s}$$ **Ans.**

(Water flow rate in aftercooler)

(ii)

Power consumption in the two compressors (c_1 and c_2)

From SFEE

$$\dot{m}_a(h_2 - h_1) = \dot{W}_{C_1}$$

$$\dot{m}_a(h_4 - h_3) = \dot{W}_{C_2}$$

Neglecting any change in KE and PE and considering perfectly insulated compressors.

$$\dot{W}_{C_1} = 12.5(c_{p_a})(T_2 - T_1)$$

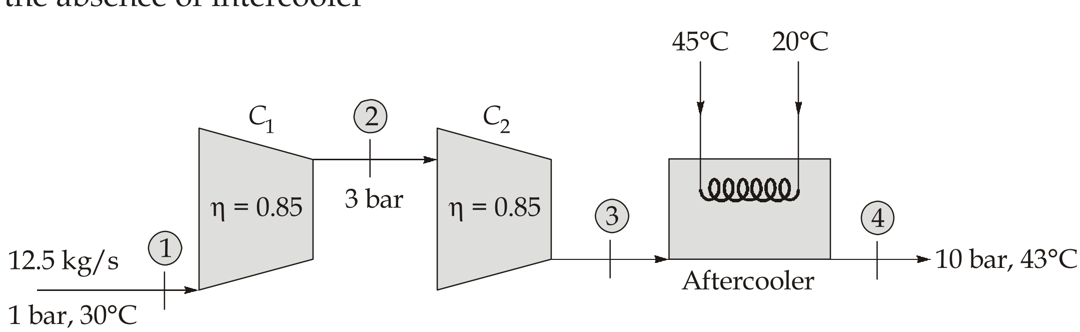
$$= 12.5 \times 1.005 \times (161.44 - 30) = 1651.25 \text{ kW}$$ **Ans.**

$$\dot{W}_{C_2} = 12.5(c_{p_a})(T_4 - T_3)$$

$$= 12.5 \times 1.005 \times (206.016 - 50) = 1959.95 \text{ kW}$$ **Ans.**

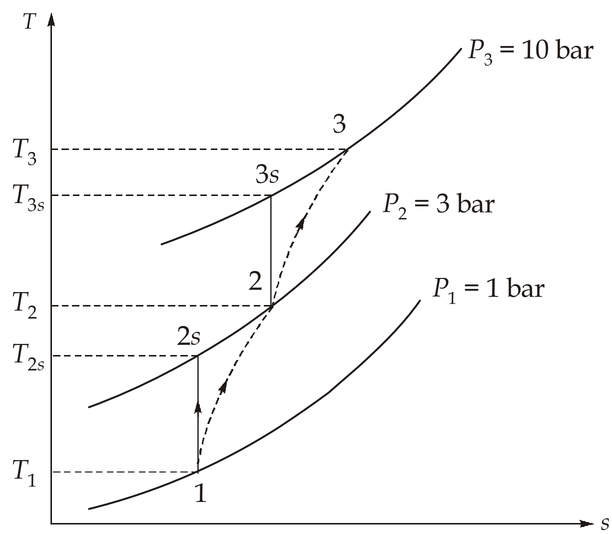
(iii)

In the absence of intercooler



Assumption:

Temperature of inlet and exit of cooling water is same in the after cooler as in previous case.



$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta} \text{ and } T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1\left[1 + \frac{r_p^{(\gamma-1)/\gamma} - 1}{\eta}\right] = (273 + 30)\left[1 + \frac{3^{0.4/1.4} - 1}{0.85}\right]$$

$$T_2 = 434.444 \text{ K} = 161.44°\text{C}$$

Similarly,

$$T_3 = T_2\left[1 + \frac{\left(\frac{10}{3}\right)^{\frac{0.4}{1.4}} - 1}{0.85}\right] = 434.444 \times 1.483$$

$$T_3 = 644.29 \text{ K} = 371.3°\text{C}$$

Overall compression work,

$$\dot{W}_{total} = \dot{m}_a c_{p_a}(T_3 - T_1)$$

$$= 12.5 \times 1.005 \times (371.3 - 30)$$

$$\dot{W}_{total} = 4287.58 \text{ kW}$$ **Ans.**

Total compression work without intercooling is greater than total compression work with intercooling.

In aftercooler

$$\dot{Q}_{rejected\,by\,air} = \dot{Q}_{gained\,by\,water}$$

$$\dot{m}_a c_{p_a} \Delta T_a = \dot{m}_w c_{p_w} \Delta T_w$$

$$\dot{m}_w = \dot{m}_a \left(\frac{c_{p_a}}{c_{p_w}}\right)\left(\frac{\Delta T_a}{\Delta T_w}\right)$$

$$\dot{m}_w = 12.5\left(\frac{1.005}{4.18}\right)\left(\frac{371.3 - 43}{45 - 20}\right)$$

$$\dot{m}_w = 39.46 \text{ kg/s}$$ **Ans.**

2. (b) Solution:

Using heat conduction equation in one-Dimension cylindrical co-ordinates, with constant heat conductivity

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{k} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t}$$

At steady state

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{k} = 0$$

$$r\frac{dT}{dr} = -\frac{\dot{q}}{2k}r^2 + C_1$$

$$\frac{dT}{dr} = -\frac{\dot{q}}{2k}r + \frac{C_1}{r}$$

$$T = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

using boundary conditions

At $r = r_1, \quad t = T_1$

At $r = r_2, \quad \frac{dT}{dr} = 0 \quad [\because$ perfectly insulated$]$

So, $0 = -\frac{\dot{q}}{2k}r_2 + \frac{C_1}{r_2} \Rightarrow C_1 = \frac{\dot{q}}{2k}r_2^2$

$$T_1 = -\frac{\dot{q}}{4k}r_1^2 + \frac{\dot{q}}{2k}r_2^2 \ln r_1 + C_2$$

$$C_2 = T_1 + \frac{\dot{q}}{2k}\left[\frac{r_1^2}{2} - r_2^2 \ln r_1\right]$$

$$\Rightarrow T = -\frac{\dot{q}}{4k}r^2 + \frac{\dot{q}}{2k}r_2^2 \ln r + T_1 + \frac{\dot{q}}{2k}\left[\frac{r_1^2}{2} - r_2^2 \ln r_1\right]$$

$$T = T_1 + \frac{\dot{q}}{2k}\left[\frac{r_1^2 - r^2}{2} + r_2^2 \ln\left(\frac{r}{r_1}\right)\right]$$ **Ans.**

Apparently the temperature distribution is parabolic.

(i) Now, Given, $r_2 = 3$ cm, $r_1 = 1$ cm; $I = 6000$ Amp/cm^2; $T_1 = 80°C$

Total volumetric heat generation= $I^2 R = I^2 \frac{\rho l}{A}$

$$\dot{q} = \frac{I^2 \frac{\rho l}{A}}{A\,l} = \rho\left(\frac{I}{A}\right)^2 = 2\times10^{-8}\times[6000\times10^4]^2 = 72\times10^6 \text{ W/m}^3$$

Max temperature will occur at outer surface i.e. insulated surface at $r = r_2$

$$T_{max} = T_1 + \frac{\dot{q}}{2k}\left[\frac{r_1^2 - r_2^2}{2} + r_2^2 \ln\frac{r_2}{r_1}\right]$$

$$= 80 + \frac{72\times10^6}{2\times380}\times\left[\frac{0.01^2 - 0.03^2}{2} + 0.03^2\times\ln\frac{0.03}{0.01}\right]$$

$= 135.77°C$ **Ans.**

(ii) Internal heat transfer can be obtained by

$$q = -kA\left.\frac{dT}{dr}\right|_{r=r_1}$$

$$q = -k\times(2\pi r_1 l)\times\left[-\frac{\dot{q}}{2k}r_1 + \frac{\dot{q}}{2k}\frac{r_2^2}{r_1}\right]$$

$$q = -380\times(2\pi\times0.01\times1)\times\left[\frac{72\times10^6}{2\times380}\left(-0.01 + \frac{0.03^2}{0.01}\right)\right]$$

$q = -180955.73$ W/m length of the conductor **Ans.**

–ve sign indicates heat flow radially inwards.

Since the outer surface is insulated the entire heat generated within the conductor must be dissipated internally. Therefore the internal heat transfer must be

$$= \text{(Volume per metre length of conductor)} \times \dot{q}$$

$$= \pi \times (0.03^2 - 0.01^2) \times 72 \times 10^6$$

= 180955.73 W/m length of conductor **Ans.**

2. (c) Solution:

In open chamber CI engines the fuel is injected in the bulk of the air: so the turbulence of air in the combustion chamber at the time of fuel injection and during the combustion process is very important. The air velocity in the combustion chamber has two components: (a) air swirl (spiral flow) and {b} air radial flow (squish). Air swirl is created during the intake stroke by inducing the air into the cylinder through a shaped intake port which is tanglement with piston as shown in: Figure(i) The induced swirl can be increased during compression by transferring the air to the recess in the piston or in the cylinder head. As the piston approaches TDC: the air flows radially inwards, i.e. towards the combustion chamber recess. It is shown in Figure(ii). The radial streams coming from the opposite sides meet and get deflected upwards into the chamber. After reaching the end of the chamber, the air flow radially outwards, i.e. towards the outer walls, then downwards, i.e. towards the open end. Here the air is met by air flowing radially inwards between the cylinder and piston, and is carried around again, producing a toroidal movement within the combustion chamber.

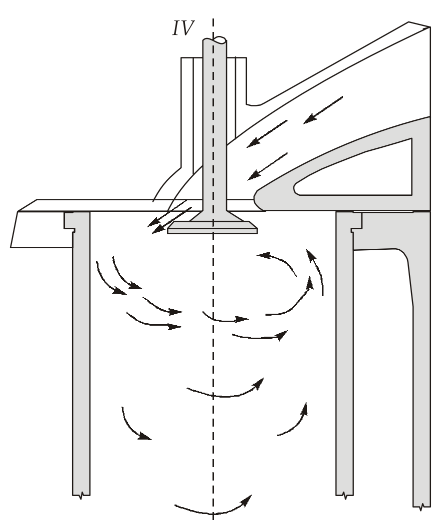


Figure (I) Induction induced air-swirl motion

A schematic of the spray pattern which results when a fuel jet is injected radially outwards into a swirling flow is shown in Figure (iii) The structure of the jet is complex because there is relative motion in both the radial and tangential directions between the initial jet and the air. The small droplets are carried away with the air and form the leading edge of the spray. The relatively large droplets are concentrated in the core and the trailing edge of the spray. The average distance between the droplets changes with the location in the spray: being the greatest in the leading edge of the spray.

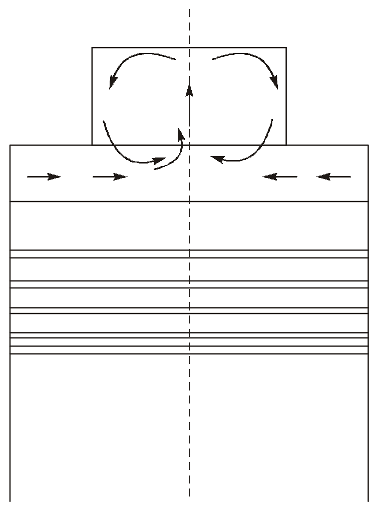


Figure (ii): Air radial flow i.e. squish motion

A plot of the local air fuel ratio on a mass basis versus the angle from the centre line of the injector hole is also shorn in Figure(iii). the fuel-air distribution varies with the radial distance from the nozzle hole. The leading edge of the spray always contains the smallest droplets, which are the first to evaporate.

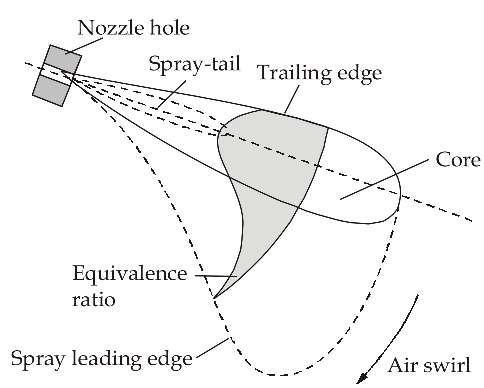


Figure (iii): Fuel spray injected into air swirl

The smaller the droplet diameter the faster it will be carried away from the core by the swirling air. Thus the mixture near the leading edge of the spray may be assumed to consist of premised fuel-vapour and air before ignition. In the core the big droplets are concentrated, and they are expected to be in the liquid phase at the start of ignition.

The spray may be divided into several regions depending upon the fuel-air distribution and mechanism of combustion in each region. These regions are: (a) the lean flame region, (b) the lean flameout region, (c) the spray core: (d) the spray tail (e) the after-injection, and (i) the fuel deposited on the walls.

3. (a) Solution:

(i)

The velocity distribution for flow between parallel plates is given by

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + Ay + B$$

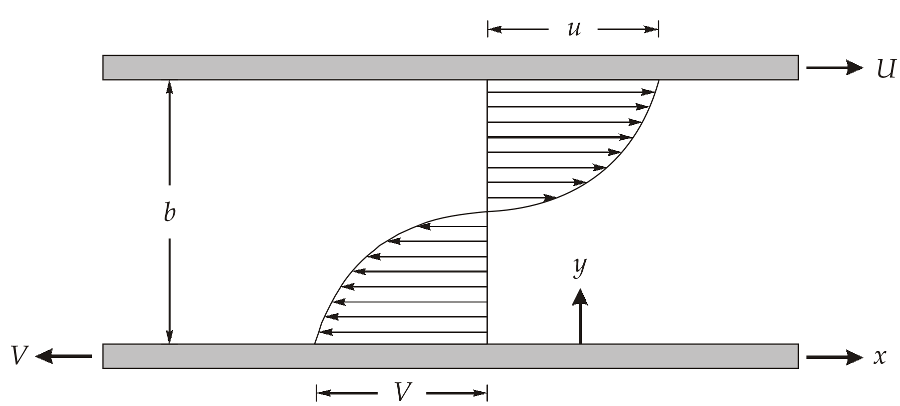
The constants of A and B are to be evaluated for the boundary conditions of the problem, that is

1. At $y = 0$, $u = -V$
2. At $y = b$, $u = U$

Substituting these boundary conditions, we obtain

$$-V = \frac{1}{2\mu}\frac{dp}{dx} \times 0 + A \times 0 + B,$$

$$B = -V$$ **Ans.**



and $$U = \frac{1}{2\mu}\frac{dp}{dx}b^2 + Ab + (-V)$$

which yields,

$$A = \frac{U}{b} + \frac{V}{b} - \frac{1}{2\mu}\frac{dp}{dx}b$$ **Ans.**

and the resulting velocity distribution equation is

$$u = (U+V)\frac{y}{b} - \frac{1}{2\mu}\frac{dp}{dx}(by - y^2) - V$$ **Ans.**

(ii)

The distance y at which the velocity u is zero may be obtained as follows:

$$(U+V)\frac{y}{b}-\frac{1}{2\mu}\frac{dp}{dx}(by-y^2)-V=0$$

or
$$\frac{1}{2\mu}\frac{dp}{dx}y^2+\left(\frac{U+V}{b}-\frac{1}{2\mu}\frac{dp}{dx}b\right)y-V=0$$

Solving this quadratic equation

$$y=\frac{-\left\{\frac{U+V}{b}-\frac{1}{2\mu}\frac{dp}{dx}b\right\}\pm\sqrt{\left(\frac{U+V}{b}-\frac{1}{2\mu}\frac{dp}{dx}b\right)^2+4\frac{1}{2\mu}\frac{dp}{dx}V}}{2\frac{1}{2\mu}\frac{dp}{dx}}$$

which simplifies to

$$y=\frac{-\left\{\frac{U+V}{b}-\frac{1}{2\mu}\frac{dp}{dx}b\right\}\pm\sqrt{\left(\frac{V+U}{b}\right)^2+\frac{1}{4\mu^2}\left(\frac{dp}{dx}\right)^2b^2-\frac{(U-V)}{\mu}\frac{dp}{dx}}}{\frac{1}{\mu}\frac{dp}{dx}}$$ **Ans.**

This equation gives two values of y of which only one, which is positive and less than b, will be physically meaningful and the other one may, therefore, be discarded.

(iii)

The discharge per unit width of plates may be obtained as under

$$q=\int_0^b u\,dy=\int_0^b\left\{(U+V)\frac{y}{b}-\frac{1}{2\mu}\frac{dp}{dx}(by-y^2)-V\right\}dy$$

$$q=(U-V)\frac{b}{2}-\frac{1}{12\mu}\frac{dp}{dx}b^3$$ **Ans.**

(iv)

The shear stress distribution may be expressed using Newton's law of viscosity as

$$\tau=\mu\frac{du}{dy}=\mu\left\{\frac{U+V}{b}-\frac{1}{2\mu}\frac{dp}{dx}(b-2y)\right\}$$

$$\tau=\mu\frac{U+V}{b}-\frac{1}{2}\frac{dp}{dx}(b-2y)$$

$$\tau=(U+V)\frac{\mu}{b}-\frac{dp}{dx}\left(\frac{b}{2}-y\right)$$

The shear stress will be zero at a distance y,

$$0 = (U+V)\frac{\mu}{b} - \frac{dp}{dx}(b/2 - y)$$

$$\therefore \qquad y = \frac{b}{2} - \frac{(U+V)\mu}{b(dp/dx)}$$

$$= \frac{b}{2} - \frac{\mu}{b}\left(\frac{U+V}{dp/dx}\right)$$ **Ans.**

3. (b) Solution:

Given : $\dot{m}_{CO_2} = \dot{m}_h = \frac{88000}{3600} = 24.44$ kg/s; $t_{h1} = 520°C; t_{h2} = 340°C; t_{c1} = t_{c2} = 250°C$;

$d_i = 28$ mm $= 0.028$ m; $d_o = 28 + 2 \times 2 = 32$ mm $= 0.032$ m.

Number of tubes (N) and length of each tube (L) :

Considering the flow of CO_2 through tubes, we have

Mass flow, m = A.V.ρ (where V is the velocity of CO_2 in the tubes).

$$\therefore \qquad G = \frac{m}{A} \text{ (mass flow per unit area per unit time)}$$

$$G = \frac{AV\rho}{A} = V\rho$$

Reynolds number, $Re = \frac{\rho V d}{\mu} = \frac{Gd}{\mu}$ (where G is in kg/m^2.s and μ in Ns/m^2)

$$Re = \frac{(320000/3600) \times 0.028}{0.0000298} = 83519.7$$

Prandtl number, $Pr = \frac{\mu c_p}{k} = \frac{0.0000298 \times (1.172 \times 1000)}{0.043} = 0.812$

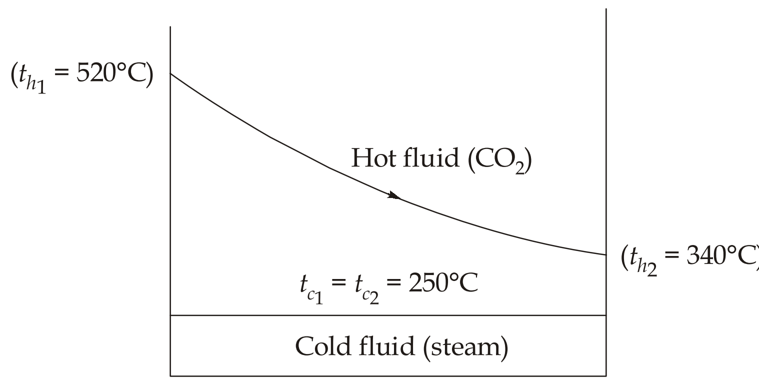
Nusselt number, $Nu = \frac{h_i d_i}{k} = 0.023(Re)^{0.8}(Pr)^{0.33}$

or $$h_i = \frac{k}{d_i} \times 0.023(Re)^{0.8}(Pr)^{0.33}$$

$$= \frac{0.043}{0.028} \times 0.023 \times (83519.7)^{0.8} \times (0.812)^{0.33}$$

$$= 285.54 \text{ W/m}^2\text{°C}$$

The temperature distribution during the flow is shown in figure.



It is assumed that the water enters into the boiler at saturated condition and comes out as saturated steam, so the temperature during the generation of steam remains constant at 250°C. The logarithmic mean temperature difference (LMTD) is given by

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln[(t_{h1} - t_{c1})/(t_{h2} - t_{c2})]}$$

$$= \frac{(520 - 250) - (340 - 250)}{\ln[(520 - 250)/(340 - 250)]} = \frac{270 - 90}{\ln(270/90)} = 163.84°\text{C}$$

Further,
$$Q = \dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2})$$

$$= 24.44 \times (1.172 \times 10^3)(520 - 340) = 5.147 \times 10^6 \text{ W}$$

Neglecting the steam side thermal resistance, we can write

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i}{k_{copper}} \ln(r_o / r_i)$$

$$= \frac{1}{285.54} + \frac{0.014}{384} \ln(0.0160 / 0.0140) = 3.507 \times 10^{-3}$$

$$U_i = \frac{1}{0.003507} = 285.14 \text{ W/m}^2\text{°C}$$

$$Q = U_i A_i \theta_m$$

$$5.156 \times 10^6 = 285.14 \times A_i \times 163.84$$

$$A_i = 110.37 \text{ m}^2$$

Again,
$$G = \rho V = \frac{320000}{3600} \quad \text{(Given)}$$

∴
$$V = \frac{320000}{3600} \times \frac{1}{3.26} = 27.267 \text{ m/s}$$

$$(\because \rho = 3.26 \text{ kg/m}^3 \ldots\text{given})$$

The mass flow rate of CO_2 through the tubes is given by

$$\dot{m} = A_i V\rho = \frac{\pi}{4} d_i^2 \times V \times \rho \times N \quad \text{(where, N number of tubes)}$$

or $$\frac{88000}{3600} = \frac{\pi}{4} \times (0.028)^2 \times 27.267 \times 3.26 \times N$$

or $$N = 446.6 \simeq 447$$

Now, $$A_i = (\pi d_i L) \times N$$

$$110.37 = \pi \times 0.028 \times L \times 446.6$$

or, $$L = \frac{110.37}{\pi \times 0.028 \times 446.6} \simeq 2.81 \text{ m}$$ **Ans.**

3. (c) Solution:

Given : N = 450 rpm; brake load = 90 kg; imep = 3.5 bar; $\dot{m}_f$ = 8.2 kg/h; Δt_w = 40°C;

$\dot{m}_w$ = 435 kg/h; D = 1.23 m; d = 0.24 m; l = 0.28 m; C.V. = 43 MJ/kg; A/F = 30

$$bp = \frac{2\pi NT}{60000} = \frac{2\pi \times 450 \times 90 \times 9.81}{60000} \times \frac{1.23}{2} = 25.587 \text{ kW}$$

$$ip = \frac{p_{im} LAn}{60000}$$

$$= \frac{3.5 \times 10^5 \times 0.28 \times \frac{\pi}{4} \times 0.24^2 \times 450}{60000} = 33.25 \text{ kW}$$

$$\eta_{ith} = \frac{33.25 \times 60}{\frac{8.2}{60} \times 43000} \times 100 = 33.94\%$$ **Ans.**

$$bsfc = \frac{8.2 \times 1000}{25.587} = 320.47\, g/kWh$$

$$V_s = \frac{\pi}{4} D^2 Ln = \frac{\pi}{4} \times 0.24^2 \times 0.28 \times 450 = 5.70\, \text{m}^3/\text{min}$$

$$\dot{m}_a = A/F \times \dot{m}_f = 30 \times \frac{8.2}{60} = 4.1 kg/min$$

$$\rho = \frac{P}{RT} = \frac{1 \times 10^5}{287 \times 292} = 1.193 kg/m^3$$

$$\text{Air consumption in } m^3 = \frac{4.1}{1.193} = 3.43 m^3/min$$

$$\eta_v = \frac{3.43}{5.70} \times 100 = 60.28\%$$

$$\text{Heat input} = \frac{8.2}{60} \times 43000 = 5876.6 kJ / min = 100\%$$

Heat equivalent of bp = 25.587 × 60 = 1535.2 kJ/min = 26.12%

$$\text{Heat lost to cooling water} = \frac{435}{60} \times 4.18 \times 40 = 1212.2 \text{ kJ/min} = 20.63\%$$

$$\text{Heat carried away by dry exhaust gas} = \frac{8.2}{60} \times (30 + 1 - 9 \times 0.15) \times 1 \times (350 - 19)$$

$$= 1341.27 \text{ kJ/min} = 22.82\%$$

$$\text{Heat carried away by steam} = 9 \times 0.15 \times \frac{8.2}{60} \times (3250 - 4.18 \times 19) = 584.97 \text{ kJ/min} = 9.95\%$$

Unaccounted loss (by difference) = 5876.67 – 1535.2 – 1212.2 – 1341.27 – 584.97

= 1203.03 kJ/min = 20.47% **Ans.**

Heat input (per minute)	%	Heat expenditure (per minute)	%
Heat supplied by fuel	100	1. Heat equivalent to bp	26.12
		2. Heat lost to cooling water	20.63
		3. Heat carried away by dry exhaust	22.82
		4. Heat lost in steam	9.95
		5. Unaccounted losses	20.47

4. (a) Solution:

Given data : m_1 = 15 kg; m_2 = 8 kg

At 40°C, R-134a (saturated liquid)

$$P_1 = 10.166 \text{ bar}$$

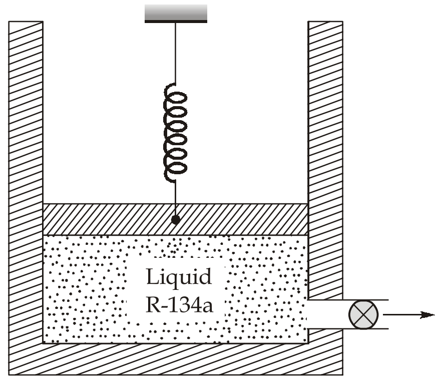
$$v_1 = \frac{1}{1146.7} = 8.7206 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$h_1 = 256.41 \text{ kJ/kg}$$

$$u_1 = h_1 - p_1 v_1$$

$$u_1 = 256.41 - \frac{10.166 \times 10^5 \times 8.7206 \times 10^{-4}}{10^3}$$

$$u_1 = 255.523 \text{ kJ/kg}$$



$$v_1 = 15 \times 8.726 \times 15^4 \text{ m}^3$$
$$= 13.0809 \text{ litre}$$

At 16°C, $P_2 = 5.0425$ bar

$$v_2 = \frac{1}{1239.8} = 8.065 \times 10^{-4} \text{m}^3/\text{kg}$$
$$h_2 = 221.87 \text{ kJ/kg}$$
$$u_2 = h_2 - p_2 v_2 = 221.87 - \frac{5.0425 \times 10^5 \times 8.065 \times 10^{-4}}{10^3}$$
$$u_2 = 221.46 \text{ kJ/kg}$$
$$v_2 = 7 \times 8.065 \times 10^{-4} \text{ m}^3$$
$$= 5.645 \times 10^{-3} \text{ m}^3$$
$$= 5.645 \text{ litre}$$

$$\text{Movement of piston} = \frac{V_1 - V_2}{\text{Area}} = \frac{(13.0809 - 5.645) \times 10^3 \text{cm}^3}{350 \text{ cm}^2}$$
$$= 21.24 \text{ cm}$$

During this movement, the cylinder pressure drops from 10.166 bar to 5.0425 bar. This corresponds to a change in resistance force of the spring by

$$= (10.166 - 5.0425) \times 10^5 \times 350 \times 10^{-4} \times 10^{-3} \text{ kN}$$
$$= 17.93 \text{ kN}$$

$$\text{Spring constant, } k = \frac{\text{Change in force}}{\text{Change in length}}$$
$$= \frac{17.93}{0.2124} = 84.427 \text{ kN/m}$$

Now, considering the control volume as the volume of cylinder at t = 0, we can analyze the transient process by making appropriate assumptions. We approximate the process by assuming that liquid is flowing out with a constant value of enthalpy equal to the average of initial and final value i.e.

$$h_e = \frac{h_1 + h_2}{2} = \frac{256.41 + 221.87}{2}$$

$$h_e = 239.14 \text{ kJ/kg}$$

Now, work transfer to the control volume during this process is the work done through movement of spring and can be calculated as

$$W_{cv} = \int F dx = \int_{V_1}^{V_2} -P dV \quad \text{...(i)}$$

Since, spring is linear pressure can be written as

$$P = a + bV$$

At V = 13.0809 litre; P = 10.166 bar

At V = 5.645 litre; P = 5.0425 bar

So, $$10.166 = a + b \times 13.0809$$

$$5.0425 = a + b \times 5.645$$

On solving, $$a = 1.163$$

$$b = 0.688$$

$$P = 1.163 + 0.688\,V$$

From equation (i), $$W_{cv} = -\int_{5.645}^{13.0809} (1.163 + 0.688V)dV$$

$$W_{cv} = 56.547 \text{ bar litre}$$

$$W_{cv} = 56.547 \times 10^5 \times 10^{-3} \text{ J}$$

$$W_{cv} = 5.655 \text{ kJ}$$

By neglecting the change in kinetic and potential energy, energy equation for control volume can be written as

$$U_2 - U_1 = \cancelto{0}{m_i h_i} + Q_{cv} - m_e h_e - W_{cv}$$

$$m_2 u_2 - m_1 u_1 = Q_{cv} - m_e h_e - W_{cv}$$

$$7 \times 221.46 - 15 \times 255.523 = Q_{cv} - (15 - 7) \times 239.14 - (-5.655)$$

On solving, $Q_{cv} = -375.16$ kJ

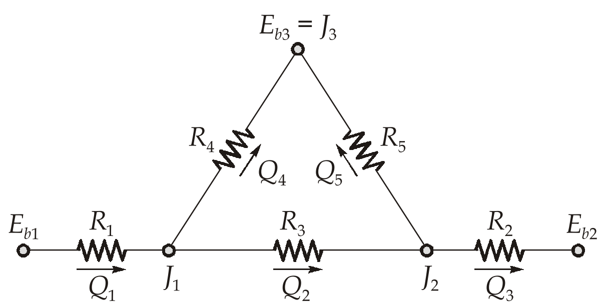
i.e. there is a heat transfer to surroundings of 375.16 kJ.

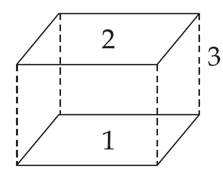
4. (b) Solution:

Given: $A_1 = A_2 = 1\ m^2$, $L = 0.5$, $T_3 = 27°C = 300$ K, $T_1 = 900°C = 1173$ K, $T_2 = 400°C = 673$ K, $\in_1 = 0.2$, $\in_2 = 0.5$, $F_{12} = 0.42$

Here, suffix 1, 2 and 3 denotes plate 1, plate 2 and room respectively.

For three surfaces, radiation network is shown in figure.





Now, $F_{1-2} + F_{1-3} = 1$

$$F_{1-3} = 1 - F_{1-2} = 1 - 0.42 = 0.58$$

and $F_{2-1} + F_{2-3} = 1$

$$F_{2-3} = 1 - F_{2-1} = 1 - 0.42 = 0.58 \qquad [\because F_{1-2} = F_{2-1}]$$

The values of the resistance are:

$$R_1 = \frac{1 - \in_1}{\in_1 A_1} = \frac{1 - 0.2}{1 \times 0.2} = 4$$

$$R_2 = \frac{1 - \in_2}{\in_2 A_2} = \frac{1 - 0.5}{1 \times 0.5} = 1$$

$$R_3 = \frac{1}{A_1 F_{1-2}} = \frac{1}{1 \times 0.42} = 2.38$$

$$R_4 = \frac{1}{A_1 F_{1-3}} = \frac{1}{1 \times 0.58} = 1.72 = R_5$$

For large room, $\frac{1 - \in_3}{A_3 \in_3} = 0$ and $E_{b3} = J_3$

Node J_1: $\frac{E_{b1} - J_1}{4} + \frac{J_2 - J_1}{2.38} + \frac{E_{b3} - J_1}{1.72} = 0 \quad \ldots \text{(i)}$

Node J_2: $\frac{J_1 - J_2}{2.38} + \frac{E_{b3} - J_2}{1.72} + \frac{E_{b2} - J_2}{1} = 0$ ... (ii)

Where,

$$E_{b1} = \sigma T_1^4 = 5.67 \times 11.73^4 = 107.34 \text{ kW/m}^2$$
$$E_{b2} = \sigma T_2^4 = 5.67 \times 6.73^4 = 11.63 \text{ kW/m}^2$$
$$J_3 = E_{b3} = \sigma T_3^4 = 5.67 \times 3^4 = 0.46 \text{ kW/m}^2$$

Substituting the values of E_{b1}, E_{b2} and E_{b3} into the above equation for nodes J_1 and J_2 we get,

$$J_1 = 25.46 \text{ kW/m}^2, J_2 = 11.29 \text{ kW/m}^2$$

∴ Total heat lost by plate 1,

$$Q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{107.34 - 25.46}{4} = 20.47 \text{ kW}$$ **Ans.**

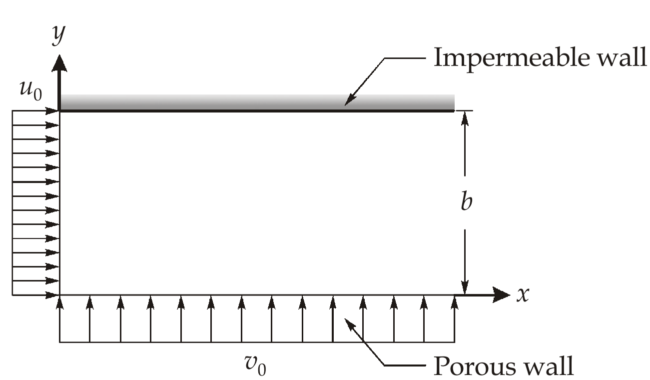
Total heat lost by plate 2,

$$Q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{11.63 - 11.29}{1} = 0.34 \text{ kW}$$ **Ans.**

Total heat received/absorbed by room,

$$Q_{room} = Q_4 + Q_5$$
$$= \frac{J_1 - J_3}{R_4} + \frac{J_2 - J_3}{R_5}$$
$$= \frac{25.46 - 0.46}{1.72} + \frac{11.29 - 0.46}{1.72} \quad [\because E_{b3} = J_3]$$
$$Q_{room} = 20.83 \text{ kW}$$

4. (c) Solution:



Given: $u = f(x)$ only

$$u_{x=0} = u_0$$

$$v = f(y) \text{ only}$$

$$v_{y=0} = v_0$$

(i) Mass conservation principle for incompressible 2-D flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

Since, $u = f(x)$ only and $v = f(y)$ only

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= f(x) \text{ only} \\ -\frac{\partial v}{dy} &= f(y) \text{ only} \end{aligned}\right\}$$ can be same only when they are equal to same constant value (say k)

i.e.,
$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = k \quad \text{...(i)}$$

from (i)

$$\frac{\partial u}{\partial x} = k$$

$$u = kx + c_1 \quad \text{...(ii)}$$

$$\frac{\partial v}{dy} = -k$$

$$v = -ky + c_2 \quad \text{...(iii)}$$

Using boundary conditions:

$$u_{x=0} = u_0$$

$$v_{y=0} = v_0$$

$$v_{y=b} = 0$$

$$C_1 = u_0$$

$$C_2 = v_0$$

$$k = \frac{v_0}{b}$$

Substituting these in (ii) and (iii),

$$\left.\begin{aligned} u &= u_0 + \left(\frac{v_0}{b}\right)x \\ v &= v_0 - \left(\frac{v_0}{b}\right)y \end{aligned}\right\} \quad \textbf{Ans.}$$

(ii) Stream function and velocity potential functions:

Stream function is given by

$$\frac{\partial\psi}{\partial x} = v \text{ and } \frac{\partial\psi}{\partial y} = -u$$

$$\partial\psi = v\partial x = \left(v_0 - \frac{v_0 y}{b}\right)dx$$

$$\psi = \left(v_0 - \frac{v_0 y}{b}\right)x + f(y) = v_0 x - \frac{v_0 xy}{b} + f(y) \quad \text{...(iv)}$$

$$\partial\psi = -u\partial y = -\left(u_0 + \frac{v_0 y}{b}\right)\partial y$$

$$\psi = -u_0 y - \frac{v_0 xy}{b} + f(x) \quad \text{...(v)}$$

$$\psi = v_0 x - u_0 y - \frac{v_0 xy}{b} \quad \textbf{Ans.}$$

Velocity potential function is given by

$$\frac{\partial\phi}{\partial x} = -u \text{ and } \frac{\partial\phi}{\partial y} = -v$$

$$\partial\phi = -udx = -\left(u_0 + \frac{v_0}{b}x\right)\partial x$$

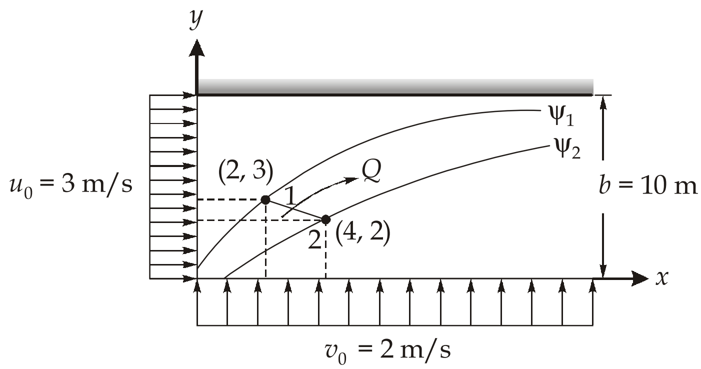
$$\phi = -u_0 x - \frac{v_0}{b}\frac{x^2}{2} + f(y) \quad \text{...(v)}$$

$$\partial\phi = -v\partial y = -\left(v_0 - \frac{v_0}{b}y\right)dy$$

$$\phi = -v_0 y + \frac{v_0}{b}\frac{y^2}{2} + f(x) \quad \text{...(vi)}$$

$$\phi = -v_0 y - u_0 x + \frac{v_0}{2b}\left(y^2 - x^2\right) \quad \textbf{Ans.}$$

Now; assuming points, (2, 3) and (4, 2) lies in flow regime.



$$\psi_{(2,\ 3)} = 2v_0 - 3u_0 - \frac{6v_0}{b} = 4 - 9 - \frac{12}{10} = -6.2$$

$$\psi_1 \ : \ v_0x - u_0y - \frac{v_0xy}{b} = -6.2$$

$$\psi_1 \ : \ 2x - 3y - 0.2xy = -6.2$$

$$\psi_{(4,\ 2)} = 4v_0 - 2u_0 - \frac{8v_0}{b} = 8 - 6 - \frac{16}{10} = 0.4$$

$$\psi_2 \ : \ v_0x - u_0y - \frac{v_0xy}{b} = 0.4$$

$$\psi_2 \ : \ 2x - 3y - 0.2xy = 0.4$$

Discharge per unit width

$$\frac{Q}{W} = |\psi_1 - \psi_2| = |-6.2 - 0.4| = |-6.6|$$

$$\frac{Q}{W} = 6.6 \text{ m}^2/\text{s}$$ **Ans.**

Section : B

5. (a) Solution:

A capillary tube is used as a refrigerant control devices in refrigerating systems because of various advantages:

1. It is inexpensive.
2. It does not have any moving parts; hence, it does not require maintenance.
3. It is found to be more advantageous with on-off control because of its unloading characteristics.

4. When the compressor stops, the refrigerant continues to flow from the high-pressure side to the low-pressure side until the pressure is equalized. This requires less starting torque to start the compressor, so a low starting torque motor (low-cost motor) can be used with these units.
5. Systems using this device do not require a receiver.
6. Ideal for hermetic compressor-based systems, which are critically charged and factory assembled.

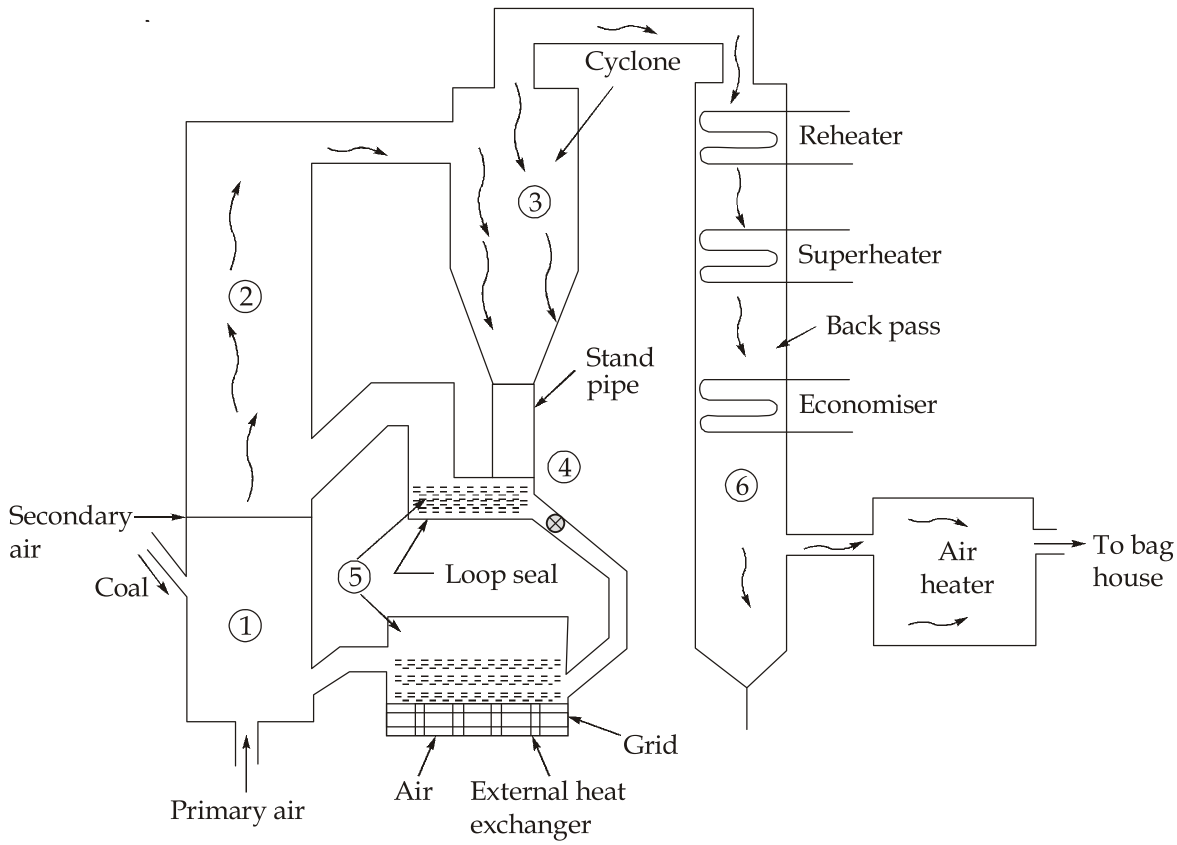
Disadvantages of capillary tube:

Once a capillary tube is selected, it will be suitable only for that design pressure and flow. It cannot satisfy the flow requirements with changing condenser and evaporator pressures.

5. (b) Solution:

The CFB boiler is said to be the second generation fluidized bed boiler. It is divided into two sections. The first section consists of (a) furnace or fast fluidized bed, (b) gas-solid separator (cyclone), (c) solid recycle device (loop seal or L-valve), and (d) external heat exchanger (optional). These components form a solid circulation loop in which fuel is burned. The furnace enclosure of a CFB boiler is generally made of water tubes as in pulverized coal-fired (PC) boilers. A fraction of the generated heat is absorbed by these heat transferring tubes. The second section is the back-pass, where the remaining heat from the flue gas is absorbed by the reheater, superheater, economizer, and air preheater surfaces.

The lower part of the first section (furnace) is often tapered. Its walls are lined with refractory up to the level of secondary air entry. Beyond this the furnace walls are generally cooled by evaporative, superheater, or reheater surfaces. The gas-solid separator and the non-mechanical valve are also lined with refractory. In some designs, a part of the hot solids recycling between the cyclone and the furnace is diverted through an external heat exchanger, which is a bubbling fluidized bed with beat transfer surfaces immersed in it to remove heat from the hot solids. Coal is generally injected into the lower section of the furnace. It is sometimes fed into the loop-seal, from which it enters the furnace along with returned hot solids. Limestone is fed into the bed in a similar manner. Coal burns when mixed with hot bed solids.



1. *Furnace (below secondary air level) - Turbulent/bubbling fluidized bed.*
2. *Furnace (above secondary air level) - Fast fluidized bed.*
3. *Cyclone - Swirl flow.*
4. *Return leg (stand pipe) - moving packed bed.*
5. *Loop seal/external heat exchanger - bubbling fluidized bed.*
6. *Back pass - pneumatic transport.*

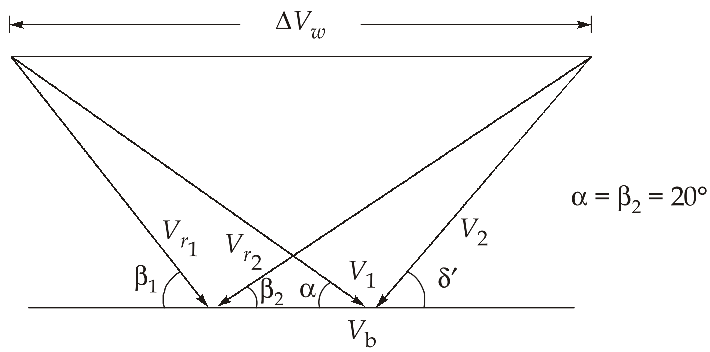
Figure : Flow regimes in a CFB boiler

The primary combustion air enters the furnace through an air distributor or grate at the furnace floor. The secondary air is injected at some height above the grate to complete the combustion. Bed solids are well mixed throughout the height of the furnace. Thus, the bed temperature is nearly uniform in the range 800-900°C, though heat is extracted along its height. Relatively coarse particles of sorbent (limestone) and unburned char, larger than the cyclone cut-off size, are captured in the cyclone and are recycled back near the base of the furnace. Finer solid residues (ash and spent sorbents) generated during combustion and desulphurization leave the furnace, escaping through the cyclones, but they are collected by a bag-house or electrostatic precipitator located further downstream.

5. (c) Solution:

Given : $V_{r_2} = V_1, V_{r_1} = V_2, \alpha = \beta_2 = 20°; V_b = 110$ m/s; $\frac{V_b}{V_1} = 0.58$; $h_b = 26$ mm,

$v = 0.68$ m^3/kg, $N = 3100$ rpm



$$V_b = 110 \text{ m/s}, \; V_b / V_1 = 0.58$$

$$V_1 = \frac{110}{0.58} = 189.65 \text{ m/s} = V_{r_2}$$

$$V_b = \frac{\pi D_m N}{60} = \frac{\pi D_m \times 3100}{60} = 110 \text{ m/s}$$

$$D_m = 0.6776 \text{ m}$$

$$m_s v = \pi D_m h_b k_{tb} V_1 \sin\alpha$$

$$\therefore \quad m_s = \frac{\pi \times 0.6776 \times 26 \times 1 \times 189.65 \times \sin 20°}{1000 \times 0.68} = 5.28 \text{ kg/s}$$

$$= 19006.2 \text{ kg/h}$$

$$\tan\beta_1 = \frac{V_1 \sin\alpha}{V_1 \cos\alpha - V_b} = \frac{189.65 \sin 20°}{189.65 \cos 20° - 110}$$

$$\beta_1 = 43.55°$$

$$V_{r_1} = \frac{V_1 \sin\alpha}{\sin\beta_1} = \frac{189.65 \sin 20°}{\sin 43.55°} = 94.14 \text{ m/s}$$

$$\Delta h_{mb} = \frac{1}{2}(V_{r_2}^2 - V_{r_1}^2) = \frac{(V_{r_2} + V_{r_1})(V_{r_2} - V_{r_1})}{2}$$

$$= \frac{(189.65+94.14)(189.65-94.14)}{2} = 13552.39 = 13.55 \text{ kJ/kg}$$

$\therefore$ $$\Delta h_{stage} = \Delta h_{fb} + \Delta h_{mb} = 2\Delta h_{mb} = 2\times 13.55 = 27.10 \text{ kJ/kg}$$

$$= 2 \times 13.55 = 27.10 \text{ kJ/kg}$$

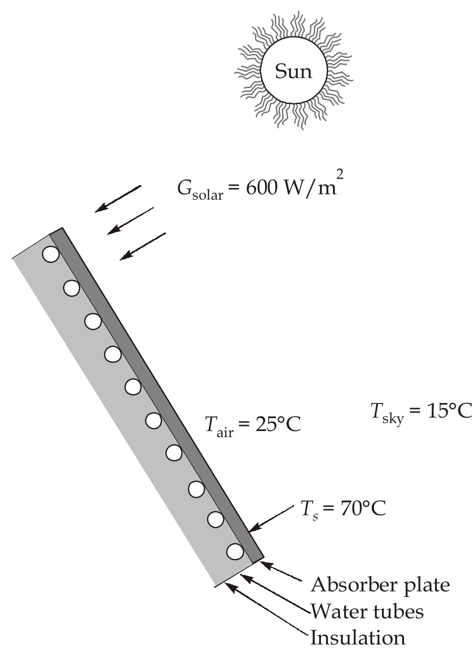
For 5 pairs of blades, $(\Delta h)_{total} = 5 \times 27.10 = 135.52$ kJ/kg

$\therefore$ Diagram power $= 5.28 \times 135.52 = 715.54$ kW

5. (d) Solution:

Given: Solar absorptivity, $\alpha_s = 0.87$

Plate emissivity, $\in = 0.09$



The net rate of solar energy delivered by the absorber plate to the water behind it can be determined from an energy balance to be given as

$$Q_{net} = Q_{gain} - Q_{loss}$$

$$Q_{gain} = \alpha_s G_{solar}$$

$$= 0.87 \times 600 = 522 \text{ W/m}^2$$

Heat loss due to convection, $Q_{convection} = h(T_s - T_{air})$

$$Q_{conv.} = 10 \times (70 - 25) \quad \text{(for unit area)}$$

$$= 450 \text{ W/m}^2$$

Absorber plate temperature, $T_s = 70°C = 70 + 273 = 343$ K

sky temperature, $T_{sky} = 15°C = 15 + 273 = 288$ K

Heat loss due to radiation, $Q_{radiation} = \in\sigma(T_s^4 - T_{sky}^4)$

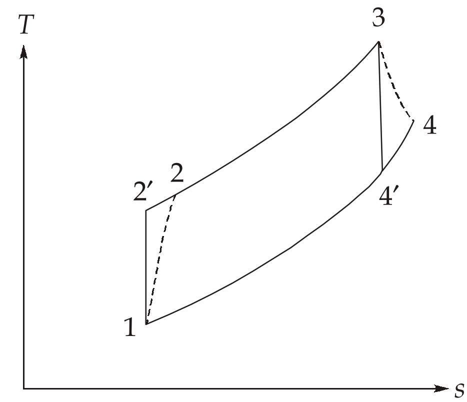
$$Q_{rad.} = 0.09 \times 5.67 \times 10^{-8}(343^4 - 288^4)$$
$$= 32.525 \text{ W/m}^2$$

Total heat loss, $Q_{loss} = Q_{conv} + Q_{rad.} = 450 + 35.525 = 485.525$ W/m^2

$$Q_{net} = 522 - 450 - 32.525 = 36.475 \text{ W/m}^2$$

Therefore, heat is gained by the plate and transferred to water at a rate of 36.475 W per m^2 of surface area.

5. (e) Solution:



Given : $r_p = 11$; $Q = 475$ kJ/kg

$\eta_T = 72\%$; $T_1 = 300$ K

$$W_{net} = 0$$
$$W_T - W_C = 0$$
$$W_T = W_C$$
$$C_p(T_3 - T_4) = C_p(T_2 - T_1)$$
$$T_3 - T_4 = T_2 - T_1$$
$$T_3 - T_2 = T_4 - T_1 \quad \text{...(i)}$$
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$\frac{T_{2'}}{T_1} = (11)^{\frac{1.4-1}{1.4}} = 1.984$$
$$T_{2'} = 300 \times 1.984 = 595.2 \text{ K}$$

Also,
$$\frac{T_3}{T_{4'}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$
$$\frac{T_3}{T_{4'}} = (11)^{\frac{1.4-1}{1.4}} = 1.984$$

Given that heat added = 475 kJ/kg of air

$$C_p(T_3 - T_2) = 475$$
$$T_3 - T_2 = \frac{475}{1.005} = 472.63 \quad \text{...(ii)}$$

$$\text{Turbine efficiency, } \eta_T = \frac{T_3 - T_4}{T_3 - T_{4'}}$$

$$0.72 = \frac{1 - \dfrac{T_4}{T_3}}{1 - \dfrac{T_{4'}}{T_3}}$$

$$0.72 = \frac{1 - \dfrac{T_4}{T_3}}{1 - \dfrac{1}{1.984}}$$

$$\frac{T_4}{T_3} = 0.643 \quad \text{...(iii)}$$

From equation (i) and (ii),

$$T_4 = T_1 + (T_3 - T_2)$$
$$= 300 + 472.63 = 772.63 \text{ K}$$

From equation (iii), $T_3 = \dfrac{772.63}{0.643} = 1201.6$ K

$$\text{Temperature ratio, } t = \frac{T_{max}}{T_{min}} = \frac{1201.6}{300} = 4.005$$

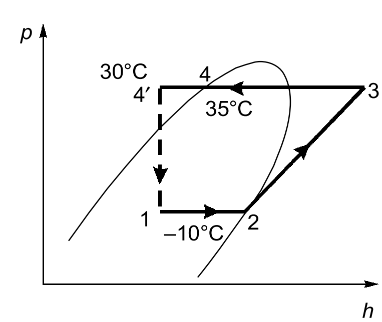
From equation (ii), $T_2 = 1201.6 - 472.63 = 728.97$ K

Now, compressor efficiency,

$$\eta_c = \frac{T_{2'} - T_1}{T_2 - T_1} = \frac{595.2 - 300}{728.97 - 300}$$
$$= 0.6882 \text{ or } 68.82\%$$

6. (a) Solution:

for isentropic compression process 2 - 3, we have :



$$S_2 = S_3$$

$$1.733 = 1.715 + 1.1\ln\left(\frac{t_3 + 273}{35 + 273}\right)$$

$$\frac{t_3 + 273}{308} = \exp\left(\frac{1.733 - 1.715}{1.1}\right)$$

$$t_3 + 273 = 308 \times 1.0165$$

$$t_3 = 40.08°C$$

$$h_3 = h_g + c_{pg}(t_3 - t_{condensation})$$

$$h_3 = 417.6 + 1.1(40.08 - 35) = 423.19 \text{ kJ/kg}$$

$$h_2 = 392.4 \text{ kJ/kg}$$

$$h_1 = h_{f4'} = h_{f4} - c_{pl}(35 - 30) = 249.1 - 1.458 \times 5$$

$$= 241.81 \text{ kJ/kg}$$

$$v_{g2} = 0.0994 \text{ m}^3\text{/kg}$$

Volumetric efficiency, $\eta_{vol} = 1 + c - c\left(\frac{P_2}{P_1}\right)^{1/n}$

$$= 1 + 0.03 - 0.03\left(\frac{8.87}{2.014}\right)^{1/1.12} = 0.9173$$

Volume of refrigerant admitted at state '2'

$$= 269.4 \times 10^{-6} \times 0.9173 = 247.12 \times 10^{-6} \text{ m}^3\text{/cycle}$$

Assuming the compressor to be single-acting, mass of air circulated

$$\dot{m} = \frac{247.12 \times 10^{-6} \times 2800}{0.0994} = 6.961 \text{ kg/min}$$

(i) Refrigeration capacity $= \frac{\dot{m}(h_2 - h_1)}{60 \times 3.5} = \frac{6.961(392.4 - 241.81)}{60 \times 3.5}$

$= 5$ tonnes **Ans.**

(ii) Power input $= \frac{\dot{m}(h_3 - h_2)}{60} = \frac{6.961}{60} \times (423.19 - 392.4)$

$= 3.572$ kW **Ans.**

(iii) $$\text{COP} = \frac{\text{Refrigerating effect}}{\text{Power}} = \frac{\dot{m}(h_2 - h_1)}{60 \times p}$$

$$= \frac{6.961}{60} \times \frac{1}{3.572} \times (392.4 - 241.81) = 4.89$$ **Ans.**

(iv) Heat rejected to condenser $= \dot{m}(h_3 - h_{4'})$

$$= 6.961(423.19 - 241.81) = 1262.6 \text{ kJ/min}$$

$$= 21.05 \text{ kW}$$ **Ans.**

(v) $$(\text{COP})_{\text{Carnot}} = \frac{273 - 10}{35 + 10} = 5.844$$

Ratio of actual to Carnot COP $= \dfrac{4.89}{5.844} = 0.837$ **Ans.**

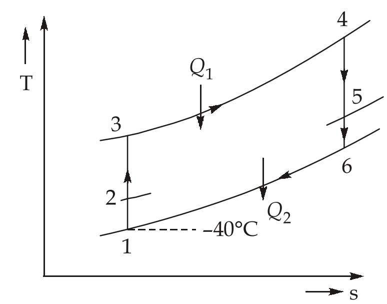
6. (b) Solution:

Using SFEE for diffuser,

$$Q_{1-2} - W_{1-2} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

(As $W_{1-2} = 0$, $Q_{1-2} = 0$ and $V_2 = 0$ for diffuser)

$$h_2 - h_1 = \frac{V_1^2}{2}$$



$$T_2 - T_1 = \frac{V_1^2}{2c_p}$$

$$= 233 + \frac{(350)^2}{2 \times 1.005} \times 10^{-3} = 233 + 60.94$$

$$\boldsymbol{T_2} = 293.945 \text{ K}$$

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$P_2 = 0.35 \times 10^2 \left(\frac{293.945}{233}\right)^{\frac{1.4}{0.4}} = 78.93 \text{ kPa}$$

$$P_3 = r_p \times P_2 = 10 \times 78.93 = 789.3 \text{ kPa}$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\gamma-1/\gamma} = (10)^{0.4/1.4} = 1.93$$

$$T_3 = 293.945 \times 1.93 = 567.518 \text{ K}$$

$$W_C = W_T$$

$$h_3 - h_2 = h_4 - h_5$$

$$T_3 - T_2 = T_4 - T_5$$

$$T_5 = T_4 - T_3 + T_2$$

$$= 1373 - 567.518 + 293.945 = 1099.43 \text{ K}$$ **Ans.**

$$P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\gamma/\gamma-1} = 789.3\left(\frac{1099.43}{1373}\right)^{3.5}$$

$[P_3 = P_4 = 789.3 \text{ kPa}]$

$$P_5 = 362.64 \text{ kPa}$$ **Ans.**

For isentropic expansion of gases in the nozzle,

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{\gamma-1}{\gamma}} = 1099.63\left(\frac{35}{362.64}\right)^{0.286} = 563.704 \text{ K}$$

Neglecting the kE of gas at nozzle inlet,

$$V_6 = (2c_p(T_5 - T_6) \times 1000)^{1/2}$$

$$= [2 \times 1005(1099.43 - 563.704)]^{1/2} = 1037.69 \text{ m/s}$$

The propulsive efficiency of a turbojet engine is the ratio of the propulsive power developed $\dot{W}_P$ to the total heat transfer to the fluid.

$$\dot{W}_P = \dot{m}[V_{exit} - V_{inlet}]V_{aircraft}$$

$$= 60[1037.69 - 350] \times 350 = 14.45 \text{ MW}$$

$$Q = \dot{m}[h_4 - h_3] = \dot{m}c_p[T_4 - T_3]$$

$$= 60 \times 1.005(1373 - 567.518) = 48.57 \text{ MW}$$

$$\eta_{propulsive} = \frac{14.45}{48.583} = 0.2975 \text{ or } 29.75\%$$ **Ans.**

6. (c) (i) Solution:

Volume of the basin, AH = $0.78 \times 10^6 \times 7\ m^3 = 5.46 \times 10^6\ m^3$

$$\text{Average discharge, } Q = \frac{AH}{t} = \frac{0.78 \times 10^6 \times 7}{4 \times 3600} = 379.17\ m^3/s$$

$$\text{Power at any point of time, } P = \frac{379.17 \times 1025 \times 7}{75} \times 0.736 \times 0.88$$

$$= 234.93 \times 10^2\ kW$$

Energy generated per tidal cycle = $234.93 \times 10^2 \times 4$ kWh

$= 939.75 \times 10^2$ kWh

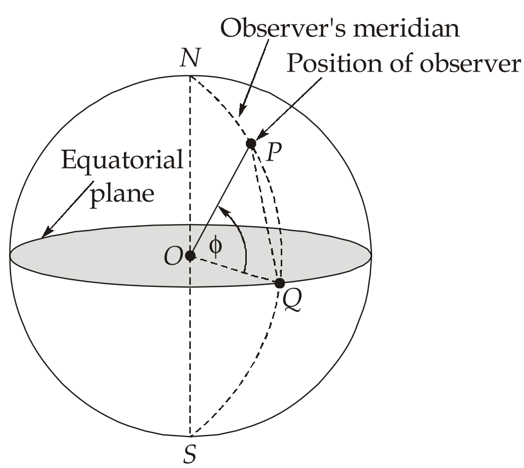
Total number of tidal cycles in a year = 705

∴ Yearly energy generation = $939.75 \times 10^2 \times 705$ kWh

$= 662.5 \times 10^5$ kWh

6. (c) (ii) Solution:

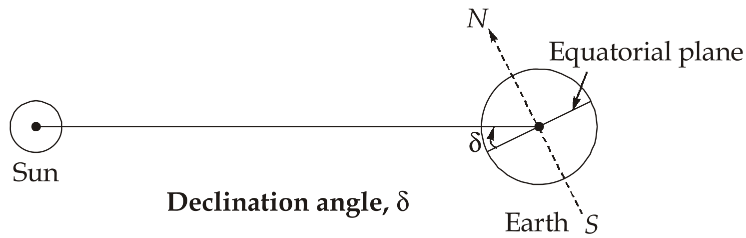
1. **Latitude (Angle of Latitude), (ϕ):** The latitude of a location on earth's surface is the angle made by radial line, joining the given location to the center of the earth, with its projection on the equator plane as shown below. The latitude is positive for northern hemisphere and negative for southern hemisphere.



2. **Declination,(δ):** It is defined as the angular displacement of the sun from the plane of earth's equator as shown in Fig. It is positive when measured above equatorial plane in the northern hemisphere. The declination δ can be approximately determined from the equation:

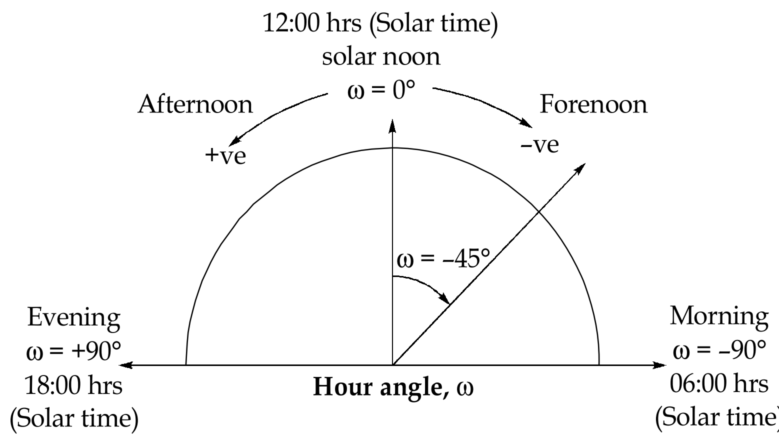
$$\delta = 23.45 \times \sin\left[\frac{360}{365}(284 + n)\right] \text{degrees}$$

where n is day of the year counted from 1st January.



Declination angle, δ

3. **Hour angle,(ω):** The hour angle at any moment is the angle through which the earth must turn to bring the meridian of the observer directly in line with sun's rays.



Hour angle, ω

In other words, at any moment, it is the angular displacement of the sun towards east or west of local meridian (due to rotation of the earth on its axis). The earth completes one rotation in 24 hours. Therefore, one hour corresponds to 15° of rotation. At solar noon, as sunrays are in line with local meridian, hour angle is zero. It is positive in the forenoon and negative in the afternoon. Thus at 06:00 hrs it is –90° and at 18:00 hrs it is +90° as shown above. We adopt the convention of measuring it from noon based on LAT, being positive in the morning and negative in the afternoon.

It can be calculated as:

$$\omega = [\text{Solar time} - 12{:}00] \text{ (in hours)} \times 15 \text{ degrees}$$

7. **(a) Solution:**

Given : $\beta_1 = 44°$; $\beta_2 = 12°$; $u = 215$ m/s; $r_p = 5$; $\phi = 0.88$; $\eta_{isen} = 0.86$

$$T_{02s} = T_{01}(r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\eta_{pc} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)$$

$$\frac{n-1}{n} = \frac{\gamma-1}{\gamma} \times \frac{1}{\eta_{pc}}$$

$$T_{02} = T_{01}(r_p)^{\frac{n-1}{n}} = T_{01}(r_p)^{\left(\frac{\gamma-1}{\gamma}\right)\times\frac{1}{\eta_{pc}}}$$

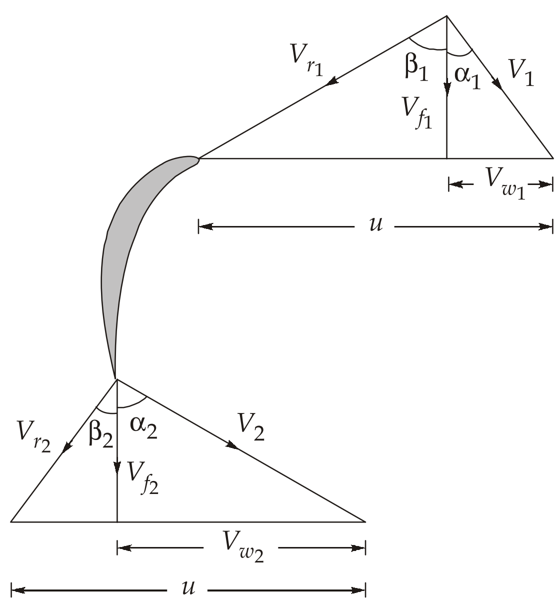
$$\eta_{isen} = \frac{T_{02s}-T_{01}}{T_{02}-T_{01}} = \frac{T_{01}(r_p)^{\frac{\gamma-1}{\gamma}} - T_{01}}{T_{01}(r_p)^{\frac{\gamma-1}{\gamma}\times\frac{1}{\eta_{pc}}} - T_{01}}$$

$$0.86 = \frac{5^{\frac{1.4-1}{1.4}} - 1}{5^{\frac{1.4-1}{1.4}\times\frac{1}{\eta_{pc}}} - 1}$$

On solving, $\eta_{pc} = 0.8875$ or 88.75%

Since blades are symmetrical, degree of reaction is 50%.

$$\alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1$$



From the velocity triangle,

$$u = V_{f_1}(\tan\alpha_1 + \tan\beta_1)$$

For axial flow compressor, $V_{f_1} = V_{f_2} = V_f$

$$V_f = \frac{u}{\tan\alpha_1 + \tan\beta_1}$$

$$V_f = \frac{215}{\tan 44 + \tan 12} = 182.47 \text{ m/s}$$

Now, for stage temperature rise,

$$c_p \Delta T_s = \phi \Delta V_w \cdot u$$

$$\Delta T_s = \frac{\phi u}{c_p} V_f (\tan\alpha_2 - \tan\alpha_1)$$

$$\left[\text{if } \gamma = 1.4, \text{ then } c_p = \frac{\gamma R}{\gamma - 1};\ c_p = 996.1 \text{ J/kgK}\right]$$

$$= \frac{0.88 \times 215}{996.1} \times 182.47 \times (\tan 44° - \tan 12°) = 26.102 \text{ K}$$

$$T_{02} = T_{01}(r_p)^{\frac{\gamma-1}{\gamma} \times \frac{1}{\eta_{pc}}} = 291 \times (5)^{\frac{1.4-1}{1.4} \times \frac{1}{0.8875}} = 488.55 \text{ K}$$

Total temperature rise, $T_{02} - T_{01} = 488.55 - 291 = 197.55$ K

$$\text{Total number of stages} = \frac{\text{Total temperature rise}}{\text{Temperature rise in one stage}}$$

$$= \frac{197.55}{26.102} = 7.57 \simeq 8 \text{ stages}$$

$$\text{Mach number at inlet, } M_{a_1} = \frac{V_{r_1}}{\sqrt{\gamma R T_1}}$$

$$V_{r_1} = \frac{V_f}{\cos\beta_1} = \frac{182.47}{\cos 44°} = 253.66 \text{ m/s}$$

$$V_1 = \frac{V_f}{\cos\alpha_1} = \frac{182.47}{\cos 12°} = 186.55 \text{ m/s}$$

$$T_1 = T_{0_1} - \frac{V_1^2}{2c_p} = 291 - \frac{186.55^2}{2 \times 996.1} = 273.53 \text{ K}$$

$$M_{a_1} = \frac{253.66}{\sqrt{1.4 \times 284.6 \times 273.53}} = 0.768$$

Ans.

7. **(b) Solution:**

Following are the requirements of electrolyte and electrode:

Electrolyte:

1. It should be conductive to ions.
2. It should be electrically non-conductive.
3. Ions should be free to move through the electrolyte
4. The composition of electrolyte should not get changed during operation.

Electrode:

1. It should be electrically conductive.
2. It should not react with electrolyte to prevent corrosion.
3. It should be able to withstand high temperature.
4. It should also act as a catalyst to convert hydrogen and oxygen molecules.

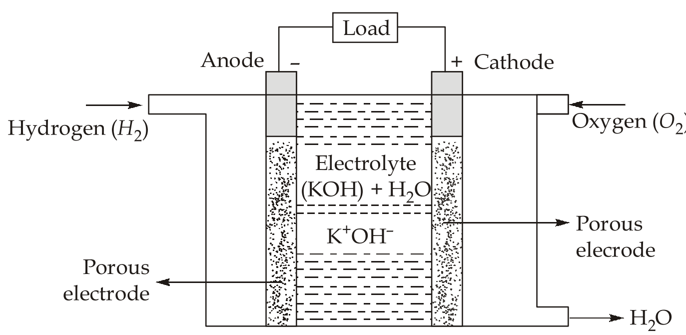


Figure: H_2–O_2 (Alkaline) Fuel Cell

In the H_2-O_2 alkaline fuel cell, hydrogen and oxygen are used as the 'fuel' and 'oxidant' respectively as these elements are most reactive with least complications. The 'electrolyte' is potassium hydroxide (20 to 40% concentration) which has high electrical conductivity and is less corrosive than acids.

Working: The gases diffuse through the electrodes, undergoing reactions as shown below:

$$KOH \rightarrow K + OH^-$$

Anode: $H_2 + 2(OH)^- \rightarrow 2H_2O + 2e^-$

The electrons so produced build up a negative potential and move towards the cathode through an externally connected circuit.

Cathode: At cathode, the electrons are picked up by oxygen atoms available there, react with water present in the electrolyte to form hydroxide $(OH)^-$ ions, the reaction being:

$$\frac{1}{2}O_2 + H_2O + 2e^- \rightarrow 2(OH)^-$$

$(OH)^-$ ions combine with hydrogen ion (H^+) to form water as per the following reactions,

$$H^+ + OH^- \rightarrow H_2O$$

Overall cell reaction:

$$2H_2 + O_2 \rightarrow 2H_2O + \text{electrical energy}$$

The water formed is drawn off from the side. The electrolyte provides the $(OH)^-$ ions needed for the reaction, and remains unchanged at the end, since these are regenerated. The electrons liberated at the anode find their way to the cathode through the external circuit. This transfer is equivalent to the flow of current from the cathode to anode.

7. (c) (i) Solution:

Combined cycle is the typical process which uses both gas (gas turbine) and steam (steam turbine) to generate the power which is 50% higher than the normal process.

Combined cycle is a term applied to gas turbine generators in which the exhaust heat from the gas turbine is used to produce steam (in a heat recovery steam generator-HRSG), which is then fed to a steam turbine. The steam turbine may be on the same shaft as the gas turbine generator, or it may be a completely separate steam turbine & electrical generator.

Applications

- Large thermal power stations
- Base load power generation
- Utility power plants Combined Cycle Power Plant

Co-generation Cycle is the method where the consumption of steam required for the process (like sugar plant or paper mill), but the consumption rate is meager and the max portion of steam used to generate the power using steam turbine for their power requirement. most of the captive power plant works in this co-generation terminology.

Cogeneration is when the heat produced from a combustion process is split between electrical generation and industrial process steam.

The 'combustion process' can be either a boiler or a gas turbine with HRSG. The 'industrial process' steam can be for truly industrial chemical processes, or it could be for non-industrial steam plants, such as campus heating and cooling.

Applications

- Sugar industries

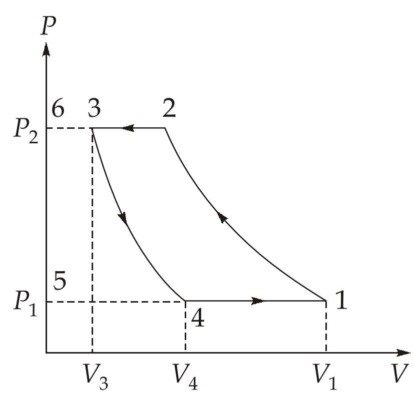
- Paper mills
- Chemical plants
- Refineries
- Captive power plants

"Combined cycle" refers specifically to a gas turbine generator with an exhaust-heated steam turbine generator to increase overall power plant efficiency. "Cogeneration" is making heat through whatever means, and dividing the heat energy between electric generation and other process needs.

7. (c) (ii) Solution:

The volumetric efficiency of a compressor is the ratio of refrigerant delivered to the displacement of the compressor. It is also the ratio of effective swept volume to the swept volume.

$$\text{Effective swept volume} = V_1 - V_4$$
$$\text{Swept volume} = V_1 - V_3 = V_s$$
$$\text{Clearance volume} = V_3 = V_c$$
$$\text{Total volume} = V_1$$



$$\text{Volumetric efficiency} = \frac{\text{Effective swept volume}}{\text{Swept volume}} = \frac{V_1 - V_4}{V_1 - V_3}$$

$$\text{Clearance ratio, } C = \frac{\text{Clearance volume}}{\text{Swept volume}} = \frac{V_3}{V_1 - V_3} = \frac{V_c}{V_s}$$

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} = \frac{(V_1 - V_3) + (V_3 - V_4)}{(V_1 - V_3)}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3} \times \frac{V_3}{V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3}\left[1 - \frac{V_4}{V_3}\right] = 1 + C - C\frac{V_4}{V_3}$$

$$P_3 V_3^n = P_4 V_4^n \text{ or } \frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

Hence, $$\eta_{vol} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{1/n} \quad [P_3 = P_2 \text{ and } P_4 = P_1]$$

8. (a) Solution:

(i)

For (U × R) to be minimum and the figure of merit to be maximum

we have, $$x = \frac{A_p L_n}{A_n L_p} = \sqrt{\frac{k_n \rho_p}{k_p \rho_m}}$$

We have, $$L_p = L_n, k_p = k_n \text{ and } \sigma = \frac{1}{\rho}$$

Therefore, $$\frac{A_p}{A_n} = \sqrt{\frac{\rho_n k_n}{\rho_p k_p}} = \sqrt{\frac{1500}{1000}} = 1.225$$

$$A_p = \frac{\pi(0.01)^2}{4} = 7.85 \times 10^{-5} \text{m}^2$$

∴ $$A_n = \frac{A_p}{1.225} = 6.41 \times 10^{-5} \text{m}^2$$

$$d_n = \sqrt{\frac{4A_n}{\pi}} = \sqrt{\frac{4 \times 6.41 \times 10^{-5}}{\pi}}$$

$$d_n = 9.034 \times 10^{-3} \text{ m}$$

with $$L_p = L_n, k_p = k_n$$

We get, $$U = \frac{(A_p + A_n)k}{L} = \frac{(7.85 \times 10^{-5} + 6.41 \times 10^{-5}) \times 1.2}{0.01}$$

$$= 0.0171 \text{ W/K}$$

$$R = \frac{L\left(\frac{1}{\sigma_p A_p} + \frac{1}{\sigma_n A_n}\right)}{100}$$

$$= \frac{0.01\left(\frac{1}{1000 \times 7.85 \times 10^5} + \frac{1}{1500 \times 6.41 \times 10^{-5}}\right)}{100}$$

$$R = 2.314 \times 10^{-3} \text{ Ohm}$$

$$U \times R = 0.0171 \times 2.314 \times 10^{-3}$$

$$= 3.96 \times 10^{-5} \text{ W-Ohm/K}$$

$$\alpha_{pn} = \alpha_p - \alpha_n = 0.00035 \text{ V/K}$$

$$z = \frac{\alpha_{pn}^2}{UR} = \frac{0.00035^2}{3.96 \times 10^{-5}} = 3.09 \times 10^{-3} \text{k}^{-1}$$

$$T_m = \frac{T_h + T_c}{2} = \frac{258 + 313}{2} = 285.5 \text{ K}$$

(ii)

For the case of maximum COP (ξ_{max})

$$\xi_{max} = \frac{T_c\left[\sqrt{1 + zT_m} - \frac{T_h}{T_c}\right]}{(T_h - T_c)\left[\sqrt{1 + zT_m} + 1\right]}$$

$$= \frac{258\left[\sqrt{1 + 3.09 \times 10^{-3} \times 285.5} - \left(\frac{313}{258}\right)\right]}{(313 - 258)\left[\sqrt{1 + 3.09 \times 10^{-3} \times 285.5} + 1\right]}$$

$$= \frac{258 \times (1.372 - 1.2132)}{55 \times (1.372 + 1)} = 0.3141$$

Also,

$$I_{max} = \frac{\alpha_{pn}\Delta T}{R\left[\sqrt{1 + zT_m} - 1\right]} = \frac{0.00035 \times 55}{2.314 \times 10^{-3}(1.372 - 1)}$$

$$= 22.36 \text{ amps}$$

$$(Q_c)_{\text{per pair}} = \alpha_{pn} I_{max} T_c - \left(I_{max}^2 \frac{R}{2}\right) - U(T_h - T_c)$$

$$= 0.00035 \times 22.36 \times 258 - \left(\frac{22.36^2 \times 2.314 \times 10^{-3}}{2}\right) - 0.0171 \times 55$$

$$(Q_c)_{\text{per pair}} = 0.5001 \simeq 0.5 \text{ W per pair}$$

For 20 W cooling, the number of thermocouple pairs required $= \frac{20}{0.5} = 40$ pairs

$$\therefore \qquad \text{Power} = \frac{Q_c}{COP} = \frac{20}{0.3141} = 63.67 \text{ W}$$

For the case of maximum cooling,

We have,
$$\xi_{\max} = \frac{\left(\frac{zT_c^2}{2}\right) - (T_h - T_c)}{zT_hT_c}$$

$$= \frac{\left(\frac{3.09 \times 10^{-3} \times (258)^2}{2}\right) - 55}{3.09 \times 10^{-3} \times 313 \times 258} = 0.192$$

Also,
$$I_{\max} = \frac{\alpha_{pn}T_c}{R} = \frac{0.00035 \times 258}{2.314 \times 10^{-3}} = 39.023 \text{ amps}$$

$$(Q_{c,\,\max})_{\text{per pair}} = U\left[\frac{zT_c^2}{2} - (T_h - T_c)\right]$$

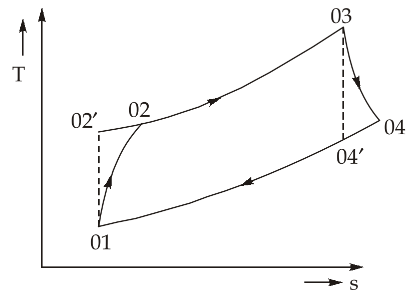
$$(Q_{c,\,\max})_{\text{per pair}} = 0.0171\left[\frac{3.09 \times 10^{-3} \times (258)^2}{2} - (55)\right]$$

$$(Q_{c,\,\max})_{\text{per pair}} = 0.82 \text{ W per pair}$$

For 20 W cooling, the number of thermocouple pairs required $= \frac{20}{0.82} = 24.39 \simeq 25$ pairs

$$\text{Power} = \frac{Q_c}{COP} = \frac{20}{0.192} = 104.17 \text{ W}$$

8. (b) (i) Solution:



Lowest temperature, $T_{01} = 35°C$

$$T_{01} = 273 + 35 = 308 \text{ K}$$

Maximum temperature, $T_{max} = T_{03}$

$$T_{03} = 680°C = 273 + 680 = 953 \text{ K}$$

$$t = \frac{T_{max}}{T_{min}} = \frac{T_{03}}{T_{01}} = \frac{953}{308} = 3.094$$

For maximum specific work, optimum pressure ratio is given as

$$(r_p)_{opt} = \left(\frac{\eta_T \eta_C C_{pg} T_3 n}{C_{P_a} T_1 m}\right)^{\frac{1}{m+n}}$$

where, $m = \dfrac{\gamma_a - 1}{\gamma_a}$ and $n = \dfrac{\gamma_g - 1}{\gamma_g}$

$$(r_p)_{opt} = \left(\frac{0.78 \times 0.8 \times 1.147 \times 953 \times 0.33 \times 1.4}{1.005 \times 308 \times 0.4 \times 1.33}\right)^{\frac{1}{\frac{0.4}{1.4}+\frac{0.33}{1.33}}}$$

$$(r_p)_{opt} = 3.37$$ **Ans.**

$$\frac{T_{02}'}{T_{01}} = (3.37)^{\frac{\gamma_a - 1}{\gamma_a}}$$

$$T_{02}' = 435.915 \text{ K}$$

$$T_{02} = T_{01} + \frac{T_{02'} - T_{01}}{\eta_c} = 308 + \frac{435.915 - 308}{0.8} = 467.89 \text{ K}$$

$$T_{04}' = \frac{T_{03}}{(r_p)^{\gamma_g - 1/\gamma_g}} = \frac{953}{(3.37)^{0.248}} = 704.98 \text{ K}$$

$$\text{Isentropic efficiency, } \eta_T = \frac{T_{03} - T_{04}}{T_{03} - T_{04'}}$$

$$0.78 = \frac{953 - T_{04}}{953 - 704.98}$$

$$T_{04} = -(953 - 704.98)0.78 + 953$$

$$T_{04} = 759.54 \text{ K}$$

$$\text{Thermal efficiency, } \eta_{th} = \frac{C_{pg}(T_{03} - T_{04}) - C_{pa}(T_{02} - T_{01})}{C_{pg}(T_{03} - T_{02})}$$

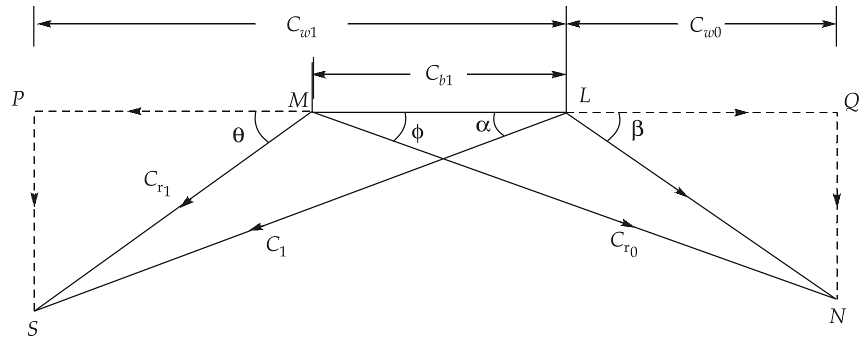
$$= \frac{1.147(953 - 759.54) - 1.005(467.89 - 308)}{1.147(953 - 467.89)}$$

$= 0.110$ or 11% **Ans.**

8. (b) (ii) Solution:

The rotor of the turbine can be of drum type or disc type. Disc type construction is difficult (complicated) to make, but lighter in weight. Hence the centrifugal stresses are lower at a particular speed. On the other hand drum type construction is simple in construction, and it is easy to attach airfoil shape blades. Further it is easier to design for tip leakage reduction which is a major problem in reaction turbines. Moreover due to small pressure drop per stage (larger number of stages) in reaction turbines, their rotational speeds are lower and so the centrifugal stresses are not very high (even the reaction blades are lighter). Therefore drum type construction is preferred over disc type in reaction turbines.

To accommodate increase in specific volume at lower pressures the drum diameter is stepped up which allows greater area without increasing blade height. The increased drum diameter also increases the torque due to steam pressure.



8. (c) Solution:

Condition I: $P_1 = 1 \text{ atm} = 101.325 \text{ kPa}, T_1 = 30 + 273 = 303\,\text{K}$

$$u_1 = 24 \text{ kmph} = 24 \times \frac{5}{18} = 6.667 \text{ m/s}$$

Now, Density of air, $\rho_1 = \frac{P_1}{RT_1} = \frac{101.325}{0.287 \times 303}$

$$\rho_1 = 1.165 \text{ kg/m}^3$$

Power developed, $P_1 = \eta_0 \times \left(\frac{1}{2}\rho_1 A u_1^3\right)$

$$1500 = 0.45 \times (0.5 \times 1.165 \times A \times (6.667)^3)$$

Cross-sectional area of rotor, $A = 19.31 \text{ m}^3$

Condition II: $P_2 = 0.88\, P_{atm} = 0.88 \times 101.325 = 89.166 \text{ kPa}$

$$T_2 = 10 + 273 = 283\,\text{K}$$

$$u_2 = 30 \text{ kmph} = \left(30 \times \frac{5}{18}\right) = 8.333 \text{ m/s}$$

Density of air, $\rho_2 = \frac{P_2}{RT_2} = \frac{89.166}{0.287 \times 283} = 1.0978 \text{ kg/m}^3$

$$\rho_2 = 1.0978 \text{ kg/m}^3$$

Power developed, $P_2 = \eta_0 \times \left(\frac{1}{2}\rho_2 A u_2^3\right)$

$$= 0.45 \times 0.5 \times 1.0978 \times 19.31 \times (8.333)^3$$

$$P_2 = 2759.94 \text{ Watt}$$

% change in power output = $\left(\frac{P_2 - P_1}{P_1}\right) \times 100\%$

$$= \left(\frac{2759.94 - 1500}{1500}\right) \times 100\%$$

%Change in power output = 83.99% **Ans.**

Solidity: Solidity is defined as the ratio of the blade area to the circumference of the rotor. It determines the quantity of blade material required to intercept a certain wind area.

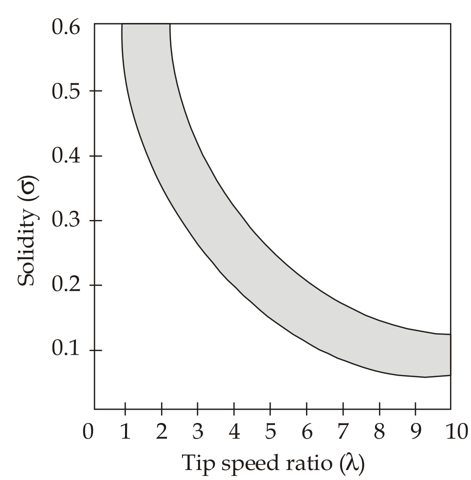
$$\text{Solidity, } \sigma = \frac{Nb}{2\pi R}$$

where, N = Number of blades
b = Blade width
R = Blade radius

It represents the fraction of the swept area of the rotor which is covered with metal.

i. A two or three-blade turbine has a low solidity and so needs to rotate faster to intercept and capture wind energy. Otherwise the major part of wind energy would be lost through the large gaps between the blades. High speed wind turbines have a low starting torque.

ii. Rotors having a high value of solidity, operates at low tip speed ratio. Such rotors need a high starting torque.



A high solidity rotor rotates slowly and uses the drag force while a low solidity rotor uses lift forces.