Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Civil Engineering Test No : 12

Section - A

1. (a) Solution:

$\sigma_x = 80$ MPa, $\sigma_y = 0$, $\sigma_z = 150$ MPa, $E = 70$ GPa, $\mu = \dfrac{1}{3}$

Hooke's Law: $\sigma_y = 0$, we find the strain in each of the coordinate directions.

$$\in_x = +\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

$$\Rightarrow \quad \in_x = \frac{1}{70\times10^3}\left[(80) - 0 - \frac{1}{3}(150)\right] = +\,0.4285 \times 10^{-3} \text{ mm/mm}$$

$$\in_y = \frac{-\mu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

$$\Rightarrow \quad \in_y = \frac{1}{70\times10^3}\left[-\frac{1}{3}(80) + 0 - \frac{1}{3}(150)\right] = -\,1.095 \times 10^{-3} \text{ mm/mm}$$

$$\in_z = -\frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\Rightarrow \quad \in_z = \frac{1}{70\times10^3}\left[-\frac{1}{3}(80) - 0 + (150)\right] = 1.762 \times 10^{-3} \text{ mm/mm}$$

(i) Diameter AB: The change in length is $\delta_{AB} = \in_x d$

$$\delta_{AB} = \in_x d = (0.4285 \times 10^{-3} \text{ mm/mm}) \times (200 \text{ mm})$$

$$\Rightarrow \quad \delta_{AB} = +\,0.0857 \text{ mm}$$

(ii) Diameter CD:

$$\delta_{CD} = \in_z d = (1.762 \times 10^{-3} \text{ mm/mm}) \times (200 \text{ mm})$$

$$\Rightarrow \quad \delta_{CD} = +0.3524 \text{ mm}$$

(iii) Thickness: $t = 20$ mm, we have

$$\delta_t = \in_y t = (-1.095 \times 10^{-3} \text{ mm/mm}) \times (20 \text{ mm})$$

$$\Rightarrow \quad \delta_t = -0.0219 \text{ mm}$$

(iv) Volume of the plate:

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = (0.4285 - 1.095 + 1.762)10^{-3} = 1.0955 \times 10^{-3}$$

$$\Delta V = \varepsilon_v = 1.0955 \times 10^{-3} [(350 \text{ mm}) \times (350 \text{ mm}) \times (20 \text{ mm})]$$

$$= 2683.975 \text{ mm}^3$$

1. **(b) Solution:**

Seasoning is the process of removing moisture from freshly felled timber to make it suitable for use in construction or manufacturing. The goal is to bring the moisture content close to the average humidity of the area where it will be used.

Objectives of seasoning:

1. Reduce shrinkage and warping after placement in structure.
2. Increase strength, durability and workability.
3. Reduce weight for easier handling.

Preservation is the process of physical or chemical treatment of timber in order to protect it from fungi, insects, moisture, decay etc.

Difference between seasoning and preservation:

Aspect	Seasoning	Preservation
Purpose	To reduce the moisture content in timber	To protect timber from decay, fungi and other harmful factors.
Main benefit	Improves strength, durability, and workability.	Increases service life by resisting biological attack and moisture degradation.
When applied	Immediately after felling	Usually after seasoning or before exposure

Methods of seasoning:

(i) Natural or Air Seasoning: The log of wood is sawn into planks of convenient sizes and stacked under a covered shed in crosswise direction in alternate layers so as to permit free circulation of air. The duration for drying depends upon the type of wood and the size of planks. The rate of drying is however very slow. Air seasoning reduces the moisture content of the wood to 12-15 percent. It is used very extensively in drying ties and the large size structural timbers.

(ii) Water Seasoning: The logs of wood are kept completely immersed in running stream of water, with their larger ends pointing upstream. Consequently, the sap, sugar, and gum are leached out and are replaced by water. The logs are then kept out in air to dry. It is a quick process but the elastic properties and strength of the wood get reduced.

Methods of preservation:

(i) Chemical treatment: Preservatives like creosote, copper based salts are forced into the timber under pressure. It offers deep and lasting protection against rots and insects.

(ii) Hot and Cold Process ensures sterilisation against fungi and insects. The timber is submerged in the preservative solution, which is then heated to about 90° to 95°C and maintained at this temperature for a suitable period depending on the charge. It is then allowed to cool until the required absorption is obtained. During the heating period, the air in the timber expands and is partially expelled. While cooling, the residual air in the timber contracts and creates a partial vacuum which causes the preservative to be sucked into the timber.

1. (c) Solution:

Given data:

Span of the beam, $L = 400$ mm

Young's Modulus $E = 200$ GPa $= 200 \times 10^3$ N/mm^2

Compressive strain at A $(\in_A) = 50 \times 10^{-3}$

Distance of gage A from the top of the section = 3 mm

Centroid and Moment of Inertia

Let the bottom of the flange be the reference axis

Total area:

$$A_{total} = (12 \times 4) + (4 \times 12) = 96 \text{ mm}^2$$

Centroid location:

$$\bar{y} = \frac{(12\times4\times2)+(4\times12\times10)}{96} = 6 \text{ mm}$$

The neutral axis is 6 mm from the bottom.

Since the total height is 16 mm, the neutral axis is 10 mm from the top.

Moment of inertia about the neutral axis:

$$I = \left[\frac{12\times4^3}{12}+(12\times4)(6-2)^2\right]+\left[\frac{4\times12^3}{12}+(4\times12)(10-6)^2\right]$$

$$I = 2176 \text{ mm}^4$$

Stress and Bending Moment at *A*

Using Hooke's Law:

$$\sigma_A = E \times \varepsilon_A = 200000 \times 0.05 = 10000 \text{ N/mm}^2$$

Distance from neutral axis to gage *A*:

$$y_A = 10 - 3 = 7 \text{ mm}$$

Using flexure formula $\sigma = \dfrac{My}{I}$

$$M_A = \frac{\sigma_A \times I}{y_A} = \frac{10000 \times 2176}{7}$$

$$M_A = 3108571.429 \text{ N-mm}$$

Reactions and Internal Moment at *A*

Taking moments about the right support:

$$R_1 \times 400 = (P \times 300) + (P \times 200) + (3P \times 100)$$

$$\Rightarrow \quad R_1 \times 400 = 800P$$

$$\Rightarrow \quad R_1 = 2P$$

Point *A* is located at 250 mm from the left support

Bending moment at *A*:

$$M_A = (R_1 \times 250) - (P \times 150) - (P \times 50)$$

$$\Rightarrow \quad M_A = M_A = (2P \times 250) - 150P - 50P = 300P$$

Calculation of Force *P*

$$300P = 3108571.429 \text{ N-mm}$$

$$P = 10361.905 \text{ N} = 10.361 \text{ kN}$$

1. (d) Solution:

Length of wire (L_w) = 5 m = 5000 mm

Area of wire (A_w) = 160 mm^2

Modulus of Elasticity of wire (E_w) = 200 GPa = 200 × 10^3 N/mm^2

Thermal expansion coefficient (α) = 12 × 10^{-6} /°C

Temperature drop (ΔT) = 50°C

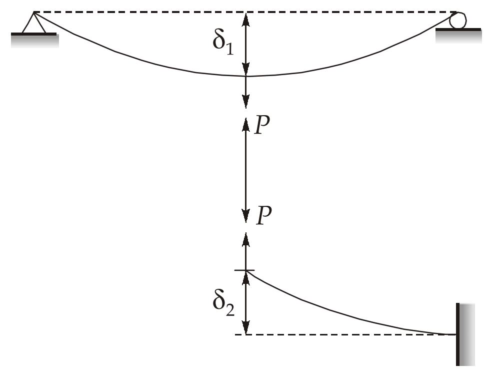
Length of simple beam (L_1) = 0.8 + 0.8 = 1.6 m = 1600 mm

Length of cantilever beam (L_2) = 0.8 m = 800 mm

Moment of Inertia for beams (I) = 10 × 10^6 mm^4

Modulus of Elasticity for beams (E_b) = 10 GPa = 10 × 10^3 N/mm^2

Let P be the tension developed in the wire due to the temperature drop. This force P acts downward on the simple beam and upward on the cantilever.



Deflection of the midpoint of the simple beam (δ_1):

$$\delta_1 = \frac{PL_1^3}{48E_bI}$$

$$\Rightarrow \quad \delta_1 = \frac{P \times 1600^3}{48 \times 10000 \times 10000000}$$

$$\Rightarrow \quad \delta_1 = 0.000853333 \times P \ (\text{mm})$$

Deflection of the free end of the cantilever beam (δ_2):

$$\delta_2 = \frac{PL_2^3}{48E_bI}$$

$$\Rightarrow \quad \delta_2 = \frac{P \times 800^3}{3 \times 10000 \times 10000000}$$

$$\Rightarrow \quad \delta_2 = 0.001706667 \times P \ (\text{mm})$$

Deformation of the wire (δ_w):

The total change in length of the wire is the thermal contraction minus the elastic stretch due to tension P.

$$\delta_w = (\alpha \times \Delta T \times L_w) - \frac{PL_w}{A_wE_w}$$

$$\Rightarrow \quad \delta_w = (12 \times 10^{-6} \times 50 \times 5000) - \frac{P \times 5000}{160 \times 200000}$$

$$\Rightarrow \quad \delta_w = 3 - 0.00015625 \times P \text{ mm}$$

Compatibility Equation:

The contraction of the wire must be equal to the sum of the deflections of both beams it is connected to.

$$\delta_w = \delta_1 + \delta_2$$

$$\Rightarrow \quad 3 - 0.00015625 \times P = 0.000853333 \times P + 0.001706667 \times P$$

$$\Rightarrow \quad 3 = 0.00271625 \times P$$

$$\Rightarrow \quad P = 1104.464 \text{ N}$$

Final Deflection of the Cantilever End:

$$\delta_2 = 0.001706667 \times 1104.464$$

$$\delta_2 = 1.885 \text{ mm}$$

1. (e) Solution:

$$\text{Width of section } A\text{-}A,\ b_A = 250 \text{ mm}$$

$$\text{Depth of section } A\text{-}A = 400 - \left(\frac{400-200}{3000}\right) \times 1000$$

$$\Rightarrow \quad D_A = 333.33 \text{ mm} \simeq 334 \text{ mm}$$

$$\text{Effective depth, } d_A = 334 - 40 = 294 \text{ mm}$$

$$\text{Area of tension steel, } A_{st} = 2 \times \left(\frac{\pi}{4} \times 18^2 + \frac{\pi}{4} \times 22^2\right) = 1269.203 \simeq 1270 \text{ mm}^2$$

$$p_t\% = \frac{100 \times A_{st}}{bd} = \frac{100 \times 1270}{250 \times 294} = 1.728\%$$

By interpolation:

$$\tau_c = 0.72 + \frac{(0.75-0.72)}{(1.75-1.5)}(1.728-1.5)$$

$$\Rightarrow \quad \tau_c = 0.747 \text{ N/mm}^2 \simeq 0.75 \text{ N/mm}^2$$

Maximum shear stress: $\tau_{c\,max} = 0.63\sqrt{f_{ck}} = 2.8 \text{ N/mm}^2$ (for M20)

Nominal shear stress,

$$\tau_v = \frac{V_u \pm \left(\frac{M_u}{d}\right)\tan\beta}{bd}$$

$$V_u = 1.5 \times 120 = 180 \text{ kN}$$

$$M_u = 1.5 \times 160 = 240 \text{ kNm}$$

$$\tan\beta = \frac{400-200}{3000} = \frac{1}{15}$$

$$\tau_v = \frac{180 \times 1000 - \left(\frac{240 \times 10^6}{294}\right) \times \frac{1}{15}}{250 \times 294}$$

$$\tau_v = 1.708\ \text{N/mm}^2 < 2.80\ \text{N/mm}^2 \qquad \text{(Ok)}$$

Design shear stress, $\tau_{us} = \tau_v - \tau_c$

$$= 1.708 - 0.75 = 0.958\ \text{N/mm}^2$$

Assuming 8 mm - 2 legged vertical shear stirrups of Fe415 grade,

$$A_{sv} = 2 \times \frac{\pi}{4} \times (8)^2 = 100.53\ \text{mm}^2$$

Spacing of vertical shear stirrups,

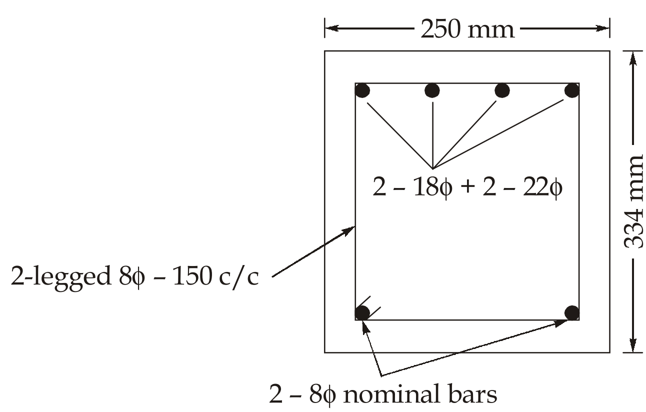
$$S_v = \frac{0.87 f_y A_{sv}}{\tau_{us} b}$$

$$\Rightarrow \quad S_v = \frac{0.87 \times 415 \times 100.53}{0.955 \times 250} = 152\ \text{mm}$$

$$\text{Maximum permitted spacing} = \min \begin{cases} 152\ \text{mm} \\ 0.75\,d = 0.75 \times 294 = 220.5\ \text{mm} \\ 300\ \text{mm} \end{cases}$$

$$= 152\ \text{mm}$$

Hence, providing 2-legged vertical shear stirrups of 8 mm at 150 mm c/c spacing.



Section *A-A*

Note: 2-8 mm ϕ nominal bars are provided to hold the stirrups only.

2. (a) Solution:

Step-1: It is essential that the total load must be transferred at the base of column without reinforcement. For that, the bearing resistance of concrete must be greater than the total factored load P_u.

$$\because \quad P_u = 1.5 \times 400 = 600 \text{ kN}$$

Permissible bearing stress of concrete,

$$\sigma_{br} = 0.45 \times f_{ck}\left(\frac{A_1}{A_2}\right)^{1/2}$$

$$A_1 = A_2 = 400 \times 400$$

$$= 160000 \text{ mm}^2 \qquad (\because \text{ Column is square})$$

$$\therefore \quad \sigma_{br} = 0.45 \times 20 = 9 \text{ N/mm}^2$$

$\therefore$ Permissible bearing force,

$$P_{br} = 9 \times 160000 \text{ N}$$

$$= 1440 \text{ kN} > P_u \ (= 600 \text{ kN}) \qquad \text{(OK)}$$

Step-2: Size of the footing

$$\text{Footing area required} = \frac{1.15 \times 400}{300} = 1.53 \text{ m}^2$$

Provide square footing

$$\therefore \text{ Size of square footing,} \quad a = \sqrt{1.53} = 1.238 \text{ m}$$

Provide 1250 mm × 1250 mm size footing

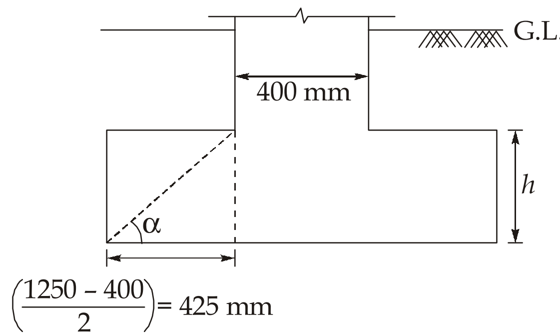
$$\text{Bearing pressure} = \frac{400 \times 1.15}{1.25 \times 1.25} = 294.4 \text{ kN/m}^2 = 0.2944 \text{ N/mm}^2$$

Step-3: Thickness of footing

$$\tan\alpha \geq 0.9\sqrt{\left[\frac{100 q_a}{f_{ck}} + 1\right]}$$

$$\Rightarrow \quad \tan\alpha \geq 0.9\sqrt{\frac{100 \times 0.2944}{20} + 1}$$

$$\Rightarrow \quad \tan\alpha \geq 1.415$$



From figure $\tan\alpha = \frac{h}{425}$

$\Rightarrow$ $h = 1.415 \times 425 = 601.375$ mm

$\therefore$ Provide $h = 650$ mm

So provide 1250 mm × 1250 mm × 650 mm block of plain concrete

Step-4: Minimum reinforcement

The plain concrete block shall be provided with minimum reinforcement of 0.12% for temperature, shrinkage and tie action.

Min $A_{st} = 0.0012 \times 1250 \times 650 = 975\ \text{mm}^2$

Provide 9 bars of 12 mm diameter (= 1018 mm^2) both ways

As shown in figure below

$$\text{Clear spacing between the bars} = \frac{1250 - \underbrace{2\times 50}_{\text{side cover}} - \underbrace{9\times 12}_{\text{total cover}}}{9-1} = 130.25\ \text{mm}$$

Provide the bars @ 130 mm clear spacing

Step-5: Check for gross base pressure

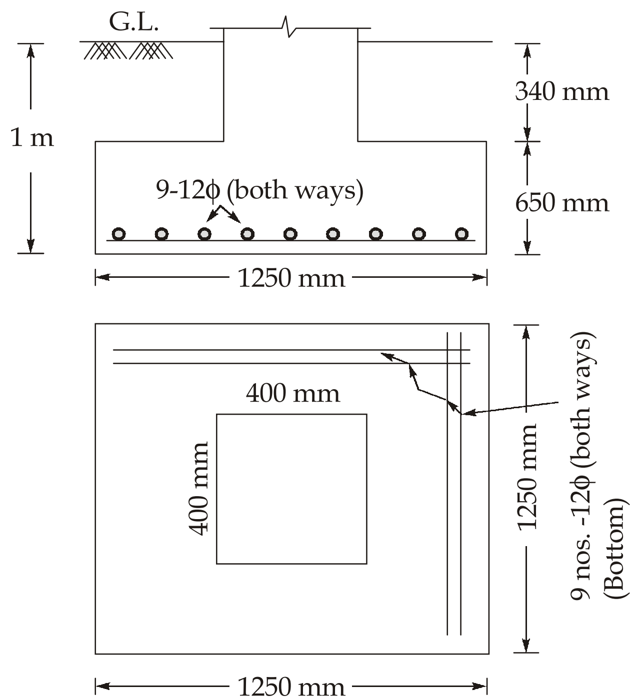
Service load = 400 kN

Weight of footing = 0.65 × 1.25 × 1.25 × 24 = 24.375 kN

Weight of soil = (1 – 0.65) × 1.25 × 1.25 × 20 = 10.9375 kN

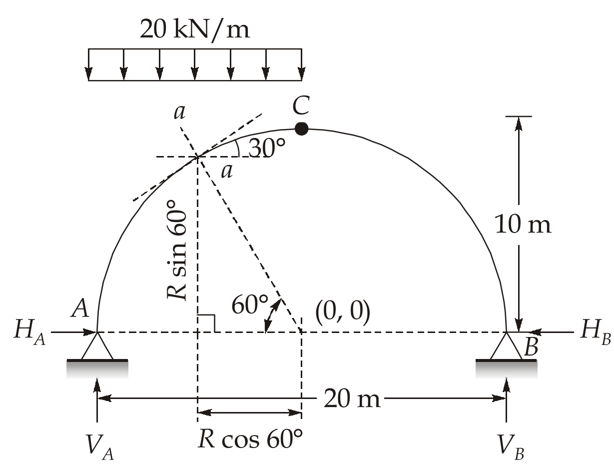
Total load on footing base = 400 + 24.375 + 10.9375 = 435.3125 kN

$$q_a = \frac{435.3125}{1.25\times 1.25} = 278.6\ \text{kN/m}^2 < 300\ \text{kN/m}^2 \quad \text{(OK)}$$



Reinforcement detailing

2. (b) Solution:



Support Reactions

Taking moments about the right support (B):

$$\Sigma M_B = 0$$

$$\Rightarrow \quad V_A \times 20 - (20 \times 10) \times 15 = 0$$

$$\Rightarrow \quad V_A = 150 \text{ kN } (\uparrow)$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad V_A + V_B - (20 \times 10) = 0$$

$$\Rightarrow \quad V_B = 50 \text{ kN} (\uparrow)$$

Taking moments about the crown hinge (C) from the right side:

$$\Sigma M_C = 0$$

$$V_B \times 10 - H_B \times 10 = 0$$

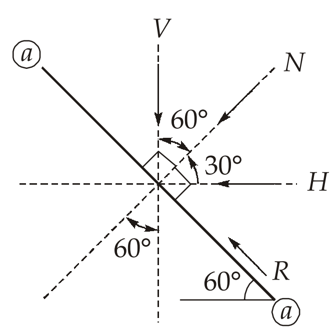
$$H_B = 50 \text{ kN} (\leftarrow)$$

Internal Forces at Section a – a

The coordinates of section a – a relative to the center of the circle:

$$x = R\cos(60°) = 5 \text{ m (from center, so 5 m from left support)}$$

$$y_{\text{arch}} = R\sin(60°) = 8.660 \text{ m}$$



$$\text{Vertical shear at } a\text{ - }a = 5 \text{ m:}$$

$$V_a = V_A - (w \times 5) = 150 - (20 \times 5) = 50 \text{ kN}$$

Bending Moment at section a - a

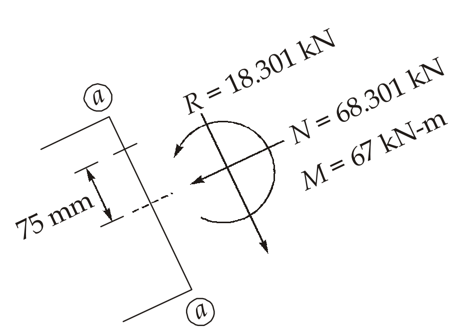
$$M = V_A \times 5 - (w \times 5) \times 2.5 - H \times y_{\text{arch}}$$

$$\Rightarrow \quad M = 150 \times 5 - 100 \times 2.5 - 50 \times 8.66 = 67 \text{ kNm}$$

Normal Force (N) and Radial shear (R) at the section:

$N = H\sin 60° + V_a\cos(60°) = 50 \times \cos 30° + 50 \times \cos 60° = 68.301$ kN

$R = V_a\sin(60°) - H\cos 60° = 50 \times \sin 60° - 50 \times \cos 60° = 18.301$ kN



The normal stress at the given point is calculated using direct stress and bending stress.

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

$$\Rightarrow \quad \sigma = \frac{68.301 \times 10^3}{150 \times 300} + \frac{67 \times 10^6 \times 75}{\left(\frac{150 \times 300^3}{12}\right)}$$

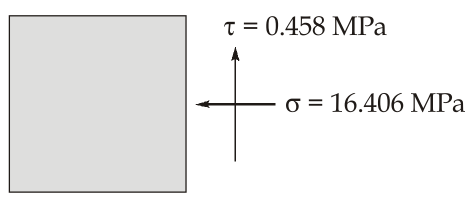
$$\sigma = 16.406 \text{ MPa (Compressive)}$$

The shear stress at the given point is obtained using the shear formula.

$$\tau = \frac{VQ}{Ib}$$

$$\Rightarrow \quad \tau = \frac{18.301 \times 10^3 \times 75 \times 150 \times 112.5}{\frac{150 \times 300^3}{12} \times 150}$$

$$\Rightarrow \quad \tau = 0.458 \text{ MPa}$$



The principal stresses are calculated using the principal stress equation.

$$\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,2} = \frac{+16.406}{2} \pm \sqrt{\left(\frac{-16.406}{2}\right)^2 + (0.458)^2}$$

$$\sigma_1 = +16.419 \text{ MPa}$$

$$\sigma_2 = -0.012 \text{ MPa}$$

2. (c) (i) Solution:

Equilibrium Equation

Let R_A be the reaction at the left support and R_B be the reaction at the right support. Assuming both reactions act to the left:

$$\Sigma F_x = 0$$

$$-R_A + P - R_B = 0$$

$$R_A + R_B = P \quad \text{...(i)}$$

Compatibility Equation

Since the bar is fixed at both ends, the total elongation must be zero:

$$\delta_{total} = \delta_1 + \delta_2 = 0$$

Let P_1 be the internal force in the tapered section and P_2 be the internal force in the uniform section. From a free-body cut:

In section 1: $P_1 = R_A$ (Tension)

In section 2: $P_2 = R_A - P = -R_B$ (Compression)

$$\delta_1 + \delta_2 = 0 \quad \text{...(ii)}$$

Calculating Displacements

For Segment 1 (Tapered):

The width $w(x)$ at any point x from point C is given by:

$$w(x) = 30 + \frac{10-30}{L/2} \times x = 30 - \frac{40x}{L}$$

The area is $A(x) = t \times w(x) = t\left(30 - \frac{40x}{L}\right)$

The displacement δ_1 is:

$$\delta_1 = \int_0^{L/2} \frac{R_A}{E \times A(x)} dx = \frac{R_A}{Et} \int_0^{L/2} \frac{1}{30 - \frac{40x}{L}} dx$$

$$\Rightarrow \quad \delta_1 = \frac{R_A}{Et}\left[-\frac{L}{40}\ln\left(30 - \frac{40x}{L}\right)\right]_0^{L/2}$$

$$\Rightarrow \quad \delta_1 = \frac{R_A L}{40Et}\ln\left(\frac{30}{10}\right) = \frac{R_A L \ln(3)}{40Et}$$

For Segment 2 (Uniform):

The area is $A_2 = 10t$.

$$\delta_2 = \frac{P_2 L_2}{A_2 E} = \frac{(R_A - P)(L/2)}{10tE} = \frac{(R_A - P)L}{20Et}$$

Solving for Reactions

Substitute δ_1 and δ_2 into the compatibility equation:

$$\frac{R_A L \ln(3)}{40Et} + \frac{(R_A - P)L}{20Et} = 0$$

$$\Rightarrow \quad \frac{R_A \ln(3)}{2} + R_A - P = 0$$

$$R_A\left(1 + \frac{\ln(3)}{2}\right) = P$$

$$R_A = 0.645P(\leftarrow)$$

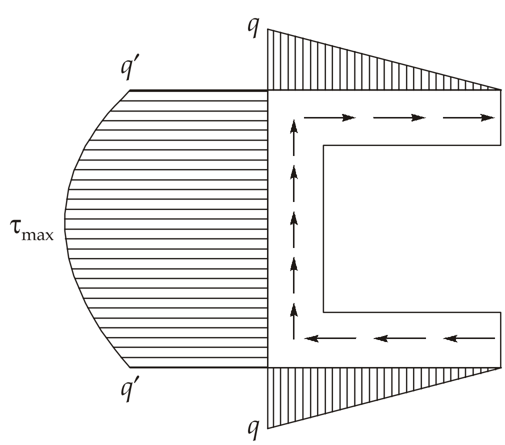
Using equation (i),

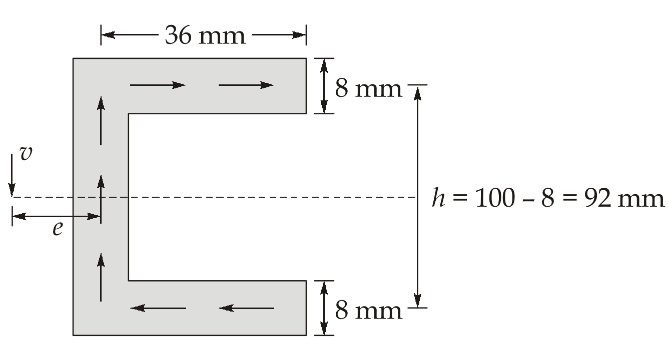
$$R_B = P - 0.645\,P$$

$$R_B = 0.355\,P\,(\leftarrow)$$

2. (c) (ii) Solution:

Shear flow diagram is as follows





Position of shear center from mid line of web is given by,

$$e = \frac{b^2h^2t}{4I}$$

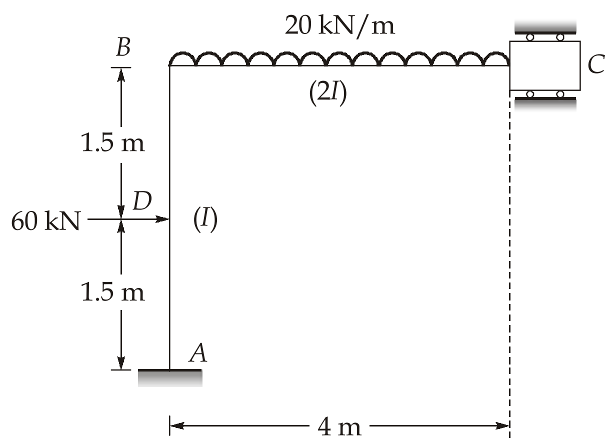
$$I = \frac{8\times(100)^3}{12} + 2\left(\frac{32\times 8^3}{12} + 32\times 8\times 46^2\right)$$

$$\Rightarrow \quad I = 1752789.333 \text{ mm}^4$$

$$\therefore \quad e = \frac{36^2\times 92^2\times 8}{4\times 1752789.333} = 12.52 \text{ mm}$$

3. (a) Solution:

The unknowns are θ_B are δ



Fixed End Moments (*FEMs*)

For member *AB* (L = 3): $M_{FAB} = \frac{-60\times 3}{8} = -22.5$ kN-m

$$M_{FBA} = \frac{+60\times 3}{8} = 22.5 \text{ kN-m}$$

For member *BC* $(L = 4,\ w = 20)$:

$$M_{FBC} = -\frac{20\times 4^2}{12} = -26.67 \text{ kN-m}$$

$$M_{FCB} = \frac{20\times 4^2}{12} = +26.67 \text{ kN-m}$$

Slope Deflection Equations

For span AB:

$$M_{AB} = -22.5 + \frac{2EI}{3}\left(\theta_B - \frac{3\delta}{l}\right) = -22.5 + \frac{2EI}{3}\left(\theta_B - \frac{3\delta}{3}\right)$$

$$M_{AB} = -22.5 + 0.67EI\theta_B - 0.67EI\delta$$

$$M_{BA} = 22.5 + \frac{2EI}{3}(2\theta_B - \frac{3\delta}{l}) = 22.5 + 1.33EI\theta_B - 0.67EI\delta$$

For span BC:

$$M_{BC} = -26.67 + \frac{2E(2I)}{4}(2\theta_B) = -26.67 + 2EI\theta_B$$

$$M_{CB} = 26.67 + \frac{2E(2I)}{4}(\theta_B) = 26.67 + EI\theta_B$$

Equilibrium Equations

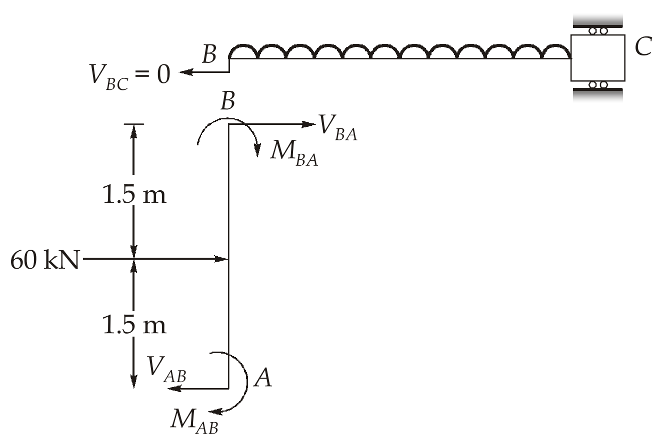
Joint B Equilibrium:

$$M_{BA} + M_{BC} = 0$$

$$(22.5 + 1.33EI\theta_B - 0.67EI\delta) + (-26.67 + 2EI\theta_B) = 0$$

$$3.33EI\theta_B - 0.67EI\delta = 4.17 \quad \text{...(i)}$$

Shear Equation:



Here, $V_{BA} = V_{BC} = 0$

$\Sigma M_A = 0 \quad \Rightarrow \quad M_{AB} + M_{BA} + 60 \times 1.5 + V_{BA} \times 3 = 0$

$$M_{AB} + M_{BA} = -90$$

Substituting:

$$(-22.5 + 0.67EI\theta_B - 0.67EI\delta) + (22.5 + 1.33EI\theta_B - 0.67EI\delta) = -90$$

$$2EI\theta_B - 1.34EI\delta = -90 \quad \text{...(ii)}$$

Solving equation (i) and (ii)

$$\theta_B = \frac{21.103}{EI} \text{ rad (Clockwise)}$$

$$\delta = \frac{98.66}{EI} \text{ m (To the right)}$$

Final Moments

$$M_{AB} = -22.5 + 0.67(21.103 - 98.66) = -74.46 \text{ kN-m}$$

$$M_{BA} = 22.5 + 0.67(2 \times 21.103 - 98.66) = -15.35 \text{ kN-m}$$

$$M_{BC} = -26.67 + 2(21.103) \simeq 15.35 \text{ kN-m}$$

$$M_{CB} = 26.67 + 21.103 = 47.773 \text{ kN-m}$$

Support Reactions

At Support *A*:

Horizontal Reaction: $H_A = 60 \text{ kN (Rightward)}$

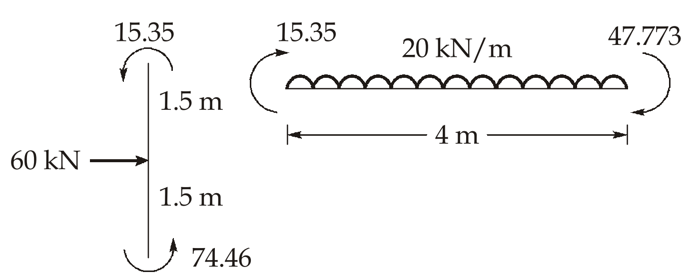
Vertical Reaction: $V_A = \frac{20 \times 4}{2} + \frac{15.35 - 47.773}{4} = 31.89 \text{ kN (Upward)}$

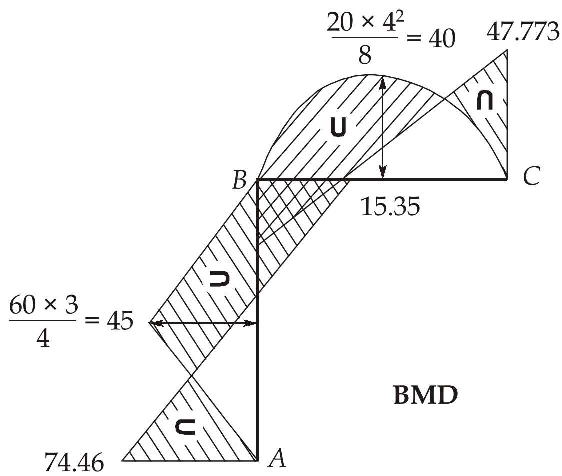
Moment Reaction: $M_A = 74.46 \text{ kN-m (Counter-clockwise)}$

Horizontal reaction: $H_C = 0 \text{ kN}$

Vertical Reaction: $V_C = 20 \times 4 - 31.89 = 48.11 \text{ kN (Upward)}$

Moment Reaction: $M_C = 47.742 \text{ kN-m (Clockwise)}$

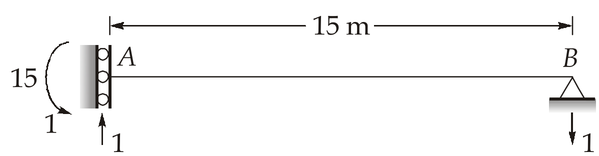




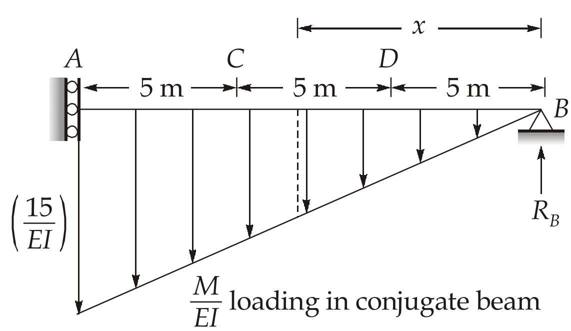
3. (b) Solution:

The vertical restraint at A is removed by inserting a slider at A. The slider will resist moment but will release vertical restraint. A unit load is applied at A. When a unit load is applied at A, reaction at B = 1 downwards.

Now deflections of beam is calculated corresponding to above loading using conjugate beam method.



Conjugate Beam of the Above Real Beam is as shown below.



$$R_B = \frac{1}{2} \times \frac{15}{EI} \times 15$$

$\Rightarrow$
$$R_B = \frac{112.5}{EI} (\uparrow)$$

d_{AA} = deflection at *A* due to unit load at A in real beam is given by Bending Moment (BM) at A in conjugate beam:

$$\Rightarrow \qquad d_{AA} = \frac{112.5}{EI}\times 15 - \frac{1}{2}\times\frac{15}{EI}\times 15\times 5$$

$$\Rightarrow \qquad d_{AA} = \frac{1125}{EI}\text{[(+)ve value of deflection means it is upward]}$$

Now at point D,

$$d_{DA} = \frac{112.5}{EI}\times 5 - \frac{1}{2}\times\frac{5}{EI}\times 5\times\frac{5}{3} = \frac{541.67}{EI}$$

At point C,

$$d_{DA} = \frac{112.5}{EI}\times 10 - \frac{1}{2}\times\frac{10}{EI}\times 10\times\frac{10}{3} = \frac{958.333}{EI}$$

In general, at a distance x from end B, deflection is given by:

$$d_{XA} = \frac{112.5}{EI}x - \frac{15x}{15EI}\times\frac{1}{2}\times x\times\frac{x}{3}$$

$$d_{XA} = \frac{112.5x}{EI} - \frac{x^3}{6EI}$$

Thus, influence line ordinate at a distance x from end B, is given $\frac{\delta_{XA}}{\delta_{AA}}$

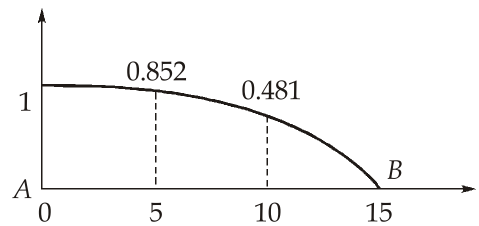
$$V_A(x) = \frac{\left(112.5x - \frac{x^3}{6}\right)/EI}{\frac{1125}{EI}}$$

$$\Rightarrow \qquad V_A(x) = \frac{112.5x}{1125} - \frac{x^3}{6\times 1125}$$

Then for various different location of x the *ILD* ordinate for V_A is given by

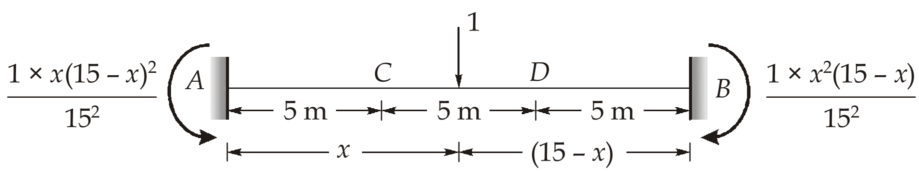
$x(in\ \text{m})\text{from}B$	$V_A(x)$
0	0
5	0.481
10	0.852
15	1

Hence, *ILD* for V_A is as shown below.

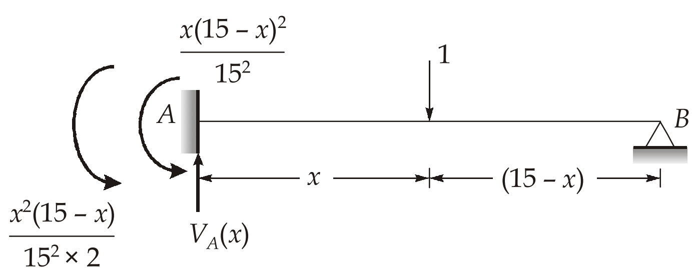


ILD for R_A

Alternate Solution:



But in question at point B, hinge is given



Taking moment about *B*

$$\Sigma M_B = 0$$

$$\Rightarrow \quad V_A(x) \times 15 = \frac{x^2(15-x)}{15^2 \times 2} + \frac{x(15-x)^2}{15^2} + 1(15-x)$$

$$V_A(x) = \frac{x^2(15-x)}{2 \times 15^3} + \frac{x(15-x)^2}{15^3} + \frac{(15-x)}{15}$$

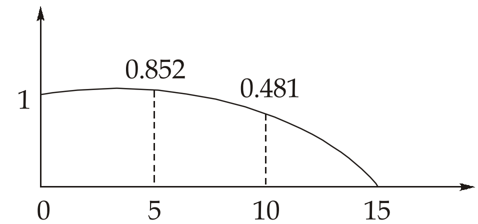
Put $x = 0$ m, $\quad V_A(x) = 1$

Put $x = 5$ m, $\quad V_A(5) = \frac{5^2 \times 10}{2 \times 15^3} + \frac{5(10)^2}{15^3} + \frac{10}{15} = 0.852$

Put $x = 10$ m, $\quad V_A(10) = \frac{10^2 \times 5}{2 \times 15^3} + \frac{10(5)^2}{15^3} + \frac{5}{15} = 0.481$

Put $x = 15$ m, $\quad V_A(15) = 0$

ILD for V_A



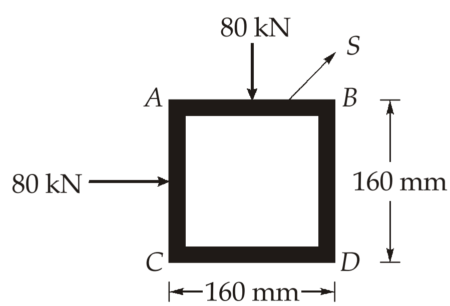
ILD for R_A

3. (c) Solution:

Assume, $f_u = 410 \text{ N/mm}^2$

Factored Load acting $= 80$ kN

Assume size of weld be 'S'



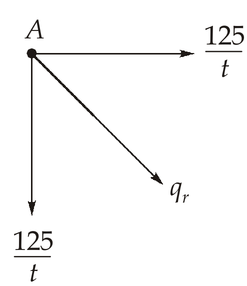
$$\text{Shear stress due to direct load } (q) = \frac{P}{t \times l}$$

$$t = \text{Throat thickness} = (0.7)S$$

$$q = \frac{80 \times 10^3}{t \times (160 \times 4)} = \frac{125}{t} \text{ N/mm}^2$$

At point A, the resultant act in 45° with the horizontal.

This shear will act in horizontal as well in vertical direction.



$$q_r = \sqrt{2} \times \frac{125}{t} = \frac{176.776}{t} \text{ (N/mm}^2\text{)}$$

Moment of Inertia of Welding

$$I_{xx} = I_{yy} = 2 \times t \times \frac{160^3}{12} + 2\left[\frac{160 \times t^3}{12} + 160 \times t \times 80^2\right]$$

$$\Rightarrow \quad I_{xx} = I_{yy} = 2{,}730{,}666.667\, t \text{ [neglecting '}t^3\text{' for being too small]}$$

Moments, $M_{xx} = M_{yy} = 80 \times 10^3 \times 150 = 12 \times 10^6$ N-mm

Bending stress, $f = \dfrac{My}{I}$

$$\Rightarrow \quad f = \frac{12 \times 10^6 \times 80}{2730666.667t} \text{ N/mm}^2 = \frac{351.562}{t} \text{ N/mm}^2$$

The point A of the weld will be in tension due to both loads and the combined tension will be

$$\Rightarrow \quad f_t = \frac{2 \times 351.562}{t} = \frac{703.125}{t} \text{ N/mm}^2$$

Maximum resultant of stresses (q_r and f_t):

$$\Rightarrow \quad f_e = \sqrt{3q_r^2 + f_t^2} < \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

$$\Rightarrow \quad \sqrt{3 \times \left(\frac{176.776}{t}\right)^2 + \left(\frac{703.125}{t}\right)^2} < \frac{410}{\sqrt{3} \times 1.25}$$

$$\Rightarrow \quad \frac{766.898}{t} < \frac{410}{\sqrt{3} \times 1.25}$$

$$\Rightarrow \quad t > 4.050 \text{ mm}$$

$$\therefore \quad \text{Size of weld, } S = \frac{4.050}{0.7} = 5.785 \text{ mm}$$

Provide size of weld = 6 mm

4. (a) Solution:

Unsupported length of the column,

$$L_0 = 5.5 \text{ m}, \quad L_{\text{eff}} = 0.65 \times 5.5 = 3.575 \text{ m}$$

Slenderness ratio:

$$SR_{X-X} = \frac{L_{\text{eff }x}}{D} = \frac{3.575}{500} = 7.15 < 12$$

$$SR_{Y-Y} = \frac{L_{\text{eff } y}}{B} = \frac{3.575}{335} = 10.67 < 12$$

∴ The given column is a short column.

Design values:

$$P_u = 1500 \text{ kN}$$

$$M_{ux} = P_u \cdot e_x = 1500 \times 0.08 = 120 \text{ kN-m}$$

$$M_{uy} = P_u \cdot e_y = 1500 \times 0.06 = 90 \text{ kN-m}$$

Calculate minimum design moments:

About major axis: $$(e_{\min})_x = \frac{L_{x0}}{500} + \frac{D}{30} = \frac{5500}{500} + \frac{500}{30} = 27.67 \text{ mm}$$

$$(M_{ux})_{\min} = P_u \cdot (e_{\min})_x$$

⇒ $$(M_{ux})_{\min} = 1500 \times \frac{27.67}{1000} = 41.505 \text{ kN-m} < (M_{ux} = 120 \text{ kN-m})$$

OK

About minor axis: $$(e_{\min})_y = \frac{L_{y0}}{500} + \frac{B}{30} = \frac{5500}{500} + \frac{335}{30} = 22.17 \text{ mm}$$

$$(M_{uy})_{\min} = P_u \cdot (e_{\min})_y$$

⇒ $$(M_{uy})_{\min} = 1500 \times \frac{29.5}{1000} = 44.25 \text{ kN-m} < (M_{ux} = 90 \text{ kN-m})$$ OK

$$A_{sc} = 12 \times \frac{\pi}{4} \times 20^2 = 3769.91 \text{ mm}^2$$

$$A_g = 335 \times 500 = 167{,}500 \text{ mm}^2$$

$$A_c = A_g - A_{sc} = 163730.09 \text{ mm}^2$$

Now, $$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

⇒ $$P_{uz} = (0.45 \times 20 \times 163730.09 + 0.75 \times 415 \times 3769.91) \text{ N}$$

⇒ $$P_{uz} = 2646.955 \text{ kN}$$

Assume effective column = 50 mm

$$\frac{P_u}{f_{ck} bD} = \frac{1500 \times 10^3}{20 \times 335 \times 500} = 0.448$$

$$\frac{d'}{D} = \frac{50}{500} = 0.1$$

$$\text{Percentage of steel, } p = \frac{3769.91}{167,500} \times 100 = 2.251\%$$

$$\frac{p}{f_{ck}} = \frac{2.251}{20} = 0.112$$

Referring to chart-44, SP : 16,

For, $\frac{p}{f_{ck}} = 0.112$

and $\frac{P_u}{f_{ck}bD} = 0.448$

$$\frac{M_u}{f_{ck}bD^2} = 0.15$$

$\Rightarrow$ $M_{ux1} = 0.15 \times 20 \times 335 \times 500^2 \text{ N-mm} = 251.25 \text{ kN-m}$

Here, $\frac{d'}{b} = \frac{50}{335} = 0.149 \text{ say } 0.15$

Referring to chart-45, SP:16,

For $\frac{p}{f_{ck}} = 0.112$

and $\frac{P_u}{f_{ck}bD} = 0.448$

$$\frac{M_u}{f_{ck}bD^2} = 0.116$$

$\Rightarrow$ $M_{uy1} = 0.116 \times 20 \times 500 \times 335^2 \text{ N-mm} = 130.841 \text{ kN-m}$

Here, $\frac{P_u}{P_{uz}} = \frac{1500}{2646.955} = 0.567$ (lies between 0.2 and 0.8)

$$\alpha_n = 1 + \frac{(2-1)}{(0.8-0.2)} \times (0.567 - 0.2) = 1.612$$

Now, $\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} = \left(\frac{120}{251.25}\right)^{1.612} + \left(\frac{90}{130.841}\right)^{1.612} = 0.851 < 1$ (safe)

Hence, the column is safe under the given loading condition.

4. (b) Solution:

Given data

Temperature increase in BC, $\Delta T = 85°C$

Modulus of elasticity, $E = 210 \text{ kN/mm}^2$

Coefficient of thermal expansion, $\alpha = 1.05 \times 10^{-5}/°C$

Solving for S-system of forces

The internal member forces due to external loading are obtained using joint equilibrium.

At joint D,

$$\Sigma F_y = 0 \quad \Rightarrow \quad S_{AD} - 600 = 0$$
$$S_{AD} = 600 \text{ kN}$$
$$\Sigma F_x = 0 \quad \Rightarrow \quad -S_{CD} = 0$$
$$S_{CD} = 0$$

At joint A,

$$\Sigma F_y = 0 \quad \Rightarrow \quad -S_{AD} - S_{AC}\sin\theta = 0$$
$$-600 - 0.8S_{AC} = 0$$
$$S_{AC} = -750 \text{ kN}$$
$$\Sigma F_x = 0 \quad \Rightarrow \quad 500 - S_{AB} - S_{AC}\cos\theta = 0$$
$$500 - S_{AB} - 0.6(-750) = 0$$
$$500 - S_{AB} + 450 = 0$$
$$S_{AB} = 950 \text{ kN}$$

At joint C,

$$\Sigma F_y = 0 \quad \Rightarrow \quad S_{BC} + S_{AC}\sin\theta = 0$$
$$S_{BC} + 0.8(-750) = 0$$
$$S_{BC} = 600 \text{ kN}$$

Thus, forces in members due to actual loading are:

Member	S(kN)
AB	950
BC	600
CD	0
AD	600
AC	–750

Solving for k-system of forces-

Now apply a unit vertical load of 1 kN downward at joint *A*.

At joint *D*,

$$\Sigma F_y = 0 \quad \Rightarrow \quad k_{AD} = 0$$

$$\Sigma F_x = 0 \quad \Rightarrow \quad -k_{CD} = 0$$

$$k_{CD} = 0$$

A Joint *A*,

$$\sum F_y = 0 \Rightarrow -k_{AC}\sin\theta - 1 = 0$$

$$-0.8k_{AC} - 1 = 0$$

$$k_{AC} = -1.25$$

$$\sum F_x = 0 \Rightarrow -k_{AB} - k_{AC}\cos\theta = 0$$

$$-k_{AB} - 0.6(-1.25) = 0$$

$$-k_{AB} + 0.75 = 0$$

$$k_{AB} = 0.75$$

At joint *C*,

$$\sum F_y = 0 \Rightarrow k_{BC} + k_{AC}\sin\theta = 0$$

$$k_{BC} + 0.8(-1.25) = 0$$

$$k_{BC} = 1$$

Thus, forces in members due to unit load are:

Member	k
AB	0.75
BC	1
CD	0
AD	0
AC	–1.25

Now compute displacement using unit load method:

Member	L (mm)	$A(mm^2)$	S(kN)	k	$\frac{S \times k \times L}{A}$
AB	1800	1150	950	0.75	1115.217
BC	2400	1150	600	1	1252.174
CD	1800	1150	0	0	0
AD	2400	1150	600	0	0
AC	3000	1200	–750	–1.25	2343.75

$$\sum\left(\frac{S \times k \times L}{A}\right) = 4711.141 \text{ kN/mm}$$

$$\Delta_{\text{load}} = \frac{1}{E}\sum\left(\frac{S \times k \times L}{A}\right)$$

$$\Rightarrow \quad \Delta_{\text{load}} = \frac{4711.141}{210}$$

$$\Rightarrow \quad \Delta_{\text{load}} = 22.34 \text{ mm } (\downarrow) \text{ (deflection due to external load)}$$

Thermal expansion of member BC is:

$$\delta L_{th} = L \times \alpha \times \Delta T$$

$$\delta L_{th} = 2400 \times 1.05 \times 10^{-5} \times 85 = 2.142 \text{ mm}$$

$$\Delta_{temp} = k_{BC} \times \delta L_{th}$$

$$\Delta_{th} = 1 \times 2.142 = 2.142 \text{ mm } (\downarrow) \text{ (deflection due to temperature)}$$

Total vertical deflection at point A.

$$(\Delta_{\text{Total}})_A = \Delta_{\text{load}} + \Delta_{\text{temp}}$$

$$\Rightarrow \quad (\Delta_{\text{Total}})_A = 22.434 + 2.142$$

$$\Rightarrow \quad (\Delta_{\text{Total}})_A = 24.576 \text{ mm } (\downarrow)$$

The total vertical displacement of joint A is 24.576 mm downward.

4. (c) (i) Solution:

Given that

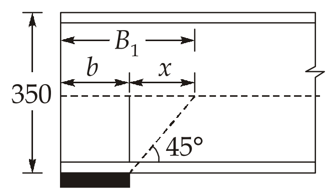
ISLB 350, simply supported at both ends.

$$\text{Stiff bearing length} = 100 \text{ mm}$$
$$\text{Grade of steel} = E\,250$$

Properties of ISLB 350

$$t_w = 7.4 \text{ mm}$$
$$t_f = 11.4 \text{ mm}$$
$$R = 16 \text{ mm}$$

For web buckling strength



$$x = 350 \times \frac{1}{2} = 175 \text{ mm}$$

$$\text{Stiff bearing length} = 100 \text{ mm}$$
$$\text{Area of bearing } (A_b) = B_1 t_w = (b + x)\, t_w = (100 + 175) \times 7.4 = 2035 \text{ mm}^2$$
$$\text{Effective length of web} = 0.7\,[350 - 2\,(11.4 + 16)] = 206.64 \text{ mm}$$
$$I_{eff} \text{ of web} = 8.33 t_w^3 = 8.33 \times 7.4^3 = 3375.52 \text{ mm}^4$$
$$A_{eff} \text{ of web} = 100 \times 7.4 = 740 \text{ mm}^2$$
$$\text{Radius of gyration } (r) = \sqrt{\frac{I_{\text{eff}}}{A_{\text{eff}}}} = \sqrt{\frac{3375.52}{740}} = 2.136 \text{ mm}$$
$$\text{Slenderness ratio } (\lambda) = \frac{KL}{r} = \frac{206.64}{2.136} = 96.75$$

From table

$$f_{cd} = 121 - \frac{(121-107)}{(100-90)}(96.75-90)$$

$$\Rightarrow \quad f_{cd} = 111.52 \text{ N/mm}^2$$

Capacity of web section = $A_b f_{cd} = 2035 \times 111.52 = 227 \times 10^3 \text{ N} = 227 \text{ kN}$

For Web crippling strength:

$$f_w = (b + n_1)\frac{t_w f_y}{\gamma_{m0}}$$

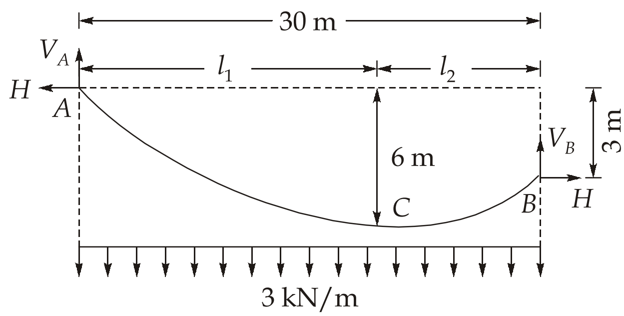
$$n_1 = 2.5 \times (11.4 + 16) = 68.5 \text{ mm}$$

$$F_w = (100 + 68.5) \times 7.4 \times \frac{250}{1.1}$$

$$F_w = 283.39 \times 10^3 \text{ N} = 283.39 \text{ kN}$$

4. (c) (ii) Solution:

Considering equilibrium of part AC of the cable,



$$l_1 = l\left(\frac{\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}\right) = 30\left(\frac{\sqrt{6}}{\sqrt{6} + \sqrt{3}}\right) = 17.57 \text{ m}$$

$$l_2 = l\left(\frac{\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}\right) = 30\left(\frac{\sqrt{3}}{\sqrt{6} + \sqrt{3}}\right) = 12.43 \text{ m}$$

$\Sigma f_y = 0$

$\Rightarrow$ $$V_A + V_B = 3 \times 30 = 90 \quad \text{...(i)}$$

Taking moments about C, (considering left portion)

$$V_A \times 17.57 - H \times 6 - 3 \times \frac{17.57^2}{2} = 0$$

$\Rightarrow$ $$V_A = \left(\frac{6H}{17.57} + \frac{3 \times 17.57}{2}\right) \quad \text{...(ii)}$$

Taking moment aboutC, (Considering right portion)

$$V_B \times 12.43 - H \times 3 - 3 \times \frac{12.43^2}{2} = 0$$

$\Rightarrow$ $$V_B = \left(\frac{3H}{12.43} + \frac{3 \times 12.43}{2}\right) \quad \text{...(iii)}$$

On putting the value of V_A and V_B in equation (i), we get

$$\left(\frac{6H}{17.57} + \frac{3 \times 17.57}{2}\right) + \left(\frac{3H}{12.43} + \frac{3 \times 12.43}{2}\right) = 90$$

$$\Rightarrow \quad 0.5828H = 45$$

$$\Rightarrow \quad H = 77.21 \text{ kN}$$

From equation (ii)

$$V_A = \frac{6 \times 77.21}{17.57} + \frac{3 \times 17.57}{2} = 52.72 \text{ kN}$$

From equation (iii)

$$V_B = \frac{3 \times 77.21}{12.43} + \frac{3 \times 12.43}{2} = 37.28 \text{ kN}$$

Maximum tension occur at highest point i.e. (A)

$$T_{max} = \sqrt{V_A^2 + H^2}$$

$$\Rightarrow \quad T_{max} = \sqrt{(52.72)^2 + (77.21)^2} = 93.50 \text{ kN}$$

Minimum tension occur at lowest point i.e. (C)

$$T_{min} = H = 77.21 \text{ kN}$$

Section B

5. (a) Solution:

The assumptions made while analyzing the reinforced concrete beam using Limit State Method as per IS 456:2000 Code are as follows:

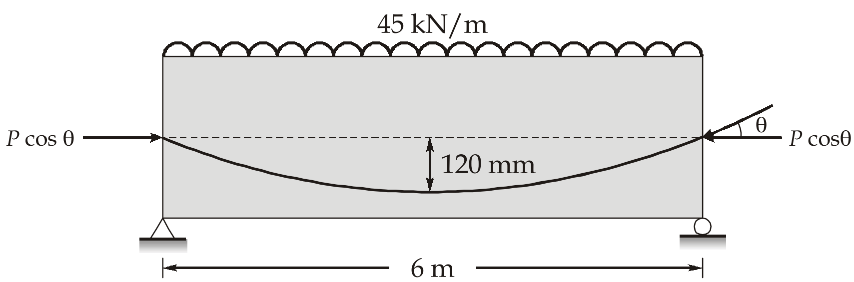
1. Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section. Thus, strain variation diagram is linear.
2. The maximum compressive strain in concrete at the outermost fiber (ε_{cu}) is taken as 0.0035, regardless of whether the beam is under-reinforced or over-reinforced, because collapse invariably occurs by the crushing of concrete.
3. IS 456: 2000 allows the use of any other possible shape of the stress-strain curve of concrete which results in substantial agreement with the results of the tests on reinforced concrete.
4. For design purposes, compressive strength of concrete may be assumed as 0.67 times the characteristic strength of concrete. The partial safety factor of γ_c = 1.5 shall be applied in addition to this.
5. The tensile strength of concrete is ignored i.e. not taken into account. Cl. B-1.3(b) of IS 456: 2000 states that all tensile stresses are to be taken up by reinforcement and none by concrete, except as otherwise specifically permitted.

6. The stress in reinforcement is derived from representative stress-strain curve for the type of steel used.
7. For design purpose, the partial safety factor for steel is taken as γ_s = 1.15 i.e. design stress of steel = $\dfrac{f_y}{1.15} = 0.87\, f_y$.
8. The maximum strain (ε_{st}) in the tension reinforcement at the level of centroid of reinforcement steel at the ultimate limit state shall not be less than ε_{st} which is defined as:

$$\varepsilon_{st} = \frac{0.87 f_y}{E_s} + 0.002$$

where, E_s = Young's modulus of elasticity of steel.

5. (b) Solution:



$$\tan\theta = \frac{4e}{l} = \frac{4\times 0.12}{6} = 0.08$$

$$\theta = 4.574°$$

$$\cos\theta = 0.9968$$

Dip of the tendon at the centre, e = 0.120 m

Upward uniformly distributed pressure provided by the cable

$$w_p = \frac{8Pe\cos\theta}{l^2} = \frac{8\times 1000\times 0.9968\times 0.120}{6^2} = 26.58 \text{ kN/m}$$

Net downward load on the beam = 45 – 26.58 = 18.42 kN/m

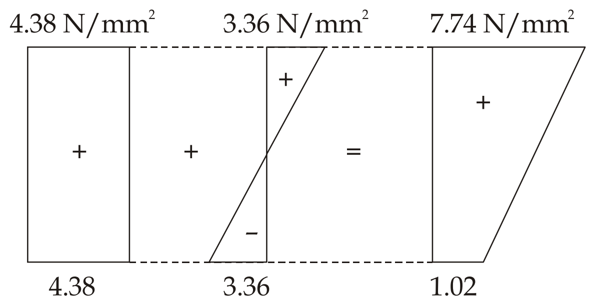
$$\text{Maximum } B.M. \text{ at mid span, } M = \frac{18.42\times 6^2}{8} = 82.89 \text{ kN/m}$$

$$\text{Extreme stresses for the mid section} = \frac{P\cos\theta}{A} \pm \frac{M}{Z} = \frac{996.8\times 10^3}{350\times 650} \pm \frac{82.89\times 10^6}{\dfrac{350\times 650^2}{6}} \text{ N/mm}^2$$

$$= 4.38 \pm 3.36 \text{ N/mm}^2$$

$$\text{Extreme stress at top} = +4.38 + 3.36 = +7.74 \text{ N/mm}^2 \text{ (c)}$$

and $$\text{Extreme stress at bottom} = +4.38 - 3.36 = +1.02 \text{ N/mm}^2 \text{ (c)}$$



5. (c) Solution:

Given Data:

Height of columns (L) = 4 m = 4000 mm

Flexural rigidity of first column (EI) = 22×10^{12} N-mm^2

Flexural rigidity of second column ($2EI$) = 44×10^{12} N-mm^2

Total load on the beam (W) = 80 kN = 80000 N

Initial displacement (x_0) = 40 mm

Initial velocity (v_0) = 32 mm/s

Time (t) = 3 sec

Acceleration due to gravity (g) = 9.81 m/sec^2

Calculation of Mass (m):

$$m = \frac{W}{g}$$

$$\Rightarrow \quad m = \frac{80000}{9.81} = 8154.944 \text{ kg}$$

Calculation of Lateral Stiffness (k):

The total stiffness for the two fixed-fixed columns in parallel is:

$$k = \frac{12EI}{L^3} + \frac{12(2EI)}{L^3} = \frac{36EI}{L^3}$$

$$\Rightarrow \quad k = \frac{36 \times 22 \times 10^{12}}{4000^3} = 12375 \text{ N/mm} = 12375000 \text{ N/m}$$

Natural Frequency (ω_n):

$$w_n = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad w_n = \sqrt{\frac{12375 \times 10^3}{8154.944}} = 38.955 \text{ rad/s}$$

Amplitude of Vibration (A):

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$$

$$\Rightarrow \quad A = \sqrt{40^2 + \left(\frac{32}{38.955}\right)^2} = 40.008 \text{ mm}$$

Displacement at t = 3 sec (x):

The displacement equation for undamped free vibration is:

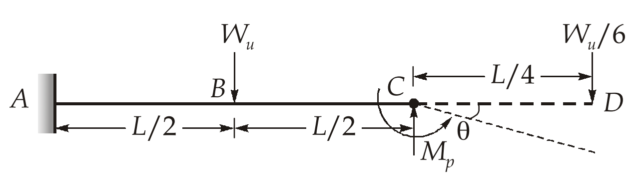
$$x(t) = x_0 \cos(\omega_n t) + \frac{v_0}{\omega_n}\sin(\omega_n t)$$

$$\Rightarrow \quad x(t = 3 \text{ sec}) = 40 \times \cos(\underbrace{38.955 \times 3}) + \frac{32}{38.955} \times \sin(\underbrace{38.955 \times 3}_{Radians})$$

$$\Rightarrow \quad x(t = 3 \text{ sec}) = -32.895 \text{ mm}$$

5. (d) Solution:

End A is fixed and end D is free, so a plastic hinge will form at C only.



$$\text{External work done} = \frac{W_u}{6} \times \frac{L}{4}\theta = \frac{W_u L}{24}\theta$$

$$\text{Internal work done} = M_p\theta$$

By principle of virtual work,

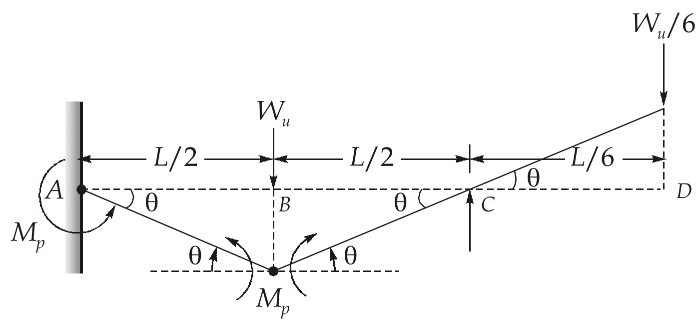
External work done = Internal work done

$$\frac{W_u L}{24}\theta = M_p\theta$$

$$\Rightarrow \quad W_u = \frac{24M_p}{L} \quad \text{...(i)}$$

Span AC

Since the end C is propped, the end D will lift up when a mechanism is formed and a negative work is done by the load acting at D. Plastic hinges will form at A and B only. At C no hinge will be formed since it is propped.



External work done = work done by load W + negative work done by load $W_u/6$

$$= W_u \frac{L}{2}\theta + \left(-\frac{W_u}{6} \times \frac{L}{4}\theta\right) = \frac{11}{24} W_u L\theta$$

$$\text{Internal work done} = M_p\theta + M_p(\theta + \theta) = 3M_p\theta$$

By principle of virtual work,

$$\text{External work done} = \text{Internal work done}$$

$$\Rightarrow \quad \frac{11}{24} W_u L\theta = 3M_p\theta$$

$$\Rightarrow \quad W_u = 6.545 \frac{M_p}{L} \quad \text{...(ii)}$$

Hence, the collapse load for the propped cantilever is least of (i) and (ii) *i.e.* $\frac{6.545 M_p}{L}$

5. (e) Solution:

For ISA $65 \times 65 \times 8$, Gross area $A_g = (65 + 65 - 8) \times 8 = 976 \text{ mm}^2$

Strength governed by yielding,

$$A_g = \frac{f_y}{\gamma_{mo}} A_g$$

$$= \left[976 \times \frac{250}{1.1}\right] \times 10^{-3} = 221.82 \text{ kN}$$

$$A_{nc} = \text{area of connected leg}$$

$$= (65 - 4 - 22) \times 8 = 312 \text{ mm}^2$$

$$A_{go} = (65 - 4) \times 8 = 488 \text{ mm}^2$$

Strength governed by rupture of critical section.

$$T_{dn} = 0.9 f_u A_{nc} / \gamma_{m1} + \beta A_{go} f_y / \gamma_{m0}$$

$$\beta = 1.4 - 0.076 \times \left(\frac{65}{8}\right)\left(\frac{250}{410}\right)(92)/(3\times 60)$$

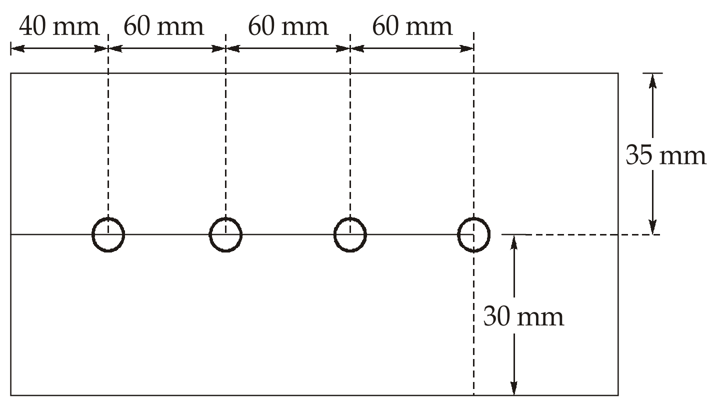
$$= 1.208 > 0.7 \qquad \text{OK}$$

Here, $$\frac{f_u \gamma_{mo}}{f_y \gamma_{m1}} = 1.4432 > 1.208$$

$$T_{dn} = \left[0.9\times 410\times\frac{312}{1.25} + 1.208\times 488\times\frac{250}{1.10}\right]\times 10^{-3}$$

$$= 226.08 \text{ kN}$$

Strength governed by block shear



$$A_{vg} = 8\times(3\times 60+40) = 1760 \text{ mm}^2$$

$$A_{vn} = 8\times(3\times 60+40-3.5\times 22) = 1144 \text{ mm}^2$$

$$A_{tg} = 8\times 30 = 240 \text{ mm}^2$$

$$A_{tn} = 8\times(30-0.5\times 22) = 152 \text{ mm}^2$$

$$T_{db1} = \left[1760\times 250/(\sqrt{3}\times 1.1) + 0.9\times 410\times\frac{152}{1.25}\right]\times 10^{-3}$$

$$= 275.81 \text{ kN}$$

$$T_{db2} = \left[0.9\times 410\times\frac{1144}{\sqrt{3}\times 1.25} + 250\times\frac{240}{1.10}\right]\times 10^{-3}$$

$$= 249.52 \text{ kN}$$

Tension capacity of the angle =221.82 kN > 170 kN

Hence the angle is safe

Q.6 (a) Solution:

(i) Types of loss of prestress:

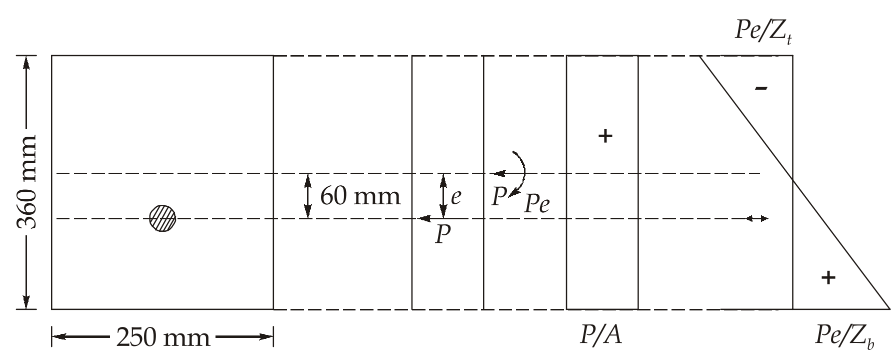
(a) In pretensioned member:

S.No	Loss due to
1.	Elastic deformation of concrete
2.	Relaxation of stress in steel
3.	Shrinkage of concrete
4.	Creep of concrete

(b) In post tensioned member:

S.No	Loss due to
1.	No losses due to elastic deformation if all the wires are simultaneously tensioned. If the wires are successively tensioned than there will be loss of prestress due to elastic deformation of concrete.
2.	Relaxation of stress in steel
3.	Shrinkage of concrete
4.	Creep of concrete
5.	Friction
6.	Anchorage slip

(ii)



Given, Initial prestress = 1250 N/mm^2

Area of steel wires = 350 mm^2

∴ Prestressing force,

$$P = \frac{1250 \times 350}{1000} \text{ kN} = 437.5 \text{ kN}$$

Stress in concrete at the level of steel,

$$= \frac{P}{A} + \frac{Pe^2}{I}$$

$$= \frac{437.5 \times 10^3}{250 \times 360} + \frac{437.5 \times 10^3 \times (60)^2}{\frac{250 \times (360)^3}{12}} = 6.48 \text{ N/mm}^2$$

Modular ratio $m = \frac{E_s}{E_c} = \frac{210}{35} = 6$

(a) Loss of stress for the pretensioned beam:

(i) Loss of stress due to elastic shortening of concrete

$$= m \times f_c = 6 \times 6.48 = 38.88 \text{ N/mm}^2$$

(ii) Loss of stress due to creep of concrete

$$= 45 \times 10^{-6} \times 6.48 \times 210 \times 10^3 = 61.24 \text{ N/mm}^2$$

(iii) Loss of stress due to shrinkage of concrete

$$= 300 \times 10^{-6} \times 210 \times 10^3 = 63 \text{ N/mm}^2$$

(iv) Loss of stress due to relaxation of steel stress

$$= \left(\frac{5}{100} \times 1250\right) = 62.50 \text{ N/mm}^2$$

∴ Total loss of stress = 225.62 N/mm²

∴ Percentage loss of stress

$$= \frac{225.62}{1250} \times 100 = 18.05\%$$

(b) Loss of stress for the post tensioned beam:

(i) Loss of stress due to elastic shortening of concrete = 0

(ii) Loss of stress due to creep of concrete

$$= 22 \times 10^{-6} \times 210 \times 10^3 \times 6.48 = 29.94 \text{ N/mm}^2$$

(iii) Loss of stress due to shrinkage of concrete

$$= 215 \times 10^{-6} \times 210 \times 10^3 = 45.15 \text{ N/mm}^2$$

(iv) Loss due to relaxation of stress in steel

$$= \frac{5}{100} \times 1250 = 62.50 \text{ N/mm}^2$$

(v) Loss of stress due to a anchorage slip

$$= \frac{1.25}{12 \times 1000} \times 210 \times 10^3 = 21.88 \text{ N/mm}^2 \quad (\because \alpha = 0)$$

(vi) Loss of stress due to friction effect

$$= f_0\,(kx) = 1250 \times 0.0015 \times 12 = 22.5 \text{ N/mm}^2$$

$\therefore$ Total Loss of stress = 181.97 N/mm^2

$\therefore$ Percentage loss of stress

$$= \frac{181.97}{1250} \times 100 = 14.56\%$$

6. (b) Solution:

Step-1: Determination of natural period

Given that infill panels are provided.

Fundamental natural period, $T = \dfrac{0.09h}{\sqrt{d}} = \dfrac{0.09 \times (3 \times 3.45)}{\sqrt{7.5}} = 0.340 \text{ sec}$

Step-2: Other important factors

For $T = 0.340$ sec, damping of 5% and for hard rock, $\dfrac{S_a}{g} = 2.5$, for Bhuj located in zone V, zone factor, $z = 0.36$, since the building is used as a hospital building, the importance factor $I = 1.5$.

For a special moment resisting beam,

The response reduction factor, $R = 5.0$.

Step-3: Determination of design horizontal seismic coefficient

The design horizontal seismic coefficient,

$$A_h = \left(\frac{Z}{2}\right)\left(\frac{I}{R}\right)\left(\frac{S_a}{g}\right)$$

$$= \left(\frac{0.36}{2}\right)\left(\frac{1.5}{5}\right)(2.5) = 0.135$$

Step-4: Determination of seismic weight

Weight of one storey = Total weight of beams + Slab + Columns + Walls + Live load

$$= 120 + 250 + 50 + 550 + 145$$

$$= 1115 \text{ kN}$$

$\therefore$ Weight of I and II floor = 1115 kN

Weight of terrace floor = 650 kN

Total weight of building, $W = 2 \times 1115 + 650 = 2880 \text{ kN}$

Step-5: Determination of base shear

Design base shear, $V_B = A_h W$

$= 0.135 \times 2880 = 388.80$ kN

Step-6: Distribution of equivalent lateral load

$$Q_i = V_B\left(\frac{W_i h_i^2}{\sum_{i=1}^{n} W_i h_i^2}\right)$$

where, h_i is calculated from base

For 1st floor, $Q_1 = V_B\left(\frac{W_1 h_1^2}{W_1 h_1^2 + W_2 h_2^2 + W_3 h_3^2}\right)$

$\Rightarrow$ $Q_1 = 388.80\left(\frac{1115\times 3.45^2}{1115\times 3.45^2 + 1115\times 6.90^2 + 650\times 10.35^2}\right)$

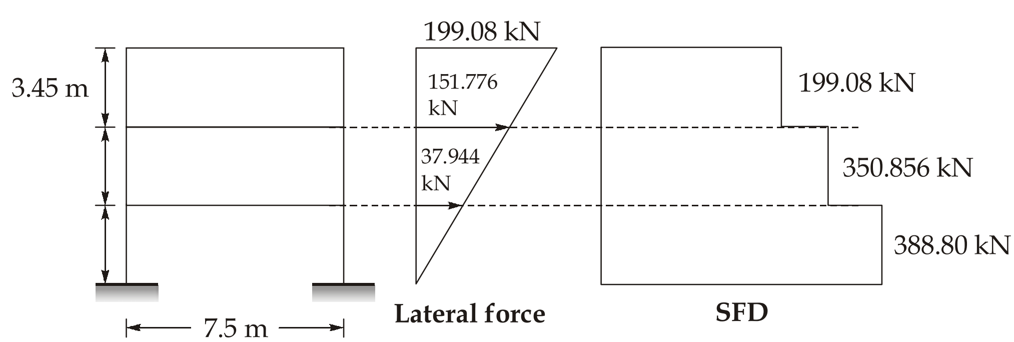
$\Rightarrow$ $Q_1 = 37.944$ kN

For 2nd floor, $Q_2 = 388.80\left(\frac{1115\times 6.90^2}{1115\times 3.45^2 + 1115\times 6.90^2 + 650\times 10.35^2}\right)$

$\Rightarrow$ $Q_2 = 151.776$ kN

For 3rd floor, $Q_3 = 388.80\left(\frac{650\times 10.35^2}{1115\times 3.45^2 + 1115\times 6.90^2 + 650\times 10.35^2}\right)$

$\Rightarrow$ $Q_3 = 199.08$ kN



Check: $V_B = Q_1 + Q_2 + Q_3 = 388.80$ kN

6. (c) Solution:

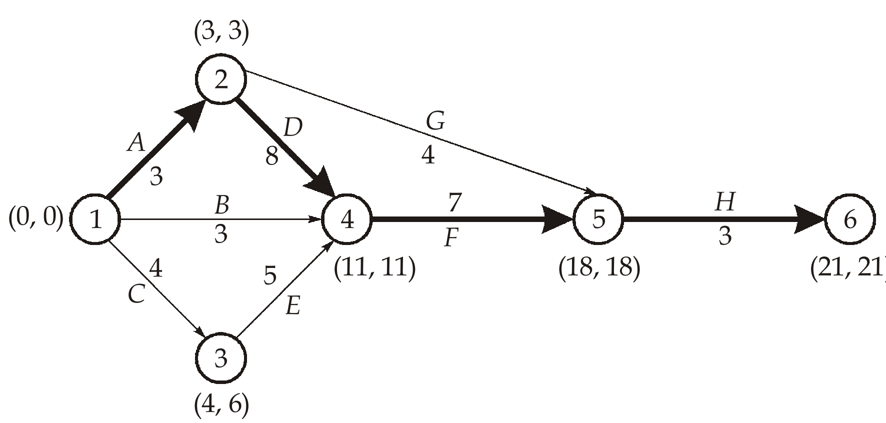
Expected time can be calculated by using the following expression,

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

where,
t_e = Optimistic time
t_m = Most likely time
t_p = Pessimistic time

Standard deviation, $\sigma = \left(\frac{t_p - t_o}{6}\right)$

Activity	Expected time (t_e)	σ
A	3	4/6
B	3	2/6
C	4	2/6
D	8	8/6
E	5	2/6
F	7	8/6
G	4	4/6
H	3	6/6



(i) Total Project duration, t_e = 21 days

$$\sigma = \sqrt{(4/6)^2 + (8/6)^2 + (8/6)^2 + (6/6)^2}$$
$$= 2.236 \text{ days}$$

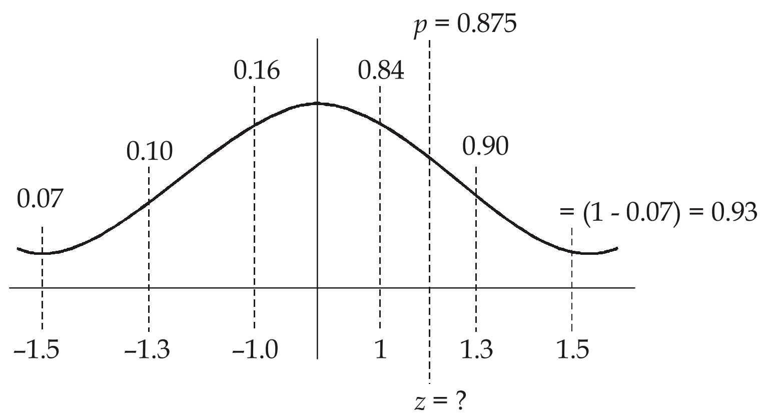
for, t = 18 days

$$Z = \frac{t - t_e}{\sigma} = \frac{18 - 21}{2.236} = -1.342$$

From table, $p = 0.10 - \frac{0.10 - 0.07}{0.2} \times (1.342 - 1.3)$

$\Rightarrow$ $p = 0.0937 = 9.37\%$

(b) For $p = 87.5\%$ $\Rightarrow$ $t = ?$



$$Z = 1.0 + \frac{1.3 - 1.0}{0.90 - 0.84} \times (0.875 - 0.84)$$

$$= 1.175$$

Now, $\frac{t - 21}{2.236} = 1.175$

$$t = 23.627 \text{ days}$$

7. **(a) Solution:**

Given: $\gamma = 16 \text{ kN/m}^3$ $\phi = 30°$

$\gamma_c = 25 \text{ kN/m}^3$ M20, Fe 415

$f_{ck} = 20 \text{ N/mm}^2$; $f_y = 415 \text{ N/mm}^2$

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1}{3}$$

Eccentricity of resultant force, $e = 0.334 \text{ m}$

Summation of vertical forces:

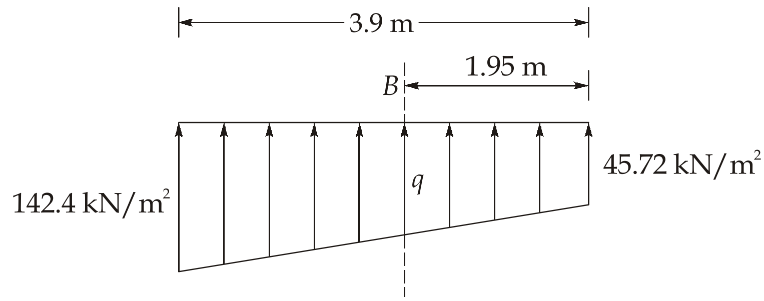
$$\Sigma F_v = 366.8 \text{ kN/m} \text{ (Given)}$$

Soil pressure at footing base:

$$(\sigma_v)_{\text{Toe}} = \frac{\Sigma F_v}{B}\left(1 + \frac{6e}{B}\right) = \frac{366.8}{3.9}\left(1 + \frac{6 \times 0.334}{3.9}\right) = 142.4 \text{ kN/m}^2$$

$$(\sigma_v)_{Heel} = \frac{\Sigma F_v}{B}\left(1-\frac{6e}{B}\right) = \frac{366.8}{3.9}\left(1-\frac{6\times0.334}{3.9}\right) = 45.72 \text{ kN/m}^2$$

Base pressure distribution:



$$q = 45.72 + \frac{142.4-45.72}{3.9}\times1.95 = 94.06 \text{ kN/m}^2$$

Design of heel slab:

The distributed loading acting downward on the heel slab is given by

(i) Weight of soil (over burden) above heel slab

$$= 16 \times (7.75 - 0.62) \times 1.95 = 222.456 \text{ kN/m}$$

(ii) Weight of heel slab

$$= (1.95 \times 0.62 \times 25) = 30.225 \text{ kN/m.}$$

∴ The total downward load acting on heel slab

$$W = 252.681 \text{ kN/m}$$

Upward pressure on heel slab, U

$$= \frac{1}{2}\times(45.72+94.06)\times1.95 = 136.2855 \text{ kN/m}$$

Location of this upward pressure from B,

$$\bar{x} = \frac{2\times45.72+94.06}{45.72+94.06}\times\frac{1.95}{3} = 0.863 \text{ m}$$

∴ Shear force at B,

$$V_B = W - U = 252.681 - 136.2855 = 116.4 \text{ kN}$$

∴ Factored shear force, $V_u = 1.5 \times 116.4 = 174.6$ kN

Bending moment at B, $BM = 252.681\times\frac{1.95}{2} - 136.2855\times0.863 = 128.75$ kN-m

∴ Factored bending moment

$$BM_u = 1.5 \times 128.75 = 193.125 \text{ kNm}$$

As, Clear cover = 75 mm, diameter of bar = 16 mm

∴ Effective depth, $d = 620 - 75 - 16/2 = 537$ mm

Also for Fe415, $BM_{u,\,lim} = 0.138\, f_{ck} bd^2$

$$= 0.138 \times 20 \times 1000 \times (537)^2 \text{ Nmm} = 795.898 \text{ kNm}$$

As $BM_u < \text{BM}_{\text{u,lim}}$

Section is under-reinforced and thus,

$$A_{st} = \frac{0.5 \cdot f_{ck}\, bd}{f_y}\left(1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck} bd^2}}\right)$$

$$\Rightarrow \quad A_{st} = \frac{0.5 \times 20 \times 1000 \times 537}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 193.125 \times 10^6}{20 \times 1000 \times 537^2}}\right)$$

$$\Rightarrow \quad A_{st} = 1038.24 \text{ mm}^2$$

$$\therefore \quad \text{Spacing of 16 mm bars} = \frac{1000 \times \frac{\pi}{4}(16)^2}{1038.24} = 193.66 \text{ mm c/c}$$

Provide 16 mm ϕ bars @ 180 mm c/c

$$\therefore \quad (A_{st})_{\text{provided}} = \frac{1000 \times \frac{\pi}{4}(16)^2}{180} = 1117.01 \text{ mm}^2 > (A_{st})_{\text{required}}$$

Distribution steel, $A_{st} = 0.12\% \text{ of } A_g = \frac{0.12}{100} \times 1000 \times 620 = 744 \text{ mm}^2$

Using 10 mm ϕ bars,

$$\text{Spacing} = \frac{1000 \times \frac{\pi}{4} \times (10)^2}{744} = 105.56 \text{ mm}$$

$\therefore$ Provide 10 mm ϕ bars @ 100 mm c/c.

Check for shear:

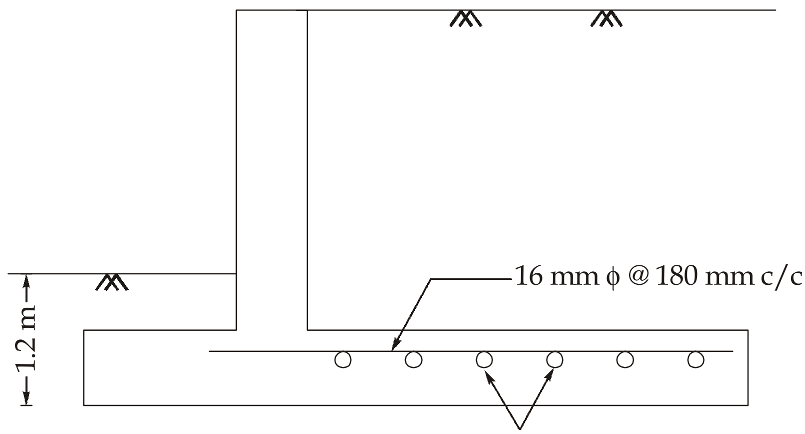
Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{174.6 \times 10^3}{1000 \times 537} = 0.325 \text{ N/mm}^2$

$$P_t(\%) = \frac{A_{st}}{bd} \times 100 = \frac{1117.01}{1000 \times 537} \times 100 = 0.21\%$$

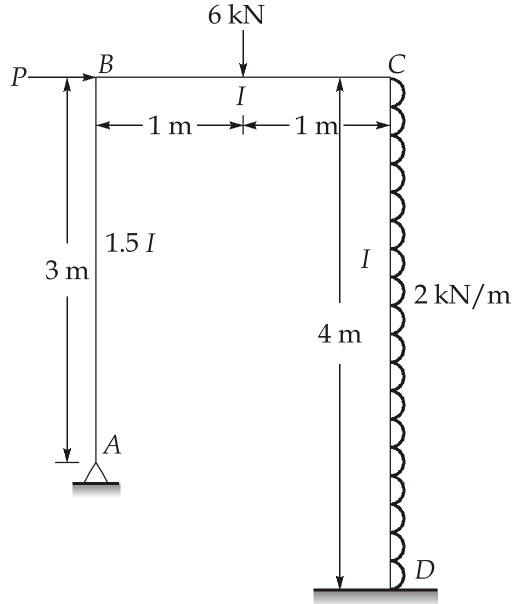
From table given

$$\tau_c = 0.28 + \frac{0.36 - 0.28}{0.25 - 0.15}(0.21 - 0.15) = 0.328 \text{ N/mm}^2$$

$\therefore \quad \tau_v < \tau_c$ (OK)



7. (b) Solution:



Distribution Factor:

Joint	Member	Stiffness	T.S.	D.F.
B	BA	$\frac{3(1.5\,EI)}{3} = \frac{3EI}{2}$	$\frac{7EI}{2}$	$\frac{3}{7}$
	BC	$\frac{4EI}{2} = EI$		$\frac{4}{7}$
C	CB	$\frac{4EI}{2} = 2EI$	$3EI$	$\frac{2}{3}$
	CD	$\frac{4EI}{4} = EI$		$\frac{1}{3}$

Fixed and Moments:

$$M_{FBC} = \frac{-6 \times 2}{8} = -1.5 \text{ kNm}$$

$$M_{FCB} = +1.5 \text{ kNm}$$

$$M_{FCD} = -\frac{2 \times 4^2}{12} = -2.67 \text{ kNm}$$

$$M_{FDC} = 2.67 \text{ kNm}$$

Moment Distribution:

Assuming a horizontal force P is to be applied at the joint B to prevent the side sway. The moment distribution, neglecting the sway will then be carried out.

	A	B		C		D
		$\frac{3}{7}$	$\frac{4}{7}$	$\frac{2}{3}$	$\frac{1}{3}$	
FEM	0	0	–1.50	+1.50	–2.67	+2.67
Balance	-	+0.64	+0.85	+0.78	+0.39	–
C.O.	-	–	+0.39	+0.43	–	+0.20
Balance	-	–0.17	–0.22	–0.29	–0.14	–
C.O.	-	–	–0.15	–0.11	–	–0.07
Balance	-	+0.06	+0.09	+0.07	+0.04	–
C.O.	-	–	+0.04	+0.05	–	+0.02
Balance & C.O.	-	–0.02	–0.02	–0.03	–0.02	
Final Moments	0	+0.51	–0.51	+2.40	–2.4	2.82

$$\text{Horizontal reaction at } A = \frac{0.51}{3} = 0.17 \text{ kN} (\rightarrow)$$

For horizontal reaction at D i.e., H_D taking moments about C for equilibrium of CD,

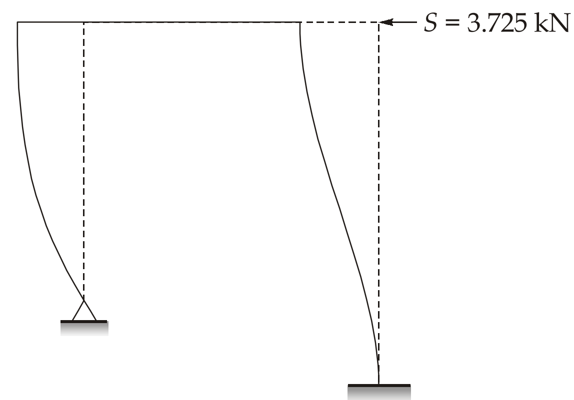
$$H_D \times 4 = 2 \times 4 \times 2 + 2.82 - 2.4$$

$\Rightarrow$ $$H_D = 4.105 \text{ kN}$$

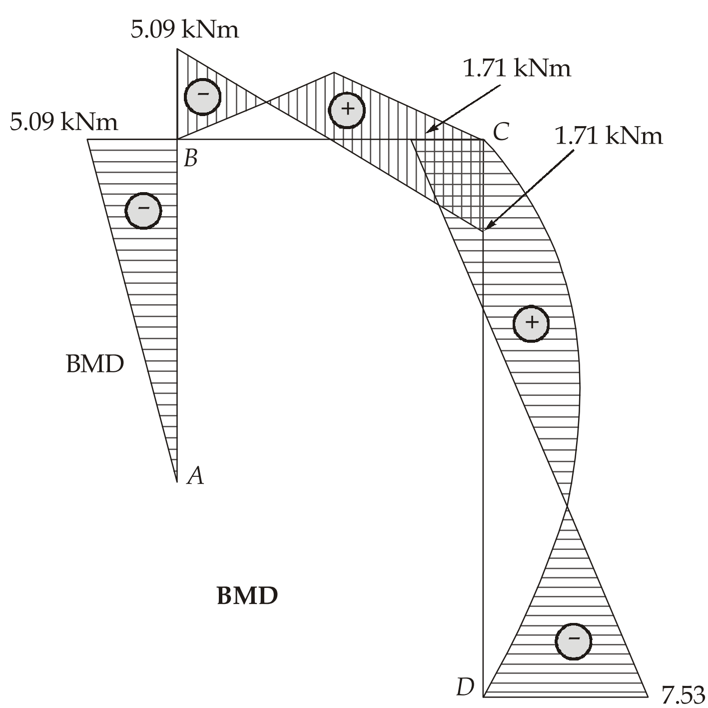
$\therefore$ $$P = 8 - 0.17 - 4.105 = 3.725 \text{ kN} (\rightarrow)$$

Side Sway:

As actually there is no force like P acting at joint B, so, an equal but opposite force S = 3.73 kN ($\leftarrow$) at joint C is applied to neutralize the effect of P.

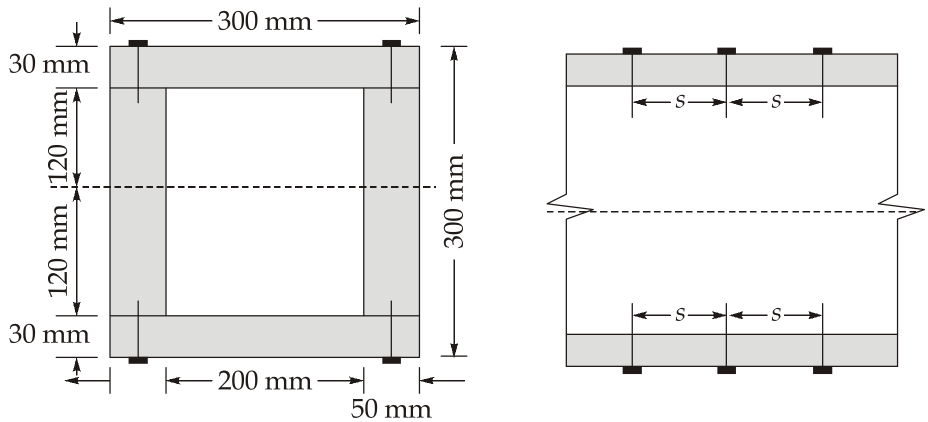


	A		B		C	D
1. Sway = 1 kN (←)	0	1.227	–1.227	–1.101	+1.101	1.263
2. Sway = 3.73 kN (←)	0	+4.58	–4.58	–4.11	+4.11	+4.71
3. Non sway moments	0	+0.51	–0.51	+2.40	–2.40	+2.82
4. Final moments	0	+5.09	–5.09	–1.71	+1.71	+7.53



BMD

7. (c) (i) Solution:



The spacing of the lag screws is s.

$$I = \frac{1}{12}\times 2\times 50(240)^3 + 2\left[\frac{1}{12}\times 300(30)^3 + 30\times 300(135)^2\right] = 4446\times 10^5 \text{ mm}^4$$

$$\text{OR, } I = \frac{300^4}{12} - \frac{200\times 240^3}{12} = 4446 \times 10^5 \text{ mm}^4$$

$$M = fZ = 8\times\frac{4446\times 10^5}{150} = 2371.2\times 10^4 \text{ N-mm} = 23.712 \text{ kN-m}$$

But $$M = \frac{WL}{4}$$

$\Rightarrow$ $$W = \frac{4M}{L} = \frac{4\times 23.712}{2.5} = 37.9392 \text{ kN}$$

Hence $$V = \frac{W}{2} = \frac{37.9392}{2} = 18.9696 \text{ kN}$$

Now, shear flow at the junction, 30 mm from the top (or the bottom) is

$$q' = \frac{VA\bar{y}}{I} = \frac{18.9696\times 10^3}{4446\times 10^5}(30\times 300\times 135) = 51.84 \text{ N/mm}$$

Since there are two screws in each row, total shear transmitted by each screw

$$= \frac{1}{2}(s\times 51.84) = 25.92\, s\,(\text{N})$$

Equating this to shear value of each screw, we have

$$25.92\, s = 2800$$

$\Rightarrow$ $$s = 108.02 \text{ mm center to center}$$

7. (c) (ii) Solution:

Given data:

$$\sigma_1 = 100 \text{ MN/m}^2$$
$$\sigma_2 = 60 \text{ MN/m}^2$$
$$\sigma_3 = -40 \text{ MN/m}^2$$
$$E = 210 \times 10^3 \text{ MN/m}^2$$
$$\mu = 0.25$$

(i) Total strain energy (U):

$$U = \frac{1}{2\times E}\times[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)]$$

$$\Rightarrow \quad U = \frac{1}{2\times 210\times 10^3}\times[100^2+60^2+(-40)^2-2\times 0.25\times(100\times 60+60\times(-40)+(-40)\times 100)]$$

$$\Rightarrow \quad U = 0.036667 \text{ MN/m}^2 = 36.667 \text{ kJ/m}^3$$

(ii) Volumetric strain energy (U_v):

$$U_v = \frac{1}{2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)\frac{(\sigma_1+\sigma_2+\sigma_3)}{E}(1-2u)$$

$$\Rightarrow \quad U_v = \frac{3\times(1-2\mu)}{2E}\times\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)^2$$

$$\Rightarrow \quad U_v = \frac{3\times(1-2\times 0.25)}{2\times 210\times 10^3}\times\left(\frac{100+60+(-40)}{3}\right)^2$$

$$U_v = 5.714 \text{ kJ/m}^3$$

(iii) Shear strain energy (U_s):

$$U_s = \frac{(1+\mu)}{6E}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]$$

$$U_s = \frac{1+0.25}{6\times 210\times 10^3}\times[(100-60)^2+(60-(-40))^2+(-40-100)^2]$$

$$U_s = 30.952 \text{ kJ/m}^3$$

OR

$$U_s = U - U_v = 36.667 - 5.714$$

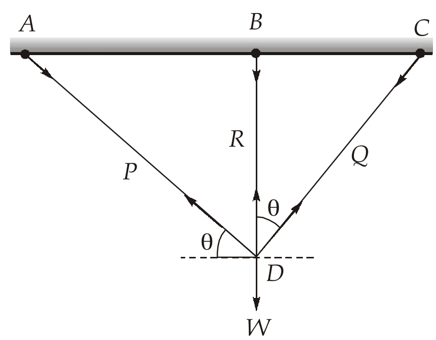
$$\Rightarrow \quad U_s = 30.953 \text{ kJ/m}^3$$

8. (a) **Solution:**

It is easily seen that ADC is a right-angled triangle. Let the tensions in *DA*, *DC* and *DB* be *P*, *Q* and *R* respectively.

Let *DC* be inclined at θ with the vertical then *AD* will be inclined at θ with the horizontal.

For the equilibrium of the joint *D*, resolving the forces at *D* horizontally, we have $P\cos\theta = Q\sin\theta$, $P = Q\tan\theta$



$$\tan\theta = \frac{3}{4}$$

$$\therefore \quad \sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

$$\Sigma H = 0$$

$$\Rightarrow \quad P\cos\theta = Q\sin\theta$$

$$\Rightarrow \quad P\left(\frac{4}{5}\right) = Q\left(\frac{3}{5}\right)$$

$$\Rightarrow \quad P = \frac{3}{4}Q$$

Resolving the forces at *D* vertically, we have,

$R + P\sin\theta + Q\cos\theta = W$

$$\Rightarrow \quad R + \frac{3}{4}Q\times\frac{3}{5} + Q\frac{4}{5} = W$$

$$\Rightarrow \quad R + \frac{5}{4}Q = W$$

$$\Rightarrow \quad Q = \frac{4}{5}(W - R)$$

$$\text{But} \quad P = \frac{3}{4}Q$$

$$\therefore \qquad P = \frac{3}{5}(W-R)$$

$$\text{Strain energy stored, } W_t = \frac{P^2(AD)}{2AE}+\frac{Q^2(DC)}{2AE}+\frac{R^2(BD)}{2AE}$$

$$\Rightarrow \qquad W_t = \frac{1}{2AE}\left[5P^2+3.75Q^2+3R^2\right]$$

At equilibrium, strain energy is minimum.

$$\therefore \qquad \frac{\partial W_i}{\partial R} = 0$$

$$\therefore \qquad \frac{\partial W_i}{\partial R} = \frac{1}{2AE}\left[10P\frac{dP}{dR}+7.5Q\frac{dQ}{dR}+6R\right]=0$$

$$P = \frac{3}{5}(W-R)$$

$$\text{and} \qquad Q = \frac{4}{5}(W-R)$$

$$\therefore \qquad \frac{dP}{dR} = -\frac{3}{5} \text{ and } \frac{dQ}{dR}=-\frac{4}{5}$$

$$\therefore \qquad 10\times\frac{3}{5}(W-R)\left(-\frac{3}{5}\right)+7.5\times\frac{4}{5}(W-R)\left(-\frac{4}{5}\right)+6R = 0$$

$$\Rightarrow \qquad -\frac{18}{5}(W-R)-\frac{24}{5}(W-R)+6R=0$$

$$\Rightarrow \qquad \frac{72}{5}R = \frac{42}{5}W$$

$$\therefore \qquad R = \frac{42}{72}W=\frac{7}{12}W$$

$$\therefore \qquad P = \frac{3}{5}(W-R)=\frac{3}{5}\left(W-\frac{7}{12}W\right)=\frac{W}{4}$$

$$\text{and} \qquad Q = \frac{4}{5}(W-R)=\frac{4}{5}\left(W-\frac{7}{12}W\right)=\frac{W}{3}$$

$$\text{Horizontal component of the extension of } DA = \frac{P}{AE}\times 5\cos\theta=\frac{W}{4AE}\times5\times\frac{4}{5}=\frac{W}{AE}$$

$$\text{Horizontal component of the extension of } DC = \frac{Q}{AE}\times 3.75\sin\theta$$

$$= \frac{W}{3AE} \times 3.75 \times \frac{3}{5} = \frac{0.75W}{AE}$$

∴ Horizontal movement of D,

= Difference between the horizontal components of the extensions of DA and DC

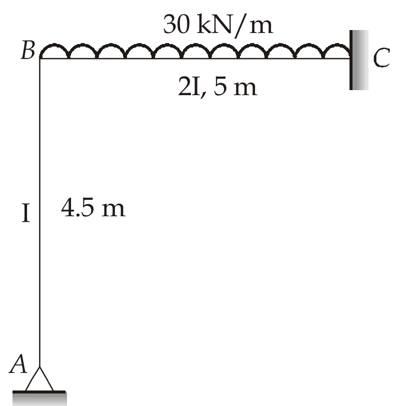
$$= \frac{W}{AE} - \frac{0.75W}{AE} = \frac{W}{4AE}$$

$$\text{Extension of } BD = \frac{R}{AE} \times 3 = \frac{7}{12}\frac{W}{AE} \times 3 = \frac{7W}{4AE}$$

$$\therefore \quad \text{Horizontal movement of } D = \frac{1}{7} \text{ of the extension of } BD.$$

8. (b) (i) Solution:

Due to symmetry there will be no horizontal reaction at D. Also there will be zero slope on C so, C can be treated as fixed joint.



Fixed End moments:

For AB:

$$M_{FAB} = 0$$

$$M_{FBA} = 0$$

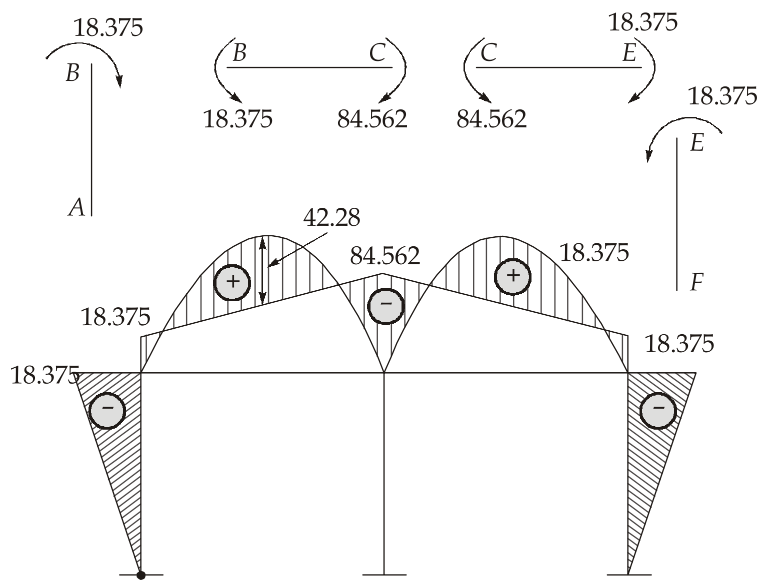
For BC:

$$M_{FBC} = \frac{-30 \times 5^2}{12} = -62.5 \text{ kN-m}$$

$$M_{FCB} = \frac{30 \times 5^2}{12} = 62.5 \text{ kN-m}$$

Joint	Member	Stiffness	T.S.	D.F.
B	BA	$\frac{3EI}{4.5} = \frac{2EI}{3}$	$\frac{34EI}{15}$	0.294
	BC	$\frac{4(2EI)}{5} = \frac{8EI}{5}$		0.706

	AB	BA (B)	BC (B)	CB
		0.294	0.706	
FEM	0	0	–62.5	62.5
Balance		18.375	44.125	
COM				22.0625
Final M	0	18.375	–18.375	84.562



BMD (All value in kN-m)

$$\text{Moment at middle} = \frac{30 \times 5^2}{8} - \left(\frac{18.375 + 84.562}{2}\right) = 42.28 \text{ kN-m}$$

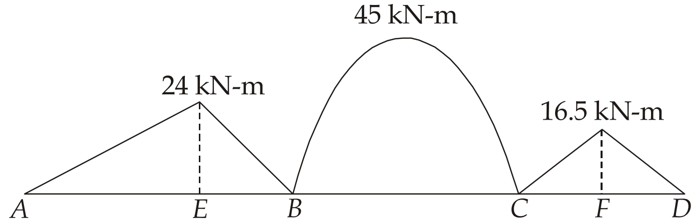
8. **(b) (ii) Solution:**

Thermal cracking in mass concrete occurs when the heat generated by the hydration of cement causes a significant temperature differential between the hot interior and the cooler surface of the structure. To prevent these cracks, the following steps can be taken:

1. **Use of Low-Heat Cement:** Utilize cements with low heat of hydration, such as Portland Pozzolana Cement (PPC), Slag cement, or specialized low-heat Portland cement to reduce the initial temperature rise.
2. **Replacement with Admixtures:** Replace a portion of the cement with supplementary cementitious materials (SCMs) like fly ash or ground granulated blast-furnace slag (GGBS), which react more slowly and generate less heat.
3. **Pre-cooling of Ingredients:** Lower the temperature of the concrete mix before placement by cooling the aggregates with water sprays or using chilled water and chipped ice instead of normal mixing water.

4. **Post-cooling Systems:** Install a network of pipes within the concrete mass through which cool water is circulated to extract heat directly from the interior during the curing process.
5. **Reducing Cement Content:** Optimize the mix design to use the minimum amount of cement necessary for the required strength, often by increasing the maximum size of the aggregate.
6. **Control of Pouring Temperature:** Schedule concrete placement during cooler times of the day (evening or night) or during cooler seasons to maintain a low placement temperature.
7. **Thermal Insulation:** Apply insulation to the surface of the concrete to reduce the temperature gradient between the core and the atmosphere, allowing the entire mass to cool more uniformly.
8. **Lift Thickness Control:** Cast the concrete in thinner layers (lifts) to allow more heat to dissipate from the surface before the next layer is poured.

8. (c) Solution:



Free BMD

Area of free *BMD* on $AB = \frac{1}{2} \times 6 \times 24 = 72 \text{ kN-m}^2$

Area of free *BMD* on $BC = \frac{2}{3} \times 6 \times 45 = 180 \text{ kN-m}^2$

Area of free *BMD* on $CD = \frac{1}{2} \times 3 \times 16.5 = 24.75 \text{ kN-m}^2$

Now, Centroidal distance of free *BMD* on *AB* from *A*,

$$\bar{x}_1 = \left(\frac{4+6}{3}\right) = 3.33 \text{ m}$$

Centroidal distance of free *BMD* on *BC* from *C*,

$$\bar{x}_2 = 3 \text{ m}$$

Centroidal distance of free *BMD* on *CD* from *D*,

$$\bar{x}_3 = 1.5 \text{ m}$$

Now, applying three moment equation on spans *AB* and *BC*,

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{-6a_1\bar{x}_1}{l_1} - \frac{6a_2\bar{x}_2}{l_2}$$

Here, +ve moment (sagging) and –ve moment (Hogging)

$$\Rightarrow \quad 0 + 2M_B(6+6) + M_C \times 6 = \frac{-6\times72\times3.33}{6} - \frac{6\times180\times3}{6}$$

$$\Rightarrow \quad 24M_B + 6M_C = -779.76 \quad \text{...(i)}$$

Applying three moment equation for span *BC* and *CD*,

$$M_B l_2 + 2M_C(l_2 + l_3) + M_D \times l_3 = \frac{-6a_2\bar{x}_2}{l_2} - \frac{6a_3\bar{x}_3}{l_3}$$

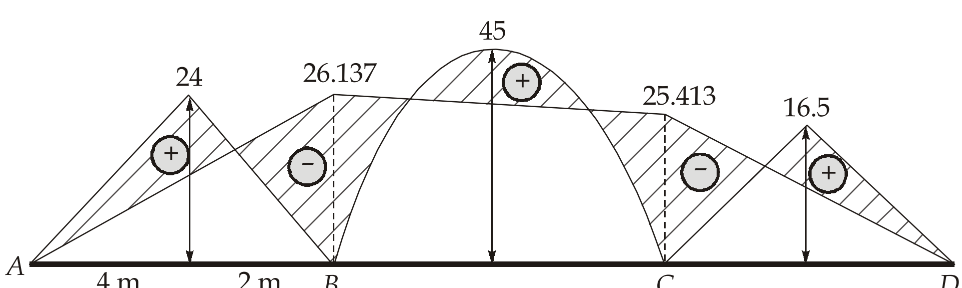
$$\Rightarrow \quad 6M_B + 2M_C(6+3) + 0\times3 = \frac{-6\times180\times3}{6} - \frac{6\times24.75\times1.5}{3}$$

$$\Rightarrow \quad 6M_B + 18M_C = -614.25 \quad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$M_B = -26.137 \text{ kN-m}$$

$$M_C = -25.413 \text{ kN-m}$$



BMD diagram

(Value in kN-m)

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