Detailed Solutions

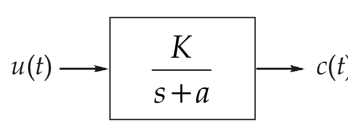
ESE-2026 Mains Test Series

E & T Engineering Test No : 11

Full Syllabus Test (Paper-II)

Section A

Q.1 (a) Solution:



We have,
$$C(s) = H(s).R(s),$$
where $R(s)$ is the Laplace Transform of input

$$C(s) = \frac{K}{s+a} \times \frac{1}{s} \qquad \left(\because\ u(t) \rightleftharpoons \frac{1}{s}\right)$$

$$C(s) = \frac{\left(\frac{K}{a}\right)}{s} + \frac{\left(-\frac{K}{a}\right)}{(s+a)}$$

Taking inverse Laplace Transform,

$$c(t) = \frac{K}{a}(1-e^{-at})u(t)$$

For a first-order system, the tangent drawn to the step response at $t = 0$ intersects the final steady-state value at a time equal to the time constant τ. From Figure II, we have $\tau = 0.2$ sec. Since the time constant can be determined as the negative reciprocal of the pole of the system,

$\therefore$ Time constant, $\tau = \dfrac{1}{a} = 0.2$

$a = 5$

Using the final value theorem,

$$c(\infty) = \lim_{s\to 0} sC(s) = \lim_{s\to 0} s\times\frac{K}{s(s+a)} = \frac{K}{a}$$

$\because$ $\dfrac{K}{a} = c(\infty) = 2.0$

$\therefore$ $K = 2.0 \times a = 10$

Thus, $K = 10; \quad a = 5$

Q.1 (b) Solution:

(i) Given, $h(t) = e^{-|t|}$

$x(t) = 3\cos(4t + 0.5)$

For an LTI system with impulse response $H(j\omega)$ and sinusoidal input $A\cos(\omega_0 t + \phi)$, the output is given as

$$y(t) = A\,|H(j\omega_0)|\,\cos(\omega_0 t + \phi + \angle H(j\omega_0))$$

For the given inpt $x(t) = 3\cos(4t + 0.5)$, we have

$$A = 3;\ \omega_0 = 4 \text{ rad/sec};\ \phi = 0.5$$

The Fourier transform of $h(t)$ is

$$e^{-|t|} \overset{FT}{\longleftrightarrow} \frac{2}{1+\omega^2} \qquad \left\{\because\ e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+\omega^2}\right\}$$

$\therefore$ $$H(j\omega) = \frac{2}{1+\omega^2}$$

$$|H(j\omega)|_{\text{at } \omega = 4} = \frac{2}{1+4^2} = \frac{2}{1+16} = \frac{2}{17}$$

Clearly the $H(j\omega)$ is real and positive

$\Rightarrow$ $\angle H(j\omega) = 0$

Thus, $$y(t) = 3\times\frac{2}{17}\cos(4t + 0.5 + 0)$$

$$y(t) = \frac{6}{17}\cos(4t + 0.5)$$

(ii) Power attenuation (in dB),

$$= 10\log_{10}\left(\frac{P_i}{P_0}\right)$$

$$= -10\log_{10}\left[|H(j4)|^2\right]$$

$$= -10\log_{10}\left(\frac{2}{17}\right)^2$$

$$= -10\log_{10}\frac{4}{289} = 18.6 \text{ dB}$$

(iii) Given condition for essential bandwidth,

$$|H(j\omega)| \geq 0.01\,|H(j0)|$$

where, $$|H(0)| = \frac{2}{1+0} = 2$$

1% of $|H(j0)| = 0.02$

$\therefore$ required condition for essential BW,

$$\frac{2}{1+\omega^2} \geq 0.02$$

$$\Rightarrow \qquad \frac{2}{0.02} \geq 1 + \omega^2$$

$$\omega^2 \leq 99$$

$$\Rightarrow \qquad \omega \leq 9.95 \text{ rad/sec}$$

Thus,

$$\text{Essential Bandwidth} = 2f = 2\times\frac{\omega}{2\pi}$$

$$= 2\times\frac{9.95}{2\pi}$$

$$= 2 \times 1.58 \text{ Hz}$$

$$= 3.16 \text{ Hz}$$

Q.1 (c) Solution:

(i) Address format:

← 32 bits →		
Tag	Set Index	Block offset

Given, Block size = 32 B

$\therefore$ Bits needed for block offset = $\log_2 32$ = 5 bits

Number of tag bits = 16

Since the address is of 32 bits, hence

$$\text{Bits for set index} = 32 - (16 + 5) = 32 - 21 = 11$$

For a K-way set associative cache,

$$\text{Number of sets} = \frac{\text{Number of cache lines}}{\text{K-way}}$$

$$\text{Number of cache lines} = \frac{\text{Cache size}}{\text{Block size}} = \frac{256\text{ kB}}{32\text{ B}} = \frac{2^{18}}{2^5} = 2^{13}$$

$$\text{Number of sets} = \frac{2^{13}}{K} = 2^{11}$$

$$K = \frac{2^{13}}{2^{11}} = 4$$

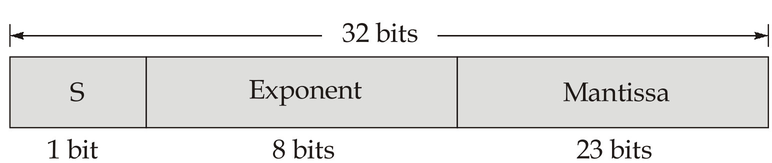
Hence, the cache is 4-way set associative.

(ii) 1. Comparison of Single and Double Precision

The IEEE 754 standard ensures portability and consistency in floating-point arithmetic across different computer architectures.

Feature	Single Precision (Binary 32)	Double Precision (Binary 64)
1. Total Bits	32 bits	64 bits
2. Sign Bit	1 bit (0 for +, 1 for –)	1 bit
3. Exponent	8 bits (Bias = 127)	11 bits (Bias = 1023)
4. Mantissa	23 bits	52 bits
5. Precision	~7 decimal digits	~15 to 17 decimal digits
6. Range	$\pm 1.0 \times 2^{-126}$ to $+2.0 \times 2^{127}$ $\approx \pm 1.2 \times 10^{-38}$ to $\pm 3.4 \times 10^{38}$	$\pm 1.0 \times 2^{-1022}$ to $+2.0 \times 2^{1023}$ $\approx \pm 2.2 \times 10^{-308}$ to $\pm 1.8 \times 10^{308}$
7. Usage	Graphics, mobile apps, and scenarios where memory is limited.	Scientific simulations, financial modeling, and high-accuracy physics.

2. Format of single precision floating point is



Representation of given number:

0	1 0 0 0 0 1 1 1	101000000000...00

$$\text{Value} = 1.M \times 2^{E-127}, \text{ where } E = (10000111)_2 = 135_{10}$$
$$= 1.1010 \times 2^{135-127}$$
$$= (1.1010)_2 \times 2^8$$
$$= 1.625 \times 2^8 = (416)_{10}$$

Converting decimal into octal,

8	416	
8	52	0
	6	4

Thus, Octal representation of the given number is $(640)_8$.

Q.1 (d) Solution:

(i) **1.** We know that, for a band pass signal,

$$m(t)\cos\omega_c t \xrightarrow{HT} m(t)\cdot HT\{\cos\omega_c t\}$$
$$= m(t)\cdot\sin\omega_c t$$

$\therefore$ For $x(t) = \text{sinc } 3t \cdot \cos 9\pi t$, Hilbert transform will be,

$$\hat{x}(t) = \text{sinc } 3t \,.\, \sin 9\pi t$$

2. Pre-envelope, $x_+(t) = x(t) + j\hat{x}(t)$

$$= \text{sinc}3t\,(\cos 9\pi t + j\sin 9\pi t)$$
$$x_+(t) = \sin c3t\, e^{j9\pi t}$$

Complex envelope, $\tilde{x}(t) = x_+(t)e^{-j\omega_c t} = \sin c3t\, e^{j9\pi t}\cdot e^{-j9\pi t}$

$\therefore$ Complex envelope = sinc $3t$

(ii)
$$x_1(t) = \frac{3}{4}x(t) + \frac{1}{4}j\hat{x}(t)$$

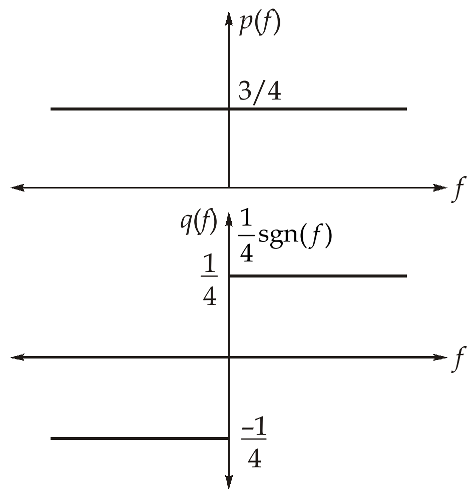
The Fourier Transform of Hilbert Transform of a signal $x(t)$ is given by

$$F\{H(\{x(t)\}\} = F\{\hat{x}(t)\} = -j\,\text{sgn}(f)X(f)$$

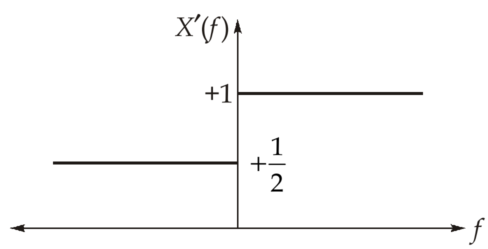
$\therefore$
$$X_1(f) = \frac{3}{4}X(f) + \frac{1}{4}j(-j\,\text{sgn}(f))X(f)$$
$$= \left[\frac{3}{4} + \frac{1}{4}\text{sgn}(f)\right]X(f)$$

Let, $X'(f) = \frac{3}{4} + \frac{1}{4}\text{sgn}(f) = p(f) + q(f)$

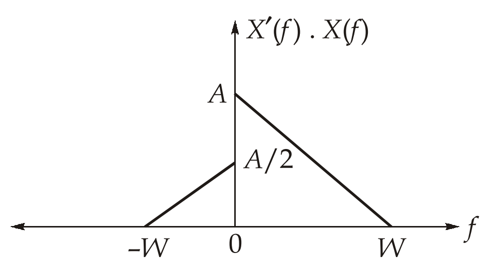
$\therefore$ $X_1(f) = X'(f) \cdot X(f)$



We have, $X'(f) = p(f) + q(f)$



$\therefore$ $X_1(f) =$



Q.1 (e) Solution:

For a satellite to be in the fixed orbit, the centrifugal force and contripetal force acting on the satellite must be equal.

Thus, centripetal force = centrifugal force

$$mg = \frac{mV^2}{r}$$

$$V^2 = rg$$

For circular motion, $V = r\omega$

Thus,
$$r\omega^2 = g$$
$$\omega^2 = \frac{g}{r}$$
$$\omega = \sqrt{\frac{g}{r}}$$
$$\omega = \sqrt{\frac{g}{R+H}} \quad [\because r = R + H]$$

and we know that $\omega = \frac{V}{R}$. Thus,
$$\frac{V}{R} = \sqrt{\frac{g}{R+H}}$$
$$V = R\sqrt{\frac{g}{R+H}}$$

Q.2 (a) Solution:

(i) Given,

DELAY	:	MVI A, 00 H	7T
LOOP	:	INR A	4T
		CPI 6D H	7T
		JNZ LOOP	10 (T) / 7 (F)

Initial value of A = 00 H and is incremented by 1 in each iteration of loop.

Loop continues until A = 6D H

Number of iterations, N = 6D H = 109 (decimal)

For first 108 iterations, the condition for JNZ instruction is True, hence total number of T-states per iteration is
$$4 + 7 + 10 = 21 \text{ T-states per iteration}$$
$$\text{total} = 108 \times 21 = 2268 \text{ T-states}$$

For last 109th iteration, the condition for JNZ is false, hence number of T-states
$$= 4 + 7 + 7 = 18 \text{ T-states}$$

MVI Instruction is executed once and take 7 T-states. Thus, Total number of T-states
$$= 7 + 2268 + 18 = 2293 \text{ T-states}$$

Given, Crystal oscillator frequency,
$$f = 5 \text{ MHz}$$

In the 8085 microprocessor, the internal clock frequency is half of the external crystal frequency. Thus,

Time per T-state, $$T = \frac{1}{2.5\times10^6} = 0.4\ \mu\text{sec}$$

Time delay introduced by the loop

$$= 2293 \times 0.4\ \mu\text{sec}$$

$$= 917.2\ \mu\text{sec}$$

(ii) Given, chip select signal is active for A_7 to A_2 as 100101.

8253 uses A_1 to A_0 for internal selection:

A_1	A_0	**Selected Register**
0	0	Counter 0
0	1	Counter 1
1	0	Counter 2
1	1	Control Word Register

The counter and control port-addresses are thus obtained as below,

A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	
1	0	0	1	0	1	0	0	⇒ 94 H (counter 0)
1	0	0	1	0	1	0	1	⇒ 95 H (counter 1)
1	0	0	1	0	1	1	0	⇒ 96 H (counter 2)
1	0	0	1	0	1	1	1	⇒ 97 H (Counter word register)

Q.2 (b) Solution:

(i) As we know that

$$S = \frac{I}{A}\times\frac{t}{q}$$

where S = dose; I = current; A = area; t = time; q = charge of electron

Given : $S = 5 \times 10^{15}\ \text{cm}^{-2}$

$I = 2\ \text{mA}$

$A = 120\ \text{cm}^2$

$q = 1.6 \times 10^{-19}\ \text{C}$

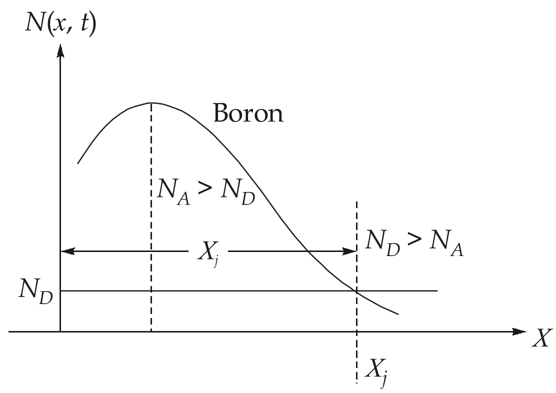
We can write, $$t = S\times\frac{A}{I}\times q$$

$$= 5\times10^{15}\times\frac{120}{2\times10^{-3}}\times1.6\times10^{-19}$$

$$t = 48 \text{ sec}$$

Thus, the implantation should be carried out for 48 sec to realize dose of 5×10^{15} cm^{-2}.

(ii) The implanted Boron concentration varies with distance according to Gaussian distribution as below.



At the junction X_j, the concentration of the implanted Boron is equal to the background concentration of the substrate i.e.

At junction, $X = X_j$,

$$N(X) = N_D$$

At 0.15 MeV Projected Range,

$$R_p = 4\ \mu m/MeV \times 0.15\ MeV$$

$$R_p = 0.6\ \mu m$$

Standard Deviation, $\Delta R_p = 0.4R_p = 0.4 \times 0.6$

$$\Delta R_p = 0.24\ \mu m$$

The peak concentration occurs at $x = R_p$ and is given by,

$$N_p = \frac{S}{\sqrt{2\pi}\times\Delta R_p}$$

$$N_p = \frac{5\times10^{15}}{\sqrt{2\pi}\times0.24\times10^{-4}}$$

$$N_p = 8.33 \times 10^{19}\ cm^{-3}$$

For Gaussian implantation,

$$N(X) = = N_p e^{-(X-R_p)^2/2\Delta R_p^2}$$

At $X = X_j$,
$$\frac{N_D}{N_P} = \exp\left[\frac{-(X_j - R_P)^2}{2\Delta R_P^2}\right]$$

$$\ln\left(\frac{N_D}{N_P}\right) = \frac{-(X_j - R_P)^2}{2\Delta R_P^2}$$

$$-(X_j - R_P)^2 = 2\Delta R_P^2 \ln\left(\frac{N_D}{N_P}\right)$$

$$X_j = R_P + \sqrt{2\Delta R_P^2 \ln\left(\frac{N_P}{N_D}\right)}$$

$$X_j = 0.6\times10^{-4} + \sqrt{2\times(0.24\times10^{-4})^2 \ln\left(\frac{8.33\times10^{19}}{10^{16}}\right)}$$

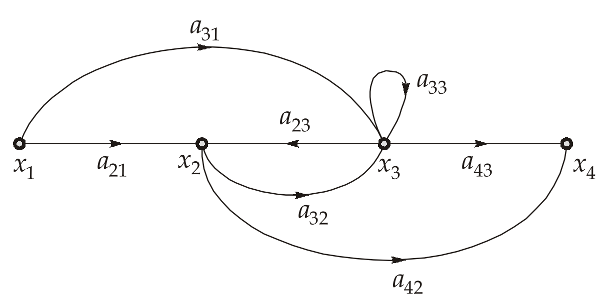
$$= 0.6 \times 10^{-4} + 1.02 \times 10^{-4}$$

$$X_j = 1.62 \times 10^{-4} \text{ cm}$$

$$X_j = 1.62\ \mu\text{m}$$

Q.2 (c) Solution:

(i) From the given equations, the signal flow graph can be drawn as below:



Now, Forward paths, $P_1 = a_{21}a_{42}$

$P_2 = a_{21}a_{32}a_{43}$

$P_3 = a_{31}a_{43}$

$P_4 = a_{31}a_{23}a_{42}$

Individual loops, $L_1 = a_{23}a_{32}$

$L_2 = a_{33}$

Using Mason's gain formula, we can write

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

Here, $\Delta = 1 - (L_1 + L_2)$, $\Delta_1 = 1$, $\Delta_2 = 1$, $\Delta_3 = 1$, $\Delta_4 = 1$. Thus,

$$T = \frac{a_{21}a_{42} + a_{21}a_{32}a_{43} + a_{31}a_{43} + a_{31}a_{23}a_{42}}{1 - a_{23}a_{32} - a_{33}}$$

(ii) Input Velocity $= \frac{1}{2}$ revolution per second

$$= \frac{1}{2} \times 2\pi = \pi = 3.14 \text{ rad/sec}$$

Therefore, the ramp input, $R = 3.14$ rad/sec

The steady-state error $e_{ss} = 0.2° = \frac{0.2 \times \pi}{180}$ rad

For a ramp input of R units, steady state error is given by

$$e_{ss} = \frac{R}{K_V}$$

$$K_V = \frac{R}{e_{ss}} = \frac{3.14 \times 180}{0.2 \times 3.14} = 900$$

But $K_V = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} s \frac{10K}{s(1+0.1s)} = 10K$

$\therefore$ $10K = 900$

$K = 90$

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{900}{s(1+0.1s)}}{1 + \frac{900}{s(1+0.1s)}} = \frac{9000}{s^2 + 10s + 9000}$$

Comparing this with the standard form of the transfer function of a second-order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \text{ we get } \omega_n^2 = 9000$$

Therefore, the natural frequency,

$$\omega_n = \sqrt{9000} = 94.87 \text{ rad/sec}$$

$$2\xi\omega_n = 10$$

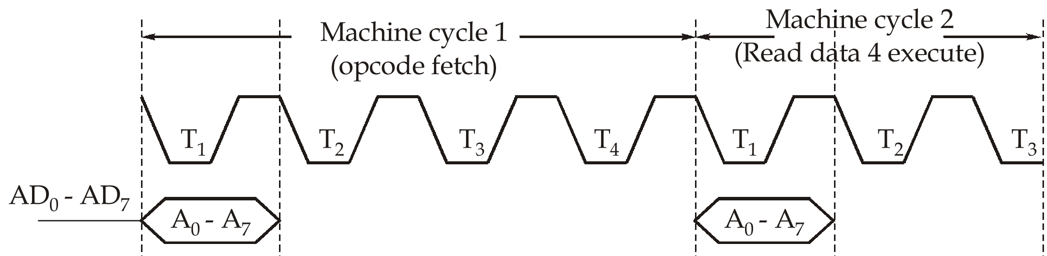
and, the damping ratio,

$$\xi = \frac{10}{2\omega_n} = \frac{10}{2\times 94.87} = 0.053$$

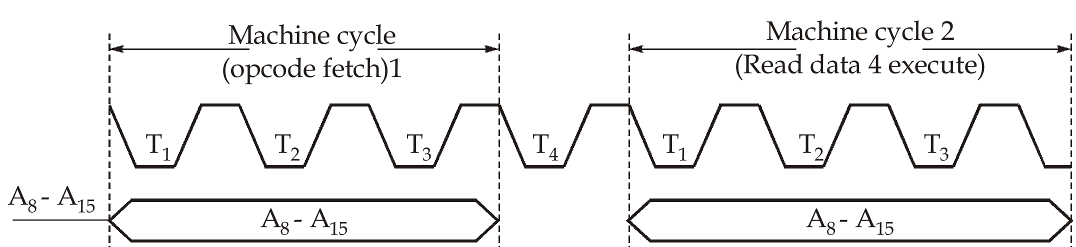
Q.3 (a) Solution:

(i) The lower byte of address (AD_0-AD_7) is available on the multiplexed address/data bus during T_1 state of each machine cycle, except during the bus idle machine cycle. Address Latch Enable (ALE) is a control signal used to demultiplex the lower-order address and data bus in the 8085 microprocessor. ALE is a positive-going pulse generated at the beginning of every machine cycle and remains high for one T-state. During this interval, the lower-order address is latched from the multiplexed AD_0–AD_7 lines.

The higher byte of address (A_8-A_{15}) is available during T_1 to T_3 states of each machine cycle except during the bus idle machine cycle.

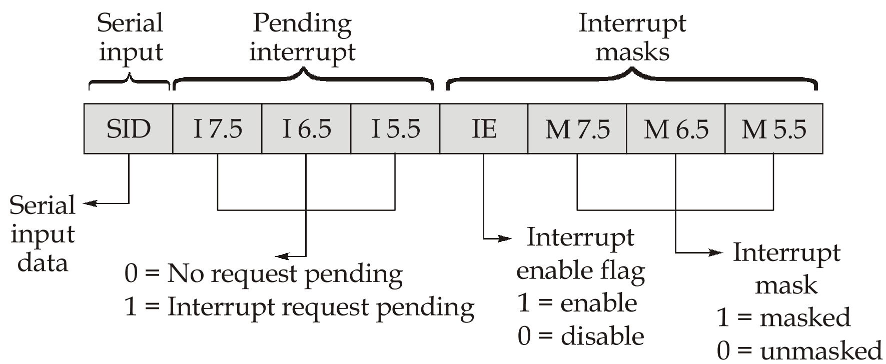


Lower byte address on the multiplexed bus (AD_0-AD_7)



Higher byte address on A_8-A_{15}

(ii) RIM stands for 'Read Interrupt Mask'. The instruction is used to read the status of RST 7.5, RST 6.5 and RST 5.5 interrupts and serial input data bit. The contents of the Accumulator after the execution of the RIM instruction provide the information in the format as below:



The RIM instruction format

When RIM instruction is executed , the status of SID, pending RST 7.5, RST6.5 and RST 5.5 interrupts and interrupt masks are loaded in the accumulator. Thus, their status can be monitored. For instance, it may so happen that when one interrupt is being serviced, other interrupt(s) may occur. The status of these pending interrupts can be monitored by RIM instruction.

(iii) Assume 16-bit number is present at memory locations 2501 H (LSB) and 2502H (MSB)

LXI H, 2501 H	;	Load HL pair with address 2501 H where 16-bit data is stored.
MOV A, M	;	Data stored at location indicated by HL pair (LSB of 16-bit number) moved into accumulator
CMA	;	Complement the data stored in accumulator
INR A	;	Add '1' to the complement of LSB to obtain 2's complement
STA 2503 H	;	Store the data of accumulator (LSB of 2's complement) at address 2503 H
INX H	;	Increment the content of HL pair
MOV A, M	;	Data stored at location indicated by HL pair (MSB of 16-bit number) moved into accumulator
CMA	;	Complement the data stored in accumulator
ACI 00H	;	Add the carry obtained from LSB
STA 2504 H	;	Store the data of accumulator (MSB of 2's complement) at address 2504 H
HLT	;	Stop

Q.3 (b) Solution:

(i) Periodic time

We have, Eccentricity, $e = 0.15$

Semi major axis, $a = 9000$ km

Radius, $R = 6370$ km

Using Kepler's Third Law,

$$\text{Time period, } T = 2\pi\sqrt{\frac{a^3}{\mu}} \text{, where } \mu = GM \approx 4 \times 10^{14} \text{ m}^3/\text{s}^2$$

$$T = 2\pi\sqrt{\frac{(9000\times10^3)^3}{4\times10^{14}}}$$

$$T = 8482.3 \text{ sec}$$

(ii) Apogee height

Apogee distance from Earth center,

$$r_a = a(1 + e)$$

$$r_a = 9000(1 + 0.15)$$

$$r_a = 10350 \text{ km}$$

Apogee height above earth,

$$h_a = r_a - R = 10350 - 6370$$

$$h_a = 3980 \text{ km}$$

(iii) Perigee Height

Perigee distance from Earth center,

$$r_p = a(1 - e)$$

$$r_p = 9000(1 - 0.15)$$

$$r_p = 7650 \text{ km}$$

$$h_p = r_p - R$$

Perigee height above earth,

$$h_p = 7650 - 6370$$

$$h_p = 1280 \text{ km}$$

(iv) Velocity at Apogee:

$$V_a = \sqrt{\mu\left[\frac{2}{r_a} - \frac{1}{a}\right]}$$

$$V_a = \sqrt{4\times10^5\left[\frac{2}{10350}-\frac{1}{9000}\right]} = 5.73 \text{ km/s}$$

(v) Velocity at Perigee:

$$V_p = \sqrt{\mu\left[\frac{2}{r_p}-\frac{1}{a}\right]}$$

$$V_p = \sqrt{(4\times10^5)\left[\frac{2}{7650}-\frac{1}{9000}\right]} = 7.75 \text{ km/s}$$

Q.3 (c) Solution:

(i)

$$V(t) = 10[1 + 0.5 \cos \omega t + 0.2 \cos 2\omega t] \cos\omega_0 t$$

$$= 10[1 + M(t)]\cos\omega_0 t$$

$$\text{Maximum value} = 10[1 + M(t)_{max}]$$

$$\text{Minimum value} = 10[1 + M(t)_{min}] \quad [\text{Assuming } \omega_0 >> \omega]$$

Now, $M(t) = 0.5 \cos \omega t + 0.2 \cos 2\omega t$

$$M(t)_{max} = 0.5 + 0.2 \text{ [at } t = 0] = 0.7$$

Now, to calculate minimum value, consider,

$$\frac{dM(t)}{dt} = 0$$

$$\Rightarrow \quad -0.5 \sin \omega t - 0.4 \sin 2\omega t = 0$$

$$\Rightarrow \quad 5 \sin \omega t + 8 \sin \omega t \cos \omega t = 0$$

$$\Rightarrow \quad 5 + 8 \cos \omega t = 0$$

$$\cos \omega t = \frac{-5}{8}$$

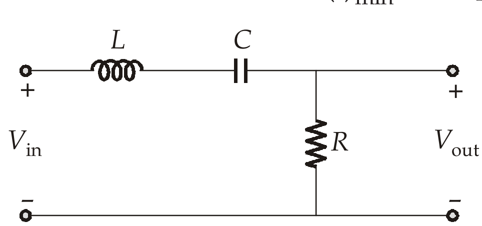
Using the identity, $\cos 2\omega t = 2\cos^2\omega t - 1$

So, $$M(t)_{min} = 0.5\left(\frac{-5}{8}\right)+0.2\left[2\left(\frac{-5}{8}\right)^2-1\right] = -0.35625$$

So, $$V(t)_{max} = 10[1 + 0.7] = 17$$

$$V(t)_{min} = 10[1 - 0.35625] = 6.4375$$

(ii)



Consider the bandpass RLC circuit as shown in figure.

The transfer function relating the output voltage to input voltage can be given as,

$$H(f) = \frac{R}{R + j2\pi fL + \dfrac{1}{j2\pi fC}}$$

Now, resonant frequency, $f_c = \dfrac{1}{2\pi\sqrt{LC}}$

and $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$$\therefore \quad H(f) = \frac{1}{1 + jQ\left[\dfrac{f}{f_c} - \dfrac{f_c}{f}\right]}$$

Since, it is given in the question that the filter is a narrow band filter, hence Q will be very high. Thus, $H(f)$ can be written as,

$$H(f) = \begin{cases} \dfrac{1}{1 + 2jQ\left(\dfrac{f - f_c}{f_c}\right)}; & f_c > 0 \\ \dfrac{1}{1 + 2jQ\left(\dfrac{f + f_c}{f_c}\right)}; & f_c < 0 \end{cases} \quad (\because \text{High } Q)$$

Now, since the input noise spectral density is given as $G_n(f) = \dfrac{\eta}{2}$.

Thus, the output power spectral density of noise,

$$S_N(f) = G_n(f) \cdot |H(f)|^2$$

$$S_N(f) = \begin{cases} \dfrac{\frac{\eta}{2}}{1 + 4Q^2 \dfrac{(f - f_c)^2}{f_c^2}}; & f_c > 0 \\ \dfrac{\frac{\eta}{2}}{1 + 4Q^2 \dfrac{(f + f_c)^2}{f_c^2}}; & f_c < 0 \end{cases}$$

Now, the inphase and the quadrature phase component can be written as,

$$S_{NC}(f) = S_{NQ}(f) = S_N(f - f_c) + S_N(f + f_c)$$

$$S_{NC}(f) = \frac{\eta}{1+\left(\frac{2Qf}{f_c}\right)^2}$$

∴ Total output noise power,

$$P_{out} = \int_{-B/2}^{B/2} \frac{\eta}{1+\left(\frac{2Qf}{f_c}\right)^2} df$$

Let, $$\frac{2Qf}{f_c} = X$$

$$df = \frac{f_c}{2Q} dX$$

Thus, $$P_{out} = \frac{\eta f_c}{2Q} \int_{-\frac{BQ}{f_c}}^{+\frac{BQ}{f_c}} \frac{1}{1+X^2} dX$$

$$= \frac{\eta f_c}{2Q} * 2\tan^{-1}\left[\frac{QB}{f_c}\right]$$

$$P_{out} = \frac{\eta f_c}{Q} \tan^{-1}\left[\frac{QB}{f_c}\right]$$

Q.4 (a) Solution:

(i) Comparing the given characteristic equation with $1 + G(s)H(s) = 0$, the open loop transfer function is obtained as,

$$G(s)H(s) = \frac{K}{(1+s)(1.5+s)(2+s)}$$

We want all the roots to satisfy Re(s) < –1. Using the extended Nyquist criterion, we substitute $s = -1 + j\omega$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(1.5-1+j\omega)(2-1+j\omega)}$$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(0.5+j\omega)(1+j\omega)}$$

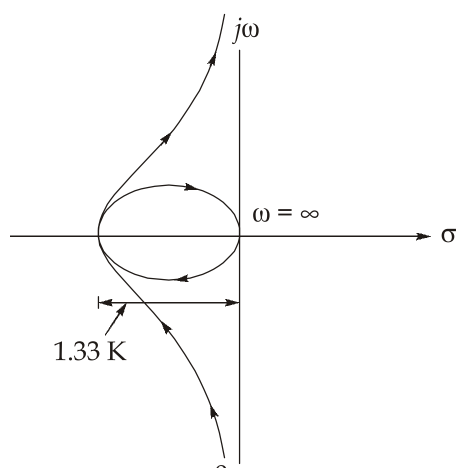
$$\text{Magnitude, } M = |GH(-1+j\omega)| = \frac{K}{\omega\left(\sqrt{1+\omega^2}\right)\left(\sqrt{0.25+\omega^2}\right)}$$

$$M = \frac{2K}{\omega\left(\sqrt{1+4\omega^2}\right)\left(\sqrt{1+\omega^2}\right)}$$

Phase angle, $\angle GH(-1 + j\omega) = \phi = -90° - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$

ω	M	ϕ
0	∞	–90°
∞	0	–270°
0.5	2.53K	–161.56°
0.1	19.5K	–107°
1	0.63K	–198°
10	$9.94 \times 10^{-4} K$	–261°
0.707	1.33K	–180°

Nyquist plot for the system is thus obtained as below:



Now, the frequency at which phase is –180° is given by

$$-180° = -90° - \tan^{-1}\omega - \tan^{-1}(2\omega)$$

$$\frac{\omega + 2\omega}{1 - 2\omega^2} = \frac{1}{0}$$

$$\omega = \frac{1}{\sqrt{2}} \text{ rad/sec}$$

At $\omega = \frac{1}{\sqrt{2}}$ rad/sec, the value of M is given as,

$$M = \frac{2K}{\frac{1}{\sqrt{2}} \times \sqrt{1+4\times\frac{1}{2}} \times \sqrt{1+\frac{1}{2}}} = \frac{4}{3}K$$

From the Nyqusit criterion, $Z = P - N$

Here, Z = No. of closed-loop poles in the right half of $s = -1$ plane.

P = No. of open-loop poles in the right half of $s = -1$ plane.

N = No. of encirclements around $s = -1$.

Since the roots of the characteristic equation have real parts less than –1, thus

$$P = 0,$$

∴ For system to be stable, N should be zero.

For N to be zero, $1.33K < 1$

$$K < \frac{1}{1.33}$$

$$K < 0.75$$

Hence, $K = 0.75$ is the largest value of K satisfying the given condition.

(ii) From the given characteristic equation, the Routh table is formulated as follows:

s^4	1	6	8
s^3	2	8	0
s^2	$\frac{2\times6-1\times8}{2} = 2$	$\frac{2\times8-1\times0}{2} = 8$	
s^1	$\frac{2\times8-2\times8}{2} = 0$		
s^0			

All the elements in the s^1 row are zero. That means there are symmetrically located roots of the characteristic equation with respect to the origin of the s-plane. So the system can be unstable or marginally stable.

To determine the location of the roots form the auxiliary equation $A(s)$ by using the coefficients of the row just above the row of zeros, i.e.,

$$A(s) = 2s^2 + 8 = 0$$

Take the first derivative of the auxiliary equation, i.e,

$$\frac{dA(s)}{ds} = 4s + 0 = 0$$

Replace the row of zeros with the coefficients of the first derivative of the auxiliary equation and complete the formation of the Routh table,

$$\begin{array}{c|ccc} s^4 & 1 & 6 & 8 \\ s^3 & 2 & 8 & \\ s^2 & 2 & 8 & \\ s^1 & 4 & 0 & \\ s^0 & 8 & & \end{array}$$

There are no sign changes in the elements of the first column of the Routh array and hence there are no roots of the characteristic equation in the right-half of the s-plane. There must be roots on the imaginary axis of the s-plane which can be determined by solving the auxiliary equation,

$$2s^2 + 8 = 0$$

$$s = \pm j2$$

This shows that there is a pair of roots at $s = \pm j2$, and so the system oscillates and the frequency of sustained oscillations is $\omega = 2$ rad/sec

To determine the other two roots, factorize the characteristic equation,

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = (s^2 + 4)\,(s^2 + 2s + 2) = 0$$

$$s^2 + 2s + 2 = (s + 1 + j1)\,(s + 1 - j1) = 0$$

The other two roots are a pair of complex conjugate roots in the left-half of the s-plane given by $s = -1 \pm j1$.

Q.4 (b) Solution:

(i) From Maxwell's equations:

$$\vec{\nabla}\times\vec{H} = \vec{J}+\frac{\partial\vec{D}}{\partial t}$$

But, $$\vec{J} = \sigma\vec{E} = 0 \qquad (\because\ \sigma = 0 \text{ given})$$

$\therefore$ $$\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t} = \in\frac{\partial\vec{E}}{\partial t}$$

But, $$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

Given, $$\vec{H} = 5\cos(10^9 t - \beta z)\hat{y}\ \text{A/m}$$

i.e., $H_y = 5\cos(10^9 t - \beta z),\ H_x = H_z = 0$

$$\therefore \quad \vec{\nabla}\times\vec{H} = -\frac{\partial H_y}{\partial z}\hat{x} = \in \frac{\partial E_x}{\partial t}$$

$$\Rightarrow \quad -\frac{\partial H_y}{\partial z} = -\in \frac{\partial E_x}{\partial t} \qquad \text{...(i)}$$

Also from Maxwell's equations:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu\frac{\partial \vec{H}}{\partial t} = -\mu\frac{\partial H_y}{\partial t}$$

Since x-component of E is non-zero and a function of z, we get

$$\vec{\nabla}\times\vec{E} = \frac{\partial E_x}{\partial z}\hat{y}$$

$$\therefore \quad \frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t}$$

From equation (i),

$$\frac{\partial^2 H_y}{\partial z^2} = \frac{\partial}{\partial z}\left(\in\frac{\partial E_x}{\partial t}\right) = -\in\frac{\partial}{\partial t}\left(\frac{\partial E_x}{\partial z}\right)$$

$$= -\in\frac{\partial}{\partial t}\left(-\mu\frac{\partial H_y}{\partial t}\right) = \in\mu\frac{\partial^2 H_y}{\partial t^2} \qquad \text{...(ii)}$$

where $\in = \in_r\in_0 = 20 \times 8.854 \times 10^{-12} = 1.77 \times 10^{-10}$ F/m

$\mu = \mu_r\mu_0 = 1 \times 4\pi \times 10^{-7} = 4\pi \times 10^{-7}$ H/m

We have,

$$\frac{\partial^2 H_y}{\partial z^2} = \frac{\partial}{\partial z}\left[\frac{\partial}{\partial z}\left[5\cos(10^9 t - \beta z)\hat{y}\right]\right]$$

$$= \frac{\partial}{\partial z}\left[-\beta\times 5(-\sin(10^9 t - \beta z))\right]$$

$$= 5\beta \times -\beta\,(\cos\,(10^9 t - \beta z))$$

$$\therefore \quad \frac{\partial^2 H_y}{\partial z^2} = -5\beta^2\cos\,(10^9 t - \beta z)$$

$$\in\mu\frac{\partial^2 H_y}{\partial t^2} = -1.77 \times 10^{-10} \times 4\pi \times 10^{-7} \times 5 \times 10^{18}\cos\,(10^9 t - \beta z)$$

$$= -1.11 \times 10^{3}\cos\,(10^9 t - \beta z)$$

Substituting values in equation (ii),

$$-5\beta^2 \cos(10^9 t - \beta z) = -1.11 \times 10^3 \cos(10^9 t - \beta z)$$

$$\Rightarrow \quad \beta^2 = 222.54 \text{ m}^2$$

$$\therefore \quad \beta = 14.92 \text{ m}^{-1} \quad (\text{as } \beta > 0)$$

(ii) The displacement current density,

$$\vec{J_d} = \vec{\nabla} \times \vec{H} = \frac{\partial H_y}{\partial z}\bar{x}$$

$$= -\frac{\partial}{\partial z}\left[5\cos(10^9 t - \beta z)\right]\bar{x}$$

$$= -\frac{\partial}{\partial z}\left[5\cos(10^9 t - 14.92z)\bar{x}\right]$$

$$= (-5)\times(-14.92)(-\sin(10^9 t - 14.92z))\bar{x}$$

$$= 74.59\sin(10^9 t - 14.92z)\vec{x} \text{ A/m}^2$$

At $z = 0$, $$\vec{J_d} = -74.59\sin(10^9 t)\bar{x} \text{ A/m}^2$$

(iii) Total displacement current crossing the surface,

$$x = 0.5d;\ 0 < y < b;\ 0 < z < 0.1 \text{ m in the } \bar{x}\text{-direction is}$$

$$I = \iint (\vec{J_d}\cdot\bar{x})\,dS = \int_0^b \int_0^{0.1} \vec{J_d}\cdot\vec{x}\,dy\,dz$$

$$= \int_0^b dy \int_0^{0.1} J_d\,dz = \int_0^b dy \int_0^{0.1} -74.59\sin(10^9 t - 14.92z)\,dz$$

$$= b\times\left[-74.59\frac{\cos(10^9 t - 14.92z)}{14.92}\right]_0^{0.1}$$

$$= \frac{-0.04\times 74.59}{14.92}\left[\cos(10^9 t - 14.92\times 0.1) - \cos(10^9 t - 0)\right]$$

$$I = -0.2[\cos(10^9 t - 14.92) - \cos 10^9 t]\ A$$

Q.4 (c) Solution:

(i) $$y[n] = x[n] * h[n]$$

Taking discrete time Fourier transform,

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$$Y(e^{j\omega}) = \frac{1}{(1-0.8e^{-j\omega})}\cdot\frac{1}{(1-0.2e^{-j\omega})} \quad \text{...(i)}$$

For $Y(e^{j\pi})$, put $\omega = \pi$ rad/sec. We get,

$$Y(e^{j\pi}) = \frac{1}{[1-0.8(-1)]}\cdot\frac{1}{[1-0.2(-1)]} = \frac{1}{1.8}\cdot\frac{1}{1.2}$$

$$Y(e^{j\pi}) = \frac{100}{18\times 12} = 0.462$$

Using the partial fraction expansion, we can write equation (i) as,

$$Y(e^{j\omega}) = \frac{(4/3)}{1-0.8e^{-j\omega}} + \frac{(-1/3)}{1-0.2e^{-j\omega}}$$

Taking inverse DTFT, we get

$$y(n) = (4/3)(0.8)^n u(n) - (1/3)(0.2)^n u(n)$$

(ii)

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y(t) = x(t) - \frac{2dx}{dt}$$

Taking Laplace Transform,

$$(s^2 + s - 2)Y(s) = X(s) - 2s\,X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1-2s}{(s^2+s-2)}$$

$$H(s) = \frac{1-2s}{(s+2)(s-1)}$$

$$H(s) = \frac{-5}{3(s+2)} - \frac{1}{3(s-1)}$$

Possible ROCs,

$\sigma > -2, \sigma > 1$ i.e., $\sigma > 1$

$\sigma < -2, \sigma < 1$ i.e., $\sigma < -2$

$\sigma > -2, \sigma < 1$ i.e., $-2 < \sigma < 1$

For the system to be stable, ROC should include the $j\omega$ axis.

$$H(s) = \frac{-5}{3(s+2)} - \frac{1}{3(s-1)} \quad \text{ROC: } -2 < \sigma < 1$$

Taking the inverse Laplace Transform, we get

$$h(t) = \frac{-5}{3}e^{-2t}u(t) + \frac{1}{3}e^{t}u(-t)$$

Initial value of impulse response,

$$\lim_{t \to 0} h(t) = \frac{-5}{3} + \frac{1}{3} = \frac{-4}{3} = -1.33$$

Section B

Q.5 (a) Solution:

Processor to memory communication:

The following sequence of events takes place when information is transferred from memory to the processor:

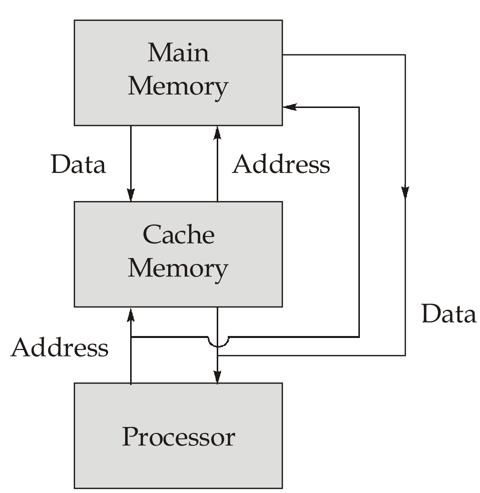
1. The processor places the address in MAR (Memory Address Register) through the address bus.
2. The processor issues a READ command through the control bus.
3. The memory places retrieved data on the data bus, which is then transferred to processor.

Similarly, the following sequence of events takes place when information is written into the memory:

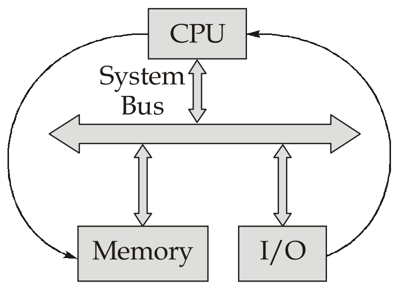
1. The processor places the address in MAR through the address bus.
2. The processor transmits the data to be written in memory using the data bus.
3. The processor issues a WRITE command to memory by control bus.
4. The data is written in memory at address specified in MAR.

The main concern in processor-memory communication is the speed mismatch between the memory and processor.

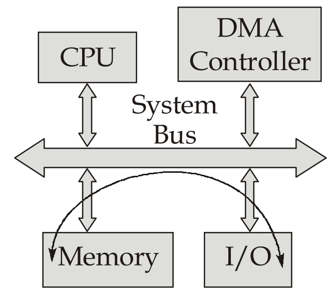
Memory access time is generally slower than the CPU's access time. Hence to eliminate speed mismatch, fast memory as an intermediate buffer between processor and memory called cache memory is used.



Processor to I/O communication:



Without DMA Transfer



With DMA Transfer

I/O units can be connected to the computer system through the system bus. Here, the CPU facilitates the data transfer between I/O and Memory. There is no direct communication between I/O and Memory. When a large amount of data are to be transferred, a DMA controller can be used. Each I/O device in a computer system is first met with controller, called DMA (Direct Memory Access) Controller which controls the operation of that device.

The controller is connected to the buses to perform a sequence of data transfer on behalf of the CPU. It is capable of taking over control of the system bus from the CPU, which is required to transfer data to and from memory over the system bus. A DMA controller can use the system bus only when the CPU doesn't require it or it should suspend the operations currently being processed by CPU.

DMA allows I/O unit exchange data directly with memory without going through CPU except at beginning (to issue the command) and at end (to clean up after the command is processed). While the I/O is being performed by the DMA, the CPU can start execution of some other part of the same program or can start executing some other program.

Q.5 (b) Solution:

Given,

Waveguide dimensions, $a = 6 \text{ cm} = 0.06 \text{ m}$

$b = 4 \text{ cm} = 0.04 \text{ m}$

Distance between successive antinodes (maxima),

$$d = 4.55 \text{ cm} = 0.0455 \text{ m}$$

The distance between two successive nodes or two successive antinodes in a standing wave is half the guided wavelength. Thus,

$$d = \frac{\lambda_g}{2} \Rightarrow \lambda_g = 2d = 2 \times 0.0455$$

$$\therefore \quad d = 0.091 \text{ m}$$

The dominant mode in rectangular waveguide is TE10. Thus, cutoff wavelength for TE_{10},

$$\lambda_c = 2a = 2 \times 0.06 = 0.12 \text{ m}$$

We know that,

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

$$\frac{1}{(0.091)^2} = \frac{1}{\lambda^2} - \frac{1}{(0.12)^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{(0.091)^2} + \frac{1}{(0.12)^2} = 190.2$$

$$\therefore \quad \lambda = \frac{1}{\sqrt{190.2}} = 0.0725 \text{ m}$$

Transmitted frequency, $f = \frac{c}{\lambda} = \frac{3\times10^8}{0.0725}$

$$\therefore \quad f \simeq 4.14 \times 10^9 \text{ Hz}$$

Q.5 (c) Solution:

The network layer of the OSI model is responsible for the delivery of data packets from the source to the destination across multiple networks. It handles several critical functions to ensure effective and efficient communication. Below are the primary functions of the network layer:

1. Routing
2. Logical addressing
3. Path determination
4. Packet forwarding.

1. Routing:

Routing is the process by which data packets are forwarded from one network to another. The network layer is responsible for determining the best path through the network to ensure that data reaches its intended destination efficiently.

Routing Tables: Routers maintain routing tables that contain information about various paths through the network. These tables are used to make forwarding decisions.

Routing protocols: Protocols like OSPF (Open Shortest Path First), BGP (Border Gateway Protocol) and RIP (Routing Information Protocol) are used to exchange routing information between routers and update routing tables dynamically.

Route Selection: The network layer uses metrics such as hop count, bandwidth, delay, and cost to select the optimal path for data transmission.

2. **Logical addressing:**

   Logical addressing is used to uniquely identify each device on a network. The network layer assigns a unique address, known as an IP address, to each device. This allows for the correct delivery of packets to the destination. Key components of logical addressing include:

   - **IP addressing:** IP addresses are assigned to devices to facilitate identification and communication. IPv4 and IPv6 are the two versions of IP addressing currently in use.
   - **Hierarchical addressing:** IP addresses are structured hierarchically, allowing for efficient routing and management. This hierarchy typically includes a network portion and a host portion.
   - **Subnetting:** Subnetting divides a larger network into smaller, manageable sub-networks improving routing efficiency and network organization.

3. **Path determination:**

   Path determination involves selecting the most appropriate path for data to travel from the source to the destination. This function ensures that data packets take the optimal route through the network. Components of path determination include:

   - **Routing Algorithms:** Algorithms such as distance-Vector and link-state are used to calculate the best path for data transmission.

     Distance -vector algorithms use hop count as a metric, while link-state algorithms use a map of the network to determine the shortest path.
   - **Metrics and Policies:** Various metrics (e.g. bandwidth, latency, reliability) and routing policies (e.g. administrative preferences) are considered to determine the best path for data packets.

4. **Packet Forwarding:-**

   Packet forwarding is the actual process of moving packets from one network interface to another within a router or between routers based on the destination address. The key aspects include:

   - **Forwarding Decisions:** Routers examine the destination IP address of incoming packets and look up their routing tables to determine the next hop for each packet.
   - **Switching:** The network layer performs packet switching, directing packets to the appropriate output interface to continue their journey to the destination.

- **Encapsulation:** Data is encapsulated in packets with appropriate headers and trailers that contain information necessary for routing and delivery.
- **Fragmentation:** When packets are too large to pass through a network segment, the network layer fragments the packet into smaller units that can be reassembled at the destination.

Q.5 (d) Solution:

Comparing with the state space representation

$$\dot{x} = A_x + B_u, \text{ we have}$$

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s+2 & -1 & 0 \\ 0 & s+2 & -1 \\ 0 & 0 & s+2 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{\text{adj}[sI-A]}{\det|sI-A|} = \frac{\begin{bmatrix} (s+2)(s+2) & (s+2) & 1 \\ 0 & (s+2)(s+2) & (s+2) \\ 0 & 0 & (s+2)(s+2) \end{bmatrix}}{(s+2)(s+2)(s+2)}$$

$$= \begin{bmatrix} \frac{1}{(s+2)} & \frac{1}{(s+2)^2} & \frac{1}{(s+2)^3} \\ 0 & \frac{1}{(s+2)} & \frac{1}{(s+2)^2} \\ 0 & 0 & \frac{1}{(s+2)} \end{bmatrix}$$

We have, $L\{t^n u(t)\} = \frac{n!}{s^{n+1}}$ and $L\{e^{-at}u(t)\} = \frac{1}{s+a}$.

The state transition matrix can be obtained using Inverse Laplace Transform as below:

$$\phi(t) = e^{At} = L^{-1}(sI-A)^{-1} = \begin{bmatrix} e^{-2t} & te^{-2t} & \frac{t^2}{2}e^{-2t} \\ 0 & e^{-2t} & te^{-2t} \\ 0 & 0 & e^{-2t} \end{bmatrix}$$

Q.5 (e) Solution:

Surface layer of Silicon Oxide (SiO_2) is formed through oxidation process. The ability to form a chemically stable protective layer of SiO_2 at the surface of silicon is one of the main reasons that makes silicon the most widely used semiconductor material. The SiO_2 layer is a high quality electrically insulating layer on the silicon surface serving as a dielectric in numerous devices and can also be a preferential masking layer in many steps during device fabrication.

SiO_2 is often thermally formed in the presence of oxygen compounds at a temperature in the range of 900°C to 1300°C. There are two basic means to supply oxygen into reaction chamber:

1. Gaseous pure oxygen (dry oxidation) through the reaction: $Si + O_2 \rightarrow SiO_2$.
2. In the form of water vapour (wet oxidation) through the reaction: $Si + 2H_2O \rightarrow SiO_2 + 2\,H_2$.

For both means of oxidation, the high temperature allows the oxygen to diffuse easily through SiO_2.

The silicon is consumed as the oxide grows, and with a total oxide thickness of X, about 0.45 X lies below the original surface of Si wafer and 0.55 X lies above it as shown in the figure below:

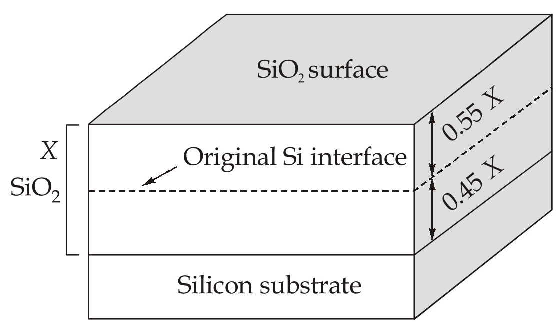


Fig: Growth of SiO_2

A typical oxidation growth cycle consists of dry-wet-dry oxidations, where most of the oxide is grown in the wet oxidation phase. Dry oxidation is slower and result in more dense, high quality oxides. This type of oxidation methods is mostly used in metal-oxide-semiconductor (MOS) gate oxides. Wet oxidation results in much more rapid growth and is used mostly for thicker masking layer. Dry oxidation tends to be used when oxides of high quality is required whereas wet oxidation is favoured for thick oxide layer.

Before thermal oxidation, the silicon is usually preceded by a cleaning sequence to remove all contaminants. The process of thermal oxidation is performed with the wafers sitting in the boat loaded into a furnace where temperature is carefully controlled. The quartz tube inside furnace is enclosed around heating coils which are controlled by electrical current in the coil. A cross-section of a typical oxidation furnace is as below:

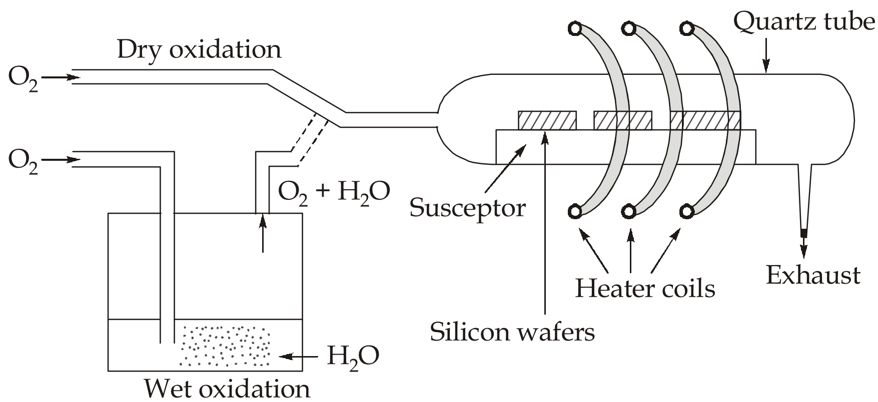


Fig: Cross-section of an oxidation furnace

The furnace is suitable for both dry and wet oxidation. The following factors determine the oxidation rate:

1. Type of oxidation -Wet oxidation has higher growth rate than dry oxidation due to the high solubility of the water vapor.
2. Orientation dependence: The oxidation rate depends on the total number of available Si atom per unit area for oxidation at the oxide-silicon interface. For example, the oxidation rate for (111) oriented silicon is faster than that for (100) oriented Si initially in linear region.
3. Pressure is proportional to the number of oxidants and is directly proportional to both linear and parabolic rate constants. An increase in pressure results in a slower growth rate.
4. Impurity effect.

Q.6 (a) Solution:

Given,

$$\omega = 10^6 \text{ rad/sec}$$
$$\alpha = 0.921 \text{ Np/m}$$
$$\beta = 1 \text{ rad/m}$$
$$Z_0 = 60 + j40\ \Omega$$
$$V_s = 10\angle 0° \text{ V}; Z_g = 40\ \Omega$$
$$Z_L = 20 + j50\ \Omega$$
$$l = 2 \text{ m}$$

Input impedance, $Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}\right]$

where, propagation constant,

$$\gamma = \alpha + j\beta = 0.921 + j1$$

$$\gamma l = (0.921 + j1) \times 2 = 1.842 + j2$$

$$\tan(\gamma l) = \tanh(1.842 + j2)$$

We have, $\tan(x + jy) = \frac{\sinh(2x) + j\sin(2y)}{\cosh(2x) + \cos(2y)}$

Using $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$, we get

$$\tanh(1.842 + j2) = \frac{\sinh(2\times1.842) + j\sin(4)}{\cosh(2\times1.842) + \cos(4)} = \frac{19.905 - j0.7568}{19.930 - 0.6536}$$

$$= 1.033 - j0.039$$

$$Z_{in} = (60 + j40)\left[\frac{(20 + j50) + (60 + j40)(1.033 - j0.039)}{(60 + j40) + (20 + j50)(1.033 - j0.039)}\right]$$

$$Z_{in} = (60 + j40)\left[\frac{83.54 + j88.98}{82.61 + j90.87}\right] = \frac{1453.2 + j8680.4}{82.61 + j90.87}$$

$\therefore$ $$Z_{in} \simeq 60.2 + j38.8\ \Omega$$

Sending-end current, $I_s = \frac{V_s}{Z_{total}}$

where, $Z_{total} = Z_g + Z_{in} = 40 + (60.2 + j38.8)$

$$= 100.2 + j38.8\ \Omega = 107.5\ \angle 21.2°\ \Omega$$

$\therefore$ $$I_s = \frac{10\angle 0°}{100.2 + j38.8}$$

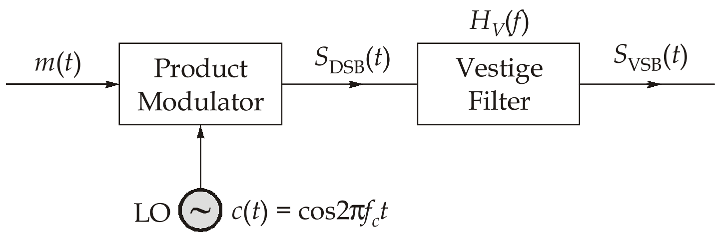
$$I_s = \frac{10\angle 0°}{107.5\angle 21.2°}$$

$\therefore$ $$I_s = 0.093\ \angle -21.2° \text{A}$$

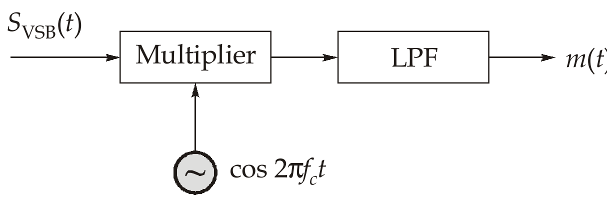
(or) $$I_s = 93\angle -21.2°\ \text{mA}$$

Q.6 (b) Solution:

(i) VSB Generation: Assume $m(t)$ as the message signal and $c(t)$ as the carrier signal.



Demodulation of VSB:



Let's find the condition to be satisfied by $H_V(f)$ to recover $m(t)$ from transmitted VSB signal $S_{VSB}(t)$.

For VSB generation, a DSB-SC signal is generated as

$$S_{DSB}(t) = m(t)c(t) = m(t).\cos(2\pi f_c t)$$

$$S_{DSB}(f) = \frac{M(f-f_c)+M(f+f_c)}{2} \quad ...(1)$$

To generate VSB signal $S_{VSB}(t)$, DSB-SC signal is passed through a filter having transfer function $H_V(f)$. Hence,

$$S_{VSB}(f) = S_{DSB}(f)\cdot H_V(f) \quad ...(2)$$

At demodulator:

$$(\text{Multiplier})_{o/p} = S_{VSB}(t)\cdot\cos(2\pi f_c t) \longleftrightarrow \frac{S_{VSB}(f-f_c)+S_{VSB}(f+f_c)}{2}$$

From (2),

$$(\text{Multiplier})_{o/p} \longleftrightarrow \frac{S_{DSB}(f-f_c)H_V(f-f_c)+S_{DSB}(f+f_c)H_V(f+f_c)}{2}$$

The output of Low pass filter is given by,

From (1),

$$(\text{Multiplier})_{o/p} \longleftrightarrow \frac{1}{4}[[M(f-2f_c)+M(f)]H_V(f-f_c)+[M(f)+M(f+2f_c)]\,H_V(f+f_c)]$$

$$(\text{LPF})_{o/p} = \frac{1}{4}M(f)[H_V(f-f_c)+H_V(f+f_c)]$$

To recover $M(t)$:

$$H_V(f-f_c)+H_V(f+f_c) = 4 \text{ (i.e., constant)}$$
$$= 2H(f_c), \text{ where } H(f_c) \text{ is constant}$$

(ii)
$$f_{XY}(x,y) = \begin{cases} Ae^{-(2x+y)}; & x,y \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

For a valid joint pdf,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_0^{\infty}\int_0^{\infty} Ae^{-(2x+y)}\,dx\,dy = 1$$

$$\Rightarrow A\int_0^{\infty} e^{-2x}dx \int_0^{\infty} e^{-y}dy = 1$$

$$\Rightarrow \left.\frac{Ae^{-2x}}{-2}\right|_0^{\infty} \cdot \left.\frac{e^{-y}}{-1}\right|_0^{\infty} = 1$$

$$\Rightarrow A\left[\frac{1}{2}\cdot 1\right] = 1$$

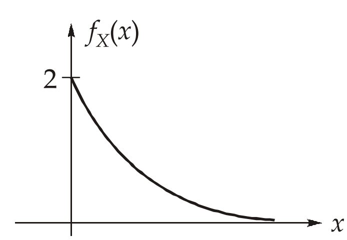
$$A = 2$$

Marginal probability distribution function,

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$

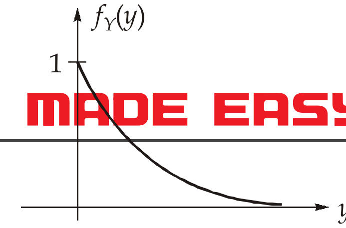
$$= \int_0^{\infty} 2e^{-(2x+y)}dy$$

$$= 2e^{-2x}\int_0^{\infty} e^{-y}dy = 2e^{-2x}$$



$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$$

$$= \int_{-\infty}^{\infty} 2e^{-(2x+y)}dx$$



$$= e^{-y}\int_0^{\infty} e^{-2x}dx = e^{-y}$$

Q.6 (c) Solution:

Let us assume, the small loop antenna carries a uniform current, I_0 and area, A.

We know that, the far electric field (in ϕ-direction) is:

$$E_\theta = j\eta\frac{\beta^2 I_0 A}{4\pi r}e^{-j\beta r}\sin\theta$$

where, $$\beta = \frac{2\pi}{\lambda};\ \eta = 120\pi\,\Omega$$

Time average power density, $$S = \frac{|E_\phi|^2}{2\eta}$$

Total radiated power, $$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} S\cdot r^2\sin\theta\, d\theta\, d\phi$$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi}\frac{|E_\phi|^2}{2\eta}\times r^2\sin\theta\, d\theta\, d\phi$$

$$= \frac{\eta\beta^4 I_0^2 A^2}{32\pi^2}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\, d\theta\, d\phi$$

where, $$\int_0^{\pi}\sin^3\theta\, d\theta = \frac{4}{3};\ \int_0^{2\pi} d\phi = 2\pi$$

$$P_{\text{rad}} = \frac{\eta\beta^4 I_0^2 A^2}{32\pi^2}\times\frac{4}{3}\times 2\pi = \frac{\eta\beta^4 I_0^2 A^2}{12\pi}$$

We have, $$P_{\text{rad}} = \frac{1}{2}I_0^2 R_r$$

where R_r is the radiation resistance

$\therefore$ $$R_r = \frac{2P_{\text{rad}}}{I_0^2} = \frac{2\times\eta\beta^4 I_0^2 A^2}{12\pi\times I_0^2} = \frac{\eta\beta^4 A^2}{6\pi}$$

But, $$\beta = \frac{2\pi}{\lambda}$$

$$\therefore \qquad R_r = \frac{\eta\left(\frac{2\pi}{\lambda}\right)^4 A^2}{6\pi}$$

For a circular loop with radius a and N turns,

$$A = N \times \pi a^2$$

Thus,

$$R_r = \frac{\eta\times 16\times\pi^4\times\pi^2\times a^4\times N^2}{6\pi\times\lambda^4} = \frac{120\pi\times 16\times\pi^4\times a^4\times N^2}{\lambda^4\times 6\pi}$$

$$R_r \approx 31200\left(\frac{N\pi a^2}{\lambda^2}\right)^2$$

Q.7 (a) Solution:

For a system with the state space representation $\dot{x} = Ax + Bu$ and $y = Cx + D$, the transfer function is given by

$$G(s) = C[sI - A]^{-1}B$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & -4 & -1 \\ -1 & -6 & -2 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} s+1 & 4 & 1 \\ 1 & s+6 & 2 \\ 1 & 2 & s+3 \end{bmatrix}$$

$$\text{Adj}[sI - A] = \begin{bmatrix} (s+6)(s+3)-4 & -\{4(s+3)-2\} & 8-(s+6) \\ -\{(s+3)-2\} & (s+1)(s+3)-1 & -\{2(s+1)-1\} \\ 2-(s+6) & -\{2(s+1)-4\} & (s+1)(s+6)-4 \end{bmatrix}$$

$$= \begin{bmatrix} s^2+9s+14 & -(4s+10) & -(s-2) \\ -(s+1) & s^2+4s+2 & -(2s+1) \\ -(s+4) & -(2s-2) & s^2+7s+2 \end{bmatrix}$$

$$|sI - A| = s + 1\{(s + 6)(s + 3) - 4\} - \{4(s + 3) - 2\} + [8 - (s + 6)]$$

$$= (s + 1)\{s^2 + 9s + 14\} - \{4s + 10\} + \{2 - s\}$$

$$= s^3 + 9s^2 + 14s + s^2 + 9s + 14 - 4s - 10 + 2 - s$$

$$= s^3 + 10s^2 + 18s + 6$$

$$G(s) = C[sI - A]^{-1} B$$

$$= [1 \quad 1 \quad 1]\frac{1}{s^3+10s^2+18s+6}$$

$$\times\begin{bmatrix} s^2+9s+14 & -(4s+10) & -(s-2) \\ -(s+1) & s^2+4s+2 & -(2s+1) \\ -(s+4) & -(2s-2) & s^2+7s+2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= [1 \quad 1 \quad 1]\frac{1}{s^3+10s^2+18s+6}\begin{bmatrix} -4s-10-s+2 \\ s^2+4s+2-2s-1 \\ -2s+2+s^2+7s+2 \end{bmatrix}$$

$$= [1 \quad 1 \quad 1]\frac{1}{s^3+10s^2+18s+6}\begin{bmatrix} -5s-8 \\ s^2+2s+1 \\ s^2+5s+4 \end{bmatrix}$$

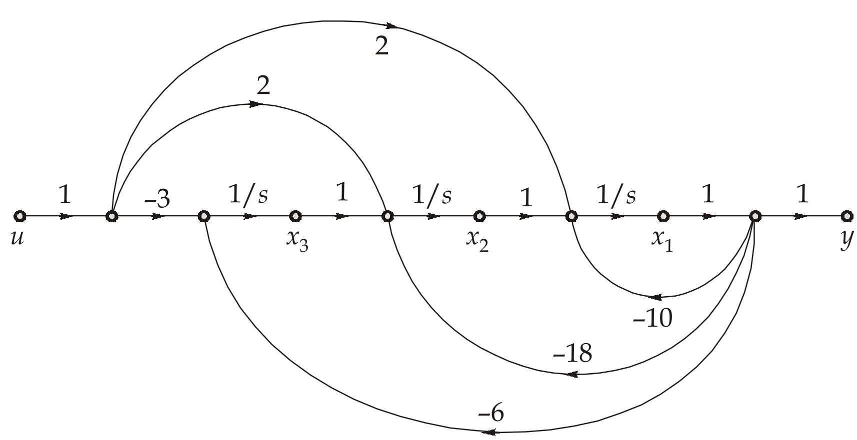
$$= \frac{-5s-8+s^2+2s+1+s^2+5s+4}{s^3+10s^2+18s+6}$$

$$= \frac{2s^2+2s-3}{s^3+10s^2+18s+6} = \frac{\frac{2}{s}+\frac{2}{s^2}-\frac{3}{s^3}}{1-\left(-\frac{10}{s}-\frac{18}{s^2}-\frac{6}{s^3}\right)}$$

Using Mason's Gain Formula, Signal flow graph is obtained as shown in fig. below:

$$T(s) = \frac{P_1\Delta_1+P_2\Delta_2+P_3\Delta_3}{1-(L_1+L_2+L_3)} = \frac{\frac{2}{s}+\frac{2}{s^2}-\frac{3}{s^3}}{1-\left(-\frac{10}{s}-\frac{18}{s^2}-\frac{6}{s^3}\right)}$$

where P_1, P_2 and P_3 are the forward path gains and L_1, L_2 , L_3 are Loop gains.



Q.7 (b) Solution:

(i) Delays in a packet switched network can be divided into four types:

1. Transmission Delay
2. Propagation Delay
3. Processing Delay
4. Queuing Delay

1. Transmission Delay: A source host or a router can not send a packet instantaneously. A sender needs to put the bits in a packet on the line one by one. If the first bit of the packet is put on the line at time t_1 and the last bit is put on the line at time t_2, transmission delay of the packet is $(t_2 - t_1)$. Definitely, the transmission delay is longer for a longer packet and shorter if the sender can transmit faster. In other words, the transmission delay is

$$(\text{Delay})_{tr} = (\text{Packet length})/\text{Transmission rate}$$

2. Propagation Delay: Propagation Delay is the time it takes for a bit to travel from point A to point B in the transmission media. The propagation delay for a packet switched network depends on the propagation delay of each network (LAN or WAN). The propagation delay depends on the propagation speed of the media, which is 3×10^8 m/sec in vacuum and normally much less in wired medium; it also depends on the distance of the link. In other words, propagation delay is,

$$(\text{Delay})_{pg} = (\text{Distance})/(\text{Propagation speed})$$

3. Processing Delay: The processing delay is the time required for a router or a destination host to receive a packet from its input port, remove the header, perform an error detection procedure, and deliver the packet to the output port (in the case of a router) or deliver the packet to the upper layer protocol. The processing delay may be different for each packet, but normally is calculated as an average.

$(\text{Delay})_{pr}$ = Time required to process a packet in a router or a destination host.

4. Queuing Delay: Queuing delay can normally happen in a router. The queuing delay for a packet in a router is measured as the time a packet waits in the input queue and output queue of a router.

$(\text{Delay})_{qu}$ = The time a packet waits in input and output queues in a router.

Total Delay: Assuming equal delays for the sender, routers and receiver, the total delay a packet encounters can be calculated if we know the number of routers 'n' in the whole path.

Total Delay = $(n + 1)[(\text{Delay})_{tr} + (\text{Delay})_{pg} + (\text{Delay})_{pr}] + n(\text{Delay})_{qu}$

If we have n routers, we have $(n + 1)$ links. Therefore, we have

1. $(n + 1)$ transmission delays related to n routers and the source.
2. $(n + 1)$ propagation delays related to $(n + 1)$ links.
3. $(n + 1)$ processing delays related to n routers and the destination.
4. Only 'n' queuing delays related to 'n' routers.

(ii) 1. Given Data: $n = 4.5$, $S/I = 13$ dB

We know that, $$\frac{S}{I} = \frac{1}{6}(Q)^n$$

Let's consider $Q = \sqrt{3N}$ for $N = 3$

$$Q = \sqrt{3\times 3} = 3$$

$$\therefore \quad \frac{S}{I} = \frac{1}{6}(3)^{4.5}$$

$$\left(\frac{S}{I}\right) = 23.38$$

$$\left(\frac{S}{I}\right)_{dB} = 13.6889 \text{ dB}$$

For $N = 3$, the calculated S/I is greater than the minimum required S/I i.e., 13 dB.

Hence, $N = 3$ can be used.

∴ Permissible reuse factor,

$$Q = \sqrt{3N} = \sqrt{3\times 3}$$

$$Q = 3$$

2. Now, Given data: $n = 3$; $\frac{S}{I} = 13$ dB

Let's consider $Q = \sqrt{3N}$ for $N = 3$

$$Q = \sqrt{3\times 3} = 3$$

$$\frac{S}{I} = \frac{1}{6}(3)^3 = \frac{1}{6}\times 27$$

$$\left(\frac{S}{I}\right) = 4.5$$

$$\left(\frac{S}{I}\right)_{dB} = 6.53 \text{ dB}$$

For $N = 3$, the calculated S/I is less than the minimum required S/I i.e., 13 dB.

Hence $N = 3$ can't be used. We need to use a larger N.

Next possible value of $N = 7$. (for $i = 2, j = 1$ as $N = i^2 + ij + j^2$)

Now, $$Q = \sqrt{3 \times N} = \sqrt{3 \times 7} = \sqrt{21}$$

$$\therefore \quad \frac{S}{I} = \frac{1}{6}(Q)^3 = \frac{1}{6}\left(\sqrt{21}\right)^3$$

$$\left(\frac{S}{I}\right) = 16.039$$

$$\left(\frac{S}{I}\right)_{dB} = 12.05 \text{ dB}$$

For $N = 7$, the calculated S/I is less than the minimum required S/I i.e., 13 dB.

Hence $N = 7$ can't be used. We need to use a larger N.

Next possible value of $N = 12$ ($i = 2, j = 2$)

Now $Q = \sqrt{3 \times 12} = \sqrt{36} = 6$

$$\frac{S}{I} = \frac{1}{6}(Q)^3 = \frac{1}{6}(6)^3 = 36$$

$$\left(\frac{S}{I}\right)_{dB} = 15.56 \text{ dB}$$

For $N = 12$, the calculated $\frac{S}{I}$ is greater than the minimum required $\frac{S}{I}$ i.e., 13 dB.

Hence, $N = 12$ can be used.

$\therefore$ Permissible reuse factor

$$Q = \sqrt{3N} = \sqrt{3 \times 12} = 6$$

Q.7 (c) Solution:

(i) ER Modeling in Embedded Communication Systems

In resource-constrained environments like IoT gateways, the ER model acts as the architectural blueprint that balances data accessibility with hardware limitations (CPU, RAM, and Flash memory).

1. **The Role of ER Modeling:**
   - **Managing Dynamic Data:** ER models allow designers to decouple Static Data (Device ID, MAC address) from Volatile Data (Sensor readings, RSSI values). By defining a SENSORS entity and a READINGS entity separately, we ensure the database doesn't store redundant device metadata for every single reading.
   - **Network Topology:** ER diagrams help visualize complex parent-child relationships in mesh or star topologies. For example, a NODE entity can have a self-referencing relationship ("Manages") to represent a Cluster Head overseeing sub-nodes.
2. **Challenges in Resource-Constrained Environments:**
   - **Storage Overhead:** Standard relational databases (like SQLite) add metadata overhead that can exhaust the small flash memory of an embedded node.
   - **Write Amplification:** Frequent updates to "Dynamic Status" attributes can wear out NAND flash memory.
   - **Concurrency:** Low-power MCUs often lack the processing power to handle complex JOIN operations or high-frequency ACID-compliant transactions.
3. **ER-to-Relational Optimizations**
   - **Denormalization for Performance:** While 3NF (Third Normal Form) is ideal, in IoT, we sometimes merge entities (e.g., merging Device_Status into Device) to reduce the number of JOINs, which are computationally expensive.
   - **Vertical Partitioning:** Split a table into "Frequently Accessed" (e.g., Current_Power_Level) and "Rarely Accessed" (e.g., Manufacturer_Details) to keep the primary working set in the limited RAM.
   - **Data Archiving via Relationships:** Use a 1:N relationship between Device and Logs where logs are periodically offloaded to a cloud gateway to keep the local table size constant.

(ii) 1. In vertical microprogramming, Control signals are encoded. Vertical Micro-programmed Control Word Format: It includes the bits required for control signals, flag selection, and next address sequencing.

Flags	Control signal	Next Address Field
$\log_2 18$ = 5 bit	$\log_2 85$ = 7 bit	$\log_2 (450 \times 20)$ = 14 bit

Length of control word = Flag bits + Control signal bits + Address bits

= 5 bit + 7 bit + 14 bit = 26 bits

Total Vertical Control Word Size = 26 bits

Horizontal Micro-programmed Control Word Size:

In horizontal micro-programming, each control signal is assigned a dedicated bit.

- Flags Field: 5 bits
- Control Signal Field: 85 bits (1 bit per signal)
- Next Address Field: 14 bits

Total Horizontal Control Word Size = 5 + 85 + 14 = 104 bits

2. Comparison of Control Units

Feature	Horizontal Micro-Programming	Vertical Micro-Programming
1. Control Word Length	Longer: Each control signal has a dedicated bit, leading to a wider control word.	Shorter: Control bits are encoded resulting in a narrow compact control word.
2. Execution Speed	Faster: No decoding is required; signals directly control the hardware.	Slower: Requires decoding logic before signals can be used.
3. Hardware Complexity	Low: Simple architecture but requires more storage (ROM) space.	High: Requires additional decoders, which increases gate count.
4. Flexibility	Higher: Can trigger multiple independent signals simultaneously.	Lower: Encoding limits the ability to trigger certain signals at the same time.

Q.8 (a) Solution:

(i) A cyclic code is given by

$$c(x) = d(x)\,g(x)$$

where, $c(x)$: code polynomial

$d(x)$: data polynomial

$g(x)$: generator polynomial

For x bit codeword, $d(x) \cdot g(x)$ will have maximum order n.

If $g(x)$ is a polynomial of degree $(n - k)$ and is a factor of $x^n + 1$,then $g(x)$ generates an (n, k) cyclic code.

In this case, $n = 7$

So, $g(x)$ would be factor of $x^7 + 1$,

$$x^7 + 1 = (x + 1)(x^3 + x^2 + 1)(x^3 + x + 1)$$

So, to generate constant 7 bit cyclic codeword, $g(x)$ can be choosen as $(x^3 + x^2 + 1)$ or $(x^3 + x + 1)$.

Both will provide similar codewords. Thus the two given generator polynomial are equivalent.

For cyclic codes, if two generator polynomials are reciprocals of each other, then the corresponding cyclic codes are equivalent because reversing every codeword coordinate transforms one code into the other.

Here: $g_1(P) = P^3 + P^2 + 1$

The reciprocal polynomial of $g_1(P)$ is

$$g_1^*(P) = P^{n-k} g_1(P^{-1}) = P^3 g_1(P^{-1})$$

$$g_1^*(P) = P^3(P^{-3} + P^{-2} + 1)$$

$$g_1^*(P) = P^3 + P + 1$$

$$g_1^*(P) = g_2(P)$$

Thus $g_2(P)$ is the reciprocal polynomial of $g_1(P)$. Therefore, the two (7, 4) cyclic codes $g_1(P)$ and $g_2(P)$ are equivalent codes.

(a) $D_1 : [0\ 0\ 1\ 1]$; $d(P) = P + 1$

$$C_1(P) = g_1(P)\, d(P) \quad \text{[Modulo-2 Arithmetic]}$$

So, $C_1(P) = (P^3 + P^2 + 1)(P + 1) = P^4 + P^2 + P + 1$

[In Modulo-2 Arithmetic, $P^3 + P^3 = 0$]

$C_1 : [0\ 0\ 1\ 0\ 1\ 1\ 1]$

(b) $D_2 : [0\ 1\ 0\ 1]$; $d_2(P) = P^2 + 1$

$$C_2 = (P^2 + 1)(P^3 + P^2 + 1) = P^5 + P^4 + P^3 + 1$$

$C_2 : [0\ 1\ 1\ 1\ 0\ 0\ 1]$

(c) $D_3 : [1\ 0\ 1\ 0]$; $d_3(P) = P^3 + P$

$$C_3(P) = (P^3 + P)(P^3 + P^2 + 1) = P^6 + P^5 + P^4 + P$$

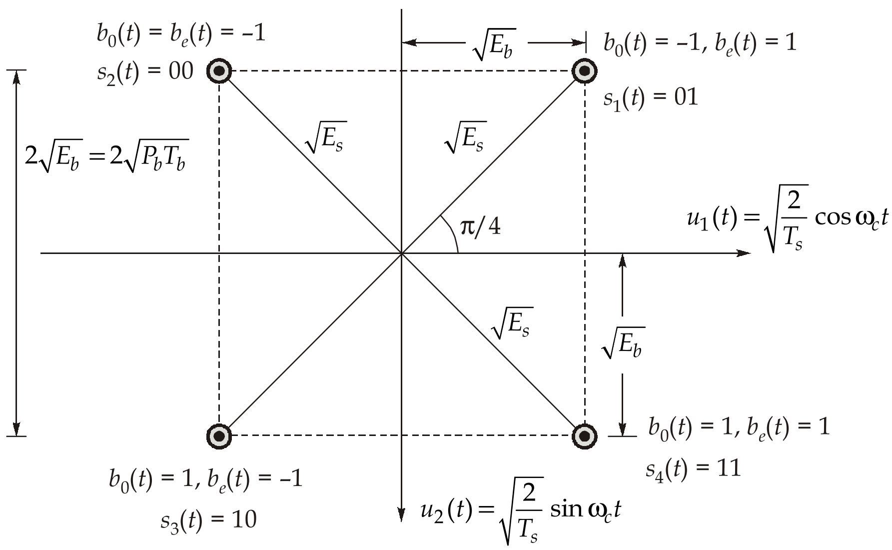
$C_3 : [1\ 1\ 1\ 0\ 0\ 1\ 0]$

(d) D_4 : [1 1 0 1]; $d_4(P) = P^3 + P^2 + 1$

$$C_4(P) = (P^3 + P^2 + 1)\,(P^3 + P^2 + 1) = P^6 + P^4 + 1$$

C_4 : [1 0 1 0 0 0 1]

(ii) Expression for the signal set for QPSK system and signal space diagram:



$$V_{QPSK}(t) = \sqrt{E_b} \times b_e(t) \times u_1(t) + \sqrt{E_b} \times b_0(t) \times u_2(t) \quad ...(1)$$

where, E_b = Energy per bit

$$b_e(t) = \sqrt{2}\cos\left[(2m+1)\frac{\pi}{4}\right] \quad \text{(where, } m = 0, 1, 2, 3)$$

$$b_0(t) = \sqrt{2}\sin\left[(2m+1)\frac{\pi}{4}\right]$$

Ortho-normal basis functions are used as carriers as below:

$$u_1(t) = \sqrt{\frac{2}{T_s}}\cos\omega_c t\,;$$

$$u_2(t) = \sqrt{\frac{2}{T_s}}\sin\omega_c t$$

The QPSK signals are given by:

$$s_1(t) = \sqrt{\frac{2E_b}{T_s}}\cos(\omega_c t) - \sqrt{\frac{2E_b}{T_s}}\sin(\omega_c t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(\omega_c t + \frac{\pi}{4}\right)$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_s}}\cos(\omega_c t) - \sqrt{\frac{2E_b}{T_s}}\sin(\omega_c t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(\omega_c t + \frac{3\pi}{4}\right)$$

$$s_3(t) = -\sqrt{\frac{2E_b}{T_s}}\cos(\omega_c t) + \sqrt{\frac{2E_b}{T_s}}\sin(\omega_c t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(\omega_c t + \frac{5\pi}{4}\right)$$

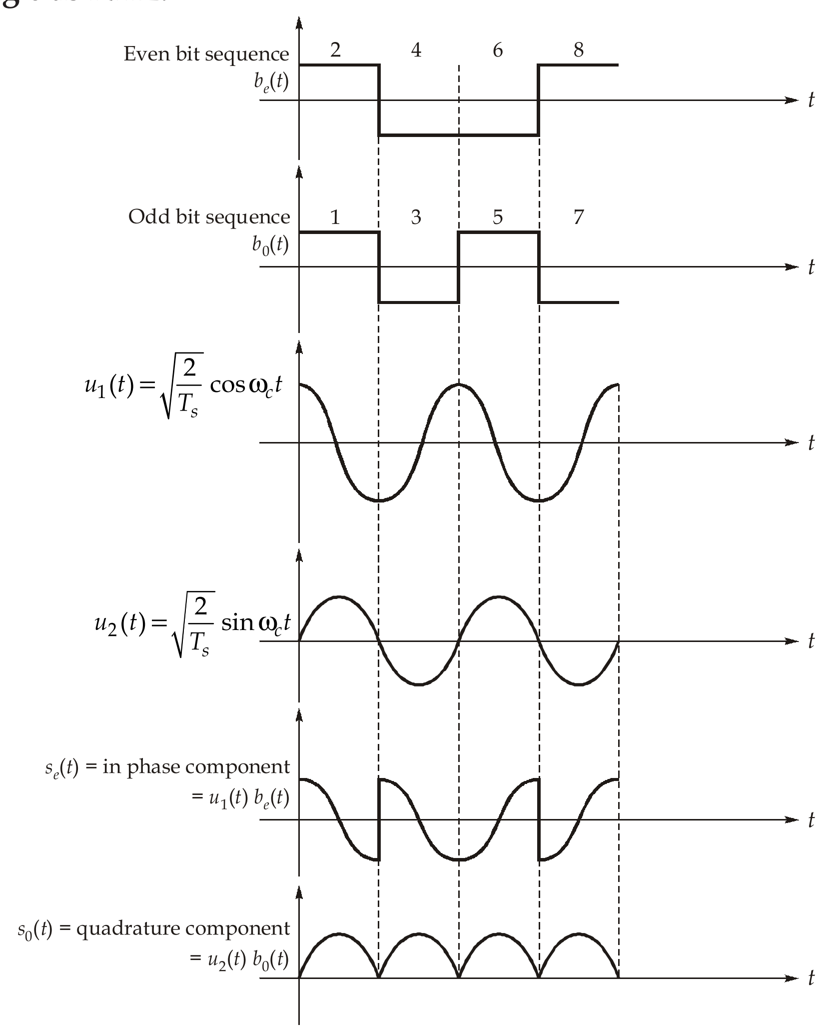
$$s_4(t) = \sqrt{\frac{2E_b}{T_s}}\cos(\omega_c t) + \sqrt{\frac{2E_b}{T_s}}\sin(\omega_c t) = \sqrt{\frac{2E_b}{T_b}}\cos\left(\omega_c t + \frac{7\pi}{4}\right)$$

Sketch of inphase and quadrature components of modulated QPSK signal:

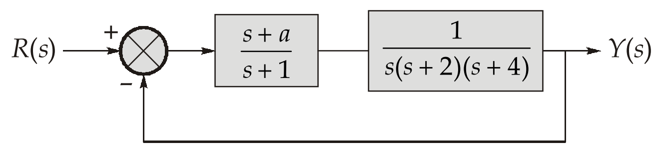
In equation (1) we assigned that even bit, $b_e(t)$ will modulate $u_1(t)$, i.e., $\sqrt{\frac{2}{T_s}}\cos\omega_c t$ and odd bit $b_0(t)$ will modulate $\sqrt{\frac{2}{T_s}}\sin\omega_c t$.

Input binary sequence	1	1	0	0	1	0	0	1
Bit number	1	2	3	4	5	6	7	8

Assuming 0 as NRZ:



Q.8 (b) Solution:



The characteristic equation of unity negative feedback system:

$$1 + G(s)H(s) = 0$$

$$s(s+1)(s+2)(s+4) + (s+a) = 0$$

$$s^4 + 6s^3 + 8s^2 + s^3 + 6s^2 + 8s + s + a = 0$$

$$s^4 + 7s^3 + 14s^2 + 9s + a = 0$$

Forming routh array:

$$\begin{array}{c|ccc} s^4 & 1 & 14 & a \\ s^3 & 7 & 9 & \\ s^2 & \frac{89}{7} & a & \\ s^1 & \frac{114.43 - 7a}{12.71} & & \\ s^0 & a & & \end{array}$$

For the system to be stable, there should be no sign change in the first column of the Routh array.

From s^1 :
$$\frac{114.43 - 7a}{12.71} > 0$$

$$a < 16.35$$

and
$$a > 0$$

Thus, for the system to be stable, $0 < a < 16.35$,

For critical stability, s^1 row should be zero implying the poles existing on imaginary axis. Hence, for critical stability,

$$114.43 - 7a = 0 \Rightarrow a = 16.35$$

At critical stability, the transfer function of compensator will be $\frac{s + 16.35}{s + 1}$

Since the pole is closer to the origin than the zero, thus the compensator is lag compensator.

Q.8 (c) Solution:

(i) Typical processing steps for fabrication of poly-silicon gate self-aligning *n* MOS technology are given below. It is for fabrication of a single enhancement type transistor.

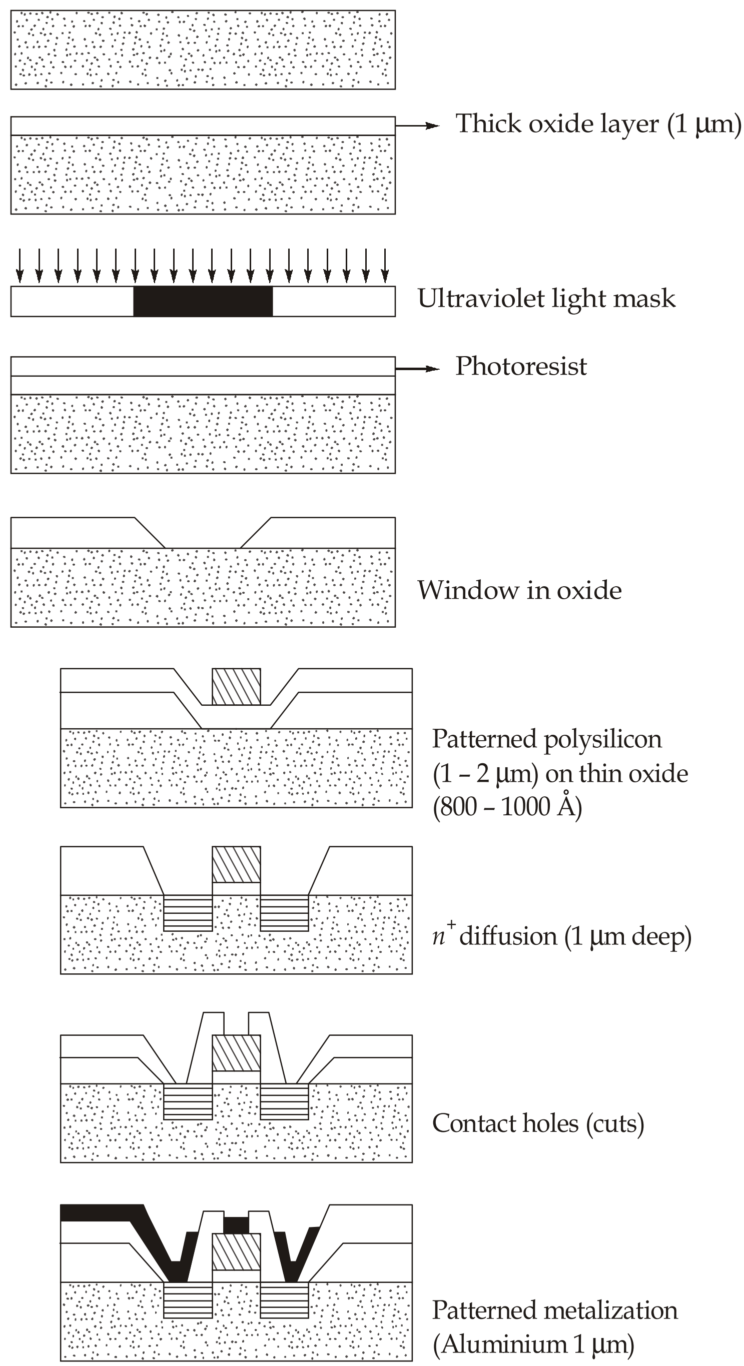


Fig: Step-by-step n MOS fabrication

Step 1: First step is to grow a thick SiO_2 layer, typically of 1 μm thickness all over the wafer surface using wet oxidation technique. This oxide layer will acts as a barrier to dopant during subsequent processing and provide an insulating layer on which other patterned layer can be formed.

Step 2: Some regions are defined on SiO_2 layer where transistor are to be formed. This is done by photolithographic process. At the end of this step, the wafer surface is exposed in those areas where diffusion regions along with a channel are to be formed to create a transistor.

Step 3: A thin layer of SiO_2 (0.1 μm) is deposited all over the entire wafer surface and on the top of this poly-silicon layer is deposited. The poly-silicon of 1.5 μm, which consists of heavily doped poly-silicon is deposited using Chemical Vapor Deposition (CVD) technique.

Step 4: Again by using another mask and photolithographic process, the poly-silicon is patterned. By this process, poly gate structures and inter connection to poly gate are formed.

Step 5: Thin oxide layer is removed to expose areas where *n*-diffusions are to take place to obtain source and drain. With the poly-silicon and underlying thin oxide layer as the protective mask, the diffusion process is performed.

Step 6: A thick oxide layer is grown all over again and holes are made at selected areas of poly-silicon gate, drain and source region by using a mask and photolithographic process.

Step 7: A metal (Aluminium) layer of 1 μm is deposited on entire surface by CVD process. The metal layer is then patterned with the help of mask and photolithographic process. Necessary interconnections are provided with the help of metal layer.

Step 8: The entire wafer is again covered with thick SiO_2, this is known as glassing. It acts an protective layer against environment.

(ii) The statement is correct.

A number of components are fabricated on the same IC chip. It become necessary to provide electrical isolation between different components and interconnections. Various types of isolation techniques have been developed. However, the two commonly used techniques are

1. *p*-*n* junction isolation
2. Dielectric isolation

In *p*-*n* junction technique, isolation islands are formed.

***p-n* junction isolation:** In this technique, p^+ type impurities are selectively diffused into the *n*-type epitaxial layer so as to reach *p*-type substrate as shown below:

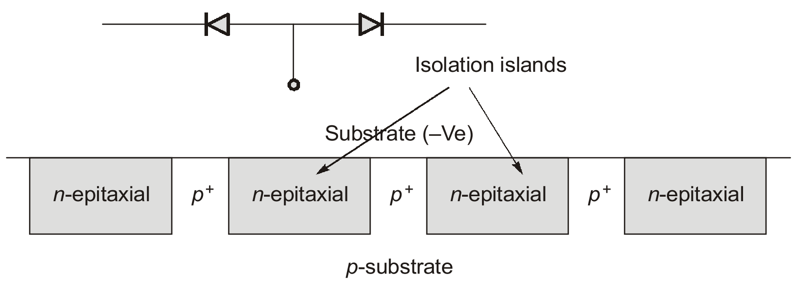


Fig: p-n junction isolation

This produces islands surrounded by *p*-type moats. It can be seen that these regions are separated by two back-to-back *p-n* junction diodes. If the *p*-type substrate material is held at the most negative potential in the circuit, diodes will be reversed biased providing electric isolation between these islands. The different components are fabricated in these islands. The concentration of the acceptor atoms in the region between isolation islands is usually kept much higher (p^+) than the *p*-type substrate. This prevents the depletion region of reverse biased diode from penetrating more into p^+ region and possibly connecting the isolation islands.

One drawback of this isolation technique is that the presence of transition capacitance at the isolating pn junction which cause an inevitable capacitor coupling between components and substrate.

These parasitic capacitances limit the performance of the circuit at higher frequencies. But being economical, this technique is commonly used for general purpose ICs.