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Leading Institute for ESE, GATE & PSUs

Detailed Solutions

# ESE-2026 Mains Test Series

# Electrical Engineering Test No : 11

## Section-A

**Q.1 (a) Solution:**

Given, kVA rating of transformers,

$$S = 10 \text{ kVA}$$

**(i)**

$$P_{out} = 10 \times 0.8 = 8 \text{ kW}$$

$$P_{cu, fL} = 60 \times (2)^2 = 240 \text{ W}$$

$$P_{core} = 100 \text{ W}$$

$$\% \eta = \frac{P_{out}}{P_{out} + P_{losses}} \times 100$$

$$\% \eta = \frac{8000}{8000 + 240 + 100} \times 100 = 95.92\%$$

**(ii)** Per unit rating at which the transformer efficiency is maximum

$$x = \sqrt{\frac{P_{core}}{P_{cu, fL}}} = \sqrt{\frac{100}{240}} = 0.6455 \text{ pu}$$

$$\eta_{max} = \frac{S \times x \times \text{p.f.}}{S \times x \times \text{p.f.} + 2P_{core}} \times 100$$

$$= \frac{10 \times 10^3 \times 0.6455 \times 0.8}{10 \times 10^3 \times 0.6455 \times 0.8 + 2 \times 100} \times 100 = 96.27\%$$

**(iii)** Total power output in 24 hrs,

$$E_{out} = 0 + (10 \times 0.7 \times 0.8 \times 10) + (10 \times 0.9 \times 0.9 \times 8)$$
$$= 120.8 \text{ kWhr}$$
$$E_{core} = 100 \times 24 = 2.4 \text{ kWhr}$$
$$E_{cu} = 240 \times (0.7)^2 \times 10 + 240 \times (0.9)^2 \times 8$$
$$= 2.7312 \text{ kWhr}$$
$$\% \, \eta_{\text{all day}} = \frac{120.8}{120.8 + 2.4 + 2.7312} \times 100 = 95.93\%$$

**Q.1 (b) Solution:**

Inductance per phase, $L = \frac{X_L}{2\pi f} = \frac{5}{2\pi \times 50} = 0.0159 \text{ H}$

Capacitance per phase, $C = 0.01 \, \mu\text{F} = 10^{-8} \text{ F}$

**(i)** Maximum value of recovery voltage (phase to neutral)

$$E_{max} = \sqrt{2} \times \frac{11}{\sqrt{3}} = 8.98 \text{ kV}$$

∴ Peak re-striking voltage $= 2\,E_{max}$

$$= 2 \times 8.98$$
$$= 17.96 \text{ kV}$$

**(ii)** Frequency of oscillations is

$$f_n = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.0159 \times 10^{-6}}}$$
$$= 12{,}628 \text{ Hz}$$

**(iii)** Peak re-striking voltage occurs at a time $t$ given by:

$$t = \frac{1}{2 f_n} = \pi\sqrt{LC}$$
$$= \pi\sqrt{0.0159 \times 10^{-8}}$$
$$= 39.6 \times 10^{-5} \text{ sec}$$
$$= 39.6 \, \mu\text{sec}$$

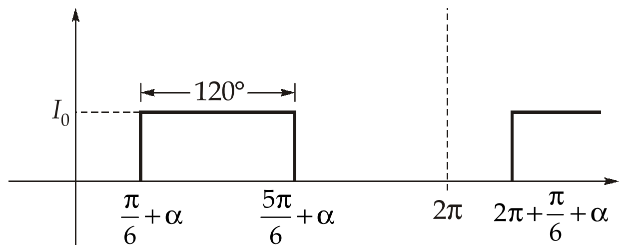
∴ Average rate of rise of re-striking voltage

$$= \frac{\text{Peak re-striking voltage}}{\text{Time upto first peak}} = \frac{17.96 \text{ kV}}{39.6 \, \mu\text{sec}}$$
$$= 453 \times 10^3 \text{ kV/sec}$$

**Q.1 (c) Solution:**

There will be no freewheeling action if $\alpha \le 30°$.

Source current waveform is shown below:




To find displacement factor and distortion factor, we should know Fourier coefficients for $n = 1$.

Fourier series expression is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\alpha x) + b_n \sin(n\alpha x)]$$

Trigonometric expression is $f(t) = \frac{a_0}{2} + \sqrt{a_n^2 + b_n^2} \sum_{n=1}^{\infty} \sin(n\alpha x + \theta)$

Where $\theta = \tan^{-1}\frac{a_n}{b_n}$

If $n = 1$,

$$a_1 = \frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_s \cos\omega t d\alpha x = -\frac{\sqrt{3}}{\pi}I_a \sin\alpha$$

$$a_1 = \frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_s \sin\omega x\, d\alpha x = -\frac{\sqrt{3}}{\pi}I_a \cos\alpha$$

Displacement angle, $\theta = \tan^{-1}\frac{a_1}{b_1} = -\alpha$

Displacement factor, DPF = cos α

RMS value of fundamental source current,

$$I_{s1} = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}} = \frac{I_0}{\pi}\sqrt{\frac{3}{2}}$$

From the waveform, RMS value of source current,

$$I_s = \frac{I_s}{\sqrt{3}}$$

Distortion factor, $$DF = \frac{I_{s1}}{I_s} = \frac{3}{\pi\sqrt{2}}$$

Power factor, $$\text{DF} \times \text{DPF} = \frac{3}{\pi\sqrt{2}}\cos\alpha = \frac{3}{\pi\sqrt{2}}\cos 15° = 0.6522$$

**Q.1 (d) Solution:**

**(i)** 
$$\bar{A}\bar{C} + ABC + A\bar{C} = \bar{C}(\bar{A} + A) + ABC$$
$$= ABC + \bar{C}$$
$$= (\bar{C} + C)(\bar{C} + AB)$$
$$= \bar{C} + AB$$

**(ii)** 
$$(\overline{\bar{x}\,\bar{y} + z}) + z + xy + wz = \overline{\bar{x}\,\bar{y}} \cdot \bar{z} + z + xy + wz$$
$$= (x + y)\bar{z} + z + xy$$
$$= (z + \bar{z})[z + (x + y)] + xy$$
$$= x + y + z + xy$$
$$= x + y + z$$

**(iii)** 
$$\bar{A}B(\bar{D} + \bar{C}D) + B(A + \bar{A}CD) = \bar{A}B(\bar{D} + D)(\bar{D} + \bar{C}) + B(A + \bar{A})(A + CD)$$
$$= \bar{A}B(\bar{D} + \bar{C}) + B(A + CD)$$
$$= B[\bar{A}\bar{D} + \bar{A}\bar{C} + A + CD]$$
$$= B[(A + \bar{A})(A + \bar{D}) + \bar{A}\bar{C} + CD]$$
$$= B[A + \bar{D} + \bar{A}\bar{C} + CD]$$
$$= B[(A + \bar{A})(A + \bar{C}) + (\bar{D} + D)(\bar{D} + C)]$$
$$= B[A + \bar{C} + \bar{D} + C]$$
$$= B[A + \bar{D} + 1]$$
$$= B$$

**(iv)** 
$$(\bar{A} + C)(\bar{A} + \bar{C})(A + B + \bar{C}D) = (\bar{A} + \bar{A}\bar{C} + \bar{A}C + C\bar{C})(A + B + \bar{C}D)$$
$$= (\bar{A} + \bar{A}C)(A + B + \bar{C}D)$$
$$= \bar{A}(A + B + \bar{C}D)$$
$$= \bar{A}(B + \bar{C}D)$$

**(v)** $AB\overline{C}D + \overline{A}BD + ABCD = ABD + \overline{A}BD$

$= BD$

**(vi)** $(\overline{a}+\overline{c})(a+\overline{b}+\overline{c}) = a\overline{a}+\overline{a}(\overline{b}+\overline{c})+\overline{c}a+\overline{c}(\overline{b}+\overline{c})$

$= \overline{a}\,\overline{b}+\overline{a}\,\overline{c}+\overline{c}\,a+\overline{c}\,\overline{b}+\overline{c}$

$= \overline{a}\,\overline{b}+\overline{a}\,\overline{c}+\overline{c}+\overline{c}\,\overline{b}$

$= \overline{a}\,\overline{b}+\overline{c}+\overline{c}\,\overline{b}$

$= \overline{a}\,\overline{b}+\overline{c}$

**Q.1 (e) Solution:**

The open loop transfer function is,

$$G(s) = \left(K_p + \frac{K_I}{s}\right)\frac{25}{s^2+10s+25}$$

$$G(s) = \frac{(K_p s + K_I)25}{s(s^2+10s+25)}$$

The system is 'type-1' system, so the error constant is velocity error constant.

$$\therefore \quad K_V = \lim_{s\to 0} s \cdot G(s)$$

$$100 = \lim_{s\to 0} s \cdot \frac{(K_p s + K_I)25}{s(s^2+10s+25)}$$

$$\therefore \quad K_I = 100$$

The characteristic equation,

$$1 + G(s)H(s) = 0$$

$$1+\left(\frac{K_p s+100}{s}\right)\left(\frac{25}{s^2+10s+25}\right) = 0$$

$s^3 + 10s^2 + 25s + 25K_p s + 2500 = 0$

$s^3 + 10s^2 + (25K_p + 25)s + 2500 = 0$

By using Routh-Hurwitz table,

| | | |
|---|---|---|
| $s^3$ | $1$ | $(25+25K_p)$ |
| $s^2$ | $10$ | $2500$ |
| $s^1$ | $25+25K_p-250$ | |
| $s^0$ | $2500$ | |

Since the system is stable.

So, $25 + 25K_p - 250 \geq 0$

$$\therefore \quad K_p \geq \frac{225}{25}$$

$$\therefore \quad K_p \geq 9$$

The critical value of $K_p = 9$, at which the system is marginally stable.

**Q.2 (a) (i) Solution:**

The expression can be generated using single op-amp with summing integrator circuit. The output of summing integrator with three inputs is given by,

$$V_{\text{out}} = -\int_0^t \left( \frac{1}{R_1 C_F} V_{\text{in}1} + \frac{1}{R_2 C_F} V_{\text{in}2} + \frac{1}{R_3 C_F} V_{\text{in}3} \right) dt$$

Comparing this with the given expression we get,

$$\frac{1}{R_1 C_F} = 1$$

$$\frac{1}{R_2 C_F} = 2$$

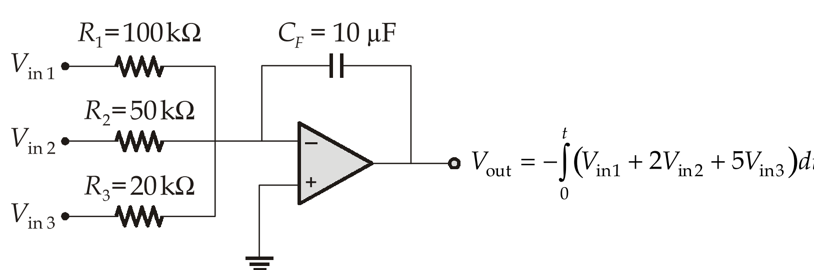
$$\frac{1}{R_3 C_F} = 5$$

Let us choose $C_F$ as 10 μF,

Then, $R_1 = 100\ \text{k}\Omega$,

$R_2 = 50\ \text{k}\Omega$,

$R_3 = 20\ \text{k}\Omega$




**Q.2 (a) (ii) Solution:**

The gain of an instrument amplifier is given by,

$$A = \frac{V_{\text{out}}}{V_{\text{in}2} - V_{\text{in}1}} = \left(1 + \frac{2R_F}{R_G}\right)\left(\frac{R_2}{R_1}\right)$$

In the circuit given, we have,

$$R_1 = 200\ \Omega,$$
$$R_2 = 100\ \Omega,$$
and $$R_F = 100\ \text{k}\Omega$$

$R_G$ is the series combination of 100 Ω and potentiometer of 100 kΩ.

Let potentiometer resistance is 0 Ω at start,

Hence, $$R_G = 100 + 0 = 100\ \Omega$$

Gain, $$A = \left(1 + \frac{2 \times 100 \times 10^3}{100}\right)\left(\frac{100}{200}\right) = 1000.5$$

When potentiometer resistance is at its maximum value, then

$$R_G = 100 + (100 \times 10^3) = 100.1\ \text{k}\Omega$$

Therefore, $$\text{Gain, } A = \left(1 + \frac{2 \times 100 \times 10^3}{100.1 \times 10^3}\right)\left(\frac{100}{200}\right) = 1.5$$

∴ For all practical purpose, the gain can be varied from 1.5 to 1000.5

**Q.2 (b) Solution:**

**(i)** Given: Closed loop transfer function,

$$T(s) = \frac{C(s)}{R(s)} = \frac{ks + a}{s^2 + bs + a}$$

We know that, $$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$[1 + G(s)]\,T(s) = G(s)$$

$$G(s) = \frac{T(s)}{1 - T(s)}$$

$$G(s) = \frac{(ks + a)/(s^2 + bs + a)}{1 - \dfrac{(ks + a)}{s^2 + bs + a}}$$

$$= \frac{ks + a}{s^2 + bs + a - ks - a} = \frac{ks + a}{s^2 + (b - k)s} = \frac{ks + a}{s(s + b - k)}$$

∴ The open loop transfer function,

$$G(s) = \frac{ks + a}{s(s + b - k)}$$

We know that, for ramp input,

Steady state error, $e_{ss} = \frac{1}{k_v}$ (where, $k_v$ = velocity error coefficient)

where, $k_v = \lim_{s\to 0} s\cdot G(s) = \lim_{s\to 0} s\cdot\frac{ks+a}{s(s+b-k)}$

$\therefore$ $k_v = \frac{a}{b-k}$

Steady state error, $e_{ss} = \frac{1}{k_v} = \frac{1}{\frac{a}{(b-k)}} = \frac{b-k}{a}$

$\therefore$ $e_{ss} = \frac{b-k}{a}$

**(ii)** Given: steady state error, $e_{ss} = 15\% = \frac{15}{100}$

$\therefore$ $e_{ss} = \frac{b-k}{a} = \frac{15}{100}$

$20(b-k) = 3a$

$b-k = \frac{3}{20}a$

$\therefore$ $k = b - \frac{3}{20}a$

**Q.2 (c) Solution:**

Checking for discontinuity of inductor current, if $L < L_{min}$ then converter is working in discontinous mode.

Now, boundary conditions

$$L\frac{\partial I_L}{\partial t} = V_0 \quad \text{(Force charging time i.e. S is OFF)}$$

At boundary $L = L_{min}$

$$L\frac{\Delta I_L}{(1-D)T_S} = V_0 \quad \text{... (i)}$$

Now, $I_{min} = 0$ at boundary

$$0 = (I_{L\,avg}) - \frac{\Delta I_L}{2} \quad \text{... (ii)}$$

$$(I_{L\,avg}) = I_0 + I_S = I_0 + \frac{D}{1-D} I_0$$

$$(I_{L\,avg}) = \left(\frac{1}{1-D}\right) I_0 \qquad \text{... (iii)}$$

From equation (ii) and (iii) we get

$$\left(\frac{1}{1-D}\right) I_0 = \frac{\Delta I_L}{2}$$

$$\Delta I_L = \left(\frac{2}{1-D}\right) I_0$$

Again from equation (i)

$$L_{min} \frac{\Delta I_L}{(1-D)T_S} = I_0 R$$

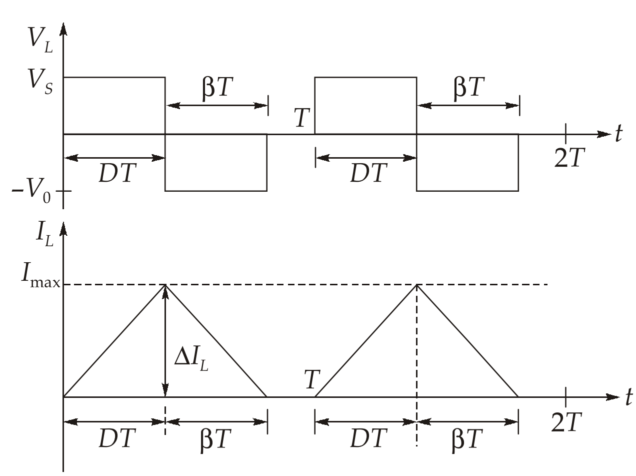
$$L_{min} \frac{2 I_0}{(1-D)^2 T_S} = I_0 R$$

$$L_{min} = \frac{(1-D)^2 R}{2f}$$

Now $$L_{min} = \frac{(1-0.4)^2 \times 5}{2 \times 100 \times 10^3} = 9 \mu H$$

Now as $L < L_{min}$ So converter is in discontinuous mode. Now for discontinuous mode of Buck-Boost converter,

$$V_0 = \frac{D}{\beta} V_S \qquad \text{... (iv)}$$



For discontinuous mode

$$(I_{L\,avg}) = \frac{1}{2}\times\Delta I_L\times(D+\beta) \quad \text{... (v)}$$

As average capacitor current is zero,

$$(I_{L\,avg}) = I_S + I_0 = \left(\frac{D}{\beta}I_0 + I_0\right)$$

$$= \frac{I_0}{\beta}(D+\beta) \quad \text{... (vi)}$$

From equation (v) and (vi)

$$\frac{1}{2}\times\Delta I_L\times(D+\beta) = \frac{I_0}{\beta}(D+\beta)$$

$$\frac{1}{2}\Delta I_L = \frac{I_0}{\beta}$$

$$\Delta I_L = \frac{2I_0}{\beta}$$

Again,
$$L\frac{di_L}{dt} = V_0$$

$$L\frac{\Delta I_L}{\beta T_S} = I_0 R$$

$$L\frac{2I_0/\beta}{\beta T_S} = I_0 R$$

$$\frac{2L}{\beta^2 T_S} = R$$

$$\beta^2 = \frac{2Lf_s}{R}$$

$$\beta = \sqrt{\frac{2Lf_s}{R}} = \sqrt{\frac{2\times 6\times 10^{-6}\times 100\times 10^3}{5}} = 0.49$$

Now from equation (iv),

$$V_0 = \frac{D}{\beta}V_S = \frac{0.4}{0.49}\times 24$$

**(i)** $V_0 = 19.6$ Volts

**(ii)** $I_0 = \frac{V_0}{R} = \frac{19.6}{5}\text{ V} = 3.91\text{ Amp}$

**(iii)** Average inductor current $(I_{L\,avg}) = I_S + I_0 = \frac{I_0(D+\beta)}{\beta}$

$$(I_{L\,avg}) = \frac{3.91(0.4+0.49)}{0.49} = 7.11 \text{ Amp}$$

**(iv)** Average switch current, $S$ = Average source current

$$I_S = \frac{D}{\beta} I_0 = 3.19 \text{ Amp}$$

**Q.3 (a) Solution:**

Given that,

Message signal, $m(t) = \cos(2000\pi t) + 2\sin(2000\pi t)$

Carrier signal, $c(t) = 100\cos(2\pi f_c t);\ f_c = 100$ kHz

**(i)** The standard time domain expression for the LSSB-AM signal can be given as,

$$s(t) = A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t)$$

$\hat{m}(t)$ is the Hilbert transform of the message signal $m(t)$.

$$\hat{m}(t) = \sin(2000\pi t) - 2\cos(2000\pi t)$$

So,

$$s(t) = 100\,[\cos(2000\pi t) + 2\sin(2000\pi t)]\cos(2\pi f_c t) + 100\,[\sin(2000\pi t) - 2\cos(2000\pi t)]\sin(2\pi f_c t)$$

$$= 100\,[\cos(2\pi f_c t)\cos(2000\pi t) + \sin(2\pi f_c t)\sin(2000\pi t)] + 200\,[\cos(2\pi f_c t)\sin(2000\pi t) - \sin(2\pi f_c t)\cos(2000\pi t)]$$

$$s(t) = 100\cos[2\pi(f_c - 1000)t] - 200\sin[2\pi(f_c - 1000)t]$$

**(ii)** By taking the Fourier transform of the above time domain expression of LSSB-AM signal $s(t)$, we get,

$$S(f) = 50[\delta(f - f_c + 1000) + \delta(f + f_c - 1000)] + j100[\delta(f - f_c + 1000) - \delta(f + f_c - 1000)]$$

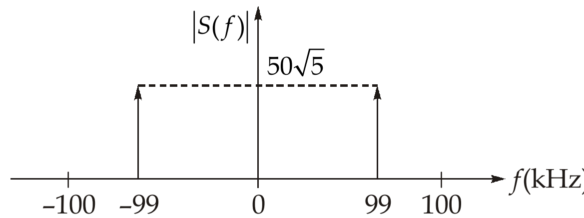
$$= (50 + j100)\,\delta(f - f_c + 1000) + (50 - j100)\,\delta(f + f_c - 1000)$$

Hence, the magnitude spectrum of the signal can be given as,

$$|S(f)| = \sqrt{(50)^2 + (100)^2}\,[\delta(f - f_c + 1000) + \delta(f + f_c - 1000)]$$

$$= 50\sqrt{5}\,[\delta(f - f_c + 1000) + \delta(f + f_c - 1000)]$$

The magnitude spectrum can be plotted, by taking $f_c$ = 100 kHz, as follows




**Q.3 (b) Solution:**

Given, impulse response, $h[n] = \begin{cases} a^n; & n \geq 0 \\ 0; & n < 0 \end{cases}$

Also we can write,

$$h[n] = a^n u[n]$$

$$x[n] = \begin{cases} 1; & 0 \leq n \leq N-1 \\ 0; & \text{otherwise} \end{cases}$$

Also we can write,

$$x[n] = u[n] - u[n-N]$$

by taking $z$-transform,

$$H(z) = \frac{1}{1-az^{-1}}; \quad |z| > |a|$$

$$X(z) = \frac{1-z^{-N}}{1-z^{-1}}; \qquad \text{All } z \text{ (by shifting property)}$$

Therefore, $Y(z) = X(z)H(z)$

$$= \frac{1}{1-az^{-1}} \cdot \frac{1-z^{-N}}{1-z^{-1}} = \frac{1-z^{-N}}{(1-z^{-1})(1-az^{-1})}$$

$$= \frac{1}{(1-z^{-1})(1-az^{-1})} - \frac{z^{-N}}{(1-z^{-1})(1-az^{-1})}$$

The ROC is $|z| > |a|$. Consider

$$P(z) = \frac{1}{(1-z^{-1})(1-az^{-1})} = \frac{A}{1-z^{-1}} + \frac{B}{1-az^{-1}}$$

For, $A = \left.\frac{1}{1-az^{-1}}\right|_{z^{-1}=1} = \frac{1}{1-a}$

$$B = \left.\frac{1}{1-z^{-1}}\right|_{z^{-1}=\frac{1}{a}} = \frac{1}{1-a^{-1}}$$

$\therefore$ $P(z) = \dfrac{\frac{1}{1-a}}{1-z^{-1}} + \dfrac{\frac{1}{1-a^{-1}}}{1-az^{-1}}$

Therefore, $$p(n) = \frac{1}{1-a}u[n] + \frac{1}{1-a^{-1}}a^n u[n]$$

Now, note that,

$$Y(z) = P(z)\left[1 - z^{-N}\right]$$

by taking inverse $z$-transform

$$y[n] = p(n) - p[n-N]$$

$$y[n] = \frac{1}{1-a}\{u[n] - u[n-N]\} + \frac{1}{1-a^{-1}}\left\{a^n u[n] - a^{n-N}u[n-N]\right\}$$

This may write as,

$$y[n] = \begin{cases} 0 & ; \quad n < 0 \\ \dfrac{(a^n - a^{-1})}{(1-a^{-1})}; & 0 \le n \le N-1 \\ \dfrac{a^n(1-a^{-N})}{(1-a^{-1})}; & n > N-1 \end{cases}$$

**Q.3 (c) Solution:**

**Given :** $$P(x_1) = 0.4\,; \quad P(x_2) = 0.19\,; \quad P(x_3) = 0.16\,;$$
$$P(x_4) = 0.15\,; \quad P(x_5) = 0.1$$

**(i) Shannon fano :**

| Symbol $(x_i)$ | Probabilities $P(x_i)$ | Stage-1 | Stage-2 | Stage-3 | Codeword | Length $(n_i)$ |
|---|---|---|---|---|---|---|
| $x_1$ | 0.4 | 0 | 0 | | 00 | 2 |
| $x_2$ | 0.19 | 0 | 1 | | 01 | 2 |
| $x_3$ | 0.16 | 1 | 0 | | 10 | 2 |
| $x_4$ | 0.15 | 1 | 1 | 0 | 110 | 3 |
| $x_5$ | 0.1 | 1 | 1 | 1 | 111 | 3 |

Efficiency, $$\eta = \frac{H(x)}{\bar{L}}$$

where, $H(x)$ = Entropy of source and

$$\bar{L} = \text{Average codeword length}$$

$$H(x) = \sum_{i=1}^{5} P(x_i)\log_2 \frac{1}{P(x_i)} = -\sum_{i=1}^{5} P(x_i)\log_2 P(x_i)$$

$$H(x) = -\left[0.4\log_2(0.4) + 0.19\log_2(0.19) + 0.16\log_2(0.16) + 0.15\log_2(0.15) + 0.1\log_2(0.1)\right]$$

$$= 2.15 \text{ bits/symbol}$$

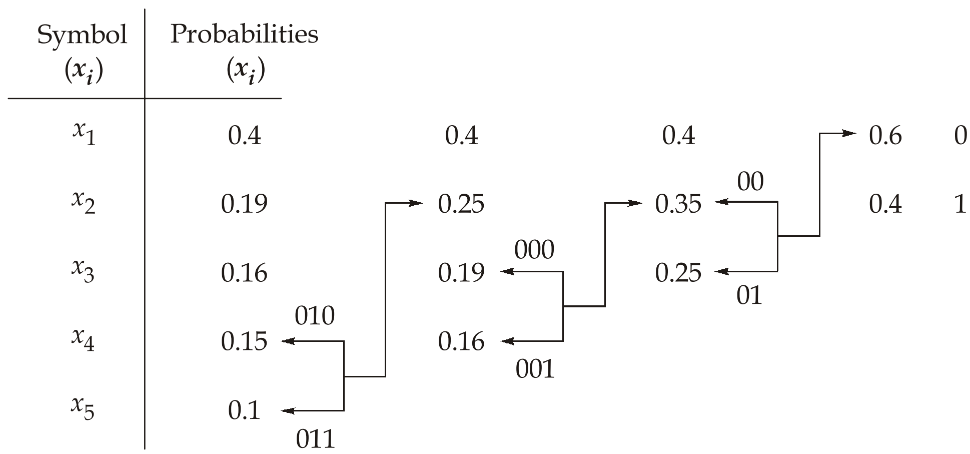
$$\bar{L} = \sum_{i=1}^{5} n_i P(x_i)$$

$$\bar{L} = 2 \times 0.4 + 2 \times 0.19 + 2 \times 0.16 + 3 \times 0.15 + 3 \times 0.1 = 2.25$$

Efficiency,
$$\eta = \frac{H(x)}{\bar{L}} = \frac{2.15}{2.25} = 0.95555$$

$$\eta = 95.56\% \quad \text{...(i)}$$

**(ii)** Huffman code




| Symbol ($x_i$) | Probabilities ($x_i$) | Codeword | Length ($n_i$) |
|---|---|---|---|
| $x_1$ | 0.4 | 1 | 1 |
| $x_2$ | 0.19 | 000 | 3 |
| $x_3$ | 0.16 | 001 | 3 |
| $x_4$ | 0.15 | 010 | 3 |
| $x_5$ | 0.1 | 011 | 3 |

$$H(x) = 2.15 \text{ bits/symbol}$$

Average codeword length,
$$\bar{L} = \sum_{i=1}^{5} n_i P(x_i)$$

$$\bar{L} = 1 \times 0.4 + 3 \times 0.19 + 3 \times 0.16 + 3 \times 0.15 + 3 \times 0.1 = 2.2$$

Efficiency,
$$\eta = \frac{H(x)}{\bar{L}} = \frac{2.15}{2.2} = 0.9773$$

$$\eta = 97.73\% \quad \text{...(ii)}$$

From equations (i) and (ii), we can say that the efficiency of Huffman coding is more than that of Shannon fano coding.

**Q.4 (a) Solution:**

Given, Window size, $M = 7$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & , \dfrac{-3\pi}{4} \le \omega \le \dfrac{3\pi}{4} \\ 0 & , \dfrac{3\pi}{4} < |\omega| \le \pi \end{cases}$$

The filter coefficients are given by

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$h_d(n) = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{-j3\omega}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{+j(n-3)\omega}d\omega$$

$$= \frac{1}{2\pi}\left\{\frac{e^{j(n-3)\omega}}{j(n-3)}\right\}_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \frac{1}{2\pi j(n-3)}\left[e^{j\frac{3\pi}{4}(n-3)} - e^{-j\frac{3\pi}{4}(n-3)}\right]$$

$$= \frac{1}{\pi(n-3)}\sin\left(\frac{3\pi(n-3)}{4}\right);\ n \ne 3$$

Using L-Hospital's rule,

$$h_d(3) = \frac{\frac{3\pi}{4}\cos\frac{3\pi}{4}(n-3)}{\pi} = \frac{3}{4} = 0.75$$

The filter coefficients,

$$h_d(0) = \frac{-1}{3\pi}\sin\left(\frac{-9\pi}{4}\right) = 0.075$$

$$h_d(1) = \frac{-1}{2\pi}\sin\left(\frac{-6\pi}{4}\right) = -0.159$$

$$h_d(2) = -\frac{1}{\pi}\sin\left(\frac{-3\pi}{4}\right) = 0.225$$

$$h_d(4) = \frac{1}{\pi}\sin\left(\frac{3\pi}{4}\right) = 0.225$$

$$h_d(5) = \frac{1}{2\pi}\sin\left(\frac{6\pi}{4}\right) = -0.159$$

$$h_d(6) = \frac{1}{3\pi}\sin\left(\frac{9\pi}{4}\right) = 0.075$$

The Hamming window function is,

$$W(n) = \begin{cases} 0.54 - 0.46\cos\dfrac{2\pi n}{M-1} & ;\ 0 \le n \le M-1 \\ 0 & ;\ \text{otherwise} \end{cases}$$

For $M = 7$, $$W(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right);\ 0 \le n \le 6$$

$W(0) = 0.08$, $W(1) = 0.31$, $W(2) = 0.77$, $W(3) = 1$, $W(4) = 0.77$, $W(5) = 0.31$, $W(6) = 0.08$

The filter coefficients of the resultant filter,

$$h(n) = h_d(n)W(n);\ 0 \le n \le M-1$$

$$h(0) = h(6) = 0.075 \times 0.08 = 0.006$$

$$h(1) = h(5) = -0.159 \times 0.31 = -0.0493$$

$$h(2) = h(4) = 0.225 \times 0.77 = 0.1733$$

$$h(3) = 0.75 \times 1 = 0.75$$

The frequency response of designed filter,

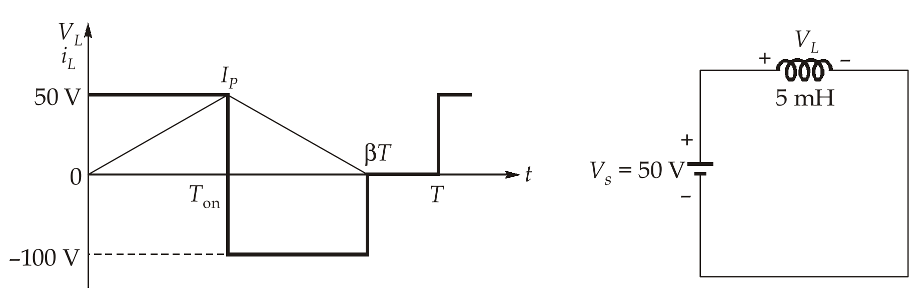
$$H(e^{j\omega}) = \sum_{n=0}^{6} h(n)e^{-j\omega n}$$

$$H(e^{j\omega}) = h(0) + h(1)e^{-j\omega} + h(2)e^{-j2\omega} + h(3)e^{-j3\omega} + h(4)e^{-j4\omega} + h(5)e^{-j5\omega} + h(6)e^{-j6\omega}$$

$$= \left\{\left(h(0)e^{j3\omega} + h(6)e^{-j3\omega}\right) + \left(h(1)e^{j2\omega} + h(5)e^{-j2\omega}\right) + \left(h(2)e^{j\omega} + h(4)e^{-j\omega}\right) + h(3)\right\}e^{-j3\omega}$$

$$H(e^{j\omega}) = e^{-j3\omega}[0.75 + 0.3466\cos\omega - 0.0986\cos 2\omega + 0.012\cos 3\omega]$$

**Q.4 (b) (i) Solution:**



During $T_{on}$ the circuit behaves as,

$$V_s = L\frac{di}{dt}$$

$$di = \frac{V_s}{L}dt$$

Integrating on both sides, we get

$$I_P = \frac{V_s}{L}T_{on}$$

$$T_{on} = \alpha T = \frac{\alpha}{f} = \frac{0.4}{5\times10^3} = 80\times10^{-6}\text{ s}$$

$$I_P = \frac{50}{5\times10^{-3}}\times(80\times10^{-6}) = 0.8\text{ A}$$

During $T_{off}$, it is $T_{on} \le t \le \beta T$

Apply KVL in the circuit,

$$V_L = V_s - V_0$$

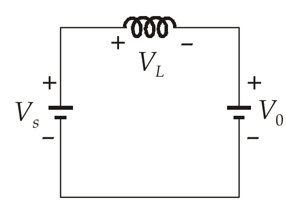
$$(V_L)_{avg} = 0$$

$$V_sT_{on} + (V_s - V_0)(\beta T - T_{on}) = 0$$

$$V_s\,\beta T = V_0(\beta T - T_{on})$$

$$\frac{V_0}{V_s} = \frac{\beta}{\beta-\alpha}$$

$$\beta = 0.6$$




From the graph of $I_L$,

$$I_{L(avg)} = \frac{\frac{1}{2}\times b\times h}{T}$$

$$= \frac{\frac{1}{2}\times\beta T\times I_P}{T} = \frac{1}{2}\times0.6\times0.8 = 0.24\text{ A}$$

**Q.4 (b) (ii) Solution:**

Given, rotor induced emf at standstill = 700 V line

$$\text{Per phase, } E_2 = \frac{700}{\sqrt{3}}\text{ V}$$

$$\text{Slip, } s = \frac{1500-1200}{1500} = 0.2$$

Voltage drop in diode, $V_{Diode} = 0.7$ V

Voltage drop in thyristor, $V_{th} = 1.5$ V

DC voltage across the diode rectifier is

$$V_d = \frac{3\sqrt{6}sE_2}{\pi} - 2\times V_{Diode} = \frac{3\sqrt{2}\times 0.2\times 700}{\pi} - 1.4$$

$$= 187.67 \text{ V}$$

With no voltage drop in inductor,

$$V_{dc} = V_d$$

$$\text{Inverter DC voltage} = -\left(\frac{3\sqrt{6}\times V_s}{\pi}\cos\alpha - V_{th}\times 2\right)$$

DC voltage across the diode rectifier without voltage drop in inductor = Inverter DC voltage

$$-\frac{3\sqrt{2}\times 415}{\pi}\cos\gamma + 3 = \frac{3\sqrt{2}\times 0.2\times 700}{\pi} - 1.4$$

$$\gamma = \cos^{-1}\left[\frac{-184.67\times\pi}{3\sqrt{2}\times 415}\right] = 109.24°$$

Firing advance angle of inverter = 180° – γ

180° – 109.24° = 70.76°

**Q.4 (c) Solution:**

**(i)** Condition for the desired response

1. system must be controllable.
2. system must be observable.

The state transition matrices,

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

For controllability

$$|Q_c| \neq 0$$

We know that, $Q_c = \begin{bmatrix} B & AB \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$|Q_c| = \begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix}$$

$\therefore \quad |Q| = 0 - 3 \neq 0$ so, system is controllable.

For observability, $\quad |Q_0| \neq 0$

We know that, $\quad Q_0 = \begin{bmatrix} C \\ CA \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \end{bmatrix}$$

$$|Q_0| = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}$$

$\therefore \quad |Q_0| = 3 - 0 \neq 0$ so, the system is observable.

So, the desired response is possible.

**(ii)** Given: Settling time, $T_s = 0.5$ sec, damping frequency $\omega_d = 6$ rad/sec

We know that, $\quad T_s = \dfrac{4}{\xi\omega_n} \Rightarrow \xi\omega_n = \dfrac{4}{0.5} = 8$

We know that, $\quad \omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{\omega_n^2 - (\omega_n\xi)^2}$

$$6 = \sqrt{\omega_n^2 - 8^2} \Rightarrow \omega_n = 10 \text{ rad/sec}$$

$$\therefore \quad \xi = \frac{8}{\omega_n} = \frac{8}{10} = 0.8$$

The second order characteristic equation,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + 16s + 100 = 0 \quad \text{...(i)}$$

Let the observer gain matrix be,

$$K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$$

The desired response is given by,

$$\dot{x} = (A - KC)x + Bu$$

Characteristic equation,

$$q(s) = |sI - (A - KC)| = 0$$

$$(A - KC) = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$(A - KC) = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} K_1 & 0 \\ K_2 & 0 \end{bmatrix} = \begin{bmatrix} 2-K_1 & 3 \\ -1-K_2 & 4 \end{bmatrix}$$

$$[sI - (A - KC)] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2-K_1 & 3 \\ -1-K_2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} s-2+K_1 & -3 \\ 1+K_2 & s-4 \end{bmatrix}$$

Characteristic equation,

$$q(s) = |sI-(A-KC)| = \begin{vmatrix} s-2+K_1 & -3 \\ 1+K_2 & s-4 \end{vmatrix}$$

$$(s - 2 + K_1)(s - 4) + 3(1 + K_2) = 0$$

$$s^2 - 4s + (-2 + K_1)s + 8 - 4K_1 + 3 + 3K_2 = 0$$

$$s^2 + (-6 + K_1)s + (11 - 4K_1 + 3K_2) = 0 \quad \text{...(ii)}$$

On comparing equation (i) and (ii),

$$-6 + K_1 = 16$$

$$\therefore \quad K_1 = 16 + 6 = 22$$

$$11 - 4K_1 + 3K_2 = 100$$

$$11 - 4 \times 22 + 3K_2 = 100$$

$$3K_2 = 100 - 11 + 88 = 177$$

$$\therefore \quad K_2 = 59$$

The observer gain matrix,

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 22 \\ 59 \end{bmatrix}$$

## Section-B

**Q.5 (a) Solution:**

M.D = 20 MW, LF = 75%, PCF = 50%

Plant use factor = 80%

Average load = MD × LF = 20 × 0.75 = 15 MW

**(i)** Daily Energy Generates = 15 × 24 = 360 MWh

**(ii)** Plant Reserve capacity = Plant capacity – Max. Demand

$$\text{Plant capacity} = \frac{\text{Avg. load}}{PCF} = \frac{15}{0.5} = 30 \text{ MW}$$

Plant reserve = P.C – M.D = 30 – 20 = 10 MW

**(iii)** Max. Energy that can be produced daily if the plant running all the time

$$= \frac{\text{Actual Energy Generated}}{\text{PCF}} = \frac{360}{0.5} = 720 \text{ MWh}$$

**Q.5 (b) Solution:**

Let $X_1$, $X_2$ and $X_3$ be the positive negative and zero sequence reactances respectively of the alternator.

For 3-phase fault,

$$\text{Fault current} = \frac{E_{ph}}{X_1}$$

$$2000 = \frac{11000/\sqrt{3}}{X_1}$$

$$\therefore \quad X_1 = \frac{11000}{\sqrt{3}\times 2000} = 3.175\ \Omega$$

For line-to-line fault, we have

$$\text{Fault current} = \frac{\sqrt{3}E_{ph}}{X_1 + X_2}$$

$$2600 = \frac{\sqrt{3}\times\dfrac{11000}{\sqrt{3}}}{X_1 + X_2}$$

$$X_1 + X_2 = \frac{11000}{2600} = 4.231\ \Omega$$

$$X_2 = 4.231 - X_1 = 4.231 - 3.175 = 1.056\ \Omega$$

For line-to-ground fault, we have,

$$\text{Fault current} = \frac{3E_{ph}}{X_1 + X_2 + X_0}$$

$$4200 = \frac{3\times\dfrac{11000}{\sqrt{3}}}{X_1 + X_2 + X_0}$$

$$X_1 + X_2 + X_0 = \frac{3\times 11000}{\sqrt{3}\times 4200} = 4.536\ \Omega$$

$$X_0 = 4.536 - X_1 - X_2$$

$$= 4.536 - 3.175 - 1.056 = 0.305\ \Omega$$

**Q.5 (c) Solution:**

Assembly language program for the given problem is as follows:

```
       LXI H,   XX60H ;  source pointer
       LXI D,   XX80H ;  Destination pointer.
       MVI C,   N     ;  Number of readings
LOOP:  INX H          ;  Skip high byte
       MOV A,   M     ;  Skip low byte
       XCHG
       MOV A,   M     ;  Store low byte
       XCHG
       INX H          ;  Move to next reading
       INX D          ;  Mark Next Destination
       DCR C
       JNZ LOOP
       HLT
```

**Q.5 (d) Solution:**

**(i)** We may redraw the block diagram as shown in Figure (1).

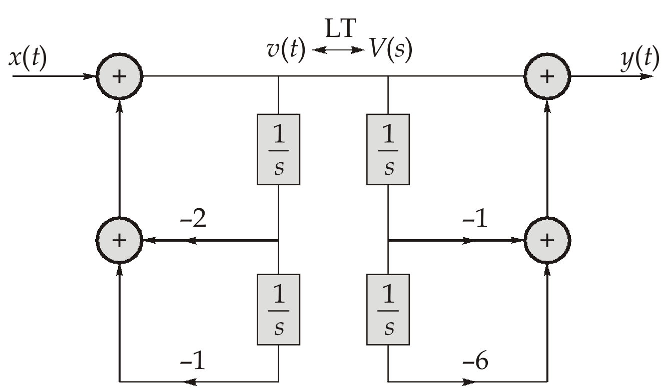



**Figure (1)**

Let $H_1(s) = \dfrac{V(s)}{X(s)}$ and $H_2(s) = \dfrac{Y(s)}{V(s)}$

$\therefore$ $H(s) = H_1(s) \cdot H_2(s)$ ...(i)

Now, consider the left half section of the Figure (1).

$$V(s) = X(s) + \frac{1}{s}V(s)(-2) + (-1)\frac{V(s)}{s^2}$$

$$V(s)\left[1+\frac{2}{s}+\frac{1}{s^2}\right] = X(s)$$

$$\frac{V(s)}{X(s)} = \frac{s^2}{(s^2+2s+1)}$$

$$\frac{V(s)}{X(s)} = H_1(s) = \frac{s^2}{s^2+2s+1} \quad \text{...(ii)}$$

Now, from the right half section of Figure (1).

$$Y(s) = V(s)+\frac{1}{s}V(s)(-1)+\frac{1}{s^2}V(s)(-6)$$

$$Y(s) = V(s)\left[1-\frac{1}{s}-\frac{6}{s^2}\right]$$

$$\frac{Y(s)}{V(s)} = H_2(s) = \frac{(s^2-s-6)}{s^2} \quad \text{...(iii)}$$

Now, from equation (i), (ii) and (iii), we get

$$H(s) = \frac{s^2}{(s^2+2s+1)} \times \frac{(s^2-s-6)}{s^2}$$

$$H(s) = \frac{(s^2-s-6)}{(s^2+2s+1)} = \frac{Y(s)}{X(s)}$$

$$Y(s)[s^2+2s+1] = X(s)[s^2-s-6]$$

The inverse Laplace transform of the above equation yields the differential equation of the system relating $x(t)$ and $y(t)$.

$$\frac{d^2y(t)}{dt^2}+\frac{2dy(t)}{dt}+y(t) = \frac{d^2x(t)}{dt^2}-\frac{dx(t)}{dt}-6x(t)$$

**(ii)**

$$H(s) = \frac{(s^2-s-6)}{(s+1)^2} = \frac{(s-3)(s+2)}{(s+1)^2}$$

We know that for a system to be stable, all the poles must lie in the left half of the $s$-plane. The two poles of the system are at $s = -1$. We may conclude that this system is stable.

**Q.5 (e) Solution:**

Armature circuit resistance, $r_a = \frac{xz}{a^2}$

Here, $x$ = individual resistance of conductor

$z$ = number of conductors

$a$ = number of parallel paths

$$r_a = 1000 \times 2 \times 10^{-3} \times \frac{1}{2^2} = 0.5\,\Omega$$

Let $I_f$ be shunt field current.

The generated emf at 1055 rpm is given by

$$E_{a1} = V_t + (I_L + I_f) \times 0.5$$
$$= 100 + (10 + I_f) \times 0.5$$

At 1105 rpm $E_{a2} = 100 + (20 + I_f) \times 0.5$

But Generated emf $\propto$ field current × speed

and $E_{a1} \propto I_f \times 1055$

$E_{a2} \propto I_f \times 1105$

$\therefore$ $I_f \times 1055 \propto 100 + (10 + I_f) \times 0.5$

and $I_f \times 1105 \propto 100 + (20 + I_f) \times 0.5$

$$\frac{I_f \times 1055}{I_f \times 1105} = \frac{100 + (10 + I_f) \times 0.5}{100 + (20 + I_f) \times 0.5}$$

Its solution gives $I_f$ = 1 A.

$\therefore$ $E_{a1} = 100 + (10 + 1) \times 0.5$

= 105.5 volts at 1055 rpm

Now $E_{a1} = \frac{\phi ZnP}{a}$

or $105.5 = \frac{\phi \times 1000 \times 1055 \times 2}{60 \times 2}$

$$\phi = \frac{105.5 \times 60}{1000 \times 1055} = 0.006 \text{ Wb}$$

$\therefore$ Field circuit resistance $= \frac{V_t}{I_f} = \frac{100}{1} = 100\,\Omega$

Flux per pole $= \frac{6}{2} = 3$ mWb

**Q.6 (a) (i) Solution:**

$$P_1 + P_2 = 210 \quad ...(i)$$

For ELD without losses $I_{C1} = I_{C2}$

$$0.15P_1 + 50 = 0.2P_2 + 40$$

$$0.15P_1 - 0.02P_2 = -10$$

$$\Rightarrow \quad 15P_1 - 20P_2 = -1000 \quad ...(ii)$$

By solving to the equations $P_1 = 91.4$ MW, $P_2 = 118.6$ MW

Total cost or production with ELD.

$$I_{C1} = (0.15 \times 91.4 + 50) + (0.2 \times 118.6 + 40) = 127.43 \text{ Rs/hr}$$

Total cost of production when $P_1 = P_2 = 105$ MW

$$I_{C2} = (0.15 \times 105 + 50) + (0.2 \times 105 + 40)$$

$$C_1 = 0.075P_1^2 + 50P_1 + K_1$$

$$C_2 = 0.1P_2^2 + 40P_2 + K_2$$

$$F_1 = C_1 + C_2 = (0.075(91.4)^2 + 50 \times 91.4 + K_1) + (0.1(118.6)^2 + 40(118.6) + K_2)$$

$$= 5196.547 + K_1 + 6150.596 + K_2 = 11347.136 + K_1 + K_2$$

$$F_2 = C_1' + C_2' = (0.075(105)^2 + 50 \times 105 + K_1 + 0.1\,(105)^2 + 40(105) + K_2)$$

$$= 6076.875 + K_1 + 5302.5 + K_2 = 11379.375 + K_1 + K_2$$

Additional cost with the scheduling of $P_1 = P_2 = 105$ is

$$= (11379.375 + K_1 + K_2) - (11347.136 + K_1 + K_2)$$

$$= 32.24 \text{ Rs/hr}$$

**Q.6 (a) (ii) Solution:**

$$\text{Load power rating } P_L = \sqrt{3}V_L I_2 \cos\phi$$

$$P_L = \sqrt{3} \times 500 \times 20 \times 0.8 = 13.856 \text{ kW}$$

$$\phi_1 = \cos^{-1}(0.8) = 36.80°, \quad \phi_2 = \cos1(1.0) = 0.0$$

$$\text{KVAR Raised} = P(\tan\phi_1 - \tan\phi_2)$$

$$= 13.856(\tan 36.86 - \tan 0) = 10388.26 \text{ KVAR}$$

$$\text{Power rating of motor} = \frac{10 \times 746}{0.8} = 9.325 \text{ kW}$$

$$\text{The KVA rating of the motor} = \sqrt{9.325^2 + 10.388^2} = 13.9 \text{ kVA}$$

$$\phi = \tan^{-1}\left(\frac{10.388}{9.325}\right) = 48.08°$$

$$\text{P.F} = \cos 48.08 = 0.67 \text{ lead}$$

**Q.6 (b) Solution:**

**State table:**

| Present state | | | Next state | | | F.F Inputs | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | $B$ | $C$ | $A^+$ | $B^+$ | $C^+$ | $J_A$ | $K_A$ | $T_B$ | $S_C$ | $R_C$ |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | X | 1 | X | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | X | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | X | 1 | 1 | 0 | X |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | X | 1 | 0 | X |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | X | 0 | 1 | 0 |

**K-map for $J_A$:**

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | X | X | X | X |

$$J_A = BC$$

**K-map for $K_A$:**

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | X | X | X | X |
| 1 | 1 | X | X | X |

$$K_A = 1$$

**K-map for $T_B$:**

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | X | X | X |

(Groups: C, B, A)

$$T_B = A + B + C$$

**K-map for $S_C$:**

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | X | 0 | 0 |
| 1 | 0 | X | X | X |

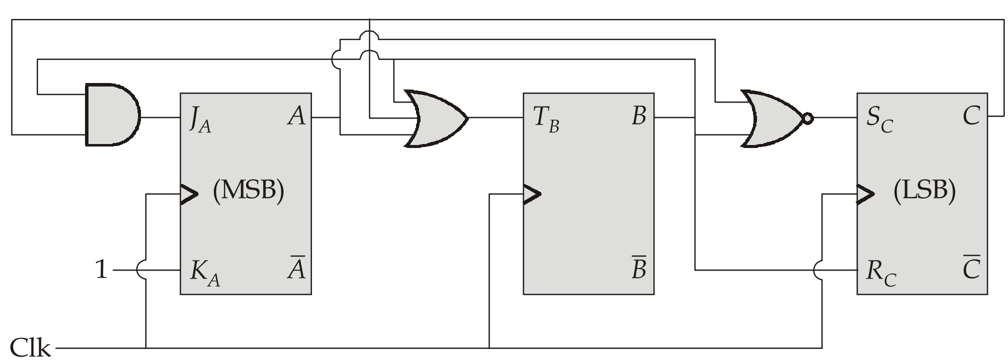
$$S_C = \bar{A}\bar{B} \text{ or } S_C = \overline{A+B}$$

**K-map for $R_C$:**

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | X |
| 1 | X | X | X | X |

$$R_C = B$$

**Circuit**

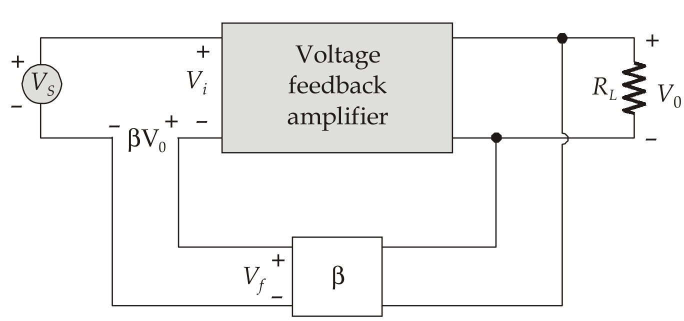



**Q.6 (c) Solution:**

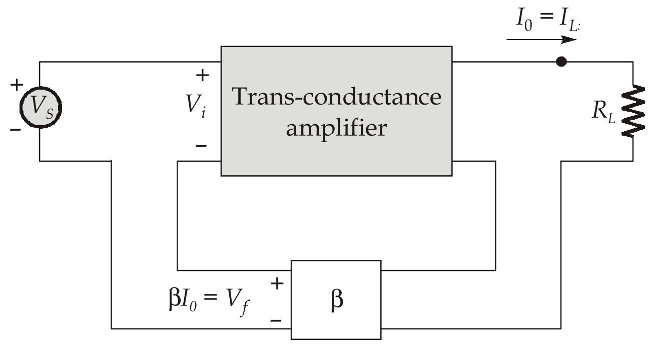
There are four feedback amplifier topologies:

1. Voltage series feedback
2. Current series feedback
3. Current shunt feedback
4. Voltage shunt feedback

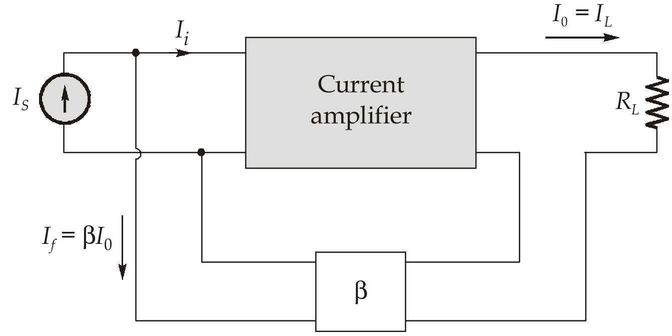
**1. Voltage series feedback:**



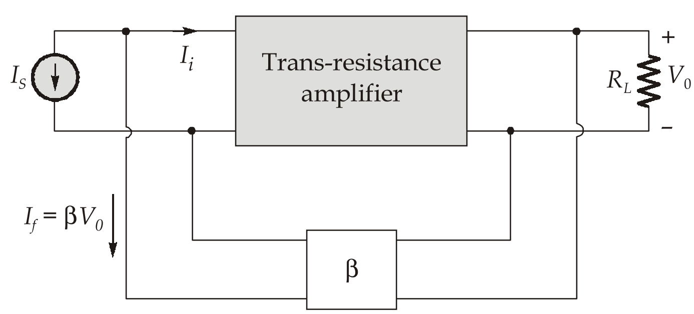
**2. Current series feedback:**



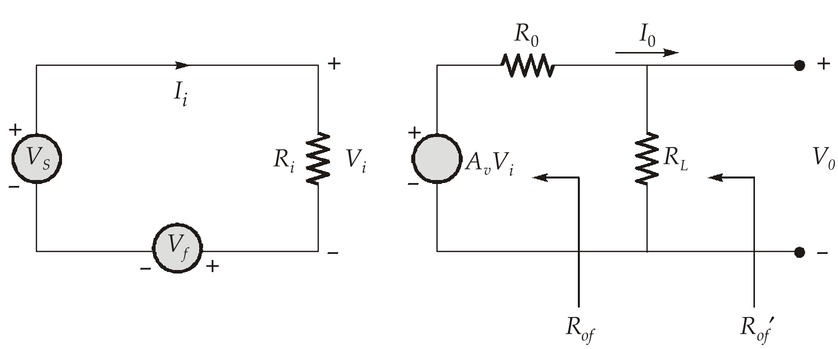

**3. Current shunt feedback:**




**4. Voltage shunt feedback:**




Voltage series feedback circuit is shown in figure below:



$$V_s = I_iR_i + V_f = I_iR_i + \beta V_0 \quad \text{...(i)}$$

and
$$V_0 = \frac{A_v V_i R_L}{R_0 + R_L} = A_v I_i R_i \quad \text{...(ii)}$$

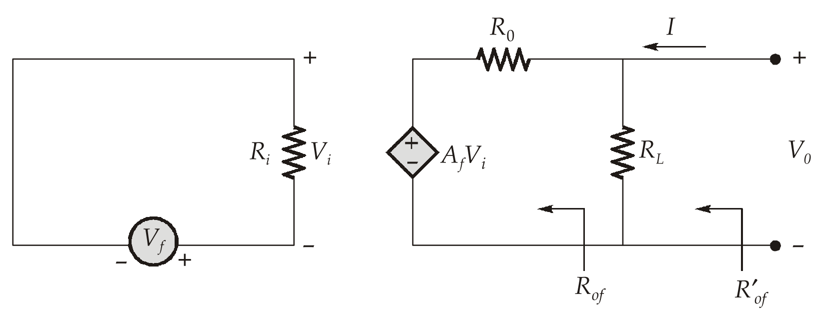
So,
$$A_{vf} = \frac{A_v R_L}{R_0 + R_L} \quad \text{...(iii)}$$

From equations (i) and (ii),

$$V_s = I_iR_i + \beta(A_v I_i R_i)$$

$$\frac{V_s}{I_i} = R_{if} = R_i(1+\beta A_v)$$

For $R_{of}$ taking $V_s = 0$ means $V_i = -V_f = -\beta V_0$




We have,
$$I = \frac{V_0}{R_L} + \frac{V_0 - A_v V_i}{R_0}$$

$$I = V_0\left[\frac{1}{R_0} + \frac{1}{R_L}\right] + \frac{A_v V_f}{R_0} \quad \text{...(iv)}$$

$$= V_0\left[\frac{1}{R_0} + \frac{1}{R_L}\right] + \frac{A_v \beta V_0}{R_0} = V_0\left[\frac{1}{R_L} + \frac{1 + A_v\beta}{R_0}\right]$$

$$\frac{1}{R'_{of}} = \frac{I}{V_0} = \frac{1}{R_L} + \frac{1}{\dfrac{R_0}{1 + A_v\beta}}$$

$$R'_{of} = R_L || R_{of} = R_L || \frac{R_0}{1 + A_v\beta} = \frac{R_L || R_0}{1 + A_v\beta} = \frac{R'_0}{1 + A_v\beta}$$

where,
$$R_{of} = \frac{R_0}{1 + A_v\beta}$$

$$R'_0 = R_0 || R_L$$

**Q.7 (a) Solution:**

Given,
$$S_{G1} = P_{G1} + jQ_{G1}$$
$$S_{D1} = P_{D1} + jQ_{D1} = 2 + j1$$
$$S_{G2} = P_{G2} + jQ_{G2} = 0.5 + j1$$
$$S_{D2} = P_{D2} + jQ_{D2} = 0 + j0$$

Using nominal π-method for transmission line

$$y_{\text{series}} = \frac{1}{0.02 + j0.08} = 2.941 - j11.764$$
$$= 12.13\angle{-75.96°} \text{ p.u.}$$

∴ Each off diagonal term = –2.941 + $j$11.764 p.u.

$$\text{Each self term} = (2(2.941 - j11.764) + j0.01)$$
$$= 5.882 - j23.528$$
$$= 24.23\angle{-75.95°} \text{ p.u.}$$

$$Y_{\text{bus}} = \begin{bmatrix} 24.23\angle -75.95° & 12.13\angle 104.04° & 12.13\angle 104.04° \\ 12.13\angle 104.04° & 24.23\angle -75.95° & 12.13\angle 104.04° \\ 12.13\angle 104.04° & 12.13\angle 104.04° & 24.23\angle -75.95° \end{bmatrix} \mho$$

Also given, $V_2^0 = 1 + j0$ and $\delta_3^0 = 0$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} + \delta_1 - \delta_2) + |V_2^2||y_{22}|\cos(\theta_{22})$$
$$+ |V_2||V_3||Y_{23}|\cos(\theta_{23} + \delta_3 - \delta_2)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} + \delta_1 - \delta_3)$$
$$+ |V_3||V_2||Y_{32}|\cos(\theta_{32} + \delta_2 - \delta_3) + V_3^2|Y_{33}|\cos(\theta_{33})$$

$$Q_2 = \left[-|V_2||V_1||Y_{21}|\sin(\theta_{21} + \delta_1 - \delta_2) - V_2^2|Y_{22}|\sin(\theta_{22})\right.$$
$$\left. - |V_2|V_3||Y_{23}|\sin(\theta_{23} + \delta_3 - \delta_2)\right]$$

$$P_2^0 = [(1.04)(1)(12.13)\cos(104.04 + 0 - 0) + (1)^2(24.23)\cos(-75.95°)$$
$$+ (1)(1.04)(12.13)\cos(104.04° + 0 - 0)]$$
$$= -3.064 + 5.8822 - 3.0604$$
$$= -0.2385 \text{ p.u.}$$

$$P_3^0 = [(1.04)(1.04)(12.13)\cos(104.04 + 0 - 0) + (1.04) \times 12.13(\cos 104.04)$$
$$+ (1.04)^2(24.23)\cos(-75.95)]$$

$$= -3.18285 - 3.0604 + 6.3622$$
$$= 0.119 \text{ p.u.}$$

Similarly, $Q_2^0 = -0.9715$ p.u.

Checking for $Q_3$ range

$$Q_3 = -|V_3|V_1Y_{31}\sin(\theta_{31} + \delta_1^{(0)} - \delta_3^{(0)}) - |V_3|^2 Y_{33}\sin(\theta_{33})$$
$$-V_3V_2Y_{32}\sin(\theta_{32} + \delta_2^{(0)} - \delta_3^{(0)})$$
$$= -12.7278 + 25.423 - 12.238 = 0.2686$$

$$Q_{G3}^{(0)} = Q_{D3} + \Delta Q_3^{(0)} = 0.6 + 0.2686 = 0.8686 \text{ p.u.}$$

Power residual,

$$\Delta P_2^0 = P_2 \text{ (specified)} - \Delta P_2^0 \text{(Calculated)} = 0.5 - (-0.235)$$
$$= 0.7385 \text{ p.u.}$$

$$\Delta P_3^0 = -1.5 - (0.119) = -1.619 \text{ p.u.}$$

$$\Delta Q_2^0 = 1 - (-0.9715) = 1.9715 \text{ p.u.}$$

So matrix equations for the solution of load flow by FDLF method,

$$\begin{bmatrix} \dfrac{\Delta P_2^{(0)}}{|V_2^{(0)}|} \\ \dfrac{\Delta P_3^{(0)}}{|V_3^{(0)}|} \end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix}$$

and
$$\begin{bmatrix} \dfrac{\Delta Q_2^{(0)}}{|V_2^{(0)}|} \end{bmatrix} = [-B_{22}]\left[\Delta |V_2^{(0)}|\right]$$

Here $B_{22}$ and $B_{33}$ are the imaginary parts of $Y_{22}$ and $Y_{33}$

$$\begin{bmatrix} \dfrac{0.7385}{1} = 0.7385 \\ \dfrac{-1.619}{1.04} = -1.5567 \end{bmatrix} = \begin{bmatrix} 23.528 & -11.764 \\ -11.764 & 23.528 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix}$$

Solving above equation,

$$\Delta\delta_2^{(0)} = -0.0023 \text{ rad}$$

$$\Delta\delta_3^{(0)} = -0.0673 \text{ rad}$$

$$\delta_2^{(1)} = 0 - 0.0023 = -0.0023 \text{ rad or } -0.13178°$$

$$\delta_3^{(1)} = 0 - 0.0673 = -0.0673 \text{ rad or } -3.856°$$

$$Q_2^{(0)} = \left(-1.04 \times 1 \times 12.13 \sin\left(104.04° + 0 + 0.0023 \times \frac{180}{\pi}\right) - (24.23)\sin(-75.95°)\right.$$

$$\left. -(1.04)(12.13 \sin\left(104.04 + 0.0023 \times \frac{180}{\pi} - 0.0673 \times \frac{180}{\pi}\right)\right)$$

$$= -12.231 + 23.50 - 12.411 = -1.14227 \text{ p.u.}$$

$$\Delta Q_2^{(0)} = 1 - (-1.14227) = -2.14227 \text{ p.u.}$$

$$-2.14227 = 23.528\left(\Delta V_2^{(0)}\right)$$

$$\Delta\left|V_2^0\right| = 0.091$$

$$\left|V_2^{(1)}\right| = \left|V_2^{(0)}\right| + \Delta\left|V_2^{(0)}\right| = 1 + 0.091 = 1.091 \text{ p.u.}$$

**Q.7 (b) Solution:**

Let 7500 kVA be the base kVA.

% Reactance of generator *A* on the base kVA

$$= 7 \times \frac{7500}{3000} = 17.5\%$$

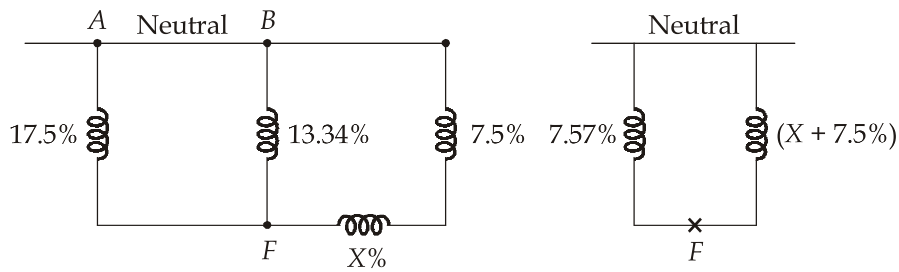
% reactance of generator *B* on the base kVA

$$= 8 \times \frac{7500}{4500} = 13.34\%$$

% Reactance of transformer on the base kVA

$$= 7.5 \times \frac{7500}{7500} = 7.5\%$$

Let the percentage reactance of the bus-bar reactor be $X$%. Then for 3-phase short-circuit fault on an outgoing feeder, the reactance diagram at the selected base kVA will be as shown in figure




Total % reactance from generator neutral to fault point $F$

$$= 7.57\% \parallel (X + 7.5)\%$$

$$= \frac{7.57(X+7.5)}{X+15.07}\%$$

$$\text{Short-circuit kVA} = 7500 \times 100 \times \frac{X+15.07}{7.57(X+7.5)}$$

But the short-circuit kVA should not exceed $150 \times 10^3$ kVA, the rupturing capacity of the breaker.

$$150 \times 10^3 = \frac{7500 \times 100 \times (X+15.07)}{7.57(X+7.5)}$$

$$7.57(X + 7.5) = 5(X + 15.07)$$

$$7.57X + 56.77 = 5X + 75.35$$

$$X = \frac{75.35 - 56.77}{7.57 - 5} = 7.23\%$$

The %age reactance can be converted into reactance in ohm by the following expression:

$$\%X = \frac{(\text{kVA})X}{10(\text{kV})^2}$$

$$7.23 = \frac{7500X}{10(3.3)^2}$$

$$X = \frac{7.23 \times 10 \times (3.3)^2}{7500} = 0.105\ \Omega$$

**Q.7 (c) Solution:**

**(i)** Given differential equation,

$$x''(t) - 2\,x'(t) + 4\,x(t) = u(t)$$

Taking Laplace transform,

$$s^2X(s) - sx(0^-) - x'(0^-) - 2[sX(s) - x(0^-)] + 4X(s) = \frac{1}{s}$$

$$X(s)[s^2 - 2s + 4] - 4 = \frac{1}{s}$$

$$X(s)[s^2 - 2s + 4] = \frac{1}{s} + 4 = \frac{(4s+1)}{s}$$

$$X(s) = \frac{(4s+1)}{s(s^2 - 2s + 4)}$$

Using partial fraction expansion,

$$X(s) = \frac{A}{s} + \frac{Bs + C}{(s^2 - 2s + 4)} \quad \text{...(i)}$$

$$A = \left.\frac{(4s+1)}{(s^2 - 2s + 4)}\right|_{s=0} = \frac{1}{4}$$

$$\therefore \quad A = \frac{1}{4}$$

and $$(4s + 1) = A(s^2 - 2s + 4) + (Bs + C)s$$

$$(4s + 1) = (A + B)s^2 + (C - 2A)s + 4A$$

Comparing coefficients of power of '$s$' on both sides, we get

$$A + B = 0 \quad , \quad C - 2A = 4$$

$$B = -A = -\frac{1}{4}, \quad C - \frac{2}{4} = 4$$

$$\therefore \quad B = -\frac{1}{4} \qquad C = 4 + \frac{1}{2}$$

$$\therefore C = \frac{9}{2}$$

Substituting all the values in equation (i), we get

$$X(s) = \frac{1}{4s} + \frac{\left(-\frac{1}{4}s + \frac{9}{2}\right)}{(s^2 - 2s + 4)}$$

$$X(s) = \frac{1}{4s} - \frac{s}{4(s^2 - 2s + 4)} + \frac{9}{2(s^2 - 2s + 4)}$$

$$= \frac{1}{4s} - \frac{s}{4[(s-1)^2 + (\sqrt{3})^2]} + \frac{9}{2[(s-1)^2 + (\sqrt{3})^2]}$$

$$= \frac{1}{4s} - \frac{(s-1)}{4[(s-1)^2 + (\sqrt{3})^2]} - \frac{1}{4[(s-1)^2 + (\sqrt{3})^2]} + \frac{9}{2[(s-1)^2 + (\sqrt{3})^2]}$$

$$= \frac{1}{4s} - \frac{(s-1)}{4[(s-1)^2 + (\sqrt{3})^2]} + \frac{17 \times \sqrt{3}}{4[(s-1)^2 + (\sqrt{3})^2](\sqrt{3})}$$

Taking inverse Laplace transform,

$$x(t) = \frac{1}{4}u(t) - \frac{1}{4}e^t \cos(\sqrt{3}t) + \frac{17}{4\sqrt{3}}e^t \sin(\sqrt{3}t);\ t \geq 0$$

$$x(t) = \frac{1}{4}\left[1 - e^t \cos(\sqrt{3}t) + \frac{17}{\sqrt{3}}e^t \sin(\sqrt{3}t)\right]u(t)$$

**(ii)** Given, frequency, $f = 2$ Hz

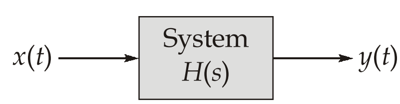
$\therefore$ $\omega = 2\pi f = 2\pi \times 2$

$= 4\pi$ rad/sec

Given input, $x(t) = \cos[\omega t]u(t) = \cos(4\pi t)u(t)$

Given transfer function,

$$H(s) = \frac{10}{(s+10)} \quad ..(i)$$




$$y(t) = |H(j\omega)|_{\omega=4\pi} \cos\left[4\pi t + \angle H(j\omega)|_{\omega=4\pi}\right] \quad ...(ii)$$

Now put, $s = j\omega$ in equation (i)

$$H(j\omega) = \frac{10}{(j\omega + 10)}$$

$$|H(j\omega)| = \frac{10}{\sqrt{\omega^2 + 10^2}}$$

$$|H(j\omega)|_{\omega=4\pi} = \frac{10}{\sqrt{(4\pi)^2 + 10^2}} = 0.623$$

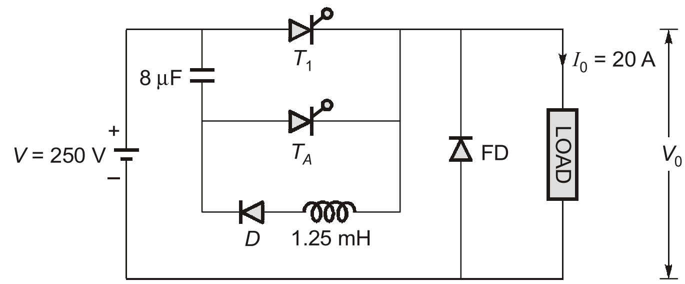
$$\angle H(j\omega)|_{\omega=4\pi} = -\tan^{-1}\left(\frac{\omega}{10}\right)\Big|_{\omega=4\pi} = -\tan^{-1}\left(\frac{4\pi}{10}\right) = -0.899 \text{ rad}$$

Substituting all the values in equation (ii), we get

$$y(t) = 0.623 \cos(4\pi t - 0.899); \quad t > 0$$

**Q.8 (a) (i) Solution:**

Voltage commutated chopper circuit,




Commutation circuit turnoff time,

$$t_c = \frac{CV_s}{I_0} = \frac{8 \times 10^{-6} \times 250}{20} = 1 \times 10^{-4} \text{ s}$$

The minimum on period for this chopper is,

$$t_1 = \frac{\pi}{\omega_0} = \pi\sqrt{LC}$$

$$= \pi\sqrt{1.25 \times 10^{-3} \times 8 \times 10^{-6}} = 3.1416 \times 10^{-4} \text{ s}$$

The minimum average output voltage of the circuit is,

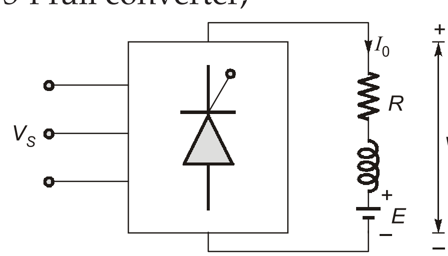
$$V_{0,\,min} = fV_s[t_1 + 2t_c]$$

$$= 250 \times 250\,[3.1416 \times 10^{-4} + 2 \times 10^{-4}]$$

$$V_{0,\,min} = 32.135 \text{ V}$$

**Q.8 (a) (ii) Solution:**

Average output voltage of 3-f full converter,




3-ϕ full converter

$$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha = E + I_0 R$$

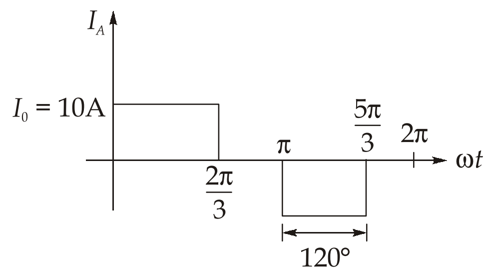
$$\frac{3 \times \sqrt{2} \times 220}{\pi}\cos\alpha = 110 + (10 \times 0.2)$$

$$297.1043 \cos\alpha = 112$$

$$\alpha = \cos^{-1}\left(\frac{112}{297.1043}\right)$$

Firing angle, $\alpha = 67.85°$

SCR conducts for 120°.




For constant load current, $I_0 = 10$ A,

Supply current $i_A$ is of square wave of amplitude 10 A. As $i_A$ flows for 120° over every half cycle of 180°, the rms supply current, $I_s$ is ,

$$I_S = 10\sqrt{\frac{120°}{180°}} = 8.1649 \text{ A}$$

$$\text{Power delivered to the load} = P_0 = V_0 I_0$$

$$= \left[\frac{3\times\sqrt{2}\times 220}{\pi}\cos(67.85°)\right]\times 10 = 1120.18 \text{ W}$$

$$\text{Input supply power factor} = \frac{V_0 I_0}{\sqrt{3}V_s I_s} = \frac{1120.18}{\sqrt{3}\times 220\times 8.1649} = 0.36 \text{ (lag)}$$

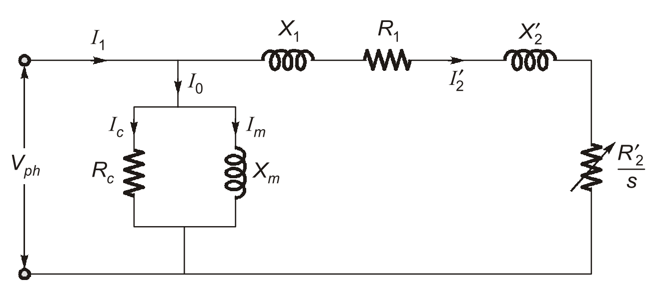
or input supply power factor for 3-ϕ full converter

$$= \frac{3}{\pi}\cos\alpha = \frac{3}{\pi}\cos(67.85°) = 0.36 \text{ (lag)}$$

**Q.8 (b) Solution:**

Given, The phase voltage, $V_{ph} = \frac{440}{\sqrt{3}} = 254$ V

and slip, $s = 0.025$




Equivalent per phase circuit diagram of induction motor

From equivalent circuit diagram, the rotor per phase current referred to stator side is

$$\bar{I}_2' = \frac{V_{ph}}{R_1 + \left(\frac{R_2'}{s}\right) + j(X_1 + X_2')}$$

$$\bar{I}_2' = \frac{254}{0.25 + \left(\frac{0.25}{0.025}\right) + j(0.75 + 0.75)}$$

$$= \frac{254}{10.25 + j1.50} = 24.52\angle -8.33° \text{ A}$$

$$\bar{I}_2' = 24.52\angle{-8.33°} = (24.26 - j3.55) \text{ A}$$

The no-load current is

$$I_c = \frac{254}{100} = 2.54 \text{ A}$$

$$I_m = \frac{254}{j1000} = -j0.254 \text{ A}$$

$$\bar{I}_0 = 2.54 - j0.254 \text{ A}$$

The input line current is,

$$\bar{I}_1 = \bar{I}_0 + I_2'$$

$$= (2.54 - j0.254) + (24.26 - j3.55)$$

$$= 26.8 - j3.804 = 27.07\angle{-8.1°} \text{ A}$$

The power factor is $\cos 8.01° = 0.99$

Synchronous speed can be given as

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{6} = 1000 \text{ RPM}$$

The rotational speed is, $$\omega_s = \frac{2\pi N_s}{60} = \frac{2\pi \times 1000}{60} = 104.72 \text{ rad/s}$$

The developed torque is, $$T = \frac{P_g}{\omega_s} = \frac{3I_2'^2 R_2'}{s\omega_s} = \frac{3 \times (24.52)^2 \times 0.25}{0.025 \times 104.72} = 172.24 \text{ Nm}$$

The power output is, $$P_0 = P_{mech} - \text{Rotational loss}$$

$$= (1 - s)\frac{3I_2'^2 R_2'}{s} - \text{Rotational loss}$$

$$= (1 - 0.025)\frac{3 \times (24.52)^2 \times 0.25}{0.025} - 450 = 17136 \text{ W}$$

$$\text{Rotor copper loss} = 3I_2'^2 \times R_2'$$

$$= 3 \times (24.52)^2 \times 0.25 = 450.92 \text{ W}$$

The efficiency can be given as

$$\eta = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{\text{Output}}{\text{Output} + \Sigma \text{ Losses}} \times 100$$

$$= \frac{17136 \times 100}{17136 + 800 + 450 + 450.92} = 90.97\%$$

**Q.8 (c) Solution:**

**(i)** Given:

Open loop transfer function,

$$G(s)H(s) = \frac{8s}{(s-1)(s-2)}$$

Open loop poles : $s = 1$, $s = 2$

There are two open loop poles on the right half of $s$-plane i.e., $P = 2$.

There are no open loop poles on the $j\omega$-axis. So, the Nyquist contour includes $j\omega$ axis.

Put $s = j\omega$,

$$G(j\omega)H(j\omega) = \frac{j8\omega}{(j\omega-1)(j\omega-2)}$$

$$|G(j\omega)H(j\omega)| = \frac{8\omega}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$\angle G(j\omega)H(j\omega) = 90° - (180° - \tan^{-1}(\omega)) - \left(180° - \tan^{-1}\left(\frac{\omega}{2}\right)\right)$$

$$= 90° + \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$G(j\omega)H(j\omega) = \frac{8\omega}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}} \angle 90° + \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right)$$

| $\omega$ | $\lvert G(j\omega)H(j\omega)\rvert \angle G(j\omega)H(j\omega)$ |
|---|---|
| 0 | $0 \angle 90°$ |
| 0.1 | $0.3975 \angle 98.57°$ |
| 1 | $2.53 \angle 161.56°$ |
| 1.5 | $2.662 \angle 183.18°$ |
| 2 | $2.53 \angle 198.44°$ |
| 10 | $0.78 \angle 252.98°$ |
| $\infty$ | $0 \angle 270°$ |

Intercept with real-axis will occur at ω where,

$$\angle G(j\omega)H(j\omega) = -180°$$

$$-180° = 90° + \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right)$$

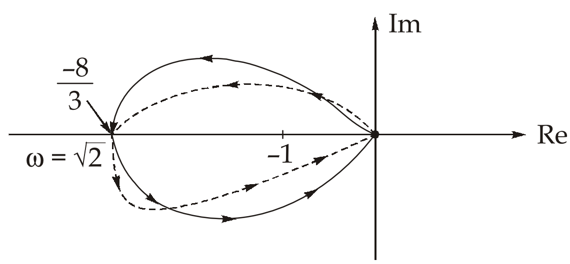
$$-90° = \tan^{-1}\left(\frac{\omega + \frac{\omega}{2}}{1 - \frac{\omega^2}{2}}\right)$$

$$1 - \frac{\omega^2}{2} = 0$$

$\Rightarrow \quad \omega = \sqrt{2}$ rad/sec

$$\therefore \quad G(j\omega)H(j\omega)\big|_{\omega = \sqrt{2}} = \frac{8}{3}\angle 180°$$

Nyquist plot,




There are two encirclements of (–1, 0) in anti-clockwise.

i.e., $N = +2$

We know, $N = P - Z \quad \Rightarrow \quad Z = P - N$

$Z = 2 - 2 = 0$

Hence, there are no closed loop poles on right side.

So, the system is stable.

**(ii)** Given, open loop transfer function,

$$G(s)H(s) = \frac{8s}{(s-1)(s-2)}$$

Characteristic equation,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{8s}{(s-1)(s-2)} = 0$$

$$s^2 - 3s + 2 + 8s = 0$$

$$s^2 + 5s + 2 = 0$$

**R-H Table:**

$$\begin{array}{c|cc} s^2 & 1 & 2 \\ s^1 & 5 & \\ s^0 & 2 & \end{array}$$

Since all the coefficients of 1st column of R-H table is positive. So, the system is stable.

❍❍❍❍