MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

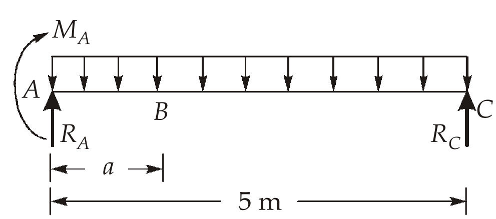
ESE-2026 Mains Test Series

Mechanical Engineering Test No : 11

Section : A

1. (a) Solution:

Drawing FBD



As there is hinge at B, $M_B = 0$

Considering BC,

$$M_B = w(5-a)\times\frac{(5-a)}{2} - R_c\times(5-a)$$

$$\Rightarrow \quad 0 = \frac{w(5-a)}{2} - R_c$$

$$\Rightarrow \quad R_c = \frac{20(5-a)}{2}\text{kN}$$

$$= (50 - 10a)\text{ kN} \quad \text{...(1)}$$

Considering AB,

$$M_B = M_A + R_A a - wa\times\frac{a}{2}$$

$$\Rightarrow \qquad 0 = M_A + R_A a - 10a^2 \qquad ...(2)$$

Considering vertical forces,

$$R_A + R_C = w \times 5 = 100 \text{ kN}$$

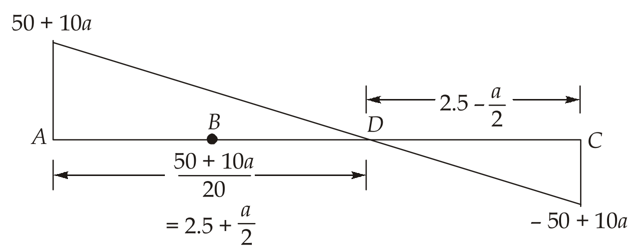
$$R_A = 100 - 50 + 10a \qquad \text{(from eq. (1))}$$

$$= 50 + 10a$$

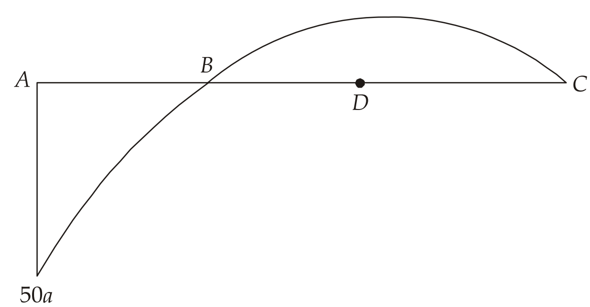
From eq. (2)

$$M_A = 10a^2 - 50a - 10a^2 = -50a$$

Drawing SFD



Drawing BMD



(i) Maximum moment will be either at A or D. For minimum magnitude of maximum bending moment we can equate:

$$|M_A| = |M_D| \qquad ...(3)$$

$$M_D = R_C \times \left(2.5 - \frac{a}{2}\right) - \frac{w\left(2.5 - \frac{a}{2}\right)^2}{2}$$

$$= (50 - 10a)\frac{(5-a)}{2} - \frac{20}{2}\frac{(5-a)^2}{4}$$

$$= (25 - 5a)(5 - a) - 2.5(5 - a)^2$$

$$= 125 - 25a - 25a + 5a^2 - 2.5(25 + a^2 - 10a)$$

$$= 125 - 50a + 5a^2 - 62.5 - 2.5a^2 + 25a$$

$$= 2.5a^2 - 25a + 62.5$$

from eq. (3)

$$50a = 2.5a^2 - 25a + 62.5$$

$$\Rightarrow \quad 2.5a^2 - 75a + 62.5 = 0$$

$$a = 29.142, 0.858$$

So, for minimum bending moment, $a = 0.858$ m

So, $$M_{max} = 50a = 42.90 \text{ kNm}$$

(ii) $$\sigma_{max} = \frac{My}{I}$$

$$= \frac{42.90\times10^3\times\frac{40}{2}\times10^{-3}}{\frac{1}{12}(20)(40)^3\times10^{-12}} = 8.044 \text{ GPa}$$

1. (b) Solution:

Given data : m = 80 kg; k = 130 mm; N = 250 rpm; α = 18°; θ = 45°

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times250}{60} = 26.179 \text{ rad/s}$$

$$\text{Maximum acceleration} = \frac{-\omega_1^2\cos\alpha\sin^2\alpha\sin2\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

$$\alpha = \frac{-26.179^2\times\cos18°\times\sin^2 18°\times\sin90°}{(1-\sin^2 18°\cos^2 45°)^2}$$

$$\alpha = -68.639 \text{ rad/s}^2$$

–ve sign indicates that it is retardation at the instant.

Torque required for retardation of the driven shaft = $I\alpha = mk^2\alpha$

$$= 80 \times 0.13^2 \times (-68.639)$$

$$= -92.80 \text{ Nm}$$

Total torque required on the driven shaft,

$$T_2 = \text{Steady torque + Accelerating torque}$$

$$= 180 + (-92.80)$$

$$= 87.199 \text{ Nm}$$

Now, as $$P = T_1\omega_1 = T_2\omega_2$$

$$T_1 = T_2\frac{\omega_2}{\omega_1} = T_2\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$$

$$T_1 = 87.199 \times \frac{\cos 18°}{1 - \sin^2 18° \cos^2 45°} = 87.09 \text{ Nm}$$

$$\text{Maximum variation} = \frac{1 - \cos^2 \alpha}{\cos \alpha} = \frac{\omega_{2max} - \omega_{2min}}{\omega_{mean}}$$

$$= \frac{1 - \cos^2 \alpha}{\cos \alpha} = \frac{16}{250} = 0.064$$

$$1 - \cos^2\alpha - 0.064 \cos\alpha = 0$$

$$\cos^2\alpha + 0.064 \cos\alpha - 1 = 0$$

$$\cos\alpha = 0.9685, -1.0325$$

$$\alpha = 14.419°$$

Ans.

1. (c) Solution:

Given : $P = 100$ kN, $\tau = 120$ N/mm^2

Let 't' be the throat thickness of weld.

$$\text{Primary shear stress, } \tau_1 = \frac{P}{A} = \frac{100 \times 10^3}{2 \times 400 \times t}$$

or

$$\tau_1 = \frac{125}{t} \text{N/mm}^2$$

$$\text{Bending stress, } \sigma_b = \frac{M_b y}{I},$$

The moment of inertia of two welds about the x-axis is given by

$$I = 2 \times \left[\frac{t(400)^3}{12}\right] = 10.66 \times 10^6 \, t \text{ mm}^4$$

$$\therefore \quad \sigma_b = \frac{(100 \times 10^3 \times 300) \times 200}{10.66 \times 10^6 \times t} = \frac{562.85}{t} \text{N/mm}^2$$

Now, maximum shear stress in the weld,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_1^2} = \sqrt{\left(\frac{562.85}{2t}\right)^2 + \left(\frac{125}{t}\right)^2}$$

$$\tau_{max} = \frac{307.93}{t} \text{N/mm}^2$$

$\therefore$ For size of weld, (h),

$$(\tau_{max})_{ind} \le \tau_{per}$$

$$\frac{307.93}{t} \leq 120$$

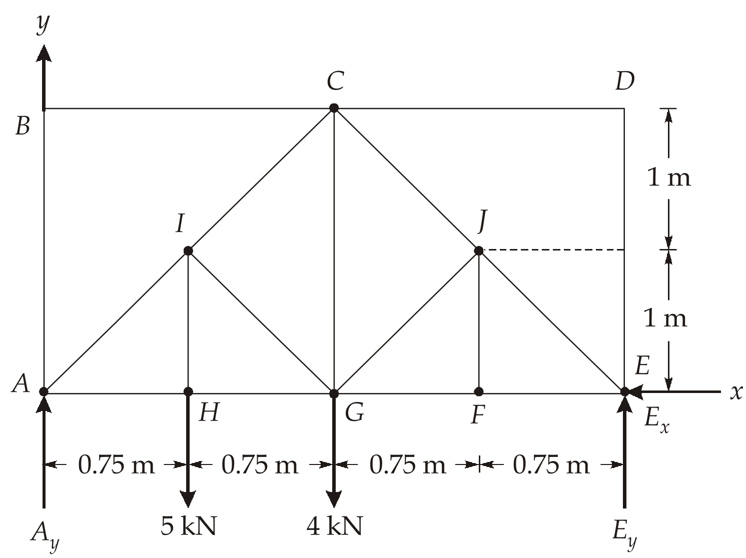
$\Rightarrow$ $t \geq 2.56$ mm

$\therefore$ $h = \dfrac{2.56}{0.707}$

$\Rightarrow$ $h = 3.62$ mm **Ans.**

1. (d) Solution:

Free body diagram:



Equilibrium equation for reactions,

$\Sigma F_y = 0$

$\Rightarrow$ $A_y + E_y = 9$ kN

$\Sigma M_A = 0$

$\Rightarrow$ $5 \times 0.75 + 4 \times 1.5 = E_y \times 3$

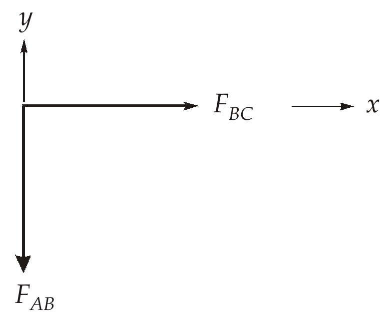
$\Rightarrow$ $E_y = 3.25$ kN

$\therefore$ $A_y = 5.75$ kN

$\Sigma F_x = 0$

$\Rightarrow$ $E_x = 0$

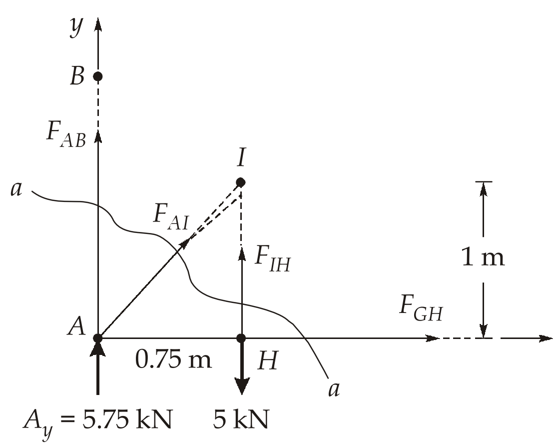
Joint B



$\Sigma F_y = 0$

$\Rightarrow \quad F_{AB} = 0 \quad$ ...(i)

By the method of section, taking a section aa through members AB, AI, IH, and GH.



$\cos\theta = \frac{1}{1.25} = \frac{4}{5}$

Assumed all members in tension,

Equilibrium equation,

$\Sigma M_A = 0$

$\Rightarrow \quad F_{IH} \times 0.75 - 5 \times 0.75 = 0$

$F_{IH} = 5 \text{ kN (Tensile)}$ **Ans.**

$\Sigma F_y = 0$

$\Rightarrow \quad F_{AB} + F_{AI}\cos\theta + A_y + F_{IH} = 5$

$\therefore$ From equation (i), $F_{AB} = 0$

We get, $\quad F_{AI} \times \frac{4}{5} + 5.75 + 5 = 5$

$\Rightarrow \quad F_{AI} = -7.1875 \text{ kN}$

or $\quad F_{AI} = 7.1875 \text{ kN (compressive)}$

Force of GH

$\Sigma F_x = 0$

$\Rightarrow \quad F_{AI} \sin\theta = F_{AH}$

$$F_{AH} = 7.1875 \times \frac{3}{5}$$

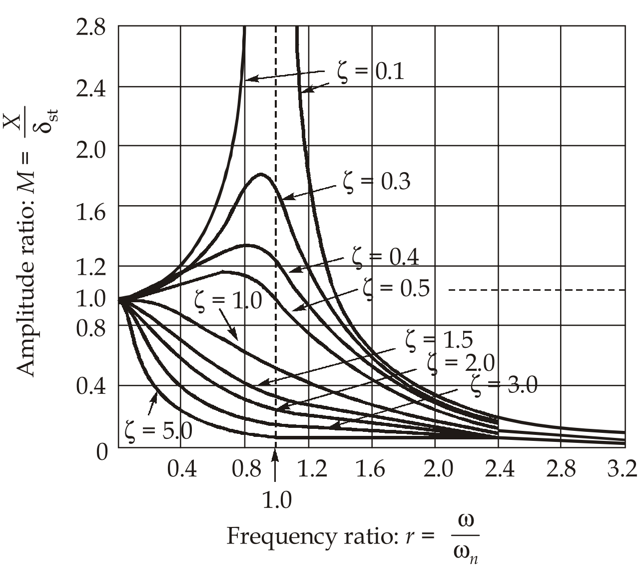
$$F_{AH} = 4.3125 \text{ kN (tensile)}$$

$\Rightarrow \quad F_{GH} = 4.3125 \text{ kN (tensile)}$

1. (e) Solution:

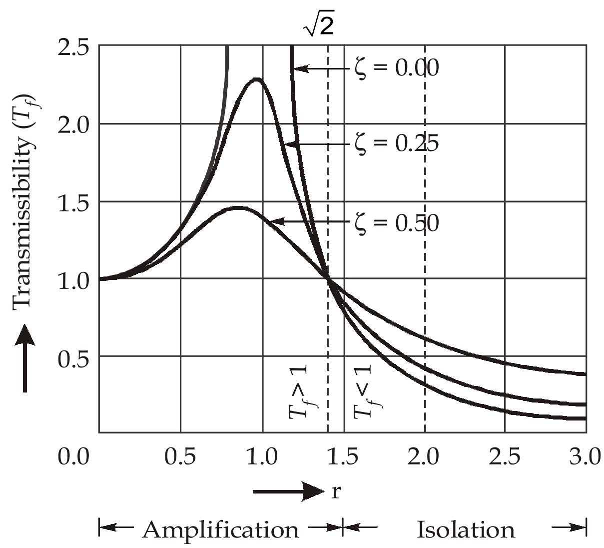
Magnification factor : The ratio of amplitude of the steady state response to the static deflection under the action of force F_0 is known as the magnification factor (MF).

$$\text{MF} = \frac{1}{\sqrt{\left(\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$



Transmissibility : It is defined as the ratio of force transmitted (to the foundation) to force applied. It is measure of the effectiveness of vibration isolating material.

$$\in \; = \; \frac{f_1}{f_0} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

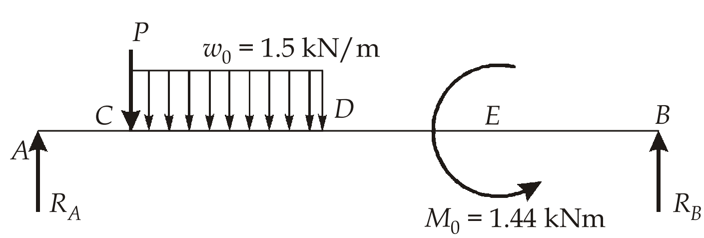


Variation of Transmissibility Ratio with frequency ratio

Transmissibility is 1 at a frequency ratio of $\frac{\omega}{\omega_n} = 0$ (static condition) because the applied force is transmitted directly to the foundation without amplification or attenuation by the dynamic components (spring/damper).

2. (a) Solution:

FBD:



Taking moment about B,

$$R_A \times 3.6 - 1.2 \times 3 - 1.5 \times 1.2 \times 2.4 - 1.44 = 0$$

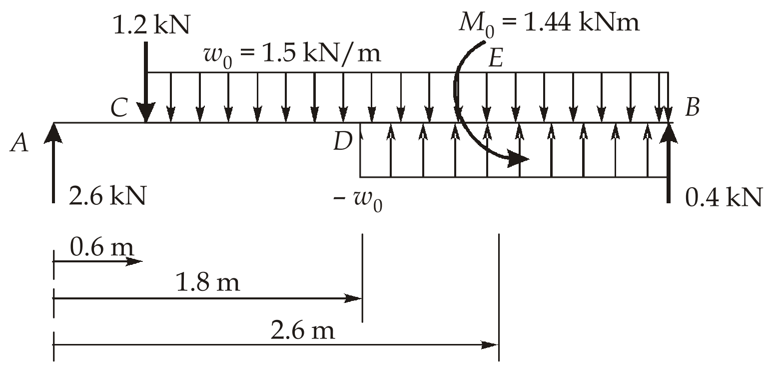
$$\Rightarrow \qquad R_A = 2.6 \text{ kN}$$

Taking moment about A,

$-1.2 \times 0.6 - 1.5 \times 1.2 \times 1.2 + 1.44 + R_B \times 3.6 = 0$

$$R_B = 0.4 \text{ kN}$$

Using singularity functions



$$M(x) = 2.6x - 1.2(x - 0.6) - \frac{1.5(x-0.6)^2}{2} + \frac{1.5(x-1.8)^2}{2} - 1.44(x-2.6)^0$$

$$EI\frac{d^2y}{dx^2} = 2.6x - 1.2(x - 0.6) - 0.75(x - 0.6)^2 + 0.75(x - 1.8)^2 - 1.44(x - 2.6)^0$$

$$EI\frac{dy}{dx} = \frac{2.6x^2}{2} - \frac{1.2(x-0.6)^2}{2} - \frac{0.75}{3}(x-0.6)^3 + \frac{0.75}{3}(x-1.8)^3 - 1.44(x - 2.6) + C_1$$

$$EIy = \frac{1.3x^3}{3} - \frac{0.6(x-0.6)^3}{3} - \frac{0.25(x-0.6)^4}{4} + \frac{0.25(x-1.8)^4}{4} - \frac{1.44(x-2.6)^2}{2} + C_1x + C_2$$

at $x = 0$, $y = 0$

$x = 3.6$ m; $y = 0$

So, $0 = 0 + 0 + C_2 \Rightarrow C_2 = 0$

and $0 = \frac{1.3\times3.6^3}{3} - \frac{0.6(3)^3}{3} - \frac{0.25\times3^4}{4} + \frac{0.25\times1.8^4}{4} - \frac{1.44\times1^2}{2} + C_1\times3.6$

$$C_1 = -2.692$$

$$\Rightarrow \left(\frac{200\times10^9\times6.87\times10^{-6}}{10^3}\right)y = \frac{1.3x^3}{3} - 0.2(x-0.6)^3 - \frac{0.25(x-0.6)^4}{4}$$

$$+\frac{0.25(x-1.8)^4}{4} - 0.72(x - 2.6)^2 - 2.692x$$

For deflection at D, $x = 1.8$ m

So, $$y_D = \frac{-2.7936}{1374} = -2.03 \text{ mm}$$

2. (b) Solution:

Given : $a_w = 0.84$, Pressure angle, $\phi = \cos^{-1}(0.94)$

$$\phi = 19.948°$$

(i) For gear ratio one

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$= \frac{2 \times 0.84}{\sqrt{1+\frac{1}{1}\left(\frac{1}{1}+2\right)\sin^2 19.948} - 1} = 10.399 \simeq 11 \text{ teeth}$$

$\because$ $$G = 1;\ T = t = 11$$

$$r = R = \frac{mT}{2} = \frac{11\,m}{2} = 5.5\,m$$

$$r_a = R_a = R + a_w m = 5.5\,m + 0.84m = 6.34\,m$$

Path of approach = Path of recess

$$= \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$$

$$= \sqrt{(6.34m)^2 - (5.5m\cos 19.948°)^2} - 5.5m\sin 19.948°$$

$$= 1.793\,m$$

Path of contact = $2 \times 1.793\,m$

$$= 3.586\,m$$

$$\text{Length of arc of contact (AoC)} = \frac{\text{Path of contact (PoC)}}{\cos\phi}$$

$$\text{AoC} = \frac{3.586m}{\cos 19.948°} = 3.815\,m$$

(ii) For gear ratio four ($G = 4$)

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$T = \frac{2 \times 0.84}{\sqrt{1 + \frac{1}{4}\left(\frac{1}{4} + 2\right)\sin^2 19.948^\circ} - 1}$$

$$T = 52.146 \simeq 53$$

$$t = \frac{T}{G} = \frac{53}{4} = 13.25 \simeq 14$$

So, $T = 4 \times 14 = 56$

$$R = \frac{mT}{2} = \frac{56m}{2} = 28\ m$$

$$R_a = R + a_w$$
$$= 28\ m + 0.84\ m = 28.84\ m$$

$$r = \frac{mt}{2} = \frac{14m}{2} = 7\ m$$

$$r_a = 7m + 0.84m = 7.84\ \text{m}$$

$$\text{Path of contact} = \sqrt{(28.84m)^2 - (28m\cos 19.948^\circ)^2} - 28m\sin 19.948^\circ$$
$$+\sqrt{(7.84m)^2 - (7m\cos 19.948^\circ)^2} - 7m\sin 19.948^\circ$$
$$= 2.237\ m + 1.874\ m$$
$$= 4.111\ m$$

$$\text{AoC} = \frac{\text{PoC}}{\cos\phi} = \frac{4.111m}{\cos 19.948^\circ} = 4.373m$$

(iii) For pinion with a rack

$$t \geq \frac{2a_r}{\sin^2\phi}$$

$$t \geq \frac{2 \times 0.84}{\sin^2 19.948^\circ}$$

$$t \geq 14.433 \simeq 15$$

$$r = \frac{mt}{2} = \frac{15m}{2} = 7.5\ m$$

$$r_a = 7.5\ m + 0.84\ m = 8.34\ m$$

As addendum of pinion and rack are equal,

$$\text{PoC} = \sqrt{(r_a)^2 - (r^2\cos^2\phi} + \frac{a_r}{\sin\phi} - r\sin\phi = 4.36\ m$$

$$= 3.793\ \text{m}$$

$$\text{AoC} = \frac{PoC}{\cos\phi} = \frac{4.36m}{\cos 19.948°} = 4.637\ m$$

2. (c) Solution:

Given : $N = 1500$ rpm, $\mu = 0.2$, $R_m = 2b$, $\alpha = 12.5°$, $p_{\text{per}} = 0.1\ \text{N/mm}^2$, $k = 0.25$ m, $m = 200$ kg; $t = 50$ sec

Initially, $\omega_1 = 0$

$$\omega_2 = \frac{2\pi N}{60} = \frac{2\pi \times 1500}{60} = 157.08\ \text{rad/s}$$

or $\omega_2 = \omega_1 + \alpha t$

$$\therefore \quad \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{157.08 - 0}{50}$$

$$\alpha = 3.14\ \text{rad/s}^2$$

Resisting torque, $T = I\alpha$

$$T = 200 \times 0.25^2 \times 3.14$$

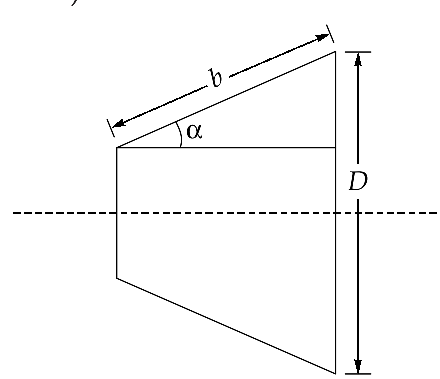
$$= 39.26\ \text{Nm}$$

From uniform wear theory,

$$T = \frac{\pi\mu P_{per} d(D^2 - d^2)}{8\sin\alpha}$$

$$\Rightarrow \quad 39.26 \times 10^3 = \frac{\pi \times 0.2 \times 0.1 \times d \times (D^2 - d^2)}{8\sin(12.5)}$$

$$\Rightarrow \quad d(D^2 - d^2) = 1082.198 \times 10^3 \quad \text{...(i)}$$



$$\sin\alpha = \frac{D-d}{2b}$$

$\Rightarrow$ $$D - d = 2b\sin\alpha \quad \text{...(ii)}$$

Also mean radius = 2 × face width

$$\frac{D+d}{4} = 2b \quad \text{...(iii)}$$

From equation (ii) and (iii),

$$\frac{D+d}{D-d} = \frac{8b}{2b\sin\alpha} = \frac{4}{\sin\alpha}$$

or $$\frac{D}{d} = \frac{4+\sin\alpha}{4-\sin\alpha} = \frac{4+\sin(12.5)}{4-\sin(12.5)}$$

$\therefore$ $$\frac{D}{d} = 1.1144$$

$\therefore$ From equation (i),

$$d[(1.1144d)^2 - d^2] = 1082.198 \times 10^3$$

or $$0.2418d^3 = 1082.198 \times 10^3$$

$\therefore$ $$d = 164.8 \text{ mm}$$ Ans. (i)

and $$D = 1.1144d = 1.1144 \times 164.8$$

$\therefore$ $$D = 183.65 \text{ mm}$$ Ans. (i)

$$\text{Face width, } b = \frac{D+d}{8} = \frac{183.65+164.8}{8} = 43.55 \text{ mm}$$ Ans. (ii)

Force required to engage clutch,

$$F = \frac{4T\sin\alpha}{\mu(D+d)} = \frac{4\times 39.26\times 10^3 \times \sin(12.5)}{0.2(183.65+164.8)}$$

$$F = 487.6 \text{ N}$$ Ans. (iii)

Now, $$\omega_{avg} = \frac{\omega_1+\omega_2}{2} = \frac{157.08}{2} = 78.54 \text{ rad/s}$$

$$\text{Angle turned, } \theta = \omega_{avg} \times t$$
$$= 78.54 \times 50$$
$$= 3927 \text{ rad}$$

$\therefore$ Heat generated during one engagement = Work done by frictional engagement torque

$$H_g = T\theta$$

$= 39.26 \times 3927$

$= 154.17$ kJ Ans. (iv)

3. (a) Solution:

Given: $k'_c = 2k'_b$, $P_i = 12$ kN, P = 7 kN, $\sigma_y = 400$ MPa, $A = 95$ mm^2,

(i) We know that, net force on the bolt is

$$P_b = P_i + P\left(\frac{k'_b}{k'_b + k'_c}\right)$$

or $$P_b = 12 \times 10^3 + 7 \times 10^3 \times \frac{1}{3} \quad [\because k'_c = 2k'_b]$$

$$P_b = 14.33 \times 10^3 \text{ N}$$

Stress on the bolt $$\sigma = \frac{P_b}{A} = \frac{14.333 \times 10^3}{95} = 150.877 \text{ MPa}$$

So, factor of safety (N), $$N = \frac{\sigma_y}{\sigma} = \frac{400}{150.877} = 2.6511$$ **Ans.**

(ii) Test for leak proof joint, $P_i > P(1 - k)$

Where, $$k = \frac{k'_b}{k'_b + k'_c} = \frac{1}{3}$$

$$12 > \left[7\left(1 - \frac{1}{3}\right) = 4.66\right]$$

So, as the condition is satisfied, the joint is leak proof. **Ans.**

(iii) Test for separation,

(a) 1st condition, $P_i > k_p$

$$12 > \left[\frac{1}{3} \times 7 = 2.33\right] \quad \text{(Satisfied)}$$

(b) 2nd condition, $P_i < A \times \sigma_y$

$$P_i < \left(\frac{95 \times 400}{1000} = 38\right) \quad \text{(satisfied)}$$ **Ans.**

Hence, as both the conditions are satisfied, the joint separation does not takes place.

3. (b) (i) Solution:

The following assumptions are made in the Lame's theory:

1. The material of the cylinder is homogeneous, isotropic and obeys Hooke's law. [The stresses are within proportionality limit]

2. Plane transverse sections of the cylinder remain plane under the action of pressure. This assumption is nearly true at a considerable distance from the ends of the cylinder. As a consequence of this assumption, the longitudinal strain remains constant at all points in the cylinder wall, i.e. it is independent of the cylinder radius.
3. All the fibres of the materials are stressed independently without being constrained by the adjacent fibres.

3. (b) (ii) Solution:

Let us use suffix '1' for the shaft and '2' for the collar. Due to radial pressure 'P' at the junction, the shaft will have compressive stress throughout.

Let σ_{h_i} be the hoop stress at the junction.

$$\therefore \quad \sigma_{h_i} = \frac{P(R^2 + r^2)}{R^2 - r^2}$$

$$\sigma_{h_i} = \frac{P(150^2 + 75^2)}{150^2 - 75^2}$$

$$\sigma_{h_i} = 1.666P \qquad \text{...(i)}$$

For steel collar,

$$\in_2 = \frac{1}{E}\left(\sigma_{h_i} + \mu P\right)$$

or

$$\frac{0.2}{150} = \frac{1}{2 \times 10^5}(1.666P + 0.3P)$$

$$\therefore \quad P = \frac{2 \times 10^5 \times 0.2}{150 \times 1.96} = 135.64 \text{ MPa}$$

$\therefore$ The radial pressure between the collar and the shaft,

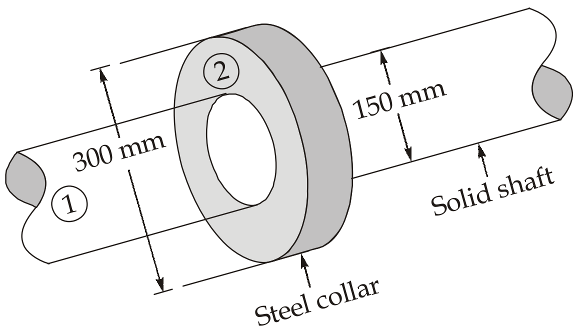
$$\sigma_r = -P = -135.64 \text{ MPa}$$

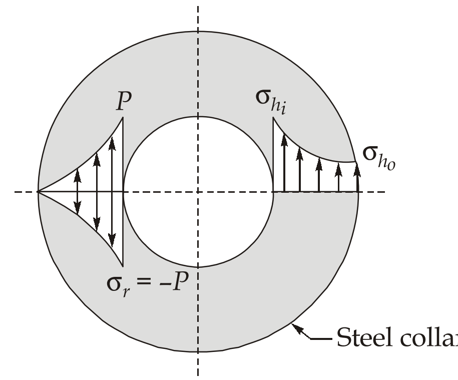
$$= 135.64 \text{ MPa (Compressive)} \qquad \textbf{Ans.}$$

From equation (i),

$$\sigma_{h_i} = 1.666 \times 135.65$$

$$= 225.97 \text{ MPa (Tension)} \qquad \textbf{Ans.}$$





For shaft, circumferential strain,

$$\epsilon_1 = \frac{\delta d_1}{d_1} = \frac{P}{E} - \mu\frac{P}{E}$$

$$\frac{\delta d_1}{d_1} = \frac{P}{E}(1-\mu)$$

$$\therefore \qquad \delta d_1 = \frac{135.64 \times 150}{2 \times 10^5}(1-0.3)$$

$$\delta d_1 = 0.0712 \text{ mm}$$ **Ans**

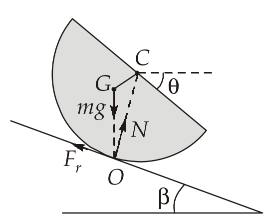
3. (c) (i) Solution:

1.

If the given hemisphere is in equilibrium then moment about any point will be zero.

For hemisphere, $CG = \frac{3r}{8}$

Drawing FBD of hemisphere.



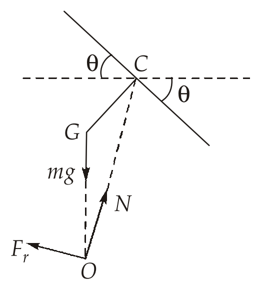
please note that mg will pass through point of contact otherwise hemisphere will rotate about it and normal reaction will pass through center of hemisphere as it is also normal to hemispherical surface.

Balancing forces in two perpendicular directions

$$mg\cos\beta = \text{N}$$

$$mg\sin\beta = F_r$$

Now, taking moment about C



$$M_c = mgCG\cos(90 - \theta) - F_r OC + N \times 0$$

$\Rightarrow$ $$0 = mg \times \frac{3r}{8}\sin\theta - mgr\sin\beta + 0$$

$\Rightarrow$ $$\sin\beta = \frac{3\sin\theta}{8}$$

$\Rightarrow$ $$(\sin\beta)_{max} = \frac{3\times 1}{8}$$

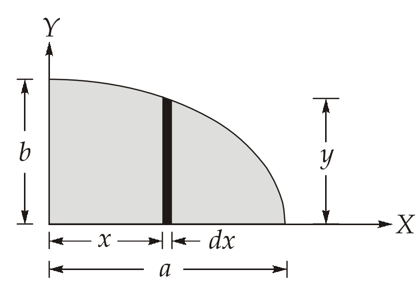
$\Rightarrow$ $$\beta_{max} = 22.024°$$

2. If $\beta = \dfrac{\beta_{max}}{2} = 11.012°$

then $$\sin\theta = \frac{8}{3}\sin\beta$$

$$\theta = 30.622°$$

3. (c) (ii) Solution:



From the equation of the ellipse, we get the expression for y as

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

Consider a thin strip of infinitesimal thickness dx parallel to the Y-axis at a distance x from the origin. Its area dA is then given as

$$dA = ydx = \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

Hence, the area of the quadrant of the ellipse is obtained by integrating the infinitesimal area between the limits:

$$A = \int_0^a dA = \int_0^a ydx$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

Upon integration by parts, we get

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a = \frac{\pi ab}{4}$$

Taking the first moment of the infinitesimal area about the Y-axis, we have

$$dM_y = xdA = x\frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

Hence, the first moment of the entire area about the Y-axis is obtained as

$$M_y = \int_0^a xdA = \frac{b}{a}\int_0^a x\sqrt{a^2 - x^2}\,dx$$

Let, $a^2 - x^2 = t$

$$dt = -2xdx$$

$$= \frac{b}{-2a}\int \sqrt{t}\,dt$$

$$= \frac{-b}{2a} \times \frac{1}{3/2}\left[\left(a^2 - x^2\right)^{3/2}\right]_0^a$$

$$= -\frac{b}{a}\left[\frac{\left(a^2 - x^2\right)^{3/2}}{3}\right]_0^a = \frac{a^2 b}{3}$$

Therefore, the *x*-coordinate of the centroid of the quadrant of the ellipse is given as

$$\bar{x} = \frac{M_y}{A} = \frac{\frac{a^2 b}{3}}{\frac{\pi ab}{4}} = \frac{4a}{3\pi}$$

In a similar manner, we can consider a thin horizontal strip and obtain the y-coordinate of the centroid is

$$\bar{y} = \frac{4b}{3\pi}$$

4. (a) Solution:

Given: Internal diameter, $d = 100$ mm

Thickness of tube, $t = 6$ mm

Internal pressure, $P = 8\ \text{N/mm}^2$

Torque applied, $T = 2.1 \times 10^6$ N-mm

Let, σ_h = Hoop stress

σ_l = Longitudinal stress

Now,
$$\sigma_h = \frac{Pd}{2t} = \frac{8 \times 100}{2 \times 6} = 66.67\ \text{N/mm}^2$$

$$\sigma_l = \frac{Pd}{4t} = \frac{8 \times 100}{4 \times 6} = 33.33\ \text{N/mm}^2$$

Now, the stresses σ_h and σ_l are tensile stresses. But the cylindrical vessel is also subjected to torque. Due to torque, shear stress will be produced in the tube. As the thickness of the wall of tube is small, the shear stress is assumed to be uniform throughout the thickness.

Now, Shear force = Shear stress × Area

$$= \tau \times \pi dt = \tau \times \pi \times 100 \times 6 = 600\ \pi\tau\ \text{N}$$

and
$$\text{Torque, } T = \text{Shear force} \times \frac{d}{2}$$

$$= 600\pi\tau \times \frac{100}{2} = 30000\pi\tau\ \text{N-mm}$$

But torque of 2.1×10^6 N.mm is given, therefore,

$$2.1 \times 10^6 = 30000\ \pi\tau$$

$$\tau = 22.28\ \text{N/mm}^2$$

Hence, the material is subjected to two tensile stresses (i.e. $\sigma_1 = 66.67\ \text{N/mm}^2$ and $\sigma_2 = 33.33\ \text{N/mm}^2$) accompanied by a shear stress ($\tau = 22.28\ \text{N/mm}^2$).

Now,Maximum principal stress = $\dfrac{\sigma_1+\sigma_2}{2}+\sqrt{\left(\dfrac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$

$$= \frac{66.67+33.33}{2}+\sqrt{\left(\frac{66.67-33.33}{2}\right)^2+(22.28)^2}$$

$$= 50+\sqrt{774.287} = 50+27.826$$

= 77.83 MPa (Tensile) **Ans.**

Now, Minimum principal stress = $\dfrac{\sigma_1+\sigma_2}{2}-\sqrt{\left(\dfrac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$

$$= \frac{66.67+33.33}{2}-\sqrt{\left(\frac{66.67-33.33}{2}\right)^2+(22.28)^2}$$

$$= 50-\sqrt{774.287} = 50-27.826$$

= 22.17 MPa (Tensile) **Ans.**

Now, Maximum shear stress = $\left|\dfrac{\text{Max. principal stress} - \text{Min. principal stress}}{2}\right|$

$$= \left|\frac{77.83-22.17}{2}\right| = 27.83 \text{ MPa}$$ **Ans.**

4. (b) Solution:

Given : m = 0.5 kg; M = 1.1 kg; r_1 = 85 mm; N_1 = 215 rpm, N_2 = 235 rpm

Let, P be the force in N exerted at the sleeve due to spring action.

Then total load on the sleeve,

$$Mg = P + 1.1 \times 9.81$$
$$= (P + 10.791) \text{ N}$$

when r = 85 mm, N = 215 rpm

$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi\times 215}{60} = 22.514 \text{ rad/s}$$

$$a = \sqrt{110^2 - (85-35)^2} = 97.98 \text{ mm}$$

$$\tan\theta = \frac{85-35}{97.98} = 0.5103$$

Now,
$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg}{2}(1+k)\right]$$

$\because$ $k = 1$

$$mr\omega^2 = \tan\theta[mg + Mg] \quad \text{...(i)}$$

$$0.5 \times 0.085 \times (22.514)^2 = 0.5103 \times [0.5 \times 9.81 + (P + 10.791)]$$

$$P = 26.52 \text{ N}$$

Taking moment about O.

$$S_1 \times 55 = P \times 135$$

$$S_1 = 26.52 \times \frac{135}{55}$$

$$S_1 = 65.09 \text{ N}$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 235}{60} = 24.609 \text{ rad/s}$$

As sleeve raised by 12 mm, height AQ is shortened by 12 mm.

$$a = 97.98 - \frac{12}{2} = 91.98 \text{ mm}$$

$$r = \sqrt{110^2 - 91.98^2} + 35 = 95.33 \text{ mm}$$

$$\tan\theta = \frac{60.329}{91.98} = 0.656$$

Using equation (i)

$$0.5 \times 0.09533 \times 24.609^2 = 0.656 \times (0.5 \times 9.81 + (P + 10.791))$$

$$P = 28.31 \text{ N}$$

Taking moments about O,

$$S_2 \times 55 = P \times 135$$

$$S_2 = 28.31 \times \frac{135}{55} = 69.488 \text{ N}$$

$$\text{Spring displacement, } \delta = 12 \times \frac{55}{135} = 4.89 \text{ mm}$$

$$\text{Spring stiffness} = \frac{S_2 - S_1}{\delta} = \frac{69.488 - 65.09}{4.89}$$

$$\text{Spring stiffness} = 0.899 \text{ N/mm}$$

Tension in the upper links when the sleeve begins to rise (at 215 rpm)

$$\text{Reaction at } Q = \text{Total downward force}$$

$$= \frac{1}{2}(P + \text{Weight of sleeve} + \text{Weight of balls})$$

$$= \frac{1}{2}(26.52 + 10.791 + 0.5 \times 9.81)$$

$$= 21.109 \text{ N}$$

Let, T be the tension in the upper arm,

then $T\cos\theta = 21.109$ N

$$T \times \frac{97.98}{110} = 21.109$$

$$T = 23.69 \text{ N}$$

4. (c) Solution:

Power transmits = 7.5 kW

Speed, N = 360 rpm

Diameter of pulley 1 = 250 mm

Diameter of pulley 2 = 500 mm

Mass of pulley 1 = 10 kg

Mass of pulley 2 = 30 kg

Tension ratio of tight side to slack side = 2.5

(σ_{yt}) = 380 N/mm^2

FOS = 3

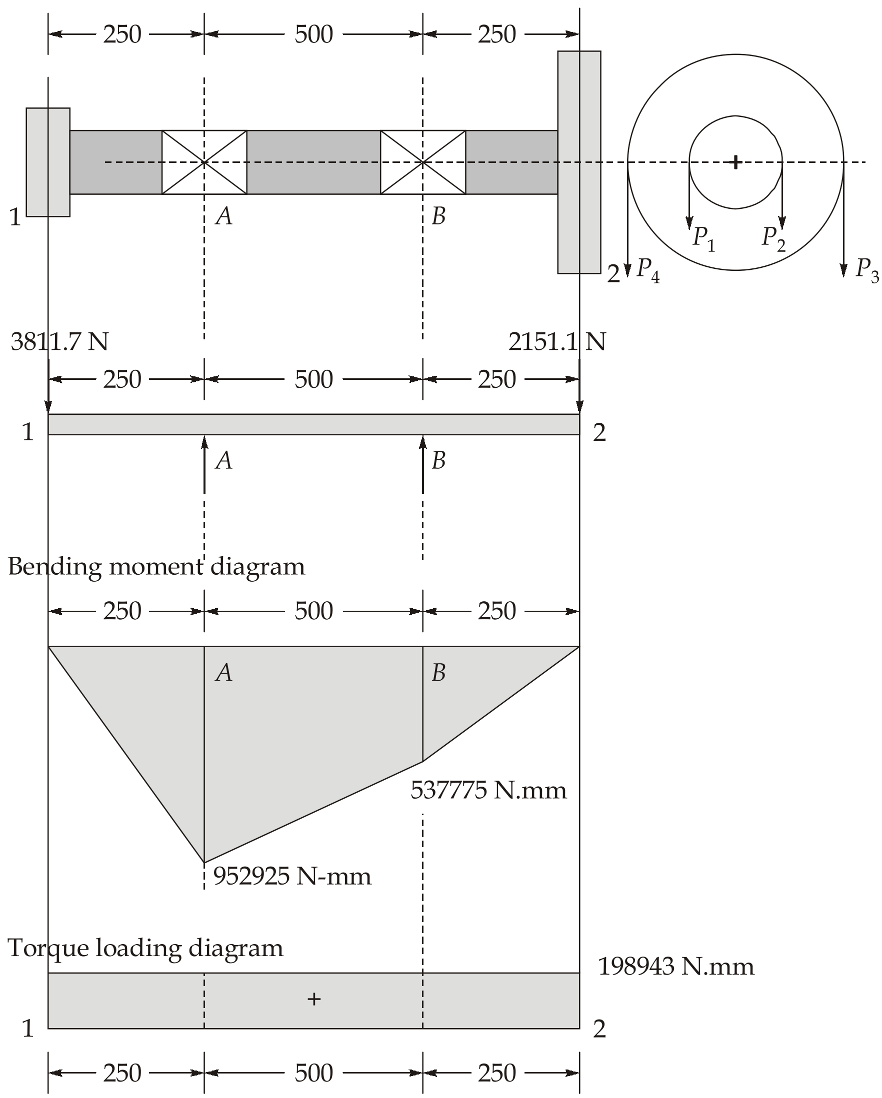
Suitable diameter, d = ?

G = 79300 N/mm^2

Permissible angle of twist, θ = 0.5°/m

$$\text{Torsional moment, } T = \frac{60 \times 10^3 \times 7.5}{2\pi \times 360}$$

$= 198.943$ N-m $= 198943$ N-mm



$$(P_1 - P_2) \times 125 = 198943 \text{ N.mm}$$

$$(P_1 - P_2) = \frac{198943}{125} = 1591.544 \text{ N}$$

$$\frac{P_1}{P_2} = 2.5$$

$$P_1 = 2.5\,P_2$$
$$(2.5\,P_2 - P_2) = 1591.544 \text{ N}$$
$$P_2 = 1061.03 \text{ N}$$
$$P_1 = 2652.57 \text{ N}$$

The weight of the pulley 1 is given by

$$W_1 = 10 \times 9.81 = 98.1 \text{ N}$$

The total downward force acting at the centre line of pulley,

$$(P_1 + P_2 + W_1) = 2652.57 + 1061.03 + 98.1$$
$$(P_1 + P_2 + W_1) = 3811.7 \text{ N}$$

The bending moment at bearing 'A' is given by

$$(M_B)_A = 3811.7 \times 250 = 952925 \text{ N.mm}$$

For pulley 2

$$(P_3 - P_4) \times 250 = 198943$$
$$(P_3 - P_4) = 795.772$$
$$\frac{P_3}{P_4} = 2.5$$
$$2.5\,P_4 - P_4 = 795.772$$
$$P_4 = \frac{795.772}{1.5} = 530.514 \text{ N}$$
$$P_3 = 1326.285 \text{ N}$$

The weight of the pulley 2 is given by

$$W_2 = m_2 g = 30 \times 9.81 = 294.3 \text{ N}$$

The total downward force acting at the centreline of pulley '2'

$$P_3 + P_4 + W_2 = 1326.285 + 530.514 + 294.3$$
$$= 2151.1 \text{ N}$$

Bending moment at bearing 'B' is given by

$$(M_B) \text{ at } B = 2151.1 \times 250 = 537775 \text{ N.mm}$$

Critical point at support 'A'

Equivalent torque (T_e) at point 'A'

$$T_e = \sqrt{(M_B)_A^2 + (T)_A^2}$$
$$T_e = \sqrt{(952925)^2 + (198943)^2}$$

$$T_e = 973470.27 \text{ N.mm}$$

'For safe design' strength criteria

$$\text{Induced stress} \leq \tau_{permissible}$$

$$\frac{16T_e}{\pi d^3} \leq \frac{\sigma_{yt}}{2.FOS}$$

$$\frac{16 \times 973470.27}{\pi d^3} \leq \frac{380}{2 \times 3}$$

$$d \geq \sqrt[3]{\frac{6 \times 16 \times 973470.27}{\pi \times 380}}$$

$$d \geq 42.78 \text{ mm}$$ **Ans.**

From rigidity criteria, $\theta_{induced} \leq \theta_{permissible}$

$$\frac{TL}{GJ} \leq \theta_{permissibles}$$

$$\frac{32 \times 198943 \times 1000}{\pi d^4 \times 79300} \leq \frac{0.5 \times \pi}{180}$$

$$d \geq \sqrt[4]{\frac{180 \times 32 \times 198943 \times 1000}{\pi \times 79300 \times 0.5 \times \pi}}$$

$$d \geq 41.3668 \text{ mm}$$ **Ans.**

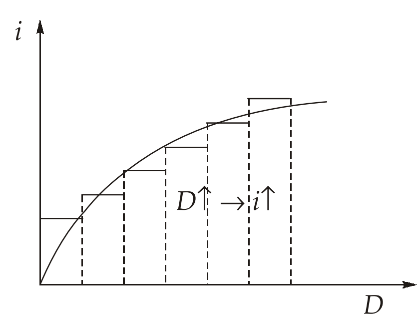
Hence best dimensions, $d = 42.78$ mm

Section : B

5. (a) Solution:

Fundamental tolerance (i)

$$i = 0.45\sqrt[3]{D} + (0.001)D$$



where; i = Fundamental tolerance (in μm)

D = Basic size (in mm)

$$D = \text{GM}(D_{min}, D_{max})$$

$$= \sqrt{D_{min} \times D_{max}} = \sqrt{18 \times 30}$$

$$\simeq 23.2379 \text{ mm}$$

$$i = 0.45\sqrt[3]{D} + \frac{D}{1000}$$

$$= 0.45\sqrt[3]{23.2379} + \frac{23.2379}{1000}$$

$$i = 1.307375 \text{ µm}$$

$$\text{Tolerance of hole} = 25i = 32.684375 \text{ µm}$$

$$\text{Tolerance on plug-gauge} = 0.003268 \text{ mm}$$

$$\text{Tolerance of shaft} = 40i = 52.295 \text{ µm}$$

$$\text{Tolerance on ring-gauge} = 0.0052295 \text{ mm}$$

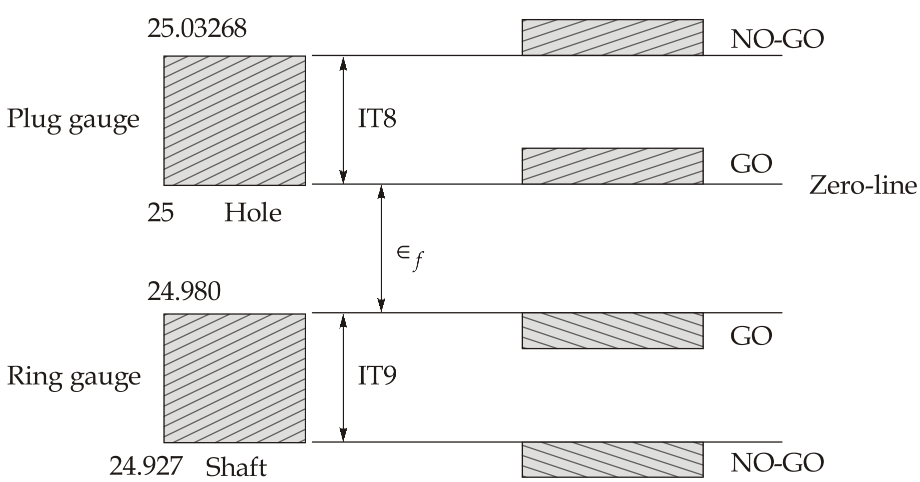
Fundamental deviation of 'f'-type shaft is

$$\in_f = -5.48d^{0.41}$$

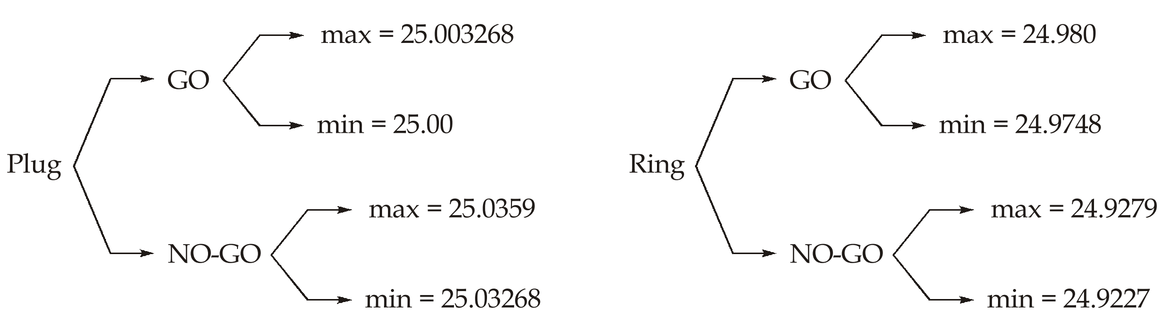
$$= -5.48(23.2379)^{0.41}$$

$$= -19.9031 \text{ µm}$$

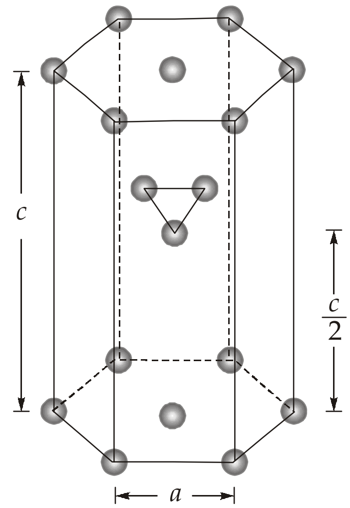
$$= -0.0199031 \text{ mm}$$



Since, work tolerance is less than 0.1 mm so no wear allowance will be given on GO-gauges.



5. (b) Solution:



HCP-unit cell

Atomic radius = r

$$\text{Atomic packing fraction} = \frac{V_{\text{atoms}}}{V_{\text{cell}}}$$

Now, total number of atom per unit cell,

$$n = 1 \times (3) + \frac{1}{6}(12) + \frac{1}{2}(2) = 6 \text{ atoms/unit cell}$$

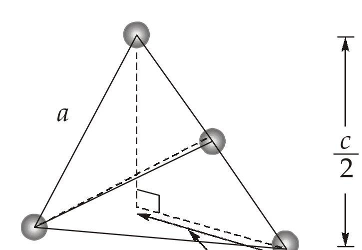
$$V_{\text{atoms}} = 6 \times \frac{4}{3}\pi r^3 = 8\pi r^3 = \pi a^3$$

$$V_{\text{cell}} = (\text{Area of base})(\text{Height of cell})$$

$$V_{\text{cell}} = \left(6 \times \frac{\sqrt{3}}{4}a^2\right)(c) = \frac{3\sqrt{3}}{2}a^2 c$$

$$\text{APF} = \frac{2\pi a^3}{3\sqrt{3}a^2 c} = \frac{2\pi}{3\sqrt{3}}\left(\frac{a}{c}\right)$$

c/a ratio



$$\frac{c}{2} = \sqrt{a^2 - \left(\frac{2}{3}\frac{\sqrt{3}}{2}a\right)^2}$$

$$= \sqrt{\left(1 - \frac{1}{3}\right)a^2} = a\sqrt{\frac{2}{3}}$$

$$\frac{c}{a} = \sqrt{\frac{8}{3}} = 1.63299$$

Ans.

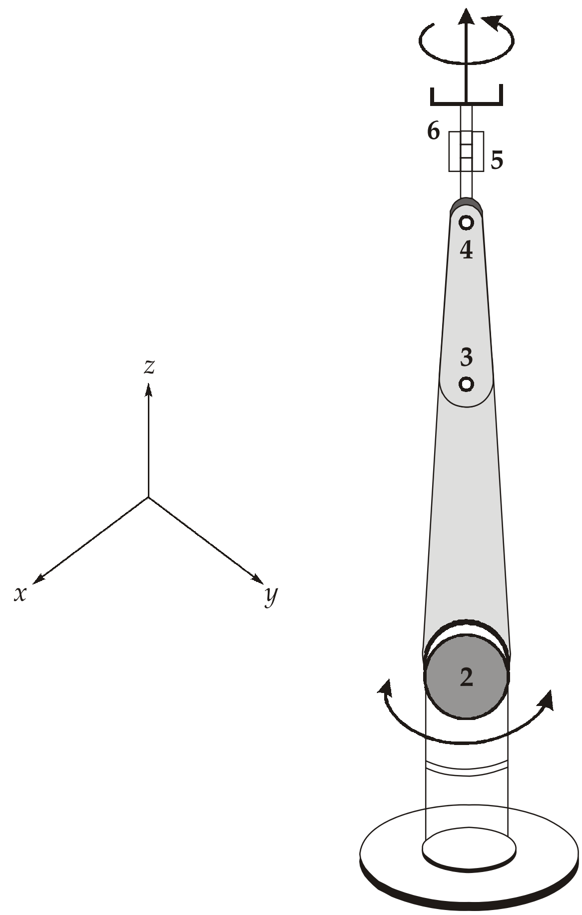
Now, APF,

$$\text{APF} = \frac{2\pi}{3\sqrt{3}}\left(\sqrt{\frac{3}{8}}\right) = \frac{2\pi}{3\sqrt{3}}\frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\pi}{3\sqrt{2}} \simeq 0.74048 \text{ i.e. } 74.048\% \qquad \textbf{Ans.}$$

5. (c) **Solution:**

(a) **Degeneracy:** Degeneracy occurs when the robot loses a degree of freedom, and therefore cannot perform as desired. This occurs under two conditions: (1) when the robot's joints reach their physical limits and as a result, cannot move any further; (2) a robot may become degenerate in the middle of its workspace if the z-axes of two similar joints become collinear. This means that, at this instant, whichever joint moves, the same motion will result, and consequently, the controller does not know which joint to move. Since in either case the total number of degrees of freedom available is less then six, there is no solution for the robot. In the case of collinear joints, the determinant of the position matrix is zero as well, show a simple robot in a vertical configuration, where joints 1 and 6 are collinear. As you can see, whether joint 1or joint 6 rotate, the end effector will rotate the same amount. In practice, it is important to direct the controller to take an emergency action; otherwise the robot will stop. Please note that this condition occurs if the two joints are similar. Otherwise, if one joint is prismatic and one is revolute (as in joints 3 and 4of the Stanford arm), although the z-axes are collinear, the robot will not be in degenerate condition. The robot will be degenerate when joints 4 and 5, or 5 and 6 are parallel, and therefore, result in similar motions.



Robot in a degenerate position

(b) **Dexterity:** We should be able to position and orientate 6-DOF robot at any desired location within its work envelope by specifying the position and the orientation of the hand. However, as the robot gets increasingly closer to the limits of its workspace, it will get to a point where, although its is possible to locate it at a desired point, it will be impossible to orientate it at desired orientation. The volume of points where we can position the robot as desired but not orientate it is called nondexterous volume.

5. **(d) Solution:**

Given data : D = 10000 units per year; c = Rs. 70 per unit; c_o = Rs. 150 per order;

c_h = (0.3 × c) Rs. per unit per year

Case I :

$$\text{EOQ} = Q^* = \sqrt{\frac{2Dc_0}{c_h}} = \sqrt{\frac{2 \times 10000 \times 150}{0.3 \times 70}}$$

$$Q^* = 377.9645 \text{ units per order} \simeq 378 \text{ units per order}$$

Case II : Price-break

Lot Size	Unit Price
1 - 200	Rs. 62
201 - 500	Rs. 60
501 and above	Rs. 55

D = 10000; c_o = Rs.150; c_h = 0.3c

Starting for lowest unit price

Search for feasibly EOQ and compare with costs at all the price-breaks.

$$c = \text{Rs.}55;\ c_h = 0.3 \times 55 = \text{Rs.}16.5$$

$$Q^* = \sqrt{\frac{2Dc_0}{c_h}} = \sqrt{\frac{2 \times 10000 \times 150}{16.5}} = 426.4$$

Now, c = 60, c_h = 18

$\Rightarrow$ $$Q^* = 408.2083 \text{ units per order}$$

$$\text{Feasibility check : } Q^* \in (201 - 500) \text{ lot-range}$$

$\therefore$ Feasible.

Now, calculating cost at EOQ and price-breaks after EOQ.

$$\text{Total cost} = DC + \left(\frac{Q}{2}\right)c_h + \left(\frac{D}{Q}\right)c_0$$

Total cost at feasible EOQ

$$TC(Q^*) = DC + \left(\frac{Q^*}{2}\right)c_h + \left(\frac{D}{Q^*}\right)c_0$$

$$= 10000 \times 60 + \left(\frac{501}{2}\right) \times 18 + \left(\frac{10000}{501}\right) \times 150$$

$$= \text{Rs.}557127.26$$

TC at next price-break i.e. (Q = 501)

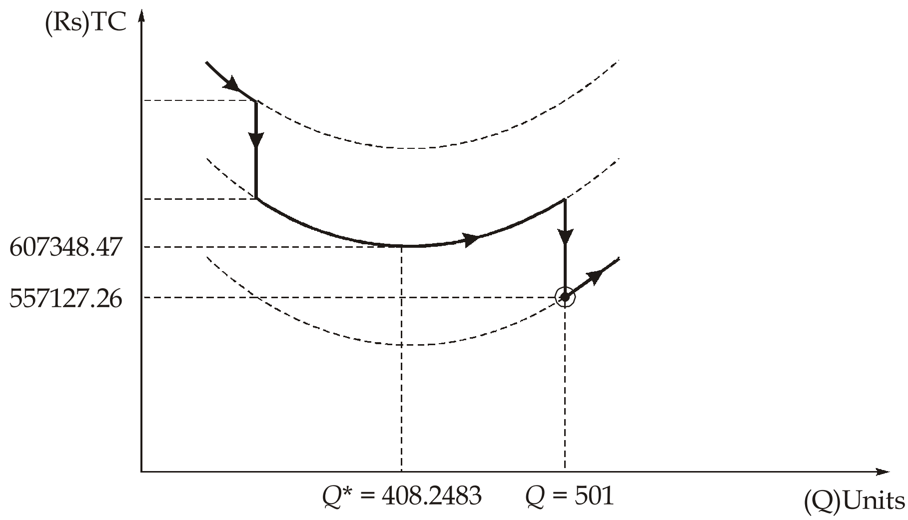
$$\text{TC}(Q = 501) = D \times C + \left(\frac{Q}{2}\right)c_h + \left(\frac{D}{Q}\right)c_0$$

$$= 10000 \times 55 + \left[\frac{501}{2}\right] \times 16.5 + \left[\frac{10000}{501}\right] \times 150$$

$$= \text{Rs.}557127.26$$

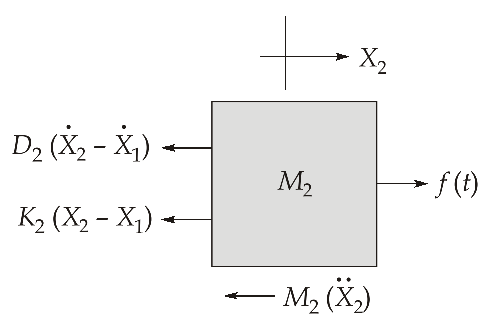
Since; $\quad TC(Q = 501) < TC(Q = Q^*)$

Hence Q = 501 is the most economical order quantity with a price of Rs.557127.26



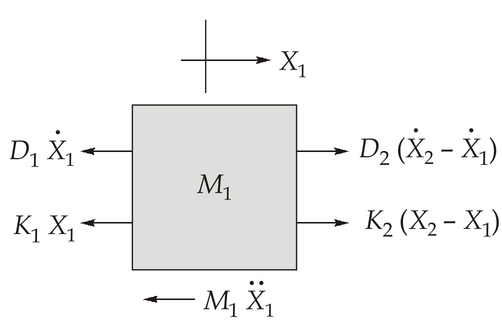
5. **(e) Solution:**

For Mass M_2



$$f(t) = M_2\ddot{X}_2 + D_2\left(\dot{X}_2 - \dot{X}_1\right) + K_2\left(X_2 - X_1\right) \quad \text{... (i)}$$

For Mass M_1



$$M_1\ddot{X}_1 + D_1\dot{X}_1 + K_1X_1 - D_2\left(\dot{X}_2 - \dot{X}_1\right) - K_2\left(X_2 - X_1\right) = 0 \quad \text{... (ii)}$$

Taking Laplace of equation (i)

$$[M_2s^2 + A_2s + K_2]X_2(s) = [K_2 + D_2s]X_1(s) + F(s) \quad \text{... (iii)}$$

Taking Laplace of equation (ii)

$$[M_1s^2 + (D_1 + D_2)s + (K_1 + K_2)]X_1(s) = [D_2s + K_2]X_2(s) \quad \text{... (iv)}$$

Substituting the value of X_2(s) from equation (iii) in equation (iv), we can relate X_1(s) and $F(s)$ as

$$\left[\frac{\{M_1s^2 + (D_1 + D_2) + (K_1 + K_2)\}(M_2s^2 + D_1s + K_2) - (K_2 + D_2s)^2}{K_2 + D_2s}\right]X_1(s) = F(s)$$

6. (a) (i) Solution:

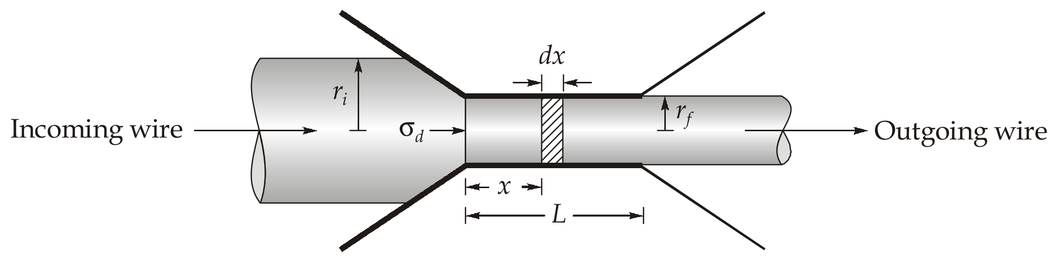
Extrusion defects

1. **Surface Cracking**
   - If extrusion temperature, friction or speed is too high, surface temperatures rise significantly and this condition may cause surface cracking and tearing.
   - Surface cracking may also occur at lower temperatures because of periodic sticking of extruded product along die-land; because of this welding (sticking), extrusion pressure increases and causes cracking.
2. **Pipe defect:** The unusual type of the metal flow pattern tends to draw surface impurities towards the centre of the billet, this defect is called pipe defect, and tailpipe or fishtailing. This can be minimized by modifying the flow pattern to more uniform. These impurities can also be removed by etching of surface oxides before extrusion. Etching will help in cleaning the surface of billet.
3. **Internal Cracking (Chevron Cracking)**
   - Extruded product centre can develop various cracks called Centre - cracking, Centre - burst, arrowhead fracture or chevron cracking. These cracks are attributed to a state of hydrostatic tensile stresses at the Centre-line in the deformation zone in the die.
   - Centre - cracking increases with increase in die-angle, increase in amount of impurities and increase in extrusion ratio of friction.

6. (a) (ii) Solution:

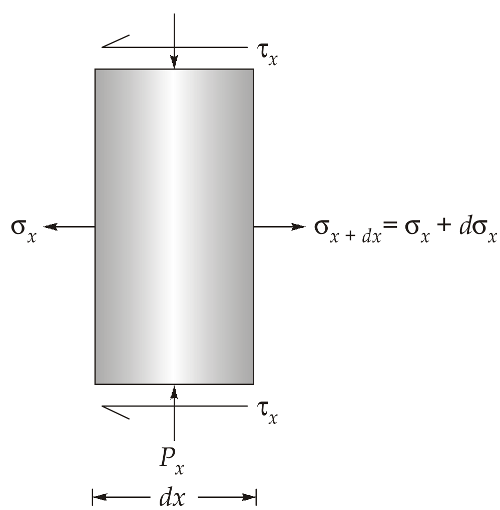
Given data : $r_i = 3.5$ mm; $r_f = 3$ mm; $L = 2$ mm; $\sigma_0 = 25$ MPa, $\alpha = 8.2°$, $\mu = 0.05$

$$B = \mu\cot\alpha = (0.05)\cot(8.2°) = 0.346976$$



Die land

Equilibrium of forces in horizontal direction



$$(\sigma_x + d\sigma_x)\pi r_f^2 - \sigma_x \pi r_f^2 - \tau_x 2\pi r_f \cdot dx = 0$$

So, $$d\sigma_x = \frac{2\tau_x}{r_f}dx = \frac{2\mu P_x}{r_f}dx \qquad [\text{Put } \tau_x = \mu P_x]$$

Considering σ_x and P_x and also due to spherical coordinate system both Von-Mises and Tresca will lead to same result.

$$\sigma_x + P_x = \sigma_0$$

$$P_x = \sigma_0 - \sigma_x$$

$$\int \frac{d\sigma_x}{\sigma_0 - \sigma_x} = \int \frac{2\mu}{r_f}dx$$

$$\ln(\sigma_0 - \sigma_x) = \frac{-2\mu}{r_f}x + c_1$$

Boundary condition at : $x = 0, \sigma_x = \sigma_d$ where

$$\sigma_d = \sigma_0\left(\frac{1+B}{B}\right)\left[1-\left(\frac{r_f}{r_i}\right)^{2B}\right]$$

So; $$\sigma_x = \sigma_0 - (\sigma_0 - \sigma_d)e^{-\frac{2\mu x}{r_f}}$$

At $x = L, \sigma_x = \sigma_t$

$B = \mu\cot\alpha = 0.05 \times \cot 8.2° = 0.346976\text{MPa}$

$$\sigma_d = 25\left(\frac{1+0.346976}{0.346976}\right)\left(1-\left(\frac{3}{3.5}\right)^{2\times 0.346976}\right)$$

$$\sigma_d = 9.845 \text{ MPa}$$

$$\sigma_t = 25-(25-9.845)e^{-\frac{2\times 0.05\times 2}{3}}$$

$$= 10.823 \text{ MPa}$$

$$\text{Pull required} = \sigma_t \cdot A_f$$

$$= \left(10.823\times 10^6\right)\left(\pi\left(\frac{3}{1000}\right)^2\right)$$

$$= 305.995 \text{ N}$$ **Ans.**

6. **(b) Solution:**

Group technology: Group technology (GT) is a methodology that seeks to take advantage of the design and processing similarities among the parts to be produced. GT suggests that major benefits can be obtained by classifying and coding the parts into families. One company found that by dis-assembling each product into its individual components and then identifying the similar parts, 90% of the parts fell into very few major families.

A pump, for example, can be broken down into its basic components, such as the motor, housing, shaft, flanges and seals. Each of these components is basically the same in terms of its design and manufacturing characteristics, consequently, all shafts, for example, can be placed in one family of shafts. Group technology becomes especially attractive because of the ever-growing variety of products, which are often produced in batches. About 75% products are manufactured by batch production.

The group technology approach becomes even more attractive because of demand of an ever larger variety of products, each in smaller quantities, thus involving batch production. Maintaining high efficiency in batch production is difficult, so overall manufacturing efficiency is compromised by a reduction in production volume. In GT same type of machines are arranged in same groups: group of lathes, milling machines, drill presses and grinders.

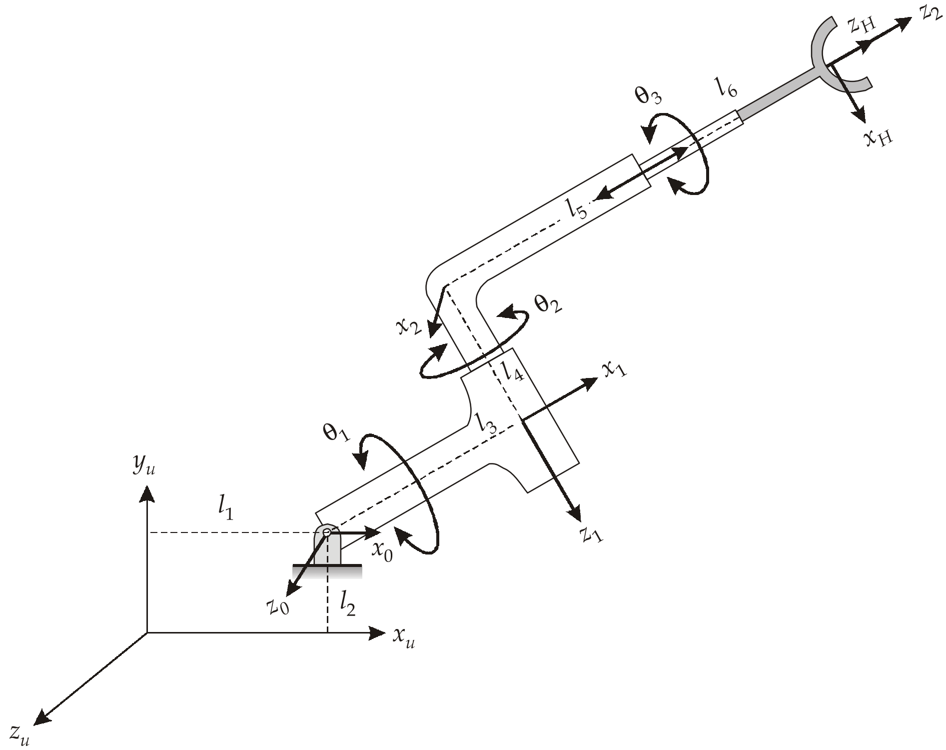
The extreme case of customer driven variety can result in batches of one product and is referred as mass customization or mass production. It is typical case of additive manufacturing, mass production requires effective enterprise resource planning (ERP), and a fairly high level of automation, generally involving manufacturing cells.

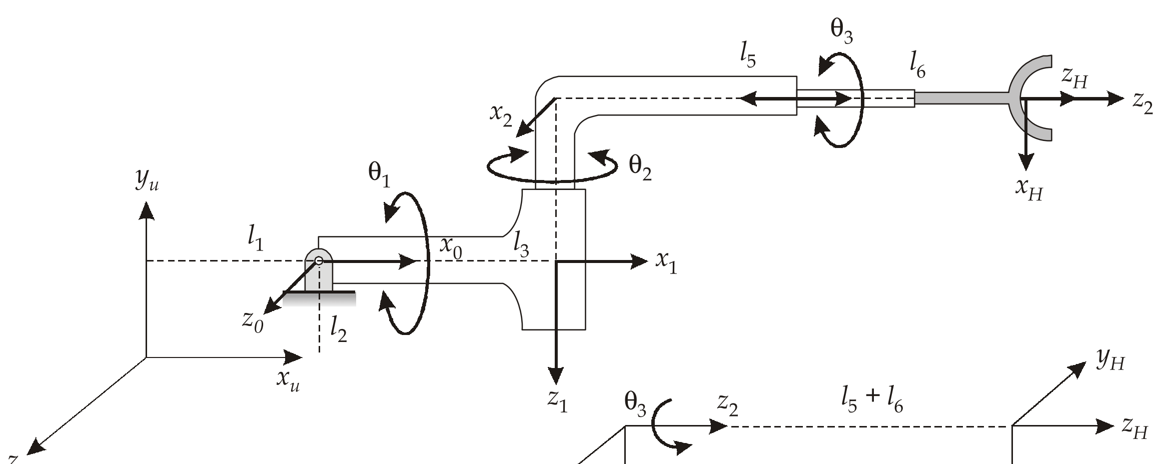
The machines in cellular manufacturing are arranged in an efficient product flow line, called group layout. The manufacturing cell layout depends on the common features in parts, thus group technology is an essential feature for designing cells and therefore implementing lean manufacturing.

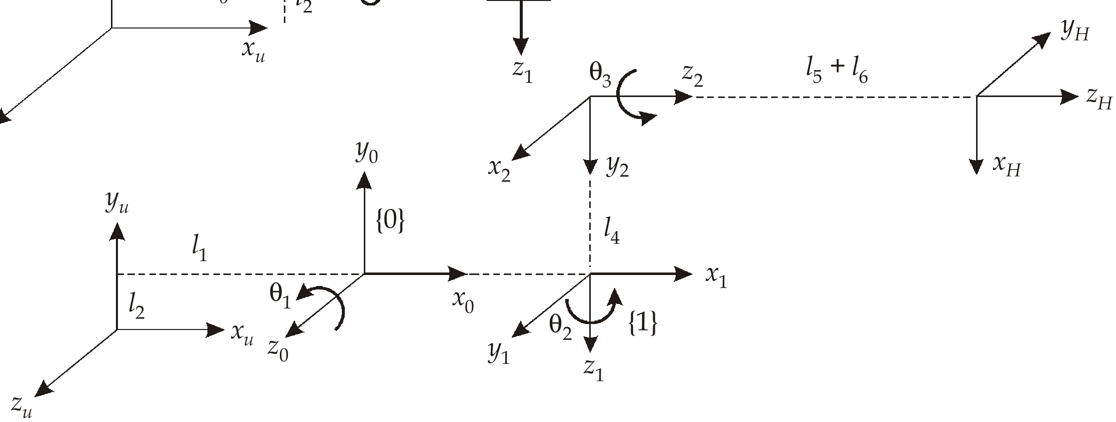
Advantages of group technology:

1. Group technology makes possible the standardization of part designs and minimization of design duplication, new part designs can be developed using already existing and similar designs, thus saving a significant amount of time and effort.
2. Data that reflect the experience of the designer and the process planner are stored in the database, a new or less experienced engineer can then quickly benefit from that experience.
3. Manufacturing costs can more easily be estimated, and the relevant statistics on materials, processes, number of parts produced, and other factors can easily be obtained.
4. Process plans are standardized and scheduled more efficiently, orders are grouped for more efficient production, machine utilization is improved, setup times are reduced, and parts are produced more efficiently and with better and more consistent product quality, similar tools, fixtures, and machinery are shared in the production of a family of parts.
5. With the implementation of CAD/CAM, cellular manufacturing and CIM, group technology is capable of greatly improving productivity and reducing costs in small batch production. Depending on the level of implementation, potential savings in each of the various design and manufacturing phases have been estimated to range from 5 to 75%.

6. (c) Solution:







Parameter Table

$^{i-1}T_i$	θ	d	a	α
$0-1$	θ_1	0	l_3	+90
$1-2$	θ_2	$-l_4$	0	+90
$2-H$	θ_3	l_5+l_6	0	0

Using generalized transformation matrix,

$$^{i\text{-}1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For 0 – 1, $\theta = \theta_1$, $\alpha = 90°$, $d = 0$, $a = l_3$

$$^{0}T_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & l_3 C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & l_3 S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For 1 – 2, $\theta = \theta_2$, $\alpha = 90°$, $d = -l_4$, $a = 0$

$$^{1}T_2 = \begin{bmatrix} C\theta_2 & 0 & S\theta_2 & 0 \\ S\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 1 & 0 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 – H, $\theta = \theta_3$, $a = 0$, $d = l_5 + l_6$, $\alpha = 0$

$$^{2}T_H = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_5+l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, $$^{U}T_H = {}^{U}T_0 \times {}^{0}T_H$$

$$^{U}T_H = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_H$$

$$^{U}T_H = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & l_3C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & l_3S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times {}^{1}T_2 \times {}^{2}T_H$$

$$= \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & l_3C\theta_1 + l_1 \\ S\theta_1 & 0 & -C\theta_1 & l_3S\theta_1 + l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} C\theta_2 & 0 & S\theta_2 & 0 \\ S\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 1 & 0 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times {}^{2}T_H$$

$$= \begin{bmatrix} C\theta_1C\theta_2 & S\theta_1 & C\theta_1S\theta_2 & -l_4S\theta_1 + l_3C\theta_1 + l_1 \\ S\theta_1C\theta_2 & -C\theta_1 & S\theta_1S\theta_2 & l_4C\theta_1 + l_3S\theta_1 + l_2 \\ S\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times {}^{2}T_H$$

$$= \begin{bmatrix} C\theta_1C\theta_2 & S\theta_1 & C\theta_1S\theta_2 & -l_4S\theta_1 + l_3C\theta_1 + l_1 \\ S\theta_1C\theta_2 & -C\theta_1 & S\theta_1S\theta_2 & l_4C\theta_1 + l_3S\theta_1 + l_2 \\ S\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_5 + l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1C\theta_2C\theta_3 + S\theta_1S\theta_3 & -C\theta_1C\theta_2S\theta_3 + S\theta_1C\theta_3 & C\theta_1S\theta_2 & (l_5 + l_6)C\theta_1S\theta_2 - l_4S\theta_1 + l_3C\theta_1 + l_1 \\ S\theta_1C\theta_2C\theta_3 & -C\theta_1S\theta_3 & S\theta_1S\theta_2 & (l_5 + l_6)S\theta_1S\theta_2 + l_4C\theta_1 + l_3S\theta_1 + l_2 \\ S\theta_2C\theta_3 & -S\theta_2S\theta_3 & -C\theta_2 & -(l_5 + l_6)C\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. (a) (i) Solution:

Pitting is a localized corrosion damage characterized by cavities. It is a particularly insidious form of corrosion, because even if one pit perforates the side of a tank, serviceability is lost until the tank is repaired. The chemical nature of the environment causes pitting in the following cases :

1. Halogen-containing solutions
2. Brackish water
3. Salt water
4. Chloride bleaches
5. Reducing inorganic acids solutions that tend to produce pitting.

Stainless steels are particularly prone to pitting. Pitting of brass conductive tubes sometimes occurs due to dezincification. This consists in the solution of the brass followed by precipitation of copper by zinc in the brass. The net result is selective removal of zinc. A localized attack frequently occurs near the inlet ends of the condenser tubes, due to impingement of air bubbles, which carry away the corrosion products. Pumps and ship propellers are liable to an attack known as cavitation, an impact caused by the collapse of vapour bubbles.

Major preventive measure are as follows:

1. Austenitic steels pit in salt water, so most designers tend to use copper alloys, bronzes, monels and other materials having lower pitting tendencies.
2. Some are carbon steels in salt water, in which corrosion rate is much higher than with stainless steels, but attack is more uniform and no pitting takes place.

7. (a) (ii) Solution:

Super alloys are nickel, cobalt or iron based alloys with excellent elevated temperature strength, creep properties and oxidation resistance.

Iron based super alloys contain	32 to 37% Iron
	15 to 22% chromium
	9 to 38% nickel
Cobalt based super alloys contain	35 to 65% cobalt
	19 to 30% chromium
	upto 35% nickel
Nickel based super alloys contain	38 to 76% nickel
	27% chromium
	20% cobalt

Properties:

- Iron based super alloys are characterized by high temperature as well as room temperature strength and resistance to creep, oxidation, corrosion and wear. Wear resistance increases with carbon content.
- Nickel based super alloys based on the formula $Ni_3(Al, Ti)$ are particularly resistant to temperature.
- Cobalt-base super alloys have excellent high-temperature creep and fatigue strengths and resistance to hot corrosion attack.

Application:

Iron based super alloys: High temperature aircraft bearings, and machinery parts subjected to sliding contact.

Nickel based super alloys: Aero engine turbine blades, turbine discs, turbo chargers.

Cobalt based super alloys: Gas turbine engines.

7. (b) Solution:

Given : t_2 = 0.8 mm, N = 400 rpm, f = 0.3 mm/rev, d = 4 mm

Tool geometry : 0°, 0°, 10°, 8°, 15°, 75°, 0 (mm)

$$F_c = P_z = 1200 \text{ N}$$

$$F_f = P_x = 800 \text{ N}$$

$$\Rightarrow \quad F_T = \frac{F_f}{\cos\phi} = \frac{800}{\cos(90-75)} = 828.221 \text{ N}$$

Since; the second last value in tool-signature is very-high and the side-cutting edge angle as per ASA can't be so high. So, ORS tool signature is given.

$$\alpha = 0°$$

$$V = \frac{\pi DN}{60} = \frac{\pi \times 400 \times 0.16}{60} \simeq 3.351 \text{ m/s}$$

$$\begin{bmatrix} F \\ N \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} F_T \\ F_c \end{bmatrix}$$

$$\begin{bmatrix} F \\ N \end{bmatrix} = \begin{bmatrix} \cos 0° & \sin 0° \\ -\sin 0° & \cos 0° \end{bmatrix} \begin{bmatrix} 828.221 \\ 1200 \end{bmatrix}$$

$$\begin{bmatrix} F \\ N \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ +0 & 1 \end{bmatrix} \begin{bmatrix} 828.221 \\ 1200 \end{bmatrix} = \begin{bmatrix} 828.221 \\ 1200 \end{bmatrix}$$

$$F = 828.221 \text{ N}$$

$$N = 1200 \text{ N}$$

$$\frac{F_s}{\left(\frac{w \cdot t_1}{\sin\phi}\right)} = \tau_s$$

$$\tan\phi = \frac{\cos\alpha}{\frac{t_2}{t_1} - \sin\alpha}$$

$$\alpha = 0°$$

$$\Rightarrow \quad \tan\phi = \frac{t_1}{t_2}$$

$$t_1 = f\cos\psi = 0.3 \times \cos 15° = 0.2898 \text{ mm}$$

$$w = \frac{d}{\cos\psi} = \frac{4}{\cos 15°} = 4.141 \text{ mm}$$

$$\tan\phi = \frac{t_1}{t_2} = \frac{0.2898}{0.8}$$

$$\phi = 19.9129°$$

$$F_s = F_c \cos\phi - F_T \sin\phi$$

$$= 1200 \times \cos 19.9129° - 828.22 \sin 19.9129°$$

$$= 846.16928 \text{ N}$$

$$\tau_s = \frac{(846.16928)\sin 19.9129°}{(0.2898)(4.141)} = 240.1525 \text{ MPa}$$

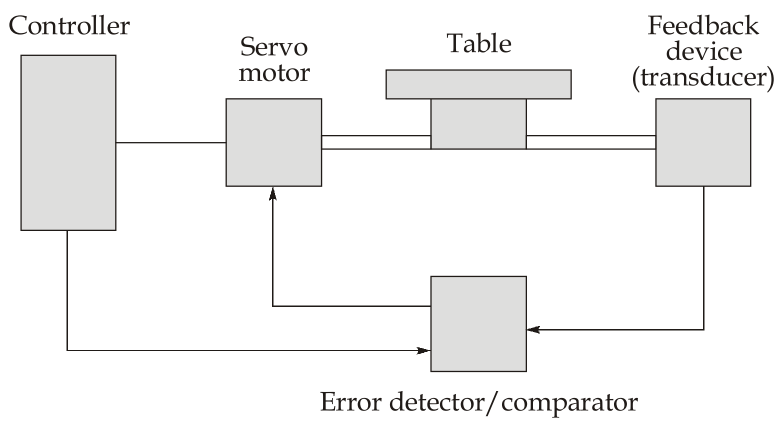
$$P_c = F_c V$$

$$= 1200 \times 3.351$$

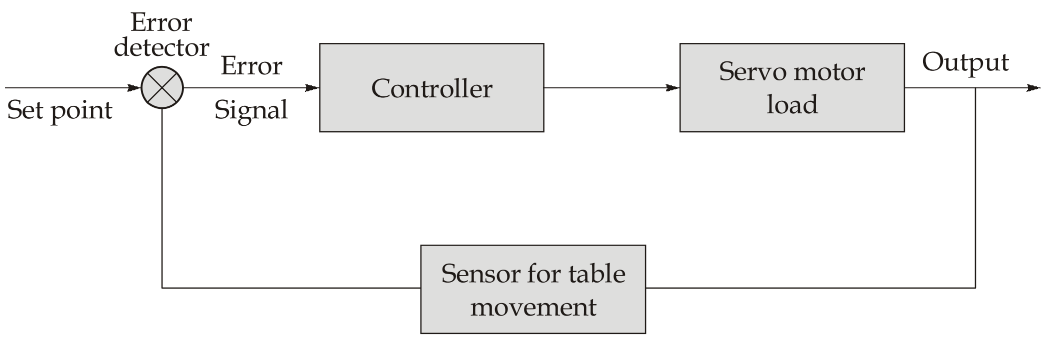
$$= 4021.2 \text{ W}$$

Ans.

7. (c) (i) Solution:



Closed-loop system



A **closed-loop control system** is one in which the output is continuously monitored and compared with the desired input (set point), and corrective action is taken automatically based on the error signal.

Working Principle:

- The input (reference value) is given to the system.
- The output is measured using a feedback device (sensor/transducer).
- The measured output is compared with the set value in an error detector (comparator).
- The difference between the desired and actual output is called the **error signal**.
- This error signal is processed by the controller, which generates a control action.
- The actuator (e.g., servo motor) acts to reduce the error.
- The process continues until the error becomes zero (desired output achieved).

Main Components and Functions:

1. **Feedback Device (Sensor/Transducer):**
   - Measures the output and converts it into an electrical signal.
2. **Error Detector (Comparator):**
   - Compares actual output with reference input and generates error signal.
3. **Controller:**
   - Processes error signal and decides corrective action.
4. **Actuator:**
   - Converts control signal into physical action (e.g., motion, rotation).
5. **Plant/System:**
   - The system whose output is being controlled.

Advantages:

1. High accuracy due to continuous feedback.
2. Automatic error correction.
3. Less sensitive to disturbances and parameter variations.
4. Widely used in automation and control applications.

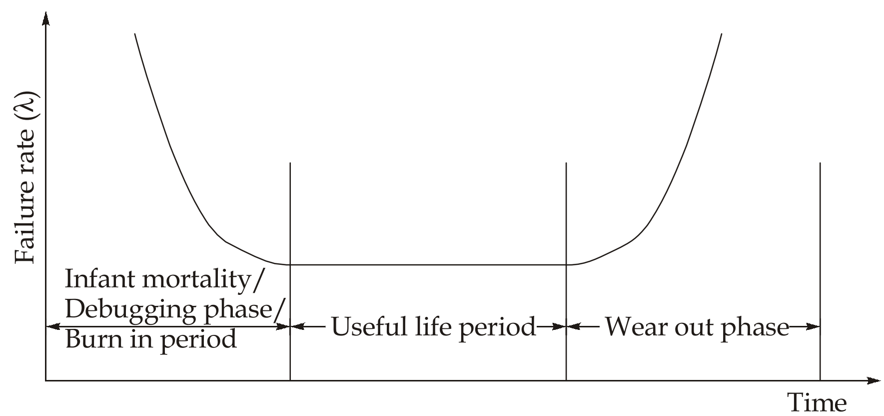
Disadvantages:

1. More complex and costly.
2. May become unstable if not properly designed.

7. (c) (ii) Solution:

Most products go through three distinct phases from product inception to wear-out. Life cycle curve for failure rate 'λ' can be represented as a function of time. This model is known as bath tub curve which has three phases.

- **Burn in period :** Also known as infant mortality phase or debugging phase. The failure rate decreases in this phase as initial problems identified are ironed out.
- **Useful life period :** In this phase, failures occur randomly and independently. Failure rate is constant.
- **Wear out phase :** At the end of useful life components age and wear out. An increase in failure rate is observed.



Bath tub curve

8. (a) (i) Solution:

Side riser :

For optimum side riser, $h = 2r$

$$V_R = 3 \times (V_{shrinkage})$$

$$V_R = 3 \times \left(\frac{8}{100}\right) \times (28)^3 \text{ cm}^3 = \pi r^2 h$$

and since $\frac{r}{h} = \frac{1}{2}$

$$V_R = 5268.48 \text{ and } h = 18.8596 \text{ cm}, \; r = \frac{h}{2} = 9.4298 \text{ cm}$$

Confirming these dimensions of riser.

$$\left(\frac{V}{A}\right)_R = \frac{\pi r^2 h}{2\pi r^2 + 2\pi r h} = \frac{2\pi r^3}{6\pi r^2} = \frac{r}{3} = 3.1433 \text{ cm}$$

$$\left(\frac{V}{A}\right)_C = \frac{(28)^3}{6\times(28)^2} = 4.667 \text{ cm}$$

$$\left(\frac{V}{A}\right)_C > \left(\frac{V}{A}\right)_R$$

$\Rightarrow$ Riser will solidify prior to casting

So, previous dimensions are not correct.

$$\left(\frac{V}{A}\right)_R \geq \left(\frac{V}{A}\right)_{Casting}$$

$$\frac{r}{3} \geq 4.667$$

$$r \geq 14 \text{ cm}$$

$$h \geq 28 \text{ cm} \qquad \{\because \quad h = 2r\}$$

Top riser:

$$V = \pi r^2 h$$

$$A = \pi r^2 + 2\pi rh;$$

($\because$ there is not heat transfer through bottom)

$$A = \pi r^2 + \frac{2V}{r}$$

For minimum area of riser

$$\frac{dA}{dr} = 2\pi r - \frac{2V}{r^2} = 0$$

$\Rightarrow$
$$r = \left(\frac{V}{\pi}\right)^{1/3} = h$$

$$\left(\frac{V}{A}\right)_{riser} = \frac{\pi r^2 h}{2\pi rh + \pi r^2} = \frac{r}{3}$$

$$V_R = 5268.48 \text{ cm}^3 = \pi r^2 h$$

$$M_R = \left(\frac{V}{A}\right)_R = \frac{r}{3} = 3.96 \qquad [M \text{ is the modulus here}]$$

$$M_C = \left(\frac{V}{A}\right)_C = \frac{28^3}{5\times 28\times 28} = 5.6$$

Again $M_R < M_C$

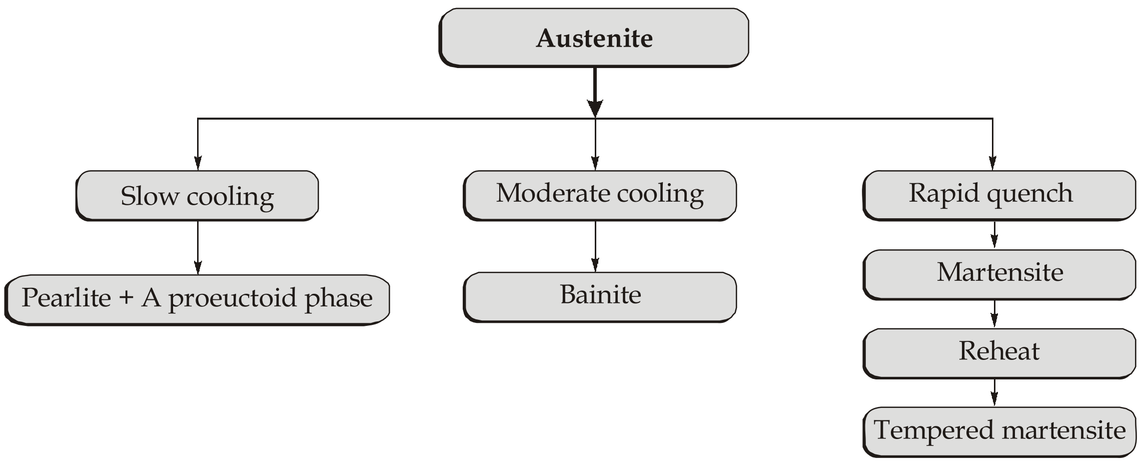
So, dimensions are not correct.

$$\frac{r}{3} \geq 5.6$$

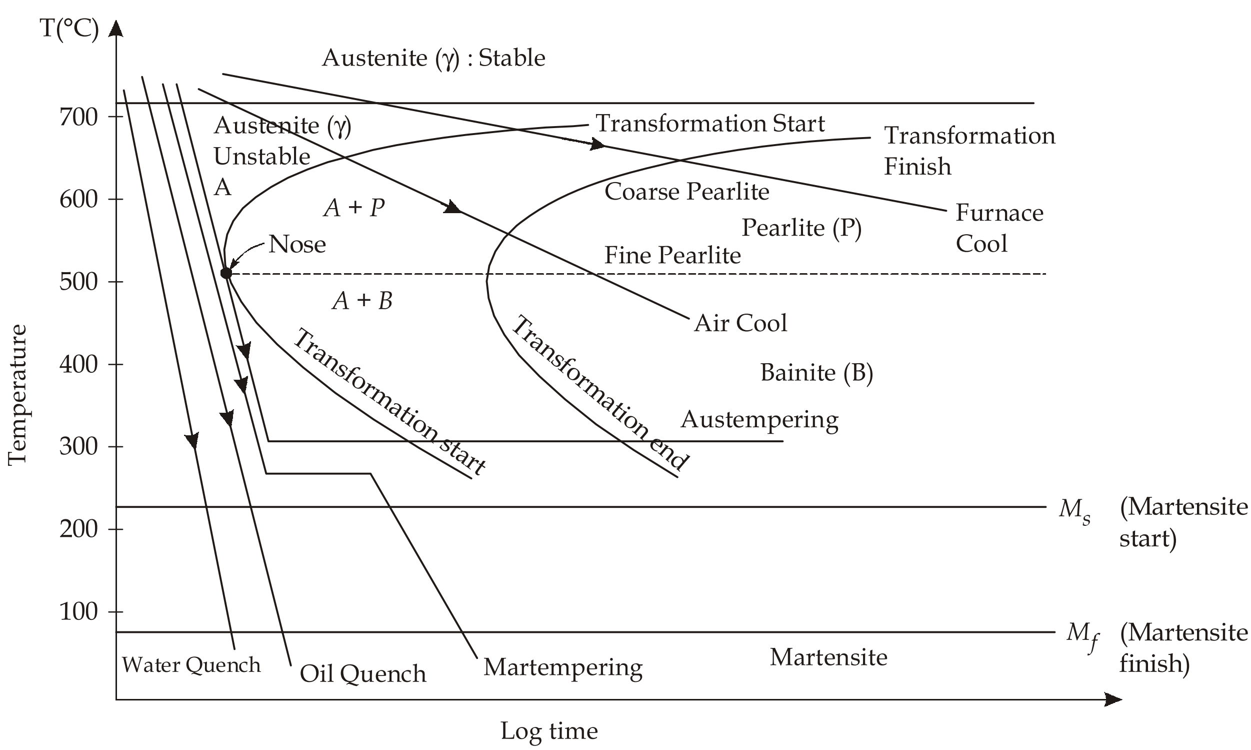
$$r \geq 16.8$$

$$r = h = 16.8 \text{ cm}$$

8. (a) (ii) Solution:



Time-Temperature-transformation diagram of transformation of Austenite (γ)



Transformation mechanism of Austenite (γ) into various structures:

(1) Pearlite:

- Slow cooling/Reconstructive transformation.
- Above 550°C, austenite transforms via-diffusion into a lamellar structure of ferrite (BCC-Low C) and cementite (Fe_3C, high C) arranged in alternate plates.
- Plate thickness ratio of a-Fe3C in pearlite.

(2) Bainite:

- Intermediate or moderate cooling rate: (Diffusion).
- Between 250°C and 550°C, austenite transforms through a combined process. It involves Bainitic ferrite plates growing-shear displacive) alongside dispersed carbide particles (Diffusion).
- Stronger than pearlite.

(3) Martensite (Rapid Quenching-Displacive):

- Rapid Quenching-displacive.
- Quenching below the Ms temperature (typically < 250°C), prevents Carbon-diffusion.
- Austenite transforms instantaneously via shear into a metastable, supersaturated, Body centered tetragonal structure.
- Carbon is trapped in solution causing high lattice strain resulting in extremely high hardness and brittleness.

(4) Tempered Martensite:

- Reheating below 723°C.
- BCT martensite breaks down into ferrite and Fe_3C.

8. **(b) (i) Solution:**

Lead Through or Teach Mode : In this mode, the robot's joints are moved with a teach pendant. When the desired location and orientation is achieved, the location is entered (taught) into the controller. During playback, the controller moves the joints to the same locations and orientations. This mode is usually point-to-point; as such, the motion between points is not specified or controlled. Only the points that are taught are guaranteed to reach.

Continuous Walk-Through Mode : In this mode, all robot joints are moved simultaneously, while the motion is continuously sampled and recorded by the controller. During playback, the exact motion that was recorded is executed. The motions are taught by an operator, either through a model, by physically moving the end-effector, or by "wearing" the robot arm and moving it through its workspace. Painting robots, for example, may be programmed by skilled painters through this mode.

Payload : Payload is the weight a robot can carry and still remain within its other specifications. As an example, a robot's maximum load capacity may be much larger than its specified payload, but at these levels, it may become less accurate, may not follow its intended trajectory accurately, or may have excessive deflections. The payload of robots compared to their own weight is usually very small.

Reach : Reach is the maximum distance a robot can reach within its work envelope. Many points within the work envelope of the robot may be reached with any desired orientation (called dexterous). However, for other points close to the limit of robot's reach capability, orientation cannot be specified as desired (called nondexterous point). Reach is a function of the robot's joints and lengths and its configuration. This is an important specification for industrial robots and must be considered before a robot is selected and installed.

Precision (Validity) : Precision is defined as how accurately a specified point can be reached. This is a function of the resolution of the actuators as well as the robot's feedback devices. Most industrial robots can have precision in the range of 0.001 inches or better. The precision is a function of how many positions and orientations were used to test the robot, with what load, at the what speed.

8. (b) (ii) Solution:

Given : $l_1 = l_2 = 1$ m

The joint angles can be determined as follows:

$$C_{12} = \cos(\theta_1 + \theta_2)$$

$$S_{12} = \sin(\theta_1 + \theta_2)$$

$$C_1 = \cos\theta_1$$

$$S_1 = \sin\theta_1$$

From the transformation matrix

$$C_{12} = -0.2924,\ S_{12} = 0.9563$$

$\therefore \quad \theta_1 + \theta_2 = \tan^{-1}\left[\dfrac{0.9563}{-0.2924}\right] = 107°$ ...(i)

Also, $l_2C_{12} + l_1C_1 = P_x = 0.6978$

$\therefore \quad C_1 = \dfrac{0.6978 - 1 \times \cos 107°}{1} = 0.99$

and $l_2S_{12} + l_1S_1 = P_y = 0.8172$

$\therefore \quad S_1 = \dfrac{0.8172 - 1 \times \sin 107°}{1} = -0.1391$

Now, $\theta_1 = \tan^{-1}\left[\dfrac{-0.1391}{0.99}\right]$

$\theta_1 = -8°$ Ans.

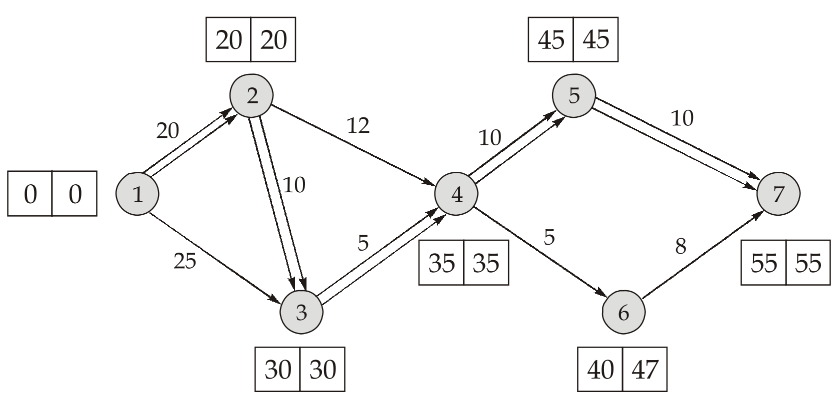
Substituting in equation (i), we have

$\theta_2 = 107 - (-8) = 115°$ Ans.

8. (c) Solution:

(i) Critical path 1 - 2 - 3 - 4 - 5 - 7

Duration = 20 + 10 + 5 + 10 + 10 = 55 days



(ii)

Activity	Normal time (days)	Crash time (days)	Normal cost (Rs.)	Crash Cost (Rs.)	Cost Slope	EST	LST	LFT	TF
1 - 2	20	17	600	720	40	0	20	0	0
1 - 3	25	25	200	200	0	0	25	5	5
2 - 3	10	8	300	440	70	20	30	20	0
2 - 4	12	6	400	700	50	20	32	23	3
3 - 4	5	2	300	420	40	30	35	30	0
4 - 5	10	5	300	600	60	35	45	35	0
4 - 6	5	3	600	900	150	35	40	42	7
5 - 7	10	5	500	800	60	45	55	45	0
6 - 7	8	3	400	700	60	40	48	47	7
			3600						

For activity $1 \rightarrow 2$

$$\text{Cost slope} = -\left[\frac{CC - NC}{CT - NT}\right] = \frac{720 - 600}{20 - 17} = 40$$

where ; CC = Crash cost; NC = Normal cost; CT = Crash time; NT = Normal time

(iii)

For activity $1 \rightarrow 2$

EST = 0; LFT = 20

$$\text{EFT} = \text{EST} + t_{1\text{-}2} = 20 - 20 = 0$$

$$\text{TF} = \text{LST} - \text{EST} = \text{LFT} - \text{EFT} = 0$$

Therefore $1 \rightarrow 2$ is critical activity

Similarly from table $2 \rightarrow 3$, $3 \rightarrow 4$, $4 \rightarrow 5$, $5 \rightarrow 7$ have TF = 0

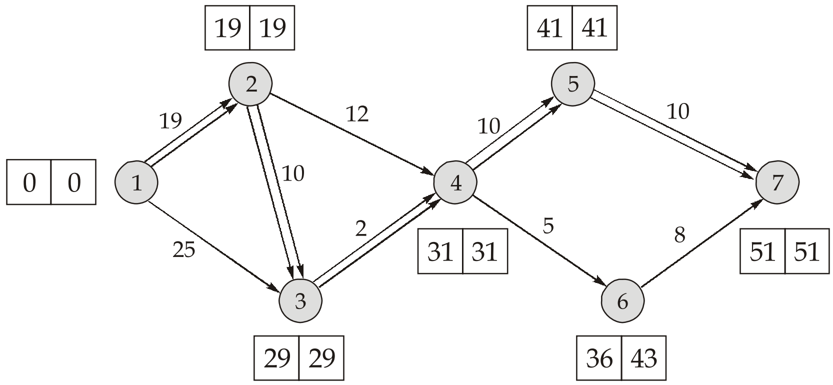
$$\text{TF}_{1 \rightarrow 3} = 5 \text{ days and } \text{TF}_{2 \rightarrow 4} = 3 \text{ days}$$

So, we can crash 3 days on activity $3 \rightarrow 4$ and 1 day on activity $1 \rightarrow 2$.

Therefore project cost = Total normal cost + 4(40)

= Rs.3760

(iv)



New critical path = 1 - 2 - 3 - 4 - 5 - 7

= 19 + 10 + 2 + 10 + 10 = 51 days