MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

# ESE-2026 Mains Test Series

# E & T Engineering Test No : 10

## Full Syllabus Test (Paper-I)

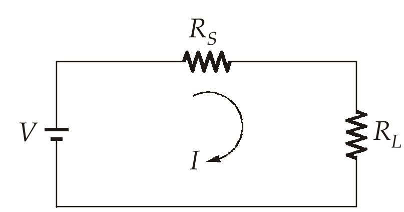
## Section A

**Q.1 (a) Solution:**

**(i)** The maximum power transfer theorem states that the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.

Consider the circuit shown below with source resistance $R_S$ and load resistance $R_L$.

We have,




$$I = \frac{V}{R_S + R_L}$$

Power delivered to the load

$$P = I^2 R_L = \frac{V^2 R_L}{(R_S + R_L)^2}$$

To determine the value of $R_L$ for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\frac{V^2}{(R_S + R_L)^2} R_L$$

$$= \frac{V^2\left[(R_S + R_L)^2 - (2R_L)(R_S + R_L)\right]}{(R_S + R_L)^4}$$

$$(R_S + R_L)^2 - 2R_L(R_S + R_L) = 0$$

$$R_S^2 + R_L^2 + 2R_S R_L - 2R_L R_S - 2R_L^2 = 0$$

$$R_S = R_L$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps to be followed in maximum power transfer theorem.

1. Remove the variable load resistor $R_L$.
2. Find the open circuit voltage $V_{Th}$ across points $A$ and $B$.
3. Find the resistance $R_{Th}$ as seen from points $A$ and $B$ with voltage source and current source replaced by internal resistance.
4. Find the resistance $R_L$ for maximum power transfer.

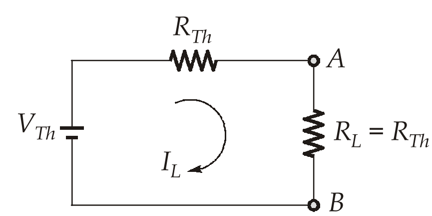
$$R_L = R_{Th}$$

5. Find the maximum power.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{V_{Th}}{2R_{Th}}$$

$$P_{max} = I_L^2 R_L$$

$$= \frac{V_{Th}^2}{4R_{Th}^2} \times R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$




**(ii) Step I:** *Calculation of $V_{Th}$*

Removing the variable resistor $R_L$ from the circuit,

Applying KVL to Mesh 1,

$$80 - 5I_1 - 10(I_1 - I_2) - 20(I_1 - I_2) - 20 = 0$$

$$35I_1 - 30I_2 = 60 \quad \text{...(1)}$$

Writing the current equation for Mesh 2,

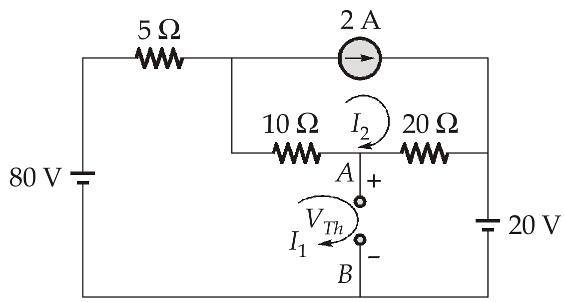
$$I_2 = 2 \text{ A} \quad \text{...(2)}$$

Solving Eqs (1) and (2), $I_1 = 3.43$ A

Applying KVL to obtain $V_{Th}$,

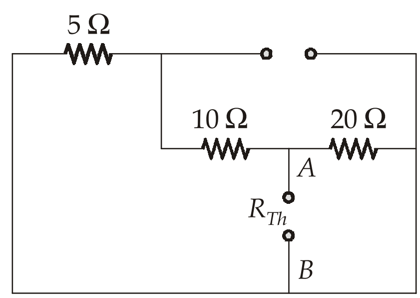
$$V_{Th} - 20(I_1 - I_2) - 20 = 0$$

$$V_{Th} = 20(3.43 - 2) + 20 = 48.6 \text{ V}$$




**Step II:** *Calculation of* $R_{Th}$

Replacing the voltage sources by short circuits and the current source by an open circuit,

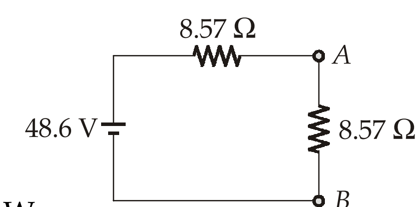



$$R_{Th} = (5 + 10) \parallel 20 = 15 \parallel 20 = 8.57\ \Omega$$

**Step III:** *Calculation of* $R_L$

For maximum power transfer,

$$R_L = R_{Th} = 8.57\ \Omega$$




**Step IV:** Calculation of $P_{max}$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(48.6)^2}{4\times 8.57} = 68.9\text{ W}$$

**Q.1 (b) Solution:**

Given,

Hysteresis loss per cycle per unit volume,

$$W = 10^3\text{ J/m}^3$$

Density of iron,

$$\rho = 7.5\text{ g/cm}^3 = 7.5 \times 10^3\text{ kg/m}^3$$

$$\text{Specific heat, } C = 100\text{ cal/kg °C}$$

We know that, $1\text{ cal} = 4.2\text{ J}$

$\therefore$ $C = 100 \times 4.2 = 420\text{ J/kg °C}$

$$\text{frequency, } f = 50\text{ Hz}$$

Thus, power loss per unit volume = $W \times f$

$$= 10^3 \times 50$$

$$= 5 \times 10^4 \text{ J/m}^3\text{/s}$$

Energy loss per minute,

$$E = 5 \times 10^4 \times 60$$

$$E = 3 \times 10^6 \text{ J/m}^3$$

Heat capacity per unit volume = $\rho C$

$$= 7.5 \times 10^3 \times 420$$

$$= 3.15 \times 10^6 \text{ J/m}^3\,°\text{C}$$

Rise in temperature per minute,

$$\Delta T = \frac{\text{Heat produced}}{\text{Heat capacity}} = \frac{3\times10^6}{3.15\times10^6}$$

$$\therefore \quad \Delta T \approx 0.95°\text{C}$$

**Q.1 (c) Solution:**

Number of turns per phase, $T = \frac{60\times8}{3\times2} = 80$

Number of slots per phase per pole,

$$m = \frac{60}{4\times3} = 5$$

$$\beta = \frac{180°}{\text{Number of slotes/pole}} = \frac{180°}{60/4} = 12°$$

The distribution factor of a 3-phase alternator is given by

$$K_d = \frac{\sin\left(\frac{mn\beta}{2}\right)}{m\sin\left(\frac{n\beta}{2}\right)}$$

where $n$ is the order of harmonic.

Fundamental Distribution factor,

$$K_{d1} = \frac{\sin\left(\frac{5\times12°}{2}\right)}{5\sin\frac{12°}{2}} = 0.956$$

Third Harmonic Distribution factor,

$$K_{d3} = \frac{\sin\left(\frac{5\times3\times12^\circ}{2}\right)}{5\sin 18^\circ} = 0.647$$

Fifth Harmonic Distribution Factor,

$$K_{d5} = \frac{\sin\left(\frac{5\times5\times12^\circ}{2}\right)}{5\sin 30^\circ} = 0.2$$

Since the coil is short-pitched by 2 slots,

$$\text{Chording Angle, } \alpha = 2 \times 12^\circ = 24^\circ$$

The pitch factor is given by

$$K_p = \cos\left(\frac{n\alpha}{2}\right), \text{ where } n \text{ is the order of harmonic}$$

Fundamental Pitch Factor, $K_{p1} = \cos\frac{24^\circ}{2} = 0.978$

Third harmonic Pitch Factor,

$$K_{p3} = \cos\left(3\times\frac{24^\circ}{2}\right) = 0.809$$

Fifth harmonic Pitch Factor, $K_{p5} = \cos\left(5\times\frac{24^\circ}{2}\right) = 0.5$

Induced emf is given by $E_{Ph} = 4.44 f K_d K_{p\phi} T$ . Thus,

$$E_{Ph1} = 4.44 \times 50 \times 80 \times 1 \times 10^{-3} \times 0.956 \times 0.978 = 16.61 \text{ kV}$$

$$E_{Ph3} = 4.44 \times 3 \times 50 \times 80 \times 0.05 \times 10^{-3} \times 0.647 \times 0.809 = 1.394 \text{ kV}$$

$$E_{Ph5} = 4.44 \times 5 \times 50 \times 80 \times 0.02 \times 10^{-3} \times 0.2 \times 0.5 = 0.177 \text{ kV}$$

**(i)** For star connection, phase voltage

$$E_{Ph} = \sqrt{E_{Ph1}^2 + E_{Ph3}^2 + E_{Ph5}^2} = 16.66 \text{ kV}$$

In both star and delta-connections, the third harmonic components of the three phases cancel out at the line terminals because they are co-phased. Hence, the line emf is composed of the fundamental and the fifth harmonic only.

$$\text{Line voltage, } E_L = \sqrt{3}\left(E_{Ph1}^2 + E_{Ph5}^2\right)^{1/2} = 28.77 \text{ kV}$$

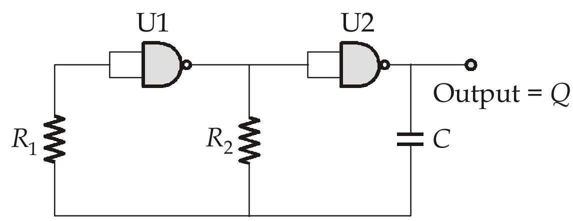
**(ii)** For delta connection, $E_{Ph} = E_L = \left(E_{Ph1}^2 + E_{Ph5}^2\right)^{1/2} = 16.67 \text{ kV}$

**(iii)** In delta-connection, third harmonic components are additive round the mesh, hence a circulating current is set up whose magnitude depends on the reactance per phase at the third harmonic frequency. The reactance per phase for the third harmonic is $X_3 = 3X_1$.

$$I_C = \frac{E_{Ph3}}{X_3} = \frac{E_{Ph3}}{3X_1} = \frac{1.394\times10^3}{3\times10} = 46.46 \text{ A}$$

**Q.1 (d) Solution:**

**(i) NAND Gate Astable multivibrator:**




Here NAND gates are used in 'NOT' gate configuration.

**Components used:** 2 NAND gates, 2 resistors $R_1$ and $R_2$, 1 capacitor.

**Working:** An astable multivibrator is a free running oscillator that has no permanent steady state but are continuously changing from One State (LOW) to other state (HIGH) and then back again. In the above circuitry, suppose initially output of U2 = HIGH (Logic '1'), then the input of U2 must be 'Low'

∴ Output of U1 = 'LOW'

∴ Input of U1 = 'HIGH'

Now the output of U2 is connected to a capacitor which is connected to input of U2 via a timing resistor '$R_2$'. As output of U2 is 'HIGH',

'$C$' starts charging at a rate determined by time constant '$R_2C$'.

Now, '$C$' is also connected to input of U1 through timing resistor '$R_1$'.

As '$C$' charges, the voltage at the junction between '$R_2$' and '$C$' decreases until the lower threshold value of U1 is reached at which point, the output of U1 changes state from LOW to HIGH.

Thus now, Input of U2 = Output of U1 = HIGH

∴ Output of U2 = LOW

Now, '$C$' starts discharging through input of U1 and charges up again in opposite direction (with time constant $R_1C$) until it reaches the upper threshold value of U1. This causes U1 to change state again and cycle repeats itself.

**(ii)** Parity generators and checkers are combinational logic circuits for error detection in digital communication, so as to ensure data integrity. A parity generator is a combinational logic circuit that generates the parity bit in the transmitter. On the other hand, a circuit that checks the parity in the receiver is called Parity Checker.

**Parity Generator :** It is a combinational circuit that accepts an $n$–1 bit data and generates the additional bit that is to be transmitted with the bit stream. This additional or extra bit is called as a Parity Bit.

In EVEN parity bit scheme, the parity bit is ‘0’ if there are even number of 1’s in the data stream, and the parity bit is ‘1’ if there are odd number of 1’s in the data stream.

In ODD parity bit scheme, the parity bit is ‘1’ if there are even number of 1’s in the data stream, and the parity bit is ‘0’ if there are odd number of 1’s in data stream.

**Parity Checker :** It is a logic circuit that checks for possible errors in transmission. This circuit can be an even parity checker or odd parity checker depending on the type of parity generated at transmission end. For even parity, If the total number of 1’s received is even, then the output indicates No Error and if the total number of 1’s received is odd, then the output indicates Error. Similarly for odd parity, If the total number of 1’s received is odd, then the output indicates No Error and if the total number of 1’s received is even, then the output indicates Error.

**Even Parity Generator :**

| 3-bit message | | | Even Parity Bit |
|---|---|---|---|
| $A$ | $B$ | $C$ | $P$ |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

| $A \backslash BC$ | $\overline{B}\overline{C}$ | $\overline{B}C$ | $BC$ | $B\overline{C}$ |
|---|---|---|---|---|
| $\overline{A}$ | (0) | 1 (1) | (3) | 1 (2) |
| $A$ | 1 (4) | (5) | 1 (7) | (6) |

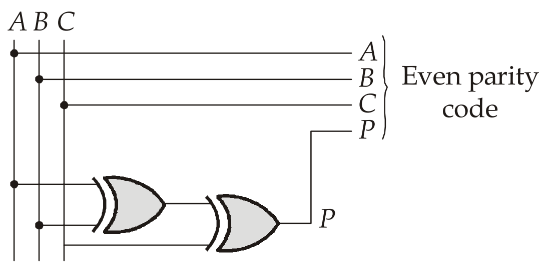
**K-Map for ‘$P$’**

$$P = \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$$

$$= \overline{A}(B \oplus C) + A(\overline{B \oplus C})$$

$$P = A \oplus B \oplus C$$

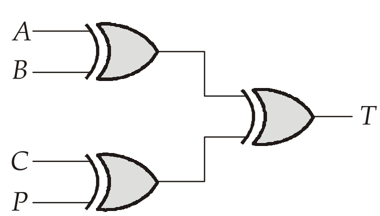
**Even Parity Checker:**




| 4-bit message received | | | | Parity check |
|---|---|---|---|---|
| *A* | *B* | *C* | *P* | *T* |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

| AB \ CP | $\bar{C}\bar{P}$ | $\bar{C}P$ | $CP$ | $C\bar{P}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | 0 | 1 (1) | 3 | 1 (2) |
| $\bar{A}B$ | 1 (4) | 5 | 1 (7) | 6 |
| $AB$ | 12 | 1 (13) | 15 | 1 (14) |
| $A\bar{B}$ | 1 (8) | 9 | 1 (11) | 10 |

$$T = (A \oplus B) \oplus (C \oplus P)$$




**Q.1 (e) Solution:**

Using the concept of virtual ground $V^- = V^+ = 0$. Applying KCL at inverting node of op-amp:

$$\frac{V_i}{R_1} + \frac{V_0}{1/sC} + \frac{V_0}{R_F} = 0$$

$$\left|\frac{V_0}{V_i}\right| = |A_V| = \frac{R_F/R_1}{(1+sCR_F)}$$

Substituting $s = j\omega$

$$\left|\frac{V_0}{V_i}\right| = |A_V| = \frac{R_F/R_1}{\sqrt{(\omega CR_F)^2+1}}$$

at $\omega = 0$; $\quad |A_V|_{\omega=0} = \left|\frac{V_0}{V_i}\right|$ gives peak value

$\therefore \quad |A_V|_{\omega=0} = \dfrac{R_F}{R_1}$

$$10 = \frac{R_F}{R_1} \qquad \ldots |A_V| = 20\text{ dB} = 10$$

$$R_F = 10R_1 \qquad \ldots\text{(i)}$$

We have corner frequency ($\omega_c$) = 10 k rad/s

Since; at corner frequency gain is reduced by 3 dB from its peak value.

Therefore, $\quad \omega_{3\text{ dB}} = \dfrac{1}{R_F C}$

$$10\times 10^3 = \frac{1}{R_F\cdot(10^{-8})}$$

$$R_F = 10\text{ k}\Omega$$

From equation (i), $\quad R_1 = 1\text{ k}\Omega$

**Q.2 (a) Solution:**

Given, illuminated diode I-V characteristics (Solar cell):

$$I(V) = (1\times10^{-12})\left(e^{V/28\text{ mV}}-1\right)-0.01\text{ A}$$

Comparing with the standard illuminated diode current equation,

$$I(V) = I_0\left(e^{V/V_T}-1\right)-I_L,$$

where $I_0$ is the reverse saturation current, $V_T$ is the thermal voltage and $I_L$ is the Light-generated current.

$\therefore \quad I_0 = 1\times10^{-12}$ A, $V_T = 28$ mV, $I_L = 0.01$ A

**(i)** Short-circuit current, $I_{SC}$:

For short-circuit $\Rightarrow V = 0$

$$I_{SC} = 1\times10^{-12}(e^0-1)-0.01$$

$\therefore \quad I_{SC} = -0.01\text{ A} = -10\text{ mA}$

**(ii)** Open circuit voltage, $V_{OC}$:

For open circuit, $I = 0$

$$0 = 1\times10^{-12}\left(e^{V_{OC}/28\,\text{mV}}-1\right)-0.01$$

$$= 10^{-12}e^{\frac{V_{OC}}{0.028}}-10^{-12}-0.01$$

Since, $0.01 >> 10^{-12}$

$$e^{V_{OC}/0.028} = 10^{10}$$

$$V_{OC} = 0.028\ln(10^{10}) = 0.028 \times 23.03$$

$\therefore$ $V_{OC} \simeq 0.645$ V

**(iii)** Maximum power point $(V_m, I_m)$

Power output, $P(V) = V \times I(V)$

To obtain maximum power condition, we put

$$\frac{dP}{dV} = 0 \quad\Rightarrow\quad I+V\frac{dI}{dV}=0$$

$$I_0(e^{V/28mV}-1)-0.01+V\frac{I_0}{28\,mV}e^{\frac{V}{28mV}} = 0$$

$$e^{\frac{V}{0.028}}\left(1+\frac{V}{0.028}\right) = 10^{10}$$

Let $\frac{V}{0.028} = y$. Thus,

$$e^y(1 + y) = 10^{10}$$

Using trial and error, we get $y \approx 20$.

Thus,

$$V_m = 0.028 \times 20 = 0.56 \text{ V}$$

Current at $V_m$,

$$I_m = 10^{-12}\left(e^{V_m/0.028}-1\right)-0.01$$

$$\approx 10^{-12}\left(e^{\frac{0.56}{0.028}}-1\right)-0.01$$

$$I_m \approx 4.85 \times 10^{-4} - 0.01 \approx -9.51 \times 10^{-3} \text{ A}$$

$\therefore$ $(V_m, I_m) \approx (0.56 \text{ V}, -9.51 \text{ mA})$

**(iv)** Load resistance for maximum power,

$$R_L = \frac{V_m}{I_m} = \frac{0.56}{9.51\times10^{-3}}$$

$$\therefore \quad R_L \approx 58.9\ \Omega$$

**(v)** Maximum output power,

$$P_{\max} = V_m \times I_m$$

$$P_{\max} = 0.56 \times 9.51 \times 10^{-3}$$

$$P_{\max} \simeq 5.3\text{ mW}$$

**(vi)** Fill factor (F.F)

$$\text{F.F} = \frac{P_{\max}}{V_{OC}\times I_{SC}}$$

We have, $V_{OC} \times I_{SC} = 0.645 \times 0.01 = 6.45 \times 10^{-3}$ W

$$\therefore \quad \text{F.F} = \frac{5.3}{6.45} \approx 0.82$$

**Q.2 (b) Solution:**

Given, a 555 astable multivibrator with,

$$R_1 = 1\text{ k}\Omega$$

$$R_w = 10\text{ k}\Omega$$

$$R_2 = 1\text{ k}\Omega$$

$$C = 0.01\ \mu\text{F}$$

As the wiper resistance connects between the two 1 kΩ resistors, so from the given diagram, the charging resistance, $R_A = R_1 + R_{wu}$, where $R_{wu}$ is the upper part of wiper resistance discharging resistance, $R_B = R_2 + R_{wl}$, where $R_{wl}$ is the lower part of wiper resistance

$$\text{Charge Time } (T_{\text{high}}) = 0.693 R_A C$$

$$\text{Discharge Time } (T_{\text{low}}) = 0.693(R_A + 2R_B)C$$

$$\text{Total time period, } T = T_{\text{high}} + T_{\text{low}} = 0.693\,(R_A + 2R_B)C$$

$$\text{frequency, } f = \frac{1}{T}$$

$$\text{Duty cycle, } D = \frac{T_{\text{high}}}{T} = \frac{R_A + R_B}{R_A + 2R_B}$$

**Case 1:** The wiper resistance at lower end [$R_{wu}$ = 10 kΩ and $R_{wl}$ = 0].

$$R_A = 1\text{ K} + 10\text{ K} = 11\text{ K}\Omega$$

$$R_B = 1\text{ K}\Omega$$

$$\text{frequency}, f = \frac{1}{T}$$

$$T = 0.693(11\text{ K} + 2 \times 1\text{ K})(0.01\text{ μF})$$

$$T = 0.693\ (13\text{ K})(10^{-8})$$

$$T = 0.693(1.3 \times 10^{-4}) = 9.009 \times 10^{-5}\text{s}$$

$$\therefore \quad f = \frac{1}{9.009\times10^{-5}} = 11.1\text{ kHz}$$

$$\text{Duty cycle}, D_1 = \frac{11\text{ K}+1\text{ K}}{11\text{ K}+2\text{ K}} = 0.923$$

$$\therefore \quad D_1 = 92.3\%$$

**Case 2:** Wiper resistance at upper end [$R_{wu}$ = 0 and $R_{wl}$ = 10 kΩ]

$$R_A = 1\text{ KΩ}$$

$$R_B = 1\text{ K} + 10\text{ K} = 11\text{KΩ}$$

$$\text{Frequency}, f = \frac{1}{T}$$

$$T = 0.693(1\text{ K} + 2 \times 11\text{ K})(0.01\text{ μF})$$

$$T = 0.693\ (23\text{ K}) \times 10^{-8}$$

$$\therefore \quad T = 1.5939 \times 10^{-4}\text{ s}$$

$$\therefore \quad f = \frac{1}{1.5939\times10^{-4}} = 6.27\text{ kHz}$$

$$\text{Duty cycle}, D_2 = \frac{1\text{ K}+11\text{ K}}{1\text{ K}+22\text{ K}} = 0.522$$

$$\therefore \quad D_2 = 52.2\%$$

**Q.2 (c) Solution:**

- Since molybdenum has BCC crystal structure, there are 2 atoms in the unit cell.
- The density can be given by,

$$\rho = \frac{\text{Mass of atoms in unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{(\text{Number of atoms in unit cell})\times(\text{Mass of one atom})}{\text{Volume of unit cell}}$$

$$= \frac{2\left(\frac{M_{\text{at}}}{N_A}\right)}{a^3}$$

So, $$a = \left(\frac{2M_{at}}{\rho N_A}\right)^{1/3} = \left(\frac{2\times 95.94}{10.22\times 6.022\times 10^{23}}\right)^{1/3} \text{cm}$$

$$= 3.147 \times 10^{-8} \text{ cm} = 3.147 \times 10^{-10} \text{ m}$$

- The atomic concentration ($n_{at}$) is 2 atoms in a cube of volume $a^3$.

So, $$n_{at} = \frac{2}{a^3} = \frac{2}{(3.147\times 10^{-10})^3} \text{m}^{-3}$$

$$= 6.415 \times 10^{28} \text{ m}^{-3}$$

- For a BCC cell, the lattice parameter $a$ and the radius of the atom $R$ are related by,

$$R = \frac{a\sqrt{3}}{4} = \frac{(3.147\times 10^{-10})\sqrt{3}}{4} \text{m}$$

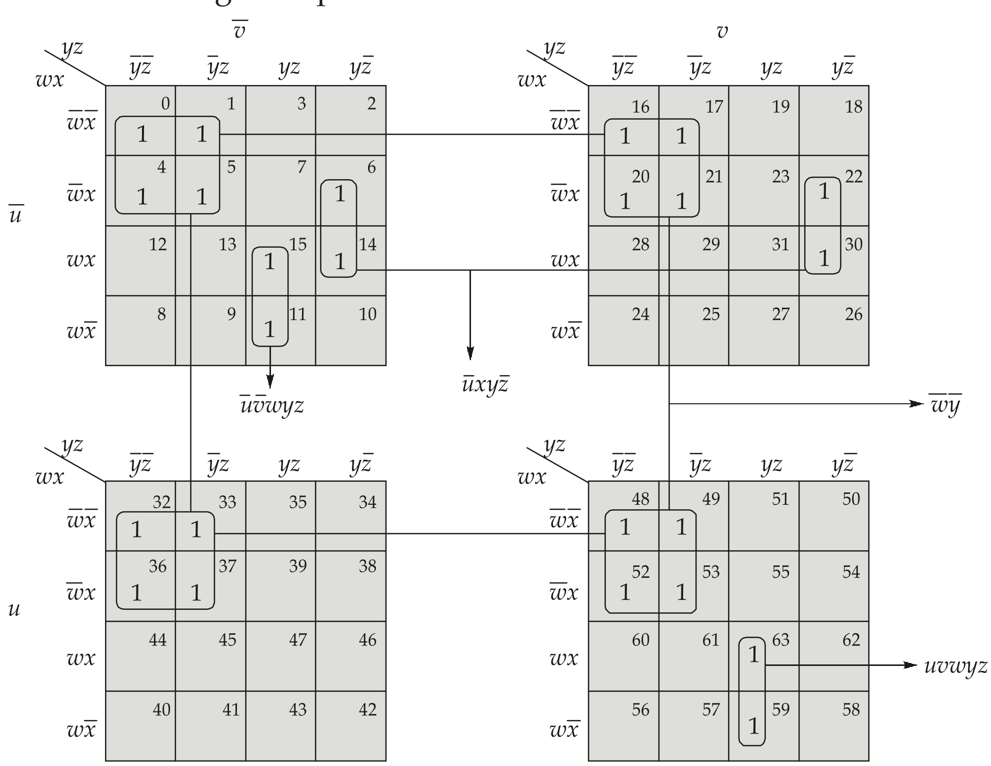
$$= 1.363 \times 10^{-10} \text{ m}$$

**Q.3 (a) Solution:**

**(i)** Given that,

**$F(uvwxyz) = \Sigma m$ (0, 1, 4, 5, 6, 11, 14, 15, 16, 17, 20, 21, 22, 30, 32, 33, 36, 37, 48, 49, 52, 53, 59, 63).**

Minimization using $K$ map:



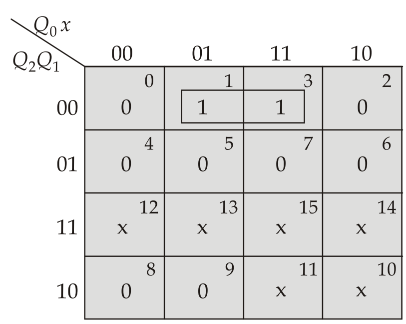

$$F = \bar{w}\bar{y} + \bar{u}xy\bar{z} + \bar{u}\,\bar{v}wyz + uvwyz$$

**(ii)** From the given state diagram, the state table can be obtained as below:

| | $Q_2$ | $Q_1$ | $Q_0$ | $x$ | $Q_2^+$ | $Q_1^+$ | $Q_0^+$ | $D_2$ | $D_1$ | $D_0$ | $y$ = output |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 6 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 8 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 10 | x | x | x | x | x | x | x | x | x | x | x |
| 11 | x | x | x | x | x | x | x | x | x | x | x |
| 12 | x | x | x | x | x | x | x | x | x | x | x |
| 13 | x | x | x | x | x | x | x | x | x | x | x |
| 14 | x | x | x | x | x | x | x | x | x | x | x |
| 15 | x | x | x | x | x | x | x | x | x | x | x |

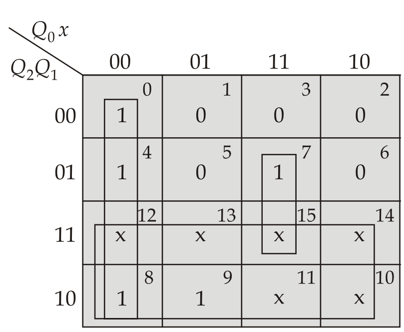
**Minimization using K-Map:**

**D2:**



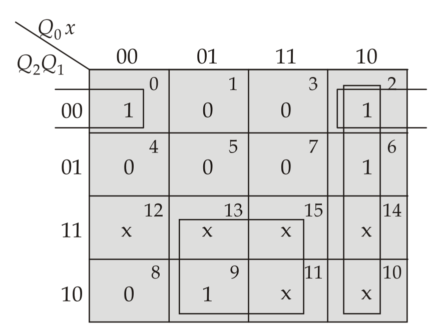

$$D_2 = \bar{Q}_2 \bar{Q}_1 x$$

**D₁:**




$$D_1 = Q_2 + \bar{Q}_0 \bar{x} + Q_1 Q_0 x$$

**D₀:**

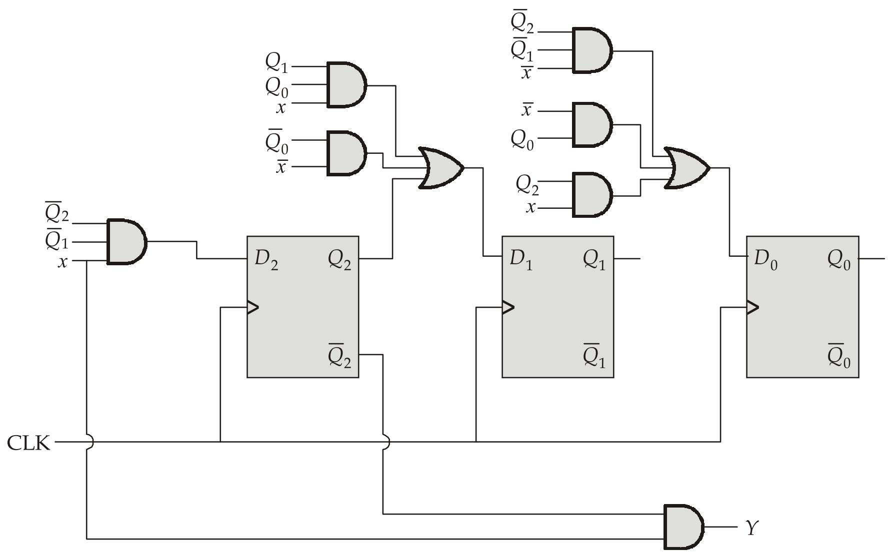



$$D_0 = \bar{Q}_2 \bar{Q}_1 \bar{x} + Q_0 \bar{x} + Q_2 x$$

**Y:**

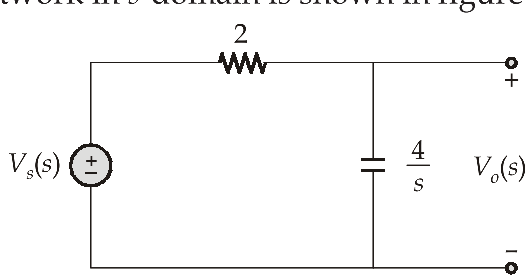
| $Q_2Q_1$ \ $Q_0x$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | (0) 0 | (1) 1 | (3) 1 | (2) 0 |
| 01 | (4) 0 | (5) 1 | (7) 1 | (6) 0 |
| 11 | (12) x | (13) x | (15) x | (14) x |
| 10 | (8) 0 | (9) 0 | (11) x | (10) x |

$$y = \bar{Q}_2\, x$$




**Q.3 (b) Solution:**

**(i)** The transformed network in $s$-domain is shown in figure below:




For $v_s(t) = \frac{1}{2}\cos t\, u(t)$, $\quad V_s(s) = \frac{1}{2}\frac{s}{s^2+1}$

By voltage-division formula,

$$V_o(s) = V_s(s) \times \frac{\frac{4}{s}}{2+\frac{4}{s}} = \frac{2V_s(s)}{s+2} = \frac{s}{(s^2+1)(s+2)}$$

By partial-fraction expansion,

$$V_o(s) = \frac{As+B}{s^2+1}+\frac{C}{s+2}$$

$$s = (As + B)(s + 2) + C(s^2 + 1)$$

$$s = s^2(A + C) + s(2A + B) + (2B + C)$$

Comparing coefficients of $s^2$, $s$ and $s^0$, we have

$$A + C = 0$$

$$2A + B = 1$$

$$2B + C = 0$$

Solving the equations, we get

$$A = 0.4$$

$$B = 0.2$$

$$C = -0.4$$

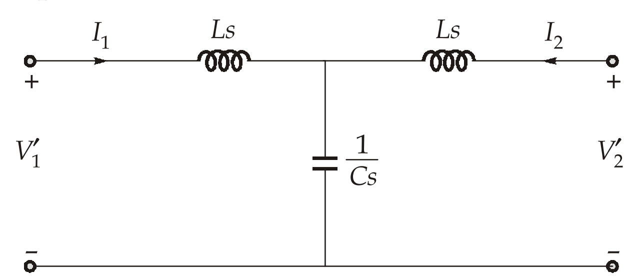
$$\therefore \quad V_o(s) = \frac{0.4s+0.2}{s^2+1}-\frac{0.4}{s+2}=\frac{0.4s}{s^2+1}+\frac{0.2}{s^2+1}-\frac{0.4}{s+2}$$

Taking the inverse Laplace transform,

$$v_0(t) = 0.4 \cos t + 0.2 \sin t - 0.4e^{-2t} \quad \text{for } t > 0$$

**(ii)** The above network can be considered as a series connection of two networks, $N_1$ and $N_2$.

**For the network $N_1$**




Applying KVL to Mesh 1,

$$V_1' = \left(Ls+\frac{1}{Cs}\right)I_1+\left(\frac{1}{Cs}\right)I_2 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2' = \left(\frac{1}{Cs}\right)I_1+\left(Ls+\frac{1}{Cs}\right)I_2 \quad \text{...(ii)}$$

Comparing Eqs (i) and (ii) with $Z$-parameter equations,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \text{ and } V_2 = Z_{21}I_1 + Z_{22}I_2$$

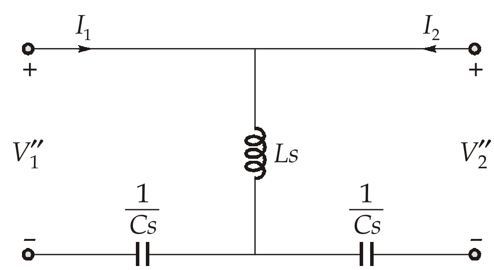
we get

$$\begin{bmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{bmatrix} = \begin{bmatrix} Ls + \frac{1}{Cs} & \frac{1}{Cs} \\ \frac{1}{Cs} & Ls + \frac{1}{Cs} \end{bmatrix}$$

**For the network $N_2$**

Applying KVL to Mesh 1,

$$V''_1 = \left(Ls + \frac{1}{Cs}\right)I_1 + (Ls)I_2 \quad \text{...(iii)}$$




Applying KVL to Mesh 2,

$$V''_2 = (Ls)I_1 + \left(Ls + \frac{1}{Cs}\right)I_2 \quad \text{...(iv)}$$

Comparing Eqs (iii) and (iv) with Z-parameter equations, we get

$$\begin{bmatrix} Z''_{11} & Z''_{12} \\ Z''_{21} & Z''_{22} \end{bmatrix} = \begin{bmatrix} Ls + \frac{1}{Cs} & Ls \\ Ls & Ls + \frac{1}{Cs} \end{bmatrix}$$

Hence, the overall $Z$-parameters of the network are,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z'_{11} + Z''_{11} & Z'_{12} + Z''_{12} \\ Z'_{21} + Z''_{21} & Z'_{22} + Z''_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 2Ls + \frac{2}{Cs} & Ls + \frac{1}{Cs} \\ Ls + \frac{1}{Cs} & 2Ls + \frac{2}{Cs} \end{bmatrix} = \left(Ls + \frac{1}{Cs}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

**Q.3 (c) Solution:**

$$T_L \propto n^2 \text{ (Given)}$$

For series motor,

before magnetic saturation, $T_L = k\phi I_a$

($\phi \propto I_a$ for series motor)

$$\therefore \quad T_L = kI_a^2$$

$$T_L \propto I_a^2$$

Given $T_L \propto n^2$, thus $I_a^2 \propto n^2$

$$I_a \propto n$$

$$\frac{I_{a1}}{I_{a2}} = \frac{n_1}{n_2}$$

$$\frac{40}{I_{a2}} = \frac{500}{600}$$

$$I_{a2} = \frac{400 \times 600}{500} = 48\ A$$

For a dc motor, $E_{b1} \propto n\phi$. Since $\phi \propto I_a$, thus $E_b \propto nI_a$. We have,

$$\frac{E_{b1}}{E_{b2}} = \frac{I_{a1}}{I_{a2}}\frac{n_1}{n_2}$$

where $E_{b1} = V_1 - I_{a1}R = 220 - 40 \times 1.3 = 220 - 52.0 = 168$ V

$$\frac{168}{E_{b2}} = \frac{40 \times 500}{48 \times 600}$$

$$E_{b2} = 241.92 \text{ V}$$

Thus, $V_2 = E_{b2} + I_{a2}R = 241.92 + 48 \times 1.3 = 304.32$ V

**Q.4 (a) Solution:**

**(i)** For a two-wattmeter method,

$$\text{Total Power, } P = W_1 + W_2$$

$$= 4600 + 2300 = 6900 \text{ W}$$

We have, $\tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \dfrac{\sqrt{3}(4600 - 2300)}{4600 + 2300} = \dfrac{1}{\sqrt{3}}$

or $\phi = 30°$

Positive sign shows that the power factor is lagging.

$\therefore$ Power factor = $\cos\phi = \cos(30°) = 0.866$ (lagging)

When $W_2 = -2300$ W

Power, $P = 4600 - 2300 = 2300$ W

and $$\tan\phi = \frac{\sqrt{3}\,(4600-(-2300))}{4600-2300} = 3\sqrt{3}$$

The power factor thus can be obtained as below,

$$\text{Power Factor} = \cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}} = \frac{1}{\sqrt{1+(-3\sqrt{3})^2}}$$
$$= 0.189 \text{ (lagging)}$$

**(ii)** Given:

Load voltage, $V = 100$ V

Load current, $I = 9$ A

Power factor $\cos\phi = 0.1 \Rightarrow \phi \approx 84.26°$ (lagging)

**Pressure coil:**

$R_p = 3000\ \Omega$

$L = 30$ mH = 0.03 H

$f = 50$ Hz

True power, $P_{true} = VI\cos\phi = 100 \times 9 \times 0.1 = 90$ W

**Pressure coil impedance:**

Inductive reactance: $X_L = 2\pi fL = 2\pi \times 50 \times 0.03 \approx 9.42\ \Omega$

Phase angle of pressure coil:

$$\alpha = \tan^{-1}\left(\frac{X_L}{R_p}\right) = \tan^{-1}\left(\frac{9.42}{3000}\right) \approx 0.18°$$

**1.** Pressure coil on load side

Due to pressure coil impedance, the wattmeter reading is proportional to: $VI\cos(\phi - \alpha)$. Thus,

$$\text{Fractional error} = \frac{\cos(\phi-\alpha) - \cos\phi}{\cos\phi} = \frac{\cos\phi\cos\alpha + \sin\phi\sin\alpha}{\cos\phi}$$

Since $\alpha$ is very small, $\cos a \approx 1$. Thus,

$$\text{Fractional error} \approx \tan\phi\sin\alpha$$

We have, $\tan\phi = \tan(84.26°) \approx 10$

$$\sin\alpha \approx \sin(0.18°) \approx 0.00314$$

Thus, $$\text{Error} \approx 10 \times 0.00314 = 0.0314 = 3.14\%$$

**2.** Pressure coil on supply side:

When the pressure coil is connected on the supply side, there is also error due to the power lost in the current coil.

Current coil resistance, $R_c = 0.1\ \Omega$

Voltage drop:

$$V_c = IR_c = 9 \times 0.1 = 0.9 \text{ V}$$

Extra power measured:

$$P_c = I^2R_c = 81 \times 0.1 = 8.1 \text{ W}$$

Percentage error:

$$\frac{8.1}{90}\times 100 = 9\%$$

Total error:

$$\approx 3.14\% + 9\% = 12.14\%$$

**Q.4 (b) Solution:**

First we determine the process transconductance parameter $k_n'$.

$$k_n' = \mu_n C_{ox}$$
$$= 450 \times 10^{-4} \times 8.6 \times 10^{-15} \times 10^{12} \text{ A/V}^2$$
$$= 387\ \mu\text{A/V}^2$$

and the transistor transconductance parameter $k_n$,

$$k_n = k_n'\left(\frac{W}{L}\right)$$
$$= 387\left(\frac{2}{0.18}\right)\times 10^{-6} = 4.3 \text{ mA/V}^2$$

**1.** With the transistor operating in saturation,

$$I_D = \frac{1}{2}k_n V_{OV}^2$$

Thus, $$100 \times 10^{-6} = \frac{1}{2}\times 4.3\times 10^{-3}\times V_{OV}^2$$

which results in

$$V_{OV} = 0.22 \text{ V}$$

Thus, $$V_{GS} = V_{tn} + V_{OV} = 0.5 + 0.22 = 0.72 \text{ V}$$

and since operation is at the edge of saturation,

$$V_{DS} = V_{OV} = 0.22 \text{ V}$$

**2.** With $V_{GS}$ kept constant at 0.72 V and $I_D$ reduced from the value obtained at the edge of saturation, the MOSFET will now be operating in the triode region, thus

$$I_D = k_n\left[V_{OV}V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$50 = 4.3\times10^3\left[0.22V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

which can be rearranged to the form

$$V_{DS}^2 - 0.44V_{DS} + 0.023 = 0$$

This quadratic equation has two solutions

$$V_{DS} = 0.06 \text{ V and } V_{DS} = 0.39 \text{ V}$$

The second answer is greater than $V_{OV}$ and thus is physically meaningless, since we know that the transistor is operating in the triode region. Thus we have

$$V_{DS} = 0.06 \text{ V}$$

**3.** For $v_{GS}$ = 0.7 V, $V_{OV}$ = 0.2 V, and since $V_{DS}$ = 0.3 V, the transistor is operating in saturation and

$$I_D = \frac{1}{2}k_nV_{OV}^2$$

$$= \frac{1}{2}\times4300\times0.04 \ \mu\text{A}$$

$$= 86 \ \mu\text{A}$$

Now when $V_{GS}$ changes by +0.01 V, for $v_{GS}$ = 0.710 V, $v_{OV}$ = 0.21 V and

$$i_D = \frac{1}{2}\times4300\times0.21^2 = 94.8 \ \mu\text{A}$$

and when $V_{GS}$ changes by +0.01 V, for $v_{GS}$ = 0.690 V, $v_{OV}$ = 0.19 V, and

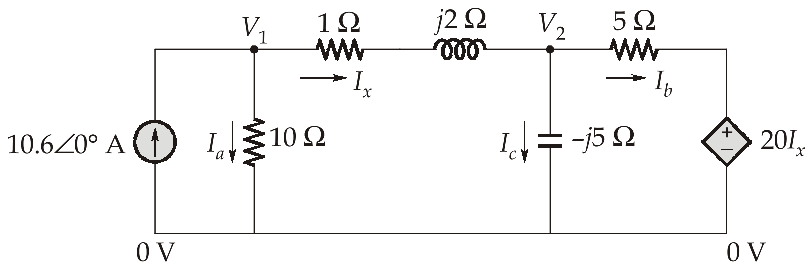
$$i_D = \frac{1}{2}\times4300\times0.19^2 = 77.6 \ \mu\text{A}$$

Thus, for $\Delta V_{GS}$ = +0.01 V, $\Delta i_D$ = 8.8 μA; and for $\Delta V_{GS}$ = –0.01 V, $\Delta i_D$ = –8.4 μA.

**Q.4 (c) Solution:**

(i)




Applying KCL at node $V_1$, we get

$$10.6\angle 0° = \frac{V_1}{10} + \frac{V_1 - V_2}{1 + j2}$$

$$\Rightarrow \quad V_1\left[\frac{1}{10} + \frac{1}{1+j2}\right] - \frac{V_2}{1+j2} = 10.6\angle 0°$$

$$(0.3 - 0.4j)V_1 - (0.2 - 0.4j)V_2 = 10.6\angle 0° \qquad \text{...(i)}$$

Also, $$I_x = \frac{V_1 - V_2}{1 + j2} \qquad \text{...(ii)}$$

Applying KCL at node $V_2$, we get

$$\frac{-V_2}{j5} + \frac{V_2 - V_1}{1+j2} + \frac{V_2 - 20I_x}{5} = 0$$

$$\frac{-V_2}{j} + \frac{5(V_2 - V_1)}{1+j2} + V_2 - 20\left[\frac{V_1 - V_2}{1+j2}\right] = 0 \qquad \text{...using equation (ii)}$$

$$V_2\left[\frac{-1}{j} + \frac{5}{1+j2} + 1 + \frac{20}{1+j2}\right] - V_1\left[\frac{5}{1+j2} + \frac{20}{1+j2}\right] = 0$$

$$-V_1(5 - 10j) + (6 - 9j)V_2 = 0$$

$$V_1 = \frac{(6 - 9j)}{(5 - 10j)}V_2$$

$$V_1 = (0.96 + 0.12j)V_2$$

From (i),

$$(0.3 - 0.4j)(0.96 + 0.12j)V_2 - (0.2 - 0.4j)V_2 = 10.6\angle 0°$$

$$V_2 = 72.80\angle -20.92° \text{ V}$$

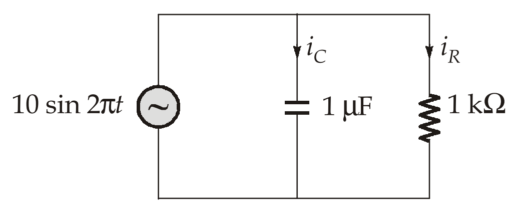
$$V_1 = 70.43\angle -13.80° \text{ V}$$

Using above, the branch currents can be obtained as below:

- $I_a = \dfrac{V_1}{10} = 7.043\angle -13.80°$ A
- $I_c = \dfrac{V_2}{-j5} = \dfrac{72.80\angle -20.92°}{-5j} = 14.56\angle 69.08°$ A
- $I_x = \dfrac{V_1 - V_2}{1+j2} = \dfrac{68.4 - j16.8 - (68 - j26)}{1+j2} = \dfrac{0.4 + j9.2}{1+j2} = 3.76 + j1.68A$
- $I_b = I_x - I_c = 3.76 + j1.68 - (5.2 + j13.6)$

  $= -1.44 - 11.92 = 12.01 \angle -96.90°$ A

**(ii)** • Current in a capacitor leads voltage by 90° and current is in phase with voltage across resistor.




$$i_c = \frac{10\sin\left(2\pi t + \frac{\pi}{2}\right)}{X_c} \qquad X_c \to \text{Capacitive reactance}$$

$$i_c = \frac{10}{X_c}\cos 2\pi t$$

Voltage across capacitor, $V_c = 10 \sin 2\pi t$

Power in capacitor, $V_c(t) = V_c(t) \times i_c(t)$

- Energy stored in capacitor $= \int_0^{0.5} V_c(t) i_c(t) dt \qquad 0 < t < 0.5$

$$E_c = \int_0^{0.5} 10\sin 2\pi t \cdot \frac{10}{X_c}\cos 2\pi t \cdot dt$$

$$E_c = \int_0^{0.5} \frac{100}{X_c}(\sin 2\pi t)(\cos 2\pi t)dt$$

$$E_c = \frac{50}{X_c}\int_0^{0.5} \sin 4\pi t\, dt$$

$$E_c = \frac{50}{X_c}\left[\frac{-(\cos 4\pi t)}{4\pi}\right]_0^{0.5}$$

$$E_c = \frac{50}{4\pi X_c}[-\cos 2\pi + \cos 0]$$

$$E_c = 0$$

Analytically also, energy stored in a capacitor over a cycle [0 < $t$ < 0.5 constitute a cycle for energy waveform] is zero.

- For resistor, $i(t) = \frac{10}{1\times10^3}\sin 2\pi t$

  Power dissipated across resistor,

$$P = i^2(t)R$$

Energy dissipated over the interval 0 < $t$ < 0.5s,

$$E_R = \int_0^{0.5} i^2(t)R\,dt$$

$$E_R = \int_0^{0.5} \frac{100}{10^3}\sin^2 2\pi t\,dt$$

$$= \frac{1}{10}\int_0^{0.5}\frac{(1-\cos 4\pi t)}{2}dt$$

$$E_R = \frac{1}{20}\int_0^{0.5}(1-\cos 4\pi t)dt$$

$$= \frac{1}{20}\left[t - \frac{\sin 4\pi t}{4\pi}\right]_0^{0.5} = \frac{1}{20}[0.50-0-0]$$

$$E_R = \frac{0.5}{20} = 25\times10^{-3}\,\text{J}$$

Energy dissipated in resistor, $E_R = 25 \times 10^{-3}$ Joule

## Section B

**Q.5 (a) Solution:**

The circuit arrangement is shown in Fig. (a) Because of the ideal diode, voltage appears across the pressure coil only during the positive half-cycle of the applied sinusoidal voltage. The rectified voltage appearing across the pressure coil is shown in Fig. (b).

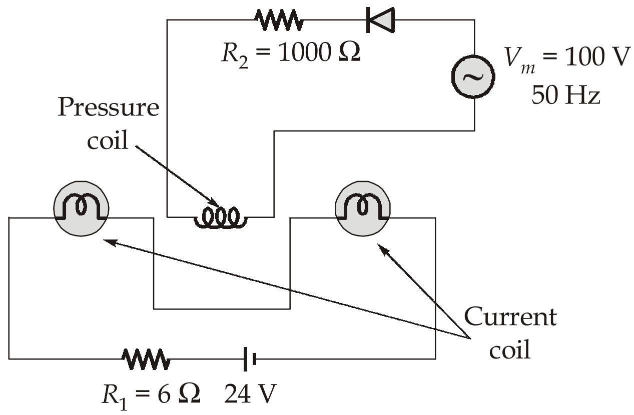



Fig. (a): Circuit arrangement

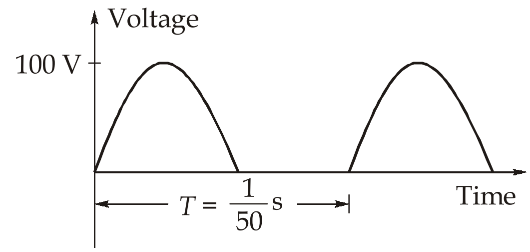



Fig. (b): Rectified voltage

The current through the current coil,

$$I = \frac{24}{6} = 4\text{ A}$$

The instantaneous value of the voltage applied to the pressure coil circuit is

$$v = 100\sin 100\pi t \quad ; \quad 0 \le t \le \frac{1}{100}s$$

$$= 0 \quad ; \quad \frac{1}{100} \le t \le \frac{1}{50}s$$

The instantaneous power,

$$p = Iv$$

The average power read by the wattmeter,

$$P = \frac{1}{T}\int_0^T vI\,dt = \frac{1}{T}\int_0^{T/2} 4\times 100\sin 100\pi t dt$$

$$= \frac{1}{\left(\frac{1}{50}\right)}\int_0^{1/100} 400\sin 100\pi t\,dt$$

$$= \frac{20000}{100\pi}[-\cos 100\pi t]_0^{1/100}$$

$$= \frac{400}{\pi} = 127.32\text{ Watt}$$

**Q.5 (b) Solution:**

Given,

$$\text{Electron concentration, } n(x) = 10^{16}\left(1-\frac{x}{L}\right)\text{cm}^{-3}; 0 \le x \le L$$

$$\text{Length, } L = 10 \text{ mm} = 1 \text{ cm}$$

$$\text{Total current density, } J = -85 \text{ A/cm}^2$$

Electron diffusion coefficient,

$$D_n = 26 \text{ cm}^2/\text{s}$$

$$\text{Electron mobility, } \mu_n = 1000 \text{ cm}^2/\text{V-s}$$

$$\text{Electron charge, } q = 1.6 \times 10^{-19} \text{ C}$$

Total electron current density,

$$J = J_{\text{diff}} + J_{\text{drift}}$$

$$J = q\mu_n nE + qD_n \frac{dn}{dx}$$

We have, $$\frac{dn}{dx} = 10^{16}\left[-\frac{1}{L}\right] = -10^{16}\text{ cm}^{-4}$$

Thus, $$-85 = (1.6 \times 10^{-19})(1000)n(x)\,E + (1.6 \times 10^{-19})(26)(-10^{16})$$

$$-85 = 1.6 \times 10^{-16} \times n(x) \times E - 0.0416$$

$$1.6 \times 10^{-16} \times n(x) \times E = -84.95$$

but $$n(x) = 10^{16}[1 - x]$$

$\therefore$ $$E(x) = \frac{-84.95}{1.6\times10^{-16}\times10^{16}(1-x)}$$

$$E(x) = \frac{-53.1}{1-x}\text{V/cm}; 0 \le x \le 1$$

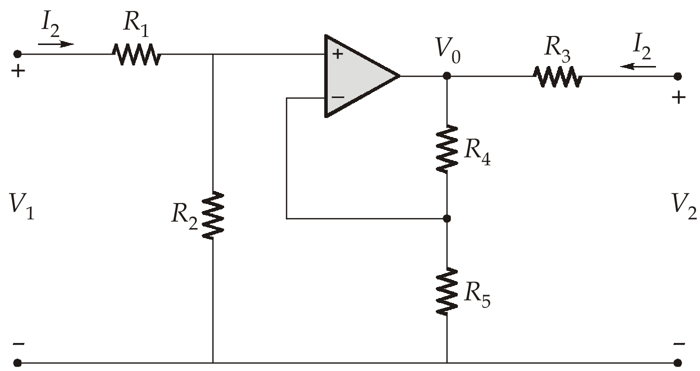
**Q.5 (c) Solution:**

For an ideal op-amp, the current into the input terminals of op-amp is zero. Applying KVL in the input loop, we get

$$V_1 = I_1(R_1 + R_2) + 0I_2 = Z_{11}I_1 + Z_{12}I_2$$

$\therefore$ $$Z_{11} = R_1 + R_2$$

$$Z_{12} = 0$$




Since it is an non-inverting op-amp amplifier, we have

$$V_0 = \left(1+\frac{R_4}{R_5}\right)\times\left(\frac{R_2}{R_1+R_2}\right)\times V_1$$

$$= \left(1+\frac{R_4}{R_5}\right)\times\left(\frac{R_2}{R_1+R_2}\right)\times(R_1+R_2)I_1$$

$$V_0 = \left(1+\frac{R_4}{R_5}\right)R_2I_1 \quad \text{...(i)}$$

$$I_2 = (V_2-V_0)\times\frac{1}{R_3}$$

$$I_2R_3 = V_2 - V_0$$

$$V_0 = V_2 - I_2R_3 \quad \text{...(ii)}$$

Put equation (ii) in equation (i),

$$V_2 - I_2R_3 = \left(1+\frac{R_4}{R_5}\right)\times R_2I_1$$

$$V_2 = \left(1+\frac{R_4}{R_5}\right)\times R_2I_1 + I_2R_3 = Z_{21}I_1 + Z_{22}I_2$$

$\therefore$
$$Z_{21} = \left(R_2+\frac{R_2R_4}{R_5}\right)$$

and
$$Z_{22} = R_3$$

Thus, the z-parameters of the network are given as

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} R_1+R_2 & 0 \\ \left(R_2+\dfrac{R_2R_4}{R_5}\right) & R_3 \end{bmatrix}$$

For a reciprocal network,

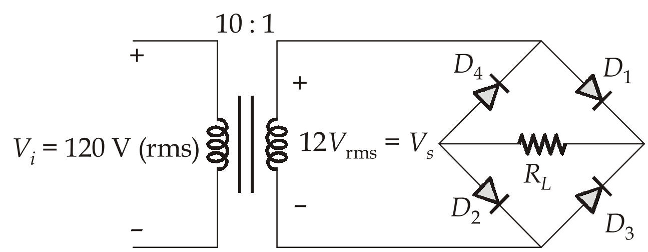
$$Z_{12} = Z_{21}$$

But for above network, $Z_{12} \neq Z_{21}$

Hence, it is not a reciprocal network.

**Q.5 (d) Solution:**

Full wave bridge rectifier,




Given data, $R_L = 1\text{ k}\Omega$, $V_i = 120$ V (rms), $\dfrac{N_1}{N_2} = \dfrac{10}{1}$, $V_D = 0.7$ V, $f = 60$ Hz.

**(i)** Voltage across the secondary winding,

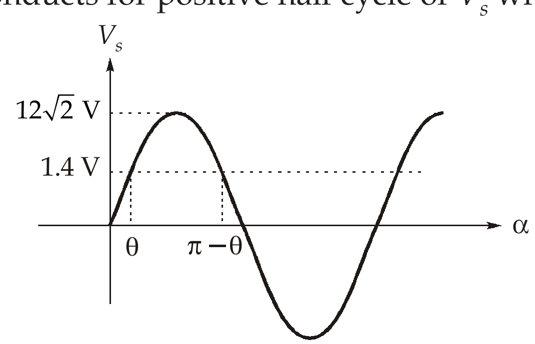
$$V_s = (N_2/N_1)V_i = 12 \text{ V(rms)}$$

Peak value of the rectified voltage across load = $V_{s(\text{peak})} - 2V_D$

$$= 12\sqrt{2} - 2 \times 0.7$$

$$= 15.57 \text{ volt}$$

**(ii)** Diode $D_1$ and $D_2$ conducts for positive half cycle of $V_s$ when $V_s > 2V_D = 1.4$ V.




$$12\sqrt{2}\sin\theta = 1.4$$

$$\sin\theta = \frac{1.4}{12\sqrt{2}}$$

$$\sin\theta = 0.0826 \quad \Rightarrow \quad \theta = 0.0826 \text{ rad}$$

Fraction of cycle for which diode $D_1$ and $D_2$ conduct is $\dfrac{\pi - 2\theta}{2\pi} \times 100 = 47.4\%$

**Note:** Diode $D_3$ and $D_4$ conduct in the negative half cycle for 47.4% of the cycle. Hence, each diode conducts for 47.4% of the cycle.

**(iii)** The relationship between the input and the output voltage is

$$V_0 = \begin{cases} |V_i| - 2V_D, \theta < \alpha < \pi - \theta \text{ and } \pi + \alpha < 2\pi - \theta \\ 0 \quad , \quad \text{otherwise} \end{cases}$$

Avg. voltage across load $R_L$ is,

$$V_{0,\text{ avg}} = \frac{2}{2\pi}\int_{\theta}^{\pi-\theta}\left[12\sqrt{2}\sin\alpha - 2V_D\right]d\alpha$$

$$= \frac{1}{\pi}\int_{\theta}^{\pi-\theta}\left[12\sqrt{2}\sin\alpha - 1.4\right]d\alpha$$

$$= \frac{1}{\pi}\left[-12\sqrt{2}\cos\alpha - 1.4\alpha\right]_{\theta}^{\pi-\theta}$$

$$= \frac{1}{\pi}\left[2\times 12\sqrt{2}\cos\theta - 1.4[\pi - 2\theta]\right]$$

$$V_{0,\text{ avg}} = 9.44 \text{ volts}$$

**(iv)** Avg. current through load $R_L$ is,

$$i_{R,\text{ avg}} = \frac{V_{0,\,avg}}{R_L} = \frac{9.44}{1\text{ k}\Omega}$$

$$i_{\text{R, avg}} = 9.44 \text{ mA}$$

**Q.5 (e) Solution:**

Given, $(h\,k\,l) = (3\,2\,6)$

$$a = 1.5\text{ Å}, b = 2\text{ Å}, c = 2\text{ Å}$$

For a crystal plane with intercepts $p$, $q$, $r$ along the crystallographic axes and lattice constants $a$, $b$, $c$, the Miller indices are defined by taking the reciprocals of the intercepts measured in units of the lattice constants. Thus,

$$h:k:l = \frac{a}{p}:\frac{b}{q}:\frac{c}{r} \Rightarrow p = \frac{na}{h}, q = \frac{nb}{k}, r = \frac{nc}{l}$$

$\therefore$ Intercept on $X$-axis $= \frac{na}{h} = \frac{n1.5\text{Å}}{3} = 0.5n\text{ Å}$

Intercept on $Y$-axis $= \frac{nb}{k} = \frac{n2\text{ Å}}{2} = n\text{ Å}$

$$\text{Intercept on } Z\text{-axis} = \frac{nc}{l} = \frac{n2\text{ Å}}{6} = \frac{n}{3}\text{ Å}$$

But actual intercept on $X$-axis is given as 1.5 Å. Thus, $n$ = 3.

∴ Intercept on $Y$-axis = 1 Å × 3 = 3 Å.

$$\text{Intercept on } Z\text{-axis} = \frac{1}{3}\text{Å} \times 3 = 1\text{ Å}$$

**Q.6 (a) Solution:**

**(i) Digital Voltmeter (DVM):**

- A digital voltmeter (DVM) displays the value of AC or DC voltage being measured directly as discrete numerals in the decimal number system.
- Numerical readout is advantageous in many applications because it reduces human reading and interpolation errors and eliminates parallax errors.
- The use of digital voltmeters increase the speed with which reading can be taken, and also the output of digital voltmeter can be fed to memory devices for storage and future computations.
- A digital voltmeter is a versatile and accurate voltmeter which has many laboratory applications.

**Types of DVMs:**

(i) Ramp type DVM. (ii) Integrating type DVM. (iii) Potentiometric type DVM. (iv) Successive approximation type DVM. (v) Continuous balance type DVM.

**Basic Function:**

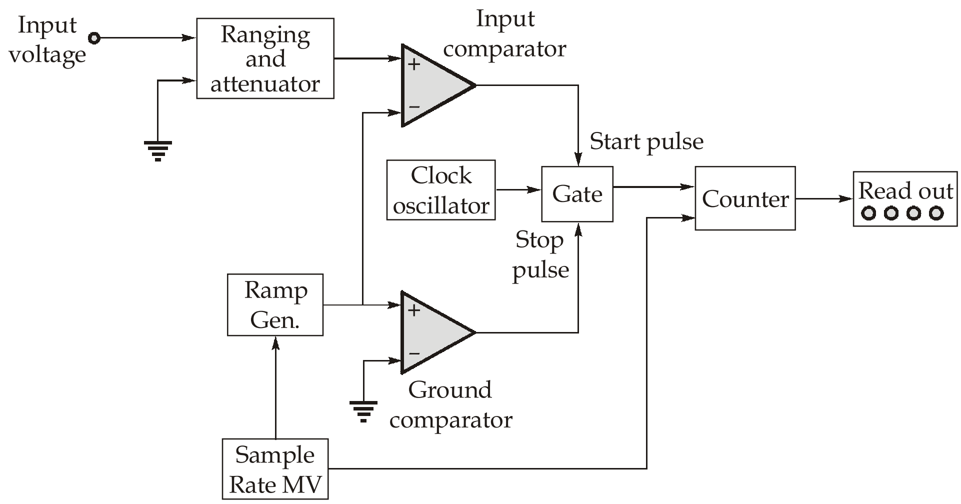
In every case, the basic function that is performed is an analog to digital (A/D) conversion.

For example, a voltage value may be changed to a proportional time interval, which starts and stops a clock oscillator. In turn the oscillator output is applied to an electronic counter which is provided with a read out in terms of voltage values.

**Ramp Type Digital Voltmeter:**

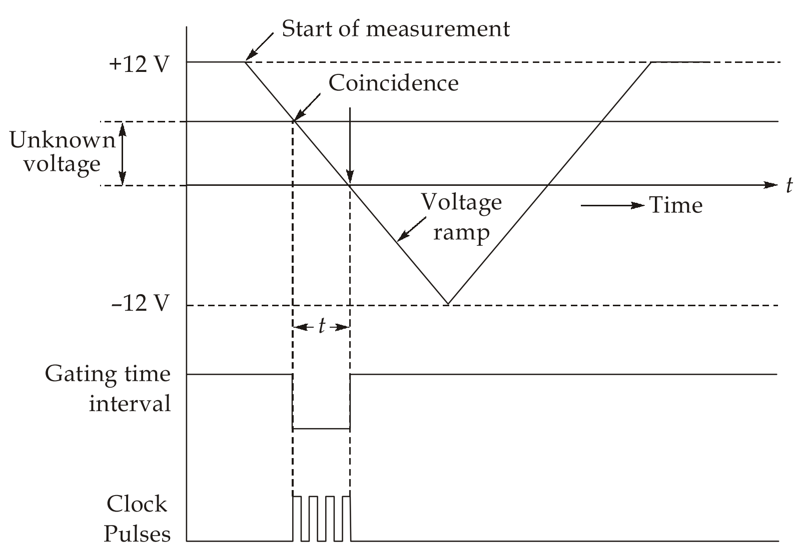
- The operating principle of a ramp type digital voltmeter is to measure the time that a linear ramp voltage takes to change from level of input voltage to zero voltage (or vice versa).This time interval is measured with an electronic time interval counter and the count is displayed as a number of digits on electronic indicating tubes of the output readout of the voltmeter.

- Block diagram of ramp type DVM is shown in figure below:




At the start of the measurement, a ramp voltage is initiated. The ramp voltage value is continuously compared with the voltage being measured (unknown voltage). At the instant, the value of ramp voltage is equal to that of unknown voltage, a coincidence circuit called an input comparator, generates a pulse which opens the Gate. The ramp voltage continues to decrease till it reaches ground level (zero voltage). At this instant, another comparator called ground comparator generates a pulse and closes the gate. The time elapsed between opening and closing of the gate is t as indicated in below Timing diagram. During this time interval, pulses from a clock pulse generator pass through the gate and are counted and displayed.

- The conversion of a voltage value of a time interval is as shown in figure below:




**(Timing diagram showing voltage to time conversion)**

- The decimal number as indicated by the read out is a measure of the value of input voltage.
- The sample rate multivibrator determines the rate at which the measurement cycles are indicated. The sample rate circuit provides an indicating pulse for the ramp generator to start its next ramp voltage. At the same time, it sends a pulse to the counter which sets all of them to 0. This momentarily removes the digital display of the readout.

**(ii)** Controlling torque at full scale deflection

$$T_c = 240 \times 10^{-6} \text{ N-m}$$

Deflecting torque at full scale deflection

$$T_d = NBAI = NB(lb)I, \quad \text{where } A = \text{Area of coil} = lb$$
$$= 100 \times 1 \times 40 \times 10^{-3} \times 30 \times 10^{-3}\, I$$
$$= 120 \times 10^{-3}\, I \text{ N-m}$$

At final steady state position,

$$T_d = T_c$$

or $$120 \times 10^{-3}\, I = 240 \times 10^{-6}$$

$\therefore$ Current at full scale deflection, $I$

$$= 2 \times 10^{-3} \text{ A} = 2 \text{ mA}$$

Let the resistance introduced in series with voltmeter be $R$

$\therefore$ Voltage across the instrument at full scale deflection = $2 \times 10^{-3} R$

This produces a deflection of 100 division

$\therefore$ $$\text{Volts per division} = 2 \times 10^{-3} R/100$$

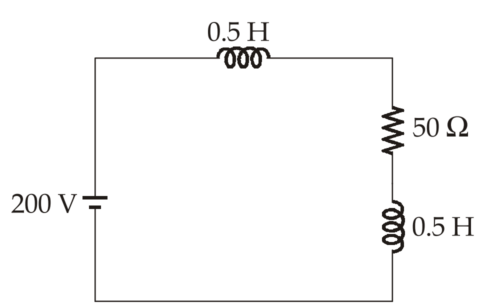
This value should be equal to 1 in order to get 1 volt per division

$\therefore$ $$2 \times 10^{-3} R/100 = 1$$

or $$R = 50000\ \Omega = 50\ \text{k}\Omega$$

**Q.6 (b) Solution:**

For current at $t$ = 10 ms after switch is closed, [Diodes $D_1$ and $D_2$ are reverse biased] the circuit will become



$$i(t) = i(\infty) + [i(0^+) - i(\infty)e^{-t/\tau}$$

where $i(0^+) = 0$ and

$$i(\infty) = \frac{200}{50} = 4\text{ A}$$

$$\tau = \frac{L_{eq}}{R} = \frac{0.5+0.5}{50} = \frac{1}{50}\text{sec}$$

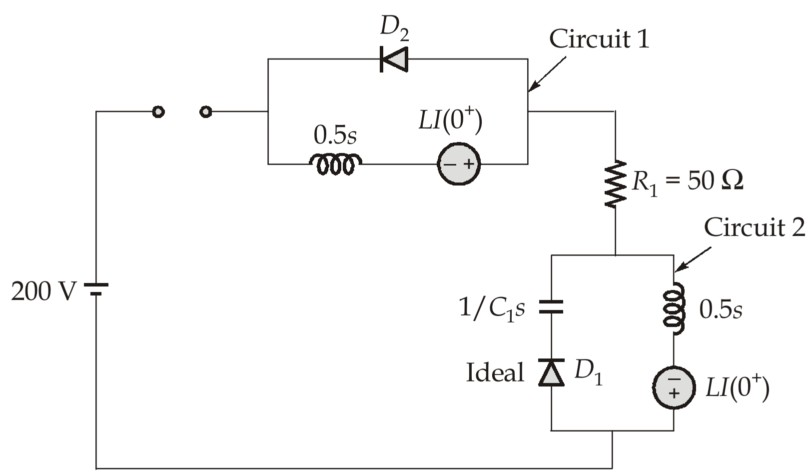
$\therefore$

$$i(t) = 4[1 - e^{-50t}]$$

At $t = 10$ ms,

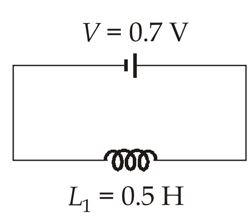
$$I_0 = i(10\text{ ms}) = 4[1 - e^{-50 \times 10\text{ ms}}]$$
$$= 1.57\text{ A}$$

After opening the switch, the circuit in $s$-domain is obtained as below:




Both the diodes will be forward biased and current will not flow in the resistance, hence current in $R_1 = 0$ A.

In circuit 1:




$$L_1\frac{di}{dt} + V = 0$$

$$\int_{I_0}^{I} di = \int_0^{8\text{ms}} \frac{-V}{L_1} dt$$

$$I - I_0 = \frac{-V}{L_1}\cdot(t)_0^{8\text{ms}}$$

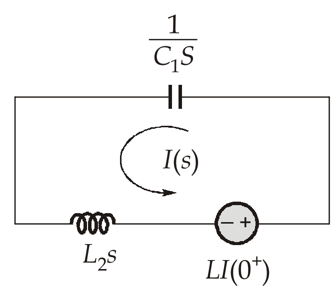
$$I = I_0 - \frac{V}{L_1} \times 8 \text{ ms}$$

Current in inductor $L_1$,

$$I = 1.57 - \left[\frac{0.7}{0.5} \times 8 \text{ ms}\right]$$

$$I = 1.558 \text{ A}$$

Analyzing circuit 2 in s-domain:




$$-L_2 I(0^+) + I(s)\frac{1}{C_1 s} + L_2 s I(s) = 0$$

$$I(s) = \frac{-L_2 I(0^+)}{\frac{1}{sC_1} + sL}$$

Solving we get,

$$I(s) = I_0 \cdot \frac{s}{s^2 + \frac{1}{L_2 C_1}}$$

Taking inverse Laplace transform,

$$i(t) = I_0 \times \cos \omega_0 t$$

where,
$$\omega_0 = \frac{1}{\sqrt{L_2 C_1}} = \frac{1}{\sqrt{0.5 \times 10 \times 10^{-6}}}$$

$$\omega_0 = 447.22 \text{ rad/sec}$$

At $t = 8$ ms,

Current in inductor $L_2$,

$$i(t) = 1.57 \times \cos(447.22 \times 8 \times 10^{-3})$$

$$= 1.57 \times \cos(3.57) = -1.42 \text{ A}$$

Voltage across $C_1$:

$$V = \frac{1}{C}\int i(t)dt$$

$$= \frac{1}{C}\int_0^{8\text{ms}} I_0 \cdot \cos(\omega_0 t)dt$$

$$= \frac{1.57}{4.4722\times10^{-3}}\times\sin(3.57)$$

$$= -145.84 \text{ V}$$

**Q.6 (c) Solution:**

From the given band diagram,

$$E_C - E_i = 0.56 \text{ eV}$$

$$E_i - E_F = 0.30 \text{ eV}$$

Metal oxide barrier potential,

$$q\phi_m = 3.15 \text{ eV}$$

Work function of semiconductor substrate,

$$q\phi_{sc} = 3.25 + 0.3 = 3.55 \text{ eV}$$

$$\text{oxide thickness, } t_{ox} = 50 \text{ nm}$$

Also given effective oxide charge density,

$$Q_{ox} = 10^{10} \text{ charge/cm}^2$$

**(i)** Work function difference, $\phi_{ms}$

$$\phi_{ms} = \phi_m - \phi_{sc}$$

$$= 3.15 - 3.55$$

$$\therefore \quad \phi_{ms} = -0.40 \text{ eV}$$

**(ii)** Surface potential, $\phi_s$

Surface potential equals band bending:

$$\phi_s = E_i(\text{surface}) - E_i(\text{bulk})$$

From the given energy band diagram, the band bending equals to 0.6 eV.

$$\therefore \quad \phi_s = 0.6 \text{ eV}$$

**(iii)** Oxide voltage, $V_{ox}$:

The oxide drop equals the difference between the oxide barriers (semiconductor and metal barriers)

$$\therefore \quad qV_{ox} = \phi_{sc} - \phi_m$$
$$= 3.55 - 3.15 = 0.4 \text{ eV}$$
$$\therefore \quad V_{ox} = 0.4 \text{ V}$$

**(iv)** Flat-band voltage, $V_{FB}$:

$$V_{FB} = \phi_{ms} - \frac{Q_{ox}}{C_{ox}}$$

where, oxide capacitance,

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9\epsilon_0}{t_{ox}}$$

$$C_{ox} = \frac{3.9\times 8.854\times 10^{-14}}{50\times 10^{-7}} = 69.06 \text{ nF/cm}^2$$

$$\therefore \quad V_{FB} = -0.40 - \frac{1.6\times 10^{-19}\times 10^{10}}{69.06\times 10^{-9}}$$
$$= -0.40 - 0.023$$
$$V_{FB} = -0.423 \text{ V}$$

**(v)** Threshold voltage, $V_T$:

Fermi potential, $\phi_F = E_i - E_F = 0.3$ V (given)

Threshold surface potential,

$$\phi_{ST} = 2\phi_F$$
$$= 0.6 \text{ V}$$

$$\therefore \quad \text{Threshold voltage, } V_T = V_{FB} + 2\phi_F = +\frac{Q_{dep}}{C_{ox}}$$
$$= -0.423 + 0.6 \qquad \text{...Neglecting the depletion charge}$$
$$V_T \simeq 0.177 \text{ V}$$

Since, $V_T > 0$, for a $p$-type substrate n-channel MOSFET, a positive gate voltage is required to form inversion.

$\therefore$ The given MOSFET is an Enhancement-mode n-channel MOSFET.

**Q.7 (a) Solution:**

**(i) 1.** 2's complement of 10010010

= 01101101 + 1 = 01101110

2's complement of 11011000

= 00100111 + 1 = 00101000

**2. Truth Table:** $Y = (A+BC)(B+\bar{C}A)$

| $A$ | $B$ | $C$ | $Y$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

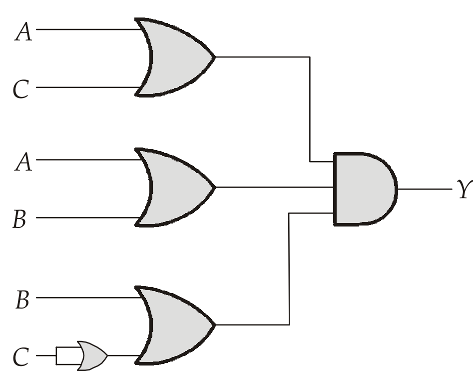
Minimization using K-map:

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 |

In POS (Product of sum) form,

$$Y = (A+C)\cdot(A+B)\cdot(B+\bar{C})$$

Using OR and AND Gates, the circuit can be implemented as below:

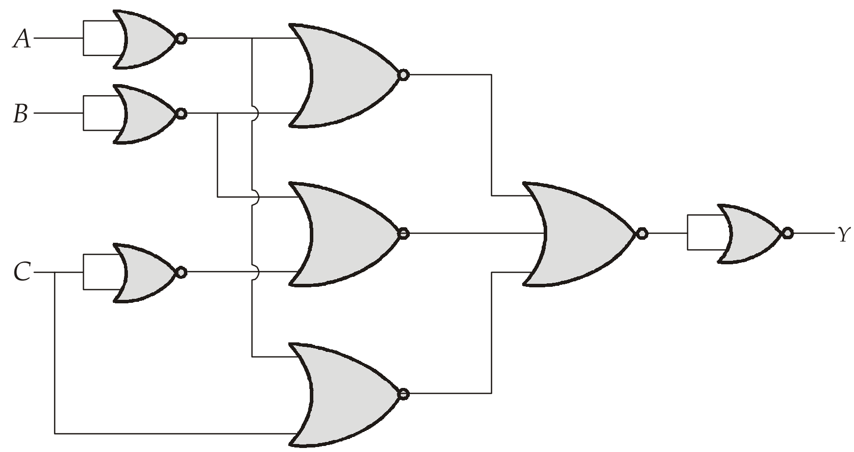



In SOP (Sum of Products) form,

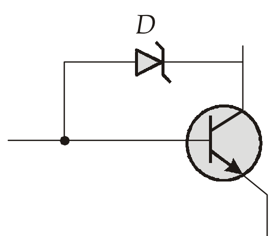
$$Y = AB + A\bar{C} + BC$$

$$Y = \overline{\bar{A}+\bar{B}} + \overline{\bar{A}+C} + \overline{\bar{B}+\bar{C}}$$

Using NOR Gates, the circuit can be implemented as below:




**3.**



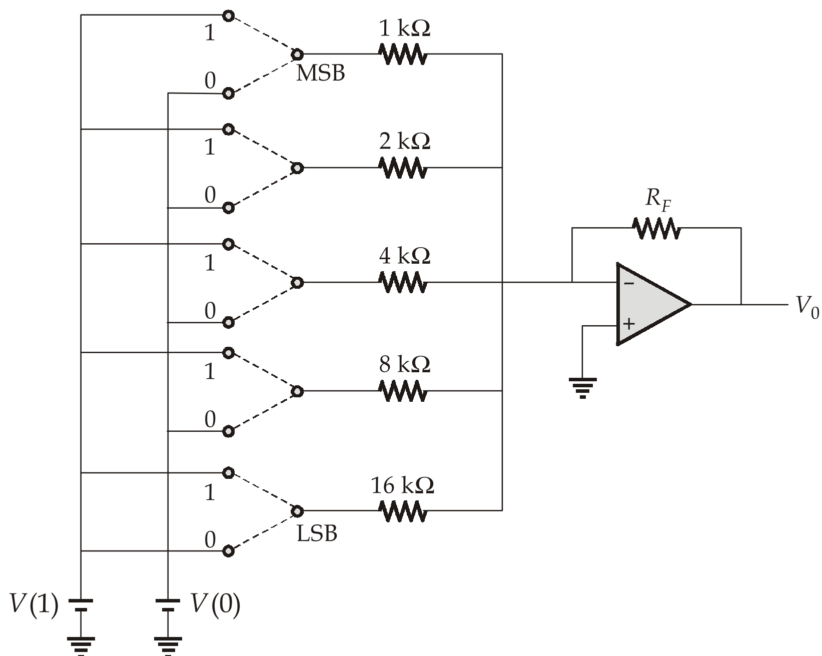

The speed limitation of TTL is mainly due to the turn off time delays involved in transistors while making transitions from saturation to cutoff.

Placing a schottky diode between base and collector prevents the transistor from entering into saturation. Thus, it makes the schottky TTL gate faster than the TTL gate.

A n-bit binary weighted resistor D/A converter uses resistor values scaled in binary powers ($R$, $2R$, $4R$, $8R$, ..., $2n$-$1R$)

Given:Lowest resistance = 1 kΩ and Highest resistance = 4 kΩT. Thus the existing resistor set is: 1 kΩ, 2 kΩ and 4 kΩ. So, the original DAC is a 3-bit DAC.

**(ii)**




If the bit length is to be increased by 2, 5 binary weighted resistors would be required. Thus, we need to add two more resistors of value 8 kΩ and 16 kΩ.

Complete circuit of the converter is shown above.

**Q.7 (b) Solution:**

**(i)** Utilizing the voltage-divider rule, we can express $V_i$ in terms of $V_s$ as follows

$$V_i = V_s \frac{Z_i}{Z_i + R_s}$$

where $Z_i$ is the amplifier input impedance. Since $Z_i$ is composed of two parallel elements, it is obviously easier to work in terms of $Y_i = 1/Z_i$. Dividing the numerator and denominator by $Z_i$, we obtain

$$V_i = V_s \frac{1}{1 + R_s Y_i}$$

$$= V_s \frac{1}{1 + R_s[(1/R_i) + sC_i]}$$

Thus, $$\frac{V_i}{V_s} = \frac{1}{1 + (R_s/R_i) + sC_iR_s}$$

This expression can be put in the standard form for a low-pass STC [Single Time constant] network by extracting $[1 + (R_s/R_i)]$ from the denominator, thus we have

$$\frac{V_i}{V_s} = \frac{1}{1+(R_s/R_i)}\frac{1}{1+sC_i[(R_sR_i)/(R_s+R_i)]} \quad ...(i)$$

At the output side of the amplifier, using the voltage-divider rule, we get

$$V_0 = \mu V_i \frac{R_L}{R_L+R_0}$$

This equation can be combined with Eq. (i) to obtain the amplifier transfer function as

$$\frac{V_0}{V_s} = \mu\frac{1}{1+(R_s/R_i)}\frac{1}{1+(R_0/R_L)}\frac{1}{1+sC_i[(R_sR_i)/(R_s+R_i)]} \quad ...(ii)$$

The last factor in this expression is a result of the input capacitance $C_i$, with the time constant being

$$\tau = C_i\frac{R_sR_i}{R_s+R_i} = C_s(R_s \parallel R_i) \quad ...(iii)$$

The DC gain can be found as:

$$K \equiv \left.\frac{V_0}{V_s}\right|_{s=0} = \mu\frac{1}{1+(R_s/R_i)}\frac{1}{1+(R_0/R_L)} \quad ...(iv)$$

The 3-dB frequency $\omega_0$ can be found from

$$\omega_0 = \frac{1}{\tau} = \frac{1}{C_i(R_s \parallel R_i)} \quad ...(v)$$

**(ii)** Substituting the numerical values given into Eq. (iv) results in

$$K = 144\frac{1}{1+(20/100)}\frac{1}{1+(200/1000)} = 100 \text{ V/V}$$

Thus, the amplifier has a dc gain of $20 \log_{10}100 = 40$ dB. Substituting the numerical values into Eq. (v) gives the 3-dB frequency as,

$$\omega_0 = \frac{1}{60 \text{ pF}\times(20 \text{ k}\Omega \parallel 100 \text{ k}\Omega)}$$

$$= \frac{1}{60\times10^{-12}\times(20\times100/(20+100))\times10^3} = 10^6 \text{ rad/s}$$

Thus, $$f_0 = \frac{10^6}{2\pi} = 159.2 \text{ kHz}$$

Since the gain falls off at the rate of –20 dB/decade, starting at $\omega_0$, the gain will reach 0 dB in two decades (a factor of 100); thus we have

Unity-gain frequency = 100 × $\omega_0$ = $10^8$ rad/s or 15.92 MHz

**(iii)** To find $v_0(t)$, we need to determine the gain magnitude and phase at $10^2$ rad/s. This can be done either approximately utilizing the Bode plots or exactly utilizing the expression for the amplifier transfer function.

$$T(j\omega) \equiv \frac{V_0(j\omega)}{V_s(j\omega)}$$

$$= (j\omega) = \frac{100}{1 + j(\omega/10^6)} = \frac{100}{\sqrt{1+(\omega/10^6)^2}} \angle -\tan^{-1}\left(\frac{\omega}{10^6}\right)$$

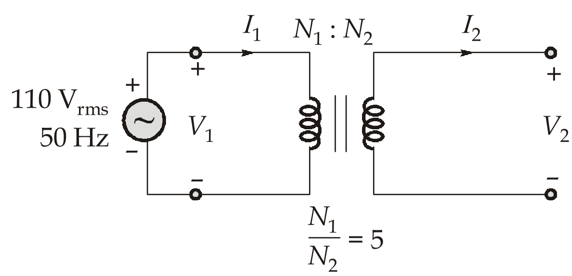
For $\omega = 10^2$ rad/s, we have $|T| \simeq 100$ and $\phi = -\tan^{-1} 10^{-4} \simeq 0°$. Thus,

$$v_0(t) = 10 \sin 10^2 t \text{ V}$$

**Q.7 (c) Solution:**

**Case-I (when switch is opened):**

The given transformer circuit can be drawn in this case as follows:




As the switch is open circuited, the current $I_2$ in the secondary side will be zero.

So,
$$I_{2\,rms} = 0$$

$$I_{1\,rms} = \frac{N_2}{N_1} I_{2\,rms} = 0$$

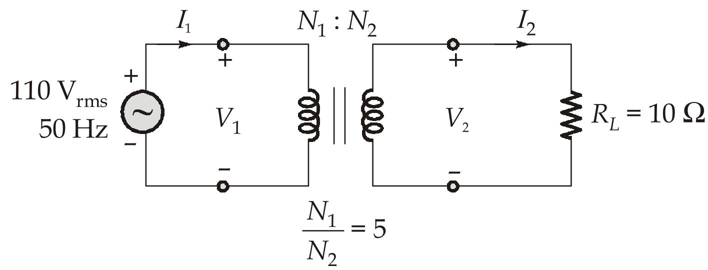
$$\frac{V_{2\,rms}}{V_{1\,rms}} = \frac{N_2}{N_1} = \frac{1}{5}$$

Given,
$$V_{1\,rms} = 110 \text{ V}$$

So,
$$V_{2\,rms} = \frac{1}{5}(110) \text{ V} = 22 \text{ V}$$

**Case-II (when switch is closed):**

The given transformer circuit can be drawn in this case as follows:




$$\frac{V_{2\text{ rms}}}{V_{1\text{ rms}}} = \frac{N_2}{N_1} = \frac{1}{5}$$

Given, $$V_{1\text{ rms}} = 110 \text{ V}$$

$\therefore$ $$V_{2\text{ rms}} = \frac{1}{5} V_{1\text{ rms}} = 22 \text{ V}$$

Thus, $$I_{2\text{ rms}} = \frac{V_{2\text{ rms}}}{R_L} = \frac{22 \text{ V}}{10\ \Omega} = 2.2 \text{ A}$$

$$\frac{I_{1\text{ rms}}}{I_{2\text{ rms}}} = \frac{N_2}{N_1} = \frac{1}{5}$$

$$I_{1\text{ rms}} = \frac{I_{2\text{ rms}}}{5} = \frac{2.2}{5} \text{ A} = 0.44 \text{ A}$$

**Q.8 (a) Solution:**

Given,

Intrinsic layer thickness, $W = 5\ \mu\text{m}$

Distance of start of intrinsic layer from surface,

$$x_1 = 1.5\ \mu\text{m}$$

Hence, intrinsic layer ends at,

$$x_2 = x_1 + W = 6.5\ \mu\text{m}$$

For a PIN photodiode, the optical intensity decays exponentially inside the semiconductor according to Beer–Lambert law as,

$$I(x) = I_0 e^{-\alpha x}$$

Fraction of photons absorbed in the intrinsic layer i.e. between $x_1$ and $x_2$,

$$F = e^{-\alpha x_1} - e^{-\alpha x_2}$$

$$F = e^{-\alpha x_1}(1 - e^{-\alpha W})$$

We must maximize $F$ w.r.t $\alpha$, To maximize absorption, differentiate $F$ with respect to a and equate to zero,

$$\frac{dF}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left[e^{-\alpha x_1}(1-e^{-\alpha W})\right] = 0$$

$$-x_1 e^{-\alpha x_1} + (W + x_1)\cdot e^{-\alpha(x_1+W)} = 0$$

$$-x_1 e^{-\alpha x_1} + x_2 e^{-\alpha x_2} = 0$$

$$x_2 e^{-\alpha x_2} = x_1 e^{-\alpha x_1}$$

$$e^{-\alpha(x_2-x_1)} = \frac{x_1}{x_2}$$

Since $$x_2 - x_1 = W$$

$$e^{-\alpha W} = \frac{x_1}{x_2}$$

$$-\alpha W = \ln\left(\frac{x_1}{x_2}\right)$$

Optical attenuation coefficient to maximize absorption

$$\alpha = \frac{1}{W}\ln\left(\frac{x_2}{x_1}\right)$$

$$\alpha = \frac{1}{5}\ln\left(\frac{6.5}{1.5}\right)$$

∴ $$\alpha \approx 0.293\ \mu m^{-1}$$

**Q.8 (b) Solution:**

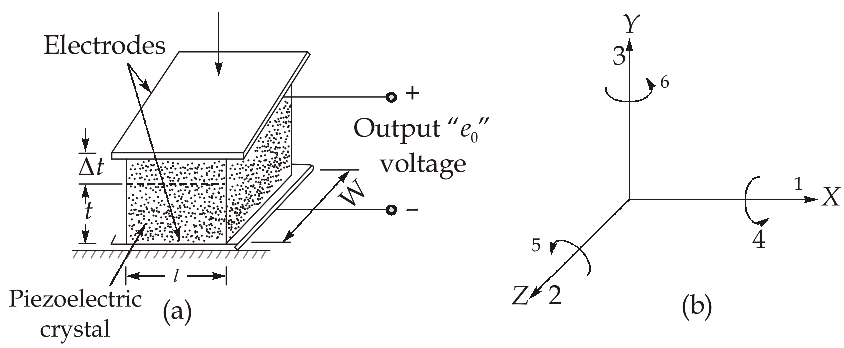
**(i) Piezo-electric Transducers:**

A piezo-electric material is one in which an electric potential appears across certain surfaces of a crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. The effect is reversible i.e., conversely, if a varying potential is applied to the proper axis of the crystal, it will change the dimensions of the crystal thereby deforming it. This effect is known as Piezo-electric effect. Elements exhibiting Piezoelectric-qualities are called as electro-resistive elements. The piezoelectric effect is very useful within many applications that involve:

- Production and detection of sound
- Generation of high voltages
- Electronic frequency generation
- Microbalances,
- Ultrafine focusing of optical assemblies
- Common Piezo-electric material include Rochelle salts, ammonium dihydrogen phosphate, lithium sulphate, dipotassium tartarate, potassium dihydrogen phosphate, quartz and ceramics $A$ and $B$.
- The materials that exhibit a significant and useful Piezo-electric effect are divided into two categories:

  (i) Natural group and (ii) Synthetic group
- Quartz and Rochelle salt belong to the natural group. Materials like lithium sulfate, ethylene diamine tartarate belong to the synthetic group.

**(ii)**




By the Piezo-electric effect,

$$Q \propto F$$

$$Q = d \cdot F \quad \text{...(i)}$$

where, $d$ = charge sensitivity

The piezo-electric crystal can be considered as a capacitor with a dielectric placed between two parallel plates. For a capacitor,

$$Q = C \times V$$

Here, $Q = C_P \times e_0$ ($C_P$ = parallel capacitor)

$$e_0 = \frac{Q}{C_P} \quad \text{...(ii)}$$

We know that, $C_P = \dfrac{\in A}{t}$

($t$ = thickness of the crystal, $A$ = area of crystal) ...(iii)

Substituting the value of $C_P$ in equation (ii), we get,

$$e_0 = \frac{Q \times t}{\in \times A}$$

But, $Q = d \times F$ as equation (i)

or, $$e_0 = \frac{d \times F \times t}{\in \times A}$$

Output voltage, $e_0 = g.p.t.$

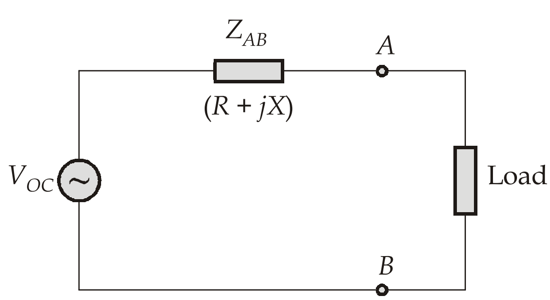
where, $\frac{d}{\in} = g$ = voltage sensitivity

$\frac{F}{A} = P$ = pressure

$t$ = thickness

**Q.8 (c) Solution:**

**(i)**

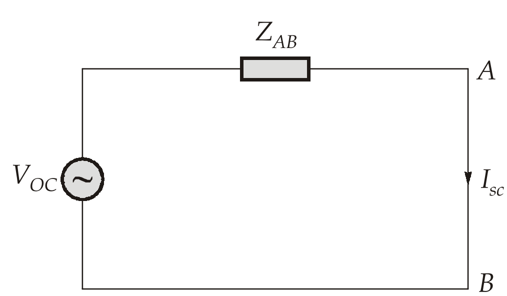



Given,

open circuit voltage, $V_{OC} = 150$ V

$V_{OC} = 150\angle 0°$ V

Now, when the circuit is short circuited around point $AB$, we get



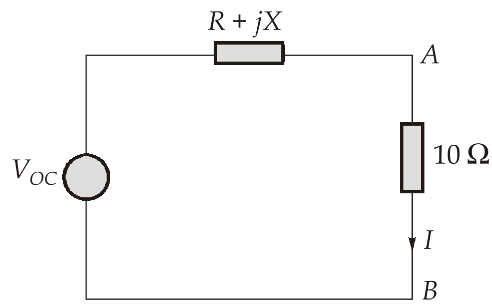

$$I_{sc} = \frac{V_{OC}}{R + jX}$$

$$|I_{sc}| = \frac{|V_{OC}|}{\sqrt{R^2 + X^2}}$$

so, $$\frac{150}{\sqrt{R^2+X^2}} = 3$$

$$\Rightarrow \quad R^2 + X^2 = 2500 \quad ...(i)$$

Now, when load $R = 10\ \Omega$ applied across $AB$




$$I = \frac{V_{OC}}{(R+10)+jX}$$

$$|I| = \frac{|V_{OC}|}{\sqrt{(R+10)^2+X^2}}$$

$$2.65 = \frac{150}{\sqrt{(R+10)^2+X^2}}$$

$$\sqrt{(R+10)^2+X^2} = \frac{150}{2.65} = 56.6$$

$$(R+10)^2 + X^2 = 3203.987 \quad ...(ii)$$

Using equations (i) and (ii), we get

$$(R+10)^2 + 2500 - R^2 = 3203.98$$

$$R^2 + 100 + 20R + 2500 - R^2 = 3203.98$$

$$20R = 3203.98 - 2500 - 100$$

$$20R = 603.98$$

$$R = 30.199\ \Omega$$

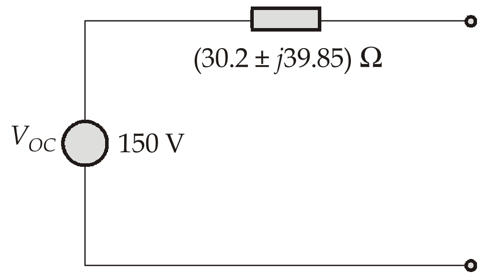
Now from equation (i),

$$R^2 + X^2 = 2500$$

$$X^2 = 2500 - (30.199)^2$$

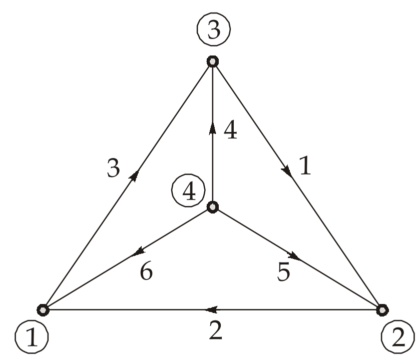
$$X = \pm 39.85\ \Omega$$

So equivalent circuit is




According to mathematical equation, 'X' may be negative or positive. Thus, 'X' may be capacitive or inductive.

**(ii) Network graph:**




Trees: A tree is a connected subgraph that includes all nodes of the graph but contains no closed loops (cycles). For graph with $N = 4$ nodes, a tree must have $N - 1 = 3$ branches. The three possible trees of the network are shown below:

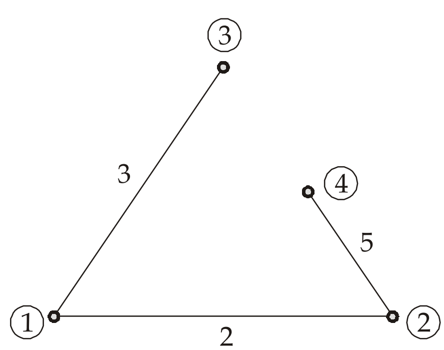



**Fig. (a)**

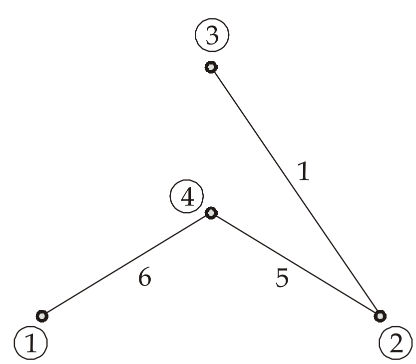



**Fig. (b)**

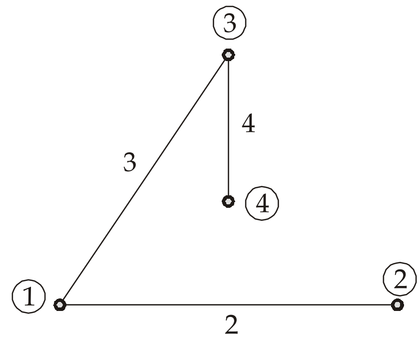



**Fig. (c)**

Given, $L$ = Number of links

$B$ = Number of branches

$N$ = Number of nodes

For any connected network graph:

- A tree always contains $(N - 1)$ branches, called twigs.
- The remaining branches of the original graph are called links (or chords).

We have, Total branches = Twigs + Links.

Thus, $B = (N-1) + L \Rightarrow L = B - (N-1)$

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