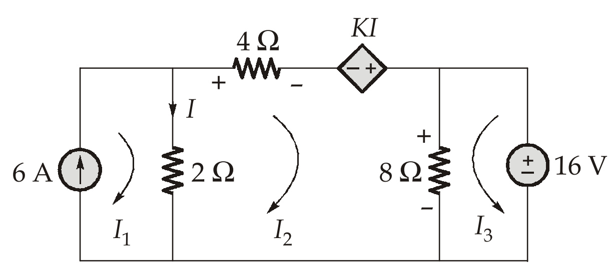
Detailed Solutions

ESE-2026 Mains Test Series

Electrical Engineering Test No : 10

Section-A

Q.1 (a) Solution:



$$I_1 = 6\text{ A} \quad \text{...(i)}$$

$$I = I_1 - I_2 \quad \text{...(ii)}$$

Apply KVL to the loops

$$-4I_2 + KI - 8I_2 - 8I_3 - 2I_2 + 2I_1 = 0$$

$$-8I_3 - 8I_2 + 16 = 0$$

$$I_2 + I_3 = 2 \quad \text{...(iii)}$$

$$(14 + K)I_2 - 8I_2 = 12 + 6K - 16$$

$$I_2 = \frac{6K-4}{6+K}$$

$$I = I_1 - I_2$$

$$= 6-\frac{(6K-4)}{6+K} = \frac{36+6K-6K+4}{6+K} = \frac{40}{6+K}$$

Now power dissipated in 2 Ω is 50 W.

$$P_{2\,\Omega} = I^2 \times R = \left[\frac{40}{6+K}\right]^2 \times 2\text{ s} = 50$$

$$\frac{40}{6+K} = \sqrt{\frac{50}{2}} = 5$$

$$6 + K = 8$$

$$\therefore \quad K = 2$$

Hence, for $K \geq 2$,

$\Rightarrow$ Power dissipation in 2 Ω resistor will not exceed 50 W.

Q.1 (b) Solution:

Curve tracing $y = x \log_e x$

Clearly, $x > 0$

For $0 < x < 1$, $x \log_e x < 0$ and for $x > 1$, $x \log_e x > 0$.

Also $x \log_e x = 0$ for $x = 1$.

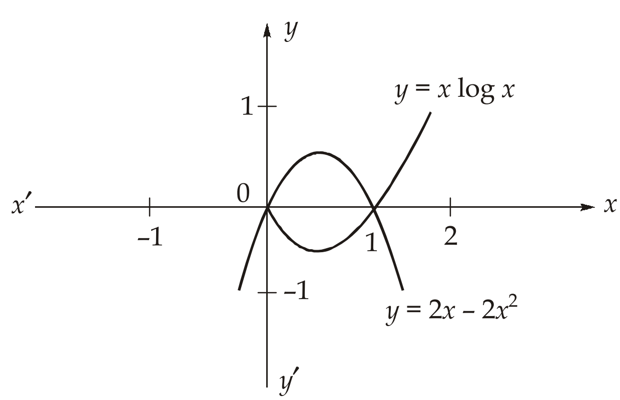
Further $\dfrac{dy}{dx} = 0$

$$1 + \log_e x = 0$$

or $x = \dfrac{1}{e}$ which is point of minima.

$$\lim_{x \to \infty} x \log_e x = \infty$$

Graph of function is as shown in the figure.



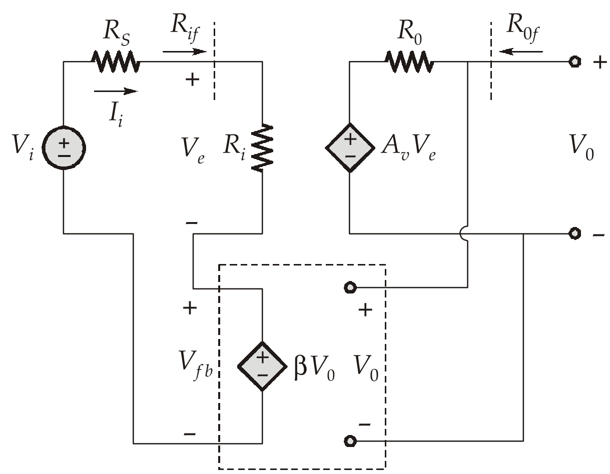
$$\text{Required area} = \int_0^1 (2x - 2x^2)\,dx - \int_0^1 x \log x \, dx$$

$$= \left[x^2 - \frac{2x^3}{3}\right]_0^1 - \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_0^1$$

$$= \left(1 - \frac{2}{3}\right) - \left[0 - \frac{1}{4} - \frac{1}{2}\lim_{x\to 0} x^2 \log x\right] = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

Q.1 (c) Solution:

Series-shunt feedback amplifier:



Input resistance including feedback is denoted by R_{if},

by applying KVL in the input loop,

$$V_i = V_e + V_{fb}$$

$$= V_e + \beta V_0 = V_e + \beta(AV_e)$$

[∵ Neglecting source resistance R_s]

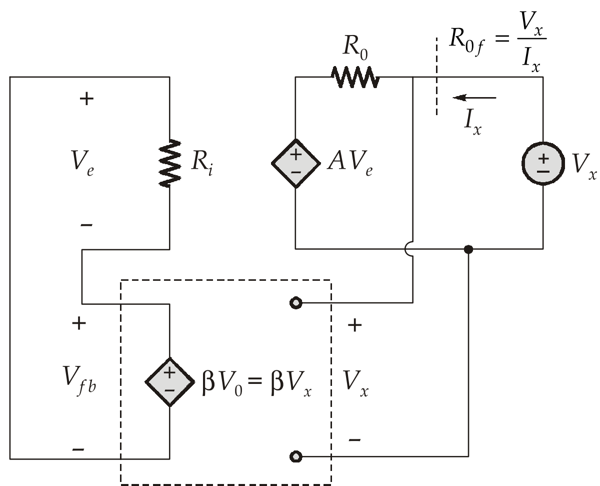
$$V_e = \frac{V_i}{1+\beta A} \quad \text{...(i)}$$

The input current, $$I_i = \frac{V_e}{R_i} = \frac{V_i}{R_i(1+\beta A)}$$

[By substituting equation (i) in above expression]

$$R_{if} = \frac{V_i}{I_i} = R_i\,(1 + \beta A)$$

Output resistance of the feedback circuit is R_{0f}, to obtain this the input signal voltage source is set equal to zero and test voltage V_x is applied to the output terminals.



$$V_e + \beta V_x = 0$$

$$V_e = -\beta V_x$$

$$I_x = \frac{V_x - AV_e}{R_0} = \frac{V_x + A\beta V_x}{R_0}$$

$$\frac{I_x}{V_x} = \frac{1 + A\beta}{R_0}$$

$$R_{0f} = \frac{R_0}{(1 + A\beta)}$$

Q.1 (d) Solution:

Voltage across instrument for full scale deflection = 100 mV.

Current in instrument for full scale deflection

$$I = \frac{V}{R} = \frac{100 \times 10^{-3}}{20} = 5 \times 10^{-3}\ A$$

Deflection torque T_d = $NBIdl = 100 \times B \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 5 \times 10^{-3}$

= $375 \times 10^{-6} B$ N-m.

∴ Controlling torque for a deflection $\theta = 120°$

$$T_c = K\theta = 0.375 \times 10^{-6} \times 120 = 44 \times 10^{-6} \text{ N-m}$$

At final steady position, $T_d = T_c$ or $375 \times 10^{-6} B = 44 \times 10^{-6}$

∴ Flux density in the air gap $B = \frac{44 \times 10^{-6}}{375 \times 10^{-6}} = 0.12 \text{ Wb/m}^2$

Resistance of winding R_c = $0.3 \times 20 = 6\ \Omega$

Length of mean turn L_{mt} = $2(l + d) = 2(30 + 25) = 110$ mm.

Let a be the area of cross-section of wire and ρ be the resistivity.

$$\text{Resistance of coil } R_c = \frac{N\rho L_{mt}}{a}$$

$$\therefore \quad \text{Area of cross-section of wire } a = \frac{100\times1.7\times10^{-8}\times110\times10^{-3}}{6} = 31.17\times10^{-9}\text{ m}^2$$

$$= 31.17 \times 10^{-3}\text{ mm}^2$$

$$\text{Diameter of wire } d = \left(\frac{4}{\pi}\times31.17\times10^{-3}\right)^{1/2} = 0.2\text{ mm}$$

Q.1 (e) Solution:

From plot,

$$I = 0.01\text{ mA/V} = 0.4\text{ V}$$

and

$$I = 10\text{ mA/V} = 0.75\text{ V}$$

Diode current equation,

$$\Rightarrow \quad I = I_0(e^{V/\eta V_\tau} - 1)$$

For forward bias $V >> \eta\ V_T$

$$\Rightarrow \quad I \approx I_0 e^{V/\eta V_T}$$

$$\Rightarrow \quad 10^{-5} = I_0 e^{0.4/0.0259\eta} \quad \text{...(i)}$$

$$10^{-2} = I_0 e^{0.75/0.0259\eta} \quad \text{...(ii)}$$

Divide (ii) by (i),

$$\Rightarrow \quad 10^3 = e^{(0.75-04)/0.0259\eta}$$

$$\frac{13.46}{\eta} = \ln(1000) = 6.91$$

$$\Rightarrow \quad \eta = 1.94$$

In forward bias, $I \approx I_0 e^{V/\eta V_T}$

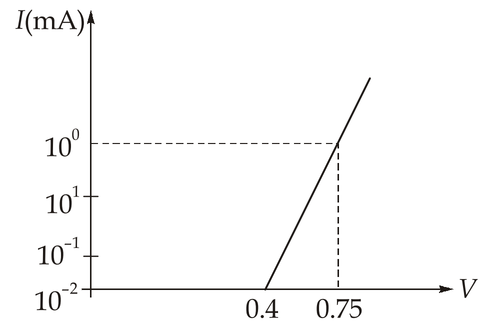
Take log of base 10

$$\log(\text{I}) = \log(I_0) + \log(e)\times\frac{V}{\eta V_T}$$

$$= \log(I_0) + \frac{0.454}{0.0259\times\eta}\times V$$

$$\Rightarrow \qquad \log(I) = \log(I_0) + \frac{17.46}{\eta} V$$

This is an equation for straight line



Q.2 (a) (i) Solution:

Let, $f(z) = (u + i\,v)$ be analytic function

Then, $\qquad i\,f(z) = (i\,u - v)$

Adding the above equations,

We get $\qquad (1 + i)\,f(z) = (u - v) + i(u + v)$

$$F(z) = U + iV \qquad \text{where } F(z) = (1 + i)\,f(z),$$

$$U = (u - v)$$

and $\qquad V = (u + v)$

$$U = (u - v) = (x - y)\,(x^2 + 4xy + y^2)$$

$$\frac{\partial U}{\partial x} = x^2 + 4xy + y^2 + (x - y)\,(2x + 4y) = 3x^2 + 6xy - 3y^2$$

$$\frac{\partial U}{\partial y} = -(x^2 + 4xy + y^2) + (x - y)\,(4x + 2y) = 3x^2 - 6xy - 3y^2$$

Put $x = z$ and $y = 0$ in above,

We get, $\qquad \dfrac{\partial U}{\partial x} = 3z^2$ and $\dfrac{\partial U}{\partial y} = 3z^2$

$$F^1(z) = \frac{\partial U}{\partial x} - i\frac{\partial U}{\partial y}$$

$$F^1(z) = 3z^2\,dz - i^3z^2\,dz$$

$$F(z) = z^3 - iz^3 + ic$$

where $c = c_1 + ic_2$ is integral complex constant

$$(1 + i)\,f(z) = (1 - i)z^3 + ic$$

$$f(z) = \frac{1-i}{1+i}z^3 + \frac{ic}{1+i}$$

$$\therefore \quad f(z) = -iz^3 + \frac{(i-1)c}{2}$$

Q.2 (a) (ii) Solution:

$$\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

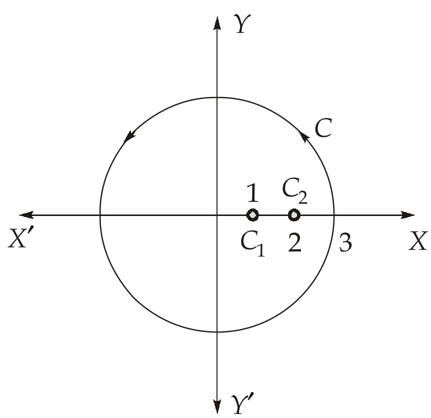
Poles of the integrated are given by putting the denominator equal to zero.

$$(z - 1)(z - 2) = 0$$

$$z = 1, 2$$

The integral has two poles at $z = 1, 2$.

The given circle $|z| = 3$ with centre at $z = 0$ and radius 3 encloses both the poles $z = 1$ and $z = 2$.



$$\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz = \int_{c_1} \frac{\frac{\sin \pi z^2 + \cos \pi z^2}{z-2}}{(z-1)} dz + \int_{c_2} \frac{\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)}}{(z-2)} dz$$

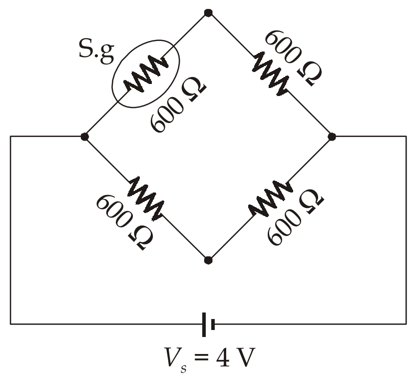
$$= 2\pi i \left[\frac{\sin \pi z^2 + \cos \pi z^2}{z-2}\right]_{z=1} + 2\pi i \left[\frac{\sin \pi z^2 + \cos \pi z^2}{z-1}\right]_{z=2}$$

$$= 2\pi i \left(\frac{\sin \pi + \cos \pi}{1-2}\right) + 2\pi i \left(\frac{\sin 4\pi + \cos 4\pi}{2-1}\right)$$

$$= 2\pi i \left(\frac{-1}{-1}\right) + 2\pi i \left(\frac{1}{1}\right) = 4\pi i$$

Which is the required value of the given integral.

Q.2 (b) (i) Solution:



$$R = 600\ \Omega \text{ (Strain gauge resistance)}$$

Given,

$$\text{G.F} = 2.5$$

$$\frac{\Delta L}{L} = 100\ \mu\text{m} = 100 \times 10^{-6}$$

So,

$$\Delta R = G.F. \times \frac{\Delta L}{L} \times R = 2.5 \times 100 \times 10^{-6} \times 600$$

$$= 0.15$$

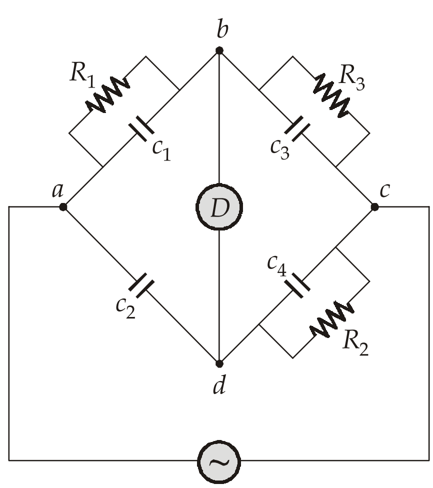
So,

$$V_B = \frac{V_s}{4} \times \frac{\Delta R}{R + 2\Delta R} = \frac{4}{4} \times \frac{0.15}{600 + 2(0.15)}$$

$$= 2.4987 \times 10^{-4} \approx 250\ \mu\text{V}$$

Q.2 (b) (ii) Solution:

The bridge circuit is shown in figure below:



For balance $$Y_1Y_4 = Y_2Y_3$$

or $$\left(\frac{1}{R_1}+j\omega C_1\right)\left(\frac{1}{R_4}+j\omega C_4\right) = (j\omega C_2)\left(\frac{1}{R_3}+j\omega C_s\right)$$

or $$\left(\frac{1}{R_1R_4}-\omega^2C_1C_4\right)+j\omega\left(\frac{C_4}{R_1}+\frac{C_1}{R_4}\right) = j\omega\frac{C_2}{R_3}-\omega^2C_2C_3$$

Equating the real and imaginary parts, we have,

$$\frac{1}{R_1R_4}-\omega^2C_1C_4 = -\omega^2C_2C_3 \quad ...(i)$$

and $$\frac{C_4}{R_1}+\frac{C_1}{R_4} = \frac{C_1}{R_3} \quad ...(ii)$$

From (i) and (ii) we have:

Now $$\omega^2C_2C_3C_4R_4^2 < \frac{C_2R_4}{R_3} \text{ and } \omega^2C_4^2R_4^2 < 1$$

Hence we can write $$C_1 = C_2\frac{R_4}{R_3}$$

When the capacitor C_1 is without specimen dielectric, let its capacitance be C_0

$$\therefore \quad C_0 = C_2\frac{R_4}{R_3} = 150\times\frac{5000}{5000} = 150 \text{ pF}$$

When the specimen is inserted as dielectric, let the capacitance be C_1.

$$\therefore \quad C_1 = C_2\frac{R_4}{R_3} = 900\times\frac{5000}{5000} = 900 \text{ pF}$$

Now $$C_0 = \epsilon_0 A/d \quad \text{and} \quad C_1 = \epsilon_r\epsilon_0 A/d$$

Hence relative permittivity of speciman

$$\epsilon_r = -\frac{C_1}{C_0} = \frac{900}{150} = 6$$

Q.2 (c) Solution:

Current, $$i = 0.5 + 0.3\sin\omega t - 0.2\sin 2\omega t$$

(i) Reading of moving coil ammeter,

$$I_{av} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} i\,dt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} (05 + 0.3\sin\omega t - 0.2\sin 2\omega t)dt$$

$$= \frac{\omega}{2\pi}\left[0.5t - 0.3\frac{\cos\omega t}{\omega} + \frac{0.2\cos 2\omega t}{2\omega}\right]_0^{\frac{2\pi}{\omega}}$$

$$= \frac{\omega}{2\pi}\times\frac{\pi}{\omega} = 0.5 \text{ ampere}$$

(ii) Reading of hot wire ammeter = I_{rms}

and $$I_{rms}^2 = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} i^2 dt$$

$$= \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} (0.25 + 0.09\sin\omega t + 0.04\sin^2 2\omega t)dt$$

$$+(0.3\sin\omega t - 0.12\sin\omega t\sin 2\omega t - 0.2\sin 2\omega t)dt$$

$$= \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\left[0.25 + 0.09\left(\frac{1-\cos 2\omega t}{2}\right)\right] + 0.04\left(\frac{1-\cos 4\omega t}{2}\right) + 0.3\sin\omega t$$

$$-0.12\left[\frac{(\cos\omega t - \cos 3\omega t)}{2} - 0.2\sin 2\omega t\right]dt$$

$$= \frac{\omega}{2\pi}\times\frac{2\pi}{\omega}\times 0.315 = 0.315\text{ A}$$

$$I_{rms} = \sqrt{0.315} = 0.561\text{ A}$$

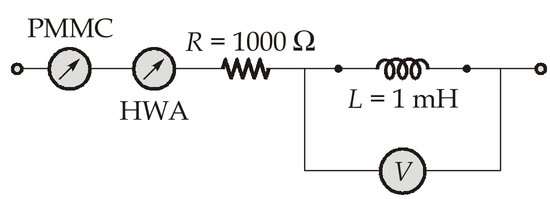
(iii) Reading of electrostatic voltmeter connected across inductance

(Voltage drop across inductance)$_{rms}$ = $\left(L\frac{di}{dt}\right)_{rms}$

Since, $$L\frac{di}{dt} = 1\times 10^{-3}\frac{d}{dt}(05 + 0.3\sin\omega t - 0.2\sin 2\omega t)$$

$$= 1 \times 10^3 \times \omega(0.3\cos\omega t - 0.4\cos 2\omega t)$$

$$\left(L\frac{di}{dt}\right)^2 = 1 \times 10^{-6} \times \omega^2(0.3\cos\omega t - 0.4\cos 2\omega t)^2$$



$$\left(L\frac{di}{dt}\right)^2 = 1\times10^{-6}\times\omega^2\times\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}(0.3\cos\omega t - 0.4\cos2\omega t)^2 dt$$

$$= \frac{\omega^3}{2\pi}\times10^{-6}\times\int_0^{\frac{2\pi}{\omega}}(009\cos^2\omega t - 0.24\cos\omega t\cos2\omega t + 0.16\cos^2 2\omega t)dt$$

$$= \frac{\omega^3}{2\pi}\times10^{-6}\times\int_0^{\frac{2\pi}{6}}\left[\frac{0.09}{2}(1+\cos2\omega t) - \frac{0.24}{2}(\cos3\omega t + \cos\omega t) + \frac{0.16}{2}(1+\cos4\omega t)\right]dt$$

$$= \frac{\omega^3}{2\pi}\times10^{-6}\times\frac{2\pi}{\omega}\times0.125 = 0.125\times10^{-6}\omega^2$$

$$= 0.125\times10^{-6}\times(10^6)^2 = 125000, \qquad \text{since } \omega = 10^6$$

or $$\left(L\frac{di}{dt}\right)_{rms} = \sqrt{125000} = 354\text{ V}$$

$\therefore$ Reading of voltmeter connected across inductance = 354 V

Q.3 (a) Solution:

$$\text{Line charge, } \rho_{L1} = 15\text{ nC/m at } y = -1, z = 0$$

$$\rho_{L2} = -15\text{ nC/m at } y = 1, z = 0$$

The radial distance between the line charge ρ_{L1} = 15 nC/m at y = –1, z = 0 and point $P(x, 0, z)$ is

$$R_1 = \sqrt{(0-(-1))^2 + (z-0)^2} = \sqrt{1+z^2}$$

Now electric field intensity, $$E_L = \frac{\rho_L}{2\pi\varepsilon_0 R_1}\hat{a}_R$$

Where, $$\hat{a}_{R1} = \frac{R_1}{|R_1|} = \frac{(0-(-1))\hat{a}_y + (z-0)\hat{a}_z}{\sqrt{1+z^2}}$$

$$\hat{a}_{R1} = \frac{\hat{a}_y + z\hat{a}_z}{\sqrt{1+z^2}}$$

$$E_{L1} = \frac{15\times10^{-9+12}(\hat{a}_y + z\hat{a}_z)}{2\pi\times8.85(1+z^2)}$$

$$= \frac{269.89\hat{a}_y + 269.89z\,\hat{a}_z}{(1+z^2)}$$

The radial distance between the line charge ρ_{L2} = –15 nC/m at y = 1, z = 0 and point $P(x, 0, z)$ is

$$R_2 = \sqrt{(0-1)^2 + (z-0)^2} = \sqrt{1+z^2}$$

Where,
$$\hat{a}_{R2} = \frac{R_2}{|R_2|} = \frac{(0-1)\hat{a}_y + (z-0)\hat{a}_z}{\sqrt{1+z^2}} = \frac{-\hat{a}_y + z\hat{a}_z}{\sqrt{1+z^2}}$$

Thus,
$$E_{L2} = \frac{-15\times 10^{-9+12}}{2\pi \times 8.85}\frac{(-\hat{a}_y + z\hat{a}_z)}{1+z^2}$$

$$= \frac{269.89\hat{a}_y - 269.89z\hat{a}_z}{1+z^2}$$

The electric field intensity at $P(x, 0, z)$

$$E(x, 0, z) = E(0, 0, z)$$

$$= E_{L1} + E_{L2}$$

$$= \frac{269.89\hat{a}_y + 269.89z\hat{a}_z}{1+z^2} + \frac{269.89\hat{a}_y - 269.89z\hat{a}_z}{1+z^2}$$

$$= \frac{539.78\hat{a}_y}{1+z^2}\text{ V/m}$$

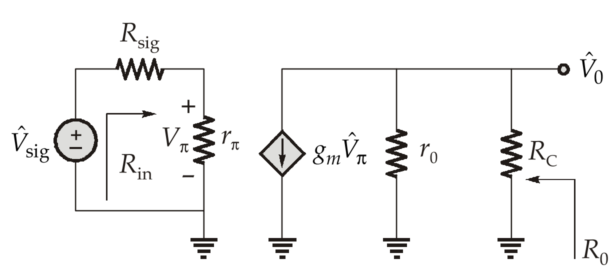
Q.3 (b) Solution:

At
$$I_C = 1\text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\text{ mA/V}} = 2.5\text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_C} = \frac{100\text{ V}}{1\text{ mA}} = 100\text{ k}\Omega$$



The amplifier characteristic parameters

$$R_{in} = r_\pi = 2.5\text{ k}\Omega$$

$$A_{V0} = -g_m \times (R_C \parallel r_0)$$

$$= -40\text{ mA/V} \times (R_c \parallel r_0)$$

$$(R_c \parallel r_0) = (5\text{k}\Omega \parallel 100\text{k}\Omega)$$

$$R_0 = \frac{5 \times 100}{105} = 4.762\text{ k}\Omega$$

$$A_{V0} = (-40\text{ mA/V}) \times 4.762\text{k}\Omega$$

$$= -190.5\text{ V/V}$$

With load resistance $R_L = 6\text{ k}\Omega$ connected the output

$$A_V = A_{V0} \times \frac{R_L}{R_L + R_0} = -190.5 \times \frac{6}{6 + 4.762} = -106.23\text{ V/V}$$

or $$A_V = -g_m(R_C \parallel R_L \parallel r_0) = -40(5 \parallel 6 \parallel 100) = -106.23\text{ V/V}$$

The overall voltage gain, $$G_V = \frac{R_{in}}{R_{in} + R_{sig}} A_V = \frac{2.5}{2.5 + 5} \times -106.23 = -35.41\text{ V/V}$$

If the maximum amplitude of $\hat{V}_\pi$ is 5 mV

Then $$\hat{V}_{sig} = \left(\frac{R_{in} + R_{sig}}{R_{in}}\right)\hat{V}_\pi = \left(\frac{2.5 + 5}{2.5}\right) \times 5 = 15\text{ mV}$$

Amplitude of signal at output, $$\hat{V}_0 = G_V \hat{V}_{sig} = 35.41 \times 0.015$$

$$= 0.5312\text{ V}$$

Q.3 (c) (i) Solution:

Given $$\chi_m = 1.4 \times 10^{-5}$$

Magnetic flux density, $$B = \mu_0 H \quad \text{...(i)}$$

When the free space is magnesium filled, then

$$B' = \mu_r \mu_0 H \quad \text{...(ii)}$$

$$\mu_r = 1 + \chi_m \quad \text{...(iii)}$$

From equation (ii) and (iii)

$$B' = (1 + \chi_m)B \quad \text{...(iv)}$$

Hence, the percentage increase in magnetic induction

$$= \frac{B' - B}{B} \times 100$$

By using equation (i) and (iv),

$$= \frac{(1+\chi_m)B - B}{B} \times 100$$

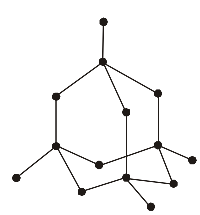
$$= \chi_m \times 100 = 1.4 \times 10^{-5} \times 100$$

$$= 0.0014\%$$

Q.3 (c) (ii) Solution:

1. **Covalent bonding of carbon in diamond:**

In diamond, every carbon atom bonds with four other adjoining atoms in a continuous network. No electrons are left unbonded. This results in very strong bonds between carbon atoms and is responsible for the great hardness of diamonds and their clear colorless appearance. The great density bend light more than other crystals do making their appearance so spectacular. Valence electrons of carbon atoms in diamonds are bond to 4 electrons in tetrahedral arrangement. The covalent bond is very strong and makes diamond to have high melting points. The covalent bond in three dimensional structure causes diamond to become the hardest material. The bonding of electrons, diamonds have dome shaped structures. Dome is one of the strongest structures. Diamond forcing more carbon atoms into a smaller dense package structure. Since there are no free electrons to wander through the structure, diamonds are excellent insulators. The brilliance and fire of cut diamonds is due to a very high index of refraction (2.42) and the strong dispersion of light, properties which are related to the structure of diamonds.

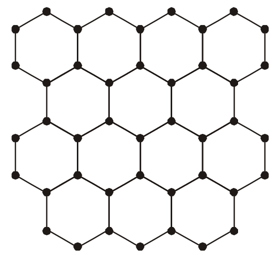


Diamond

Covalent bonding of carbon in Graphite:

In graphite, each carbon atoms shares electrons with only three neighboring carbon atoms, leaving the fourth electron relatively free to roam around from one carbon atom to another, as much the same way as metals do. The carbon atoms form a network consisting of layers of interconnected carbons able to slide against each other making in a pencil. Unlike diamond, graphite is soft, pitch black in color, and conducts electricity due to the free roaming valence electrons.

Valence electrons of graphite are only bonded to 3 valence electrons, so the covalent bond in hexagonal ring. Graphite is softer than diamond because they are held by weak intermolecular forces. Graphite sheet like array of carbon atoms joined with minimal pressure.



Graphite

2. Magnetization, $\bar{M} = 2.8$ A/m,

magnetic susceptibility, $\chi_m = 0.0025$

$$\bar{M} = \chi_m H$$

$$\therefore \quad \bar{H} = \frac{\bar{M}}{\chi_m} = \frac{2.8}{0.0025} = 1120 \text{ A/m}$$

$$\text{magnetic flux density } (B) = \mu_0(\bar{H} + \bar{M}) = 4\pi \times 10^{-7} [1120 + 2.8]$$

$$= 1.41 \times 10^{-3} \text{ Wb/m}^2$$

Magnetic field intensity = 1300 A/m

Relative permeability, $\mu_r = 1.006$

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1.006 \times 1300$$

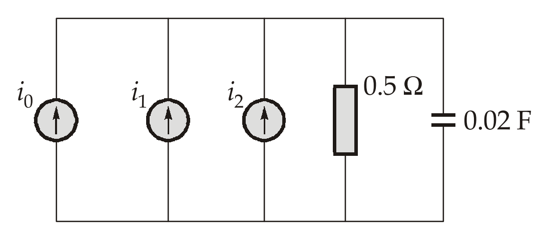
$$= 1.64 \times 10^{-3} \text{ Wb/m}^2$$

Q.4 (a) Solution:

(i) Each source is sinusoidal and hence

$$V = ZI$$

where V is the phasor voltage, I is the phasor current, and Z is the driving point impedance of the RC circuit, which is given by



$$Z = \frac{1}{Y} = \frac{1}{2 + j\omega(0.02)}$$

Using superposition theorem,

With the dc source, $V_0 = \frac{1}{2} 0.5 = 2.5$ V

With the first harmonic source,

$$\omega = 100, I_1 = 3\angle 45° \text{ A}$$

and $$V_1 = \frac{1}{2+2j} 3\angle 45° = \frac{3\angle 45°}{2.828\angle 45°} = 1.06\angle 0°$$

Thus, $$v_1(t) = 1.06 \cos 100t \text{ V}$$

With the second harmonic,

$$\omega = 200, I_2 = 2\angle -10° \text{ A}$$

and $$V_2 = \frac{1}{2+j4} 2\angle -10° = \frac{2\angle -10°}{4.472\angle 63°} = 0.447\angle -73°$$

Thus, $$v_2(t) = 0.447 \cos(200t - 73°) \text{ V}$$

By superposition, the output voltage is:

$$v(t) = 2.5 + 1.06 \cos 100t + 0.447 \cos(200t - 73°) \text{ V}$$

(ii) $$P_{av} = v(t)i(t)$$

$$= 5 \times 2.5 + 3/\sqrt{2} \times 1.06/\sqrt{2} \cos 45° + 2/\sqrt{2} \times 0.477/\sqrt{2} \cos 63°$$

$$= 12.5 + 1.59 \cos 45° + 0.477 \cos 63°$$

$$= 13.84 \text{ W}$$

Q.4 (b) (i) Solution:

By voltage division rule,

$$V_A = V\left(\frac{R}{R+R}\right) = \frac{V}{2}$$

$$V_A = V_B = V_C \text{ (By V.G. concept)}$$

$$V_A = V_B = V_C = \frac{V}{2}$$

$$V_B = V \times \left[\frac{R+\Delta R}{(2R)}\right]$$

$\therefore$ By Nodal analysis at V_C

$$\left(\frac{V_C - V_B}{R}\right) + \left(\frac{V_C - V_0}{10R}\right) = 0 \qquad \text{...(i)}$$

$$V_B = V\left[\frac{1}{2} + \frac{\Delta R}{2R}\right] = 0.05$$

Now substitute V_B value in equation (i),

$$\frac{V_c - 0.52V}{R} + \frac{V_c - V_0}{10R} = 0$$

$$\Rightarrow \quad \frac{05V - 0.52V}{R} + \frac{0.5V - V_0}{10R} = 0$$

$$V_0 = 0.3 \text{ V}$$

Q.4 (b) (ii) Solution:

When switch is on neutral side,

$$W = W_1 + W_2 + W_3$$

$$= 577.35 + 577.35 + 577.35 = 1732 \text{ W}$$

$$VA \text{ load} = 3464 = 3\ V_{ph} \times I_{ph}$$

$$V_{ph} I_{ph} = 1154.66 \text{ VA}$$

$$\text{Each wattmeter reading} = V_{ph} \cdot I_{ph} \cdot \cos(\angle V_{RN} \text{ and } I_R)$$

$$577.35 = V_{RN} \cdot I_R \cdot \cos\phi$$

$$\cos(\phi) = \frac{577.35}{1154.66} = 0.5$$

$$\phi = \cos^{-1}(0.5)$$

$$\phi = 60°$$

When switch is on Y-phase side

$$W_1 = V_{RY} \cdot I_R \cdot \cos(\angle V_{RY} \text{ and } I_R)$$

$$= \sqrt{3}V_{ph} \cdot I_{ph} \cos(30° + \phi)$$

$$= \sqrt{3} \times 1154.66 \times \cos(30° + 60°) = 0 \text{ W}$$

$$W_3 = V_{BY} \cdot I_R \cdot \cos(\angle V_{BY} \text{ and } I_B)$$

$$= \sqrt{3}V_{ph} \cdot I_{ph} \cos(30° - \phi)$$

$$= \sqrt{3} \times 1154.66 \times \cos(30° - 60°) = 1732 \text{ W}$$

$W_1 = 0$; $W_2 = 0$ and $W_3 = 1732$ W

Q.4 (c) Solution:

Assume, $\varepsilon_r = 16$ for Ge

$$n_i = 2.25 \times 10^{13}/\text{cm}^3$$

$$p_p = N_A = 10^{16} \text{ hole/cm}^3$$

$$n_p = \frac{n_i^2}{p_p} = \frac{6.25 \times 10^{26}}{10^{16}} = 6.25 \times 10^{10} \text{ electrons/cm}^3$$

$$n_n = N_D = 10^{15} \text{ electrons/cm}^3$$

$$p_n = \frac{n_i^2}{n_n} = \frac{6.25\times10^{26}}{10^{15}} = 6.25 \times 10^{11} \text{ holes/cm}^3$$

Built-in potential,

$$V_0 = V_0 = \text{kT}\ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.026 \ln\left(\frac{10^{16}\times10^{15}}{6.25\times10^{26}}\right)$$

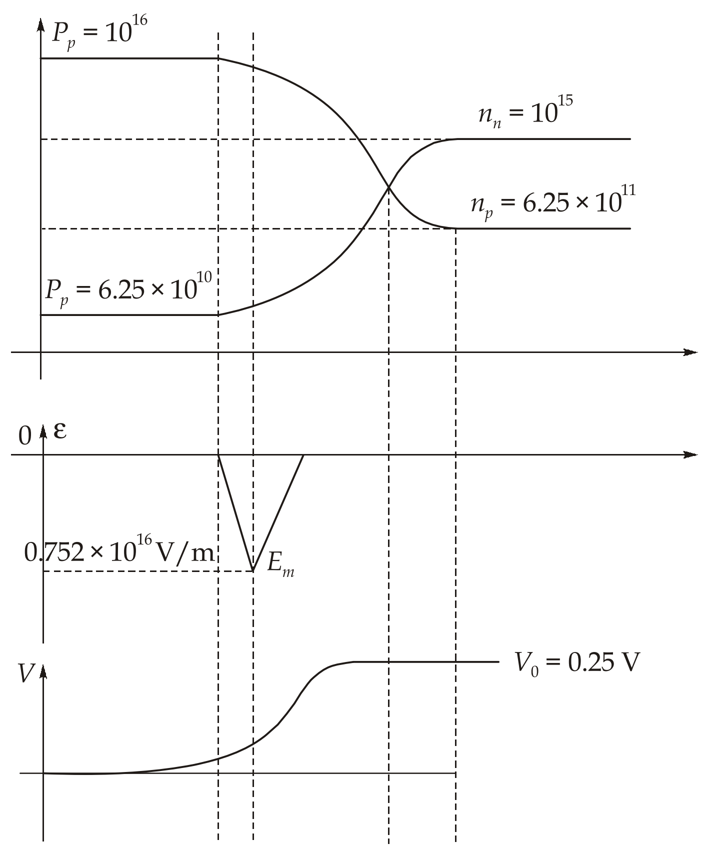
$$= 0.25 \text{ V}$$

$$E_m = \sqrt{\frac{2qN_DV_B}{\varepsilon}} = \sqrt{\frac{2\times1.6\times10^{-19}\times10^{21}\times0.25}{8.85\times10^{-12}\times16}}$$

$$N_D = 10^{21} \text{ atoms/m}^3 = 0.752 \times 10^6 \text{ V/cm}$$

$$W = \sqrt{\frac{2\varepsilon V_B}{qN_D}} = \left[\frac{2\times8.85\times10^{-12}\times16\times0.25}{1.6\times10^{-19}\times10^{21}}\right]$$

$$= 0.665 \times 10^{-6} \text{ m}$$



Section-B

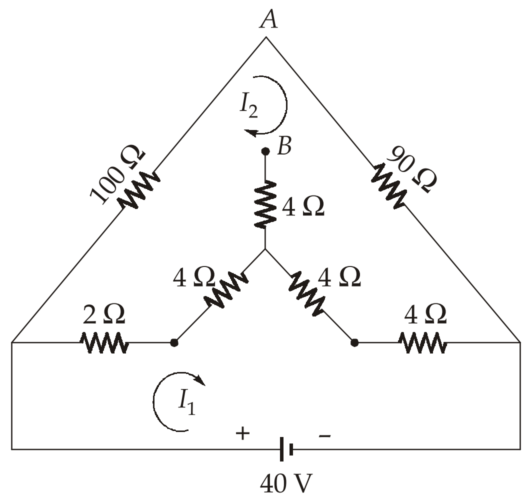
Q.5 (a) Solution:

```
# include <conio.h>
# include <stdio.h>
int main ( )
{
        int count, temp, i, j, number [30];
        print f ("HOW many numbers are you going to enter:");
        scanf ("%d", & count);
        printf ("Enter %d numbers:", count);
        for (i = 0, i < count; i++)
        scanf ("%d", number [i]);
        for (i = count – 2; i > 0; i––)
{
        for (j = 0; j < = i; j ++)
{
        if (number [j] > number [j + 1])
{
        temp = number [j];
        number [j] = number [j + 1);
        number [j + 1] = temp;
} } }
        printf ("sorted elements:")
        for (i = 0; i < count ; i++)
        printf ("%d", number [i]);
}
```

Q.5 (b) Solution:

By using Thevenin's Theorem

Convert the delta of 12 Ω each of the equivalent star.



Applying KVL to the loops,

$$-6I_1 + 6I_2 - 8I_1 + 8I_2 + 40 = 0$$

$$14I_1 - 14I_2 = 40 \quad \text{...(i)}$$

$$-6I_2 + 6I_1 - 100I_2 - 90I_2 - 8I_2 + 8I_1 = 0$$

$$14I_1 - 204I_2 = 0 \quad \text{...(ii)}$$

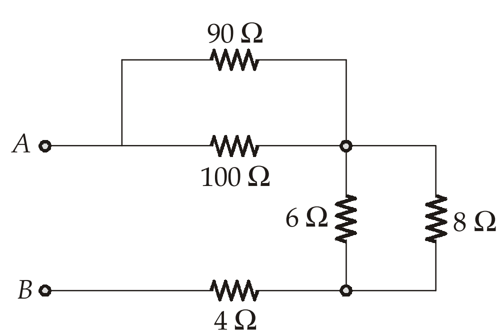
Solving equation (i) and (ii),

$$I_1 = 3.067\text{A and } I_2 = 0.21 \text{ A}$$

$$V_{AB} = -21 + 17.142$$

$$= -3.858 \text{ V}$$

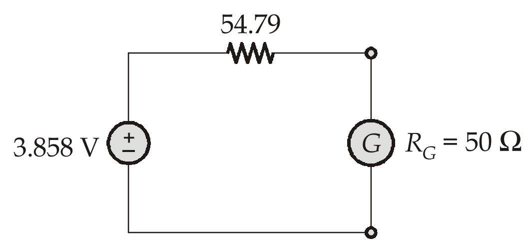
Redrawing the circuit



$$R_{eq} = (90 \,||\, 100) + (6 \,||\, 8) + 4$$

$$= 54.79 \ \Omega$$

Thevenin's equivalent



$$I_G = \frac{V_{TH}}{R_{eq}+R_G} = \frac{3.858}{54.79+50} = 36.81 \text{ mA}$$

Q.5 (c) Solution:

Let $\phi = x^2y^2z^2$

Directional derivative of ϕ

$$\Delta\phi = \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)(x^2y^2z^2)$$

$$\nabla\phi = 2xy^2z^2\hat{i}+2yx^2z^2\hat{j}+2zx^2y^2\hat{k}$$

Directional derivative of ϕ at (1, 1, –1)

$$= 2(1)(1)^2(-1)^2\hat{i}+2(1)(1)^2(-1)^2\hat{j}+2(-1)(1)^2(1)^2\hat{k}$$

$$= 2\hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$

$$= e^t\hat{i}+(\sin 2t+1)\hat{j}+(1-\cos t)\hat{k}$$

Tangent vector, $$\vec{T} = \frac{\overrightarrow{dr}}{dt} = e^t\hat{i}+2\cos 2t\hat{j}+\sin t\hat{k}$$

Tangent (at $t = 0$)

$$= e^0\hat{i}+2(\cos 0)\hat{j}+(\sin 0)\hat{k}$$

$$= \hat{i}+2\hat{j}$$

Required directional derivative along tangent

$$= (2\hat{i}+2\hat{j}+2\hat{k})\frac{(\hat{i}+2\hat{j})}{\sqrt{1+4}}$$

$$= \frac{2+4+0}{\sqrt{5}} = \frac{6}{\sqrt{5}}$$

Q.5 (d) Solution:

(i) For $\vec{B}$ to be magnetic field. Following two conditions should be satisfied

(i) $\nabla \cdot \vec{B} = 0$ (ii) $\nabla \times \vec{B} \neq 0$

$$\nabla \cdot \vec{B} = \frac{\partial}{\partial x}\left(\vec{B} \cdot \hat{a}_x\right) + \frac{\partial}{\partial y}\left(\vec{B} \cdot \hat{a}_y\right) + \frac{\partial}{\partial z}\left(\vec{B} \cdot \hat{a}_z\right)$$

So,
$$\nabla \cdot \vec{B} = \frac{\partial\left(y^2\right)}{\partial x} + \frac{\partial\left(z^2\right)}{\partial y} + \frac{\partial\left(x^2\right)}{\partial z} = 0$$

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & z^2 & x^2 \end{vmatrix}$$

$$= a_x\left[\frac{\partial(x^2)}{\partial y} - \frac{\partial(z)^2}{\partial z}\right] - a_y\left[\frac{\partial(x^2)}{\partial x} - \frac{\partial y^2}{\partial z}\right] + a_z\left[\frac{\partial(z^2)}{\partial z} - \frac{\partial(y^2)}{\partial y}\right]$$

$$= (-2z)\hat{a}_x + (-2x)\hat{a}_y + (-2y)\hat{a}_z \quad \text{...(i)}$$

Thus,
$$\nabla \times \vec{B} \neq 0$$

Hence both conditions are satisfied thus given field $\vec{B}$ is a magnetic field.

(ii) Magnetic flux through surface,

$$x = 1, 0 < y < 1, 1 < z < 4$$

$$\psi_m = \iint \vec{B} \cdot \vec{ds}$$

$$= \int_{z=1}^{4} \int_{y=0}^{1} \left[\left(y^2 \hat{a}_x\right) + \left(z^2 \hat{a}_y\right) + \left(x^2 \hat{a}_z\right)\right] dydz\hat{a}_x$$

$$= \int_{z=1}^{4} \int_{y=0}^{1} y^2 dydz$$

$$\psi_m = \left[\frac{y^3}{3}\right]_0^1 \times [4-1] = 3 \times \frac{1}{3} = 1 \text{ Wb}$$

(iii) As we know
$$\vec{J} = \nabla\times\vec{H} = \frac{1}{\mu_o}\left[\nabla\times\vec{B}\right]$$

From equation (i),
$$\nabla\times\vec{B} = -2\left(z\hat{a}_x + x\hat{a}_y + y\hat{a}_z\right)$$

So,
$$\vec{J} = \frac{1}{\mu_o}\left[\nabla\times\vec{B}\right] = -\frac{2}{\mu_o}\left[z\hat{a}_x + x\hat{a}_y + y\hat{a}_z\right]\text{A/m}^2$$

Q.5 (e) Solution:

(i)
$$\text{Net doping} = N_A - N_D$$
$$= 3\times10^{14} - 2\times10^{14} = 10^{14}\ \text{atoms/cm}^3\ \text{(p type)}$$

Since this is comparable with
$$n_i = (2.5\times10^{13}),$$
$$p = \frac{N_A}{2} + \left[\left(\frac{N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$
$$= \frac{10^{14}}{2} + \left[\left(\frac{10^{14}}{2}\right)^2 + 6.25\times10^{26}\right]^{1/2}$$
$$= 0.5\times10^{14} + [0.3125\times10^{28}]^{1/2}$$
$$= 1.058\times10^{14}\ \text{holes/cm}^3$$

$\therefore$
$$n = \frac{n_i^2}{p} = \frac{6.25\times10^{26}}{1.058\times10^{14}} = 0.059\times10^{14}\ \text{electrons/cm}^3$$

(ii) We have to first find,

n_i^2 at 400° k

$$n_i^2 = A_0T^3e^{-E_{a0}/kT}$$
$$\frac{n_i^2(400°\text{K})}{n_i^2(300°\text{K})} = \frac{400^3}{300^3}\times\frac{e^{(-0.785/8.6\times10^{-3}\times400)}}{e^{(-0.785/8.6\times10^{-5}\times300)}} = 2.3\times2011 = 4766$$
$$n_i^2(400°\text{K}) = 6.25\times10^{26}\times4766 = 2.98\times10^{30}$$

Therefore,
$$p = \frac{10^{14}}{2} + \left[\left(\frac{10^{14}}{2}\right)^2 + 2.98\times10^{30}\right]^{1/2}$$

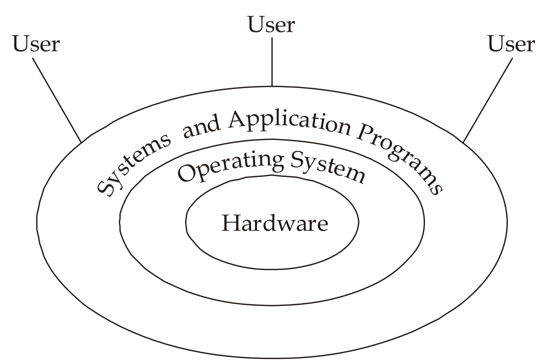
$$= 1.77 \times 10^{15} \text{ holes/cm}^3$$

$$n = \frac{n_i^2}{p} = \frac{2.98\times10^{30}}{1.77\times10^{15}} = 1.68 \times 10^{15} \text{ electrons/cm}^3$$

Since the values of n and p are almost equal, this material is essentially intrinsic at 400° K.

Q.6 (a) Solution:

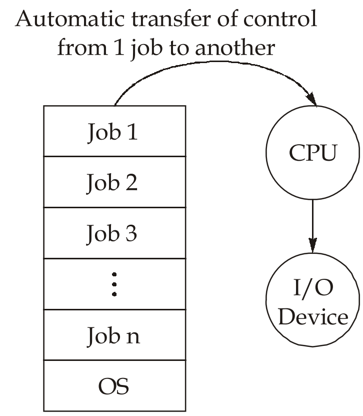
Operating System: It is a program that acts as an interface between a user (applications) and the computer hardware.



Operating System allows the computer system in more convenient to use and efficient resource utilization.

Batch operating system: The operating system in early computers was fairly simple. Its major task was to transfer control automatically from one job to the next. The operating system was always in memory. To speed up processing, jobs with similar needs were batched together and were run through the computer as a group. Thus, the programmers would leave their programs with the operator. The operator would sort programs into batches with similar requirements, and as the computer become available, would run each batch.

- Systems allowed automatic job sequencing by a resident operating system and greatly improved the overall utilization of the computer.
- In batch system there is lack of interaction between the user and the job while the job is executing.
- The CPU utilization was still low. In this execution environment the CPU is often idle. This idleness occurs because the speeds of the mechanical input/output devices are intrinsically slower than those of electronic devices.
- Batch system are appropriate for executing large jobs that need little interaction. *Example:* IBM OS/2.

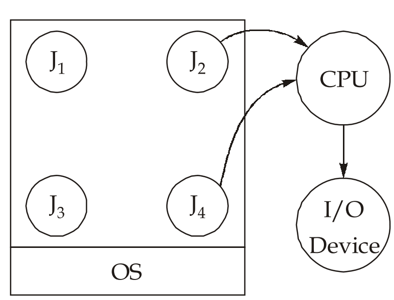


Batch System

Multi-programing operating system:

- Several jobs are kept in main memory at the same time and the CPU is multiplexed among them, to increase CPU utilization.
- A job pool on the disk consists of a number of jobs that are ready to be executed. Subsets of these jobs reside in the memory for execution.
- The operating system picks and executes one of the jobs in memory.
- When this job in execution needs an input/output operation to complete, instead of waiting for the job to complete the input/output, it switches to the subset of jobs waiting for CPU.
- In a non multi programmed system the CPU would sit idle. In multiprogramming system the operating system simple switches to and executes another job.
- If several jobs are ready to be brought into memory and there is not enough room for all of them, then the system must choose among them. Making this decision is job scheduling.

***Example*: Windows and Unix.**

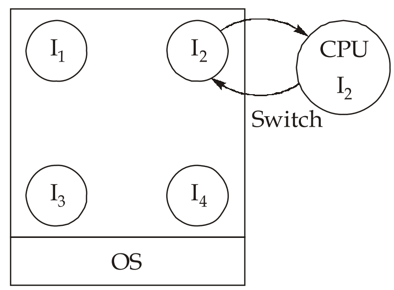


Multi-programming System

Multi-tasking operating system:

- Time sharing or multi-tasking is a logical extension of multiprogramming. Multiple jobs are executed by the CPU switching between them, but the switches occur is frequently that the users may interact with each program while it is running.

- An interactive computer system provides direct communication between the user and the system. The user gives instructions to the operating system or to a program directly and receives an immediate response.
- A time shared operating system uses CPU scheduling and multiprogramming to provide each user with a small portion of a time shared computer.
- A time shared operating system allows the many users to share the computer simultaneously. Since each action or command in a time shared system tends to be short, only a little CPU time is needed for each user. As the system switches rapidly from one user to the next, each user is given the impression that she has her own computer, whereas actually one computer is being shared among many users.



Multi-tasking System

Real time operating system: Used when there are rigid time requirements on the operating of a processor or the flow of data. Systems that control scientific experiments, medical imaging systems, industrial control systems, and some display systems are real-time systems. A real time operating system has well defined, fixed time constraints. Processing must be done within the defined constraints or the system will fail. ***Example***: RTOS

Types of Real Time Systems

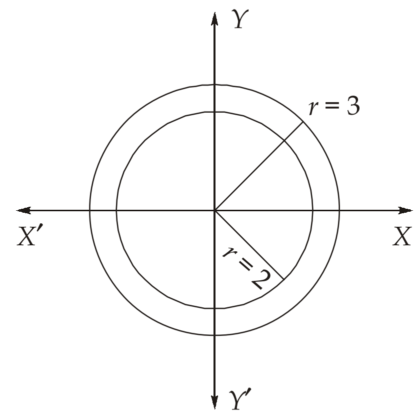
1. **Hard real time system:** Guarantees that critical tasks completed on time. This goal requires that all delays in the system be bounded from the retrieval of stored data to the time that it takes the operating system to finish any request made to it. Kernel delays need to be bounded and more restrictive. ***Example***: Satellite, Missile System.
2. **Soft real time system:** A less restrictive type of real time system is a soft real time system, where a critical real time task gets priority over other tasks, and retains that priority until it completes. Soft real time is an achievable goal that can be mixed with other types of systems. However they have more limited utility than do hard real time system. Given their late of deadline support they are risky to use for industrial control and robotics. ***Example***: Banking system.

Q.6 (b) (i) Solution:

Put $x = r\cos\theta$,

$y = r\sin\theta$

$dx\,dy = r d\theta\,dr$



$$= \int_0^{2\pi}\int_2^3 r\cdot r d\theta dr = \int_0^{2\pi} d\theta \int_2^3 r^2 dr$$

$$= \int_0^{2\pi} d\theta\left(\frac{r^3}{3}\right)_2^3 = \int_0^{2\pi} d\theta\left(9-\frac{8}{3}\right)$$

$$= \frac{19}{3}\int_0^{2\pi} d\theta = \frac{19}{3}(\theta)_0^{2\pi} = \frac{19}{3}\times 2\pi = \frac{38\pi}{3}$$

Q.6 (b) (ii) Solution:

$$A = \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1-i & 2 & 5+5i \\ -2i & 2-i & 4-2i \\ -1-i & -4 & 7 \end{bmatrix} \quad \text{...(i)}$$

$$(\bar{A})' = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$$

$$A^\theta = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$$

On adding (i) and (ii), we get

$$A + A^\theta = \begin{bmatrix} 2 & 2-2i & 4-6i \\ 2+2i & 4 & 2i \\ 4+6i & -2i & 14 \end{bmatrix}$$

$$R = \frac{1}{2}(A+A^\theta) = \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix} \quad \text{...(iii)}$$

On subtracting (ii) from (i), we get

$$A - A^{\theta} = \begin{bmatrix} 2i & 2+2i & 6-4i \\ -2+2i & 2i & 8+2i \\ -6-4i & -8+2i & 0 \end{bmatrix}$$

Let $$S = \frac{1}{2}(A - A^{\theta}) = \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix} \quad \text{...(iv)}$$

From (iii) and (iv), we have,

$$A = \underbrace{\begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix}}_{\text{Hermitian matrix}} + \underbrace{\begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}}_{\text{Skew-Hermitian matrix}}$$

Q.6 (c) (i) Solution:

Using Clausius-Mossotti equation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\varepsilon_0} \quad \text{...(i)}$$

In optical frequency range $\varepsilon_r = n^2$ and $\alpha_i = 0$.

Therefore, equation (i) becomes

$$\frac{n^2 - 1}{n^2 + 1} = \frac{N\alpha_e}{3\varepsilon_0} \quad \text{...(ii)}$$

On dividing equations (i) and (ii), we get

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\frac{n^2 + 2}{n^2 - i} = \frac{\alpha_e + \alpha_i}{\alpha_e}$$

$$1 + \frac{\alpha_e}{\alpha_i} = \frac{(4.94 - 1)(2.69 + 2)}{(4.94 + 2)(2.69 - 1)}$$

On simplifying, we get

$$\frac{\alpha_e}{\alpha_i} = 1.735$$

Q.6 (c) (ii) Solution:

Let us consider a parallel plate capacitor with a dielectric materials of dielectric constant ε_r^*, its real part $\varepsilon_r'(\omega)$ and imaginary part $\varepsilon_r''(\omega)$ are known. Let the field produced by an alternating voltage source be $E_0 \cos \omega t$. Let at any instant, $\pm Q(t)$ be the charge density per unit area on the plates. Since the flux density is numerically equal to the charge density, we must have at any instant, $D(t) = Q(t)$.

Then the current density in the capacitor,

$$J(t) = \frac{dQ(t)}{dt} = \frac{dD(t)}{dt}$$

The relation of electric flux density, $D(t)$ and electric field, $E_0 \cos \omega t$ may be expressed as

$$D(t) = \text{Re}\left\{\varepsilon_0 \varepsilon_r^* E_0 e^{j\omega t}\right\}$$

$$= \varepsilon_0 E_0 \text{Re}\left\{\varepsilon_r^* e^{j\omega t}\right\}$$

$$J(t) = \varepsilon_0 E_0 \text{Re}\left\{j\omega(\varepsilon_r' - j\varepsilon_r'')e^{j\omega t}\right\}$$

$$J(t) = \omega \varepsilon_0 \varepsilon_r'' E_0 \cos \omega t - \omega \varepsilon_0 \varepsilon_r' E_0 \sin \omega t$$

The instantaneous power per m^3 absorbed by the material is given by $J(t)\, E(t)$. So in each second the material absorbs an amount of energy per m^3 given by,

$$W(t) = \frac{1}{2\pi}\int_0^{2\pi} J(t)E(t)\, d(\omega t)$$

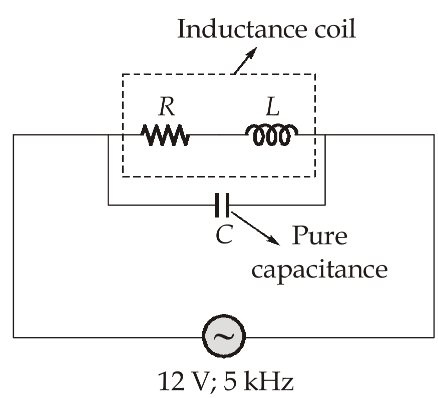
$$= \frac{1}{2\pi}\int_0^{2\pi} \omega \varepsilon_0 E^2 \left(\varepsilon_r'' \cos^2 \omega t - \varepsilon_r' \sin \omega t \cos \omega t\right) d(\omega t)$$

$$W(t) = \frac{\omega}{2}\varepsilon_0 \varepsilon_r'' E^2 \ \text{W/m}^3$$

The absorption of energy is termed as the dielectric loss which is proportional to the imaginary part of the complex dielectric constant.

Q.7 (a) Solution:

We have,



$$Y_{eq} = Y_1 + Y_2$$

$$Y_{eq} = \frac{1}{R + j\omega L} + \frac{1}{(-j/\omega C)}$$

$$Y_{eq} = \frac{1}{(R + j\omega L)} + j\omega C = \frac{R - j\omega L}{R^2 + (\omega L)^2} + j\omega C$$

$$Y_{eq} = \frac{R}{(R^2 + (\omega L)^2)} + j\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right)$$

$$Z_{\text{dyn}} = \frac{1}{Y}$$

$$Z_{dyn} = \frac{R^2 + (\omega L)^2}{R} = 13.13 \text{ k}\Omega \quad \text{...(i)}$$

$$\text{Quality factor, } Q = \frac{\omega L}{R} = 4$$

$$\frac{\omega L}{4} = R \quad \text{...(ii)}$$

Substituting (ii) in (i), we get

$$\frac{\left(\frac{\omega L}{4}\right)^2 + (\omega L)^2}{\frac{\omega L}{4}} = 13.13 \times 10^3$$

$$\frac{\omega L}{4} + 16\left(\frac{\omega L}{4}\right) = 13.13 \times 10^3$$

$$\frac{\omega L}{4}[1 + 16] = 13.13 \times 10^3$$

$$\frac{\omega L}{4} = 0.77 \times 10^3$$

$$L = 0.0983 \text{ H}$$

and equation (ii), we get

$$\frac{\omega L}{4} = R$$

$$R = \frac{2\pi \times 5000 \times 0.0983}{4} = 772.35\ \Omega$$

At resonance, $\text{Img}[Y_{eq}] = 0$

$$\omega C - \frac{\omega L}{R^2 + (\omega L)^2} = 0$$

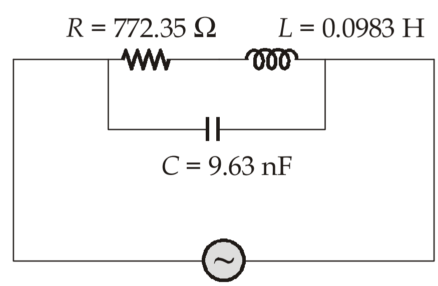
$$\omega C = \frac{\omega L}{R^2 + (\omega L)^2}$$

$$C = \frac{L}{R^2 + (\omega L)^2} = \frac{L}{(13.13 \times 10^3 \times R)}$$

$$C = \frac{0.0983}{13.13 \times 10^3 \times 772.35} = 9.63 \text{ nF}$$

Thus, we can draw equivalent circuit as,

At resonance is



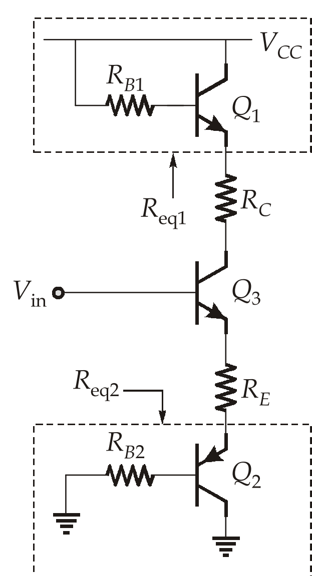
Q.7 (b) Solution:

(i) Since all transistors are matched. So,

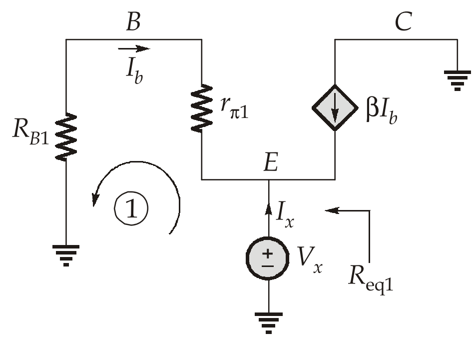
$$\beta_1 = \beta_2 = \beta_3 = \beta$$

As $V_A = \infty$

$$r_{01} = r_{02} = r_{03} = r_0 = \infty$$



The equivalent resistance seen by the transistors Q_1 and Q_2 are



$$R_{eq1} = \frac{V_x}{I_x}$$

$$I_x = -(I_b + \beta I_b) = -(\beta + 1)I_b \quad ...(1)$$

Applying KVL in loop (1),

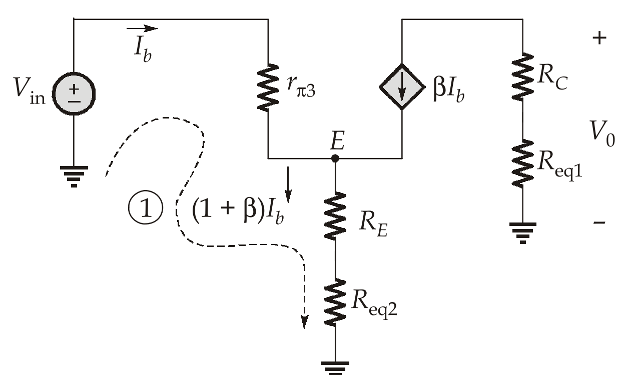
$$-V_x - I_b(r_{\pi 1} + R_{B1}) = 0$$

$$V_x = -(r_{\pi 1} + R_{B1})I_b$$

$$R_{eq1} = \frac{V_x}{I_x} = \frac{R_{B1} + r_{\pi 1}}{1+\beta} = \frac{R_{B1}}{1+\beta} + \frac{1}{g_{m1}}$$

similarly, $$R_{eq2} = \frac{R_{B2}}{1+\beta} + \frac{1}{g_{m2}} = \frac{R_{B2} + r_{\pi 2}}{1+\beta}$$

The transistor Q_3 has AC equivalent circuit as



$$V_0 = -\beta I_b(R_C + R_{eq1}) \quad ...(2)$$

Applying KVL in loop (1),

$$-V_{in} + r_{\pi 3}I_b + (R_E + R_{eq2})(1 + \beta)I_b = 0$$

$$V_{in} = \left[r_{\pi 3} + (1+\beta)(R_E + R_{eq2})\right]I_b \quad ...(3)$$

Voltage gain, $$\frac{V_0}{V_{in}} = \frac{-\beta(R_C + R_{eq1})I_b}{\left[r_{\pi 3} + (1+\beta)(R_E + R_{eq2})\right]I_b}$$

$$\frac{V_0}{V_{in}} = \frac{-\beta\left[R_C + \left(\frac{r_{\pi 1} + R_{B1}}{1+\beta}\right)\right]}{r_{\pi 3} + (1+\beta)\left[R_E + \frac{r_{\pi 2} + R_{B2}}{(1+\beta)}\right]}$$

$$\frac{V_0}{V_{in}} = \frac{-\beta[R_C(1+\beta) + r_{\pi 1} + R_{B1}]}{(1+\beta)r_{\pi 3} + R_E + r_{\pi 2} + R_{B2}}$$

(ii) Input resistance, $R_{\text{in}} = \frac{V_{in}}{I_b}$

from equation (3), $R_{\text{in}} = \frac{V_{in}}{I_b} = r_{\pi 3} + (1+\beta)(R_E + R_{eq2})$

$$R_{\text{in}} = r_{\pi 3} + (1+\beta)\left(R_E + \frac{1}{g_{m2}} + \frac{R_{B2}}{1+\beta}\right)$$

(iii) Output resistance, $R_{\text{out}} = R_C + R_{\text{eq1}} = R_C + \frac{1}{g_{m1}} + \frac{R_{B1}}{1+\beta} = R_C + \frac{R_{B1} + r_{\pi 1}}{(1+\beta)}$

Q.7 (c) Solution:

Primary turns $N_p = 1$, Secondary turns $N_s = 300$

$\therefore$ Turns ratio $n = \frac{N_s}{N_p} = 300.$

Secondary burden impedance $= \sqrt{(15)^2 + (10)^2} = 1.8\ \Omega$

Secondary circuit power factor:

$$\cos\delta = \frac{\text{resistance}}{\text{impedance}} = \frac{1.5}{1.8} = 0.833$$

and $\sin\delta = \frac{\text{resistance}}{\text{impedance}} = \frac{1.0}{1.8} = 0.555$

Secondary induced voltage, $E_s = 5 \times 1.8 = 9.0$ V

Primary induced voltage, $I_e = \frac{E_s}{n} = \frac{9}{300}$ V

Loss component of current referred to primary

$$I_s = \frac{\text{iron loss}}{E_s} = \frac{1.2}{9/300} = 40 \text{ A}$$

$$\text{Magnetizing current, } I_m = \frac{\text{magnetizing mmf}}{\text{primary winding turns}} = \frac{100}{1} = 100 \text{ A}$$

$$\text{Actual ratio, } R = n + \frac{I_e \cos\delta + I_m \sin\delta}{I_s}$$

$$= 300 + \frac{40 \times 0.833 + 100 \times 0.555}{5} = 317.6$$

In the absence of any information to the contrary we can take nominal ratio to be equal to the turns ratio,

or, $K_n = n = 300$

$$\text{Percentage ratio error} = \frac{K_n - R}{R} \times 100 = \frac{300 - 317.6}{317.6} = -5.54\%$$

We have

$$\text{Phase angle, } \theta = \frac{180}{\pi}\left(\frac{I_m \cos\delta - I_e \sin\delta}{nI_s}\right)$$

$$= \frac{180}{\pi}\left(\frac{100 \times 0.833 - 40 \times 0.555}{300 \times 5}\right) = 2.34°$$

Q.8 (a) Solution:

By resolving,

$$f(z) = \frac{1}{(z+1)(z+3)}$$

into partial fractions, we get

$$f(z) = \frac{1}{2(z+1)} - \frac{1}{2(z+3)}$$

(i) $|z| < 1$ now $|z| < 1$

$\Rightarrow |z| < 1$ and $|z| < 1 < 3$

$\therefore |z| < 1$ and $\left|\frac{z}{3}\right| < 1$

$$f(z) = \frac{1}{2(z+1)} - \frac{1}{2(z+3)} = \frac{1}{2}(1+z)^{-1} - \frac{1}{6}\left(1 + \frac{z}{3}\right)^{-1}$$

$$= \frac{1}{2}[1 - z + z^2 - z^3 + \ldots] - \frac{1}{6}\left[1 - \frac{z}{3} + \left(\frac{z}{3}\right)^2 - \left(\frac{z}{3}\right)^3 + \ldots\right]$$

$$= \left(\frac{1}{2}-\frac{1}{6}\right)-\left(\frac{1}{2}-\frac{1}{18}\right)z+\left(\frac{1}{2}-\frac{1}{54}\right)z^2-\ldots$$

$$= \frac{1}{3}-\frac{4}{9}z+\frac{13}{17}z^2-\ldots$$

(ii) $1<|z|<3$ now $1<|z|<3$

$\Rightarrow 1<|z|$ and $|z|<3$

$\therefore \frac{1}{|z|}<1$ and $\frac{|z|}{3}<1$

$$f(z) = \frac{1}{(z+1)(z+3)}=\frac{1}{2(z+1)}-\frac{1}{2(z+3)}$$

$$= \frac{1}{2z\left[1+\frac{1}{z}\right]}-\frac{1}{6\left[1+\frac{z}{3}\right]}$$

$$= \frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1}-\frac{1}{6}\left[1+\frac{z}{3}\right]^{-1}$$

$$= \frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\ldots\right]-\frac{1}{6}\left[1-\frac{z}{3}+\left(\frac{z}{3}\right)^2+\ldots\right]$$

(iii) $|z|>3$ now $|z|>3$

$\Rightarrow |z|>3>1$

$\Rightarrow |z|>3$ and $|z|>1$

$\therefore \left|\frac{1}{z}\right|<1$ and $\left|\frac{3}{z}\right|<1$

$$f(z) = \frac{1}{(z+1)(z+3)}=\frac{1}{2(z+1)}-\frac{1}{2(z+3)}$$

$$= \frac{1}{2z\left(1+\frac{1}{z}\right)}-\frac{1}{2z\left(1+\frac{3}{z}\right)}$$

$$= \frac{1}{2z}\left(1-\frac{1}{z}\right)^{-1} - \frac{1}{2z}\left(1-\frac{3}{z}\right)^{-1}$$

$$= \frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\ldots\right] - \frac{1}{2z}\left[1-\frac{3}{z}+\left(\frac{3}{z}\right)^2+\ldots\right]$$

(iv) $0 < |z+1| < 2$

Let $z + 1 = u$

Then we have $0 < |u| < 2$

$$f(z) = \frac{1}{(z+1)(z+3)} = \frac{1}{u(u+2)} = \frac{1}{2u\left(1+\frac{u}{2}\right)}$$

$$= \frac{1}{2u} - \frac{1}{4} + \frac{u}{8} - \frac{u^2}{16} + \ldots$$

$$= \frac{1}{2(z+1)} - \frac{1}{4} + \frac{(z+1)}{8} - \frac{(z-1)^2}{16} + \ldots$$

Q.8 (b) (i) Solution:

Multiplying power of shunt, $m = \frac{I}{I_m} = \frac{50}{5} = 10$

In order that the meter may read correctly at all frequencies the time constants of meter and shunt circuits should be equal. Under this condition multiplying power m is:

$$m = 1 + \frac{R}{R_{sh}}$$

∴ Resistance of shunt $R_{sh} = \frac{R}{m-1} = \frac{0.09}{10-1} = 0.01\ \Omega$. Also, $\frac{L}{R} = \frac{L_{sh}}{R_{sh}}$.

∴ Inductance of shunt $L_{sh} = \frac{L}{R} R_{sh} = \frac{90}{0.09} \times 0.01 = 10\ \mu\text{H}$

With d.c., the current through the meter for a total current of 50 A is:

$$I_m = \frac{R_{sh}}{R+R_{sh}} \times I = \frac{0.01}{0.09+0.01} \times 50 = 5.0\ \text{A}$$

With 50 Hz, the current through the meter for a total current of 50 A is:

$$I_m = \frac{R_{sh}}{\sqrt{(R+R_{sh})^2+\omega^2L^2}}\times I$$

$$= \frac{0.01}{\sqrt{(0.09+0.01)^2+(2\pi\times 50\times 90\times 10^{-6})^2}}\times 50 = 4.81\text{ A}$$

Since the meter reading is proportional to the current,

$$\text{Error} = \frac{4.81-5}{5}\times 100 = -3.8\%$$ or the meter reads 3.8% low.

Q.8 (b) (ii) Solution:

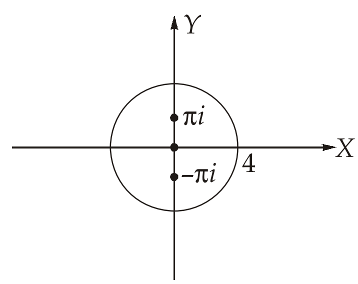
$$f(z) = \frac{1}{\sin hz}$$

To find poles, $\sin hz = 0$

$\sin iz = 0 \Rightarrow z = n\pi i$

where n is an integer.

Out of these poles $z = -\pi i$, 0 and $+\pi i$ comes inside the circle



$f(z)$ is in the form $\frac{P(z)}{Q(z)}$.

Pole at $z = m$ is given by $\frac{P(m)}{Q'(m)}$.

$$\text{Residue (at } z = 0) = \frac{1}{\cosh(0)} = \frac{1}{1} = 1$$

$$\text{Residue (at } z = -\pi i) = \frac{1}{\cosh(-\pi i)} = \frac{1}{\cos i(-\pi i)} = \frac{1}{(-1)} = -1$$

$$\text{Residue (at } z = \pi i) = \frac{1}{\cosh(\pi i)} = \frac{1}{\cos i(\pi i)} = \frac{1}{-1} = -1$$

Hence, the required integral

$$= 2\pi i(-1 + 1 - 1) = -2\pi i$$

Q.8 (c) (i) Solution:

$$Q = 20 \times 10^{-6} \text{ coulomb}$$
$$V = 10 \times 10^{3}, \varepsilon_r = 10$$
$$\varepsilon_0 = 8.854 \times 10^{-12}, d = 5 \times 10^{-4} \text{ m}$$

As $$Q = CV$$

$\therefore$ $$C = \frac{Q}{V} = \frac{20\times10^{-6}}{10\times10^{3}} = 2\times10^{-9} \text{ Farad}$$

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

$\therefore$ $$A = \frac{Cd}{\varepsilon_r \varepsilon_0} = \frac{2\times10^{-9}\times5\times10^{-4}}{10\times8.854\times10^{-2}}$$
$$= 10.294 \times 10^{-3} \text{ m}^2$$

Q.8 (c) (ii) Solution:

We know, force on charge placed in an electric field,

$$F = qE = eE \quad \text{...(i)}$$

Also, $$F = ma \quad \text{...(ii)}$$

Equating equations (i) and (ii), we get

$$ma = eE$$
$$a = \frac{eE}{m}$$

Due to collision of electron during motion and τ is the relaxation time,

The drift velocity of electron, $v_d = a \times \tau = \frac{eE}{m}\tau$

If the current flowing is I on application of electric field E and drift velocity, v_d

In time interval dt

distance travered by electron $= v_d\, dt$

So no. of electrons crossing a cross-section,

$$A = nA\, v_d\, dt$$

$\therefore$ Total charge flowing through section,

$$dq = en\, v_d\, A\, dt$$

So, $$\frac{dq}{dt} = ne\, v_d A = I$$

So, current density, $J = \frac{I}{A}$

or, $J = ne\, v_d$

We also know, $J = \sigma E$

$\sigma E = en\, v_d$

and $v_d = \mu E$

$\sigma E = ne\, \mu E$

So, $\sigma = n\, e\, \mu$ Hence proved.

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