MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

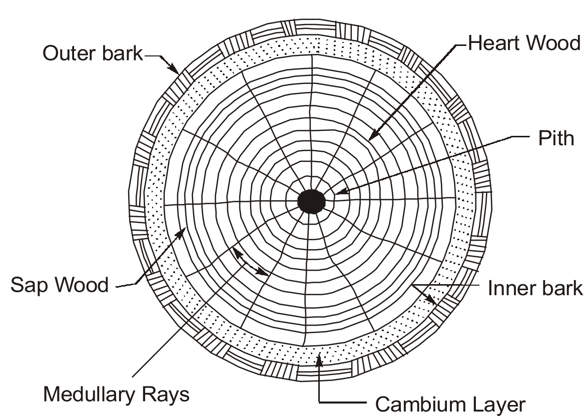
ESE-2026 Mains Test Series

Civil Engineering Test No : 10

Section - A

1. (a) Solution:

The structure of wood visible to the naked eye or at a small magnification is called the macrostructure of wood.



1. **Pith:** The innermost central portion or core of the tree is called the pith or medulla
2. **Heart Wood:** The inner annual rings surrounding the pith is known as heart wood. It is usually dark in colour. It does not take active part in the growth of tree. But it imparts rigidity to tree and hence, it provides strong and durable timber for various engineering purposes.
3. **Sap Wood:** The outer annual rings between heart wood and cambium layer is known as sap wood. It is usually light in colour and light in weight. It indicates recent growth and it contains sap.

It takes active part in the growth of tree and sap moves in an upward direction through it. Sap wood is also known as alburnum.

4. **Annual Rings:** Annual rings consist of closed cells of woody fibres and tissues arranged in distinct approximately concentric circles around pith. Every year, one such ring is formed. Hence, the total number of annual rings indicates the age of the tree. The wood near the bark is the youngest.
5. **Cambium Layer:** The thin layer of sap between sap wood and inner bark is known as cambium layer. It indicates sap which has yet not been converted into sap wood.
6. **Inner Bark:** It gives protection of cambium layer from any injury.
7. **Outer Bark:** It consists of cells of wood fibres and is also known as cortex.
8. **Medullary Rays:** The thin radial fibres extending from pith to cambium layer are known as medullary rays. The function of these rays is to hold together the annual rings of heart wood and sap wood.

1. (b) Solution:

According to IS 800: 2007, steel sections are classified into four categories based on the susceptibility of their plate elements to local buckling and their ability to develop plastic moment and rotation capacity. The classification depends on the limiting width-to-thickness ratios specified in the code.

Class 1 Plastic Sections

These sections can develop the full plastic moment capacity M_p and have sufficient plastic rotation capacity to form a plastic hinge with adequate ductility. Local buckling does not occur before the attainment of plastic moment and required rotation. These sections are suitable for plastic analysis and moment redistribution.

Design moment capacity is given by

$$M_d = \frac{\beta_b Z_p f_y}{\gamma_{m0}}$$

For plastic sections, $\beta_b = 1.0$

Behavior: The section reaches M_p and sustains large rotations without premature local buckling.

Class 2 Compact Sections

These sections can also develop the full plastic moment M_p, but their plastic rotation capacity is limited. Local buckling occurs after reaching the plastic moment but before sufficient rotation develops for plastic hinge formation required in plastic analysis.

Design moment capacity is

$$M_d = \frac{\beta_b Z_p f_y}{\gamma_{m0}}$$

For compact sections,

$$\beta_b = 1.0$$

Behavior: The section attains M_p, but local buckling restricts rotation capacity.

Class 3 Semi-Compact Sections

In semi-compact sections, the extreme fiber stress can reach the yield stress f_y, but local buckling prevents the development of the full plastic moment M_p. The section is limited to elastic moment capacity.

Design moment capacity is

$$M_d = \frac{Z_e f_y}{\gamma_{m0}}$$

Behavior: The section can reach yield stress at extreme fibers, but local buckling occurs before plastic stress distribution develops.

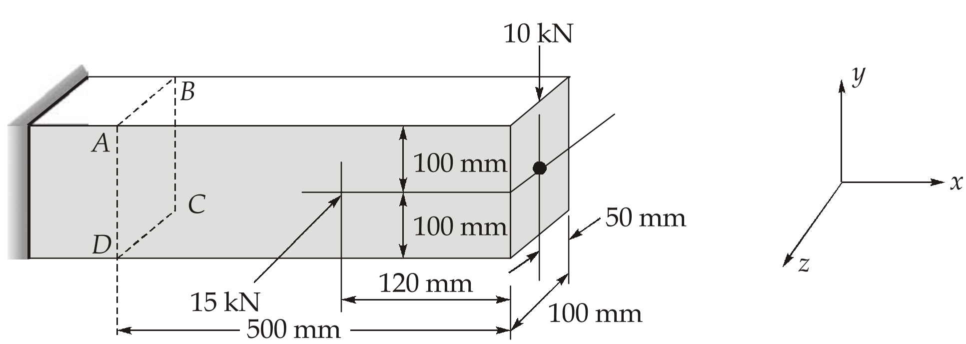
Class 4 . Slender Sections

In slender sections, local buckling occurs before the stress in compression elements reaches yield stress. Therefore, the effective cross-sectional area is reduced due to buckling effects.

Design moment capacity is calculated using effective section properties as specified in IS 800: 2007.

Behavior: Failure is governed by early local buckling of one or more plate elements before yielding.

1. (c) Solution:



Given: Vertical load, $P_y = 10 \times 10^3$ N

Horizontal load, $P_z = 15 \times 10^3$ N

Distance of section from free end, $x = 500$ mm

Distance of horizontal load from free end = 120 mm

Section width, $b = 100$ mm

Section depth, $d = 200$ mm

Moment of inertia about z-axis,

$$I_z = \frac{bd^3}{12} = \frac{100 \times 200^3}{12} = 66666666.67 \text{ mm}^4$$

Moment of inertia about y-axis,

$$I_y = \frac{db^3}{12} = \frac{200 \times 100^3}{12} = 16666666.67 \text{ mm}^4$$

First, the bending moments at the section 500 mm from the free end are determined.

The bending moment about the z-axis due to vertical load is

$$M_z = 10 \times 10^3 \times 500$$

$$M_z = 5000000 \text{ N-mm}$$

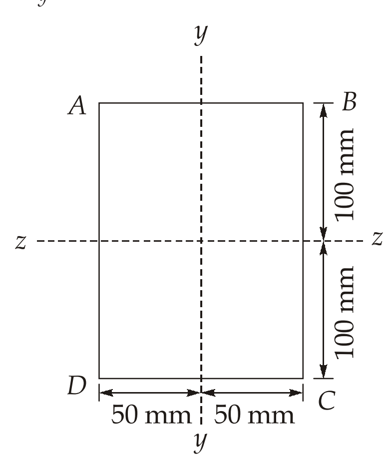
The distance of the horizontal load from the section is

$$500 - 120 = 380 \text{ mm}$$

The bending moment about the y-axis is

$$M_y = 15 \times 10^3 \times 380$$

$$M_y = 5700000 \text{ N-mm}$$



The bending stress under unsymmetrical bending is given by

$$\sigma = \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Stress at B,
$$\sigma_B = \frac{+5000000 \times 100}{66666666.67} + \left(\frac{-5700000 \times 50}{16666666.67}\right)$$

$$\sigma_B = -9.6 \text{ MPa} = 9.6 \text{ MPa (Compressed)}$$

Stress at A,
$$\sigma_A = \frac{+5000000 \times 100}{66666666.67} + \left(\frac{+5700000 \times 50}{16666666.67}\right)$$
$$\sigma_A = +24.6 \text{ MPa} = 24.6 \text{ MPa (tensile)}$$

Stress at C,
$$\sigma_C = \frac{-5000000 \times 100}{66666666.67} + \left(\frac{-5700000 \times 50}{16666666.67}\right)$$
$$\sigma_C = -24.6 \text{ MPa} = 24.6 \text{ MPa (Compressive)}$$

Stress at D,
$$\sigma_D = \frac{-5000000 \times 100}{66666666.67} + \left(\frac{+5700000 \times 50}{16666666.67}\right)$$
$$\sigma_D = +9.6 \text{ MPa} = 9.6 \text{ MPa (tensile)}$$

1. (d) Solution:

First, determine the deflection at point A using superposition.

The downward deflection at A due to the two external loads and the upward reaction R due to spring is

$$\delta_A = \frac{P\left(\frac{L}{2}\right)^3}{3EI} + \frac{P\left(\frac{L}{2}\right)^2}{2EI} \times \frac{L}{2} + \frac{PL^3}{3EI} - \frac{RL^3}{3EI}$$

$$\Rightarrow \quad \delta_A = \frac{5PL^3}{48EI} + \frac{PL^3}{3EI} - \frac{RL^3}{3EI}$$

$$\Rightarrow \quad \delta_A = \frac{7PL^3}{16EI} - \frac{RL^3}{3EI}$$

The deflection of the spring is

$$\delta_s = \frac{R}{k}$$

Substituting $k = \dfrac{EI}{2L^3}$

$$\delta_s = \frac{R}{EI/(2L^3)} = \frac{2RL^3}{EI}$$

Equating beam deflection and spring deflection,

$$\frac{7PL^3}{16EI} - \frac{RL^3}{3EI} = \frac{2RL^3}{EI}$$

$$\Rightarrow \qquad \frac{7PL^3}{16EI} = \frac{7RL^3}{3EI}$$

$$R = 0.1875\ P$$

Using vertical force equilibrium,

$$\Sigma F_y = 0 \quad \Rightarrow \quad V_B + R - P - P = 0$$

$$V_B = 2P - 0.1875\ P$$

$$V_B = 1.8125\ P$$

Taking moments about point B,

$$\Sigma M_B = 0 \quad \Rightarrow \quad M_B + RL - P(L/2) - PL = 0$$

$$M_B = 1.5PL - 0.1875\ PL$$

$$M_B = 1.3125PL$$

Hence,

Spring force, $R = 0.1875\ P$

SF at B, $V_B = 1.8125\ P$

BM at B, $M_B = 1.3125\ P$ (hogging)

1. (e) Solution:

Given

Lateral force, $F = 72.951$ kN

Initial displacement, $u_1 = 5.08$ cm

Number of cycles, $n = 4$

Time for 4 cycles, $t = 2.0$ sec

Amplitude after 4 cycles, $u_{1+4} = 2.54$ cm

Final target amplitude, $u_{1+j} = 0.508$ cm

Acceleration due to gravity, $g = 981\ \text{cm/sec}^2$

(a) Damping ratio (ξ)

Logarithmic decrement for n cycles is

$$\delta = \frac{1}{n}\ln\left(\frac{u_1}{u_{1+n}}\right)$$

$$\Rightarrow \qquad \delta = \frac{1}{4}\ln\left(\frac{5.08}{2.54}\right)$$

$$\Rightarrow \qquad \delta = 0.173$$

For small damping,

$$\xi = \frac{\delta}{2\pi}$$

$$\Rightarrow \quad \xi = \frac{0.173}{2\times\pi}$$

$$\Rightarrow \quad \xi = 0.028$$

(b) Natural period of undamped vibration T_n

Damped period,

$$T_D = \frac{t}{n}$$

$$\Rightarrow \quad T_D = \frac{2.0}{4}$$

$$\Rightarrow \quad T_D = 0.5 \text{ sec}$$

Since damping is small,

$$T_n \approx T_D = 0.5 \text{ sec}$$

(c) Effective stiffness k,

$$k = \frac{F}{u_1} = \frac{72.951\times10^3}{5.08\times10^{-2}} \text{ N/m}$$

$$k = 14.3604 \times 10^5 \text{ N/m}$$

(d) Effective weight W

Natural circular frequency,

$$\omega_n = \frac{2\pi}{T_n}$$

$$\Rightarrow \quad \omega_n = \frac{2\pi}{0.5}$$

$$\Rightarrow \quad \omega_n = 12.566 \text{ rad/sec}$$

Mass,

$$m = \frac{k}{w_n^2} = \frac{14.3604\times10^5}{(12.566)^2}$$

$$\Rightarrow \quad m = 9094.36 \text{ kN-sec}^2\text{/cm}$$

Weight,

$$W = m \times g$$

$$\Rightarrow \quad W = 9094.36 \times 9.81$$

$$\Rightarrow \quad W = 89.215 \text{ kN}$$

(e) Damping coefficient c

$$c = \xi \times \left(2\sqrt{km}\right)$$

$$\Rightarrow \quad c = 0.028 \times \left(2\sqrt{14.3604 \times 10^5 \times 9094.36}\right)$$

$$\Rightarrow \quad c = 6399.67 \text{ N-sec/m}$$

$$\Rightarrow \quad = 0.064 \text{ kN-sec/m}$$

(f) Number of cycles j for amplitude to reduce to 0.508 cm

$$j = \frac{1}{2\pi\xi}\ln\left(\frac{u_1}{u_{1+j}}\right)$$

$$\Rightarrow \quad j = \frac{1}{2 \times \pi \times 0.028}\ln\left(\frac{5.08}{0.508}\right)$$

$$\Rightarrow \quad j = 13.088 \text{ cycles}$$

To the nearest whole number,

$$j \approx 13 \text{ cycles}$$

2. (a) (i) Solution:

Primary Compression Failure (Over-Reinforced)

A primary compression failure occurs in what is known as an over-reinforced section. In this scenario, the amount of tensile steel is so high that the concrete reaches its maximum compressive strength and crushes before the steel has a chance to yield.

Mechanism: The concrete at the top of the beam (the compression zone) exceeds its strain limit (typically 0.003) while the steel remains in its elastic stage.

Behavior: It is a brittle failure. There is very little deflection or cracking to warn occupants that the structure is about to fail.

Visuals: The concrete explodes or spalls off suddenly.

Secondary Compression Failure (Under-Reinforced)–

A secondary compression failure occurs in an under-reinforced section. This is the standard sequence of failure for properly designed beams. Here, the steel reinforcement yields first, and as it stretches, the neutral axis of the beam shifts upward, eventually forcing the concrete to crush.

Mechanism: The steel reaches its yield strain $\in_y$ and continues to stretch (plastic deformation). This causes large cracks to open at the bottom of the beam. Eventually, the reduced area of concrete at the top can no longer support the load and crushes.

Behavior: It is a ductile failure. The beam undergoes significant sagging (deflection) and visible cracking long before total collapse.

Sequence: Yielding of steel → Large Deflections → Compression Crushing.

Secondary compression failure (under-reinforced design) is the only recommended "safe" failure mode. Engineering codes (like ACI or Eurocode) strictly mandate that beams be designed as under-reinforced. The goal is to ensure that if a member is overloaded, it provides a visible warning. This gives occupants time to evacuate the building or perform repairs. A primary compression failure is considered dangerous because the collapse is instantaneous and "explosive," leaving no time for intervention.

2. (a) (ii) Solution:

Given data

Width of beam (b) = 350 mm

Overall depth (D) = 650 mm

Effective cover (d') = 60 mm

Effective depth (d) = 650 – 60 = 590 mm

Area of tension steel (A_{st}) = 2500 mm^2

Area of compression steel (A_{sc}) = 900 mm^2

Characteristic strength of concrete (f_{ck}) = 20N/mm^2

Yield strength of steel (f_y) = 415 N/mm^2

Limiting Depth of Neutral Axis ($x_{u,\text{lim}}$):

$$\frac{x_{u,\text{lim}}}{d} = 0.48 \qquad \text{(For Fe 415 steel)}$$

$$\Rightarrow \quad x_{u,\text{lim}} = 0.48 \times 590$$

$$\Rightarrow \quad x_{u,\text{lim}} = 283.2 \text{ mm}$$

Actual Depth of Neutral Axis (x_u):

Equating Total Compression (C) = Total Tension (T):

$$0.36 f_{ck}\, b\, x_u + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$$

$$\Rightarrow \quad 0.36 \times 20 \times 350 \times x_u + 900 \times (f_{sc} - 0.45 \times 20) = 0.87 \times 415 \times 2500$$

$$\Rightarrow \quad 2520 \times x_u + 900 \times f_{sc} - 8100 = 902625$$

$$\Rightarrow \quad 2520 \times x_u + 900 \times f_{sc} = 910725$$

$$x_u = \frac{910725 - 900 \times f_{sc}}{2520}$$

Trial 1:

Assume f_{sc} = 350 MPa

$$x_u = \frac{910725 - 900 \times 350}{2520}$$

$$\Rightarrow \quad x_u = 236.4 \text{ mm}$$

Now, check strain in compression steel ($\in_{sc}$):

$$\in_{sc} = 0.0035 \times \left(\frac{x_u - d'}{x_u}\right)$$

$$\Rightarrow \quad \in_{sc} = 0.0035 \times \frac{236.4 - 60}{236.4}$$

$$\Rightarrow \quad \in_{sc} = 0.002611$$

From interpolation for strain 0.002611:

$$f_{sc} = 342 + \frac{(351 - 342) \times (0.002611 - 0.00241)}{(0.00276 - 0.00241)}$$

$$f_{sc} = 347.169 \text{ MPa}$$

Trial 2:

Take f_{sc} = 347.169 MPa

$$x_u = \frac{910725 - 900 \times 347.169}{2520}$$

$$\Rightarrow \quad x_u = 237.409 \text{ mm}$$

Check strain $\in_{sc}$ again:

$$\in_{sc} = 0.0035 \times \frac{237.409 - 60}{237.409}$$

$$\Rightarrow \quad \in_{sc} = 0.002615$$

f_{sc} using interpolation:

$$f_{sc} = 342 + \frac{(351 - 342) \times (0.002615 - 0.00241)}{0.00035}$$

$$f_{sc} = 347.271 \text{ MPa} \simeq (347.169 \text{ MPa})$$

$$\text{Adopt } x_u = 237.409 \text{ mm and } f_{sc} = 347.271 \text{ MPa.}$$

Since $x_u < x_{u,\lim}$(237.409 < 283.2), the section is Under-reinforced.

Ultimate Moment Capacity (M_u):

$M_u = 0.36 f_{ck} b \times x_u(d - 0.42 x_u) + A_{sc} (f_{sc} - 0.45 f_{ck}) \times (d - d')$

$\Rightarrow M_u = [0.36 \times 20 \times 350 \times 237.409 \times (590 - 0.42 \times 237.409)] + [900 \times (347.271 - 9) \times (590 - 60)]$

$\Rightarrow M_u = 454.680$ kN-m

2. (b) (i) Solution:

There are three kinds of time estimates generally made for each activity in PERT which are as indicated below:

(a) Optimistic time estimate: This is the shortest possible time in which an activity can be completed under ideal conditions. Thus, this time estimate represents the time in which the activity would be completed if everything went perfectly well without any problems or adverse conditions being developed during the execution of the activity. In arriving at this time estimate no provisions are made for delays or setbacks and better than normal conditions are assumed to prevail during the execution during the execution of the activity. This time estimate is denoted by t_0.

(b) Pessimistic time estimate: This is the maximum time that would be required to completed the activity. Thus, this time estimate represents the time required to complete the activity if everything went wrong and abnormal situations prevailed. However, this estimate does not include the possible effects of major catastrophes such as floods, fires, earthquakes, etc., or that of labour strikes or unrest etc. This time estimate is denoted by t_p.

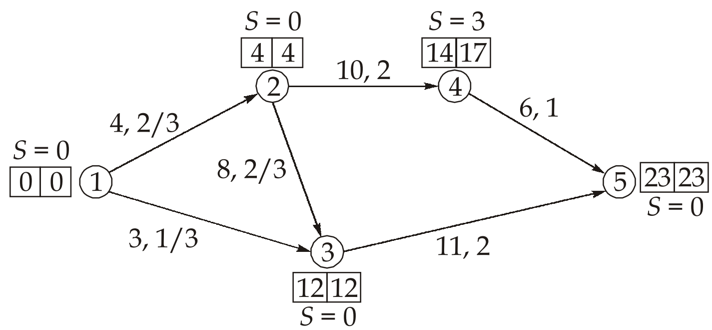
(c) Most likely time estimate: This is the time required the activity if normal conditions prevail. This time estimate lies between the optimistic and the pessimistic time estimates. Thus, this time estimate effects a situation where normal conditions are prevailing, things are as usual and there is nothing abnormal. This time estimate is denoted by t_m.

By using above time estimates, expected of an activity can be calculated as

$$\text{Expected time } (t_E) = \frac{t_0 + t_p + 4t_m}{6}$$

2. (b) (ii) Solution:

For given project, network diagram:



Activity	Time duration in days			Expected time	S.D.
	Optimistic (t_o)	Most likely (t_m)	Pessimistic (t_p)	$t_E = \frac{t_0 + t_p + 4t_m}{6}$	$\sigma = \frac{t_p - t_o}{6}$
1-2	2	4	6	4	2/3
1-3	2	3	4	3	1/3
2-3	6	8	10	8	2/3
2-4	4	10	16	10	2
3-5	7	10	19	11	2
4-5	3	6	9	6	1

From network diagram:

1. Critical path = 1-2-3-5

For which critical path duration T_c = 23 days

$$\text{Path standard deviation, } \sigma = \sqrt{\sigma_{1-2}^2 + \sigma_{2-3}^2 + \sigma_{3-5}^2}$$

$$= \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + 2^2} = 2.211 \text{ days}$$

2. For scheduled time of 28 days

$$\text{Probability factor, } Z = \frac{T_s - T_c}{\sigma} = \frac{28 - 23}{2.211} = 2.262$$

From table using

when $Z = 2.2$, $P = 98.61\%$

$Z = 2.3$, $P = 98.93\%$

$$\text{Probability of completion, } P = 98.61 + \frac{(98.93 - 98.61)}{(2.3 - 2.2)} \times (2.262 - 2.2) = 98.81\%$$

3. For probability of completion = 98.5%

From table using

when $P = 98.21\%, \quad Z = 2.1$

$P = 98.61\%, \quad Z = 2.2$

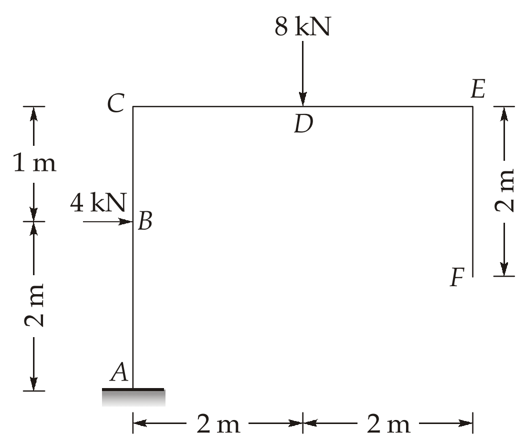
For $P = 98.5\%$

Probability factor, $$Z = 2.1 + \frac{98.5 - 98.21}{98.61 - 98.21} \times (2.2 - 2.1) = 2.1725$$

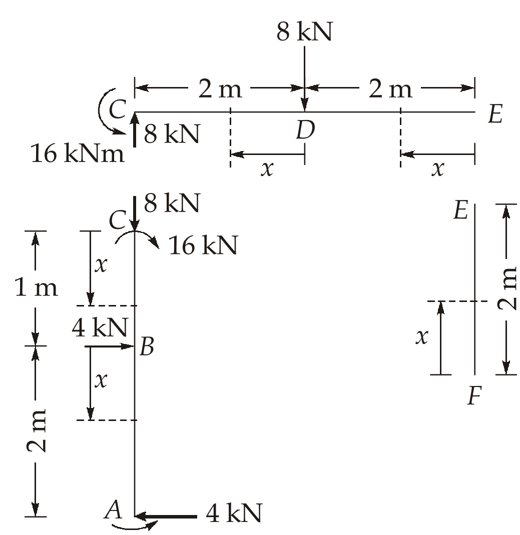
Duration for 98.5% probability, $T = T_C + \sigma \cdot Z$

$= 23 + 2.1725 \times 2.211 = 27.80 \text{ days}$

2. (c) Solution:

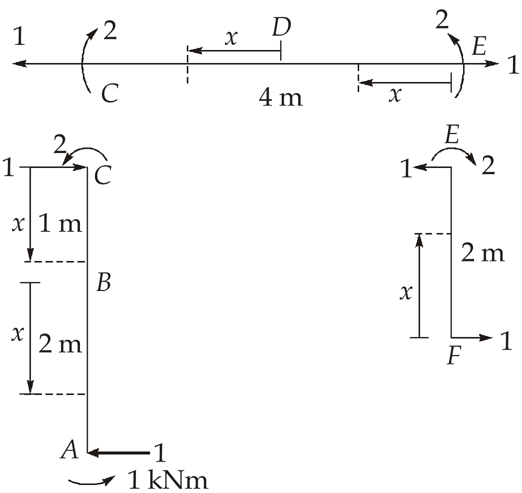


Using Unit Load Method:



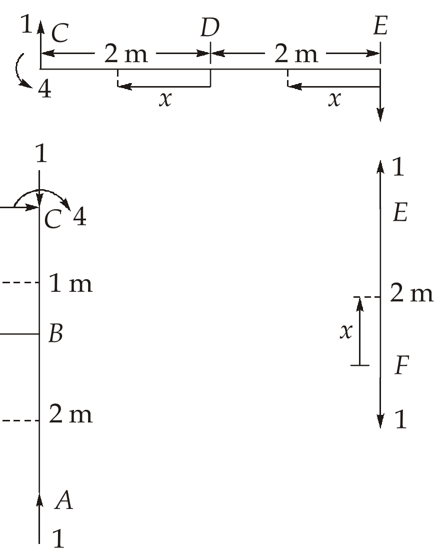
(For moment M)

Remove external load and applying unit load at point F in rightward direction.



(For moment m_1)

Remove external load and applying unit load at point F in vertical direction.



(For moment m_2)

Member	Origin	Range of x	M	m_1	m_2
FE	F	0 - 2	0	x	0
ED	E	0 - 2	0	2	$-x$
DC	D	0 - 2	$-8x$	2	$-(x+2)$
CB	C	0 - 1	-16	$2 - x$	-4
BA	B	0 - 2	$-16-4x$	$2-(1+x) = 1-x$	-4

Horizontal deflection at F

So, $$(\Delta_F)_H = \int_0^2 \frac{0 \times x dx}{EI} + \int_0^2 \frac{0 \times 2dx}{EI} + \int_0^2 \frac{-8x \times 2}{EI} dx + \int_0^1 \frac{-16 \times (2-x)}{EI} dx + \int_0^2 \frac{-(16+4x)(1-x)}{EI} dx$$

$\Rightarrow$ $$(\Delta_F)_H = \frac{-32}{EI} - \frac{24}{EI} + \frac{8}{3EI} = \frac{-160}{3EI}$$

Here negative mean that deflection is opposite to the applied unit load i.e. left wards.

So, deflection at point F is $\frac{160}{3EI}$ $(\leftarrow)$

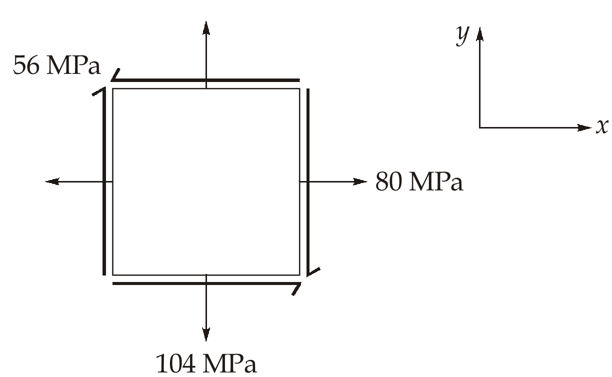
Vertical deflection at F:

$$(\Delta_F)_V = \int_0^2 \frac{(8x)(x+2)dx}{EI} + \int_0^1 \frac{(-16)(-4)dx}{EI} + \int_0^2 \frac{(16+4x)4dx}{EI}$$

$\Rightarrow$ $$(\Delta_F)_V = \frac{277.33}{EI} (\downarrow)$$

3. (a) Solution:

Given that:
$$\sigma_y = 104 \text{ MPa}$$
$$\sigma_x = 80 \text{ MPa}$$
$$\tau_{xy} = 56 \text{ MPa}$$



Principal stresses $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\Rightarrow$ $\sigma_{1,2} = \frac{80+104}{2} \pm \sqrt{\left(\frac{80-104}{2}\right)^2 + (56)^2}$

$\Rightarrow$ $\sigma_{1,2} = 92 \pm 57.27$

So, $\sigma_1 = 149.271$ MPa (tensile)

$\sigma_2 = 34.729$ MPa (tensile)

$$\tau_{max\ (in\ plane)} = \frac{\sigma_1 - \sigma_2}{2} = \frac{149.271 - 34.729}{2} = 57.271 \text{ MPa}$$

$$\tau_{max\ (absolute)} = \max\left[\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma|}{2}, \frac{|\sigma_2|}{2}\right] = 74.635 \text{ MPa}$$

For orientation:

$$\tan 2\theta_p = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \left(\frac{2 \times 56}{80 - 104}\right) = -4.667$$

$2\theta_p = -77.905°$

$\Rightarrow$ $\theta_p = -38.95°$

So, $\theta_{P_1} = 38.95°$ (Anticlockwise from vertical plane)

$\theta_{P_2} = -38.95° + 90° = 51.05°$ (clockwise from vertical plane)

Check for the maximum principal plane

$$\sigma'_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

Putting $\theta = 38.95°$

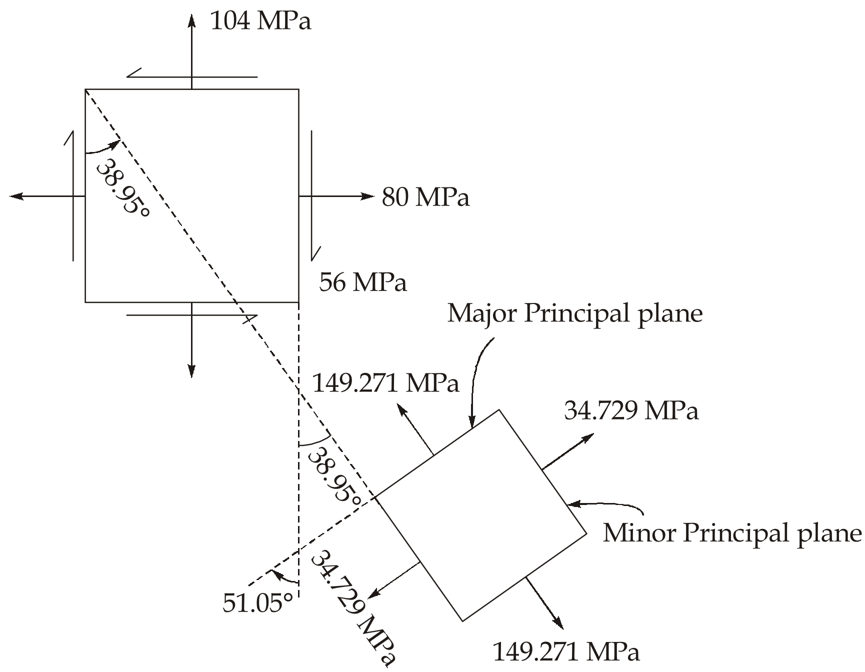
$\Rightarrow$ $\sigma'_N = \left(\frac{104+80}{2}\right) + \left(\frac{80-104}{2}\right)\cos(-77.9°) + 56\sin(-77.9°)$

$= 34.73$ MPa $= \sigma_2$ (minor principal stress) (OK)

So, the plane at angle $\theta = -38.95°$ is the minor principal plane.

and $\theta = +51.05°$ is the major principal plane

Orientation of principal planes



Orientation of max shear plane:

$$\tan(2\theta_s) = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}} = \frac{-(80-104)}{2\times 56} = 0.214285$$

$$\theta_{s_1} = 6.047° \text{ (Clockwise from vertical plane)}$$

$$\theta_{s_2} = 96.047° \text{ (clockwise from vertical plane)}$$

Check

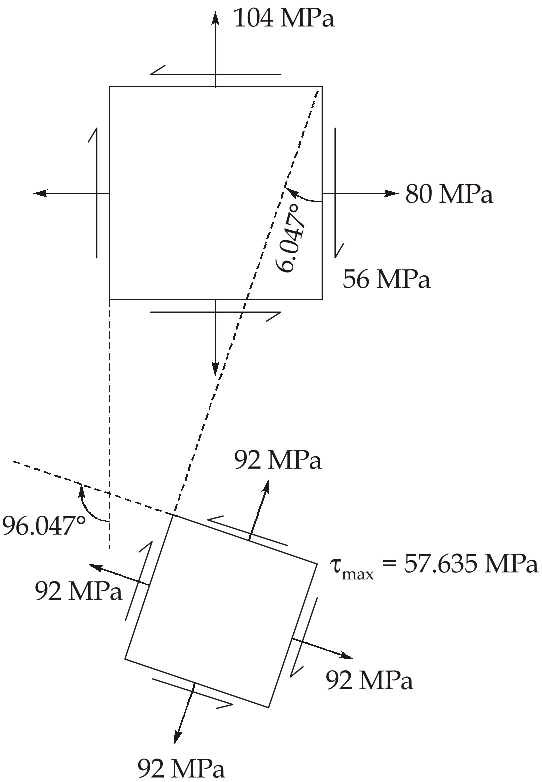
$$\tau_{x'y'} = \frac{-(\sigma_x - \sigma_y)}{20} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Put $\theta = 6.047°$

$$= \frac{-(80-104)}{2}\sin(2\times 6.047°) + 56\cos(2\times 6.047°)$$

$$= 57.271 \text{ MPa} = \tau_{max} \quad \text{(OK)}$$

Normal stress on the plane of max shear stress, $\sigma_s = \dfrac{\sigma_1 + \sigma_2}{2} = 92$ MPa



3. (b) (i) Solution:

$$\text{Diameter of bolt } (d) = 20 \text{ mm}$$

$$\text{Diameter of bolt hole } (d_h) = 20 + 2 = 22 \text{ mm}$$

Design Strength due to Yielding of Gross Section T_{dg}:

Gross Area,

$$A_g = (115 + 150 - 12) \times 12 = 3036 \text{ mm}^2$$

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}} = \frac{3036 \times 250}{1.1} \times 10^{-3} = 690 \text{ kN}$$

Design Strength due to Rupture of Critical Section T_{dn}:

Net area of connected leg (A_{nc}):

$$A_{nc} = \left(115 - \frac{12}{2} - 22\right) \times 12 = 1044 \text{ mm}^2$$

Gross area of outstanding leg (A_{go}):

$$A_{go} = \left(150 - \frac{12}{2}\right) \times 12 = 1728 \text{ mm}^2$$

Length of connection (L_c):

$$L_c = (3 - 1) \times 80 = 160 \text{ mm}$$

Width of outstanding leg (w) = 150 mm

Shear lag width (w_1): 65 mm

$$b_s = w + w_1 - t = 150 + 65 - 12 = 203 \text{ mm}$$

Now,
$$\beta = 1.4 - 0.076 \times \left(\frac{w}{t}\right) \times \left(\frac{f_y}{f_u}\right) \times \left(\frac{b_s}{L_c}\right)$$

$$\beta = 1.4 - 0.076 \times \frac{150}{12} \times \frac{250}{410} \times \frac{203}{160} = 0.663$$

Since β must be ≥ 0.7, take β = 0.7.

Also check:

$$\beta \le \frac{f_u \times \gamma_{m0}}{f_y \times \gamma_{m1}} = \frac{410 \times 1.1}{250 \times 1.25} = 1.4432$$

Design tensile strength due to rupture,

$$T_{dn} = \frac{0.9 A_{nc} f_u}{\gamma_{m1}} + \frac{\beta A_{go} f_y}{\gamma_{m0}}$$

$$\Rightarrow \quad T_{dn} = \left(\frac{0.9 \times 1044 \times 410}{1.25} + \frac{0.7 \times 1728 \times 250}{1.1}\right) \times 10^{-3}$$

$$T_{dn} = 583.097 \text{ kN}$$

Design Strength due to Block Shear T_{db}:

Shear Areas:

$$A_{vg} = (2 \times 80 + 40) \times 12 = 2400 \text{ mm}^2$$

$$A_{vn} = (200 - 2.5 \times 22) \times 12 = 1740 \text{ mm}^2$$

Tension Areas:

$$A_{tg} = 50 \times 12 = 600 \text{ mm}^2$$

$$A_{tn} = (50 - 0.5 \times 22) \times 12 = 468 \text{ mm}^2$$

Calculate block shear strengths T_{db1} and T_{db2}:

$$T_{db1} = \frac{A_{vg} f_y}{\sqrt{3}\gamma_{m0}} + \frac{0.9 A_{tn} f_u}{\gamma_{m1}}$$

$$T_{db1} = \left(\frac{2400 \times 250}{\sqrt{3} \times 1.1} + \frac{0.9 \times 468 \times 410}{1.25}\right) \times 10^{-3} = 453.071 \text{ kN}$$

$$T_{db2} = \frac{0.9 A_{vn} f_u}{\sqrt{3}\gamma_{m1}} + \frac{A_{tg} f_y}{\gamma_{m0}}$$

$$T_{db2} = \left(\frac{0.9 \times 1740 \times 410}{\sqrt{3} \times 1.25} + \frac{600 \times 250}{1.1}\right) \times 10^{-3} = 432.918 \text{ kN}$$

Block shear strength, $T_{db} = \min(T_{db1}, T_{db2}) = 432.918$ kN

Final Tensile Strength

The design tensile strength is the minimum of T_{dg}, T_{dn}, and T_{db}:

$$T_d = \min(690, 583.097, 432.918) = 432.918 \text{ kN}$$

3. (b) (ii) Solution:

Given:

Plywood web thickness, $t = 15$ mm

Allowable shear stress in plywood, $\tau_{allow} = 2 \text{ MPa} = 2 \text{ N/mm}^2$

Total beam height, $H = 240$ mm

Flange width, $b_f = 80$ mm

Flange height, $h_f = 40$ mm

Web inset into each flange, 12 mm

Neutral axis location from bottom (symmetrical section),

$$y_{NA} = 120 \text{ mm}$$

Moment of inertia of entire section,

$$I = \frac{15 \times 160^3}{12} + 2\left[\frac{80 \times 40^3}{12} + (80 \times 40)(120 - 20)^2\right]$$

$$\Rightarrow \quad I = 69{,}973{,}333.33 \text{ mm}^4$$

For balanced design, the maximum allowable shear force is governed by the allowable shear stress in the plywood web. The maximum shear stress occurs at the neutral axis.

First moment of area at the neutral axis is determined.

$$(A\bar{y})_{NA} = Q_{\text{flange}} + Q_{\text{half-web}}$$

$$\Rightarrow \quad (A\bar{y})_{NA} = (80 \times 40 \times 100) + (15 \times 80 \times 40)$$

$$\Rightarrow \quad (A\bar{y})_{NA} = 368{,}000 \text{ mm}^3$$

Using the shear formula,

$$\tau = \frac{VA\bar{y}}{Ib}$$

At NA;
$$V = \frac{\tau_{\text{allow}} Ib}{(A\bar{y})_{NA}}$$

$$\Rightarrow \quad V = \frac{2 \times 69,973,333.33 \times 15}{368,000}$$

$$\Rightarrow \quad V = 5,704.35 \text{ N}$$

Required glue strength is determined.

First moment of area at the glue interface equals the first moment of area of the flange above the glue line.

$$Q_{glue} = (80 \times 40 \times 100) - (15 \times 12 \times 86)$$

$$\Rightarrow \quad Q_{glue} = 304,520 \text{ mm}^3$$

Shear flow at the interface is

$$q = \frac{VQ_{glue}}{I}$$

$$\Rightarrow \quad q = \frac{5,704.35 \times 304,520}{69,973,333.33}$$

$$\Rightarrow \quad q = 24.825 \text{ N/mm}$$

The total glue thickness per unit length is

$$2 \times 12 + 15 = 39 \text{ mm}$$

Required glue shear stress is

$$\tau_{\text{glue}} = \frac{q}{39}$$

$$\Rightarrow \quad \tau_{\text{glue}} = 0.636 \text{ N/mm}^2$$

3. (c) Solution:

Given:

Settlement at support A (Δ) = 15 mm

Area of cross-section (A) = 1200 mm^2

Modulus of Elasticity (E) = 2×10^5 N/mm^2

Height of truss (h) = 4 m

Span, AC = 6 m, Span CE = 3 m

Geometric Properties

$$\text{Length of } BC = \sqrt{6^2 + 4^2} = 7.211 \text{ m}$$

$$\text{Length of } DE = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

For member BC:

$$\sin\theta = \frac{4}{7.211} = 0.555$$

$$\cos\theta = \frac{6}{7.211} = 0.832$$

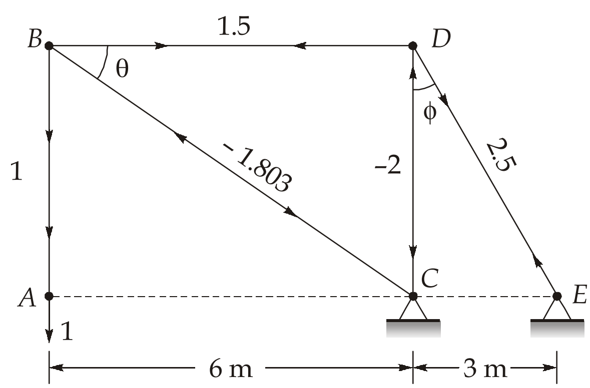
For member DE:

$$\sin\phi = \frac{3}{5} = 0.6$$

$$\cos\phi = \frac{4}{5} = 0.8$$

Force Analysis (Unit Load Method)

We select the reaction at A (R_A) as the redundant. Applying a unit load (1 N) vertically downward at A to find the internal member forces (k).



$$k_{AB} = 1$$

$$k_{BD} = \frac{1}{\tan\theta} = \frac{6}{4} = 1.5$$

$$k_{BC} = -\frac{1}{\sin\theta} = \frac{-7.211}{4} = -1.803$$

$$k_{DC} = -2$$

$$k_{DE} = \frac{1.5}{\sin\phi} = \frac{1.5\times 5}{3} = 2.5$$

Tabular Calculation for $\sum \frac{u^2 L}{AE}$

Member	Area	Length (L) (m)	k	k^2L/A
AB	A	4	1	4/A
BD	A	6	1.5	13.5/A
BC	2A	7.211	–1.803	11.716/A
DC	A	4	–2	16/A
DE	2A	5	2.5	15.625/A
Total				**60.841/A**

Calculation of Redundant Reaction (R_A)

Using the compatibility equation for settlement:

$$R_A \times \sum \frac{k^2 L}{AE} = \Delta$$

$$R_A \times \frac{60.841}{A \times E} = 15 \text{ mm}$$

$$R_A = \frac{0.015 \times 1200 \times 2 \times 10^5}{60.841}$$

$$R_A = 59170.625 \text{ N} = 59.171 \text{ kN}$$

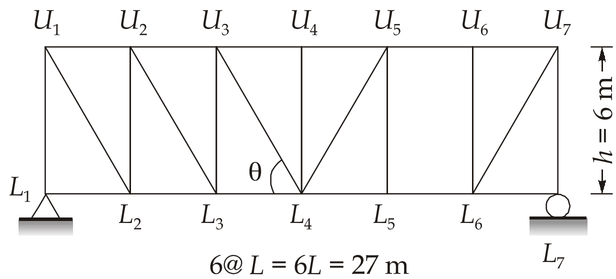
Final Member Forces

The internal force in each member is calculated as $P = R_A \times k$.

Member	k	Force $P = R_A \times k$(kN)
AB	1	59.171 (T)
BD	1.5	88.757 (T)
BC	–1.803	–106.685 (C)
DC	–2	–118.342 (C)
DE	2.5	147.928 (T)

Thus, the support reaction at A is 59.171 kN and the internal forces in the members are as listed above.

4. (a) Solution:



Given:

$$\text{Number of panels} = 6$$

$$\text{Panel length } (d) = 4.5 \text{ m}$$

$$\text{Total Span } (L) = 6 \times 4.5 = 27 \text{ m}$$

$$\text{Truss height } (h) = 6 \text{ m}$$

Distance of U_3 and L_3 from left support (x) = 9 m

Distance ofU_4 and L_4 from left support = 13.5 m

Angle of diagonal U_3L_4 with horizontal (θ):

$$\tan\theta = \frac{6}{4.5}$$

$$\theta = 53.13°$$

$$\sin\theta = 0.8$$

$$\cos\theta = 0.6$$

1. *ILD* for Top Chord Member U_3U_4

To find the force in U_3L_4, pass a vertical section through U_3U_4, U_3L_4, and L_3L_4 and take moments about L_4 (distance 13.5 m from left).

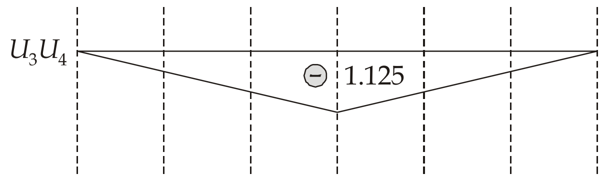
The formula for the ordinate at distance x is: $\dfrac{x(L-x)}{Lh}$

When unit load is at L_4(x = 13.5 m):

$$F_{U_3U_4} = \frac{13.5(27-13.5)}{27\times 6}$$

$$F_{U_3U_4} = 1.125 \text{ (Compression)}$$

The *ILD* is a triangle with a peak ordinate of 1.125 at L_4.



2. ***ILD* for Bottom Chord Member L_3L_4**

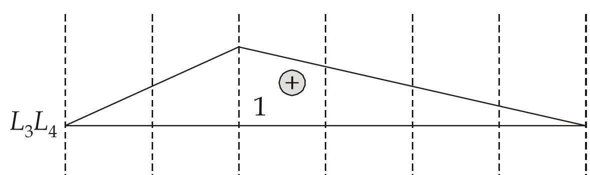
To find the force in L_3L_4, take moments about U_3(distance 9 m from left).

When unit load is at L_3(x = 9 m):

$$F_{L_3L_4} = \frac{9(27-9)}{27\times 6}$$

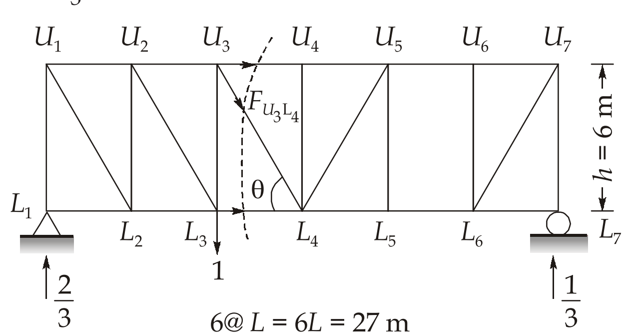
$$F_{L_3L_4} = 1\text{(Tension)}$$

The *ILD* is a triangle with a peak ordinate of 1 at L_3.



3. ***ILD* for Diagonal Member U_3L_4**

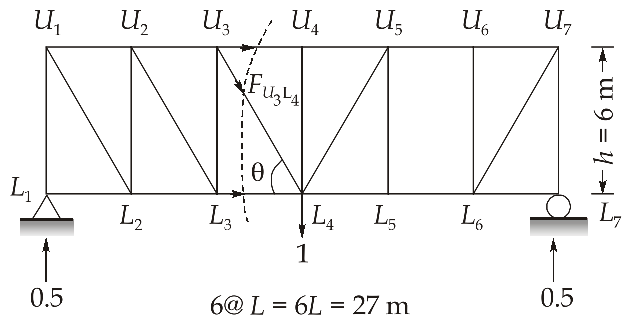
When unit load at L_3:



$\Sigma F_y = 0 \quad \Rightarrow \quad -F_{U_3L_4}\sin\theta - 1 + \frac{2}{3} = 0$

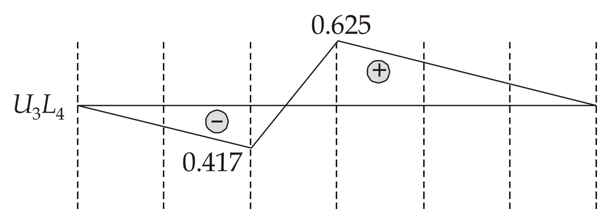
$\Rightarrow \quad F_{U_3L_4} = -0.417$ (compressive)

When unit load at L_4:



$\Sigma F_y = 0 \quad \Rightarrow \quad -F_{U_3L_4}\sin\theta + 0.5 = 0$

$\Rightarrow \quad F_{U_3L_4} = 0.625$ (tensile)



4. (b) Solution:

Given:

Slab thickness (D_f) = 120 mm

Center-to-center spacing (s) = 3500 mm

Distance between zero bending moment (L_0) = 8 m

Effective span (L_{eff}) = 8000 mm

Web width (b_w) = 300 mm

Total depth (D) = 600 mm

Effective depth (d) = 550 mm

f_{ck} = 25 N/mm^2

f_y = 500 N/mm^2

1. Effective Width of Flange (b_f)

$$b_f = \frac{L_0}{6} + b_w + 6D_f$$

$$\Rightarrow \quad b_f = \frac{8000}{6} + 300 + 6 \times 120$$

$$\Rightarrow \quad b_f = 2353.33 \text{ mm}$$

Since 2353.33 mm is less than the spacing s = 3500 mm:

$$b_f = 2353.33 \text{ mm}$$

2. Load Calculation and Factored Moment

Dead load of slab = 0.12 × 3.5 × 25 = 10.5 kN/m

Dead load of web = 0.3 × (0.6 – 0.12) × 25 = 3.6 kN/m

Live load = 4 × 3.5 = 14 kN/m

Floor finish = 1 × 3.5 = 3.5 kN/m

Total Load (w) = 31.6 kN/m

Factored Load (w_u) = 1.5 × 31.6 = 47.4 kN/m

$$\text{Factored Moment } (M_u) = \frac{w_u \times L_{eff}^2}{8} = \frac{47.4 \times 8^2}{8} = 379.2 \text{ kN-m}$$

3. Area of Tension Reinforcement (A_{st})

Assuming the neutral axis lies within the flange ($x_u < D_f$):

$$M_u = 0.36\ f_{ck} b_f x_u\ (d - 0.42x_u)$$

$$\Rightarrow \quad 379.2 \times 10^6 = 0.36 \times 25 \times 2353.33 \times x_u \times (550 - 0.42 \times x_u)$$

$$0.42\ x_u^{\ 2} - 550\ x_u + 17903.708 = 0$$

$$\Rightarrow \quad x_u = 33.4 \text{ mm, } 1276.11 \text{ mm (not OK)}$$

Since x_u < 120 mm, the assumption is correct.

$$A_{st} = \frac{M_u}{0.87 f_y \left(d - 0.42 \times x_u\right)}$$

$$\Rightarrow \quad A_{st} = \frac{379.2 \times 10^6}{0.87 \times 500 \times (550 - 0.42 \times 33.4)}$$

$$\Rightarrow \quad A_{st} = 1626.44 \text{ mm}^2$$

Providing 25 mm diameter bars:

$$\text{Number of bars} = \frac{1626.44}{\frac{\pi}{4}(25)^2} = 3.32$$

Provide 4 bars of 25 mm diameter.

$$\text{Provided } A_{st} = 4 \times \frac{\pi}{4}(25)^2 = 1963.495 \text{ mm}^2$$

4. Check for Minimum Tension Reinforcement

$$A_{st,\ min} = \frac{0.85 \times b_w \times d}{f_y}$$

$$\Rightarrow \quad A_{st,\ min} = \frac{0.85 \times 300 \times 550}{500}$$

$$\Rightarrow \quad A_{st,\ min} = 280.5\ \text{mm}^2$$

Since 1963.495 mm^2 > 280.5 mm^2, it is safe.

5. Check for Shear

$$V_u = \frac{47.4 \times 8}{2} = 189.6\ \text{kN}$$

$$\tau_v = \frac{189600}{300 \times 550} = 1.149\ \text{N/mm}^2$$

$$p_t = \frac{100 \times 1963.495}{300 \times 550} = 1.19\%$$

From given table

$$\tau_c = 0.64 + \frac{(0.7 - 0.64)}{(1.25 - 1)}(1.19 - 1)$$

$$\tau_c = 0.6856\ \text{MPa}$$

Since $\tau_v > \tau_c$, shear reinforcement is required.

$$V_{us} = V_u - (\tau_c \times b_w \times d)$$

$$\Rightarrow \quad V_{us} = 189600 - (0.6856 \times 300 \times 550)$$

$$\Rightarrow \quad V_{us} = 76476\ \text{N}$$

Using 2-legged 8 mm stirrups

$$\text{Spacing,} \quad s_v = \frac{0.87 \times 500 \times 2 \times \frac{\pi}{4}(8)^2 \times 550}{76476}$$

$$s_v = 314.5\ \text{mm}$$

6. Maximum Spacing Check

$$0.75 \times d = 0.75 \times 550 = 412.5\ \text{mm}$$

$$\text{Absolute maximum} = 300\ \text{mm}$$

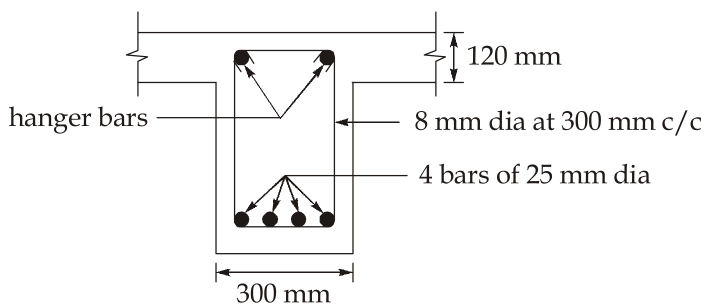
Minimum shear reinforcement:

$$s_{v,\,min} = \frac{2 \times \frac{\pi}{4}(8)^2 \times 0.87 \times 500}{0.4 \times 300}$$

$\Rightarrow$ $$s_{v,\,min} = 364.425 \text{ mm}$$

The calculated s_v exceeds the absolute maximum of 300 mm.

Provide 2-legged 8 mm ϕ stirrups @ 300 mm c/c.



4. (c) (i) Solution:

Total Area (A):

$$(200 \times 20) + (150 \times 15) + (300 \times 10) = 9250 \text{ mm}^2$$

Elastic Neutral Axis ($\bar{y}$ from the bottom):

$$\bar{y} = \frac{(150 \times 15 \times 7.5) + (300 \times 10 \times 165) + (200 \times 20 \times 325)}{9250}$$

$\Rightarrow$ $$\bar{y} = 195.878 \text{ mm from bottom}$$

Moment of Inertia (I):

$$I = \left[\frac{150 \times 15^3}{12} + 2250 \times (195.878 - 7.5)^2\right] + \left[\frac{10 \times 300^3}{12} + 3000 \times (195.878 - 165)^2\right]$$

$$+ \left[\frac{200 \times 20^3}{12} + 4000 \times (325 - 195.878)^2\right]$$

$\Rightarrow$ $$I = 172{,}069{,}946.5 \text{ mm}^4$$

Elastic Section Modulus (Z_e):

$$Z_e = \frac{I}{y_{max}} = \frac{172{,}069{,}946.5 \text{ mm}^4}{195.878 \text{ mm}}$$

$$Z_e = 878{,}454.684 \text{ mm}^3$$

Plastic Neutral Axis

The PNA divides the area into two equal halves:

$$\frac{A}{2} = 4625 \text{ mm}^2$$

Let y_p be the distance from the top.

Area of top flange = 4000 mm^2, which is less than 4625 mm^2, so the PNA lies in the web.

$$\therefore \quad 4000 + (y_p - 20) \times 10 = 4625$$

$$\Rightarrow \quad y_p = 82.5 \text{ mm}$$

Plastic Section Modulus (Z_p)

$$Z_p = \frac{A}{2} \times (\bar{y}_1 + \bar{y}_2)$$

Where $\bar{y}_1$ and $\bar{y}_2$ are the distances of the centroids of the two halves from the PNA.

$$Z_p = [200 \times 20 \times (82.5 - 10)] + \left[10 \times 625 \times \frac{62.5}{2}\right] + \left[10 \times 237.5 \times \frac{237.5}{2}\right] + [150 \times 15 \times (237.5 + 7.5)]$$

$$Z_p = 1142812.5 \text{ mm}^3$$

Shape Factor (S)

$$S = \frac{M_p}{M_y} = \frac{Z_p}{Z_e}$$

$$S = 1.301$$

4. (c) (ii) Solution:

Given:

$$L = 12 \text{ m}$$

$$W_1 = 80 \text{ kN}$$

$$W_2 = 160 \text{ kN}$$

$$\text{Load factor,} = 1.8$$

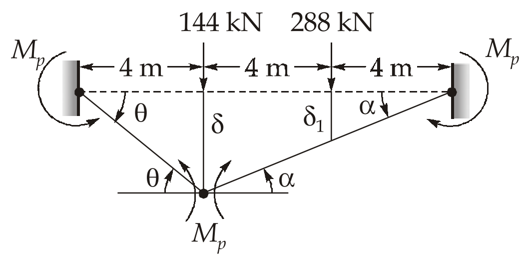
Factored Loads

$$W_{u1} = 80 \times 1.8 = 144 \text{ kN}$$

$$W_{u2} = 160 \times 1.8 = 288 \text{ kN}$$

Mechanism 1: Failure under Load W_{u1} (4 m from left)

Plastic hinges form at the fixed ends and under the load W_{u1}.



$\because \quad \delta = 4\theta = 8\alpha$

$\Rightarrow \quad \alpha = \frac{\theta}{2}$

External work done, $\quad W_e = 144 \times (4\theta) + 288 \times (4\alpha)$

$\Rightarrow \quad W_e = 144 \times 4\theta + 288 \times 4\left(\frac{\theta}{2}\right)$

$\Rightarrow \quad W_e = 1152\theta$

Internal work done, $\quad W_i = M_p\theta + M_p(\theta + \alpha) + M_p\alpha$

$\Rightarrow \quad W_i = M_p\theta + M_p(\theta + 0.5\theta) + M_p(0.5\theta)$

$\Rightarrow \quad W_i = 3M_p\theta$

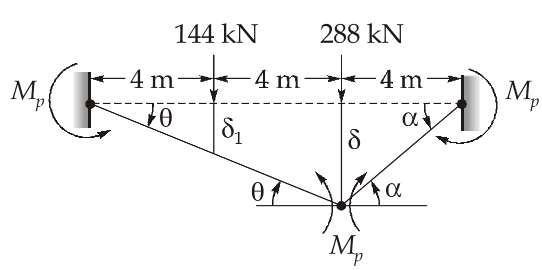
Equating external and internal work

$$1152\theta = 3M_p\theta$$

$\Rightarrow \quad M_p = 384 \text{ kNm}$

Mechanism 2: Failure under Load W_{u2}(8 m from left)

Plastic hinges form at the fixed ends and under the load W_{u2}.



$$\delta = 4\alpha = 8\theta$$

$\Rightarrow \quad \alpha = 2\theta$

External work done,

$$W_e = 144 \times (4\theta) + 288 \times (4\alpha)$$

$\Rightarrow \quad W_e = 144 \times 4\theta + 288 \times 4\,(2\theta)$

$\Rightarrow \quad W_e = 2880\theta$

Internal work done, $\quad W_i = M_p\theta + M_p(\theta + \alpha) + M_p\alpha$

$\Rightarrow \quad W_i = M_p\theta + M_p(\theta + 2\theta) + M_p(2\theta)$

$\Rightarrow \quad W_i = 6M_p\theta$

Equating external and internal work

$$2880\theta = 6M_p\theta$$

$$M_p = 480 \text{ kNm}$$

Final Plastic Moment $\quad M_p = \text{max}^m \{384, 480\}$

$$= 480 \text{ kN-m}$$

Section B

5. (a) (i) Solution:

1. Flexural Strength

When both the following conditions are met, the concrete complies with the specified flexural strength.

(a) The mean strength determined from any group of four consecutive test results exceeds the specified characteristic strength by at least 0.3 N/mm^2.

(b) The strength determined from any test result is not less than the specified characteristic strength less: 0.3 N/mm^2.

2. Characteristics compressive strength compliance requirement (Clauses 16.1 and 16.3)

Specified grade	Mean of the group of 4 Non-overlapping consecutive Test results in N/mm^2	Individual Test results in N/mm^2
(1)	(2)	(3)
M 15 and above	$\geq f_{ck}$ +0.825 × established standard deviation (rounded off to nearest 0.5N/mm^2) or f_{ck} + 3N/mm^2, whichever is greater	$\geq f_{ck}$ – 3 Nmm^2

5. (a) (ii) Solution:

Given:

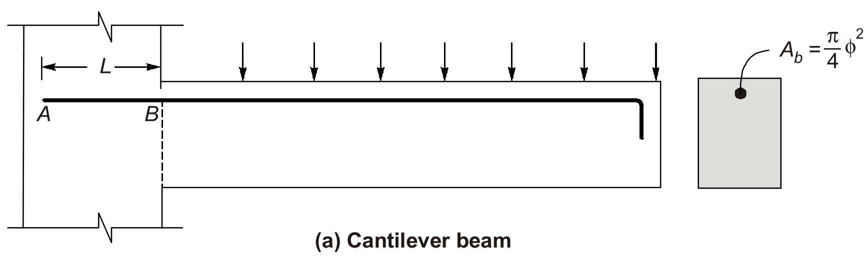
Diameter of reinforcing bar = ϕ

Stress in steel at the section considered = σ_s

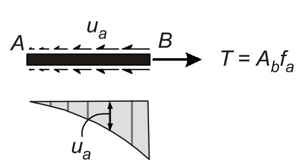
Permissible design bond stress = τ_{bd}

Embedded length of bar = L_d

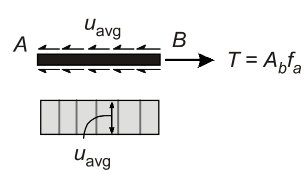
Consider a reinforcing bar of diameter ϕ embedded in concrete over a length L_d. When the bar is subjected to tensile stress, a bond stress develops between the steel and the surrounding concrete which resists the pull of the bar.



(a) Cantilever beam



(b) Probable variation of anchorage bond stress u_a



(c) Assumed uniform average bond stress u_{avg}

The tensile force developed in the reinforcing bar at the critical section is

$$T = \text{Area of bar} \times \text{stress in steel}$$

$$\Rightarrow \quad T = \frac{\pi}{4}\phi^2\sigma_s$$

The resisting force due to bond stress acts over the surface area of the embedded portion of the bar.

Surface area of bar in contact with concrete = Perimeter of bar × embedded length

$$= \pi\phi L_d$$

If the design bond stress between steel and concrete is τ_{bd}, the resisting bond force developed along the embedded length is

$$F_b = \tau_{bd} \times \pi\phi L_d$$

For equilibrium of the bar, the tensile force in the bar must be balanced by the bond resistance developed along the embedded length.

$$T = F_b$$

Substituting the expressions

$$\frac{\pi}{4}\phi^2\sigma_s = \tau_{bd}\pi\phi L_d$$

Dividing both sides by $\pi\phi$

$$\frac{1}{4}\phi\sigma_s = \tau_{bd}L_d$$

Rearranging the equation to obtain development length

$$L_d = \frac{\phi\sigma_s}{4\tau_{bd}}$$

This is the expression for development length as specified in IS 456:2000.

5. (b) Solution:

First, the cumulative mass retained on each sieve is calculated by adding the mass retained on that sieve to the masses retained on all previous sieves.

The cumulative percentage retained is obtained using the relation

$$\text{Cumulative percentage retained} = \frac{\text{Cumulative mass retained}}{\text{Total mass}} \times 100$$

The sieve analysis results are presented in the following table.

Sieve Size	Mass Retained (g)	Cumulative mass Retained (g)	Cumulative Percentage Retained (%)
4.75 mm	45	45	$\frac{45}{1000} \times 100 = 4.5$
2.36 mm	115	45 + 115 = 160	$\frac{160}{1000} \times 100 = 16$
1.18 mm	175	160 + 175 = 335	$\frac{335}{1000} \times 100 = 33.5$
600 μm	235	335 + 235 = 570	$\frac{570}{1000} \times 100 = 57$
300 μm	205	570 + 205 = 775	$\frac{775}{1000} \times 100 = 77.5$
150 μm	225	775 + 225 = 910	$\frac{1000}{1000} \times 100 = 100$

Sum of cumulative percentage retained

$$\Sigma = 288.5\%$$

The fineness modulus is calculated using the relation

$$FM = \frac{\text{Cumulative percentage retained}}{100}$$

$$\Rightarrow \qquad FM = \frac{288.5}{100}$$

$$\Rightarrow \qquad FM = 2.88$$

The obtained fineness modulus value indicates that the aggregate falls within the range of medium sand.

Therefore, the fineness modulus of the aggregate is $FM = 2.88$, and the sand is classified as medium sand.

5. (c) Solution:

Given:

The distributed torque function $T(x)$ using a linear relation:

$$T(x) = mx + c$$

At $x = 0$ m, $T(0) = 200$ so, $c = 200$ N-m

At $x = 2$ m, $T(2) = 440$

$$\therefore \qquad 440 = m(2) + 200 \quad \Rightarrow \quad m = 120 \text{ N}$$

Hence, the torque function is

$$T(x) = 120x + 200 (\text{N-m})$$

The internal torque at any section vis obtained by integrating the distributed torque from that section to the free end:

$$T(x) = \int_x^2 (120x + 200)\,dx$$

$$\Rightarrow \qquad T(x) = \left[60x^2 + 200x\right]_x^2$$

$$\Rightarrow \qquad T(x) = 60(2)^2 + 200(2) - (60x^2 + 200x)$$

$$\Rightarrow \qquad T(x) = 640 - 200x - 60x^2 (\text{N-m})$$

The differential angle of twist is

$$d\phi = \frac{T(x)}{JG}dx$$

The total angle of twist at the free end is

$$\phi_B = \int_0^2 \frac{T(x)}{JG}dx$$

$$\Rightarrow \qquad \phi_B = \frac{1}{JG}\int_0^2 (640 - 200x - 60x^2)\,dx$$

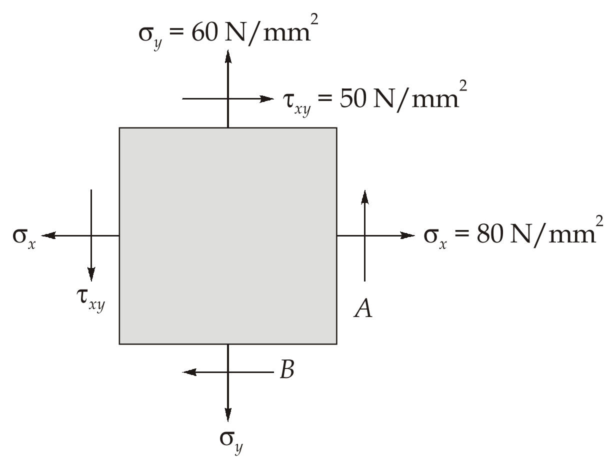
$$\Rightarrow \quad \phi_B = \frac{1}{JG}\left[640(2)-100(2)^2-20(2)^3\right]$$

$$\Rightarrow \quad \phi_B = \frac{720}{JG} \text{ radians}$$

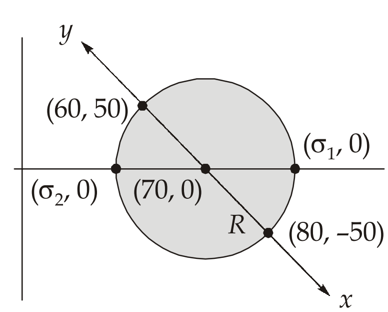
Hence, the angle of twist at the free end is

$$\phi_B = \frac{720}{JG} \text{ radian}$$

5. (d) Solution:



Representing in Mohr circle:



$$R = \sqrt{(80-70)^2+(-50-0)^2} = 50.99 \simeq 51$$

$$\Rightarrow \quad \sigma_1 = 70 + R = 70 + 51 = 121 \text{ N/mm}^2 = \text{(Maximum principal stress)}$$

$$\Rightarrow \quad \sigma_2 = 70 + R = 70 - 51 = 19 \text{ N/mm}^2 = \text{(Minimum principal stress)}$$

$$f_y = 200 \text{ N/mm}^2$$

$$\mu = 0.30$$

(i) As per maximum principal stress theory,

$$\sigma_{max} = \frac{f_y}{FOS}$$

$$\Rightarrow \quad 121 = \frac{200}{FOS}$$

$$\Rightarrow \quad FOS = 1.653$$

(ii) As per maximum principal strain theory,

$$\varepsilon_{max} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = \frac{(f_y/FOS)}{E}$$

$$\Rightarrow \quad \frac{121}{E} - \frac{0.3 \times 19}{E} = \frac{200}{E \times FOS}$$

$$\Rightarrow \quad FOS = 1.734$$

(iii) As per maximum shear stress theory,

$$\tau_{max} = \max\left\{\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1|}{2}, \frac{|\sigma_2|}{2}\right\} = \frac{f_y/FOS}{2}$$

$$\Rightarrow \quad \max\left\{\frac{121-19}{2}, \frac{121}{2}, \frac{19}{2}\right\} = \frac{200}{2 \times FOS}$$

$$\Rightarrow \quad \max\{51, 60.5, 9.5\} = \frac{100}{FOS}$$

$$\Rightarrow \quad FOS = \frac{100}{60.5} = 1.653$$

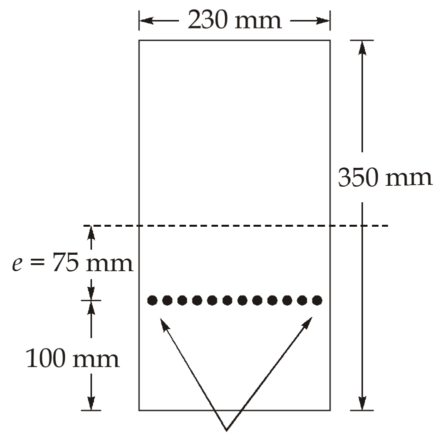
(iv) As per maximum strain energy theory,

$$U_{max} = \frac{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2}{2E} = \frac{(f_y/FOS)^2}{2E}$$

$$\Rightarrow \quad 121^2 + 19^2 - 2 \times 0.3 \times 121 \times 19 = \left(\frac{200}{FOS}\right)^2$$

$$\Rightarrow \quad FOS = 1.713$$

5. (e) Solution:



1. Elastic shortening loss = mf_c $\quad m = \frac{E_s}{E_c} = \frac{210}{35} = 6$

f_c = Stress in concrete at the level of prestressing steel = $\frac{P}{A} + \frac{(Pe)e}{I}$

$$P = 1200 \times 12 \times \frac{\pi}{4}(7)^2 = 554.18 \text{ kN}$$

Now,
$$f_c = \frac{554.18 \times 10^3}{230 \times 350} + \frac{554.18 \times 10^3 \times (75)^2}{\frac{230(350)^3}{12}}$$

$\Rightarrow \quad f_c = 10.677 \text{ N/mm}^2$

$\Rightarrow$ Elastic shortening loss = $6 \times 10.677 = 64.062 \text{ N/mm}^2$

Since elastic shortening loss is a short term loss, we will calculate the prestress after short term loss. This will then be used to calculate long term loss.

Stress after elastic shortening loss = $1200 - 64.062 = 1135.938 \text{ N/mm}^2$

f_c after elastic shortening loss = $\frac{10.632 \times 1135.938}{1200} = 10.064 \text{ N/mm}^2$

2. Loss due to creep = $mf_c \cdot \phi = 6 \times 10.064 \times 1.6 = 96.62 \text{ N/mm}^2$
3. Loss due to shrinkage = $\varepsilon_{sc} \cdot E_s = 2.5 \times 10^{-4} \times 210 \times 10^3 \text{ N/mm}^2 = 52.5 \text{ N/mm}^2$
4. Loss due to relaxation = 80 N/mm^2

Final loss of prestress = $64.062 + 96.62 + 52.5 + 80 = 293.182 \text{ N/mm}^2$

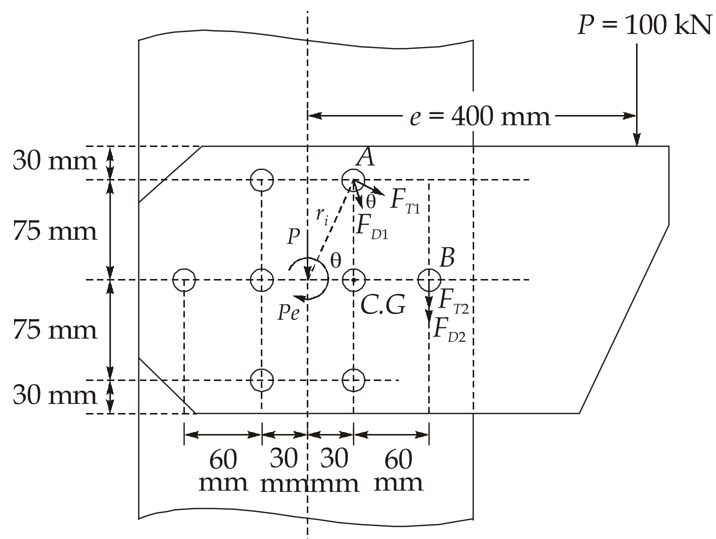
Q.6 (a) Solution:

Given: M20 bolts of grade 4.6

$$f_{ub} = 400 \text{ N/mm}^2$$
$$d = 20 \text{ mm}, \quad d_0 = 22 \text{ mm}$$

Steel of grade E250 (Fe 410)

$$f_y = 250 \text{ N/mm}^2 \; ; f_u = 410 \text{N/mm}^2$$



The two most critical bolts to be considered are A and B.

Strength of bolt $\left(V_{d_b}\right)$:

Design shear strength of bolt:

Here bolt is in single shear,

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3}\,\gamma_{mb}} \times 0.78\left(\frac{\pi}{4}\,d^2\right)$$
$$= \frac{400}{\sqrt{3} \times 1.25} \times 0.78 \times \frac{\pi}{4} \times (20)^2 \times 10^{-3} \text{ kN}$$
$$= 45.27 \text{ kN}$$

Design bearing strength of bolt:

$$V_{dpb} = 2.5\, K_b\, dt. \frac{f_u}{\gamma_{m1}}$$

$$K_b = \min.\left\{\frac{e}{3d_0}, \frac{p}{3d_0} - 0.25 \,;\, \frac{f_{ub}}{f_u} \,; 1\right\}$$

$$= \text{min.}\left\{\frac{30}{3\times 22}, \frac{75}{3\times 22} - 0.25; \frac{400}{410}; 1\right\}$$

$$= \text{min.}\{0.455, 0.88, 0.97, 1\}$$

$$= 0.455$$

$$\therefore \quad V_{dpb} = 2.5 \times 0.455 \times 20 \times 10 \times \frac{410}{1.25} \times 10^{-3} \text{ kN}$$

$$= 74.62 \text{ kN}$$

$$\therefore \quad V_{db} = \text{min.}\begin{cases} V_{dsb} \\ V_{dpb} \end{cases} = 45.27 \text{ kN}$$

Consider bolt A,

$$F_{D_1} = \text{Direct shear force}$$

$$= \frac{P}{8} = \frac{100}{8} = 12.5 \text{ kN}$$

$$F_{T_1} = \text{Torsional shear force}$$

$$= \frac{T.r_i}{\sum r_i^2} = \frac{100 \times 400 \times \sqrt{30^2 + 75^2}}{4\left(\sqrt{30^2 + 75^2}\right)^2 + 2\times 30^2 + 2\times 90^2}$$

$$F_{t_1} = 73.27 \text{ kN}$$

$\therefore$ Resultant stress on bolt A,

$$(F_R)_A = \sqrt{F_{D_1}^2 + F_{T_1}^2 + 2F_{D_1}.F_{T_1}.\cos\theta}$$

$$= \sqrt{(12.5)^2 + (73.27)^2 + 2\times 12.5 \times 73.27 \times \frac{30}{\sqrt{30^2 + 75^2}}}$$

$$= 78.77 \text{ kN}$$

Consider bolt B, F_{D_2} = Direct shear force

$$= \frac{P}{8} = \frac{100}{8} = 12.5 \text{ kN}$$

$$F_{T_2} = \text{Torsional shear force}$$

$$= \frac{Pe.r_i}{\sum r_i^2}$$

$$= \frac{100 \times 400 \times 90}{4\left(\sqrt{30^2 + 75^2}\right)^2 + 2(30)^2 + 2(90)^2} = 81.63 \text{ kN.}$$

$\therefore$ Resultant Force on bolt B.

$$(F_R)_B = F_{D_2} + F_{T_2}$$
$$= 12.5 + 81.63 = 94.13 \text{ kN.}$$

Bolt B is subjected to maximum force

For connection to be safe,

$$(F_R)_B < V_{db}$$

But here, $94.13 \text{ kN} > 45.27 \text{ kN}$

∴ The design is not safe.

6. (b) Solution:

Given:

Clear span in shorter direction, $L_x = 3.5$ m

Longer span, $L_y = 8$ m

Support thickness, $b = 230$ mm

Live load, $LL = 3 \text{ kN/m}^2$

Floor finish load, $FF = 1.25 \text{ kN/m}^2$

Characteristic compressive strength of concrete, $f_{ck} = 20 \text{ N/mm}^2$

Yield strength of steel, $f_y = 415 \text{ N/mm}^2$

Clear cover = 20 mm

The type of slab is determined by the ratio of longer span to shorter span.

$$\frac{L_y}{L_x} = \frac{8}{3.5} = 2.286 > 2$$

Since the ratio is greater than 2, the slab behaves as a one-way slab.

The effective depth is assumed based on span to depth ratio for deflection control.

$$d = \frac{L_x}{25} = \frac{3500}{25} = 140 \text{ mm}$$

Using 10 mm diameter bars, the overall depth is

$$D = d + \text{clear cover} + \frac{\phi}{2}$$

$$\Rightarrow \quad D = 140 + 20 + \frac{10}{2}$$

$$\Rightarrow \quad D = 165 \text{ mm}$$

The effective span is taken as the lesser of

$$L_{eff} = \min\begin{cases}3.5 + 0.14 = 3.64 \text{ m}\\ 3.5 + 0.23 = 3.73 \text{ m}\end{cases}$$

$$\Rightarrow \quad L_{eff} = 3.64 \text{ m}$$

Load calculation per meter width of slab.

$$\text{Self weight of slab} = 0.165 \times 25 \times 1 = 4.125 \text{ kN/m}$$

$$\text{Live load} = 3 \times 1 = 3 \text{ kN/m}$$

$$\text{Floor finish load} = 1.25 \times 1 = 1.25 \text{ kN/m}$$

Total load, $w = 4.125 + 3 + 1.25$

$$w = 8.375 \text{ kN/m}$$

Factored load, $w_u = 1.5 \times 8.375$

$$\Rightarrow \quad w_u = 12.563 \text{ kN/m}$$

Maximum bending moment for simply supported slab

$$M_u = \frac{w_u L_{eff}^2}{8}$$

$$\Rightarrow \quad M_u = \frac{12.563 \times 3.64^2}{8}$$

$$\Rightarrow \quad M_u = 20.803 \text{ kNm}$$

Maximum shear force $V_u = \dfrac{w_u L_{eff}}{2}$

$$\Rightarrow \quad V_u = \frac{12.563 \times 3.64}{2}$$

$$\Rightarrow \quad V_u = 22.865 \text{ kN}$$

Check for depth using limiting moment of resistance.

$$M_{u,\text{lim}} = 0.138 f_{ck} b d^2$$

$$\Rightarrow \quad 20.803 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$\Rightarrow \quad d = 86.841 \text{ mm}$$

Since the required depth is less than the assumed depth of 140 mm, the section is safe.

For main reinforcement, the design equation is

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}\right] bd$$

$$\Rightarrow \quad A_{st} = \frac{0.5 \times 20}{415}\left[1-\sqrt{1-\frac{4.6 \times 20.803 \times 10^6}{20 \times 1000 \times 140^2}}\right] \times 1000 \times 140$$

$$A_{st} = 440.53 \text{ mm}^2$$

Area of one 10 mm bar = 78.54 mm^2

Spacing of bars, $s = \dfrac{1000 \times 78.54}{440.53}$

$$s = 178.28 \text{ mm}$$

Provide 10 mm diameter bars at 170 mm centre to centre.

$$(A_{st})_{prov} = \frac{1000 \times 78.54}{170} = 462 \text{ mm}^2$$

Distribution reinforcement

$$A_{st,\ dist} = 0.12\% \times b \times D$$

$$\Rightarrow \quad A_{st,\ dist} = \frac{0.12}{100} \times 1000 \times 165$$

$$\Rightarrow \quad A_{st,\ dist} = 198 \text{ mm}^2$$

Area of 8 mm bar = 50.265 mm^2

Spacing of distribution bars, $s = \dfrac{1000 \times 50.265}{198}$

$$\Rightarrow \quad s = 253.864 \text{ mm}$$

Provide 8 mm diameter bars at 250 mm centre to centre.

Check for shear.

$$\tau_v = \frac{V_u}{bd}$$

$$\Rightarrow \quad \tau_v = \frac{22.865 \times 10^3}{1000 \times 140}$$

$$\Rightarrow \quad \tau_v = 0.163 \text{ N/mm}^2$$

Percentage of tension steel, $P_t = \dfrac{100 \times 462}{1000 \times 140}$

$$P_t = 0.33\%$$

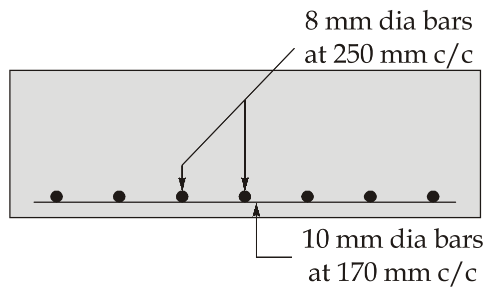
Interpolating the design shear strength

$$\tau_c = 0.36 + \frac{0.49 - 0.36}{0.5 - 0.25}(0.33 - 0.25)$$

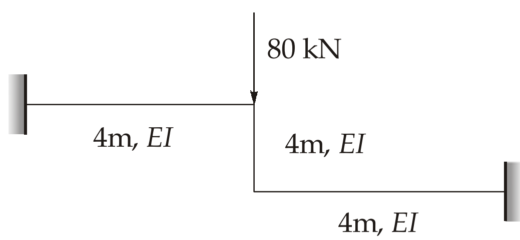
$$\tau_c = 0.4016 \text{ N/mm}^2$$

Since $\tau_v < \tau_c$

The slab is safe in shear.



6. (c) Solution:



Using moment distribution method performing sway analysis:

$$(M_F)_{AB} = (M_F)_{BA} = \frac{-6\,EI\,\Delta}{l^2} = \frac{-6\,EI\,\Delta}{4^2}$$

Assuming $$EI\,\Delta = \frac{800}{3}$$

$$\therefore \quad (M_F)_{AB} = (M_F)_{BA} = -100 \text{ kNm}$$

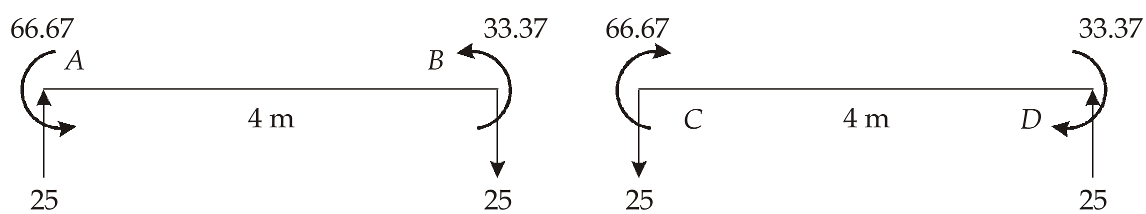
$$(M_F)_{CD} = (M_F)_{DC} = \frac{+6\,EI\Delta}{4^2} = +100 \text{ kNm}$$

Distribution Factors:

Joint	Member	Stiffness	Sum	DF
B	BA	4EI/4 = EI	2EI	0.5
	BC	EI		0.5
C	CB	EI	2EI	0.5
	CD	EI		0.5

Joint	A		B	C		D
Member	AB	BA	BC	CB	CD	DC
DF		0.5	0.5	0.5	0.5	
FEM	–100	–100	0	0	+100	+100
		50	50	–50	–50	
	25		–25	+25		–25
		+12.5	12.5	–12.5	–12.5	
	6.25		–6.25	6.25		–6.25
		3.12	3.12	–3.12	–3.12	
	1.56		–1.56	1.56		–1.56
		0.75	0.75	–0.75	–0.75	
	0.38		–0.38	0.38		–0.38
		+0.19	+0.19	–0.19	–0.19	
	0.1		–0.1	+ 0.1		– 0.1
		0.05	0.05	– 0.05	– 0.05	
	–66.67	–33.37	33.37	–33.37	33.37	–66.67

Free Body diagram:



Reaction at A due to moment,

$$= \frac{66.67 + 33.33}{4} = 25 \text{ kN } (\uparrow)$$

Reaction at D due to moment,

$$= 25 \text{ kN } (\uparrow)$$

$\therefore$ Sway force causing above moment,

$$= 25 + 25 = 50 \text{ kN } (\downarrow)$$

$$\text{Actual sway force} = 80 \text{ kN } (\downarrow)$$

Hence, Actual moment

$$M_{AB} = -66.67 \times \frac{80}{50} = -106.672 \text{ kNm}$$

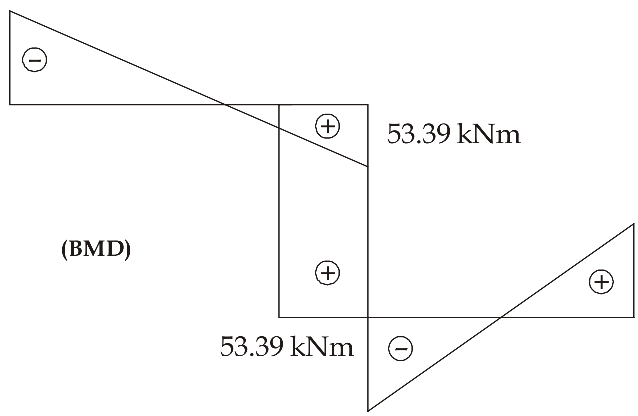
$$M_{BA} = -33.37 \times \frac{80}{50} = -53.392 \text{ kNm}$$

$$M_{BC} = 33.37 \times \frac{80}{50} = 53.392 \text{ kNm}$$

$$M_{CB} = -33.37 \times \frac{80}{50} = -53.392 \text{ kNm}$$

$$M_{CD} = 33.37 \times \frac{80}{50} = 53.392 \text{ kNm}$$

$$M_{DC} = 66.67 \times \frac{80}{50} = 106.672 \text{ kNm}$$



7. (a) Solution:

First the fixed end moments are determined for each span.

For span *AB*:

$$M_{FAB} = \frac{-wL_{AB}^2}{12} = \frac{-20 \times 4^2}{12} = -26.667 \text{ kN-m}$$

$$M_{FBA} = \frac{+wL_{AB}^2}{12} = +26.667 \text{ kN-m}$$

For span *BC*:

$$M_{FBC} = \frac{-Pab^2}{L_{BC}^2} = \frac{-40 \times 4 \times 2^2}{6^2} = -17.778 \text{ kN-m}$$

$$M_{FCB} = \frac{+Pa^2b}{L_{BC}^2} = \frac{40 \times 4^2 \times 2}{6^2} = 35.556 \text{ kN-m}$$

For the over hange span

$$M_{CD} = -P_{ext} \times L_{CD}$$

$$M_{CD} = -25 \times 2$$

$$M_{CD} = -50 \text{ kNm}$$

The slope deflection equations are written for each member.

Since support A is fixed, $\theta_A = 0$

For span *AB*:

$$M_{AB} = M_{FAB} + \frac{2EI}{L_{AB}}(2\theta_A + \theta_B)$$

$$\Rightarrow \quad M_{AB} = -26.667 + \frac{2EI}{4}(0 + \theta_B)$$

$$\Rightarrow \quad M_{AB} = -26.667 + 0.5\,EI\theta_B$$

Now, $$M_{BA} = M_{FBA} + \frac{2EI}{L_{AB}}(2\theta_B + \theta_A)$$

$$\Rightarrow \quad M_{BA} = 26.667 + \frac{2EI}{4}(2\theta_B)$$

$$\Rightarrow \quad M_{BA} = 26.667 + EI\theta_B$$

For span *BC*:

$$M_{BC} = M_{FBC} + \frac{2EI}{L_{BC}}(2\theta_B + \theta_C)$$

$$\Rightarrow \quad M_{BC} = -17.778 + \frac{2EI}{6}(2\theta_B + \theta_C)$$

$$\Rightarrow \quad M_{BC} = -17.778 + 0.667\,EI\theta_B + 0.333\,EI\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L_{BC}}(2\theta_C + \theta_B)$$

$$\Rightarrow \quad M_{CB} = 35.556 + \frac{2EI}{6}(2\theta_C + \theta_B)$$

$$\Rightarrow \quad M_{CB} = 35.556 + 0.667\,EI\theta_C + 0.333\,EI\theta_B$$

Joint equilibrium is applied.

At joint *B*:

$$M_{BA} + M_{BC} = 0$$

$$26.667 + EI\theta_B - 17.778 + 0.667\,EI\theta_B + 0.333\,EI\theta_C = 0$$

$$1.667\,EI\theta_B + 0.333\,EI\theta_C = -8.889 \qquad \text{...(i)}$$

At joint *C*:

$$M_{CB} + M_{CD} = 0$$

$$M_{CB} = 50$$

$$35.556 + 0.667\ EI\theta_C + 0.333\ EI\theta_B = 50$$

$$0.333\ EI\theta_B + 0.667\ EI\theta_C = 14.444 \quad \text{...(ii)}$$

Solving the simultaneous equations (i) and (ii)

$$EI\theta_B = -10.728$$

$$EI\theta_C = 27.011$$

Substituting these values into the moment equations

$$M_{AB} = -26.667 + 0.5(-10.728) = -32.031 \text{ kNm}$$

$$M_{BA} = 26.667 - 10.728 = 15.939 \text{ kNm}$$

$$M_{BC} = -17.778 + 0.667(-10.728) + 0.333(27.011)$$

$$M_{BC} = -15.939 \text{ kNm}$$

$$M_{CB} = 35.556 + 0.667(27.011) + 0.333(-10.728) = 50 \text{ kNm}$$

Support reactions are determined from equilibrium.

$$R_A = \frac{20 \times 4 \times 2 + 32.031 - 15.939}{4}$$

$$R_A = 44.023 \text{ kN} (\uparrow)$$

$$R_B = (20 \times 4 - 44.023) + \frac{40 \times 2 + 15.926 - 50}{6}$$

$$R_B = 35.972 + 7.656$$

$$R_B = 43.633 \text{ kN} (\uparrow)$$

$$R_C = (40 - 7.656) + 25$$

$$R_C = 57.344 \text{ kN} (\uparrow)$$

Thus the final end moments are

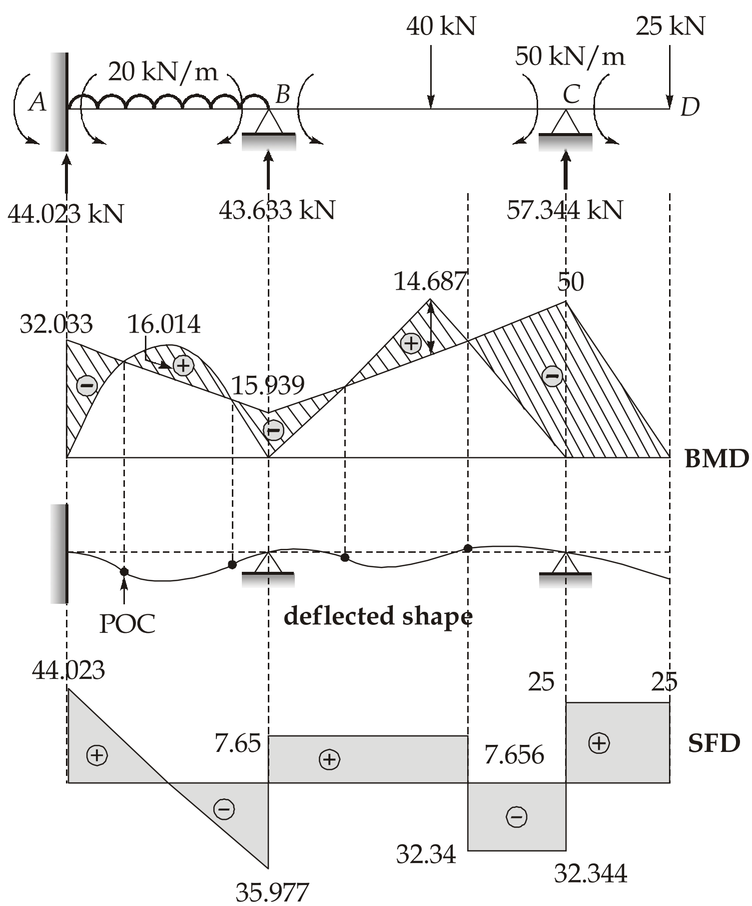
$$M_{AB} = -32.031 \text{ kNm},\ M_{BA} = 15.939 \text{ kNm}$$

$$M_{BC} = -15.939 \text{ kNm},\ M_{CB} = 50 \text{ kNm}$$

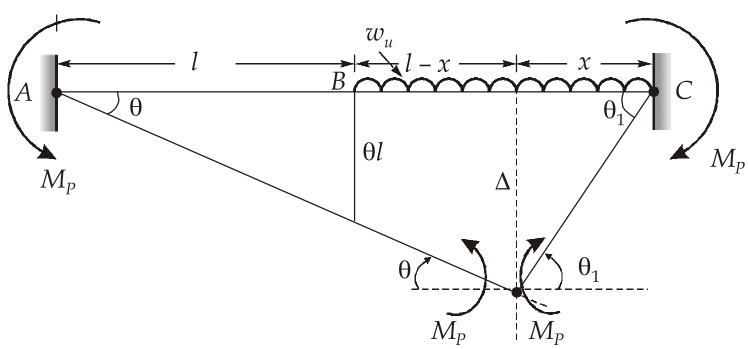
$$M_{CD} = -50 \text{ kNm}$$

The support reactions are

$$R_A = 44.023 \text{ kN},\ R_B = 43.633 \text{ kN},\ R_C = 57.344 \text{ kN}$$



7. (b) (i) Solution:



From the mechanism

$$\Delta = \theta(2l - x) = x\theta_1$$

$$\Rightarrow \quad \theta_1 = \frac{(2l-x)}{x}\theta$$

External work done = Intensity of load × Area of collapse mechanism diagram under the load.

$$\Rightarrow \quad W_e = w_u\left[\frac{1}{2}(\theta l+\Delta)(l-x)+\frac{1}{2}\Delta x\right]$$

$$\Rightarrow \quad W_e = w_u\left[\frac{1}{2}(\theta l+x\theta_1)(l-x)+\frac{1}{2}(x\theta_1)x\right]$$

$$\Rightarrow \quad W_e = w_u\left[\frac{1}{2}\left(\theta l+x\frac{(2l-x)}{x}\theta\right)(l-x)+\frac{x^2}{2}\left(\frac{2l-x}{x}\right)\theta\right]$$

$$\Rightarrow \quad W_e = \frac{w_u}{2}\left[(3\theta l-x\theta)(l-x)+x(2l-x)\theta\right]$$

$$\Rightarrow \quad W_e = \frac{w_u}{2}[3l^2-2lx]\theta$$

Now, internal workdone

$$W_i = M_p\theta + M_p(\theta+\theta_1) + M_p\theta_1$$

$$\Rightarrow \quad W_i = 2M_p(\theta+\theta_1)$$

$$\Rightarrow \quad W_i = 2M_P\left[\theta+\frac{(2l-x)}{x}\theta\right]$$

$$\Rightarrow \quad W_i = \frac{4M_P l}{x}\theta$$

By principle of virtual work

$$W_e = W_i$$

$$\Rightarrow \quad \frac{w_u}{2}(3l^2-2lx) = \frac{4M_P l}{x}$$

$$\Rightarrow \quad M_P = \frac{w_u(3l-2x)x}{8}$$

For the maximum value of M_P.

$$\frac{dM_P}{dx} = 0$$

$$\Rightarrow \quad \frac{w_u}{8}(3l-4x) = 0$$

$$\Rightarrow \quad x = \frac{3l}{4} = 0.75l$$

Hence,

$$M_P = \frac{w_u}{8}[3l-2\times 0.75l]\times 0.75l$$

$$\Rightarrow \quad M_P = 0.140625\, w_u l^2$$

$$\Rightarrow \qquad w_u = \frac{7.11M_P}{l^2}$$

$$\therefore \qquad \text{Minimum collapse load} = w_u \times l$$

$$= \frac{7.11M_P}{l}$$

7. (b) (ii) Solution:

1. **Pozzolanic Action:** Pozzolanic action is a chemical reaction that occurs in concrete when a siliceous or aluminosiliceous material (a pozzolan, like fly ash or silica fume) reacts with calcium hydroxide $Ca(OH)_2$ in the presence of water.
   - **The Mechanism:** When Portland cement hydrates, it produces Calcium Silicate Hydrate (C-S-H) – which provides strength – and Calcium Hydroxide, which is relatively weak and soluble. The pozzolan reacts with this "extra" calcium hydroxide to create additional C-S-H gel.
   - **The Benefits:**

     Increased Strength: It improves the long-term compressive strength of the concrete.

     Reduced Permeability: The additional gel fills capillary pores, making the concrete denser and more resistant to chemical attacks (like sulfates).

     Improved Durability: It reduces the risk of alkali-silica reaction (ASR) and lowers the heat of hydration.

2. **Importance of Water-Cement Ratio (*w/c*):** The water-cement ratio is the ratio of the weight of water to the weight of cement used in a concrete mix. It is arguably the most critical factor in determining the quality of the final product.
   - **Key Impacts:**

     Strength: According to Abrams' Law, the strength of concrete is inversely proportional to the w/c ratio. A lower ratio (less water) typically results in higher compressive strength.

     Durability and Permeability: Excessive water leaves behind "voids" or capillary pores once it evaporates. This makes the concrete porous, allowing water and harmful chemicals to penetrate, which can lead to corrosion of steel reinforcement.

     Workability: While a lower w/c ratio makes concrete stronger, it also makes it "stiffer" and harder to pour. This is why chemical admixtures (plasticizers) are often used to maintain workability without adding more water.

     Shrinkage and Cracking: High water content increases the likelihood of drying shrinkage, which leads to surface cracks as the concrete cures.

7. (c) (i) Solution:

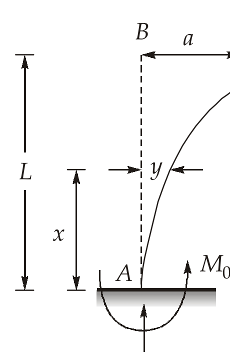
Consider a column *AB* of length *L* fixed at end *A* and free at end B as shown in figure.

Let the column buckle due to crippling load *P*.

$$a = \text{Deflection at top end } B.$$

$$M_o = \text{Moment at fixed ends.}$$

Consider a section at a distance *x* from fixed end *A* and *y* is the deflection at these section.



Bending moment at this section is,

$$EI\frac{d^2y}{dx^2} = +P(a-y)$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} + Py = Pa$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P}{EI}a$$

The solution of the above differential equation is,

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\frac{P}{EI}}\right) + a \quad \text{...(i)}$$

Where C_1 and C_2 are integration constants.

Applying boundary conditions,

(i) At. *A*, $x = 0, y = 0$

$\therefore \quad 0 = C_1 + 0 + a \quad \therefore \quad C_1 = -a$

(ii) At *A*, $x = 0, \dfrac{dy}{dx} = 0$

Slope at any section is obtained by differentiating Equation (i),

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sqrt{\frac{P}{EI}}\cos\left(x\sqrt{\frac{P}{EI}}\right)$$

$$\therefore \quad 0 = 0 + C_2\sqrt{\frac{P}{EI}}$$

$$\Rightarrow \quad C_2 = 0$$

At free end B, deflection $y = a$

(i) At B, $x = L$, $y = a$

Putting in Equation (i), $a = -a\cos\left(L\sqrt{\frac{P}{EI}}\right) + 0 + a$

$$\therefore \quad \cos\left(L\sqrt{\frac{P}{EI}}\right) = 0, \quad \Rightarrow \quad L\sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

Considering the least practical value,

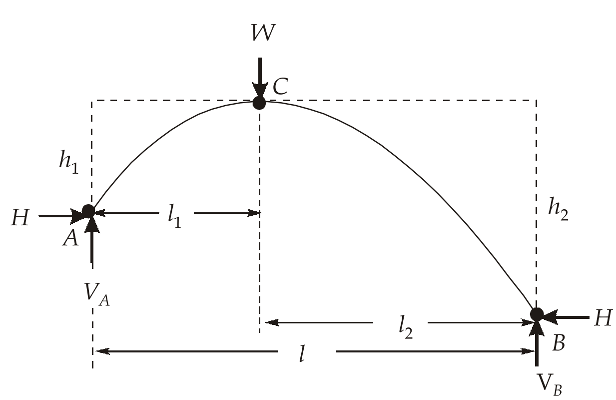
$$L\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$\Rightarrow \quad \sqrt{\frac{P}{EI}} = \frac{\pi}{2L}$$

$$\Rightarrow \quad P = \frac{\pi^2 EI}{4L^2}$$

7. (c) (ii) Solution:

Let l_1 and l_2 be the horizontal distances of the lower hinges A and B from the crown C respectively



We have,

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}} = \frac{l}{\sqrt{h_1} + \sqrt{h_2}}$$

$$l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \text{ and } l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}$$

Taking moments about C of the forces on the left-hand side of C, we have:

$$V_A l_1 = Hh_1 \quad \text{...(i)}$$

$$\Rightarrow \quad V_A = H\frac{h_1}{l_1}$$

Similarly, taking moments about C of the forces on the right-side of C, we have:

$$V_B l_2 = Hh_2 \quad \text{...(ii)}$$

$$\Rightarrow \quad V_B = H\frac{h_2}{l_2}$$

Adding equations (i) and (ii), we have:

$$V_A + V_B = H\left(\frac{h_1}{l_1} + \frac{h_2}{l_2}\right)$$

But $\quad V_A + V_B = W$

$$\therefore \quad W = H\left(\frac{h_1}{l_1} + \frac{h_2}{l_2}\right)$$

$$\Rightarrow \quad H = \frac{W}{\frac{h_1}{l_1} + \frac{h_2}{l_2}}$$

But $\quad l_1 = \dfrac{l\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}$

and $\quad l_2 = \dfrac{l\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}$

$$\therefore \quad H = \frac{W}{\frac{h_1\left(\sqrt{h_1} + \sqrt{h_2}\right)}{l\sqrt{h_1}} + \frac{h_2\left(\sqrt{h_1} + \sqrt{h_2}\right)}{l\sqrt{h_2}}}$$

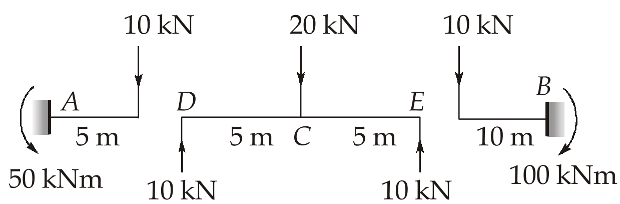
$$\Rightarrow \quad = \frac{Wl}{\frac{h_1\left(\sqrt{h_1} + \sqrt{h_2}\right)}{\sqrt{h_1}} + \frac{h_2\left(\sqrt{h_1} + \sqrt{h_2}\right)}{\sqrt{h_2}}}$$

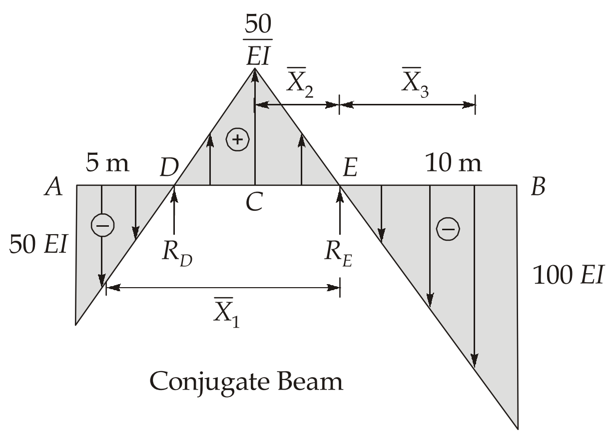
$$\Rightarrow \quad H = \frac{Wl}{\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

8. (a) Solution:

(Sign convention): ⊕, ⊖

⊕ M/EI Diagram means ↑ loading in conjugate beam and ⊖ *Ve* $\frac{M}{EI}$ mean ↓ loading in conjugate beam, ⊖ *BM* in conjugate beam means ↓ deflection ⊕ *BM* mean ↑ deflection





Conjugate Beam

In conjugate beam fixed end *A* and *B* is converted to the free and *D* and *E* of internal hinge is converted to pin support.

Calculation of reaction at *D*, taking moment about point *E* = 0.

$$\text{So} \quad R_D \times 10 - \frac{1}{2}\times\frac{50}{EI}\times 5\times\left(10+\frac{2}{3}\times 5\right)+\left(\frac{1}{2}\times\frac{50}{EI}\times 10\times 5\right)+\left(\frac{1}{2}\times\frac{100}{EI}\times 10\times\frac{2}{3}\times 10\right) = 0$$

$$\Rightarrow R_D\times 10-\frac{5000}{3EI}+\frac{1250}{EI}+\frac{10000}{3EI} = 0$$

$$\Rightarrow \qquad R_D = -\frac{291.67}{EI}$$

So taking moment about point *C*

(Bending moment at any point in conjugate beam is equal to deflection at that point in real beam)

$$\therefore \qquad \Delta_C = -\frac{291.66}{EI}\times 5+\left(\frac{1}{2}\times\frac{50}{EI}\times 5\right)\times\frac{5}{3}-\frac{1}{2}\times\frac{50}{EI}\times 5\times\left(5+\frac{2}{3}\times 5\right)$$

$$\Delta_C = \frac{2291.63}{EI}(\downarrow)$$

$$\Rightarrow \quad \Delta_C = \frac{2291.63 \times 10^3}{250000} = 9.166 \text{ mm}(\downarrow)$$

As per the sign convention of conjugate beam ⊖ BM in conjugate beam mean ↓ deflection.

8. (b) (i) Solution:

The shear flow (q) is uniformly distributed through the thickness and the shear stress is calculated using.

$$\tau = \frac{T}{2A_m t}$$

Where, T = torque

A_m = Enclosed area bounded by the median line of the section thickness

t = Wall thickness at the point of measurement

$$A_m = (40 - 5) \times (50 - 4) = 1610 \text{ mm}^2$$

Stress at A, $$\tau_A = \frac{T_A}{2_t A_m} = \frac{40 \times 10^3}{2 \times 5 \times 1610} = 2.484 \text{ MPa}$$ **Ans.**

Stress at B, $$\tau_B = \frac{T_B}{2tA_m} = \frac{25 \times 10^3}{2 \times 4 \times 1610} = 1.941 \text{ MPa}$$ **Ans.**

8. (b) (ii) Solution:

Total free elongation of bars $= L_b\alpha_b\Delta T + L_{Al}\alpha_{Al}\Delta T$

$= 0.35 \times 21.6 \times 10^{-6} \times 85 + 0.50 \times 23.2 \times 10^{-6} \times 85$

$= 1.6286 \times 10^{-3}$ m $= 1.6286$ mm

Shortening due to induced compressive force $P = 1.6286 - 0.5 = 1.1286$ mm

1. Compressive force in bars.

$$\Rightarrow \quad \frac{PL_b}{A_b E_b} + \frac{PL_{Al}}{A_{Al} \times E_{Al}} = 1.1286 \times 10^{-3}$$

$$\Rightarrow \quad P\left[\frac{0.35}{1500 \times 10^{-6} \times 105 \times 10^9} + \frac{0.50}{1800 \times 10^{-6} \times 73 \times 10^9}\right] = 1.1286 \times 10^{-3}$$

$$\Rightarrow \quad P = 187.245 \text{ kN}$$ **Ans.**

2. Charge in length of broze

$$\delta_b = L_b \alpha_b \Delta T - \frac{PL_b}{A_b E_b}$$

$$\Rightarrow \quad \delta_b = 0.35 \times 21.6 \times 10^{-6} \text{ x } 85 - \frac{187.245 \times 10^3 \times 0.35}{1500 \times 10^{-6} \times 105 \times 10^9}$$

$$\Rightarrow \quad \delta_b = 2.265 \times 10^{-4} \text{ m}$$

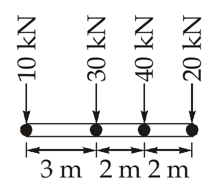
$$\Rightarrow \quad \delta_b = 0.2265 \text{ mm. (Expansion)}$$ **Ans.**

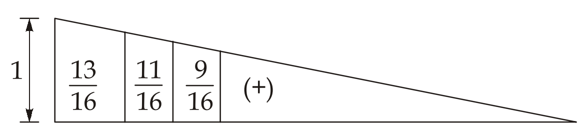
8. (c) Solution:

ILD for shear force at *A* is as shown in figure. When 10 kN load is just on *A*.

$$\text{Positive } S.F. \text{ at } A = 10 \times 1 + 30 \times \frac{13}{16} + 40 \times \frac{11}{16} + 20 \times \frac{9}{16}$$

$$= 73.125 \text{ kN}$$



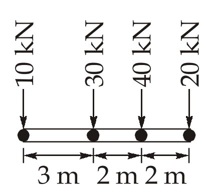


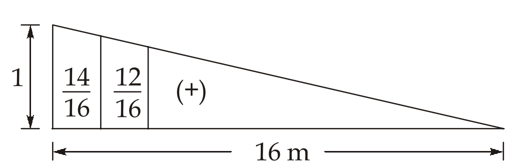
ILD for *SF* at *A*

$$\text{Maximum positive } SF \text{ at } A = 10 \times 1 + 30 \times \frac{13}{16} + 40 \times \frac{11}{16} + 20 \times \frac{9}{16}$$

$$= 73.125 \text{ kN}$$

Since, 10 kN load is lighter one more trial with 30 kN load at *A* is made. For this position.



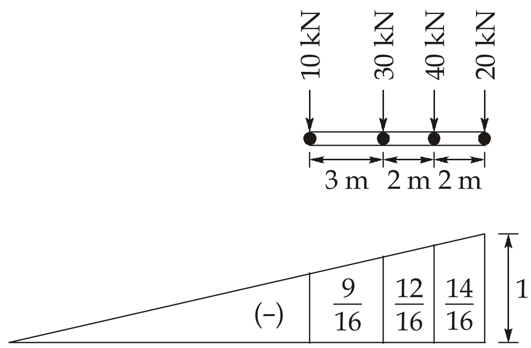


$$\text{Maximum positive S.F. at } A = 30 \times 1 + 40 \times \frac{14}{16} + 20 \times \frac{12}{16} = 80 \text{ kN}$$

Maximum positive S.F. occurs at *A* and is equal to 80 kN.

Maximum negative shear force occurs at B when leading load is on B.

∴ Maximum negative S.F.



ILD for *SF*

$$\text{Maximum negative S.F. at } B = -(20 \times 1 + 40 \times \frac{14}{16} + 30 \times \frac{12}{16} + 10 \times \frac{9}{16})$$

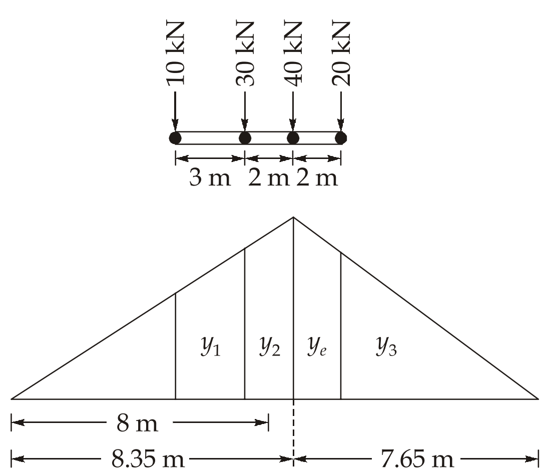
$$= -83.125 \text{ kN}$$

Since 40 kN load is heavier so check by placing 40 kN on support.

$$\text{Maximum negative S.F. at } B = -\left(1 \times 40 + \frac{14}{16} \times 30 + \frac{11}{16} \times 10\right)$$

$$= -73.125 \text{ kN}$$

So, Maximum absolute S.F. in beam = 83.125 kN



(ILD for absolute maximum moment)

For finding position for absolute maximum moment, position of C.G. of load system is to be located. Distance of C.G. for the leading load of 20 kN.

$$a = \frac{40\times2+30\times4+10\times7}{20+40+30+10} = 2.7 \text{ m}$$

It is nearest to 40 kN load and this load is heavier than another nearest load of 30 kN. Hence, maximum moment will occur under 40 kN load. Distance between this load and the resultant.

$$d = 2.7 - 2 = 0.7$$

Position of 40 kN load for maximum moment.

$$= \frac{16}{2}+\frac{0.7}{2} = 8.35 \text{ m from } A$$

For the section at 8.35 m from *A*, *ILD*

Ordinate for moment

$$y_e = \frac{z(L-z)}{L}$$

$$y_e = \frac{8.35(16-8.35)}{16} = 3.9923$$

$$\text{Absolute maximum moment} = 10\, y_1 + 30\, y_2 + 40\, y_c + 20\, y_3$$

$$= \left[10\times\frac{3.35}{8.35}+30\times\frac{6.35}{8.35}+40+20\times\frac{5.65}{7.65}\right]3.9923$$

$$= 325.762 \text{ kNm}$$

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