



MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026
Mains Test Series

Mechanical Engineering
Test No : 9

Section A : Mechatronics & Robotics [All Topics]

Section B : Machine Design-1 + Heat Transfer-2 + Power Plant-2 [Part Syllabus]

Section A : Mechatronics & Robotics

1. (a) **Solution:**
 - (a) **Manipulator or the rover:** This is the main body of the robot which consists of the links, the joints, and other structural elements of the robot. Without other elements, the manipulator alone is not a robot.
 - (b) **End effector:** The end effector is connected to the last joint (hand) of a manipulator. It generally handles objects, interfaces with other machines, or performs the required tasks. Robot manufacturers typically do not design or supply specialized end effectors; in most cases, they provide only a simple gripper. The robot hand is usually equipped with provisions for attaching specialty end effectors designed for specific applications. Examples include a welding torch, a paint spray gun, a glue-dispensing device, and a parts handler. In most cases, the action of the end effector is either controlled by the robot's controller, or the controller communicates with the end effector's control unit (such as a PLC).
 - (c) **Actuators:** Actuators are the “muscles” of the manipulators. The controller sends signals to the actuators, which, in turn, move the robot joints and links. Common types are servomotors, stepper motors, pneumatic actuators, and hydraulic actuators. Actuators are under the control of the controller.
 - (d) **Sensors:** Sensors are used to collect information about the internal state of the robot or to communicate with the outside environment. As in humans, the robot controller

needs to know the location of each link of the robot in order to know the robot's configuration. When you wake up in the morning, even without opening your eyes, or when it is completely dark, you still know where your arms and legs are. This is because feedback sensors in your central nervous system embedded in muscle tendons send information to the brain. The brain uses this information to determine the length of your muscles and, consequently, the state of your arms, legs, and so on. The same is true for robots, where sensors integrated into the robot send information about each joint or link to the controller that determines the configuration of the robot. Still similar to your major senses of sight, touch, hearing, taste, and speech, robots are equipped with external sensory devices such as a vision system, touch and tactile sensors, speech synthesizer, and the like that enable the robot to communicate with the outside world.

1. (b) Solution:

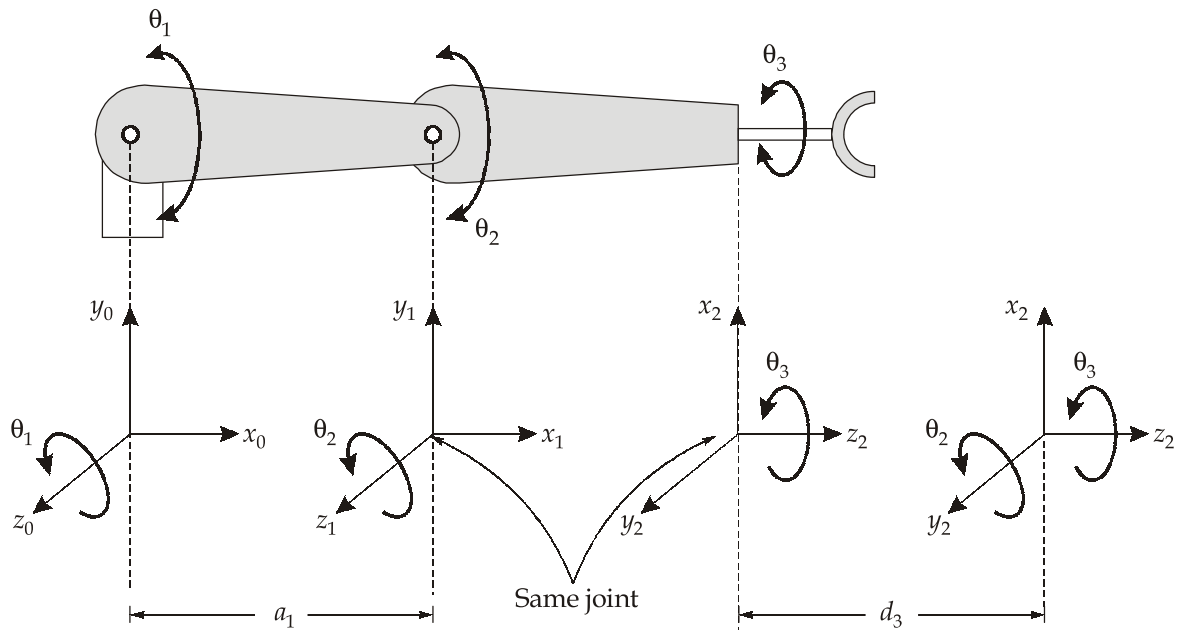
$$\begin{aligned}
 P_{xyz} &= \text{Trans}(4, -3, 7) \text{Rot}(y, 90^\circ) \text{Rot}(z, 45^\circ) P_{noa} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -5 \\ -6 \\ 3 \\ 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -5 \\ -6 \\ 3 \\ 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{11\sqrt{2}}{2} \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ -\frac{11\sqrt{2}}{2} \\ \frac{1}{\sqrt{2}} \\ 1 \end{bmatrix} \\
 P_{xyz} &= \begin{bmatrix} 7 \\ -10.778 \\ 6.293 \\ 1 \end{bmatrix}
 \end{aligned}$$

Hence, the coordinates are $P_{xyz} = (7, -10.778, 6.292)^T$

Ans.

1. (c) Solution:

Home position or reset position,



${}^i T_j$	θ	d	a	α
${}^0 T_1$	θ_1	0	a_1	0
${}^1 T_2$	$90 + \theta_2$	0	0	90
${}^2 T_3$	θ_3	d_3	0	0

$${}^0 T_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1 C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1 T_2 = \begin{bmatrix} C(90 + \theta_2) & -\sin(90 + \theta_2)\cos 90^\circ & \sin(90 + \theta_2)\sin 90^\circ & 0 \\ \sin(90 + \theta_2) & \cos(90 + \theta_2)\cos 90^\circ & -\cos(90 + \theta_2)\sin 90^\circ & 0 \\ 0 & \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1 T_2 = \begin{bmatrix} -S\theta_2 & 0 & C\theta_2 & 0 \\ C\theta_2 & 0 & S\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 \cos 0^\circ & \sin\theta_3 \sin 0^\circ & 0 \times \cos\theta \\ \sin\theta_3 & \cos\theta_3 \cos 0^\circ & -\cos\theta_3 \sin 0^\circ & 0 \times \sin\theta_3 \\ 0 & \sin 0^\circ & \cos 0^\circ & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_3 = {}^0T_1 \times {}^1T_2 \times {}^2T_3$$

$${}^1T_3 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1 C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -S\theta_2 & 0 & C\theta_2 & 0 \\ C\theta_2 & 0 & S\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_3 = \begin{bmatrix} -C\theta_1 S\theta_2 - S\theta_1 C\theta_2 & 0 & C\theta_1 C\theta_2 - S\theta_1 S\theta_2 & a_1 C\theta_1 \\ -S\theta_1 S\theta_2 + C\theta_1 C\theta_2 & 0 & S\theta_1 C\theta_2 + C\theta_1 S\theta_2 & a_1 S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

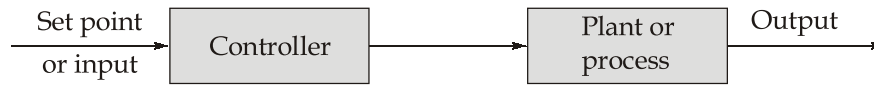
$${}^1T_3 = \begin{bmatrix} -S_{12} & 0 & C_{12} & a_1 C\theta_1 \\ C_{12} & 0 & S_{12} & a_1 S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_3 = \begin{bmatrix} -S_{12}C\theta_3 & S_{12}S\theta_3 & C_{12} & d_3 C_{12} + a_1 C\theta_1 \\ C_{12}C\theta_3 & -C_{12}S\theta_3 & S_{12} & d_3 S_{12} + a_1 S\theta_1 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. (d) Solution:

Open-loop Control Systems: An open loop control system is a system in which the control action is independent of the output. The system does not compare the actual output with the desired output. A good example of an open-loop control system is shown in figure. The input signal or command signal is given to the machine tool to perform the required machining operations. The tool moves with respect to a command signal given by the operator. There is no return signal to verify whether the tool has

moved to the correct position or not (this is why it is called non-feedback or open-loop system). Thus, the error is not found and therefore, not corrected.



Block diagram representing an open-loop system

According to the input signal, the stepping motor rotates to actuate slides. At the same time, when the gears rotate, the lead screw is turned. Thus, the slide moves. Owing to the absence of a feedback signal, the required distance moved by the slide is not correctly known. Generally, the accuracy is affected by inaccurate positioning of tools, backlash in gears and lead screw.

The controller here is the drive adjustment of a stepping motor which takes the reference input called set point. The gears and lead screw act as a plant or process.

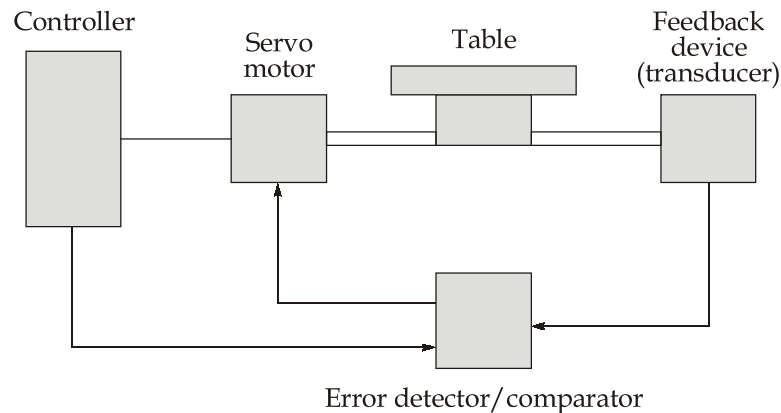
Advantages: The open-loop system has the following advantages:

1. is more suitable where accurate positioning of the tools is not required;
2. is very simple;
3. has better reliability and stability;
4. is less costly.

Disadvantages: The open-loop system has the following disadvantages:

1. It is not suitable for rough works.
2. The error cannot be corrected.
3. The control action depends upon input command.
4. The presence of nonlinearities can result in malfunctioning.

Closed-loop Control Systems: Closed-loop systems differ from open-loop systems by having a feedback device and an error detector as shown in figure. The input signal is given to the machine and the table moves. The feedback device (transducer) is connected to the table, which senses the movement and positions the table. After sensing, the transducer converts this mechanical movement into an electrical signal. This electrical signal is then fed to the error detector device for comparison with set value or reference value called an error signal (error is the difference between the measured output to the set value). Then, this error signal is sent to the controller for rectifying the error by operating the servo motor.



Closed-loop system

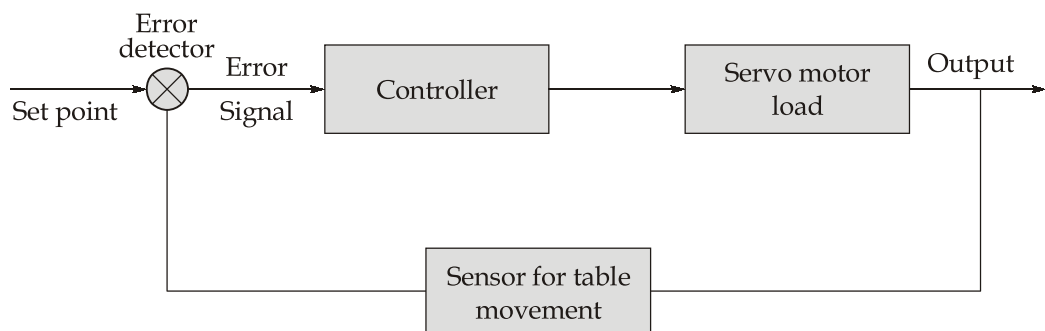
In this example, if the error signal is zero, the servo motor will stop. This indicates that the required position of the table is obtained. If the error signal is either positive or negative, the required position of the table will not be exactly obtained. So, the error signal is fed to the servo motor to correct the error signal. The process continues until the error becomes zero. The block diagram for the above system is given in figure.

Advantages: The closed-loop system has the following advantages:

1. The feedback signal controls the table position accurately.
2. The system is mostly used almost in all automation processes.
3. Change in the system is automatically taken care of.
4. Control action basically depends upon feedback.

Disadvantages: The closed-loop system has the following disadvantages:

1. The system is complicated and expensive.
2. The system may become unstable.



Block diagram representing closed-loop control system

1. (e) (i) Solution:

Given: $l = 160 \text{ mm}$, $A = 6 \text{ cm}^2$, $E = 200 \text{ GPa}$, $R = 200 \Omega$, G.F. = 2, $\Delta R = 0.014 \Omega$.

$$\text{Gauge factor, GF} = \frac{\Delta R/R}{\Delta L/L}$$

$$\Delta L = \frac{(\Delta R/R) \times L}{GF} = \frac{(0.014/200) \times 160}{2} = 5.6 \times 10^{-3} \text{ mm}$$

$$\text{Strain, } \sigma = E \epsilon = \frac{200 \times 10^9 \times 5.6 \times 10^{-6}}{160 \times 10^{-3}} \times 10^{-6} \text{ MPa} = 7 \text{ MPa}$$

$$\text{Force, } F = \sigma A = 7 \times 6 \times 100 = 4200 \text{ N} = 4.2 \text{ kN}$$

1. (e) (ii) Solution:

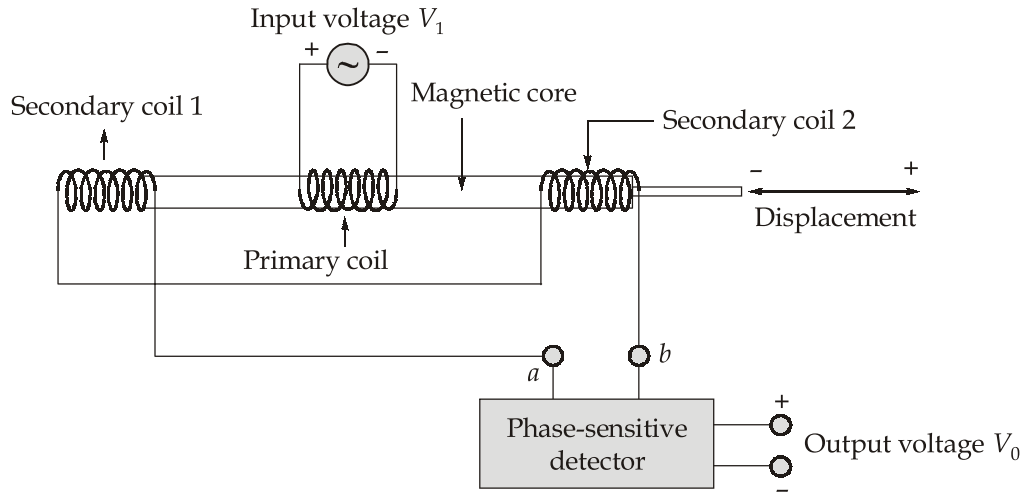
Linear Variable Differential Transformer (LVDT): The LVDT is a rugged electromagnetic transducer used to measure linear displacement. LVDT consists of an iron core which can move freely within a primary or power coil and two secondary coils as shown in figure. A movable magnetic core provides a variable coupling between windings. The secondary coils are connected in series configuration and are equally positioned with respect to the primary coil.

When the core is centrally located, the emfs generated in the secondary coils are equal and opposite and the net output voltage is zero. When the core is moved to one side, the voltage in the primary coil becomes larger and that in the other secondary coil becomes smaller. The magnitude of output voltage is proportional to the displacement of the core from the null position. The phase-sensitive detector converts the AC secondary voltage into a DC voltage, V_0 . The magnitude of the DC voltage is proportional to the amplitude of the AC voltage.

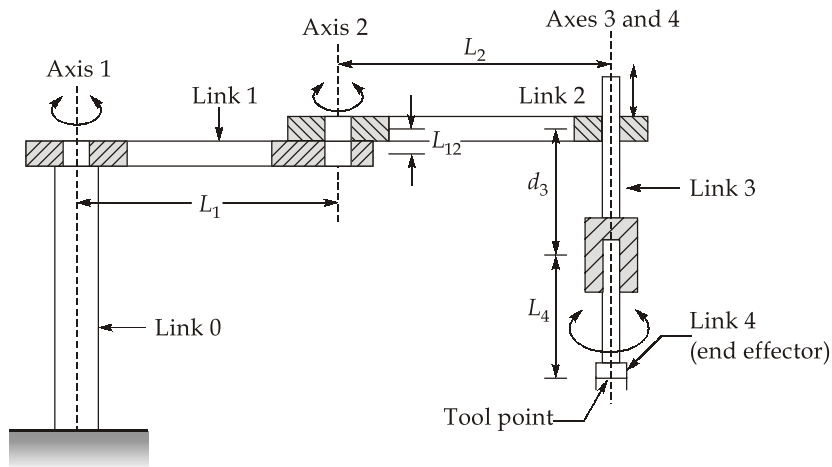
At the mid-position of the core, the induced voltage in each coil is of the same amplitude and 180° out of phase, producing a zero or null output. As the core moves from the null position, the output increases a proportional amount over a linear range around the null as shown in figure.

Advantages of LVDT

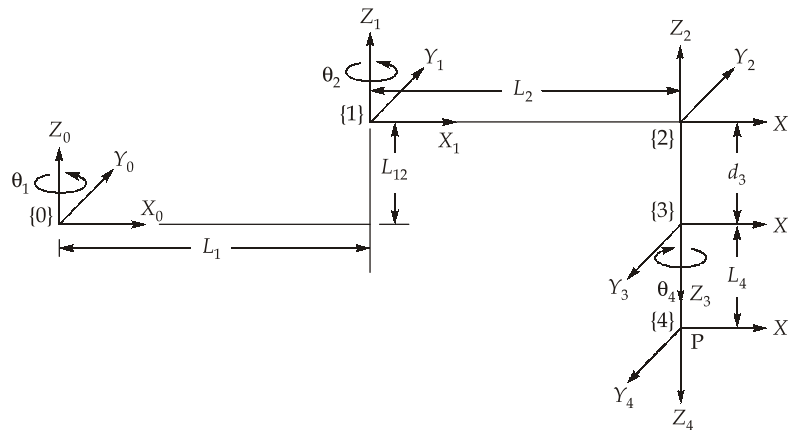
1. It has a high range, up to 1 m.
2. Friction is less and hence less is the wear problem.
3. It has low hysteresis.
4. The power consumption is less.
5. It has infinite resolution.
6. The output is highly linear and accurate.



2. (a) Solution:



Assign coordinate frame



DH Parameters table

Parameter Positions	θ	d	a	α
0 - 1	θ_1	L_{12}	L_1	0
1 - 2	θ_2	0	L_2	0
2 - 3	0	d_3	0	180
3 - 4	θ_4	L_4	0	0

$${}^0T_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & L_1C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & L_1S\theta_1 \\ 0 & 0 & 1 & L_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & L_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3T_4 = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_4 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 \times {}^3T_4$$

$${}^0T_4 = \begin{bmatrix} C\theta_1C\theta_2 - S\theta_1S\theta_2 & -C\theta_1S\theta_2 - S\theta_1C\theta_2 & 0 & L_2C\theta_1C\theta_2 - L_2S\theta_1S\theta_2 + L_1C\theta_1 \\ S\theta_1C\theta_2 + C\theta_1S\theta_2 & -S\theta_1S\theta_2 + C\theta_1C\theta_2 & 0 & L_2S\theta_1C\theta_2 + L_2C\theta_1S\theta_2 + L_2S\theta_1 \\ 0 & 0 & 1 & L_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_4 = \begin{bmatrix} C_{12} & -S_{12} & 0 & L_2C_{12} + L_1C\theta_1 \\ S_{12} & C_{12} & 0 & L_2S_{12} + L_1S\theta_1 \\ 0 & 0 & -1 & d_3 + L_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

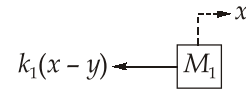
$${}^0T_4 = \begin{bmatrix} C_{12} & -S_{12} & 0 & L_2C_{12} + L_1C\theta_1 \\ S_{12} & -C_{12} & 0 & L_2S_{12} + L_1S\theta_1 \\ 0 & 0 & -1 & d_3 + L_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_4 = \begin{bmatrix} C_{12}C\theta_4 + S_{12}S\theta_4 & -C_{12}S\theta_4 + S_{12}C\theta_4 & 0 & L_2C_{12} + L_1C\theta_1 \\ S_{12}C\theta_4 - C_{12}S\theta_4 & -S_{12}S\theta_4 - C_{12}C\theta_4 & 0 & L_2S_{12} + L_1S\theta_1 \\ 0 & 0 & -1 & -L_4 + d_3 + L_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. (b) Solution:

Free body diagram for M_1 is shown in figure below. The differential equation for M_1 is

$$M_1 \frac{d^2x}{dt^2} = -k_1(x - y)$$

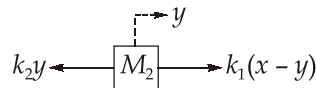


The Laplace transform of the above equation can be written as

$$(M_1 s^2 + k_1) X(s) = k_1 Y(s)$$

and
$$X(s) = \frac{k_1 Y(s)}{M_1 s^2 + k_1} \quad \dots (i)$$

Free body diagram for M_2 is shown below,



The differential equation for M_2 is

$$M_2 \frac{d^2y}{dt^2} = -k_2y + k_1(x - y)$$

or,
$$M_2 \frac{d^2y}{dt^2} + k_2y = k_1(x - y)$$

$$\therefore M_2 \frac{d^2y}{dt^2} + (k_2 + k_1)y = k_1x$$

Taking Laplace transform, we have

$$(M_2 s^2 + k_1 + k_2) \times Y(s) = k_1 X(s) \quad \dots (ii)$$

Now, substituting for $X(s)$ from equation (i) and rearranging, we have

$$(M_2 s^2 + k_1 + k_2) \times Y(s) = k_1 \times \frac{k_1 Y(s)}{M_1 s^2 + k_1}$$

$$(M_2s^2 + k_1 + k_2) \times (M_1s^2 + k_1) Y(s) = k_1^2 Y(s)$$

$$\text{or, } [M_1M_2s^4 + \{(k_1 + k_2)M_1 + k_1M_2\}s^2 + k_1k_2]Y(s) = 0$$

The final form of the equation can be written as

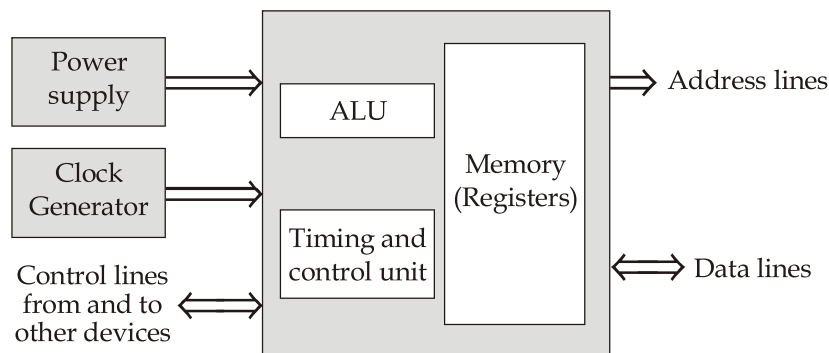
$$[M_1M_2s^4 + (M_1k_1 + M_1k_2 + k_1M_2)s^2 + k_1k_2]Y(s) = 0$$

Taking the inverse of Laplace transform, the differential equation can be derived as

$$M_1M_2 \frac{d^4y}{dt^2} + (M_1k_1 + M_1k_2 + M_2k_1) \frac{d^2y}{dt^2} + k_1k_2 \cdot y = 0$$

This is the final expression for the system output 'y'.

2. (c) (i) Solution:



A microprocessor is a VLSI-based programmable device that performs arithmetic, logic, control, and data processing operations. Its architecture mainly consists of the following components:

1. Arithmetic Logic Unit (ALU):

- Performs all arithmetic operations such as addition, subtraction, multiplication, and division.
- Executes logical operations like AND, OR, NOT, XOR, comparison, etc.
- It is the core computational unit of the microprocessor.

2. Registers (Memory Unit):

- Small, high-speed storage locations within the microprocessor.
- Used to store data, instructions, and intermediate results during execution.
- Types include general-purpose registers, accumulator, instruction register, and program counter.

- They reduce memory access time and improve processing speed.

3. Timing and Control Unit:

- Acts as the brain of the microprocessor.
- Controls the sequence of operations (fetch → decode → execute cycle).
- Generates control signals for memory and I/O devices.
- Synchronizes all operations using clock pulses.

4. Instruction Decoder:

- Decodes the fetched instruction into control signals.
- Determines the operation to be performed by the ALU.

5. Internal Buses:

- Connect various internal components (ALU, registers, control unit).
- Facilitate transfer of data and instructions inside the microprocessor.

The microprocessor follows the instruction cycle:
Fetch > Decode > Execute > Store

Instructions are fetched from memory, decoded, executed by ALU, and results are stored back.

2. (c) (ii) Solution:

1. Address Bus:

- Used to carry the address of memory locations or I/O devices.
- **Unidirectional** (from CPU to memory/I/O).
- Determines the maximum addressable memory.
- If address bus width = n bits, maximum memory locations = 2^n .

2. Data Bus:

- Transfers actual data between CPU, memory, and I/O devices.
- **Bidirectional** (data flows both ways).
- Width of data bus determines data processing capability (e.g., 8-bit, 16-bit processor).

3. Control Bus:

- Carries control and timing signals.
- Includes signals like **Read, Write, Interrupt, Clock, Reset**.
- Coordinates operations between CPU, memory, and I/O devices.

Comparison Table

Feature	Address Bus	Data Bus	Control Bus
Function	Carries address	Carries data	Carries control signals
Direction	Unidirectional	Bidirectional	Bidirectional
Purpose	Selects location	Transfers data	Controls operations
Impact	Memory capacity	Data handling capacity	System coordination

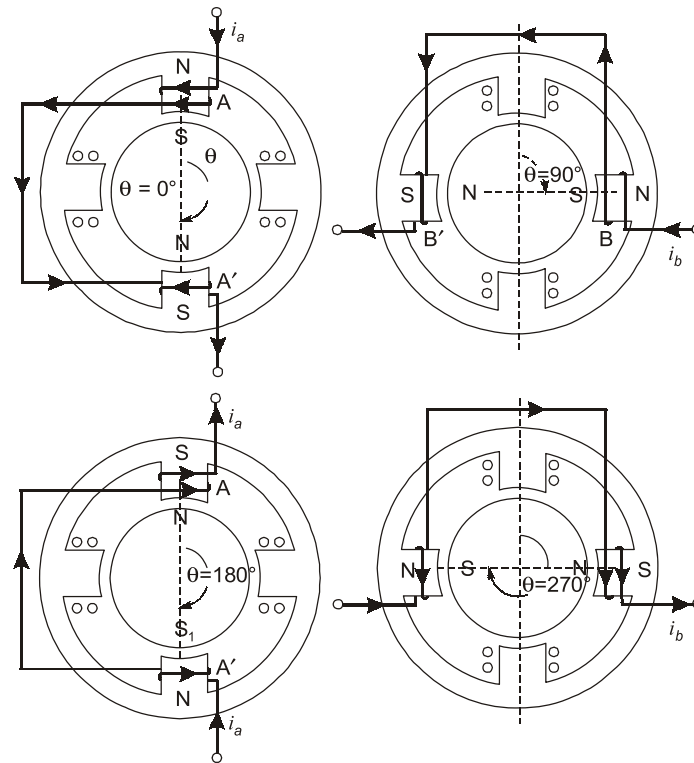
3. (a) Solution:

A stepper motor is a pulse-driven motor that changes the angular position of the rotor in steps. Due to this nature of a stepper motor it is widely used in low cost, open loop position control systems. A stepper motor is basically a brushless DC- motor whose rotor rotates in discrete angular movements when its winding is energized in a programmed manner. In stepper motor a full rotation is divided into a number of equal steps.

Permanent magnet (PM) stepper motor: The stator of permanent magnet stepper motor consists of salient poles with concentrated windings. The rotor as the name of this motor suggest, consists of permanent magnet poles. For the illustration of the working principle of PMSM an elementary form of 2-phase 4/2 pole stepper motor is considered here. The concentrated winding on diametrically opposite poles are connected in series so as to result in 2-phase winding on the stator. The rotor is magnetized to give two permanent magnets.

Working: Two coils AA' connected in series constitute phase A winding. When this winding is excited with current i_a , the stator produced poles attract the rotor permanent magnet poles so that their magnetic axis coincides. Let this exciting of phase A winding be denoted by $+A$.

Now the current in phase A winding is reduced to zero while phase B winding is excited with current i_b . Stator produced poles now attract the rotor poles, causing a CW step rotation through $\theta = 90^\circ$. Let the exciting of phase B winding be denoted by $+B$.



Internal structure of Permanent Magnet (PM) Stepper Motor

Now the phase winding A is again excited but with current opposite to i_a that is $-i_a$ this time. Now rotor poles further move through a step of 90° CW so that $\theta = 180^\circ$. This step of exciting phase winding be denoted by $-A$.

Now the phase winding B is made to carry exciting current opposite to that of i_b that is $-i_b$ this time. The rotor again executes further step of 90° CW so that $\theta = 270^\circ$. This method of exciting phase B winding be designated as $-B$.

For further 90° CW step phase winding B is de-energized and phase winding A is energized. This shows that four steps complete one revolution of the rotor movement.

So here by the application of each current pulse to the stator winding in proper sequence, the rotor can be made to execute discrete angular steps of 90° . Sequence of exciting the stator phase winding is $+A, +B, -A, -B, +A$ for CW rotor movement. For CCW rotor rotation, sequence of exciting stator phase winding is $+A, -B, -A, +B, -B$.

If both the stator windings are excited in the sequence $+A$ together with $+B$, then the resultant stator field is along the interpolar axis, the rotor therefore moves a step of 45° CW. This shows for obtaining the angular step of 45° CW the switching sequence should be as $+A, (+A +B), +B, (+B -A), -A, (-A -B), -B, (-B +A), +A$.

This method of reducing step angle to half the normal step is called half step mode of excitation.

Advantages of stepper motors:

- Low cost
- Ruggedness
- Simplicity of construction
- Low maintenance
- Less likely to stall or slip
- Will work in any environment
- Excellent start stop and reversing responses

Disadvantages of stepper motor:

- Low torque capacity compared to DC motors.
- Limited speed.
- During overloading, the synchronization will be broken. Vibration and noise occur when running at high speed.

3. (b)

We know, ${}^i_j T {}^j_k T = {}^i_k T \quad (\because {}^{i-1}_i T = {}^{i-1}_{i'} T {}^{i'}_i T)$

$${}^j_k T = {}^i_j T^{-1} {}^i_k T$$

$${}^j_k T = {}^j_i T {}^i_k T \quad (\because {}^i_j T^{-1} = {}^j_i T)$$

and, ${}^j_k T = {}^j_i T {}^k_i T^{-1} \quad (\because {}^i_k T = {}^k_i T^{-1}) \quad \dots(i)$

We have, ${}^j_i T = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 11 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and ${}^k_i T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 10 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

So, ${}^k_i T^{-1} = \left[\begin{array}{c|c} R^T & -R^T P \\ \hline 0 & 1 \end{array} \right]$

So, $R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

and,

$$-R^T P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ -20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -5\sqrt{3} + 10 \\ 5 + 10\sqrt{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.34 \\ 22.32 \end{bmatrix}$$

Then,

$${}^k_i T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 1.34 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 22.32 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting in equation (i), we get,

$${}^j_k T = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 11 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 1.34 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 22.32 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^j_k T = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & 10.33 \\ \frac{1}{2} & \frac{3}{4} & \frac{\sqrt{3}}{4} & 0.16 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 30.32 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. (c) (i) Solution:

Given: $l = 250$ mm, $b = 23$ mm, $t = 5$ mm, $F = 50$ N, $E = 210$ GPa, sensitivity of LVDT = 0.5 V/mm, Number of divisions = 100.

Moment of inertia for rectangular section,

$$I = \frac{bt^3}{12} = \frac{0.023 \times (0.005)^3}{12} = 2.395 \times 10^{-10} \text{ m}^4$$

$$1. \quad \text{Deflection, } \delta = \frac{FL^3}{3EI} = \frac{50 \times 0.25^3}{3 \times 210 \times 10^9 \times 2.395 \times 10^{-10}}$$

$$\delta = 5.175 \times 10^{-3} \text{ m} = 5.175 \text{ mm}$$

$$2. \quad \text{Deflection per unit force} = \frac{x}{F} = \frac{5.175 \times 10^{-3}}{50} \times 10^3 = 0.1035 \text{ mm/N}$$

$$\text{Total sensitivity} = 0.1035 \times 0.5 = 0.05175 \text{ V/N}$$

$$1 \text{ Scale division} = \frac{18}{100} = 0.18$$

$$\text{Resolution} = \frac{2}{10} \times 0.18 = 0.036 \text{ V}$$

$$3. \quad \text{Minimum force} = \frac{0.036}{0.05175} = 0.6956 \text{ N}$$

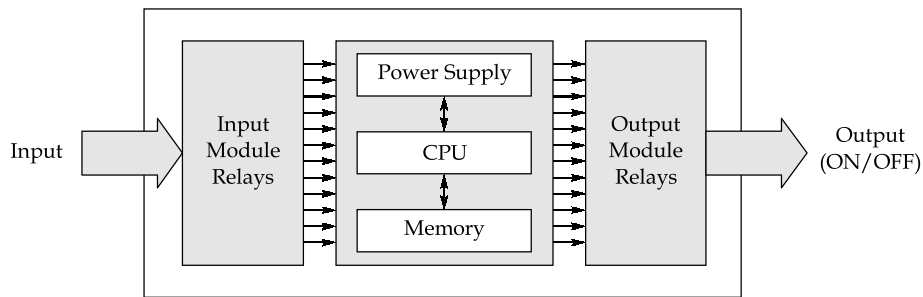
$$\text{Maximum force} = \frac{18}{0.05175} = 347.82 \text{ N}$$

3. (c) (ii) Solution:

Programmable Logic Controller abbreviated to PLC has long been used in the industrial automation platform. The primary reason for designing such a device was to facilitate communication capability and to replace the sluggish relay based machine control systems.

The PLC originated to replace traditional relay-based control implementation. It is considered as the first generation of computerized version of process control systems. The PLC based implementation is applicable where sequential logic is necessary. As per requirement the PLC has to be programmed. The desired operation is achieved when the program is executed. The PLC is programmed through a method called ladder logic technique, which means that sequences of logical actions must be established, interlinked in a synchronous and timely manner. The PLC samples the inputs and depending upon the state provides output in terms of ON or OFF. The basic blocks of the PLC are:

- Central Processing Unit
- Memory
- Input module
- Output module



Fundamentally, the PLC can be thought of as a cabinet consisting of thousands of relays, counters, timers and memory cells. The relays are categorized under three headings, namely field relays, interior relays and output relays. The field relays are realized by the use of transistors and thus physically exist. They are connected to the field devices such as sensors, switches etc. The signals from the outside world are fed to the field relays. The output relay is responsible to send on/off signal to the outside world such as solenoid, LED etc. They are also made up of transistors. The interior relays, counters and timers are realized by simulation. The register bits are used to realize the simulation. Some of the interior relays are ON state while some are OFF state. They are usually used for initializing data that was stored. The counters do not physically exist. They are called simulated counters or software counters. Through programming, the counter can count the input pulses. Essentially the software counters can count up or down. In many PLC hardwired counters are also seen. Hardware counters can provide faster counting operation.

The PLC works by repeatedly looking into the program. This repetition cycle has three steps, namely sense, decide and act. During the sense phase of the cycle, the PLC looks the current state of the process under its control in terms of looking at each field devices to conclude if it is ON or OFF state. Indeed the status of the first input, second input, third input and so on is sensed cyclically. Then it transfers the sensed data to the storage cells and records. The stored data are used during the next step during which the PLC executes the program. The programs are executed one instruction at a time, sequentially. In response, the execution of the program it turns ON or OFF the outputs of the corresponding inputs. It then stores the results of the execution for subsequent use and at the end the PLC always updates the status of the outputs. Thus it updates the outputs based on which inputs were ON or OFF during the first step and the results of executing the program during the second step. Once the final step is over the PLC goes back to step one and repeats the steps endlessly. The PLCs have been popular because of their advantages as follows.

- PLC can work in rugged environment and can work in the factory floor.

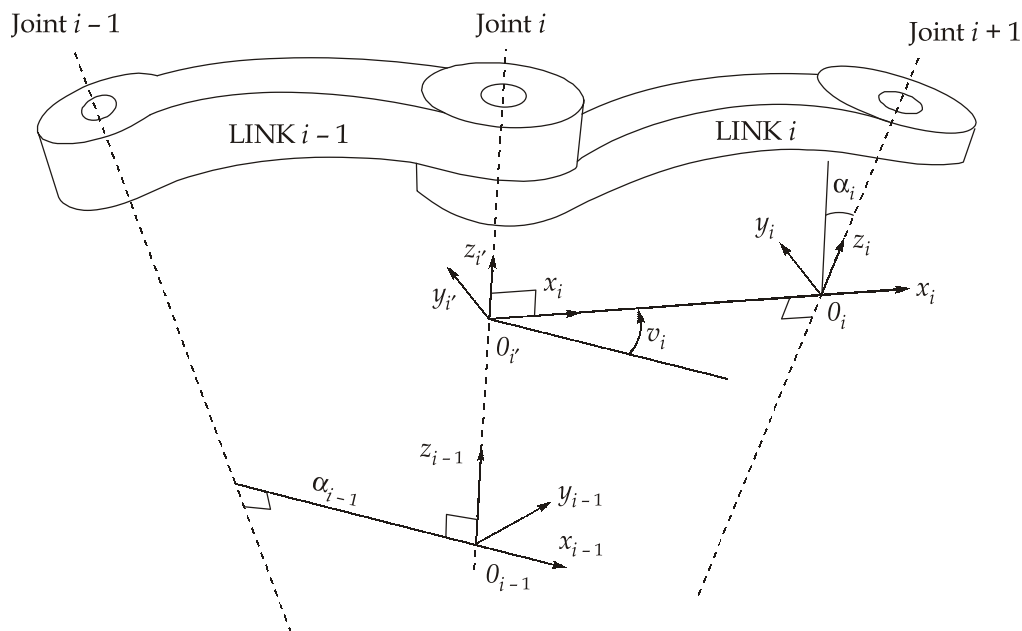
- They can operate in the adverse condition of the factory floor.
- They can withstand vibrations, interference, transients, noise etc.
- They can work from zero degrees centigrade to above forty degrees centigrade.
- PLC can replace hundreds of relays and occupy significantly less space.
- The circuit logic can be easily altered without rewiring.
- It has no moving parts or contacts to wear out.
- The down time is less than the relay circuit.
- The implementation is flexible and scalable.
- Faster response time

The major application of PLC is to replace the real world relays. The PLCs have been applied for factory automation, machine control, process control, instrumentation, data acquisition and control and many other manufacturing systems.

4. (a) Solution:

Concept of DH convention: In a serial manipulator, each link is assigned a coordinate frame such that the transformation frame $i - 1$ to frame i can be expressed using four parameters only. These parameters define the geometry of the robot in a structured manner. The transformation between two consecutive links is given as

$$T_i = \text{Rot}_z(\theta_i) \text{Trans}_z(d_i) \text{Trans}_x(a_i) \text{Rot}_x(\alpha_i)$$



The four DH parameters

- (i) **Link length (a_i)** : It is the distance between two consecutive joint axes measured along the common normal (i.e. along x_i). It represents length of the link.
- (ii) **Link twist (α_i)** : It is the angle between z_{i-1} and z_i measured about x_i . It represents twist between two joint axes.
- (iii) **Link offset (d_i)** : Distance between x_{i-1} and x_i measured along z_{i-1} . It represents offset along the joint axis.
- (iv) **Joint angle (θ_i)** : Angle between x_{i-1} and x_i measured about z_{i-1} . It represents rotation about the joint axis.

Assignment of frames to links

1. General principle

- A coordinate frame $\{i\}$ is rigidly attached to each link i .
- Frames are numbered sequentially from base (0) to end-effector (n).

2. Assignment rules for intermediate links.

(i) z-axis (z_i)

- The z_i axis is chosen along the axis of motion of joint i .
- Revolute joint \rightarrow along axis of rotation.
- Prismatic joint \rightarrow along direction of translation.

(ii) x-axis (x_i)

- x_i is taken along the common normal between z_i and z_{i+1} .
- It is directed from joint i to joint $i+1$.
- If the two z-axis intersects: x_i is perpendicular to the plane containing z_i and z_{i+1} .

(iii) Origin (O_i)

- Located at the intersection of the common normal with the joint axis z_i .
- If axes intersect \rightarrow Origin is at the point of intersection.

(iv) y-axis (y_i)

- Determine using the right hand rule:

$$y_i = z_i \times x_i$$

3. Assignment for first and last links

Base frame $\{0\}$

- Attached to the robot base (fixed frame)
- z_0 is aligned with the axis of joint 1.

End-effector frame $\{n\}$

- Assigned at the last link
- Orientation depends on application

4. (b) Solution:

Comparing the standard equation

$$T(s) = \frac{y(s)}{x(s)} = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

With the given equation

$$T(s) = \frac{361}{s^2 + 16s + 361}$$

We have $k = 1$, $\omega_n^2 = 361$ and $2\xi\omega_n = 16$

$$\omega_n^2 = 361$$

$$\Rightarrow \omega_n = \sqrt{361} = 19 \text{ rad/sec}$$

$$\text{Damping ratio, } 2\xi\omega_n = 16$$

$$\therefore \xi = \frac{16}{2 \times 19} = 0.421$$

\therefore Damped natural frequency

$$\begin{aligned} \omega_d &= \omega_n \sqrt{1 - \xi^2} \\ &= 19 \sqrt{1 - 0.421^2} = 17.23 \text{ rad/s} \end{aligned}$$

Ans.(i)

$$\text{Peak time, } T_d = \frac{\pi}{\omega_d} = \frac{\pi}{17.23} = 0.182 \text{ sec} \quad \text{Ans. (iii)}$$

$$\text{Rise time, } T_r = \frac{\pi}{2\omega_d} = \frac{\pi}{2 \times 17.23} = 0.091 \text{ sec} \quad \text{Ans. (iv)}$$

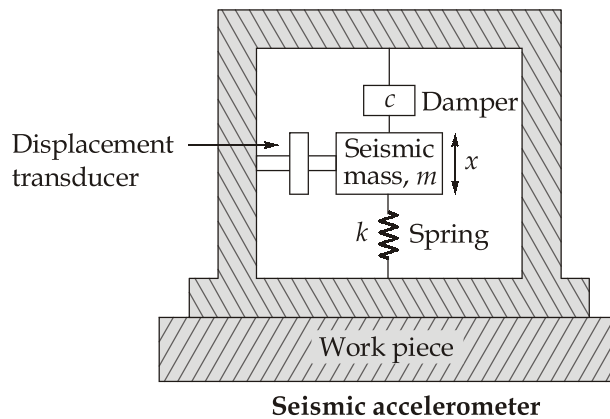
For $x = 0.02$, settling time

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.421 \times 19} = 0.5 \text{ sec} \quad \text{Ans. (ii)}$$

$$\text{Percent overshoot, \%OS} = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = e^{-\frac{\pi \times 0.421}{\sqrt{1-0.421^2}}} \times 100 = 23.3\% \quad \text{Ans. (iv)}$$

4. (c) (i) Solution:

Displacement Seismic Accelerometer: In this type accelerometer, the displacement of seismic mass is measured by displacement transducer itself. A spring-mass damper is placed inside the casing by placing the damper at the top and spring at the bottom of the casing. Again, the seismic is connected with the displacement transducer by a shaft which is already connected with the casing itself. When the acceleration or vibration is applied on the casing, the mass is displaced which is sensed by the sensor. Therefore, the displacement sensed by the transducer is directly proportional to the acceleration.



The equation of motion can be written as

$$m\ddot{x} = -c(\dot{x} - \dot{y}) - k(x - y) + F_0 \sin \omega t$$

Let $z = x - y$,

$$m\ddot{z} + c\dot{z} + kz = F_0 \sin \omega t$$

Where x is the displacement and $F_0 = m\omega^2 y$

Therefore, the relative amplitude z provided by the instrument can be evaluated as

$$z = \frac{m\omega^2 y}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = \frac{r^2 y}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$

and the position of the amplitude can be evaluated from

$$\tan \phi = \frac{2\xi r}{1 - r^2}$$

where, $r = \frac{\omega}{\omega_n}$ and $\zeta = \text{Damping ratio} = \frac{c}{2m\omega_n}$

The steady-state amplitude Z of seismic mass with respect to the frame is the measure of acceleration, when the natural frequency of the instrument is high compared to that of the vibration to be measured. This implies

$$\frac{Z}{y} = r^2 \text{ if } r \ll 1$$

$$Z = r^2 y = \frac{\omega^2 y}{\omega_n^2}$$

where $\omega^2 y$ is the acceleration.

The measured value of Z is proportional to the acceleration of the motion to be measured.

4. (c) (ii) Solution:

As per given information:

$$m = 1 \text{ kg}, k = 2 \text{ N/m}, C = 2 \text{ Ns/m}, x_{\text{in}}(t) = 10 \sin(1.25t) \text{ mm}, \omega = 1.25 \text{ rad/s},$$

$$y = 10 \text{ mm} = 0.01 \text{ m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{1}} = 1.414 \text{ rad/s}$$

$$\xi = \frac{C}{2\sqrt{km}} = \frac{2}{2\sqrt{2 \times 1}} = 0.707$$

$$\frac{z}{y} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}$$

$$\frac{z}{y} = \frac{\left(\frac{1.25}{1.414}\right)^2}{\sqrt{\left[1 - \left(\frac{1.25}{1.414}\right)^2\right]^2 + \left[2 \times 0.707 \times \frac{1.25}{1.414}\right]^2}}$$

$$\frac{z}{y} = 0.6158$$

Steady state relative amplitude,

$$z = 0.6158 \times y = 0.6158 \times 0.01 \text{ m}$$

$$z = 6.158 \text{ mm}$$

Ans.

$$\phi = \tan^{-1} \left(\frac{2\xi \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right) = \tan^{-1} \left[\left(\frac{2 \times 0.707 \times 1.25}{1.414} \right) \right]$$

$$= 80.084^\circ \text{ or } 1.3977 \text{ rad}$$

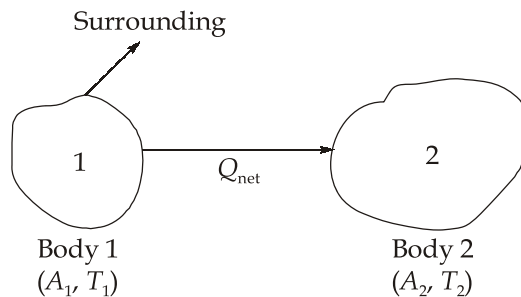
Ans.

Section B : Machine Design-1 + Heat Transfer-2 + Power Plant-2

5. (a) Solution:

Shape factor or configuration factor: The shape factor may be defined as “the fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections”.

Reciprocity theorem: Let us consider a given object 1 radiates to object 2 and to the surrounding as well as shown in figure.



For general expression of heat exchange between these two surfaces, let the shape factor be represented as

$$F_{1-2} = \text{Fraction of energy leaving 1 which reaches 2}$$

$$F_{2-1} = \text{Fraction of energy leaving 2 which reaches 1}$$

F_{i-j} is a function of geometry only

$$\text{Energy leaving body 1} = E_{b1}A_1$$

So, energy leaving body 1 and arriving at body 2 = $E_{b1}A_1F_{1-2}$

Similarly energy leaving body 2 and arriving at body 1

$$= E_{b2}A_2F_{2-1} \quad [\text{Assuming both are black bodies}]$$

The net energy interchange from body 1 to body 2 is

$$E_{b1}A_1F_{1-2} - E_{b2}A_2F_{2-1} = Q_{1-2}$$

If both surfaces are at same temperature then,

$$Q_{1-2} = 0 \text{ and } E_{b1} = E_{b2}$$

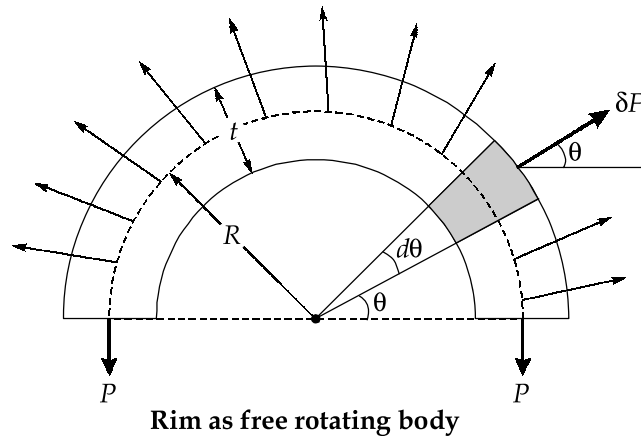
Hence,

$$A_1 F_{1-2} = A_2 F_{2-1}$$

It is known as reciprocity theorem.

5. (b) Solution:

Consider the forces acting on the half rim as shown in figure. The rim is in equilibrium under the action of inertia force which is uniformly distributed in the radial direction and the tensile force 'P' at the rim cross-section.



At an angle θ , consider an elemental section shown by the shaded area. The volume of this element is $(btRd\theta)$ and the elemental mass is given by

$$\delta m = btRd\theta \times \rho$$

where, t is the thickness, b is the width and R is the mean radius in meters. The elemental inertia force δF acting on this element is given by,

$$\delta F = \text{Mass} \times \text{Acceleration}$$

$$\delta F = (btRd\theta\rho) (R\omega^2)$$

Since,

$$v = R\omega$$

$$\delta F = bt\rho v^2 d\theta$$

Considering equilibrium of vertical forces

$$2P = \int_0^\pi bt\rho v^2 \sin\theta \, d\theta$$

$$= bt\rho v^2 \int_0^\pi \sin\theta \, d\theta = bt\rho v^2 [-\cos\theta]_0^\pi$$

$$2P = 2bt\rho v^2$$

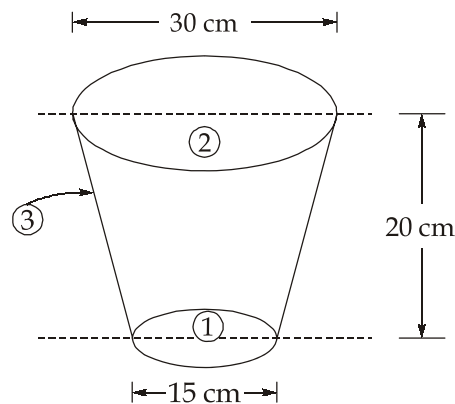
$$\therefore \sigma_t = \frac{P}{bt} = \rho v^2$$

$$\text{or } \sigma_t = \rho v^2 \quad \text{Hence proved}$$

5. (c) Solution:

Given : $r_2 = 15 \text{ cm}$; $r_1 = 7.5 \text{ cm}$; $h = 20 \text{ cm}$

Refer the figure below for configuration and nomenclature.



Area of the curved surfaces,

$$\begin{aligned} A &= \pi(r_1 + r_2)\sqrt{(r_2 - r_1)^2 + h^2} \\ &= \pi(7.5 + 15)\sqrt{(15 - 7.5)^2 + 20^2} \\ &= 1509.85 \text{ cm}^2 \end{aligned}$$

$$\text{Now, } F_{21} = 0.15 \text{ (Given)}$$

By reciprocating theorem,

$$A_1 F_{12} = A_2 F_{21}$$

$$\therefore F_{12} = \frac{A_2}{A_1} \times F_{21} = \frac{\pi \times 15^2}{\pi \times 7.5^2} \times 0.15$$

$$F_{12} = 0.6$$

$$\text{Further, } F_{11} + F_{12} + F_{13} = 1$$

$$\therefore F_{13} = 1 - F_{12} \quad (\because F_{11} = 0)$$

$$F_{13} = 1 - 0.6 = 0.4$$

Ans.

$$\text{Again, } F_{21} + F_{22} + F_{23} = 1$$

$$\therefore F_{23} = 1 - F_{21} \quad (\because F_{22} = 0)$$

$$F_{23} = 1 - 0.15 = 0.85$$

Now, by reciprocity theorem,

$$F_{32} = \frac{A_2}{A_3} F_{23}$$

$$F_{32} = \frac{\pi \times 15^2}{1509.85} \times 0.85 = 0.398$$

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{\pi \times 7.5^2}{1509.85} \times 0.4 = 0.0468$$

∴ From the identity,

$$F_{31} + F_{32} + F_{33} = 1$$

$$F_{33} = 1 - 0.0468 - 0.398$$

$$= 0.555$$

Ans.

5. (d) Solution:

Absolute pressure in condenser = 76 - 70 = 6 cm of Hg

$$= \frac{6}{100} \times 13.6 \times 1000 \times 9.81$$

$$= 8.004 \text{ kPa}$$

Applying heat balance to the condenser,

$$\dot{m}_w \times C_{pw} \times \Delta T = (h_{f \text{ at } 30^\circ} + 0.85 h_{fg \text{ at } 30^\circ} - C_{pw} T) \times \dot{m}_s$$

$$\dot{m}_w \times 4.18 \times 20 = [250.65 + 0.85 (2681.65 - 250.65) - 4.18 \times 30] \times 300$$

$$\dot{m}_w = 7864.59 \text{ kg/min}$$

$$\text{So, Volume flow rate of water} = \frac{\dot{m}_w}{\rho_w} = \frac{7864.59}{1000} = 7.86459 \text{ m}^3/\text{min}$$

$$\text{Flow surface area requirement} = \frac{\text{Volume flow rate of water}}{\text{Velocity of water flow}}$$

$$= \frac{7.86459}{50} = 0.1573 \text{ m}^2$$

$$\text{Now, Cooling surface area required} = \frac{\text{Total heat given by steam}}{\text{Heat transfer rate}}$$

$$= \frac{300(h_{f \text{ at } 30^\circ\text{C}} + 0.85 \times h_{fg \text{ at } 30^\circ\text{C}} - 4.18 \times 30) \times 60}{15 \times 10^5}$$

$$= \frac{300[250.65 + 0.85(2681.65 - 250.65) - 4.18 \times 30] \times 60}{15 \times 10^5}$$

$$\text{Cooling surface area required} = 26.3 \text{ m}^2$$

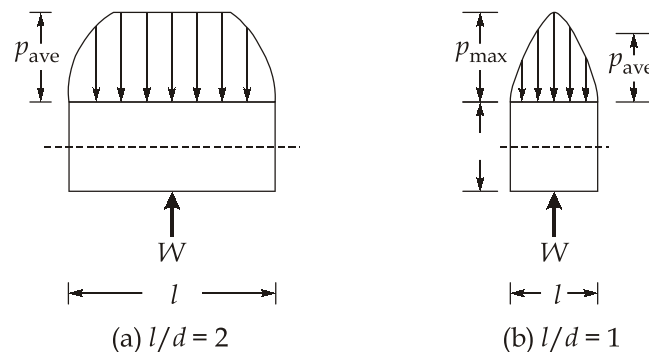
$$\text{Velocity head present} = \frac{V^2}{2g} = \frac{(50/60)^2}{2 \times 9.81} = 0.0354 \text{ m}$$

$$\begin{aligned} \text{Total head required} &= \text{Pressure head} + \text{Velocity head} \\ &= 5 + 0.0354 = 5.035 \text{ m} \end{aligned}$$

5. (e) Solution:

The design and selection of journal bearings in mechanical systems require careful consideration of several interrelated parameters to ensure optimal performance, reliability and lifespan. The key parameters are

- (i) **Length to diameter ratio :** The length to diameter ratio (l/d) affects the performance of the bearing. As the ratio increases, the resulting film pressure increases as shown in figure. A long bearing, therefore, has more load carrying capacity compared with a short bearing. A short bearing, on the other hand, has greater side flow, which improves heat dissipation. The long bearings are more susceptible to metal to metal contact at the two edges, when the shaft is deflected under load. The longer the bearing, more difficult it is to get sufficient oil flow through the passage between the journal and the bearing. Therefore, the design trend is to use (l/d) ratio as 1 or less than 1. When the shaft and the bearing are precisely aligned, the shaft deflection is within the limit and cooling of lubricant and bearing does not present a serious problem, the (l/d) ratio can be taken as more than 1. In practice, the (l/d) ratio varies from 0.5 to 2.0, but in the majority of applications, it is taken as 1 or less than 1. Following terminology is used in relation to (l/d) ratio,



Effect of (l/d) ratio on average bearing pressure

- When (l/d) ratio is more than 1, the bearing is called 'long' bearing.

- When (l/d) ratio is less than 1, the bearing is called 'short bearing'.
- When (l/d) ratio is equal to 1, the bearing is called 'square' bearing.

- (ii) **Unit Bearing Pressure** : The unit bearing pressure is the load per unit of projected area of the bearing in running condition. It depends upon number of factors, such as bearing material, operating temperature, the nature, and frequency of load and service conditions. The values of unit bearing pressure, based on past experience.
- (iii) **Start-up Load** : The unit bearing pressure for starting conditions should not exceed 2 N/mm^2 . The start-up load is static load when the shaft is stationary. It mainly consists of the dead weight of shaft and its attachments. The start-up load can be used to determine the minimum length of the bearing on the basis of starting conditions.
- (iv) **Radial Clearance** : The radial clearance should be small to provide the necessary velocity gradient. However, this requires costly finishing operations, rigid mountings of the bearing assembly, and clean lubricating oil without any foreign particles. This increases the initial and maintenance costs.
- (v) **Minimum Oil Film Thickness** : The surface finish of the journal and the bearing is governed by the value of the minimum oil film thickness selected by the designer and vice versa. There is a lower limit for the minimum oil film thickness, below which metal-to-metal contact occurs and hydrodynamic film breaks. This lower limit is given by,
- $$h_0 = (0.0002)r$$
- (vi) **Maximum Oil Film Temperature** : The lubricating oil tends to oxidize when the operating temperature exceeds 120° . Also, the surface of babbitt bearing tends to soften at 125°C (for bearing pressure of 7 N/mm^2) and at 190°C (for bearing pressure of 1.4 N/mm^2). Therefore, the operating temperature should be kept within these limits.

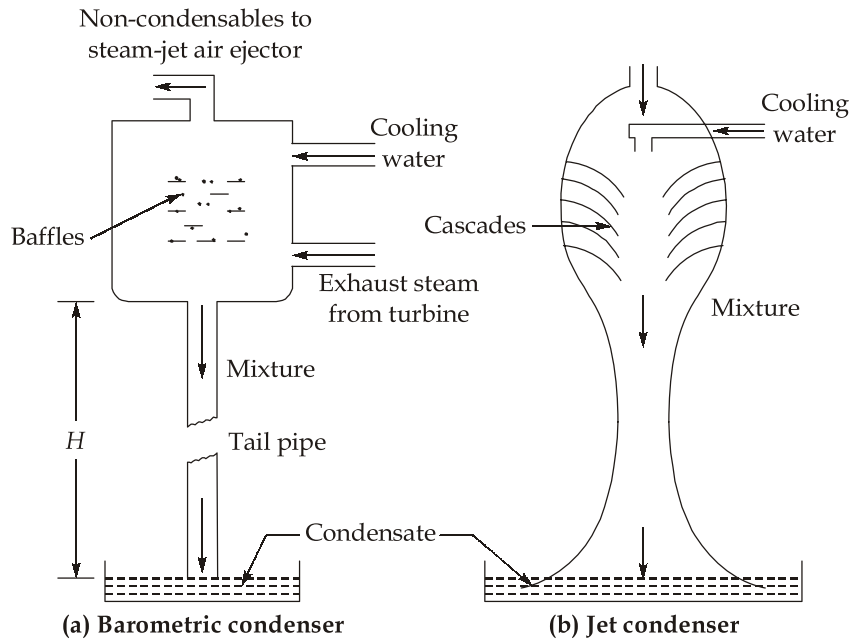
6. (a) (i) **Solution:**

Barometric Condenser: The schematic of barometric condenser is shown in figure (a). In it the cooling water is made to fall over a series of baffles to expose large surface area for the steam fed from below to come in direct contact. Condensation of steam takes place and the mixture falls in a tail pipe to the hot well below. As a consequence of its static head, the tail pipe compresses the mixture to atmospheric pressure. Thus, the height of tail pipe (H) is expressed as

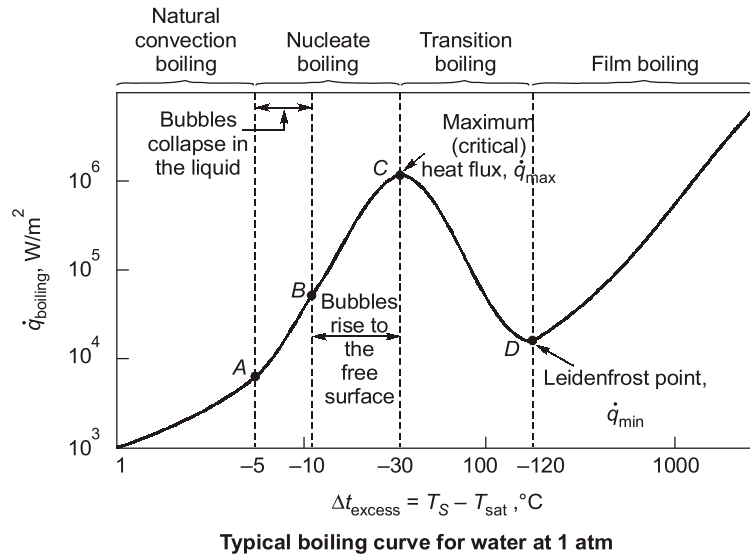
$$p_{\text{atm}} - p_{\text{cond}} + \Delta_{pf} = \rho g H$$

where, ρ is the density of mixture and Δ_{pf} is the pressure drop due to friction. For low value of Δ_{pf} , H is around 9.5 m.

Jet Condenser: Figure (b) shows the schematic diagram of a jet condenser. Baffles of barometric type are replaced by cascade and to reduce the height of tail pipe, diffuser is provided. The diffuser acts on the same principle as the diverging section of a convergent nozzle in subsonic flow. The mixture is split and cooled



6. (a) (ii) Solution:



Nucleate Boiling: The nucleate boiling can be represented between points A and C. Point A is referred to as the onset of nucleate boiling (ONB). The bubbles form at an

increasing rate at an increasing number of nucleation sites as we move along the boiling curve towards point C.

The nucleate boiling regime can be separated into two distinct regions. In region A-B, ($5^{\circ}\text{C} \leq \Delta T_{\text{excess}} \leq 10^{\circ}\text{C}$), isolated bubbles are formed at various preferential nucleation sites on the heated surface. In region B-C ($10^{\circ}\text{C} \leq \Delta T_{\text{excess}} \leq 30^{\circ}\text{C}$), the heater temperature is further increased, and bubbles form at such great rates at such a large number of nucleation sites that they form numerous continuous columns of vapor in the liquid. These bubbles move all the way up to the free surface, where they break up and release their vapor content. The large heat fluxes obtainable in this region are caused by the combined effect of entrainment and evaporation.

Film boiling: In this, region, the heater surface is completely covered by a continuous stable vapor film. Point D, where the heat flux reaches a minimum is called Leidenfrost point. The presence of a vapor film between the heater surface and the liquid is responsible for the low heat transfer rates in the film boiling region. The heat transfer rate increases with increasing excess temperature as a result of heat transfer from the heated surface to the liquid through the vapor film by radiation, which becomes significant at high temperatures.

6. (b) Solution:

Given: Number of tubes, $N = 100$, Inner diameter, $d_i = 25$ mm, Tube thickness, $t = 2$ mm, $\dot{m}_w = 8$ kg/s, $T_{c1} = 25^{\circ}\text{C}$, $T_{c2} = 75^{\circ}\text{C}$, $T_{h1} = T_{h2} = 100^{\circ}\text{C}$, $h_o = 5000$ W/m²K,

$$R_{fi} = 0.0002 \text{ m}^2\text{K/W}.$$

Now, Heat transfer rate to water,

$$Q = \dot{m}_c c_{pc} (T_{c2} - T_{c1}) = 8 \times 4175 (75 - 25) \text{ W}$$

$$\therefore Q = 1670 \text{ kW}$$

$$\text{Flow Reynolds number, Re} = \frac{\rho V d_i}{\mu}$$

$$\text{Mass flow rate inside tube, } \dot{m}_c = \rho \times \frac{\pi}{4} \times d_i^2 \times V \times N$$

$$\text{or } V = \frac{\dot{m}_c}{\rho \times \frac{\pi}{4} \times d_i^2 \times N} = \frac{8}{998 \times \frac{\pi}{4} \times 0.025^2 \times 100} = 0.163 \text{ m/s}$$

$$\therefore \text{Re} = \frac{\rho V d_i}{\mu} = \frac{998 \times 0.163 \times 0.025}{55 \times 10^{-5}} = 7394.272 \text{ (Flow is$$

turbulent)

$$\text{Prandtl number, Pr} = \frac{55 \times 10^{-5} \times 4175}{0.65} = 3.532$$

$$\text{For turbulent flow, } Nu = \frac{h_i d_i}{k} = 0.023 \times (\text{Re})^{0.8} \times (\text{Pr})^{0.4}$$

$$\therefore \frac{h_i \times 0.025}{0.65} = 0.023 \times (7394.272)^{0.8} \times (3.532)^{0.4}$$

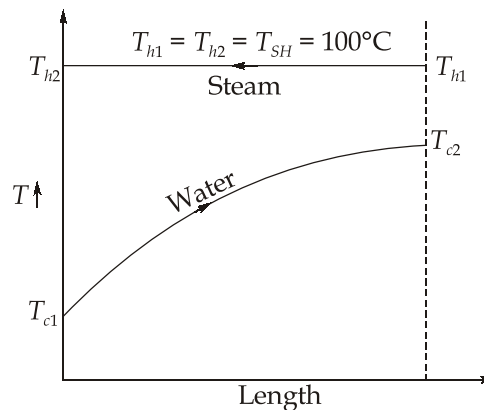
$$h_i = 1233.178 \text{ W/m}^2\text{K}$$

Overall heat transfer coefficient based on inner area.

$$\begin{aligned} \frac{1}{U_i} &= \frac{1}{h_i} + R_{F_i} + \frac{r_i}{r_o} \times \frac{1}{h_o} \\ &= \frac{1}{1233.178} + 0.0002 + \frac{0.0125}{0.0145} \times \frac{1}{5000} = 1.183 \times 10^{-3} \end{aligned}$$

$$\therefore U_i = 845.308 \text{ W/m}^2\text{K} \quad \text{Ans.}$$

Since, the temperature of one of the fluid remains constant during the flow passage. Both parallel and counter flow would give the same values of LMTD. Assuming counter flow arrangement



$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)}$$

$$\theta_1 = T_{h2} - T_{c1} = 100 - 75 = 25^\circ\text{C}$$

$$\theta_2 = T_{h1} - T_{c2} = 100 - 25 = 75^\circ\text{C}$$

$$\therefore \theta_m = \frac{25 - 75}{\ln\left(\frac{25}{75}\right)} = 45.511^\circ \text{C}$$

Now, $Q = U_i A_i \theta_m$

$$\therefore A_i = \frac{Q}{U_i \theta_m} = \frac{1670 \times 10^3}{845.308 \times 45.511} = 43.409 \text{ m}^2$$

or $N \times \pi d_i \times L = 43.409$

$$\therefore L = \frac{43.409}{100 \times \pi \times 0.025} = 5.529 \text{ m}$$

Ans.**6. (c) Solution:**

Given: $\phi = 20^\circ$, $Z_p = 24$, $Z_g = 60$, $m = 5 \text{ mm}$, $b = 50 \text{ mm}$, $N_p = 600 \text{ rpm}$,

$(S_{ut})_p = (S_{ut})_g = 650 \text{ N/mm}^2$. BHN = 230, $k_a = 1.6$, FOS = 2, $k_m = 1$.

(i) **Beam strength:** When pinion and gear both are made of same material at that time pinion is weaker than the gear in bending. Hence, it is necessary to find beam strength of pinion.

$$F_b = (bmY\sigma_b)_p$$

$$Y_p = 0.48 - \frac{2.8}{Z_p} = 0.48 - \frac{2.8}{24} = 0.3633$$

$$\therefore F_b = 50 \times 5 \times 0.3633 \times \frac{650}{3} = 19678.75 \text{ N}$$

$$= 19.678 \text{ kN}$$

Ans.

(ii) **Wear strength:**

$$F_W = D_p b Q k$$

$$D_p = m \times Z_p = 5 \times 24 = 120 \text{ mm}$$

$$Q = \frac{2Z_G}{Z_G + Z_P} = \frac{2 \times 60}{60 + 24} = 1.4285$$

$$k = 0.16 \times \left(\frac{230}{100}\right)^2 = 0.8464 \text{ N/mm}^2$$

$$\therefore F_W = 120 \times 50 \times 1.4285 \times 0.8464$$

$$= 7254.49 \text{ N} = 7.254 \text{ kN}$$

Ans.

(iii) In this case, wear strength is lower than beam strength so wear strength is the criteria

of design.

$$\therefore F_W = (\text{FOS}) \times F_{\text{eff.}}$$

$$7254.99 = 2 \times F_{\text{eff.}}$$

$$\Rightarrow F_{\text{eff.}} = 3627.2472 \text{ N}$$

$$\text{Now, } F_{\text{eff.}} = \frac{k_a k_m F_t}{C_V} \quad \dots (i)$$

$$V = \frac{\pi D_P N_P}{60} = \frac{\pi \times 120 \times 600}{60 \times 1000} = 3.77 \text{ m/s}$$

$$C_V = \frac{3}{3 + V} = \frac{3}{3 + 3.77} = 0.443$$

\therefore From equation (i)

$$3627.2472 = \frac{1.6 \times 1 \times F_t}{0.443}$$

$$\Rightarrow F_t = 1004.605 \text{ N} \quad \text{Ans.}$$

(iv) Power transmitted by gear pair:

$$\begin{aligned} P &= F_t \times V = 1004.605 \times 3.77 \\ &= 3787.36 \text{ W} = 3.787 \text{ kW} \end{aligned}$$

Ans.

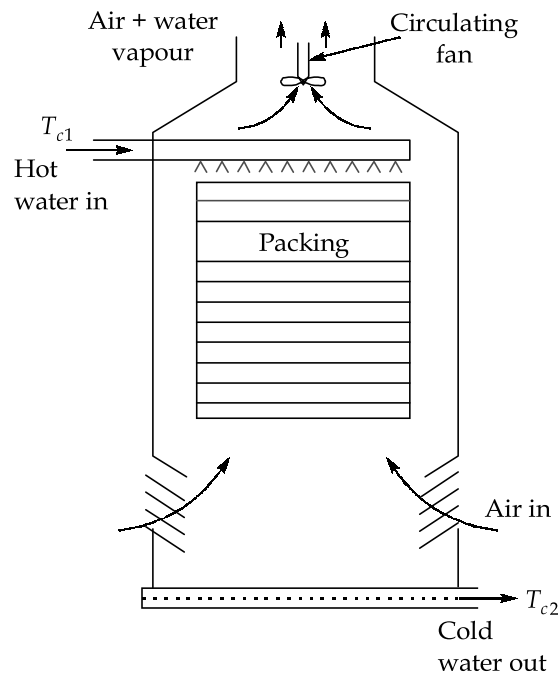
7. (a) Solution:

The purpose of cooling tower is to cool the warmed water discharged from the condenser and feed the cooled water back to the condenser. By this way, the cooling water requirement gets reduced to make-up water supply only.

The cooling tower may be classified as

1. Wet cooling towers:
 - Mechanical draught cooling tower:
 - Natural draught cooling tower:
2. Dry cooling towers

Wet Cooling Towers: Wet-cooling towers cool the hot water by dissipating heat to the environment through the mechanism of (i) addition of sensible heat to the air and (ii) evaporation of a portion of the recirculating water itself. When operated in the open mode, there is a third mechanism (iii) addition of sensible heat to the natural body of water as a result of terminal temperature difference.



Wet cooling towers employ a hot-water distribution system that showers or sprays the water evenly over a lattice work of closely set horizontal slats or bars called fill, or packing. Since the water splashes down from one fill level to the next by gravity, there is a thorough mixing of falling water with air moving through the fill. Outside air enters the tower via louvers in the form of horizontal slats on the side of the tower. The slats are arranged in such way that they usually stop downward to keep the water in. The intimate mixing between water and air results in the enhancement of heat and mass transfer (evaporation) which cools the water. The cold water gets collected in a concrete basin at the bottom of the tower from where it is pumped back to the condenser in closed system or returned to the water body in open system. The resulting hot and moist air leaves the tower at the top.

Unsaturated air enters the cooling tower and as it comes in contact with the water spray. Water continues to evaporate till it becomes saturated. From this, it follows that the minimum temperature to which water can be cooled is the adiabatic saturation or wet bulb temperature (T_{wb}) of the ambient air.

Performance Parameters: A cooling tower is characterized by three performance parameters namely (a) approach, (b) range, and (c) cooling efficiency. The approach temperature (A) is defined as the difference between the exit temperature of cooling water and the wet bulb temperature of the ambient air. Thus

$$A = T_{c2} - T_{wb}$$

where T_{wb} = the wet bulb temperature of air
 T_{c2} = cooling water exit temperature from cooling tower

This approach temperature (A) varies from 6 to 8°C.

The cooling range (R) is defined as the difference in temperature of the incoming warm water (T_{c1}) and existing cooler water (T_{c2}). Thus,

$$R = T_{c1} - T_{c2}$$

This range R varies from 8 to 10°C

The cooling efficiency is defined as the ratio of actual cooling of water to the maximum cooling possible. Thus

$$\eta_{\text{cooling}} = \frac{\text{Actual cooling}}{\text{Maximum cooling possible}} = \frac{T_{c1} - T_{c2}}{T_{c1} - T_{wb}}$$

7. (b) Solution:

Let the suffix 1, 2 and 3 refers to the inner cylinder, outer cylinder and cylindrical shield. Then

$$d_3 = \frac{d_1 + d_2}{2} = \frac{10 + 20}{2} = 15 \text{ cm}$$

Heat flow from inner cylinder to shield.

$$Q_{13} = (F_g)_{13} A_1 \sigma_b (T_1^4 - T_3^4)$$

Where,

$$(F_g)_{13} = \frac{1}{\frac{1}{\epsilon_1} + \left(\frac{1}{\epsilon_3} - 1\right) \frac{A_1}{A_3}} = \frac{1}{0.2 + \left(\frac{1}{0.3} - 1\right) \times \frac{10}{15}}$$

$$\therefore \frac{A_1}{A_3} = \frac{\pi d_1 l}{\pi d_3 l} = \frac{10}{15}, \quad F_{13} = 1$$

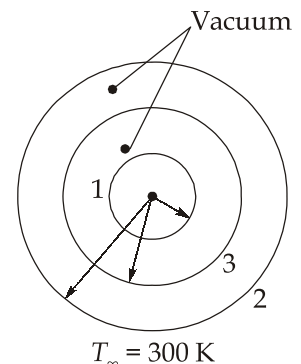
$$\therefore (F_g)_{13} = 0.152$$

$$\text{and } Q_{13} = 0.152 \times A_1 \times \sigma_b (1200^4 - T_3^4) \dots (i)$$

Likewise, the heat flow from the shield to the outer cylinder is

$$Q_{32} = (F_g)_{32} A_3 \sigma_b (T_3^4 - T_2^4)$$

$$(F_g)_{32} = \frac{1}{\frac{1}{\epsilon_3} + \left(\frac{1}{\epsilon_2} - 1\right) \frac{A_3}{A_2}}$$



$$\therefore \frac{A_3}{A_2} = \frac{\pi d_3 l}{\pi d_2 l} = \frac{15}{20}, \quad F_{32} = 1$$

$$\therefore (F_g)_{32} = \frac{1}{\frac{1}{0.5} + \left(\frac{1}{0.3} - 1\right)} \times \frac{15}{20} = 0.266$$

$$\text{and} \quad Q_{32} = 0.266 \times A_3 \sigma_b (T_3^4 - 320^4)$$

Under steady state conditions,

$$Q_{13} = Q_{32}$$

$$0.152 \times A_1 \times \sigma_b (1200^4 - T_3^4) = 0.266 \times A_3 \times \sigma_b (T_3^4 - 320^4)$$

$$0.152 \times (1200^4 - T_3^4) = 0.266 \times \frac{A_3}{A_1} \times (T_3^4 - 320^4)$$

$$\text{or,} \quad 1200^4 - T_3^4 = \frac{0.266 \times 15}{0.152 \times 10} \times (T_3^4 - 320^4)$$

$$1200^4 - T_3^4 = 2.625 T_3^4 - 2.625 \times 320^4$$

$$\therefore T_3 = \left[\frac{1200^4 + 2.625 \times 320^4}{1 + 2.625} \right]^{1/4} = 872.541 \text{ K} \quad \text{Ans.}$$

Heat loss per metre length, $Q_{13} = 0.152 \times \pi \times 0.1 \times 1 \times (5.67 \times 10^{-8}) (1200^4 - 872.541^4)$

$$Q_{13} = 4043.269 \text{ W} \quad \text{Ans.}$$

For steady flow, this heat is lost to the surrounding both by convection and radiation from the outer surface of outside cylinder.

$$\therefore 4043.269 = h_c A_2 (T_2 - T_a) + \sigma_b \epsilon_2 A_2 (T_2^4 - T_a^4)$$

$$4043.269 = h_c (\pi \times 0.2 \times 1) (320 - 300) + 5.67 \times 10^{-8} \times 0.3 \times (\pi \times 0.2 \times 1) (320^4 - 300^4)$$

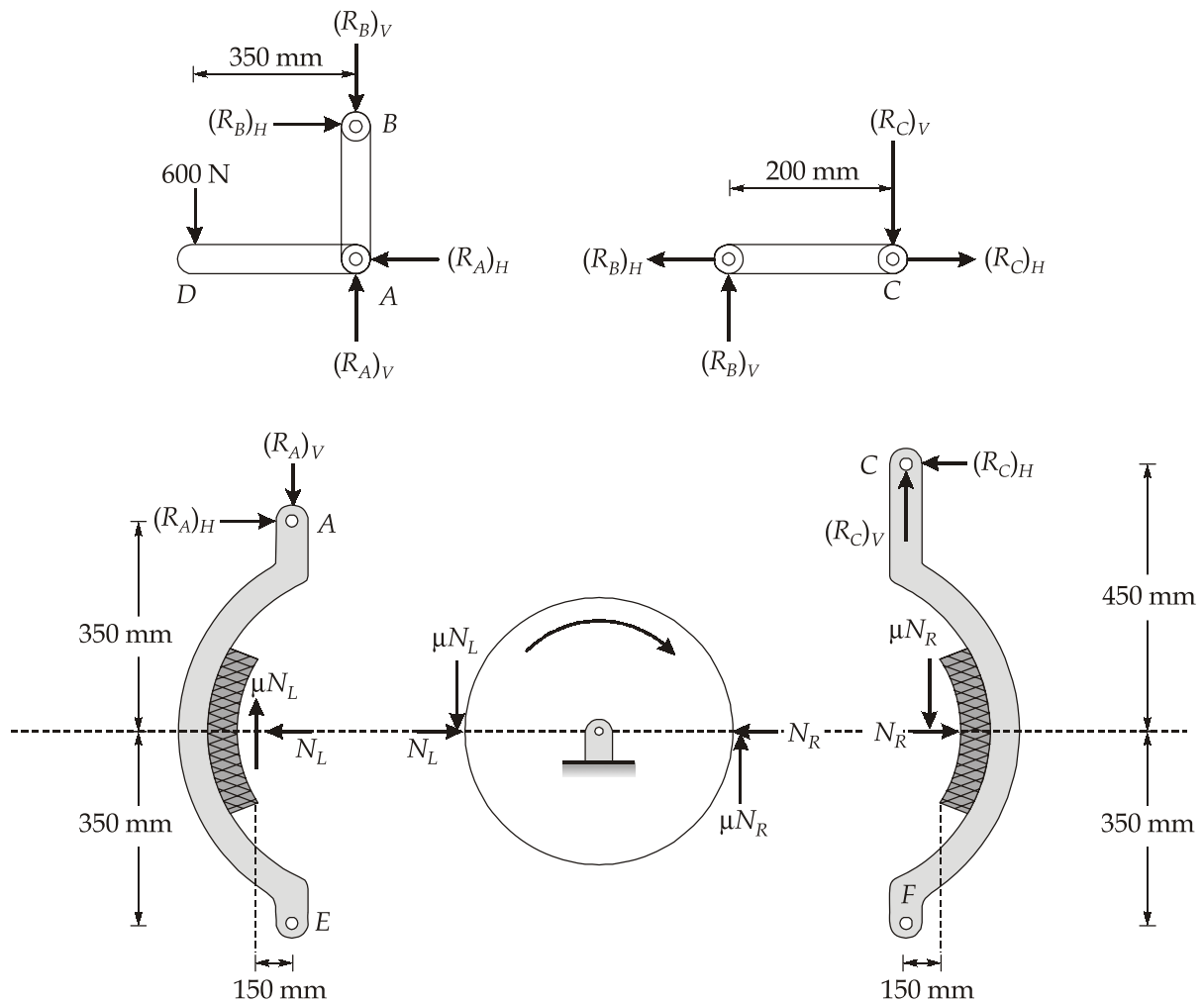
$$4043.269 = 12.56 h_c + 25.485$$

$$h_c = 319.887 \text{ W/m}^2\text{K} \quad \text{Ans.}$$

7. (c)

Given : $P = 600 \text{ N}$; $\mu = 0.30$; $R = 250 \text{ mm} = 0.25 \text{ m}$; $p = 1.2 \text{ N/mm}^2$; $l = 2 \text{ W}$

The free body diagram of forces acting on various parts as shown in the figure below.



Considering the forces acting on the link DAB and taking moments about the pin A ,

$$(R_B)_H \times 100 = 600 \times 350$$

$$(R_B)_H = 2100 \text{ N}$$

Considering equilibrium of horizontal and vertical forces on link DAB ,

We get, $(R_A)_H = (R_B)_H = 2100 \text{ N}$

and $(R_A)_V = 600 + (R_B)_V \quad \dots(i)$

Again, considering forces acting on link BC ,

$$(R_C)_H = (R_B)_H = 2100 \text{ N}$$

Taking moments about pin C ,

$$(R_B)_V \times 200 = 0$$

$$\therefore (R_B)_V = 0$$

Hence, $(R_C)_V = (R_B)_V = 0$

On putting the value of $(R_B)_V$ in equation (i)

$$(R_A)_V = 600 \text{ N}$$

Considering the free body diagram of forces acting on the right side link CF and taking moment of forces about pin F,

$$(R_C)_H (350 + 450) - N_R (350) - \mu N_R (150) = 0$$

$$2100 \times (350 + 450) - N_R (350) - 0.3 \times N_R (150) = 0$$

$$N_R = 4253.16 \text{ N}$$

Similarly, taking moments of forces about pin E of the left side link AE,

$$(R_A)_H (350 + 350) + \mu N_L (150) - N_L (350) = 0$$

$$2100 \times (700) + 0.3 \times N_L \times 150 - N_L \times 350 = 0$$

$$N_L = 4819.67 \text{ N}$$

(i) Torque absorbing capacity of brake is given by

$$T = \mu(N_R + N_L)R$$

$$= 0.3(4253.16 + 4819.67) \times 0.25$$

$$= 680.46 \text{ Nm}$$

(ii) Dimension of block

As

$$N_L > N_R$$

\therefore

$$N_L = p \times l \times w$$

$$4819.67 = 1.2 \times 2w \times w$$

$$w^2 = 2008.196$$

$$w = 44.813 \text{ mm}$$

and

$$l = 2w = 2 \times 44.813$$

$$= 89.63 \text{ mm}$$

8. (a) Solution:

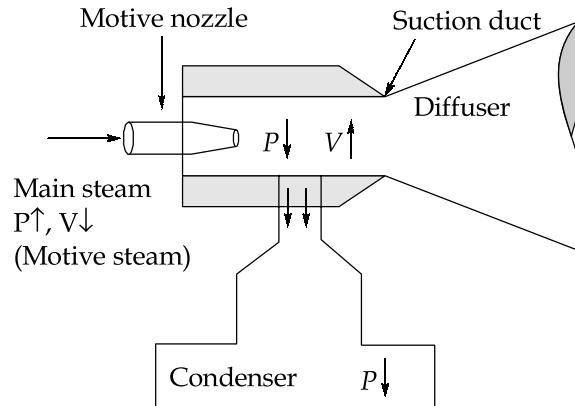
In a steam power plant, it is important to remove non-condensable gases that get accumulated in condenser. Non-condensable gas mostly include air that leaks through joints into condenser due to negative pressure. It also include gases formed due to decomposition of water into oxygen and hydrogen by thermal action and by chemical reaction between water and materials of construction.

Due to the presence of non-condensable gases, following effects are observed :

1. Condenser pressure increases ($\eta_{\text{cycle}} \downarrow$).

2. Blanket heat transfer surfaces.
3. Problem of chemical reactions like corrosion.

Working



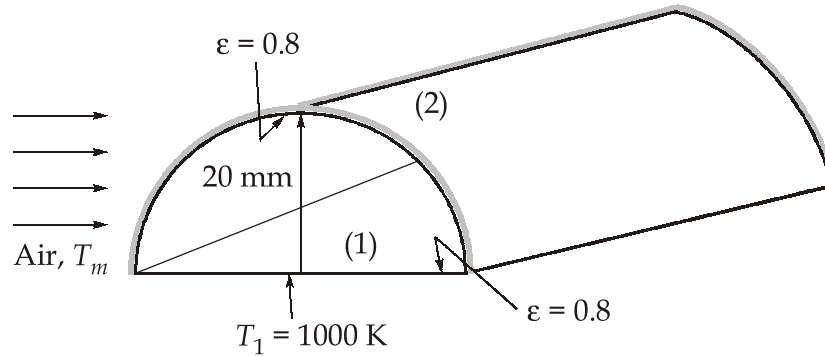
- It uses main steam at a reduced pressure that enters a driving flow nozzle in first stage ejector, from which it exits with high velocity and reduced pressure.
- This reduced pressure draws non condensable gases from condenser and it results in momentum exchange, gases are carried away by steam jet.
- The combined flow of steam and gases is now compressed in diffuser ($P\uparrow$) of the first stage of ejector and discharge into a small intercondenser, where steam is condensed. Cooling is here done by main condensate or part of feed water heating system ($\eta_{\text{cycle}}\uparrow$).
- The non-condensable gases and remaining steam are then passed to second stage of ejector where they are compressed and passed to an after condenser (known as vent condenser).
- The steam, if any, gets condensed and non-condensable gases (air) at higher pressure than atmospheric pressure is vented out.

Application of Air Ejectors :

1. **Steam Jet Air Ejector** : It is one of the types of air ejector which is used in the steam like near the condenser to remove the non condensable gases and some vapour entering into main condenser by an air ejector and it is cooled by the main condensate and released in the ejector condenser.
2. **Fresh Water Generator** : The next main application of the air ejector in marine field is in fresh water generator as it is used to remove the air and non condensable in the evaporator chamber so as to maintain the vacuum inside the chamber. Thus the efficiency of the generation increases at low temperature of the sea water.

3. **Self Priming of Centrifugal Pumps :** It is also employed in priming of the centrifugal pumps by the air ejector, which removes the air inside the casing of the pump by the suction effect created by the air ejector thus by flooding casing with the liquid so that it helps in starting of the pump.

8. (b) Solution:



Given: Tube, $r = 20\text{ mm}$, $\epsilon_1 = \epsilon_2 = 0.8$, $\dot{m}_{\text{air}} = 0.01\text{ kg/s}$, $T_m = 400\text{ K}$, $k = 0.0338\text{ W/mK}$, $\mu = 230 \times 10^{-7}\text{ kg/sm}$, $c_p = 1014\text{ J/kgK}$, $\text{Pr} = 0.69$
 For air fully developed flow,

$$\text{Re} = \frac{\rho V D_h}{\mu}$$

Hydraulic diameter, $D_h = \frac{4A_{cs}}{P}$

$$\begin{aligned} \therefore \text{Re} &= \frac{\rho \times V \times 4A_{cs}}{\mu \times P} = \frac{\rho \times V \times A_{cs} \times 4}{\mu \times (\pi r + 2r)} \\ &= \frac{\dot{m} \times 4}{\mu \times 0.02(\pi + 2)} = \frac{0.01 \times 4}{230 \times 10^{-7} \times 0.02(\pi + 2)} = 16912.37 \end{aligned}$$

$$A = \frac{\pi R^2}{2} = \frac{\pi \times (0.02)^2}{2} = 6.283 \times 10^{-4}\text{ m}^2$$

$$\begin{aligned} P &= \pi R + 2R = \pi \times 0.02 + 2 \times 0.02 \\ &= 0.10283\text{ m} \end{aligned}$$

$$D_h = \frac{4A}{P} = \frac{4 \times 6.283 \times 10^{-4}}{0.10283} = 0.02443\text{ m}$$

Nusselt number, $\text{Nu}_D = 0.023 \text{Re}_D^{0.8} \text{Pr}^{0.4}$

$$\text{Nu}_D = 0.023 \times (16912.37)^{0.8} \times (0.69)^{0.4}$$

$$\text{Nu}_D = 47.84$$

$$\frac{hD_h}{k} = 47.84$$

$$h = \frac{47.84 \times 0.0338}{0.02443} = 66.19\text{ W/m}^2\text{ K}$$

Radiation heat transfer between surface 1 and 2.

$$T_1 \bullet \begin{array}{c} R_1 \\ \text{---} \\ \frac{1 - \epsilon_1}{\epsilon_1 A_1} \end{array} \text{---} \begin{array}{c} R_2 \\ \text{---} \\ \frac{1}{A_1 F_{12}} \end{array} \text{---} \begin{array}{c} R_3 \\ \text{---} \\ \frac{1 - \epsilon_2}{\epsilon_2 A_2} \end{array} \bullet T_2$$

Here,

$$\epsilon_1 = 0.8 = \epsilon_2$$

$$L = 1 \text{ m}$$

$$A_1 = 2RL = 2 \times 0.02 \times 1 = 0.04 \text{ m}^2$$

$$A_2 = \pi R L = 3.14 \times 0.02 \times 1 = 0.0628 \text{ m}^2$$

$$R_{th} = \frac{1 - 0.8}{0.8 \times 0.04} + \frac{1}{0.04 \times 1} + \frac{1 - 0.8}{0.8 \times 0.0628}$$

$$R_{th} = 6.25 + 25 + 3.98 = 35.23$$

Since, Surface 2 is insulating and at constant temperature.

Heat received through surface 1 by radiation = Heat lost to air by convection

$$\frac{\sigma(T_1^4 - T_2^4)}{R_{th}} = hA_2(T_2 - T_m)$$

$$\frac{5.67 \times 10^{-8} \times (1000^4 - T_2^4)}{35.23} = 66.19 \times 0.0628 \times (T_2 - 400)$$

On solving,

$$T_2 = 696.22 \text{ K Ans.}$$

Total heat supplied to maintain the plane surface at 1000 K

= Heat taken by air from surface 1 + Heat taken by air from surface 2.

$$= hA_1(T_1 - T_m) + hA_2(T_2 - T_m)$$

$$= 66.19 \times 0.04 \times (1000 - 400) + 66.19 \times 0.0628 \times (696.22 - 400)$$

$$= 2819.86 \text{ W or } 2.819 \text{ kW}$$

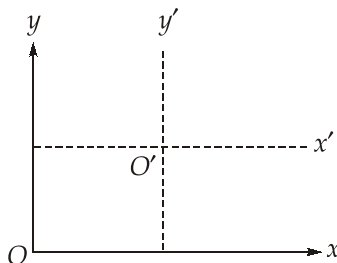
Ans.

8. (c) Solution:

Given : $P_{\max} = 2000 \text{ N}$, $N_{rpm} = 750 \text{ rpm}$, $L_{10h} = 9000 \text{ hr}$

Equivalent load for complete work cycle:

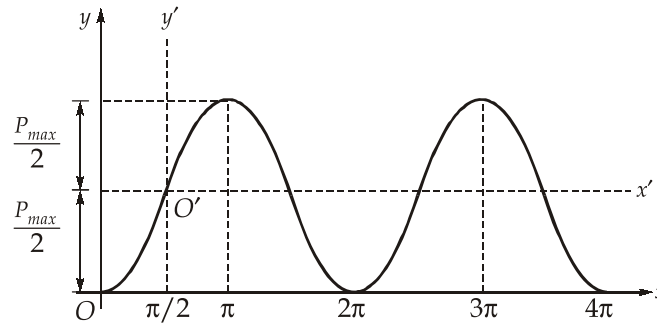
In order to determining the equation of the sinusoidal curve as show in the figure, we use the basic relationship of plane analytical geometry.



The co-ordinate of origin O' w.r.t. origin O are (x_0, y_0) . The relationships for transformation of co-ordinates with pure translation are as follows:

$$x' = x - x_0$$

$$y' = y - y_0$$



The equation of the above curve with respect to (x', y') co-ordinate system is given by

$$y' = \left(\frac{1}{2}P_{\max}\right)\sin x' \quad \dots(i)$$

Also, $x_0 = \frac{\pi}{2}$ and $y_0 = \frac{P_{\max}}{2}$

$\therefore x' = x - \frac{\pi}{2}$ and $y' = y - \frac{P_{\max}}{2}$

On putting the value of x' and y' in equation (i)

We get, $\left(y - \frac{P_{\max}}{2}\right) = \frac{P_{\max}}{2}\sin\left(x - \frac{\pi}{2}\right)$

$$\left(y - \frac{P_{\max}}{2}\right) = -\left(\frac{P_{\max}}{2}\right)\sin\left(\frac{\pi}{2} - x\right) \quad [\text{As } \sin(-\theta) = -\sin\theta]$$

$$\left(y - \frac{P_{\max}}{2}\right) = -\left(\frac{P_{\max}}{2}\right)\cos x$$

$$y = \frac{P_{\max}}{2} - \frac{P_{\max}}{2}\cos x = \frac{P_{\max}}{2}(1 - \cos x)$$

Replacing y by P and x by θ ,

We get, $P = \frac{P_{\max}}{2}(1 - \cos\theta)$

The equation for force P at an angle of rotation θ is given by $P = \frac{1}{2}P_{\max}(1 - \cos\theta)$

then considering the work cycle from $\theta = 0$ to $\theta = 2\pi$

$$\begin{aligned}
 P_e &= \left[\frac{1}{N} \int P^3 dN \right]^{1/3} \\
 &= \left[\frac{1}{2\pi} \int \frac{P_{\max}^3}{8} (1 - \cos \theta)^3 d\theta \right]^{1/3} \\
 &= \frac{P_{\max}}{2} \left[\frac{1}{2\pi} \int_0^{2\pi} (1 - \cos \theta)^3 d\theta \right]^{1/3}
 \end{aligned}$$

Also,

$$\begin{aligned}
 \int (1 - \cos \theta)^3 d\theta &= \int (1 - 3\cos \theta + 3\cos^2 \theta - \cos^3 \theta) d\theta \\
 \int (1 - \cos \theta)^3 d\theta &= \int \left(1 - 3\cos \theta + \frac{3(1 + \cos 2\theta)}{2} - \cos \theta (1 - \sin^2 \theta) \right) d\theta \\
 &= \int (2.5 - 4\cos \theta + 1.5\cos 2\theta + \cos \theta \sin^2 \theta) d\theta
 \end{aligned}$$

$$\begin{aligned}
 \int (1 - \cos \theta)^3 d\theta &= \left[2.5\theta - 4\sin \theta + 0.75\sin 2\theta + \frac{\sin^3 \theta}{3} \right] \\
 \text{Since, } \int_0^{2\pi} (1 - \cos \theta)^3 d\theta &= \left[2.5\theta - 4\sin \theta + 0.75\sin 2\theta + \frac{\sin^3 \theta}{3} \right]_0^{2\pi} \\
 &= \left[2.5 \times 2\pi - 4\sin 2\pi - 0.75\sin 4\pi + \frac{\sin^3 2\pi}{3} - 2.5 \times 0 - 4\sin 0 + 0.75\sin 0^\circ + \frac{\sin^3 0}{3} \right] \\
 \int_0^{2\pi} (1 - \cos \theta)^3 &= 5\pi
 \end{aligned}$$

$$\begin{aligned}
 \therefore P_e &= \frac{P_{\max}}{2} \left[\frac{1}{2\pi} \times 5\pi \right]^{1/3} \\
 P_e &= \frac{P_{\max} \times (2.5)^{1/3}}{2} = \frac{2000 \times (2.5)^{1/3}}{2}
 \end{aligned}$$

$$P_e = 1357.21 \text{ N}$$

As we know,

$$\begin{aligned}
 L_{10} &= \frac{60 \times N_{rpm} \times L_{10h}}{10^6} \\
 &= \frac{60 \times 750 \times 9000}{10^6} = 405 \text{ million revolution}
 \end{aligned}$$

$$\begin{aligned}
 \therefore C &= P_e (L_{10})^{1/3} = 1357.21 \times (405)^{1/3} \\
 C &= 10041.5 \text{ N}
 \end{aligned}$$

