Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 8

Section A : Control Systems + Analog Circuits

Q.1 (a) Solution:

(i) Using the concept of virtual ground, $V^- = V^+ = 0$. From the figure:

$$R_1 = R$$

R_2 can be calculated as:

$$R_2 = (R \| R) + \frac{R}{2} = R$$

$$R_3 = (R_2 \| R) + \frac{R}{2} = R$$

$$R_4 = (R_3 \| R) + \frac{R}{2} = (R \| R) + \frac{R}{2} = R$$

(ii) As, $V = -IR$

and $V_1 = -I_1R$

Thus, $I_1 = I$

So, $I' = I + I = 2I$

Using KVL in loop-1,

$$V_1 - 2I\left(\frac{R}{2}\right) = -RI_2$$

$$IR + IR = RI_2$$

So, $I_2 = 2I$

and $I'' = I_2 + I' = 4I$

Using KVL in loop-2,

$$V_2 - 4I \times \frac{R}{2} = -RI_3$$

$$R \times 2I + 4I \times \frac{R}{2} = RI_3$$

hence, $I_3 = 4I$

and $I_4 = -[4I + 4I]$

$= -8I$

(iii) $V_1 = -I_1R = -IR$

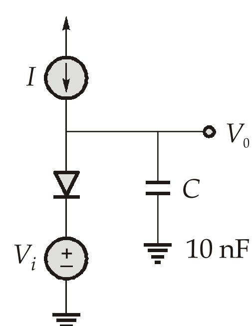
$V_2 = -I_2R = -2IR$

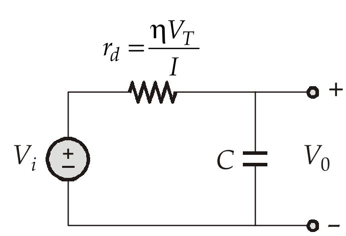
$V_3 = -I_3R = -4IR$

$$V_4 = -I_3R + I_4\frac{R}{2} = -4IR - \frac{8IR}{2} = -8IR$$

Q.1 (b) Solution:

To get small signal equivalent circuit, we consider the DC current source as open-circuit. Thus, the small signal equivalent circuit is obtained as:





From the circuit,

$$\frac{V_0}{V_i} = \frac{1/sC}{\frac{1}{sC} + r_d} = \frac{1}{1 + sCr_d}$$

$$\text{Phase shift} = -\tan^{-1}\left(\frac{\omega C r_d}{1}\right)$$

$$= -\tan^{-1}\left(2\pi \times 10^5 \times 10 \times 10^{-9} \times \frac{0.025}{I}\right)$$

As we know that the phase shift is –45°

So $$-45° = -\tan^{-1}\left(2\pi\times10^{5}\times10\times10^{-9}\times\frac{0.025}{I}\right)$$

$$2\pi\times10^{-3}\times\frac{0.025}{I} = 1$$

$$I = 157\ \mu A$$

Given that, I is varied over the range of 0.1 to 10 times of 157 μA.

Hence,Range of phase shift for I varying from 15.7 μA to 1570 μA is obtained as:

For $I = 15.7\ \mu A$,

$$\Rightarrow \quad \phi_1 = -\tan^{-1}(\omega C r_d) = -\tan^{-1}\left[2\pi\times10^{-3}\times\frac{0.025}{15.7\ \mu A}\right] = -84.3°$$

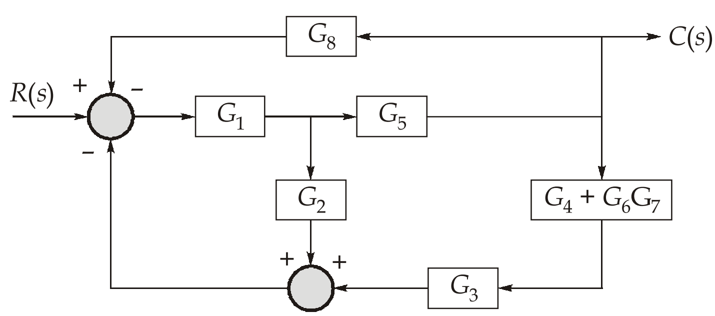
For $I = 1570\ \mu A$,

$$\Rightarrow \quad \phi_2 = -\tan^{-1}\left[2\pi\times10^{-3}\times\frac{0.025}{1570\ \mu A}\right] = -5.71°$$

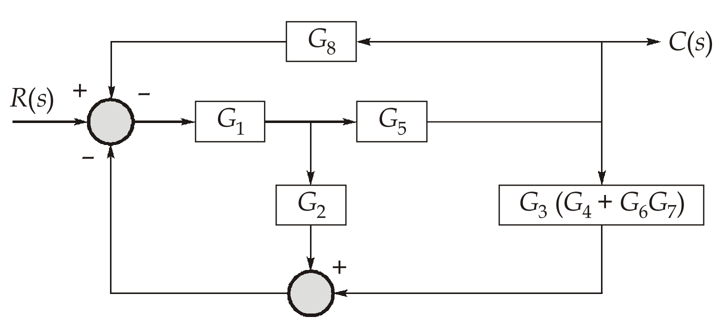
So, the range of phase shift achieved is –84.3° to –5.71°.

Q.1 (c) Solution:

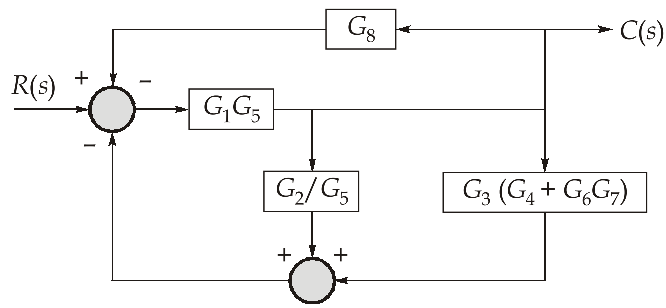
On combining blocks G_6 and G_7 and adding the forward path block G_4, we get,



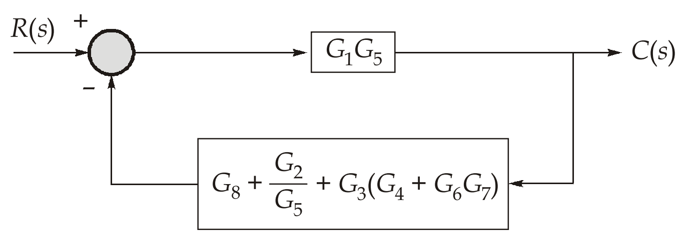
Combining G_3 and $(G_4 + G_6G_7)$ gives



Shifting block G_5 before the take-off point, we get



Adding the blocks in parallel path,



$$\frac{C(s)}{R(s)} = \frac{G_1G_5}{1+G_1G_5\left[G_8+\frac{G_2}{G_5}+G_3(G_4+G_6G_7)\right]}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_5}{1+G_1G_5G_8+G_1G_2+G_1G_5G_3G_4+G_1G_5G_3G_6G_7}$$

Q.1 (d) Solution:

Given: Open loop transfer function (OLTF) as $G(s)H(s)=\dfrac{K}{(s+2)^2\,(s+3)}$

To determine the value of K:

(i) Position error constant $K_p \geq 2$

As we know, position error constant K_p is given by

$$K_p = \lim_{s\to 0} G(s)H(s)$$

So, $$K_p = \lim_{s\to 0}\frac{K}{(s+2)^2\,(s+3)} = \frac{K}{12}$$

As given, $$K_p \geq 2$$

$\Rightarrow$ $$\frac{K}{12} \geq 2$$

So, $$K \geq 24$$

(ii) Gain margin ≥ 3

$$G(s)H(s) = \frac{K}{(s^2+4s+4)(s+3)} = \frac{K}{s^3+7s^2+16s+12}$$

Substituting $s = j\omega$,

$$\Rightarrow \quad G(j\omega)H(j\omega) = \frac{K}{-j\omega^3-7\omega^2+j16\omega+12} = \frac{K}{j(16\omega-\omega^3)+(12-7\omega^2)}$$

For imaginary part to be zero,

$$16\omega - \omega^3 = 0$$

$$\omega(16-\omega^2) = 0$$

$$\Rightarrow \quad \omega = 0, 4 \text{ rad/sec}$$

So, $\omega = 4$ rad/sec is phase cross-over frequency, ω_{pc}.

$$\text{Now, } G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}=4\text{ rad/sec}} = \frac{K}{12-7(4)^2} = -\frac{K}{100}$$

$$\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}} = \frac{K}{100}$$

Gain margin,

$$GM = \frac{1}{\left|G(j\omega)H(j\omega)\right|_{\omega=\omega_{pc}}} = \frac{1}{K/100} = \frac{100}{K}$$

As per question,

$$GM \geq 3$$

$$\Rightarrow \quad \frac{100}{K} \geq 3 \quad \Rightarrow K \leq \frac{100}{3} = 33.33$$

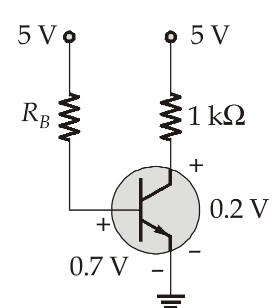
$$\Rightarrow \quad K \leq 33.33$$

So, the value of K satisfying above both conditions is given by

$$24 \leq K \leq 33.33$$

Q.1 (e) Solution:

$$\text{Over drive factor} = 10 = \frac{\beta_F}{\beta_{Forced}}$$



where β_{forced} is the ratio of collector current to the actual base current, when in saturation and β_F refers to the minimum current gain of the transistor in active region used to estimate the minimum base current required to drive the transistor into saturation.

$$\beta_{forced} = \frac{\beta_F}{10} = \frac{20}{10} = 2$$

When transistor is in saturation,

$$V_{CE\ sat} = 0.2\ \text{V}$$

By using KVL,

$$5 - I_{C\ sat} \times 1 = 0.2$$

$$I_{C\ sat} = 4.8\ \text{mA}$$

and

$$I_B = \frac{I_{C\,sat}}{\beta_{forced}} = \frac{4.8}{2} = 2.4\ \text{mA}$$

Hence,

$$R_B = \frac{5 - V_{BE}}{I_B} = \frac{5-0.7}{2.4} = 1.8\ \text{k}\Omega$$

Q.2 (a) Solution:

Given,

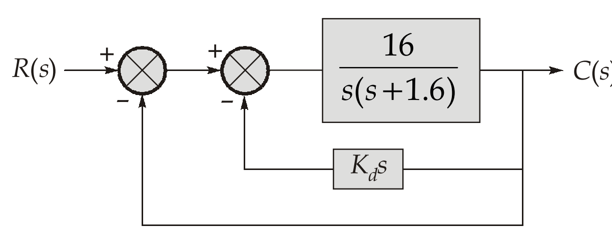
$$\frac{C(s)}{R(s)} = T(s) = \frac{16}{s^2 + 1.6s + 16}$$

Now for given control system,

OLTF:

$$G(s)\,H(s) = \frac{T(s)}{1 - T(s)} = \frac{16}{s(s+1.6)}$$

With derivative feedback constant 'K_d',
The control system can be represented as:



Now for compensated control system,
Open loop T/F:

$$\frac{\dfrac{16}{s(s+1.6)}}{1 + K_d s\left(\dfrac{16}{s(s+1.6)}\right)} = \frac{16}{s^2 + 1.6s + 16K_d s}$$

$$\therefore \quad \text{OLTF} = \frac{16}{s(s + 1.6 + 16K_d)}$$

$$\therefore \quad \left.\frac{C(s)}{R(s)}\right|_{\text{With derivative feedback}} = \frac{16}{s^2 + (1.6 + 16K_d)s + 16} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Thus, $\omega_n^2 = 16 \Rightarrow \omega_n = 4$

$$2\xi\,\omega_n = 1.6 + 16\,K_d$$

For $\xi = 0.8$ (given in problem),

$$K_d = \frac{2\times0.8\times4-1.6}{16} = 0.3$$

Without "K_d"

$$\frac{C(s)}{R(s)} = \frac{16}{s^2+1.6s+16}$$

$\therefore$ $\omega_n = 4$ rad/sec

$$\xi = 0.2$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 4\sqrt{1-0.04} = 3.92 \text{ rad/sec.}$$

Now,

(i) Rise time, $$\text{"}t_r\text{"} = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}\xi}{\omega_d} = \frac{\pi-\cos^{-1}0.2}{3.92}$$

$\therefore$ $t_r = 0.452$ sec.

(ii) Peak time, $$\text{"}t_p\text{"} = \frac{\pi}{\omega_d}$$

$\therefore$ $t_p = 0.8014$ sec.

(iii) Maximum overshoot (M_p):

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.5266$$

(iv) Steady state error for unit ramp input,

$$G(s)\,H(s) = \frac{16}{s(s+1.6)}$$

It is a type-1 system,

For ramp input, $$e_{ss} = \frac{A}{K_v}$$

where $$K_v = \lim_{s\to0} s\,G(s)\,H(s)$$

$\therefore$ $K_v = 10$

For unit ramp input, $A = 1$

$\therefore$ $$e_{ss} = \frac{1}{10}$$

With "K_d" = 0.3

$$\frac{C(s)}{R(s)} = \frac{16}{s^2+6.4s+16}$$

Here, ω_n = 4 rad/sec

ξ = 0.8

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 4\sqrt{1-0.64} = 4\times 0.6$$

= 2.4 rad/sec.

(i) Rise time, $$"t_r" = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}\xi}{\omega_d} = \frac{\pi-\cos^{-1}0.8}{2.4}$$

∴ t_r = 1.04 sec.

(ii) $$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.4} = 1.309 \text{ sec.}$$

(iii) $$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{\frac{-0.8\pi}{0.6}} = 0.01516$$

We have, $$G(s)H(s) = \frac{16}{s(s+6.4)}$$

It is a type-1 system, thus

$$K_v = \lim_{s\to 0} sG(s)H(s) = 2.5$$

For unit ramp input, A = 1

∴ $$e_{ss} = \frac{A}{K_v} = \frac{1}{2.5} = 0.4$$

Q.2 (b) Solution:

Given, β = 200

Hence $$\alpha = \frac{\beta}{\beta+1} = \frac{200}{201} = 0.995$$

$I_C = \alpha I_E$ = 0.995 × 10 mA = 9.95 mA

$V_C = I_C R_C$ = 9.95 mA × 100 = 0.995 V

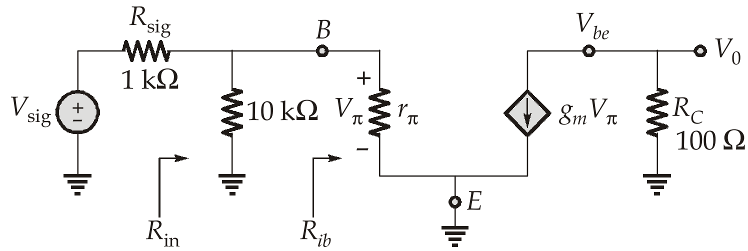
$$I_B = \frac{I_E}{\beta+1} \approx \frac{10 \text{ mA}}{200} = 0.05 \text{ mA}$$

∴ V_B = 1.5 – 10k (0.05 m) = 1 V

V_{BC} = 0.005 V = +ve

Hence, BJT is in active region

By replacing it by π-model, we get the small signal equivalent of the circuit as:



$$r_\pi = \frac{\beta}{g_m}$$

and $$g_m = \frac{I_C}{V_T} = \frac{9.95\text{ m}}{25\text{ m}} \approx 0.4 \text{ A/V}$$

So, $$r_\pi = \frac{200}{0.4} = 500\ \Omega$$

We have, $$R_{ib} = r_\pi = 500\ \Omega$$

$$R_{in} = 10k \,||\, (r_\pi)$$

$$R_{in} = 476\ \Omega$$

Using voltage division rule, $$V_\pi = V_{sig} \times \frac{R_{in}}{R_{sig} + R_{in}} = V_{sig} \times \left(\frac{476}{476 + 1000}\right)$$

$$V_\pi = 0.32\ V_{sig}$$

and $$V_0 = -g_m V_\pi R_C$$

$$= -g_m (R_C)\,[0.32\ V_{sig}]$$

$$= -0.4 \times 100 \times 0.32\ V_{sig}$$

$$V_0 = -12.8\ V_{sig} \quad \text{...(i)}$$

Thus, Voltage gain, $$A_v = \frac{V_0}{V_{sig}} = -12.8 \text{ V/V}$$

If $$V_0 = \pm 0.4 \text{ V}$$

$$V_{sig} = \frac{0.4}{12.8} = 31.25 \text{ mV}$$

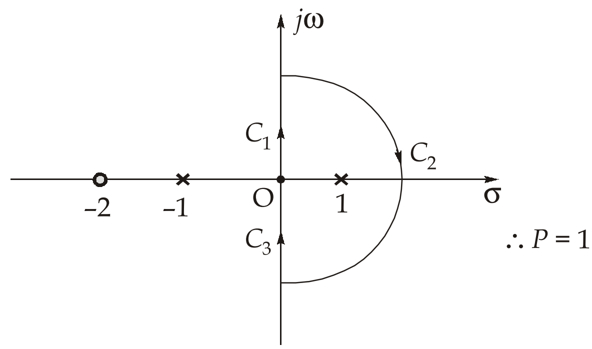
Thus, $$V_\pi = V_b = 0.32 \times (31.25 \text{ m})$$

$$V_b = 10 \text{ mV}$$

Q.2 (c) Solution:

(i)
$$G(s)\,H(s) = \frac{s+2}{(s+1)(s-1)}$$

The Nyquist contour in s-plane is



Nyquist plot in $G(s)\,H(s)$ plane is obtained as below,

Corresponding to $C_1 : s = j\omega\,;\ \omega : 0 \to \infty$

$$G(j\omega)\,H(j\omega) = \frac{j\omega+2}{(j\omega+1)(j\omega-1)}$$

We have,
$$M = \frac{\sqrt{\omega^2+4}}{\omega^2+1}$$

and
$$\phi = \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\omega) - 180^\circ + \tan^{-1}(\omega)$$

Angular frequency	M	$\angle G(j\omega)\,H(j\omega)$
$\omega = 0^+$	2	–180°
$\omega = \infty^+$	0	–90°

Corresponding to $C_3 : s = -j\omega\,;\ \omega: \infty \to 0$

$$G(-j\omega)\,H(-j\omega) = \frac{2-j\omega}{(1-j\omega)(-1-j\omega)}$$

Here,
$$M = \frac{\sqrt{4+\omega^2}}{(\omega^2+1)}$$

and
$$\phi = -\tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}(\omega) - 180^\circ - \tan^{-1}(\omega)$$

Angular frequency	M	$\angle G(j\omega)\,H(j\omega)$
$\omega = 0^-$	2	–180°
$\omega = \infty^-$	0	–270°

Corresponding to C_2 :

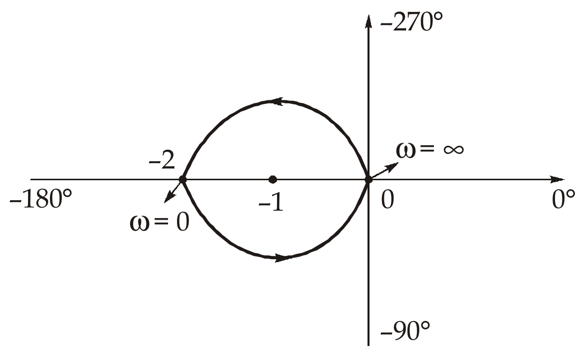
$$G(s)\,H(s) = \lim_{R\to\infty} R\,e^{j\theta};\ \theta \text{ varies from } +90° \text{ to } -90°$$

So,
$$G(s)\,H(s) = \lim_{R\to\infty} \frac{(R\,e^{j\theta}+2)}{(R\,e^{j\theta}+1)\,(R\,e^{j\theta}-1)}$$

$$G(s)\,H(s) = \lim_{R\to\infty} \frac{1}{R}e^{-j\theta}$$

Thus, $M = 0$; $\phi = -\theta$ and thus, varies from –90° to 90°.

The Nyquist plot in $G(s)\,H(s)$ plane is obtained as below:



Number of encirclement, of $(-1 + j0)$ point in anticlockwise direction $(N) = 1$

Number of open loop poles on right half of s-plane $(P) = 1$

So, $N = P - Z$

or $Z = 0$

where, Z is the number of closed loop poles on the right hand side of s-plane. Thus, the closed loop system is stable.

(ii) The observability matrix for a control system is given by,

$$Q_0 = [C^T\ A^TC^T\ \ldots(A^T)^{n-1}C^T]$$

where n = number of states = 3 (given). We have,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$A^T(A^T C^T) = \begin{bmatrix} -6 \\ -11 \\ -5 \end{bmatrix}$$

$$\text{Observability Matrix, } Q_0 = \begin{bmatrix} 1 & 0 & -6 \\ 1 & 1 & -11 \\ 0 & 1 & -5 \end{bmatrix}$$

$$|Q_0| = 1[(1 \times -5) - (1 \times -11) - 0 + (-6)(1 - 0)]$$
$$= -5 + 11 - 6 = 0$$

Since determinant is zero, rank is not 3 and hence, the system is not completely observable.

The roots of the characteristic equation are given by $|\lambda I - A| = 0$ i.e.

$$\begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$

$$\Rightarrow \quad (\lambda + 1)(\lambda + 2)(\lambda + 3) = 0$$

Thus, Eigen values are –1, –2 and –3.

The Vandermonde matrix formed by the eigen values is,

$$M = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} = \text{Vandermonde matrix}$$

$$X = CM = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 \end{bmatrix}$$

$$y = CMZ = \begin{bmatrix} 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$z_1(t)$ is not observable, a mode due to eigen value $\lambda = -1$.

Q.3 (a) Solution:

We can write,

$$K_n = \frac{\mu_n C_{ox} W_n}{L_n} = K'_n \frac{W_n}{L_n} = (120)\frac{(1.2)}{0.8} = 180\ \mu\text{A/V}^2$$

$$K_p = \frac{\mu_p C_{ox} W_p}{L_p} = K'_p \frac{W_p}{L_p} = (60)\frac{(2.4)}{0.8} = 180\ \mu\text{A/V}^2$$

$$\therefore \quad K'_n \frac{W_n}{L_n} = K'_p \frac{W_p}{L_p}$$

(i) When $V_0 = V_{OL}$ [V_0 is low], input is high ($V_I = V_{GS,n} = V_{DD}$), NMOS is ON and in the linear region. Thus,

$$R_{on,n} = \frac{1}{K'_n \left(\frac{W}{L}\right)_n (V_{DD} - V_{tn})} = \frac{1}{120\times10^{-6}\left(\frac{1.2}{0.8}\right)(3-0.7)}$$

$$= 2.4\ \text{k}\Omega$$

When $V_0 = V_{OH}$, input V_I is low, so PMOS is ON and in the linear region. Thus,

$$r_{on,p} = \frac{1}{K'_p \left(\frac{W}{L}\right)_p \left(V_{DD} - |V_{tp}|\right)} = \frac{1}{60\times10^{-6}\left(\frac{2.4}{0.8}\right)(3-0.7)}$$

$$= 2.4\ \text{k}\Omega$$

(ii) The inverter sinks current through the NMOS and sources current through the PMOS. For the output to remain within 0.1 V of ground, $V_{DS,n} \le 0.1$ V. Consider $V_{GS} = V_{DD}$ (logic 1) such that NMOS in ON and $V_{DS,n} = 0.1$ V to determine the maximum current that the inverter can sink. Since NMOS is operating in linear region, we have

$$I_D = K_n\left[(V_{GS}-V_{tn})V_{DS,n}-\frac{V_{DS,n}^2}{2}\right]$$

$$I_D = 180\times10^{-6}\left[(3-0.7)0.1-\frac{0.01}{2}\right]$$

$$I_D = 40.5\ \mu\text{A}$$

Due to symmetry ($K_n = K_p$), the maximum source current while the output remains within 0.1V of V_{DD} also remains the same.

(iii) For a matched inverter ($K_n = K_p$ and $V_{tn} = |V_{tp}| = V_t$),

$$V_{IH} = \frac{1}{8}(5V_{DD}-2V_t)$$

$$= \frac{1}{8}(5\times3-2\times0.7) = 1.7\ \text{V}$$

$$V_{IL} = \frac{1}{8}(3V_{DD}+2V_t) = 1.3\ \text{V}$$

$$NM_H = V_{OH} - V_{IH} = 3 - 1.7 = 1.3\ \text{V}$$

$$NM_L = V_{IL} - V_{OL} = 1.3\ \text{V} - 0 = 1.3\ \text{V}$$

Q.3 (b) Solution:

(i) Using Routh's array table,

$$\begin{array}{llll}
s^4 & b_0 & b_2 & b_4 \\
s^3 & b_1 & b_3 & \\
s^2 & \dfrac{b_1b_2-b_0b_3}{b_1} & b_4 & \\
s^1 & \dfrac{\left[\dfrac{b_1b_2-b_0b_3}{b_1}\right]b_3-b_1b_4}{\left[\dfrac{b_1b_2-b_0b_3}{b_1}\right]} & & \\
s^0 & b_4 & &
\end{array}$$

For system to be stable, there should be no sign change in the first column of Routh Array. Thus, the stability conditions are given by

$$b_0 > 0;\ b_1 > 0;\ b_1b_2 - b_0b_3 > 0;$$

$$(b_1b_2-b_0b_3)b_3-b_1^2b_4 > 0$$

and $$b_4 > 0$$

(ii) For the given characteristic equation, Routh's Array can be drawn as below,

s^5	1	1	4
s^4	1	1	4
s^3	0(4)	0(2)	
s^2	$\frac{4-2}{4} = \frac{1}{2}$	4	
s^1	$\frac{1-16}{0.5} = -30$		
s^0	4		

Using Auxiliary equation, we get

$$A(s) = s^4 + s^2 + 4 = 0$$

$$\frac{dA(s)}{ds} = 4s^3 + 2s$$

From the Routh-Hurtwiz table, since sign change occurs in the first column of Routh Array. Hence, the system is unstable. Further, we conclude that

- two poles are in right side of s-plane [since there are two sign changes in the first column of Routh Array].
- four poles are complex in nature [as s^3 row is zero].

Complex poles are given by:

$$s^4 + s^2 + 4 = 0 \quad \Rightarrow \quad s^2 = -0.5 \pm 1.936i = 2\angle\pm 104.48°$$

We get $s = \pm 1.414\angle\pm 52.24°$. Therefore,

$$s = (0.866 + 1.117i);\ (0.866 - 1.117i)$$

$$s = (-0.866 + 1.117i);\ (-0.866 - 1.117i)$$

Now,

the characteristic equation can be represented as

$$(s + a)(s^4 + s^2 + 4) = 0$$

$$s^5 + s^3 + 4s + as^4 + as^2 + 4a = 0$$

$$s^5 + as^4 + s^3 + as^2 + 4s + 4a = 0$$

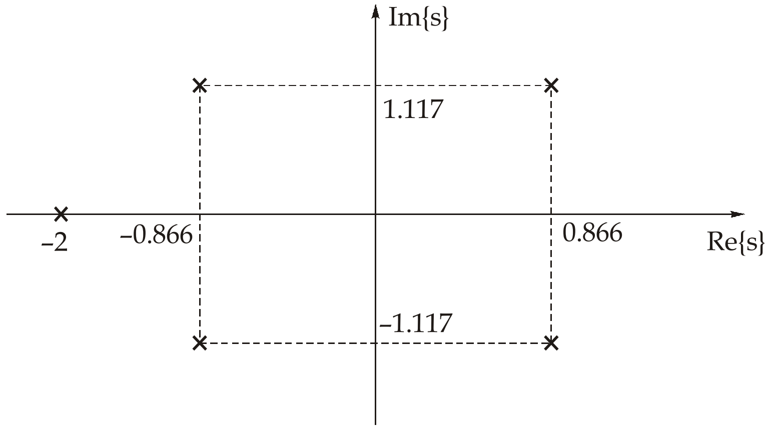
and on comparing with the given characteristic equation, we get, $a = 1$

Thus, Fifth pole lies at $s = -a$ i.e. at

$$s = -1$$

Hence, poles are at $s = (0.866 + 1.117i),\ (0.866 - 1.117i),\ (-0.866 + 1.117i),\ (-0.866 - 1.117i)$ and -1

Thus, the poles can be represented in the s-plane as below:



Q.3 (c) Solution:

Given, $$A(s) = \frac{1000}{\left(1+\frac{s}{10^4}\right)\left(1+\frac{s}{10^5}\right)^2}$$

and feedback factor β is independent of frequency

We have, $$\angle A(j\omega) = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$$

When phase shift is 180°, then

$$180° = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$$

Using: $$2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

We get, $$-180° = \tan^{-1}\left(\frac{\omega}{10^4}\right) + \tan^{-1}\left(\frac{2(\omega/10^5)}{1-(\omega/10^5)^2}\right)$$

Taking the tangent on both sides,

$$0 = \tan^{-1}\left[\frac{\frac{\omega}{10^4} + \frac{2\left(\frac{\omega}{10^5}\right)}{1-\left(\frac{\omega}{10^5}\right)}}{1-\left(\frac{\omega}{10^4}\right)\left(\frac{2\left(\frac{\omega}{10^5}\right)}{1-\left(\frac{\omega}{10^5}\right)^2}\right)}\right]$$

$$\frac{1}{10^4} + \frac{\frac{2}{10^5}}{1-\left(\frac{\omega}{10^5}\right)^2} = 0$$

$$1-\left(\frac{\omega}{10^5}\right)^2 = -0.2$$

$$\omega^2 = 1.2 \times 10^{10} \quad \Rightarrow \quad \omega = \omega_{pc} = 1.1 \times 10^5 \text{ rad/sec}$$

For oscillations to occurs

$$\left|A(j\omega_{pc})\beta\right| \geq 1$$

$$\frac{10^3\beta}{\sqrt{1+\left(\frac{\omega_{pc}}{10^4}\right)^2}\left(1+\left(\frac{\omega_{pc}}{10^5}\right)^2\right)} \geq 1$$

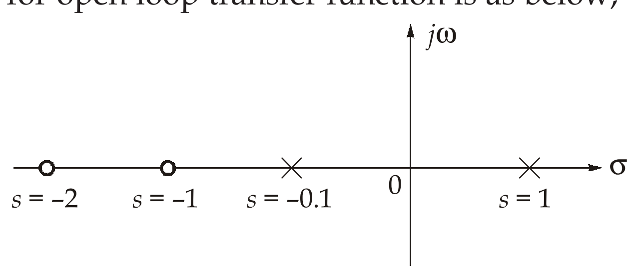
$$\frac{\beta\, 10^3}{\left(\sqrt{1+11^2}\right)\left(1+1.1^2\right)} \geq 1$$

$$\Rightarrow \qquad \beta \geq 0.0244$$

Q.4 (a) Solution:

(i) The given open-loop transfer function has two poles at $s = -0.1, 1$ and two zeros at $s = -1$, $s = -2$, therefore it will have two root locus branches, originating from open-loop poles and terminating at open-loop zeros.

The pole zero plot for open loop transfer function is as below,



(ii) The root locus on real axis will exist between $s = -2$ and -1 and between $s = 1$ and -0.1, as total number of open loop poles and zeros right to it is odd.

(iii) Intersection with imaginary axis:

Characteristic equation of the system is

$$1 + GH(s) = 0\,; \quad \text{So,} \quad 1+\frac{K(s+1)(s+2)}{(s+0.1)(s-1)} = 0$$

$(s + 0.1)(s - 1) + K(s + 1)(s + 2) = 0$

$s^2 - 0.9s - 0.1 + K(s^2 + 3s + 2) = 0$

$s^2(1 + K) + s(3K - 0.9) + 2K - 0.1 = 0$

Routh array of of the system is as given below,

$$\begin{array}{c|cc} s^2 & 1+K & 2K-0.1 \\ s^1 & 3K-0.9 & 0 \\ s^0 & 2K-0.1 & 0 \end{array}$$

Equating s^1 row to zero, we get $K = 0.3$. For $K = 0.3$, the intersection point of the root locus plot with the imaginary axis is obtained by the auxiliary equation,

$$(1 + K)s^2 + (2K - 0.1) = 0$$

$$1.3s^2 = 0.5$$

$$\Rightarrow \quad s = \pm j0.62$$

(iv) Break away point:

Break away points are found by equating $\frac{dK}{ds}$ to zero

We have, $$K = -\frac{(s+0.1)(s-1)}{(s+1)(s+2)} = -\frac{s^2-0.9s-0.1}{s^2+3s+2}$$

So, by solving $\frac{dK}{ds} = 0$, we get

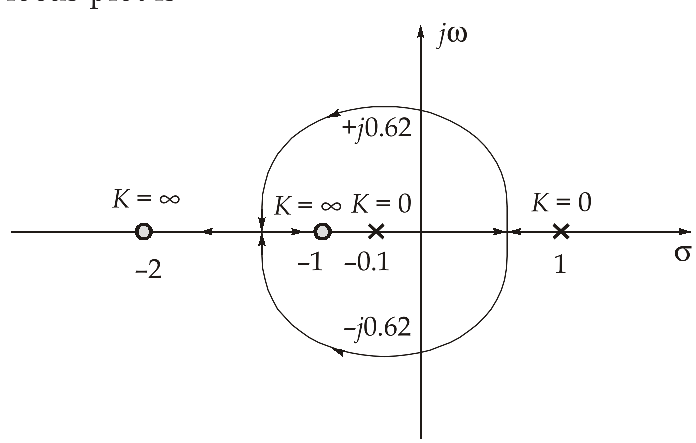
$(2s + 3)(s^2 - 0.9s - 0.1) - (2s - 0.9)(s^2 + 3s + 2) = 0$

$(2s^3 + 1.2s^2 - 2.9s - 0.3) - (2s^3 + 5.1s^2 + 1.3s - 1.8) = 0$

$$3.9s^2 + 4.2s - 1.5 = 0$$

Thus, $s = 0.282$ (break-away point), -1.36 (break-in point)

Thus the root locus plot is



The system is critically damped when both the poles are real equal and negative i.e. at $s = -1.36$.

$$\therefore \qquad K_{\text{critical}} = \frac{\text{Vector product of distance of poles}}{\text{Vector product of distance of zeros}}$$

$$= \frac{\Pi|s-p_i|}{\Pi|s-z_i|}$$

$$= \frac{2.36\times1.26}{0.36\times0.64} = 12.90 \approx 13$$

Q.4 (b) Solution:

(i) For transistor Q_1, drain is shorted to Gate, thus $V_{DS} > V_{GS} - V_t$. Hence, Q_1 is in saturation region. Thus, the drain current is given by,

$$i_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2$$

To establish a current of i_{D1}=0.2 mA in Q_1, we can write

$$0.2 \times 10^{-3} = \frac{1}{2}(200\times10^{-6})\frac{8}{0.8}\left[V_{GS_1}-0.6\right]^2$$

$$V_{GS_1}-0.6 = \sqrt{0.2}$$

$$V_{GS_1} = 1.05 \text{ V}$$

So,
$$R_1 = \frac{V_{DD}-V_{GS_1}}{i_{D1}} = \frac{3-1.05}{0.2\times10^{-3}} = 9.75 \text{ k}\Omega$$

(ii) For Q_2 to conduct at 0.5 mA and drain voltage of 1 V,

$$R_2 = \frac{V_{DD}-V_D}{i_{D2}} = \frac{3-1}{0.5} = 4 \text{ k}\Omega$$

Since the gates are connected together, hence both transistors have same V_{GS}. Thus, we have

$$\frac{i_{D2}}{i_{D1}} = \frac{\frac{\mu_n C_{ox}}{2}\left(\frac{W_2}{L_2}\right)(V_{GS_2}-V_t)^2}{\frac{\mu_n C_{ox}}{2}\left(\frac{W_1}{L_1}\right)(V_{GS_1}-V_t)^2}$$

Since $V_{GS_2} = V_{GS_1}$ and $L_2 = L_1$, thus

$$\frac{0.5\text{ mA}}{0.2\text{ mA}} = \frac{W_2}{W_1}$$

$$\frac{W_2}{W_1} = 2.5$$

thus, $$W_2 = 2.5 \times W_1$$

$$W_2 = 2.5 \times 8 = 20\ \mu\text{m}$$

Q.4 (c) Solution:

(i) The limitations of Routh-Hurwitz criterion are as:

1. It gives no information about the relative stability of the system. It only tells whether the system is stable or not, but does not provide information about how stable (degree of stability) is the system or the location of the poles.
2. It provides no information on how a system can be compensated.
3. It is applicable only to systems with real and constant coefficients in the characteristic equation.
4. If a zero appears in the first column of the Routh array, special procedures are required, which can complicate the analysis.

Nyquist Analysis: Nyquist criterion is used to identify the presence of roots of characteristic equation of a control system in a specified region of s-plane.

In this, open loop transfer function $G(s)\,H(s)$ is considered instead of closed loop characteristic equation $1 + G(s)\,H(s) = 0$.

Moreover, inspection of graphical plot of $G(s)\,H(s)$ enables to get more information than YES or NO answer of Routh-Hurwitz method pertaining to stability of control systems. This is measured through:

- **Gain Margin (GM):** The factor by which the system gain can be increased before the system becomes unstable.
- **Phase Margin (PM):** It indicates how much additional phase lag the system can tolerate before it becomes unstable.

(ii) Given, OLTF: $$G(s) = \frac{K(s+13)}{s(s+3)(s+7)}$$

1. By R-H criterion:

   The characteristic equation is given by

$$q(s) = 1 + G(s)\,H(s) = 0$$

$$1+\frac{K(s+13)}{s(s+3)(s+7)} = 0$$

$\Rightarrow s^3 + 10s^2 + 21s + Ks + 13K = 0$

$\Rightarrow s^3 + 10s^2 + (21 + K)s + 13K = 0$

Now, drawing the Routh Array,

$$\begin{array}{c|cc} s^3 & 1 & 21+K \\ s^2 & 10 & 13K \\ s^1 & \dfrac{210+10K-13K}{10} & \\ s^0 & 13K & \end{array}$$

$\therefore$ For stability of system:

$$\frac{210-3K}{10} > 0 \quad \Rightarrow \quad K < 70$$

Also, $\quad 13K > 0 \quad \Rightarrow \quad K > 0$

$\therefore$ For stability, $0 < K < 70$

2. For $K = 1$:

$$q(s) = s^3 + 10s^2 + 22s + 13 = 0 \qquad \text{...(i)}$$

Let, $\quad q(s) = (s + a)(s^2 + bs + c)$

$$= s^3 + bs^2 + cs + as^2 + abs + ac$$

$$q(s) = s^3 + (a + b)s^2 + (ab + c)s + ac \qquad \text{...(ii)}$$

Comparing (i) and (ii),

$$a + b = 10 \qquad \text{...(iii)}$$

$$ab + c = 22 \qquad \text{...(iv)}$$

$$ac = 13 \qquad \text{...(v)}$$

We can write, $\quad b = 10 - a \qquad \text{...(vi)}$

$$c = \frac{13}{a} \qquad \text{...(vii)}$$

Using (vi) and (vii) in (iv),

$$a(10-a) + \frac{13}{a} = 22$$

$$\Rightarrow \quad 10a^2 - a^3 + 13 = 22a$$

$$\Rightarrow \quad a^3 - 10a^2 + 22a - 13 = 0$$

On solving, we get

$$a = 7.19, 1.807, 1$$

Let, $\quad a = 1$

$\therefore \quad b = 9$

$$c = 13$$

$\therefore \quad q(s) = (s + 1)(s^2 + 9s + 13)$

Comparing with standard IInd order characteristic equation,

$$q(s) = s^2 + 2\xi\omega_n s + \omega_n^2$$

we get,

$$\omega_n^2 = 13$$

$$2\xi\omega_n = 9 \Rightarrow \xi = 1.248 \ ; \quad \xi\omega_n = 4.5$$

Let,

$$a = 1.807$$

$$b = 8.193$$

$$c = 7.194$$

$$q(s) = (s + 1.807)(s^2 + 8.193s + 7.194)$$

$$\omega_n^2 = 7.194$$

$$2\xi\omega_n = 8.193 \Rightarrow \xi = 1.527; \quad \xi\omega_n = 4.09$$

Let,

$$a = 7.19$$

$$b = 2.81$$

$$c = 1.808$$

$$q(s) = (s + 7.19)(s^2 + 2.81s + 1.808)$$

$\therefore$

$$\omega_n^2 = 1.808$$

$$2\xi\omega_n = 2.81 \quad \Rightarrow \quad \xi = 1.045; \quad \xi\omega_n = 1.405$$

$\therefore$ For $K = 1$, for all roots of characteristic equation,

$$\xi > 0.5$$

$$\xi\omega_n > 0.5$$

Thus, all roots have a damping factor greater than 0.5.

Section B : Advanced Electronics-1 + Electronic Measurements and Instrumentation-1 Electromagnetics-2 + Basic Electrical Engineering-2

Q.5 (a) Solution:

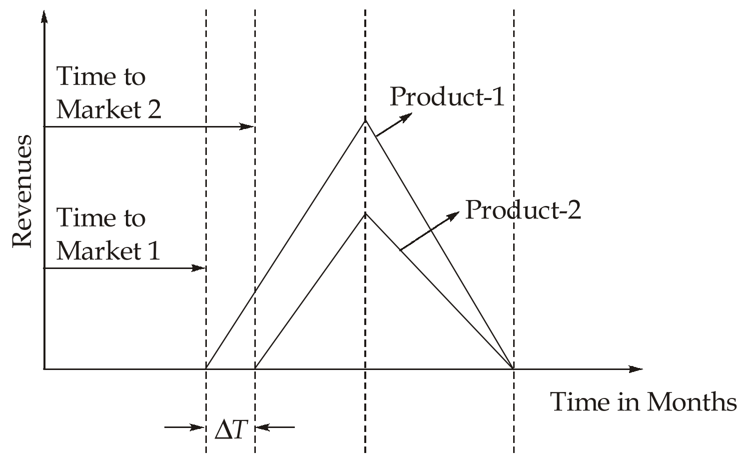
Testing : With the term integrated circuit (IC) or VLSI testing, we refer to those procedures that take place after chip fabrication in order to detect possible manufacturing defects.

Testing Necessity : Imperfections in chip fabrication may lead to manufacturing defects. Manufacturing yield (Y) is given by :

$$Y = \frac{\text{No. of acceptable parts}}{\text{Total no. of fabricated parts}}$$

The manufacturing yield (Y) depends on the used technology, the silicon area and the layout design.

The earlier a defect is detected, the less the cost for the final product.



Reliability and Time to Market :

A reliable product with small time to market will provide higher revenues than a second product with a greater time to market.

Testing procedures at the minimum cost in time and resources are required.

There are several testing methods used to verify VLSI chips:

- **Functional Testing:** Checks if the chip behaves as expected by applying input patterns and comparing outputs to expected results.
- **Structural Testing:** Focuses on the chip's internal circuits and uses techniques like scan chains to test individual components and connections.
- **Built-In Self-Test (BIST):** Incorporates testing hardware within the chip to perform self-testing without external equipment.
- **Automated Test Pattern Generation (ATPG):** Software tools generate efficient test patterns based on fault models to maximize fault coverage.

Defects are circuit failures and malfunctions due to the manufacturing process (e.g., short circuit, opens etc.)

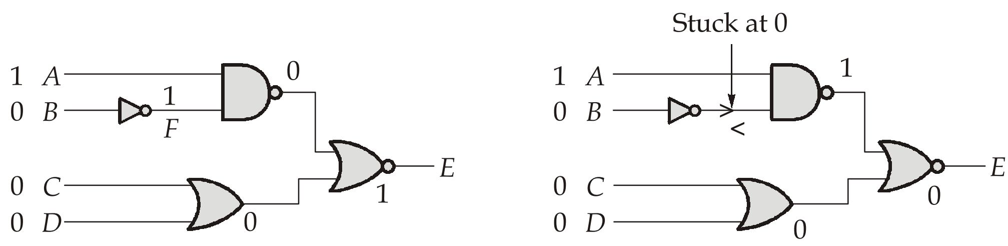
Faults model the influence of defects on the circuit operation (e.g., a line (node) is permanently stuck at "1" or "0").

Errors are the incorrect logic responses of the circuit under the presence of faults.

Testability : It is defined as a measure of the ability to detect the faults of the fault model under consideration in a circuit under test (CUT). It depends on the :

Controllability : It is a measure of ability to set a node in a predefined logic state using proper primary input values.

Observability : It is a measure of the ability to determine the logic state of a node by observing the circuit responses at the primary outputs.



Fault Free Circuit Circuit with stuck at faults

Fault detection is a process where a fault is sensitized by applying proper values at the primary inputs of the circuit, so that an error is generated at a circuit node, and afterwards this error is propagated to a primary output to be observed.

Q.5 (b) Solution:

Given,

$$\text{Dipole length, } l = 1 \text{ m}$$

$$\text{frequency, } f = 150 \text{ MHz}$$

$$\therefore \quad \text{Wavelength, } \lambda = \frac{c}{f} = \frac{3\times10^8}{150\times10^6} = 2 \text{ m}$$

$$\text{Source resistance, } R_a = 50\ \Omega$$

$$\text{Source voltage, } V = 100 \text{ V}$$

$$\text{Load resistance, } R_L = 0.625\ \Omega$$

For half-wave dipole, $R_r = 73\ \Omega$

Thus, antenna resistance, $R_{ant} = R_r + R_L = 73 + 0.625 = 73.625\ \Omega$

Total circuit resistance, $R_{\text{total}} = R_a + R_{ant}$

$$= 50 + 73.625\ \Omega$$

$$\therefore \quad R_{\text{total}} = 123.625\ \Omega$$

(i) $\therefore$ antenna current, $I_{ant} = \dfrac{V}{R_{\text{total}}}$

$$= \frac{100}{123.625}$$

$$I_{ant} = 0.809 \text{ A}$$

(ii) Power dissipated, $P_{\text{loss}} = I^2 R_L$

$$= (0.809)^2 \times 0.625$$

$$= 0.409 \text{ W}$$

(iii) Power radiated, $P_{rad} = I^2 R_r$

$= (0.809)^2 \times 73$

$= 47.8 \text{ W}$

(iv) Radiation efficiency, $\eta = \dfrac{P_{rad}}{P_{rad} + P_{Loss}} = \dfrac{47.8}{47.8 + 0.41}$

$\eta = 0.9915 \quad \text{(or)} \quad 99.15\%$

Q.5 (c) Solution:

(i) We know that, for two wattmeter method, the power factor angle ϕ is determined using the wattmeter readings W_1 and W_2 as,

$$\tan\phi = \frac{\sqrt{3}(W_2 - W_1)}{W_2 + W_1} = \frac{\sqrt{3}(4000 + 2000)}{4000 - 2000} = 3\sqrt{3}$$

i.e., $\phi = 79.1°$

$\therefore$ Power factor, $\cos\phi = \cos(79.1°) = 0.189$

(ii) Total power absorbed in the load

$$W = W_1 + W_2 = 4000 - 2000 = 2000 \text{ W}$$

Since the load is star connected. Then the phase voltage.

$$V_p = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$\text{Power per phase} = \frac{2000}{3} = 666.7 \text{ watt}$$

$$\therefore \quad \text{Phase current} = \frac{\text{Power per phase}}{V_p \times \cos\phi} = \frac{666.7}{231 \times 0.189} = 15.28 \text{ A}$$

$$\text{Phase impedance} = \frac{231}{15.28} = 15.12\ \Omega$$

Resistance of each phase = Impedance × Power factor

$= 15.12 \times 0.189 = 2.86\ \Omega$

Reactance of each phase

$$= \sqrt{(15.12)^2 - (2.86)^2} = 14.85\ \Omega$$

For the reading of $W_1 = V_L I_L \cos(30° - \phi)$ to be zero, $\phi = -60°$. Thus, the power factor of the load should be 0.5, (leading) i.e.,

$$\cos\phi = 0.5$$

Hence, the impedance per phase

$$= \frac{\text{Resistance per phase}}{\text{Power factor}} = \frac{2.86}{0.5} = 5.72\ \Omega$$

and the reactance per phase,

$$= \sqrt{(5.72)^2 - (2.86)^2} = 4.97\ \Omega$$

Therefore, the capacitive reactance

$$= 14.85 - 4.97 = 9.88\ \Omega$$

or $$2\pi fC = \frac{1}{9.88} \quad \text{or} \quad C = \frac{1}{2\pi f \times 9.88}$$

∴ $$C = \frac{1}{2\pi \times 50 \times 9.88} = 322\ \mu\text{F}$$

Q.5 (d) Solution:

The back emf in the motor is given by

$$E_b = \frac{P\phi ZN}{60A} = \frac{3.75\times10^{-3} I_a \times n \times 180}{60} \times 1$$

$$= 11.25 \times 10^{-3}\, nI_a \qquad \text{...(i)}$$

The torque developed by the motor is given by

$$T = \frac{PZ}{2\pi A}\phi I_a = \frac{1}{2\pi} \times 3.75\times10^{-3} I_a \times 180 \times I_a$$

$$= 107.4\times10^{-3} I_a^2 \qquad \text{...(ii)}$$

We have, $V = E_b + I_a R_a$.

Thus, we get $$\frac{250 - E_b}{1} = I_a \qquad \text{...(iii)}$$

Under steady conditions,

$$T = T_L$$

$$107.4\times10^{-3} I_a^2 = 10^{-4} n^2$$

or $$I_a = \frac{n}{32.77} \qquad \text{...(iv)}$$

Substituting (iii) and (iv) in (i)

$$250 - \frac{n}{32.77} = 11.25\times10^{-3}\frac{n^2}{32.77}$$

$$n^2 + \frac{10^3}{11.25}n - \frac{250 \times 32.77 \times 1000}{11.25} = 0$$

$$n^2 + 88.9n - 72.8 \times 10^4 = 0$$

$$n = \frac{-88.9 \pm \sqrt{0.79 \times 10^4 + 291.2 \times 10^4}}{2} = 810 \text{ rpm}$$

From equation (iv), current drawn by the motor,

$$I_a = \frac{810}{32.77} = 24.7 \text{ A}$$

Q.5 (e) Solution:

Given,

Voltmeter resistance, $R_V = 200 \text{ k}\Omega$

Circuit resistors are 5 kΩ and 10 kΩ.

Supply voltage, $V = 150 \text{ V}$

Total resistance of circuit, $R = 5 \text{ K} + 10 \text{ K} = 15 \text{ k}\Omega$

$$\text{Current, } I = \frac{150}{15 \text{ k}\Omega} = 0.01 \text{ A}$$

Voltage across 10 kΩ

$$V_{\text{true}} = \text{I} \times 10 \text{ K} = 0.01 \times 10000 = 100 \text{ V}$$

New resistance in the circuit due to voltmeter connection,

$$R' = 10 \text{ k}\Omega \,||\, 200 \text{ k}\Omega$$

$$R' = \frac{10 \text{ K} \times 200 \text{ K}}{10 \text{ K} + 200 \text{ K}} = 9.524 \text{ k}\Omega$$

$$\therefore \quad R_{\text{total}} = 5 \text{ K} + 9.524 \text{ k}\Omega = 14.524 \text{ k}\Omega$$

The new current due to voltmeter connection,

$$I' = \frac{150}{14.524 \text{ K}} = 0.01033 \text{ A}$$

Measured voltage,

$$V_{\text{measured}} = I' \times 9.524 \text{ K} = 0.01033 \times 9524$$

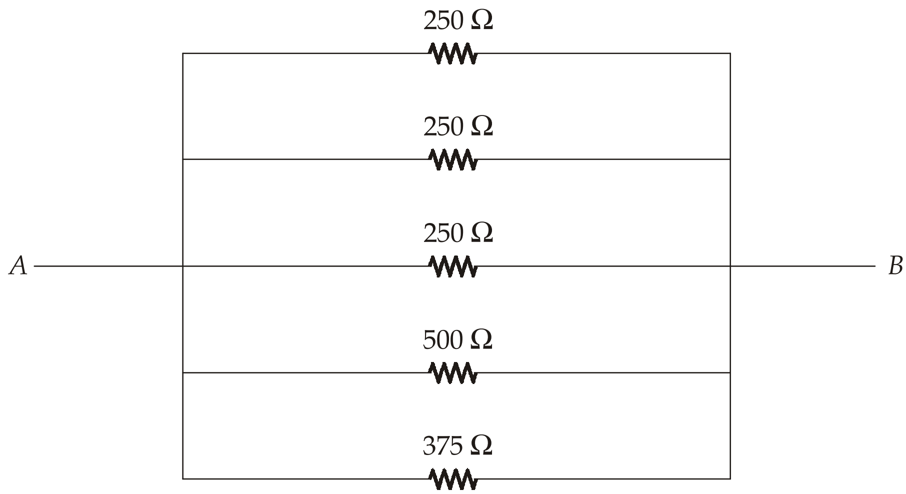
$$= 98.4 \text{ V}$$

$$\therefore \quad \text{percentage error} = \frac{V_{\text{true}} - V_{\text{meas}}}{V_{\text{true}}} \times 100$$

$$= \frac{100 - 98.4}{100} \times 100 = 1.6\%$$

Q.6 (a) Solution:

Based on the given data, we can draw circuit as,



(i) Total resistance without considering error:

Total resistance neglecting error is,

$$R_{equivalent} = \left[\left(\frac{250}{3}\right) \| 500 \| 375\right]$$

$$R_{equivalent} = 60\ \Omega$$

(ii) Total resistance with considering error:

The fractional error in $R_1 = -0.025$

$\therefore$ $$\delta R_1 = -0.025 \times 250 = -6.25\ \Omega$$

Hence, $$R_1 = 250 - 6.25 = 243.75\ \Omega$$

Similarly,

$$R_2 = 500 + 0.036 \times 500$$

$$R_2 = 518\ \Omega$$

and $$R_3 = 375 - (375 \times 0.014)$$

$$R_3 = 369.75\ \Omega$$

$\therefore$ $$R'_{equivalent} = \left[\frac{243.75}{3}\right] \| 518 \| 369.75\ \Omega$$

$$R'_{equivalent} = 59.02\ \Omega$$

(iii) Fractional error:

$$\%\in_r = \frac{M.V - T.V}{T.V} \times 100 = \frac{59.02 - 60}{60} \times 100$$

$$\%\in_r = -1.633\%$$

(iv) Current flowing through 500 Ω resistor:

Resistance 500 Ω can be written as $R = R_M + \in_r\%$

$$\in_r = 0.036 \times 500 = 18\ \Omega$$

$$\%\in_r = \frac{18}{500}\times 100 = 3.6\%$$

Thus, $$R = 500 + 3.6\%$$

Now, $$I = \frac{V}{R} = \frac{200}{500} = 0.4\text{A}$$

We have, $$\frac{\delta I}{I} = \frac{\delta V}{V} - \frac{\delta R}{R}$$

$$\text{Voltage error} = \pm 2\% \Rightarrow \frac{\delta V}{V} = \pm 0.02 \text{ and } \frac{\delta R}{R} = +0.036$$

Considering the worst case,

$$\frac{\delta I}{I} = -0.02 - 0.036 = -0.056$$

$$\delta I = -0.056 \times 0.4 = -0.0224\ \text{A}$$

Thus, $$I + \delta I = (0.4 - 0.0224)\text{A}$$

Q.6 (b) Solution:

(i) The acceleration and development of solar and wind power have been driven by a combination of technological advancements, economic factors, policy measures, and societal shifts. Here are the major factors:

1. Technological Advancements

Improved Efficiency: Advances in photovoltaic (PV) technology for solar panels and better aerodynamic designs for wind turbines have significantly increased their efficiency.

Cost Reductions: Manufacturing innovations, such as automated production processes for solar panels and larger, more efficient wind turbines, have drastically reduced costs.

Energy Storage: Developments in battery technology and other energy storage solutions have improved the ability to store and manage renewable energy, addressing intermittency issues.

Grid Integration: Enhanced grid management technologies and smart grid systems facilitate the integration of variable renewable energy sources into the electricity grid.

2. **Economic Factors**

Declining Costs: The levelized cost of energy (LCOE) for solar and wind power has decreased substantially due to economies of scale, competitive supply chains, and technological improvements.

Investment and Financing: Increased availability of investment and innovative financing mechanisms, such as green bonds and power purchase agreements (PPAs), have facilitated the deployment of renewable energy projects.

Job Creation: The renewable energy sector has become a significant source of employment, creating jobs in manufacturing, installation, and maintenance.

3. **Policy and Regulatory Support**

Government Incentives: Subsidies, tax credits, feed-in tariffs, and grants have made renewable energy projects financially attractive.

Renewable Portfolio Standards (RPS): Mandates requiring a certain percentage of energy to come from renewable sources have spurred demand.

Carbon Pricing: Mechanisms like carbon taxes and cap-and-trade systems make fossil fuels less competitive compared to renewables.

Investment in Research & Development: Government and private funding for R&D has accelerated innovation and reduced costs.

4. **Environmental and Societal Factors**

Climate Change Awareness: Increased awareness of climate change and its impacts has driven demand for cleaner energy sources.

Energy Independence: Many countries seek to reduce dependence on imported fossil fuels and enhance energy security through domestic renewable resources.

Public Support: Growing public support for sustainable energy solutions has influenced political and corporate strategies.

5. **Global Trends and Agreements**

International Agreements: Agreements like the Paris Agreement have set global targets for reducing greenhouse gas emissions, promoting the adoption of renewable energy.

Corporate Commitments: Corporations are committing to renewable energy through initiatives like RE100, which aims for 100% renewable energy usage by leading companies.

Urbanization and Electrification: Rising urban populations and the electrification of transportation and other sectors have increased overall energy demand, where renewables are seen as a sustainable solution.

6. **Economic Competitiveness**

Price Competitiveness: In many regions, solar and wind power have become competitive or cheaper than fossil fuels, leading to market-driven adoption.

Grid Parity: Achieving grid parity, where renewable energy costs equal or are lower than conventional energy costs without subsidies, has accelerated deployment.

(ii)

Basis of Difference	Synchronous motor	Induction Motor
Type of Excitation	A synchronous motor is a doubly excited machine.	An induction motor is a singly excited machine.
Supply System	Its armature winding is energized from an AC source and field winding from a DC source.	Its stator winding is energized from an AC source.
Speed	It always runs at synchronous speed. The speed is independent of load.	If the load increased, the speed of the induction motor decreases. It is always less than the synchronous speed.
Starting	It is not self starting. It has to be run up to synchronous speed by any means before it can be synchronized to AC supply.	Induction motor has self starting torque.
Operation	A synchronous motor can be operated with lagging and leading power by changing its excitation.	An induction motor operates only at a lagging power factor. At high loads, the power factor becomes very poor.
Usage	It can be used for power factor correction in addition to supplying torque to drive mechanical loads.	An induction motor is used for driving mechanical loads only.
Efficiency	It is more efficient than an induction motor of the same output and voltage rating.	Its efficiency is lesser than that of the synchronous motor of the same output and the voltage rating.
Cost	A synchronous motor is costlier than an induction motor of the same output and voltage rating	An induction motor is cheaper than the synchronous motor of the same output and voltage rating.

Q.6 (c) Solution:

$$\text{Dipole length, } l = 50 \text{ cm} = 0.5 \text{ m}$$

$$\text{Frequency, } f = 1 \text{ MHz}$$

$$\text{Current, } I_0 = 20 \text{ A}$$

$$\text{Distance, } r = 2 \text{ km} = 2000 \text{ m}$$

(i)
$$\text{Wavelength, } \lambda = \frac{c}{f} = \frac{3\times10^8}{10^6} = 300 \text{ m}$$

$$\frac{l}{\lambda} = \frac{0.5}{300} = \frac{1}{600} << 1$$

Thus, the given antenna is short dipole (i.e., Hertzian dipole)

(ii) Power density along the broadside ($\theta = 90°$),

For a short dipole, $$E_\theta = j\frac{\eta_0 I_0 l}{2r\lambda}\sin\theta$$

At broadside i.e, $\theta = 90°$

$$E_\theta = j\frac{\eta_0 I_0 l}{2r\lambda}\sin(90°)$$

$$\therefore \quad E_\theta = j\frac{\eta_0 I_0 l}{2r\lambda}$$

$$\text{Power density, } S = \frac{|E|^2}{2\eta} = \frac{1}{2\eta_0}\left(\frac{\eta_0 I_0 l}{2r\lambda}\right)^2$$

Since,

freespace impedance, $\eta_0 = 377\ \Omega$

$$\therefore \quad S = \frac{1}{2}\frac{\eta_0 I_0^2 l^2}{(2r\lambda)^2} = \frac{1}{2}\times\frac{377\times(20)^2(0.5)^2}{(2\times2000\times300)^2}$$

$$\therefore \quad S \simeq 1.31 \times 10^{-8}\ \text{W/m}^2$$

(iii) Given radiation pattern for short dipole

$$F(\theta, \phi) \propto \sin^2\theta$$

Power radiated by the antenna is given by,

$$P_{rad} = \iint F(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

the fraction of the total power radiated radiated within the sector between $\theta = 85°$ and $\theta = 90°$ is given by

$$\text{fraction of power, } \frac{P_{rad1}}{P_{rad2}} = \frac{\int_0^{2\pi}\int_{85°}^{90°}\sin^2\theta\cdot\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi}\sin^2\theta\cdot\sin\theta\, d\theta\, d\phi} = \frac{\int_0^{2\pi}\int_{85°}^{90°}\sin^3\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\, d\theta\, d\phi}$$

$$= \frac{\int_{85°}^{90°}\sin^3\theta\, d\theta}{\int_0^{\pi}\sin^3\theta\, d\theta}$$

We have, $\int \sin^3 x = \int \sin x(1-\cos^2 x)dx$

Let $\cos x = u \Rightarrow -\sin x\, dx$. We get,

$$\int \sin^3 x = -\int (1-u^2)du = -\left(u - \frac{u^3}{3}\right) + C$$

$$\int \sin^3 x = -\cos x + \frac{\cos^3 x}{3} + C$$

Thus, $\int_0^{\pi} \sin^3 \theta\, d\theta = \frac{4}{3}$;

$$\int_{85°}^{90°} \sin^3 \theta\, d\theta - (\cos 90° - \cos 85°) + \frac{1}{3}(\cos^3 90° - \cos^3 85°) = 0.087$$

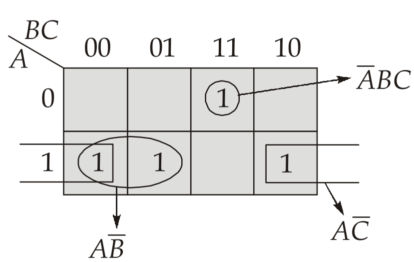
∴ Fraction of power radiated within the sector

$$= \frac{0.087}{(4/3)} = 0.06525 = 6.525\%$$

Q.7 (a) Solution:

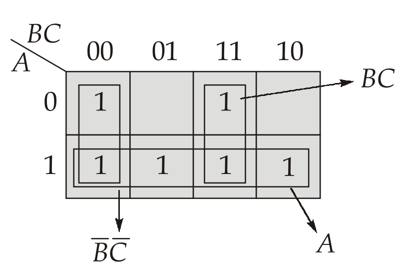
(i) PAL has programmable AND array and fixed OR array.

K-map for F_1



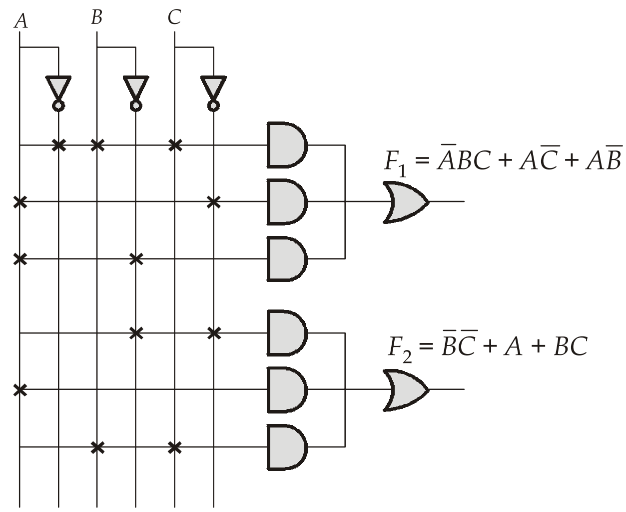
$$F_1 = A\bar{B} + A\bar{C} + \bar{A}BC$$

K-map for F_2

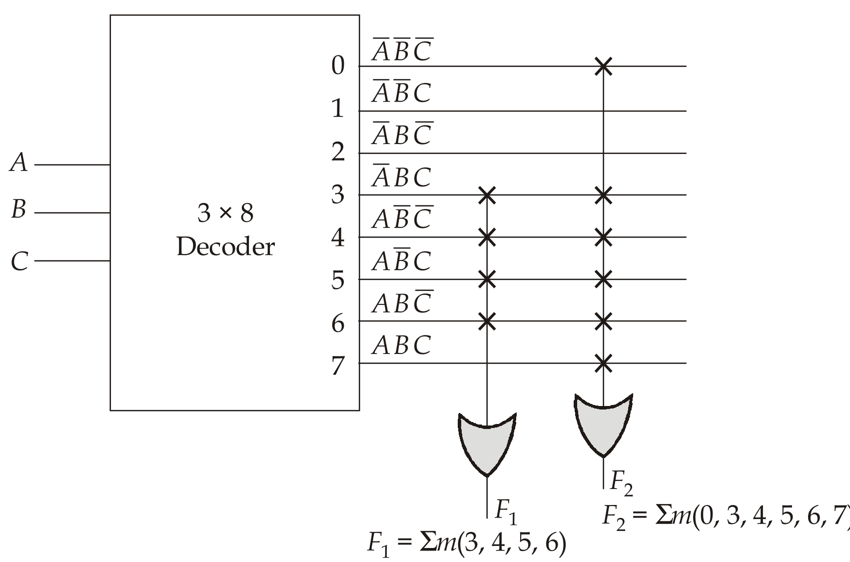


$$F_2 = A + BC + \bar{B}\bar{C}$$

Logic circuit implementation of PAL is shown as below:



(ii) PROM has fixed AND array and programmable OR array. A 3-to-8 decoder acts as a fixed AND array that generates all 8 minterms (m_0-m_7) based on three input variables (A, B, C), while a programmable OR array uses programmable connections to sum specific minterms, creating desired Boolean functions, F_1 and F_2 as shown below.



Q.7 (b) Solution:

(i) Back emf,
$$E_b = \frac{\phi n Z}{60}\left(\frac{P}{A}\right)$$

We have, $E_b = V - I_a R_a$ and given: $n = 1200$ rpm, Poles (P) = 4, No. of parallel paths (A) = 4, No. of armature conductors (Z) = 900. Substituting, we get

$$250 - 75 \times 0.2 = \frac{\phi \times 1200 \times 900}{60} \times \left(\frac{4}{4}\right)$$

or $\phi = 0.013$ Wb

(ii) Torque developed, $T = \frac{E_b I_a}{\omega}$, where $\omega = \frac{2\pi n}{60}$

$$T = \frac{1}{2\pi} \phi Z I_a \left(\frac{P}{A}\right)$$

$$= \frac{1}{2\pi} \times 0.013 \times 900 \times 75 \times 1 = 139.7 \text{ Nm}$$

(iii) Input power $= V(I_a + I_f) = 250 \times (75 + 1.5)$

$= 19.125$ kW

Mechanical power developed

$$P_g = T\omega = 139.7 \times \frac{2\pi \times 1,200}{60} = 17.56 \text{ kW}$$

Net mechanical output, $P_{out} = 15$ kW

The shaft load torque,

$$T_{shaft} = \frac{P_{out}}{\omega} = 15000\left(\frac{60}{2\pi \times 1200}\right) = 119.36 \text{ Nm}$$

(iv) Rotational loss, $P_g - P_{out} = 17.56 - 15$

$= 2.56$ kW

(v)
$$\eta = \frac{P_{out}}{P_{in}} = \frac{15}{19.125} = 0.0784 = 78.4\%$$

(vi) The field flux is reduced to 70% of its value, thus

$$\phi' = 0.7 \times 0.013 = 0.0091 \text{ Wb}$$

The torque load remains the same. We have

$$T \propto \phi I_a \Rightarrow I_a \propto \frac{1}{\phi}$$

$\therefore$
$$I'_a = 75 \times \frac{0.01}{0.007} = 107.14 \text{ A}$$

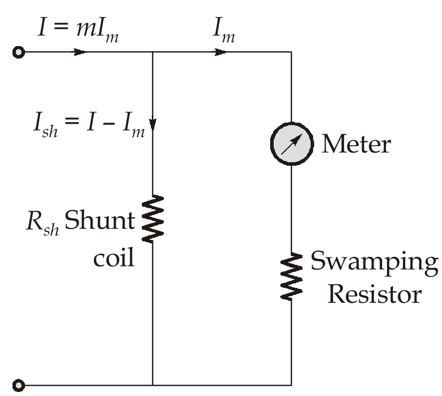
$$E'_b = V - I'_a R_a = 250 - 107.14 \times 0.2 = 228.57 \text{ V}$$

We have,
$$228.57 = \frac{\phi' n' Z}{60}\left(\frac{P}{A}\right) = \frac{0.0091 \times n' \times 900}{60} \times \left(\frac{4}{4}\right)$$

or $n' = 1674.5$ rpm

Q.7 (c) Solution:

(i) Swamping resistance is a resistor added in series with meter to reduce the error in readings caused by variation in temperature. Swamping resistor is made of materials having negligible temperature coefficient, e.g., manganin. It usually has resistance 20 to 30 times the meter coil resistance. The temperature error in ammeter occuring due to different temperature coefficients of the shunt (multiplier) and moving coil can be eliminated using swamping resistance. By connecting a swamping resistor in series with the copper coil which has a small resistance, the total resistance of the instrument circuit becomes dominated by the stable, temperature-insensitive swamping resistance. Because the swamping resistor's value remains nearly constant despite temperature fluctuations, the percentage change in the total resistance of the meter circuit is drastically reduced. This ensures that the current division in the shunted ammeter circuit remains stable, thereby improving the overall accuracy and reliability of the meter's readings.



(ii) Given,

$$I_m = 1 \text{ A}, R_m = 50 \text{ m}\Omega$$
$$L_m = 0.1 \text{ mH}, I = 10 \text{ A}$$

To extend the range of an ammeter, a low resistance shunt is connected in parallel with the meter so that most of the current bypasses the instrument. The required shunt resistance is given by

$$R_{sh} = \frac{R_m}{(m-1)}, \text{ where } m = \frac{I}{I_m}$$

Here, $m = \frac{10}{1} = 10$

$\therefore$ $$R_{sh} = \frac{50\times10^{-3}}{10-1} = 5.55 \text{ m}\Omega$$

To ensure that it works for all frequencies time constants of the shunt and meter arm should be equal.

i.e., $$\frac{\omega L_m}{R_m} = \frac{\omega L_{sh}}{R_{sh}}$$

or, $$\frac{L_m}{R_m} = \frac{L_{sh}}{R_{sh}} = \frac{0.1\times10^{-3}}{50\times10^{-3}} = 2 \text{ msec}$$

Q.8 (a) Solution:

(i) **1.** Given,
$$R = 2\ \Omega\text{m}^{-1}$$
$$L = 200\ \text{nHm}^{-1}$$
$$G = 800\ \mu\text{Sm}^{-1}$$
$$C = 80\ \text{pFm}^{-1}$$
$$f = 550\ \text{MHz}$$

We have,
$$\omega L = 2\pi f \times L = 2\pi \times 550 \times 10^6 \times 200 \times 10^{-9} \simeq 691$$
$$\omega C = 2\pi f \times C = 2\pi \times 550 \times 10^6 \times 80 \times 10^{-12} \simeq 0.276$$

since, $R << \omega L$,
$G << \omega C$

Therefore, the given transmission line is a low-loss transmission line.

$\therefore$ phase velocity, $$v_p = \frac{1}{\sqrt{LC}}$$
$$= \frac{1}{\sqrt{200\times10^{-9}\times80\times10^{-12}}}$$
$$v_p = 2.5 \times 10^8 \text{ m/s}$$

2. The amplitude of the wave decays exponentially with distance as
$$A(z) = A_0 e^{-\alpha z}$$, where α is the attenuation constant.

Let 'z' be the distance at which amplitude falls to 1%, thus
$$0.01\,A_0 = A_0 e^{-\alpha z}$$
$$z = \frac{\ln(100)}{\alpha}$$

For low-loss transmission line,
$$\alpha \approx \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$
$$\simeq \frac{2}{2}\sqrt{\frac{80\times10^{-12}}{200\times10^{-9}}} + \frac{800\times10^{-6}}{2}\sqrt{\frac{200\times10^{-9}}{80\times10^{-12}}}$$

$$\alpha \simeq 0.02 + 0.02 \simeq 0.04 \text{ Np/m}$$

$$\therefore \quad z = \frac{\ln(100)}{0.04} \simeq 115 \text{ m}$$

(ii) Given,
$$Z_0 = 50\ \Omega$$
$$Z_L = 50 - j75\ \Omega$$
$$\text{Incident power, } P_i = 100 \text{ mW}$$

Power delivered to load,

$$P_L = P_i[1 - |\Gamma|^2]$$

where,

$$\text{reflection coefficient, } \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{50 - j75 - 50}{50 - j75 + 50} = \frac{-j75}{100 - j75}$$

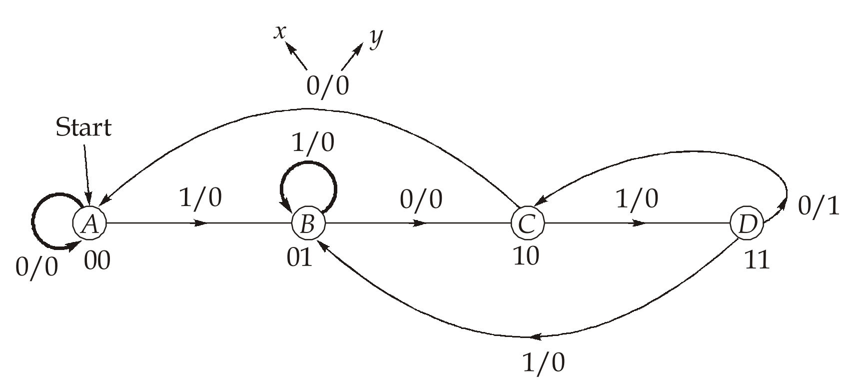
$$\therefore \quad |\Gamma| = \frac{75}{\sqrt{100^2 + 75^2}} = 0.6$$

$$\therefore \quad P_L = 100 \times 10^{-3}[1 - 0.6^2]$$
$$= 100 \times 10^{-3}[0.64] = 64 \text{ mW}$$

Q.8 (b) Solution:

First, we will draw the state diagram for given sequence detector.

State Diagram :



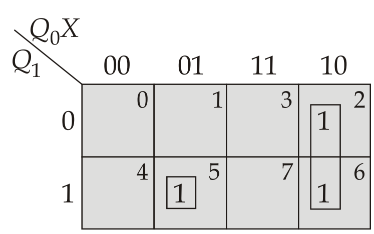
There are 4 states, so 2 flip flops will be used. Consider the states as A = 00, B = 01, C = 10 and D = 11.

State Table :

	Q_1	Q_0	X	Q_1^+ ($\uparrow D_1$)	Q_0^+ ($\uparrow D_0$)		Output (Y)
A	0	0	0	0	0	(A)	0
	0	0	1	0	1	(B)	0
B	0	1	0	1	0	(C)	0
	0	1	1	0	1	(B)	0
C	1	0	0	0	0	(A)	0
	1	0	1	1	1	(D)	0
D	1	1	0	1	0	(C)	1
	1	1	1	0	1	(B)	0

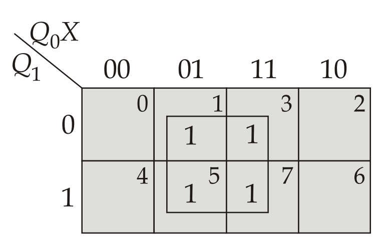
We will find the simplified expressions for D_1, D_0 and output y with the help of Karnaugh map.

For D_1 :



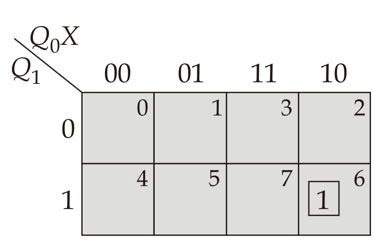
$$D_1 = Q_0\bar{X} + Q_1\bar{Q}_0X$$

For D_0 :



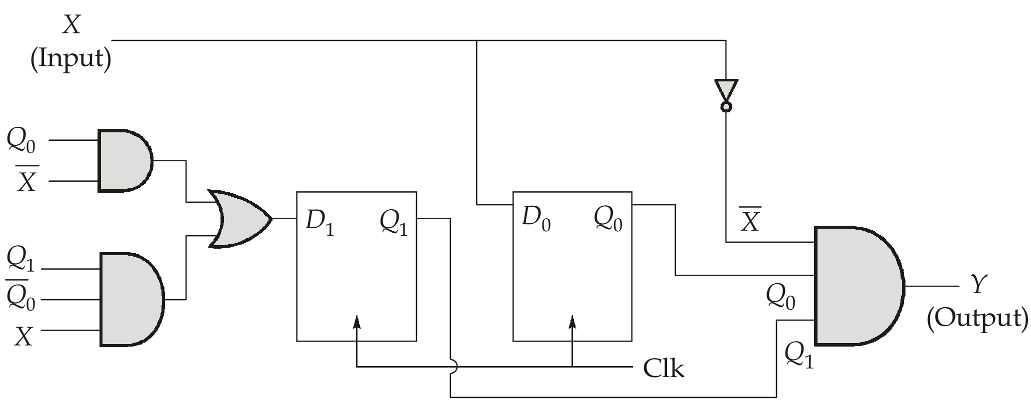
$$D_0 = X$$

For y :



$$y = Q_1Q_0\bar{X}$$

Circuit Diagram :



Q.8 (c) Solution:

We have,

$$\text{Current, } I_1 = 100 \pm 2\text{A}$$

$$I_2 = 200 \pm 5\text{A}$$

As they are flowing in parallel branches, thus, $I = I_1 + I_2$

$$I = 100 + 200 = 300 \text{ A}$$

Now,

(i) Considering the errors in I_1 and I_2 as limiting errors.

$$I = I_1 + I_2$$

Fractional error in I,

$$\frac{\delta I}{I} = \pm\left(\frac{I_1}{I}\frac{\delta I_1}{I_1} + \frac{I_2}{I}\frac{\delta I_2}{I_2}\right)$$

We have, $$\frac{\delta I_1}{I_1} = \frac{2}{100} = 0.02 \text{ and } \frac{\delta I_2}{I_2} = \frac{5}{200} = 0.025$$

Hence, fractional error

$$\frac{\delta I}{I} = \pm\left(\frac{100}{300}\times 0.02 + \frac{200}{300}\times 0.025\right)$$

$$\frac{\delta I}{I} = \pm 0.0233 = \pm 2.33\%$$

Hence, I can be written as,

$$I = 300 \pm 2.33\% = 300 \pm \frac{2.33\times 300}{100}$$

$$I \approx [300 \pm 7] \text{ A}$$

(ii) Considering the errors as standard deviations:

Standard deviation of I,

$$\sigma_I = \sqrt{\left(\frac{\partial I}{\partial I_1}\right)^2 \sigma_{I_1}^2 + \left(\frac{\partial I}{\partial I_2}\right)^2 \sigma_{I_2}^2}$$

Here, $I = I_1 + I_2$

$$\frac{\partial I}{\partial I_1} = \frac{\partial I}{\partial I_2} = 1$$

$$\sigma_I = \sqrt{2^2 + 5^2} = 5.38 \text{ A}$$

hence, $I = 300 \pm 5.38$ A

or $$I = 300 \pm \left(\frac{5.38}{300}\right) \times 100 = 300 \pm 1.8\%$$

(iii) It is clear from above calculations that limiting errors of 2 percent in I_1 and 2.5 percent in I_2 combine in this case, to give a limiting error of 2.33 percent in their sum I. While these very errors, when they are standard deviations, combine to give an error of only 1.8 per cent.

The use of standard deviation rather than limiting errors gives a more optimistic result. This is reasonable since the probability that both I_1 and I_2 are far from their respective means is small.

(iv) Considering the error as probable errors:

Now, probable error in I is

$$I_1 = \sqrt{(r_{I_1})^2 + (r_{I_2})^2}$$

$$I_1 = \sqrt{2^2 + 5^2}$$

$$I_1 = 5.38 \text{ A}$$

Thus, $I = (300 \pm 5.38)$ A

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