MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

# ESE-2026 Mains Test Series

# Mechanical Engineering Test No : 8

**Section A :** Machine Design **[All Topics]**

**Section B :** Heat Transfer-1 + Power Plant-1 **[Part Syllabus]**

Renewable Sources of Energy-2 + Industrial and Maintenance Engg.-2 **[Part syllabus]**

## Section A : Machine Design

**1. (a) Solution:**

Given : $P = \pm 60$ kN; $S_{ut} = 620$ N/mm$^2$; FOS = 1.5; $R = 90\%$; $q = 0.8$; $K_b = 0.85$, $K_c = 0.897$, $K_a = 0.77$

Endurance limit stress for the plate,

$$S_e' = 0.5 S_{ut} = 0.5 \times 620 = 310 \text{ N/mm}^2$$

$$\frac{D}{d} = \frac{100}{50} = 2 \text{ or } \left(\frac{r}{d}\right) = \frac{5}{50} = 0.1$$

From table $K_t = 2.3$

$$q = \frac{K_f - 1}{K_t - 1}$$

$\Rightarrow$ $$K_f = 2.04$$

$$K_d = \frac{1}{K_f} = \frac{1}{2.04} = 0.49$$

$$S_e = K_a K_b K_c K_d S_e'$$

$$= 0.77 \times 0.85 \times 0.897 \times 0.49 \times 310$$

$$= 89.178 \text{ N/mm}^2$$

For axial loading, $(S_e)_a = 0.8S_e$

$$= 0.8 \times 89.178$$

$$= 71.3424 \text{ N/mm}^2$$

Now, Max stress in plate $\leq$ Allowable limit

$$\frac{P}{dt} \leq \frac{(S_e)_a}{FOS}$$

$$\frac{60 \times 10^3}{50 \times t} \leq \frac{71.3424}{1.5}$$

$$t \geq 25.23 \text{ mm}$$

$$t = 25.23 \text{ mm}$$ **Ans.**

**1. (b) Solution:**

Given : $P$ = 10 kW; $N$ = 1440 rpm; $z_A$ = 40; $z_B$ = 50 and $z_C$ = 160; $m$ = 5 mm; $\phi$ = 20°

$$d_A = mz_A = 5 \times 40 = 200 \text{ mm}$$

$$d_B = mz_B = 5 \times 50 = 250 \text{ mm}$$

$$d_C = mz_C = 5 \times 160 = 800 \text{ mm}$$

($L_D$) length of arm $D$ is given by

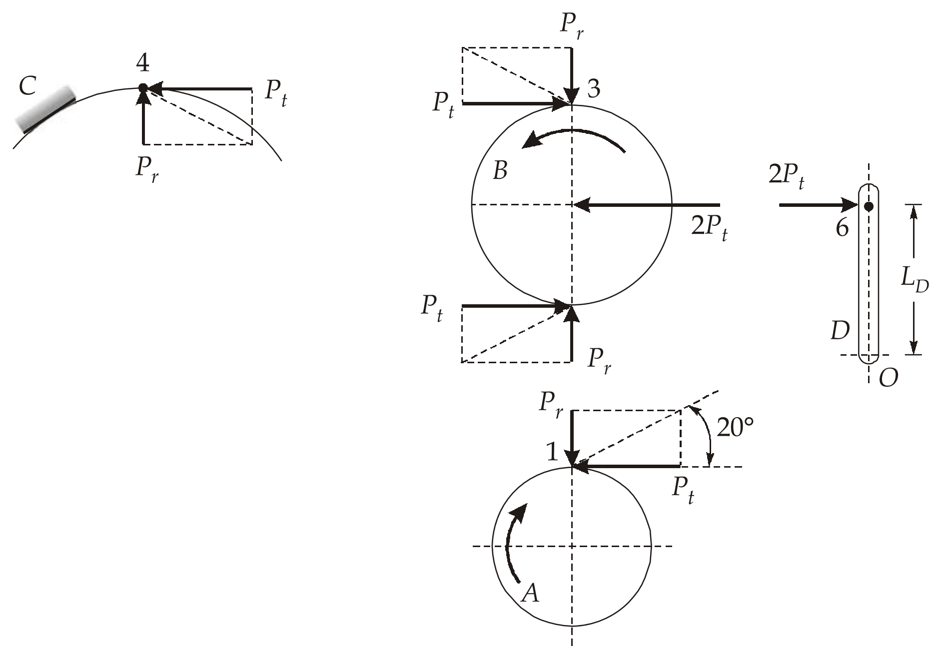
$$L_D = \frac{d_A + d_B}{2} = \frac{200 + 250}{2} = 225 \text{ mm}$$

The tangential component of the gear tooth force on the sun gear $A$ is given by

$$(M_t)_A = \frac{60 \times 10^6 \times 10}{2\pi N_A}$$

$$= \frac{60 \times 10^6 \times 10}{2\pi \times 1440} = 66314.55 \text{ Nmm}$$

$$P_t = \frac{2(M_t)_A}{d_A} = \frac{2 \times 66314.55}{200} = 663.145 \text{ N}$$

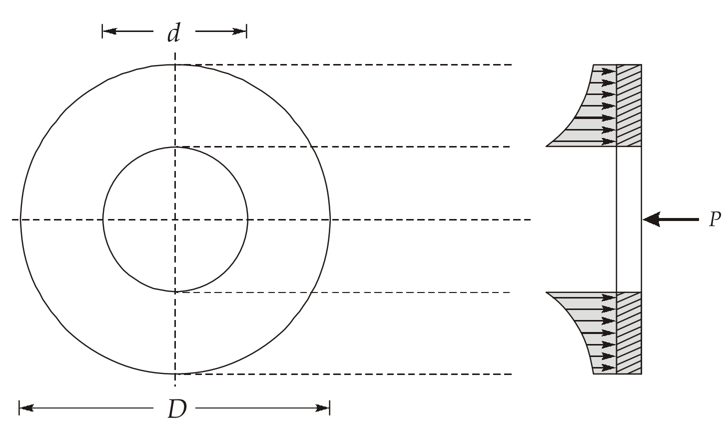



Torque that the arm $D$ can deliver to its output shaft.

$$\text{Torque} = 2P_t(L_D)$$

$$= \frac{2 \times 663.145 \times 225}{10^3} \text{Nmm} = 298.42 \text{ Nm}$$

**1. (c) Solution:**




For uniform wear theory, $p \times r$ = constant

$$P \times r = P_a\left(\frac{d}{2}\right)$$

$$\text{Torque, } M_t = 2\pi\mu \int_{d/2}^{D/2} pr^2 dr$$

$$M_t = 2\pi\mu\left(\frac{p_a d}{2}\right)\int_{d/2}^{D/2} r dr = \frac{\pi\mu p_a d}{8}\left(D^2 - d^2\right)$$

$$M_t = \frac{\pi\mu p_a}{8}\left(D^2 d - d^3\right) = \frac{\pi\mu p_a D^3}{8}\left(\frac{D^2 d}{D^3} - \frac{d^3}{D^3}\right)$$

Let, $\frac{d}{D} = x$

$$M_t = \frac{\pi\mu p_a D^3}{8}\left(x - x^3\right) \quad \text{...(i)}$$

For max torque capacity, $\frac{\partial}{\partial x}(M_t) = 0$

$$\Rightarrow \quad \frac{\partial}{\partial x}\left[x\left(1 - x^2\right)\right] = 0$$

$$1 - 3x^2 = 0$$

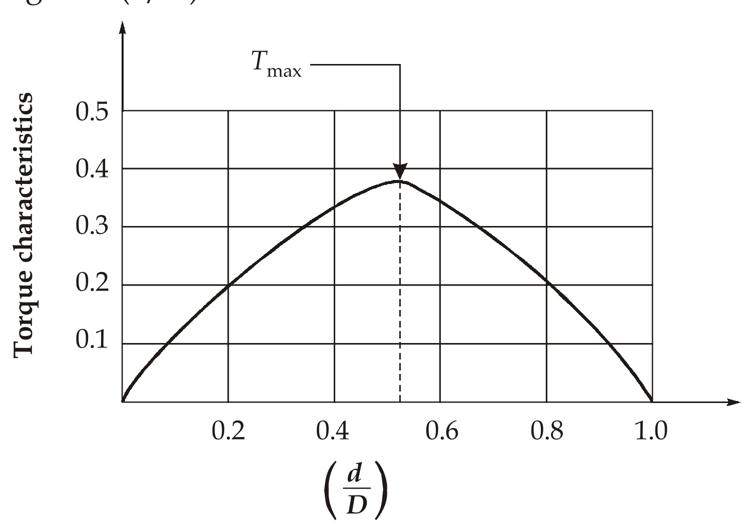
$$x = +\sqrt{\frac{1}{3}}, -\sqrt{\frac{1}{3}}$$

$$\frac{d}{D} = 0.577 \qquad \text{[Optimum ratio]}$$

Rearranging equation (i),

$$\frac{8M_t}{\pi\mu p_a D^3} = x - (1 - x^2)$$

Left hand side of the equation is called the torque characteristics. The variation of torque characteristics against $(d/D)$ is shown below:




**Variation of torque against $(d/D)$**

**1. (d) Solution:**

Backlash is defined as the amount by which the width of tooth space exceeds the thickness of the engaging tooth measured along the pitch circle. The objectives for providing backlash are as follows:

1. Backlash prevents the mating gear from jamming together.
2. Backlash compensates for machining errors.
3. Backlash compensates for thermal expansion of tooth.

Method to provide backlash are as follows:

1. The teeth of gear are cut slightly thinner.
2. The centre distance between mating gears is slightly increased.

**1. (e) Solution:**

Given : $S_{ut}$ = 600 MPa; $S_{yt}$ = 440 MPa; $S_e$ = 280 MPa; $\sigma_{max}$ = 120 MPa; $\sigma_{min}$ = 60 MPa

$$\text{Amplitude stress, } \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_a = \frac{120 - 60}{2} = 30 \text{ MPa}$$

$$\text{Mean stress, } \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_m = \frac{120 + 60}{2} = 90 \text{ MPa}$$

**(i)** Factor of safety using Gerber theory

$$\frac{N \cdot \sigma_a}{S_e} + \left(\frac{N\sigma_m}{S_{ut}}\right)^2 = 1$$

$$\frac{30N}{280} + \left(\frac{90N}{600}\right)^2 = 1$$

$$0.0225N^2 + 0.10714 \text{ N} - 1 = 0$$

$$N = 4.6981, -9.4599 \text{ (Not possible)}$$

So, $N = 4.6981$ **Ans.**

**(ii)** Factor of safety using Soderberg line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{\text{FOS}}$$

$$\frac{30}{280} + \frac{90}{440} = \frac{1}{N}$$

$$N = 3.208$$ **Ans.**

**(iii)** Factor of safety using Goodman line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{\text{FOS}}$$

$$\frac{30}{280} + \frac{90}{600} = \frac{1}{N}$$

$$N = 3.889$$ **Ans.**

**(iv)** Factor of safety against static failure

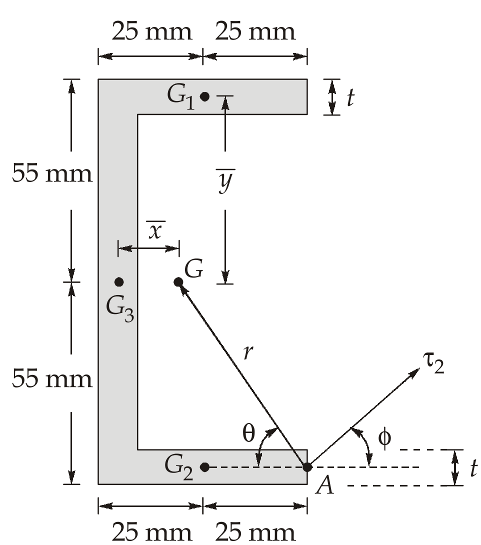
$$N = \frac{S_{yt}}{\sigma_{max}} = \frac{440}{120}$$

$$N = 3.667$$ **Ans.**

**2. (a) Solution:**

Given : $P = 60$ kN; $\tau = 110$ N/mm$^2$

Let $t$ be the thickness of the weld.




There are two horizontal welds $w_1$ and $w_2$ and one vertical welds $w_3$. By symmetry the centre of gravity $G$ of three welds is midway between two horizontal welds. Therefore

$$\bar{y} = 55 \text{ mm}$$

Taking moment of three welds about the vertical line passing through $G_3$,

$$\bar{x} = \frac{50 \times 25 + 50 \times 25 + 110 \times 0}{50 + 50 + 110}$$

$$\bar{x} = 11.904 \text{ mm}$$

Primary shear stress in the weld $(\tau_1) = \dfrac{P}{A} = \dfrac{P}{A_1 + A_2 + A_3}$

$$= \frac{60 \times 10^3}{(110 + 50 + 50)t} = \frac{285.714}{t} \text{ N/mm}^2$$

Let, $A$ be the farthest point in the weld from centre of gravity, its distance $r$ is given by

$$r = \sqrt{(50 - 11.904)^2 + (55)^2}$$

$$r = 66.905 \text{ mm}$$

$$\tan\theta = \frac{55}{50 - 11.904}$$

$$\theta = 55.29°$$

$$\phi = 90 - \theta = 90 - 55.29 = 34.708°$$

Therefore, the secondary shear stress $(\tau_2)$ is inclined at 34.708° with horizontal

$$e = 160 + (50 - 11.904) = 198.096 \text{ mm}$$

$$M = P \times e = 60 \times 10^3 \times 198.096$$

$$= 11885.76 \times 10^3 \text{ Nmm}$$

Distances of $G_1$, $G_2$ and $G_3$ from common centre of gravity $G$

$$\overline{G_1G} = \overline{G_2G} = \sqrt{(25 - 11.904)^2 + 55^2} = 56.537 \text{ mm}$$

$$r_1 = r_2 = 56.538 \text{ mm}$$

$$r_3 = \overline{G_3G} = 11.904 \text{ mm}$$

Also,

$$J_1 = J_2 = A_1\left[\frac{l_1^2}{12} + r_1^2\right]$$

$$= (50t)\left[\frac{50^2}{12} + 56.538^2\right] = 170243.938\ t \text{ mm}^4$$

$$J_3 = A_3\left[\frac{l_3^2}{12} + r_3^2\right]$$

$$= (110t)\left[\frac{110^2}{12} + 11.904^2\right] = 126504.24\ t\ \text{mm}^4$$

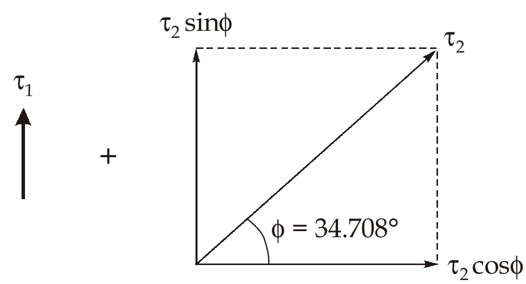
$$J = 2J_1 + J_3$$

$$= 466992.116\ t\ \text{mm}^4$$

Secondary shear stress at point $A$ is given by

$$\tau_2 = \frac{Mr}{J} = \frac{11885.76 \times 10^3 \times 66.905}{466992.116t}$$

$$\tau_2 = \frac{1702.848}{t}\text{N/mm}^2$$




Total vertical component of stress,

$$\tau_v = \tau_1 + \tau_2 \sin\phi$$

$$= \frac{285.714}{t} + \frac{1702.848}{t} \times \sin 34.708°$$

$$= \frac{1255.306}{t}\text{N/mm}^2$$

$$\text{Horizontal component, } \tau_h = \tau_2\cos\phi = \frac{1702.848}{t} \times \cos 34.708°$$

$$= \frac{1399.851}{t}\ \text{N/mm}^2$$

$$\text{Resultant stress, } \tau_R = \sqrt{\tau_v^2 + \tau_h^2} = \sqrt{\left(\frac{1255.306}{t}\right)^2 + \left(\frac{1399.851}{t}\right)^2}$$

$$= \frac{1880.259}{t}\text{N/mm}^2$$

Max stress induced $\leq$ Permissible shear stress

$$\frac{1880.259}{t} \leq 110$$

$$t \geq 17.093\ \text{mm}$$

$$t_{min} \geq 17.093 \text{ mm}$$

$$\text{Size of weld, } h = \frac{t(\text{throat})}{0.707} = \frac{t}{0.707}$$

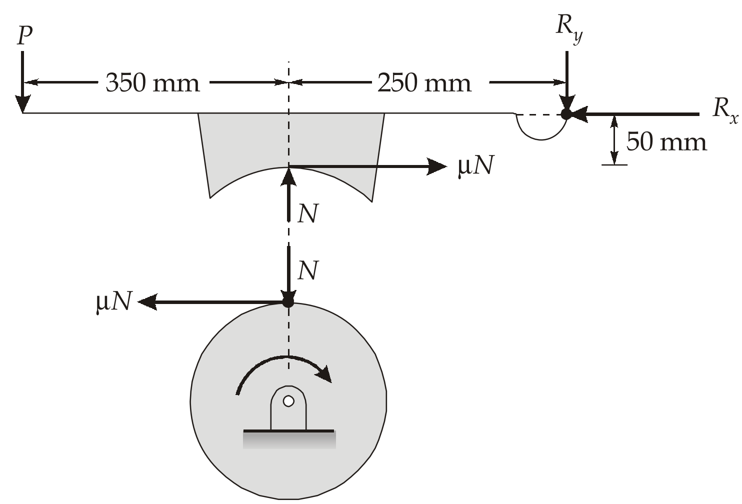
$$h = \frac{17.093}{0.707}$$

$$h = 24.18 \text{ mm}$$ **Ans.**

**2. (b) Solution:**

Given : $M_t = 300$ Nm; $n = 100$ rpm; $\mu = 0.36$; $p = 1.1$ N/mm$^2$; $l = 2 \times \omega$

**(i)**




$$T = M_t = \mu N \times R$$

$$300 \times 10^3 = 0.36 \times N \times 200$$

$$N = 4166.67 \text{ N}$$

Taking moment about hinge pin,

$$N \times 250 = P \times 600 + \mu N \times 50$$

$$4166.67 \times 250 = P \times 600 + 0.36 \times 4166.67 \times 50$$

$$P = 1611.11 \text{ N}$$

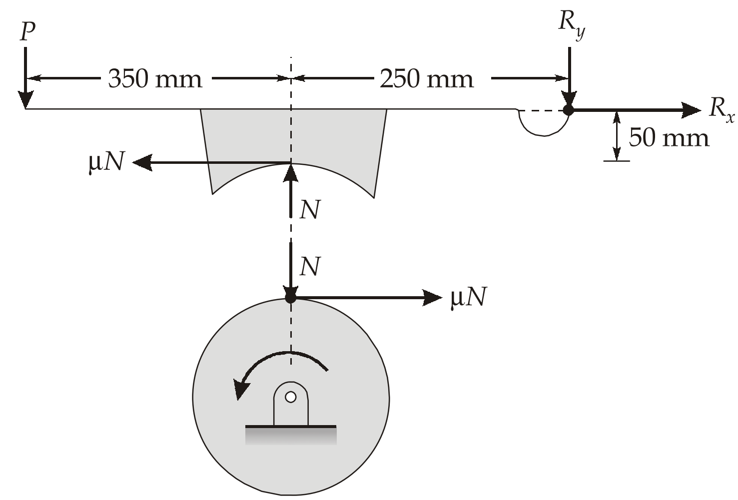
$$R_x = \mu N = 0.36 \times 4166.67 = 1500 \text{ N}$$

$$R_y = N\text{-}P = 4166.67 - 1611.11 = 2555.56 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{1500^2 + 2555.56^2}$$

$$R = 2963.26 \text{ N}$$

**(ii)**




Taking moment about hinge.

$$N \times 250 + \mu N \times 50 = P \times 600$$

$$4166.67 \times 250 + 0.36 \times 4166.67 \times 50 = P \times 600$$

$$P = 1861.11 \text{ N}$$

$$R_x = \mu N = 0.36 \times 4166.67 = 1500 \text{ N}$$

$$R_y = N - P = 4166.67 - 1861.11$$

$$= 2305.56 \text{ N}$$

$$\text{Total reaction, } R = \sqrt{1500^2 + 2305.56^2} = 2750.56 \text{ N}$$

**(iii)** Initial velocity of drum,

$$V = r\,\omega$$

$$= \frac{0.200 \times 2\pi \times 100}{60} = 2.094 \text{ m/s}$$

$$\text{Final velocity of the drum} = 0 \text{ m/s}$$

Average velocity of the drum,

$$V_{avg} = \frac{V+0}{2} = \frac{2.094+0}{2}$$

$$V_{avg} = 1.047 \text{ m/s}$$

The rate of heat generated during the braking period is equal to the rate of work done by the frictional force.

$$\text{Rate of heat generated} = \text{Frictional force} \times \text{Average velocity}$$

$$= \mu N \times V_{avg}$$

$$= 0.36 \times 4166.67 \times 1.047$$
$$= 1570.5 \text{ W}$$

**(iv)** Dimension of block, $N = p \times \text{Area}$

$$= p \times l \times w$$
$$4166.67 = 1.1 \times 2w \times w$$
$$w = 43.519 \text{ mm}$$
$$l = 2w = 2 \times 43.519$$
$$= 87.038 \text{ mm}$$ **Ans.**

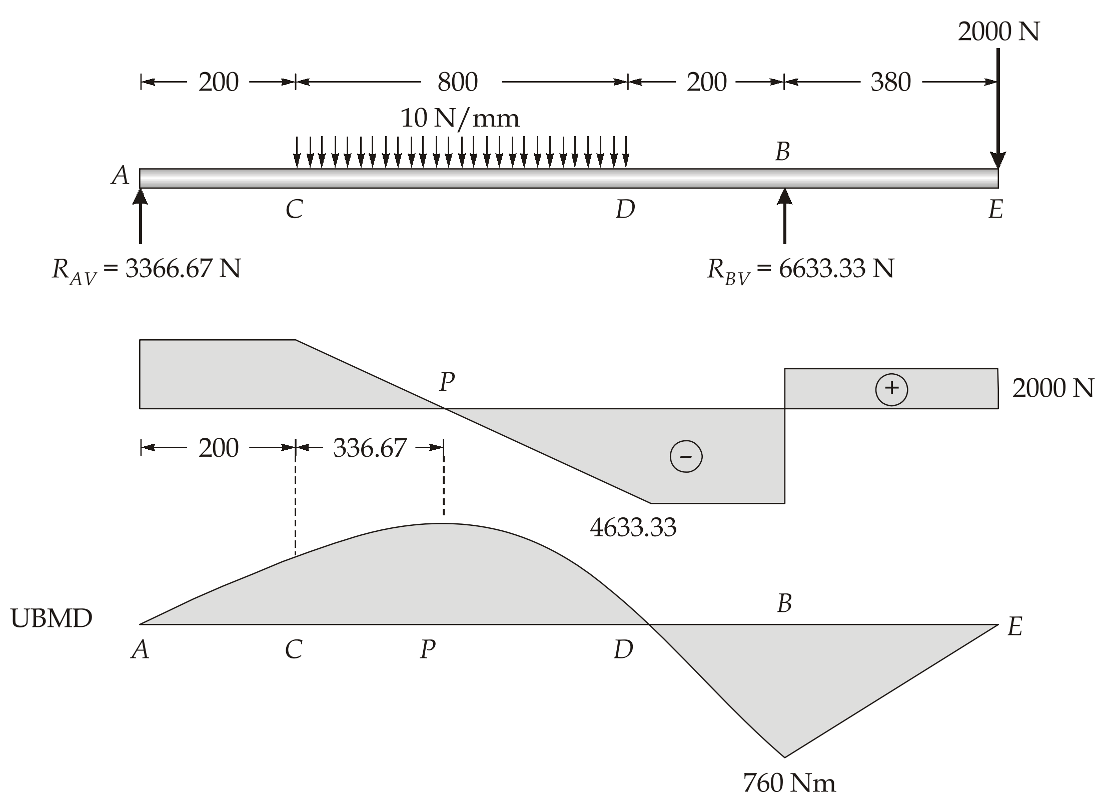
**2. (c) Solution:**

$S_{yt}$ = 560 MPa; $P$ = 50 kW; $n$ = 750 rpm, FOS = 2

Permissible shear stress, $\tau_{max} = \dfrac{S_{ys}}{FOS} = \dfrac{0.5 \times S_{yt}}{FOS} = \dfrac{0.5 \times 560}{2} = 140$ MPa

Torsional moment, $M_t = \dfrac{60 \times 10^6 \times 50}{2\pi n} = 636619.77$ Nmm

Vertical bending moment diagram



$$R_{AV} + R_{BV} = 8000 + 2000 \quad \text{...(i)}$$

$\Sigma M_A = 0;$

$2000 \times 1580 - R_{BV} \times 1200 + 8000 \times 600 = 0$

$$R_{BV} = 6633.33 \text{ N}$$

From equation (i), $R_{AV} = 3366.67$ N

$$M_C = 3366.67 \times 200 = 673334 \text{ Nmm}$$

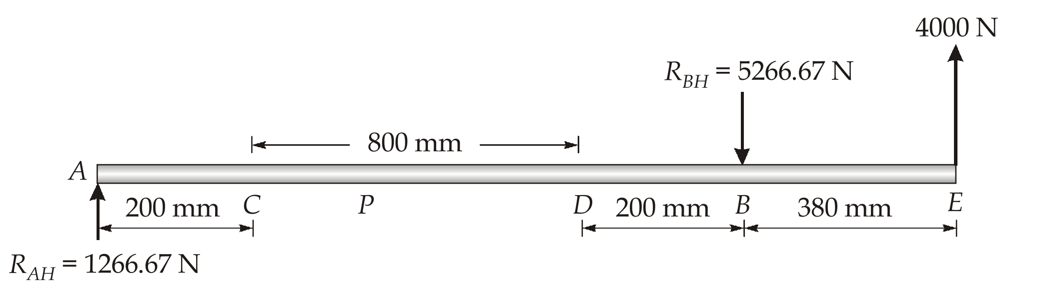
$$M_D = 3366.67 \times 1000 - 8000 \times 400 = 166670 \text{ Nmm}$$

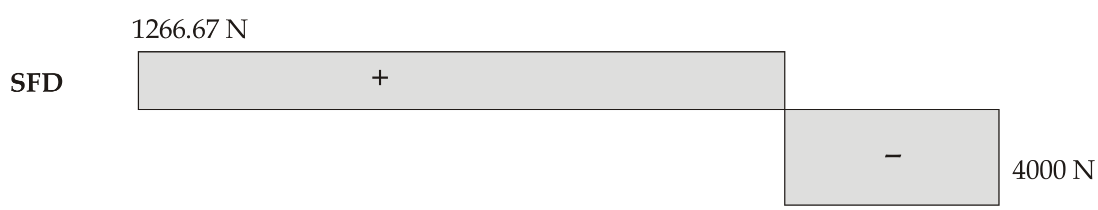
$$M_B = 2000 \times 380 = 760000 \text{ Nmm}$$

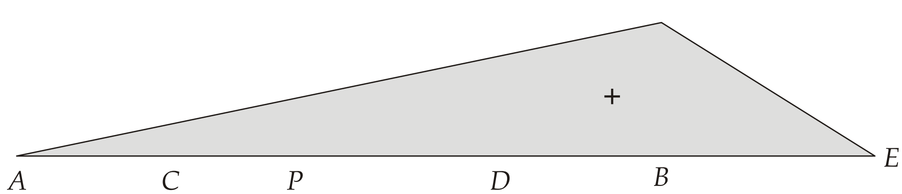
$$M_P = 3366.67 \times (200 + 336.67) + \frac{1}{2} \times 336.67 \times 336.67 \times 10$$

$$= 1240057.344 \text{ Nmm}$$

Horizontal bending moment diagram




$\Sigma F = 0$

$$R_{AH} + 4000 = R_{BH}$$

$\Sigma M_A = 0$

$$R_{BH} \times 1200 = 4000 \times 1580$$

$$R_{BH} = 5266.66 \text{ N}$$
$$R_{AH} = 1266.66 \text{ N}$$
$$M_B = 1266.66 \times 1200 = 1520000 \text{ Nmm}$$
$$M_D = 1266.66 \times 1000 = 1266670 \text{ Nmm}$$
$$M_P = 1266.67 \times 536.67 = 679783.78 \text{ Nmm}$$

$$\text{Total BM at } P, M_P = \sqrt{1240057.344^2 + 679783.78^2}$$
$$M_P = 1414159.89 \text{ Nmm}$$
$$\text{Total BM at } B, M_B = \sqrt{1520000^2 + 760000^2}$$
$$M_B = 1699411.663 \text{ Nmm}$$

Hence maximum bending moment is at $B$.

$$\text{Shaft diameter, } d^3 = \frac{16}{\pi\tau_{max}}\sqrt{(M_b)^2 + (M_t)^2}$$
$$d^3 = \frac{16}{\pi \times 140} \times \sqrt{(1699411.663)^2 \times (636619.77)^2}$$
$$d = 40.416 \text{ mm}$$ **Ans.**

**3. (a) Solution:**

As per given information

$$\text{Power, } P = 10 \text{ kW}$$
$$\text{Speed, } N = 1440 \text{ rpm}$$
$$\text{Speed reduction, } G = 4$$

Pinion and gear made of same material ($\sigma_{ut}$ = 600 N/mm$^2$)

$$\text{Factor of safety} = 1.5$$

The minimum number of teeth for 20° pressure angle is 18.

$$\text{Pinion, } Z_P = 18$$
$$\text{Gear, } Z_G = G.Z_P = 4 \times 18 = 72$$
$$\text{Torque, } T = \frac{60 \times 10^6 \times P}{2\pi N_P} = \frac{60 \times 10^6 \times 10}{2\pi \times 1440} = 66314.56 \text{ N.mm}$$

The Lewis form factor

$$Y = \pi y$$

$$Y = \pi\left(0.154 - \frac{0.912}{Z}\right) \text{ for } 20° \text{ full depth involute system}$$

$$\text{For Pinion, } Y = \pi\left(0.154 - \frac{0.912}{18}\right)$$

$$Y = 0.324$$

When both materials are same then design is done on the basis of pinion.

$$\text{Service factor, } C_S = \frac{\text{Starting torque}}{\text{Rated torque}} = 1.5$$

$$C_V = \frac{3}{3+v}$$

$$v = \frac{\pi d_P N}{60 \times 1000} = \frac{\pi m Z_P N}{60 \times 1000} = \frac{\pi \times m \times 18 \times 1440}{60 \times 1000}$$

$$v = 1.357\, m \text{ m/s}$$

To avoid failure of pinion teeth,

$$S_b > P_{\text{effective}}$$

Introducing a factor of safety,

$$S_b = P_{\text{eff}} \times \text{FOS}$$

$$P_{\text{effective}} = \frac{P_t \times C_s}{C_v}$$

$$P_{\text{effective}} = \frac{T \times 2 \times C_s}{m z_P \times C_v}$$

$$S_b = m.b \times \sigma_b y$$

$$S_b > P_{\text{effective}}$$

$$m \times b \times \frac{\sigma_{ut}}{3} \times y \geq \frac{T \times 2 \times C_s \times \text{FOS}}{m.z_p \times C_v}$$

$$m \times 10m \times \frac{\sigma_{ut}}{3} \times y \geq \frac{T \times 2 \times C_s \times 1.5}{m.z_p C_v}$$

$$m \times 10 \times m \times \frac{600}{3} \times 0.324 \geq \frac{66314.55962 \times 2 \times 1.5 \times 1.5}{m \times 18 \times \left(\frac{3}{3 + 1.357m}\right)}$$

$$m^3 \times \left(\frac{3}{3+1.357m}\right) \times 10 \times 18 \times 200 \times 0.324 \geq 66314.55962 \times 2 \times 1.5 \times 1.5$$

$$34992\, m^3 \geq (895246.555 + 404949.86\, \text{m})$$

On solving $\quad m \geq 4.202$

Using standard module of 5 mm

Width of tooth, $\quad b = 10\,m = 10 \times 5 = 50$ mm

Diameter of pinion, $\quad d_p = mz_p = 5 \times 18 = 90$ mm

Diameter of gear, $\quad d_g = mz_g = 5 \times 72 = 360$ mm

Velocity of pinion, $\quad V = m \times 1.357 = 6.785$ m/s

$$P_{\text{effective}} = \frac{C_s}{C_v} \times \frac{T \times 2}{mz_p} = \frac{1.5 \times 66314.55962 \times 2}{\left(\frac{3}{3+1.357 \times 5}\right) \times 5 \times 18}$$

$$= 7209.87 \text{ N}$$

Surface hardness of gears,

$$Q = \frac{2Z_G}{Z_G + Z_P} = \frac{2 \times 72}{72+18} = 1.6$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2 = 0.16\left(\frac{BHN}{100}\right)^2$$

$$S_w = bQd_pK \geq P_{\text{effective}} \times \text{FOS}$$

$$10 \times 5 \times 1.6 \times 90 \times 0.16\left(\frac{BHN}{100}\right)^2 \geq 7209.87 \times 1.5$$

$$BHN \geq 306.396$$

**3. (b) Solution:**

Given : $t = 25$ mm; $P = 60$ kN $= 60 \times 10^3$ N; $e = 450$ mm; $n = 7$; $\tau = 60$ MPa; $\sigma_c = 120$ MPa

First of all, find centre of gravity ($G$) of the system.

Let, $\quad \bar{x}$ = Distance of $G$ from $OY$

$\quad \bar{y}$ = Distance of G from $OX$

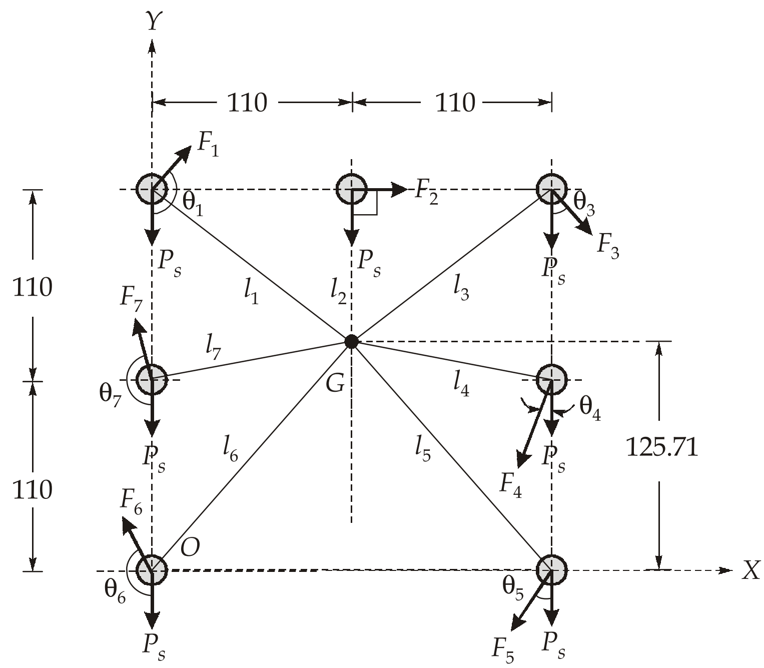
$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{110+220+220+220}{7} = 110 \text{ mm}$$

$$\bar{y} = \frac{110+110+220+220+220}{7} = 125.71 \text{ mm}$$

Direct shear on each rivet,

$$P_s = \frac{P}{n} = \frac{60 \times 10^3}{7} = 8571.42 \text{ N}$$

The direct shear load acts parallel to the direction of $P$ i.e. vertically downward.




Let, $F_1, F_2, F_3, F_4, F_5, F_6$ and $F_7$ be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at a distance $l_1, l_2, l_3, l_4, l_5, l_6$ and $l_7$ respectively from the centre of gravity of the rivet system.

From geometry,

$$l_1 = l_3 = \sqrt{110^2 + (220 - 125.71)^2} = 144.88 \text{ mm}$$

$$l_2 = 220 - 125.71 = 94.29 \text{ mm}$$

$$l_7 = l_4 = \sqrt{(125.71 - 110)^2 + 110^2} = 111.12 \text{ mm}$$

$$l_5 = l_6 = \sqrt{110^2 + 125.71^2} = 167.04 \text{ mm}$$

Let,

$$k = \frac{F_1}{l_1} = \frac{P \times e}{\Sigma l_i^2}$$

$$k = \frac{60 \times 10^3 \times 450}{144.88^2 + 94.29^2 + 111.12^2 + 144.88^2 + 111.12^2 + 167.04^2 + 167.04^2}$$

$$k = 205.525 \text{ mm}$$

$$F_3 = F_1 = kl_1 = 205.529 \times 144.88 = 29776.42 \text{ N}$$
$$F_2 = kl_2 = 205.529 \times 94.29 = 19378.92 \text{ N}$$
$$F_4 = F_7 = kl_4 = 205.529 \times 111.12 = 22837.90 \text{ N}$$
$$F_5 = F_6 = kl_5 = 205.529 \times 167.04 = 34330.85 \text{ N}$$

From geometry and calculated forces, we clearly see that rivet 3, 4 and 5 are heavily loaded.

$$\cos\theta_3 = \frac{110}{144.88} = 0.7592$$
$$\cos\theta_4 = \frac{110}{111.12} = 0.9899$$
$$\cos\theta_5 = \frac{110}{167.04} = 0.6585$$

$$\text{Resultant force, } R_i = \sqrt{P_s^2 + F_i^2 + 2P_s \times F_i \times \cos\theta_i}$$

$$R_3 = \sqrt{8571.42^2 + 29776.42^2 + 2 \times 8571.42 \times 29776.42 \times 0.7592}$$
$$R_3 = 36710.22 \text{ N}$$
$$R_4 = \sqrt{8571.42^2 + 22837.90^2 + 2 \times 8571.42 \times 22837.9 \times 0.9899}$$
$$R_4 = 31346.31 \text{ N}$$
$$R_5 = \sqrt{8571.42^2 + 34330.85^2 + 2 \times 8571.42 \times 34330.85 \times 0.6585}$$
$$R_5 = 40492.25 \text{ N}$$

Maximum resultant force is on rivet 5.

Based on shear failure,

So, $$R_5 = \frac{\pi}{4}d^2 \times \tau$$
$$40492.25 = \frac{\pi}{4} \times d^2 \times 60$$
$$d = 29.31 \text{ mm}$$

Now, based on crushing failure,

$$R_5 = d \times t \times \sigma_c$$
$$40492.25 = d \times 25 \times 120$$
$$d = 13.497 \text{ mm}$$

So, minimum diameter will be considered based on shear failure.

$$d = 29.31 \text{ mm}$$

or $$d = 30 \text{ mm}$$ **Ans.**

**3. (c) Solution:**

Given : $P = 5$ kW; $N = 750$ rpm; $r_i = 45$ mm; $r_o = 75$ mm; $\mu = 0.1$; $p = 0.35$ N/mm$^2$

Let, $n$ = Number of pairs of contact surfaces

$$T = \frac{P \times 60}{2\pi N} = \frac{5 \times 10^3 \times 60}{2\pi \times 750} = 63.66197 \text{ Nm}$$

$$T = 63661.97 \text{ Nmm}$$

For uniform wear, mean radius of contact surfaces

$$R = \frac{r_i + r_o}{2} = \frac{45 + 75}{2} = 60 \text{ mm}$$

and average axial force required,

$$W = P_{av} \times \pi\left[r_o^2 - r_i^2\right]$$

$$= 0.35 \times \pi\left[75^2 - 45^2\right] = 3958.406 \text{ N}$$

Also, torque transmitted ($T$)

$$T = n\mu WR$$

$$63661.97 = n \times 0.1 \times 3958.406 \times 60$$

$$n = 2.68$$

Since number of pairs of contact surface must be even therefore we shall use 4 pairs of contact surfaces with 3 steel disc and 2 bronze disc. [Because the number of pairs of contact surfaces is one less than the total number of discs]

**(ii)** Let, $W'$ = Actual axial force required

Since the actual number of pairs of contact surface is 4, actual torque developed for one pair of control surface,

$$T' = \frac{T}{n} = \frac{63661.97}{4} = 15915.49 \text{ N-mm}$$

$$T' = \mu W' R$$

$$15915.49 = 0.1 \times W' \times 60$$

$$W' = 2652.58 \text{ N}$$ **Ans.**

**(iii)** Actual average pressure,

$$P'_{av} = \frac{W'}{\pi\left[r_0^2 - r_i^2\right]} = \frac{2652.58}{\pi \times \left(75^2 - 45^2\right)}$$

$$P'_{av} = 0.235 \text{ N/mm}^2$$ **Ans.**

**(iv)** Let, $P_{max}$ = Actual maximum pressure

For uniform wear, $P.r = c$

Since the intensity of pressure is maximum at the inner radius,

$$\therefore \quad P_{max} \times r_i = c = P_{av} \times R$$

$$P_{max} = \frac{0.235 \times 60}{45} = 0.313 \text{ N/mm}^2$$

**4. (a) Solution:**

Given : $T_B$ = 3500 Nm = $3.5 \times 10^6$ Nmm; $d$ = 1 m; $r$ = 0.5 m = 500 mm; $\mu$ = 0.3

$$2\theta = 70° = 70 \times \frac{\pi}{180} = 1.22 \text{ rad}, \; \tau = 520 \text{ MPa}, \; C = \frac{D}{d} = 6$$

**(i)** Spring force necessary to set the brake

Let, $S$ = Spring force necessary to set the brake

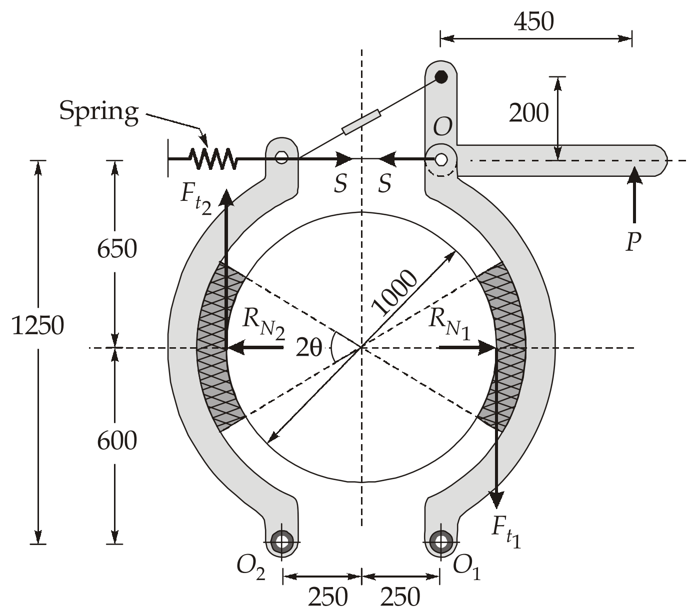
$R_{N_1}$ and $F_{t_1}$ = Normal reaction and the braking force on the right hand side shoe

$R_{N_2}$ and $F_{t_2}$ = Corresponding values for the left hand side shoe

Since the angle of contact is greater than 60°, therefore equivalent coefficient of fraction

$$\mu' = \frac{4\mu\sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.3 \times \sin 35}{2 \times \frac{7\pi}{36} + \sin 70}$$

$$\mu' = 0.319$$




(All dimensions are in mm)

Taking moment about the fulcrum $O_1$, we have

$$S \times 1250 = R_{N_1} \times 600 + F_{t_1}(500 - 250)$$

$$S \times 1250 = \frac{F_{t_1}}{0.319} \times 600 + F_{t_1}(500 - 250) \qquad \left[\because R_{N_1} = \frac{F_{t_1}}{\mu'}\right]$$

$$F_{t_1} = 0.587S \text{ N}$$

Again taking moments about the fulcrum $O_2$, we have

$$S \times 1250 + F_{t_2} \times (500 - 250) = R_{N_2} \times 600 = \frac{F_{t_2}}{0.319} \times 600$$

$$F_{t_2} = 0.766S \text{ N}$$

We know that torque capacity of the brakes ($T_{13}$)

$$3.5 \times 10^6 = \left(F_{t_1} + F_{t_2}\right) r$$

$$3.5 \times 10^6 = (0.587S + 0.766S) \times 500$$

$$S = 5173.69 \text{ N or } 5.174 \text{ kN}$$

**(ii)**

Let, $D$ = Mean diameter of the spring

$d$ = Diameter of spring wire

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6}$$

$$K_w = 1.2525$$

Since the maximum force is 1.2 times the spring force required during braking, therefore maximum spring force,

$$S_{max} = 1.2S$$

$$S_{max} = 1.2 \times 5173.69$$

$$S_{max} = 6208.43 \text{ N}$$

Now, shear stress induced in the spring,

$$\tau = K_w \frac{8S_{max} \times C}{\pi d^2}$$

$$520 = \frac{1.2525 \times 8 \times 6208.43 \times 6}{\pi d^2}$$

$$d = 15.12 \text{ mm}$$ **Ans.**

$$D = C \times d$$

$$D = 6 \times 15.12 = 90.72 \text{ mm}$$

**(iii)** Width of brake shoe

Let, $b$ = Width of brake shoes in mm, and $p_b$ = bearing pressure on the lining material of the shoes = 0.5 N/mm$^2$

$$A_b = b(2r\sin\theta)$$

$$= b \times 2 \times 500 \times \sin 35$$

$$= 573.57b \text{ mm}^2$$

$$R_{N_1} = \frac{F_{t_1}}{\mu'} = \frac{0.587 \times 5173.69}{0.319} = 9520.24 \text{ N}$$

$$R_{N_2} = \frac{F_{t_2}}{\mu'} = \frac{0.766 \times 5173.69}{0.319} = 12423.34 \text{ N}$$

Design will be based on $R_{N_2}$

$$R_{N_2} = p_b \times \text{Area of brake}$$

$$\frac{12423.34}{573.57b} = 0.5$$

$$b = 43.319 \text{ mm}$$

**(iv)** Let, $P$ = Force required to be exerted by the thruster to release the brake.

Taking moments about the fulcrum of the lever $O$, we have

$$P \times 450 + R_{N_1} \times 650 = F_{t_1} \times (500 - 250) + F_{t_2}(500 + 250) + R_{N_2} \times 650$$

Substituting,

$$F_{t_1} = 0.587 \times 5173.69 = 3036.956 \text{ N}$$

$$F_{t_2} = 0.766 \times 5173.69 = 3963.046 \text{ N}$$

$$P \times 450 + 9520.24 \times 650 = 3036.956 \times 250 + 3963.046 \times 750 + 12423.34 \times 650$$

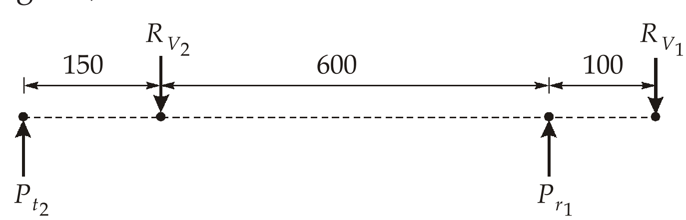
$$P = 12485.64 \text{ N}$$ **Ans.**

**4. (b) Solution:**

Given : $P$ = 60 kW; $n$ = 120 rpm; $L_{10h}$ = 10000 hr; Load factor = 1.4; $P_{r_1}$ = 6893 N;

$P_{r_2}$ = 3698 N; $P_{t_1}$ = 15917 N; $P_{t_2}$ = 9837 N

Vertical force diagram,




Taking moment about bearing $B_1$

$$P_{r_1}(100) + P_{t_2}(100 + 600 + 150) - R_{V_2}(100 + 600) = 0$$

$$\frac{6893 \times 100 + 9837 \times 850}{700} = R_{V_2}$$

$$R_{V_2} = 12929.64 \text{ N}$$

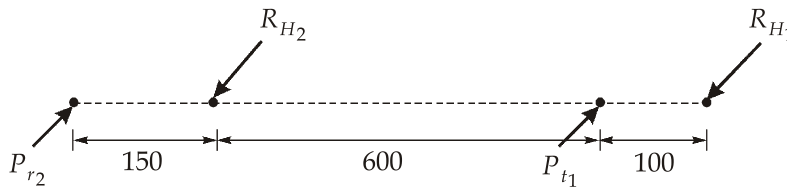
Considering equilibrium of vertical plane,

$$P_{t_2} + P_{r_1} = R_{V_2} + R_{V_1}$$

$$9837 + 6893 - 12929.64 = R_{V_1}$$

$$R_{V_1} = 3800.35 \text{ N}$$

Horizontal force diagram,




Considering forces in the horizontal plane and taking moment about bearing $B_1$,

$$P_{t_1}(100) + P_{r_2}(100 + 600 + 150) - R_{H_2}(100 + 600) = 0$$

$$\frac{15917 \times 100 + 3698 \times 850}{700} = R_{H_2}$$

$$R_{H_2} = 6764.29 \text{ N}$$

Also,
$$P_{t_1} + P_{r_2} = R_{H_1} + R_{H_2}$$

$$15917 + 3698 = R_{H_1} + 6764.29$$

$$R_{H_1} = 12850.71 \text{ N}$$

Total radial force at two bearings is given by

$$F_{r_1} = \sqrt{R_{V_1}^2 + R_{H_1}^2}$$

$$= \sqrt{3800.35^2 + 12850.71^2}$$

$$F_{r_1} = 13400.87 \text{ N}$$

$$F_{r_2} = \sqrt{R_{V_2}^2 + R_{H_2}^2}$$

$$= \sqrt{12929.64^2 + 6764.29^2}$$

$$F_{r_2} = 14592.16 \text{ N}$$

Since there is no axial thrust,

$$F_{a_1} = F_{a_2} = 0$$

$\Rightarrow$
$$P_1 = F_{r_1} \text{ and } P_2 = F_{r_2}$$

Now,
$$L_{10} = \frac{60 n L_{10h}}{10^6} = \frac{60 \times 120 \times 10000}{10^6}$$

$$L_{10} = 72 \text{ million rev.}$$

Considering load factors, the dynamic load capacities are given by,

$$C_1 = P_1(L_{10})^{1/3} \times L.F.$$
$$= 13400.87 \times (72)^{1/3} \times 1.4$$
$$= 78049.812 \text{ N}$$
$$C_2 = P_2(L_{10})^{1/3} \times L.F.$$
$$= 14592.16 \times (72)^{1/3} \times 1.4$$
$$= 84988.165 \text{ N}$$

Bearing 1, $C_1 = 78049.812$ N or 78.049 kN

Bearing 2, $C_2 = 84988.165$ N or 84.0988 kN

**4. (c) Solution:**

Given : $W = 10$ kN ; $N = 1000$ rpm; $p = 1000$ kPa; $\frac{r}{c} = 600$; $\mu = 40$ mPas; $\left(\frac{l}{d}\right) = 1$

**(i)**
$$p = 1000 \times 10^3 \text{ Pa} = 1 \text{ MPa}$$
$$p = \frac{W}{ld}$$
$$1 = \frac{10000}{d^2}$$
$$d = 100 \text{ mm} = l$$ **Ans.**

**(ii)** Sommerfeld number

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$
$$S = \frac{(600)^2 \times 40 \times 10^{-9} \times 1000}{1 \times 60} = 0.24$$ **Ans.**

**(iii)** To find $\left(\frac{r}{c}\right)f$, we need linear interpolation from the table,

$$\frac{0.24 - 0.121}{0.264 - 0.121} = \frac{\left(\frac{r}{c}\right)f - 3.22}{5.79 - 3.22}$$
$$\left(\frac{r}{c}\right)f = 5.358$$
$$f = 5.358 \times \left(\frac{c}{r}\right) = 5.358 \times \left(\frac{1}{600}\right)$$

Coefficient of friction, $f = 8.9311 \times 10^{-3}$ **Ans.**

**(iv)** Power lost in friction,

$$P = \frac{2\pi n_s f\,Wr}{10^6}$$

$$P = \frac{2\pi \times \left(\frac{1000}{60}\right) \times 8.9311 \times 10^{-3} \times 10 \times 10^3 \times 50}{10^6}$$

$$P = 0.467 \text{ kW}$$ **Ans.**

**(v)** Calculating $\frac{Q}{rcn_s l}$ for $S = 0.24$, using interpolation

$$\frac{0.24 - 0.121}{0.264 - 0.121} = \frac{\left(\frac{Q}{rcn_s\,l}\right) - 4.33}{3.99 - 4.33}$$

$$\left(\frac{Q}{rcn_s l}\right) = 4.047$$

$$Q = 4.047 \times 50 \times \frac{50}{600} \times \frac{1000}{60} \times 100$$

$$Q = 28104.603 \text{ mm}^3\text{/s or } 0.0281 \text{ Lt/s}$$ **Ans.**

**(vi)** Calculating $\frac{Q_s}{Q}$ for side leakage, using interpolation

$$\frac{0.24 - 0.121}{0.264 - 0.121} = \frac{\left(\frac{Q_s}{Q}\right) - 0.680}{0.497 - 0.680}$$

$$\left(\frac{Q_s}{Q}\right) = 0.5277$$

Side leakage, $Q_s = 0.5277 \times 28104.603$

$= 14830.79 \text{ mm}^3\text{/s or } 0.0148 \text{ Lt/s}$ **Ans.**

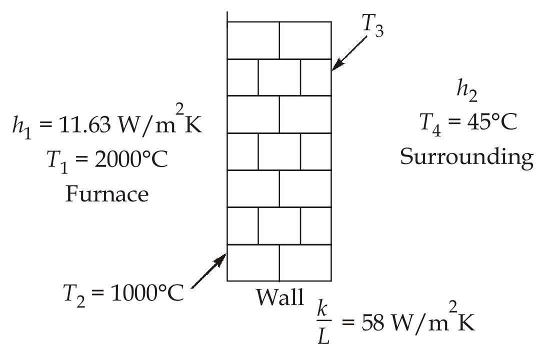
**(vii)** Temperature rise

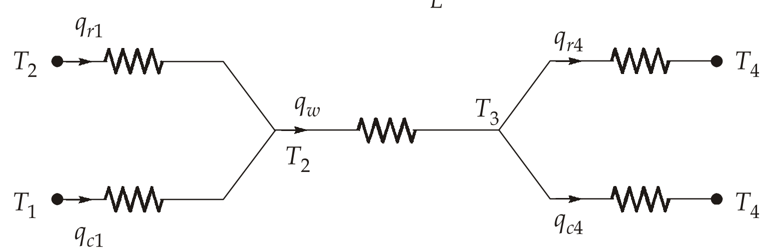
$$\Delta t = \frac{8.3 \times p \times \left(\frac{r}{c}\right) f}{\frac{Q}{rcn_s l}}$$

$$\Delta t = \frac{8.3 \times 1 \times 5.358}{4.047} = 10.988°\text{C}$$ **Ans.**

**Section B : Heat Transfer-1 + Power Plant-1**
**Renewable Sources of Energy-2 + Industrial and Maintenance Engineering-2**

**5. (a) Solution:**




$q_r \rightarrow$ Heat flow through radiation
$q_c \rightarrow$ Heat flow through convection
$q_w \rightarrow$ Total heat flow through wall

$$q_{r1} = 23.26 \text{ kW/m}^2$$
$$q_{c1} = h_1(T_1 - T_2) = 11.63 \times (2000 - 1000) \times 10^{-3} \text{ kW/m}^2$$
$$= 11.63 \text{ kW/m}^2$$
$$\text{Total heat, } q_w = q_{r1} + q_{c1} = 23.26 + 11.63$$
$$= 34.89 \text{ kW/m}^2 \quad \text{...(i)}$$

Also, from fourier law:
$$q_w = -kA\left[\frac{T_3 - T_2}{L}\right]$$
$$q_w = -58 \times 1 \times [T_3 - 1000] \qquad \left[\text{As } \frac{k}{L} = 58 \text{ W/m}^2\text{K}\right]$$

From equation (i), $34.89 \times 1000 = -58 \times [T_3 - 1000]$
$$T_3 = 398.45°\text{C or } 671.45 \text{ K}$$
$\because$
$$q_{r4} = 9.3 \text{ kW/m}^2$$
$$q_{c4} = q_w - q_{r4} = 34.89 - 9.3$$
$$= 25.59 \text{ kW/m}^2$$

Also,
$$q_{c4} = h_2(T_3 - T_4)$$

$$10^3 \times 25.59 = h_2 \times (398.45 - 45)$$

$$h_2 = 72.40 \text{ W/m}^2\text{/K}$$ **Ans.**

**5. (b) Solution:**

Given: $\dot{m}_s = 100 \times 10^3$ kg/hour, $P_s$ = 100 bar, $T_s$ = 500°C = 773 K, CV = 41 MJ/kg, $\eta_o$ = 80% = 0.80, A/F = 16 : 1, $\Delta P$ = 22 mm of water, $T_H$ = 310°C = 583 K, $T_g$ = 290°C = 563 K, $T_a$ = 30°C = 303 K, $h_1$ = 3372 kJ/kg and $h_f$ = 632.2 kJ/kg

**(i)** We know, draught produced,

$$\Delta P = gH(\rho_a - \rho_g)$$

$$= gH\frac{P_a}{R_a}\left(\frac{1}{T_a} - \frac{1}{T_g}\right)$$

$$\Rightarrow \quad 22 \times 10^{-3} \times 1000 \times g = gH \times \frac{1.01325 \times 10^5}{287}\left(\frac{1}{303} - \frac{1}{563}\right)$$

$$H = 40.885 \text{ m}$$ **Ans.**

**(ii)** Also,

$$\eta_o = \frac{\dot{m}_s(h_1 - h_f)}{\dot{m}_f(CV)}$$

$$0.80 = \frac{100 \times 10^3 \times (3372 - 632.2)}{\dot{m}_f \times 41 \times 1000}$$

$$\dot{m}_f = 8353 \text{ kg/h} = 2.32 \text{ kg/s}$$

So,

$$\dot{m}_a = 16 \times \dot{m}_f = 133648.78 \text{ kg/h}$$

$$= 37.125 \text{ kg/s}$$

So, mass flow rate of flue gases,

$$\dot{m}_{fg} = \dot{m}_a + \dot{m}_f = 37.125 + 2.32$$

$$= 39.445 \text{ kg/s}$$

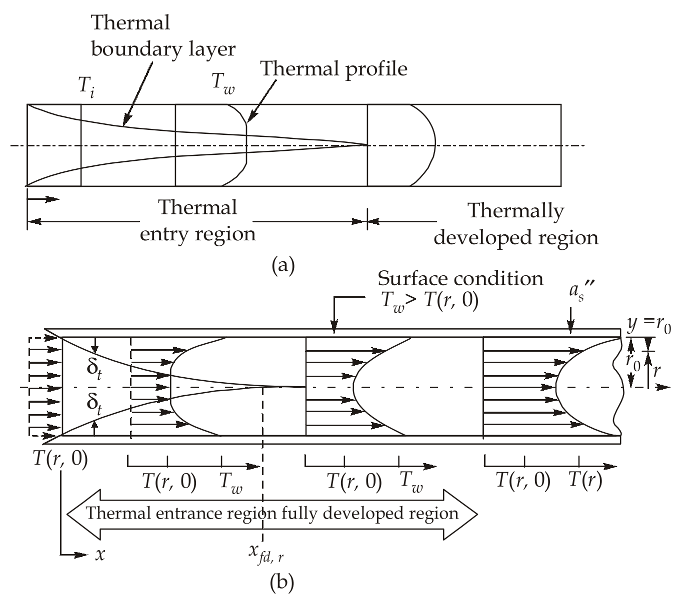
We know that, $\dot{m}_{fg} = \rho_{fg} A V_{fg}$ (Assumed velocity of gases at stack exit is negligible)

$$\Rightarrow \quad 39.445 = \frac{1.013 \times 10^5}{287 \times 583} \times \left(\frac{\pi}{4}D^2\right)\left(\sqrt{2 \times g \times 40.885}\right)$$

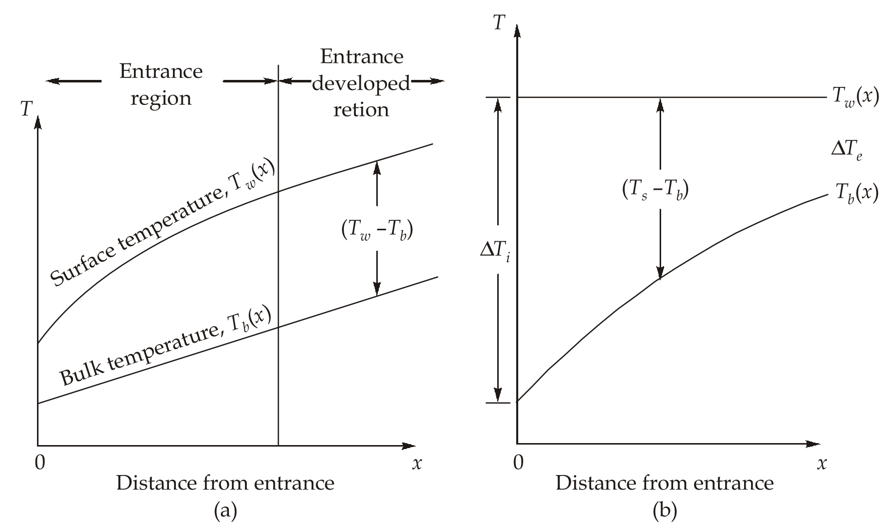
$$D^2 = 2.9282$$

$$\Rightarrow \quad D = 1.711 \text{ m}$$

**5. (c)**




Development of thermal boundary layer in a tube (a) when $T > T_w$ and (b) when $T_w > T$




Let us consider that a fluid at a uniform temperature enters a circular tube with its wall at a different temperature. The fluid particles in the layer in contact with the surface of the tube will assume the tube surface or wall temperature $T_w$. This will initiate convection heat transfer in the tube followed by development of the thermal boundary layer along the tube. The thickness of this thermal boundary layer reaches the tube center and thus fills the entire tube. The region of flow over which the thermal boundary layer develops and reaches the tube centre is called the thermal entry region. The region beyond the thermal entry region in which the temperature profile remains unchanged is called the thermally developed region/zone.

The dimensionless temperature profile $\left(\frac{T - T_w}{T_c - T_w}\right)$ does not also change upstream of thermal entry length. The zone in which the flow is both hydrodynamically and thermally developed is called the fully developed differs according to whether a uniform wall temperature ($T_w$) or a uniform heat flux is maintained. For both surface conditions, however, the amount by which fluid temperatures exceed the entrance temperature increase with increasing $x$.

Nusselt number for fully developed laminar flow in a tube is given as

$$Nu_d = \frac{hD}{k} = \frac{48}{11} = 4.364 \text{ (for constant heat flux)}$$

$$Nu_d = \frac{hD}{k} = 3.66 \text{ (for } T_w = \text{constant)}$$

**5. (d) Solution:**

Given : $R = 20$ m, $r = 5$ m, $A = 5$ km$^2$, $\eta = 0.7$, $\rho = 1025$ kg/m$^3$

$$\text{Average power potential, } P_{avg} = 0.225 \times A(R^2 - r^2) \text{ Watts}$$
$$= 0.225 \times 5 \times 10^6 \times (20^2 - 5^2)$$
$$= 421.87 \text{ MW}$$

$$\text{Average power generated} = 421.87 \times 0.7$$
$$= 295.309 \text{ MW}$$

$$\text{Energy available in single emptying} = \frac{1}{2}\rho g A(R^2 - r^2)$$
$$= \frac{1}{2} \times 1025 \times 9.81 \times 5 \times 10^6 (20^2 - 5^2)$$
$$= 9.42 \times 10^6 \text{ MJ}$$

$$\text{One ebb cycle duration} = 12\text{h } 25 \text{ min}$$
$$= 12.42 \text{ h}$$

$$\text{Number of ebb cycles in a year} = \frac{365 \times 24}{12.42}$$
$$N = 705.5$$
$$N \simeq 706$$

$$\therefore \text{ Average annual energy generation} = \frac{9.42 \times 10^6 \times 706 \times 0.7}{3.6}$$
$$= 12.93 \times 10^8 \text{ kWh}$$

**Ans.**

**5. (e) Solution:**

**Hazard Rate Function :** Hazard rate function provides an instantaneous rate of failure and is often used in reliability.

**Mathematically it is defined as** the conditional probability of failure per unit of time (failure rate).

$$\lambda(t) = \left\{\lim_{\Delta t \to 0} \frac{-[R(t+\Delta t) - R(t)]}{\Delta t}\right\}\frac{1}{R(t)}$$

$$\lambda(t) = \left\{-\frac{d}{dt}R(t)\right\}\frac{1}{R(t)} = \frac{f(t)}{R(t)}$$

where, $\lambda(t)$ is known as the instantaneous Hazard Rate or failure rate function.

The failure rate function provides an alternate way of describing a failure distribution.

Reliability in terms of Hazard-Rate function can be mathematically expressed as

$$R(t) = e^{-\int_0^t \lambda(t)dt}$$

Given data : $\lambda(t) = (5 \times 10^{-6})t$; $R(t) = 0.99$

As we know,
$$R(t) = \exp\left\{-\int_0^t \lambda(t)\,dt\right\}$$

$$\Rightarrow \quad 0.99 = \exp\left\{-\int_0^t 5\times 10^{-6}(t)\,dt\right\}$$

$$\Rightarrow \quad \ln(0.99) = -(5\times 10^{-6})\frac{t^2}{2}$$

$$\Rightarrow \quad t = \sqrt{\frac{-2\ln(0.99)}{5\times 10^{-6}}}$$

$$\Rightarrow \quad t = 63.4045 \text{ hours}$$ **Ans.**

**6. (a) Solution:**

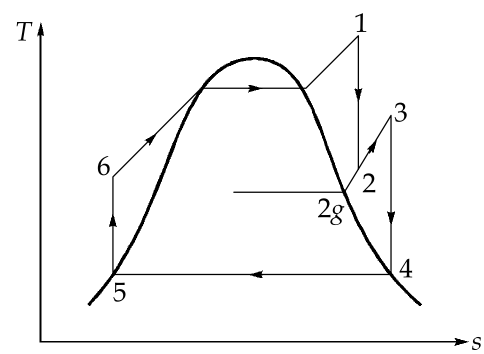
From table:

$$P_1 = 3 \text{ MPa} = 30 \text{ bar}$$
$$T_1 = 600°\text{C}, h_1 = 3682.3 \text{ kJ/kg}$$

$$S_1 = 7.5085 \text{ kJ/kgK}$$
$$S_2 = S_1 = 7.5085 \text{ kJ/kgK}$$
$$P_2 = 500 \text{ kPa} = 5 \text{ bar}$$
$$S_{2g} = 6.8213 \text{ kJ/kgk}$$

Since, $S_2 > S_{2g} \Rightarrow$ State 2 lies in superheated region,

$$T_4 = 45°\text{C}$$
$$h_4 = (h_g)_{\text{at } T = 45°\text{C}} = 2583.2 \text{ kJ/kg}$$
$$h_5 = (h_g)_{\text{at } T = 45°\text{C}} = 188.4 \text{ kJ/kg}$$
$$S_4 = (S_g)_{\text{at T} = 45°\text{C}} = 8.165 \text{ kJ/kg}$$
$$S_4 = S_3 = 8.165 \text{ kJ/kg}$$




Now, state 2 lies between temperature range of 300°C and 400°C at $P_2 = 5$ bar.

So, by interpolation,
$$h_2 = 3094.43 + \frac{7.5085 - 7.4599}{7.7938 - 7.4599}(3271.9 - 3064.2)$$
$$= 3094.43 \text{ kJ/kg}$$

and state 3, lies between temperature range of 500°C and 600°C at $P_2 = 5$ bar.

So, by interpolation,
$$h_3 = 348.9 + \frac{(8.165 - 8.0872)}{(8.3522 - 8.0872)}(3701.7 - 3483.9)$$
$$= 3547.84 \text{ kJ/kg}$$

Similarly,
$$T_3 = 500 + \frac{(8.165 - 8.0872)}{(8.3522 - 8.0872)}(600 - 500)$$
$$= 529.35°\text{C}$$ **Ans.**

Now, condenser heat transfer,

$$Q_{4\text{-}5} = 10 \text{ MW} = \dot{m}(h_4 - h_5)$$
$$\Rightarrow \quad 10 \times 10^3 = \dot{m}(2583.2 - 188.4)$$

$$\dot{m} = 4.176 \text{ kg/s}$$

and $v_5 = (v_f)_{\text{at } T = 45°C} = 0.00101 \text{ m}^3\text{/kg}$

$$P_5 = 9.593 \text{ kPa}$$

$$W_P = v_5(P_6 - P_5)$$

$$= 0.00101\,(P_1 - P_5) \qquad (\because P_1 = P_6)$$

$$= 0.00101(3000 - 9.593) = 3.02 \text{ kJ/kg}$$

Also, $W_P = h_6 - h_5$

$$3.02 = h_6 - 188.4$$

$\Rightarrow$ $h_6 = 191.42 \text{ kJ/kg}$

Now, Total turbine output,

$$P = \dot{m}\left[(h_1 - h_2) + (h_3 - h_4)\right]$$

$$= 4.176\,[(3682.3 - 3094.43) + (3547.84 - 2583.2)]$$

$$P = 6483.28 \text{ kW}$$

and Boiler heat transfer, $Q = \dot{m}\left[(h_1 - h_6) + (h_3 - h_2)\right]$

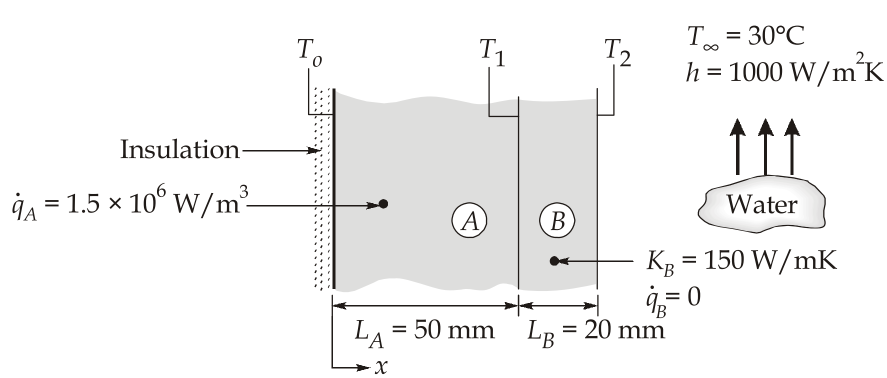
$$= 4.176\,[(3682.3 - 191.42) + (3547.84 - 3094.42)]$$

$$Q = 16471.39 \text{ kW}$$

6. **(b) Solution:**

**Assumptions:**

1. Steady-state conditions.
2. One-dimensional conduction in *x*-direction.
3. Negligible contact resistance between walls.
4. Inner surface of *A* is adiabatic.
5. Constant properties for materials *A* and *B*.

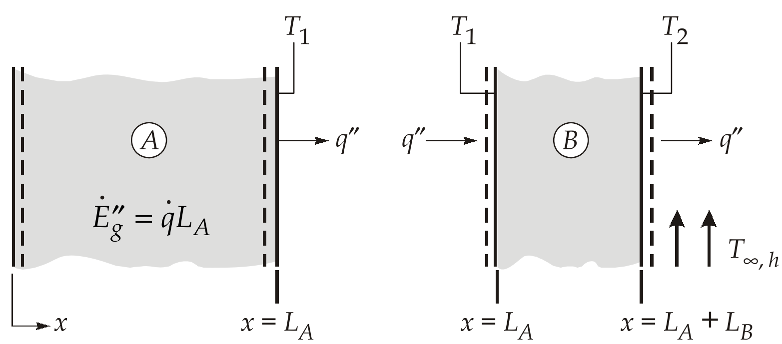


The outer surface temperature $T_2$ can be obtained by performing an energy balance on a system about material $B$. Since there is no generation in this material, it follows that, for steady-state conditions and a unit surface area, the heat flux into the material at $x = L_A$ must equal the heat flux from the material due to convection at $x = L_A + L_B$.

$\therefore$
$$q'' = h(T_2 - T_\infty) \quad \text{...(i)}$$

The surface at $x = 0$ is adiabatic, there is no inflow and the rate at which energy is generated must equal the outflow. Therefore for a unit surface area

$$q'' = \dot{q}L_A \quad \text{...(ii)}$$



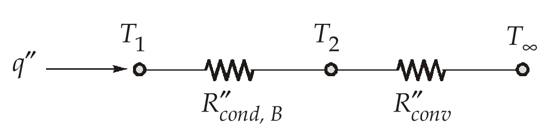
Energy balance on material $A$ Energy balance on material $B$

From equation (i) and (ii), the outer surface temperature is

$$T_2 = T_\infty + \frac{\dot{q}L_A}{h} = 30 + \frac{1.5 \times 10^6 \times 0.05}{1000} = 105°\text{C}$$

Also, the temperature at the insulated water is

$$T_0 = \frac{\dot{q}L_A^2}{2K_A} + T_1 \quad \text{...(iii)}$$



Thermal circuit representing wall B conduction and convection processes

From thermal circuit shown in figure,

$$T_1 = T_\infty + \left(R''_{cond,B} + R''_{conv}\right)q''$$

or
$$T_1 = T_\infty + \left(\frac{L_B}{k_B} + \frac{1}{h}\right)q''$$

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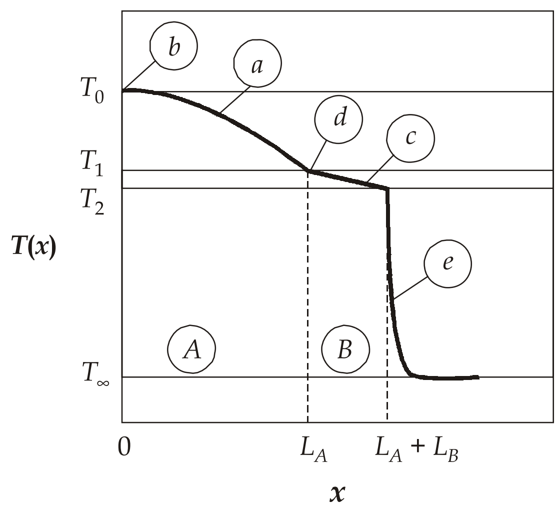
$$T_1 = 30 + \left(\frac{0.02}{150} + \frac{1}{1000}\right) \times \left(1.5 \times 10^6\right) 0.05$$

$$T_1 = 30 + 85 = 115°C$$

Substituting into equation (iii), the inner surface temperature of the composite is

$$T_0 = \frac{1.5 \times 10^6 \times (0.05)^2}{2 \times 75} + 115 = 25°C + 115°C = 140°C$$

**(ii)** The temperature distribution in the composite has the following features, as shown:




(a) Parabolic in material *A*.

(b) Zero slope at insulated boundary.

(c) Linear in material *B*.

(d) Slope change = $k_B/k_A$ at interface

The temperature distribution in the water is characterized by large gradients near the surface (e).

**6. (c) Solution:**

Given : h = 4 m, Q = 10 $m^3$/hr; $V$ = 12 m/s; $C_P$ = 0.3

Hydraulic power required to pump water,

$$\dot{m}gh = \frac{1000 \times 10}{3600} \times 9.81 \times 4 = 109 \text{ Watt}$$

$$\text{Mechanical power required} = \frac{109}{0.5 \times 0.8} = 272.5 \text{ W}$$

$$\text{Power available in wind} = \frac{1}{2}\rho_{air} A V^3$$

$$= \frac{1}{2} \times 1.2 \times \pi r^2 \times 12^3$$

$$= 3257.2\, r^2$$

$$\text{Power extracted} = C_p \times \text{Power available in wind}$$

or $$272.5 = 0.3 \times 3257.2\, r^2$$

$$r = \sqrt{\frac{272.5}{0.3 \times 3257.2}}$$

$$r = 0.528 \text{ m}$$ **Ans.**

Now, $$\text{Tip speed ratio, } \lambda = \frac{r\omega}{V}$$

$$1 = \frac{0.528 \times \omega}{12}$$

∴ $$\omega = 22.72 \text{ rad/s or N} = 217.03 \text{ rpm}$$ **Ans.**

Tip speed ratio for optimum output,

$$\lambda_0 = \frac{4\pi}{n}$$

or $$\frac{\omega_0 \times r}{V} = \frac{4\pi}{n}$$

$$\omega_0 = \frac{4\pi V}{r \times n} = \frac{4\pi \times 12}{0.528 \times 3}$$

$$\omega_0 = 95.19 \text{ rad/s}$$ **Ans.**

**7. (a) Solution:**

Given : Air flow over a flat plate,

Plate surface temperature = 80°C

Air velocity ($u_\infty$) = 50 m/s

Plate length ($L$) = 45 cm = 0.45 m

Plate width = 60 cm = 0.6 m

The transition to turbulence occurs at

$$Re_x = \frac{u_\infty x_c}{\nu} = 4 \times 10^5$$

$$x_c = \frac{Re_x \nu \times 10^5}{u_\infty} = \frac{4 \times 10^5 \times 18.1 \times 10^{-6}}{50}$$

$$x_c = 0.1448 \text{ m}$$

The Reynold number at the end on the plate is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{50 \times 0.45}{18.1 \times 10^{-6}} = 1.243 \times 10^6$$

**(i)** For the laminar region, $h_{c_L}$ is given by

$$h_{c_L} = \frac{1}{x_c}\int_0^{x_c} \frac{k}{x} 0.332 \text{Re}_x^{1/2} P_r^{1/3} dx$$

$$= \frac{k}{x_c} 0.664 \text{Re}_x^{1/2} P_r^{1/3}$$

$$h_{c_L} = \frac{0.0269}{0.1448} \times 0.664 \times \left(4 \times 10^5\right)^{1/2} (0.71)^{1/3}$$

$$h_{c_L} = 69.599 \text{ W/m}^2\text{K}$$

For the turbulent region, $h_{c_T}$ is

$$h_{c_T} = \frac{1}{L - x_c}\int_{x_c}^{L} \frac{k}{x} 0.0296 (\text{Re}_x)^{0.8} (\text{Pr})^{1/3} dx$$

$$= \frac{k}{L - x_c} 0.037\left((\text{Re}_L)^{0.8} - \left(\text{Re}_{x_c}\right)^{0.8}\right)(P_r)^{1/3}$$

$$h_{c_T} = 130.263 \text{ W/m}^2\text{K}$$

**(ii)** The total heat transfer is the sum of the heat transfer from both regions

$$q = q_{lam} + q_{turb}$$

$$= (h_{CL}A_L + h_{CT}A_T)(T_s - T_\infty)$$

$$q = [(69.599 \times 0.1448 \times 0.6) + (130.263)(0.45 - 0.1448)(0.6)][80 - 0]$$

$$= 2392.04 \text{ W}$$

For both sides, $q_{Total} = 2 \times q$

$$= 4784.08 \text{ W}$$

**7. (b) (i) Solution:**

Given : B.P. = 200 kW, $\eta_{Bth} = 0.35$, $\eta_{gasifier} = 0.6$, $(\text{C.V.})_{producer\ gas} = 12500$ kJ/kg, $(\text{C.V})_{diesel} = 45000$ kJ/kg

Brake thermal efficiency, $\eta_{Bth} = \dfrac{B.P}{H.A/\text{sec}}$

$$0.35 = \frac{200}{H.A/\text{sec}}$$

$$\therefore \quad \frac{H.A}{\text{sec}} = \frac{200}{0.35}$$

$$= 571.428 \text{ kW}$$

For 80% diesel replacement

$$0.8 \times \frac{H.A}{\text{sec}} = \dot{m}_p \times (C.V)_P$$

$$\therefore \quad 0.8 \times 571.428 = \dot{m}_p \times 12500$$

$$\therefore \quad \dot{m}_p = 131.657 \text{ kg/hr}$$

Now, gasifier efficiency,

$$\eta_{\text{gasifier}} = \frac{\dot{m}_p}{\dot{m}_s}$$

$$\therefore \quad 0.6 = \frac{131.657}{\dot{m}_s}$$

$$\therefore \quad \dot{m}_s = 219.42 \text{ kg/hr}$$ Ans.

**7. (b) (ii) Solution:**

Direct methanol fuel cell is a modified version of PEMFC in which methanol is used directly without reforming instead of pure hydrogen. Catalytic reforming process is not required. Storage of methanol is much easier than that of hydrogen as it does not need to be done at high pressures or low temperatures.

Liquid $CH_3OH$ is oxidised in the presence of water at the anode, generating $CO_2$, hydrogen ions and the electrons. The hydrogen ions travel through the electrolyte and react with oxygen from air and the electrons from the external circuit to form water at the anode.

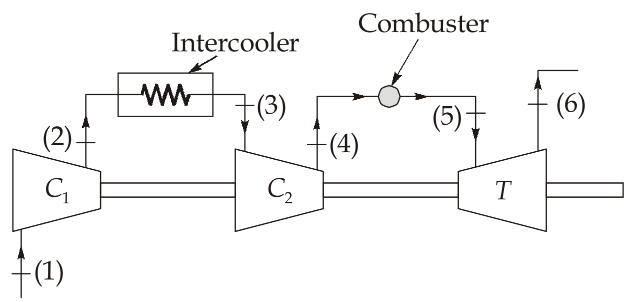
**Anode :** $CH_3OH + H_2O \rightarrow CO_2 + 6H^+ + 6e^-$

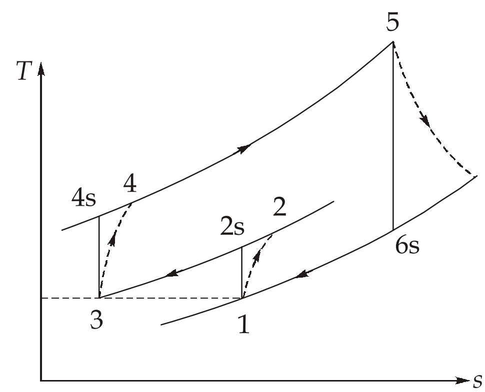
**Cathode :** $\frac{3}{2}O_2 + 6H^+ + 6e^- \rightarrow 3H_2O$

**Overall cell reaction :** $CH_3OH + \frac{3}{2}O_2 \rightarrow CO_2 + 2H_2O$

They have low operating temperatures 50-120°C and can produce small amount of power over a long period of time. DMFC have potential application in small to mid-sized gadgets like cellular phones, digital cameras, laptop computers etc. Their batteries can be instantly recharged by replacing disposal fuel cartridge.

**7. (c) Solution:**




Given: $T_{max} = T_5$, $T_{min} = T_1 = T_3$

Isentropic efficiencies of compressors = $\eta_c$

Isentropic efficiency of turbine = $\eta_T$

For process. 1-2.
$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{... (i)}$$

Let,
$$\frac{P_2}{P_1} = r_p$$

For maximum work output, perfect intercooling then,
$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = r_p$$

and overall pressure ratio, $\dfrac{P_4}{P_1} = \dfrac{P_4}{P_2} \times \dfrac{P_2}{P_1} = r_p^2 = R$

So, from equation (i), $\frac{T_{2s}}{T_1} = \left(\sqrt{R}\right)^{\frac{\gamma-1}{\gamma}} = (R)^{\frac{\gamma-1}{2\gamma}}$ ... (ii)

Similarly, for process 3-4,

$$\frac{T_{4s}}{T_3} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{4s}}{T_1} = \left(\sqrt{R}\right)^{\frac{\gamma-1}{\gamma}} \qquad (\because T_3 = T_1)$$

$$\frac{T_{4s}}{T_1} = (R)^{\frac{\gamma-1}{2\gamma}} \qquad \text{... (iii)}$$

and for process, 5-6

$$\frac{T_5}{T_{6s}} = (R)^{\frac{\gamma-1}{\gamma}}$$

$\Rightarrow$ $$T_{6s} = \frac{T_5}{(R)^{\frac{\gamma-1}{\gamma}}} \qquad \text{... (iv)}$$

Now, Compressor work, $W_C = W_{C1} + W_{C2}$

$$= c_p(T_2 - T_1) + c_p(T_4 - T_3) \qquad \text{... (v)}$$

Also, we know, $\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{T_{4s} - T_3}{T_4 - T_3}$

So, $$T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_c}$$

$$T_4 - T_3 = \frac{T_{4s} - T_3}{\eta_c}$$

Putting values in equation (v), we get

$$W_C = c_p\left[\frac{T_{2s} - T_1}{\eta_c} + \frac{T_{4s} - T_3}{\eta_c}\right]$$

$$= \frac{c_p T_1}{\eta_c}\left[\left(\frac{T_{2s}}{T_1} - 1\right) + \left(\frac{T_{4s}}{T_1} - 1\right)\right] \qquad (\because T_3 = T_1)$$

Putting values from equation (ii) and (iii) we get,

$$W_C = \frac{c_p T_1}{\eta_c}\left[(R)^{\frac{\gamma-1}{2\gamma}} - 1 + (R)^{\frac{\gamma-1}{2\gamma}} - 1\right]$$

$$= \frac{2c_p T_1}{\eta_c}\left[(R)^{\frac{\gamma-1}{2\gamma}} - 1\right] \quad \text{... (vi)}$$

Now, Turbine work $= c_p(T_5 - T_6)$ ... (vii)

From, Isentropic efficiency, $\eta_T = \dfrac{T_5 - T_6}{T_5 - T_{6s}}$

$$T_5 - T_6 = \eta_T(T_5 - T_{6s})$$

Putting value in equation (vii) we get,

$$W_T = c_p(\eta_T)(T_5 - T_{6s})$$

$$= c_p \times \eta_T\left(T_5 - \frac{T_5}{R^{\gamma-1/\gamma}}\right)$$

$$= c_p \times (\eta_T)T_5\left(1 - \frac{1}{R^{\gamma-1/\gamma}}\right) \quad \text{... (viii)}$$

Now, Net work, $W = W_T - W_C$

From equation (vi) and (vii) we get,

$$W = c_p(\eta_T)T_5\left(1 - \frac{1}{R^{\gamma-1/\gamma}}\right) - \frac{2c_p T_1}{\eta_c}\left(R^{\frac{\gamma-1}{2\gamma}} - 1\right)$$

For maximum work output,

$$\frac{dW}{dR} = 0$$

$$\Rightarrow c_p \eta_T T_5\left(\frac{\gamma-1}{\gamma}\right)(R)^{\frac{1-2\gamma}{\gamma}} - \frac{2c_p T_1}{\eta_c}\left(\frac{\gamma-1}{2\gamma}\right)(R)^{\frac{-1-\gamma}{2\gamma}} = 0$$

$$\Rightarrow \quad \eta_T T_5\left(\frac{\gamma-1}{\gamma}\right)(R)^{\frac{1-2\gamma}{\gamma}} = \frac{2T_1}{\eta_c}\left(\frac{\gamma-1}{2\gamma}\right)(R)^{\frac{-1-\gamma}{2\gamma}}$$

$$\Rightarrow \quad \eta_T T_5 \frac{\eta_c}{T_1} = (R)^{\frac{-1-\gamma}{2\gamma} - \left(\frac{1-2\gamma}{\gamma}\right)}$$

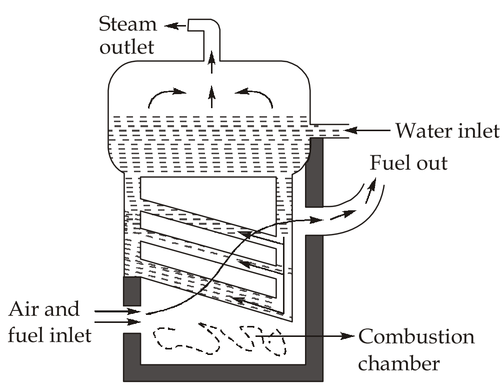
$$\Rightarrow \quad (R)^{\frac{-1-\gamma-2+4\gamma}{2\gamma}} = \frac{T_5}{T_1}\eta_c\eta_T$$

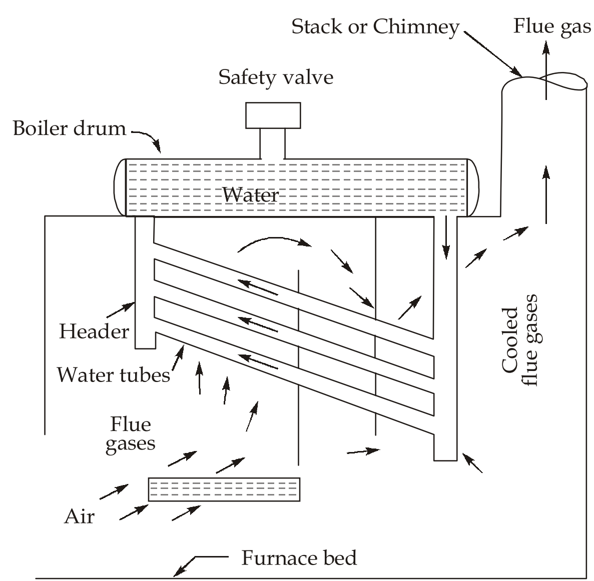
$$\Rightarrow \quad (R)^{\frac{3\gamma-3}{2\gamma}} = \frac{T_5}{T_1}\eta_c\eta_T$$

$$\Rightarrow \quad R = \left[\frac{T_5}{T_1}\eta_c\eta_T\right]^{\frac{2\gamma}{3(\gamma-1)}}$$

$$R = \left[\frac{T_{max}}{T_{min}}\eta_c\eta_T\right]^{\frac{2\gamma}{3(\gamma-1)}} \qquad \because T_{max} = T_5 \text{ and } T_{min} = T_1]$$

**8. (a) (i) Solution:**




| Fire Tube Boilers | Water Tube Boilers |
|---|---|
| (i) Hot gases inside the tubes and water outside the tubes | (i) Water inside the tubes and hot gases outside the tubes |
| (ii) Generally internally fired | (ii) Externally fired |
| (iii) Operating pressure limited to 16 bar | (iii) Can work under as high pressure as 100 bar |
| (iv) Lower steam production rate | (iv) Higher rate of steam production |
| (v) Various parts not so easily accessible for cleaning, repair and inspection | (v) Various parts are more accessible |
| (vi) Difficult in construction | (vi) Simple in construction |

**8. (a) (ii) Solution:**

A forced circulation boiler was first introduced in 1925 by La Mont. The arrangement of water circulation and different components are shown in figure below.

The feed water from hot well is supplied to a storage and separating drum (boiler) through the economizer. Most of the sensible heat is supplied to the feed water passing through the economizer. A pump circulates the water at a rate 8 to 10 times the mass of steam evaporated. This water is circulated through the evaporator tubes and the part of the vapour is separated in the separator drum. The large quantity of water circulated (10 times that of evaporation) prevents the tubes from being overheated.

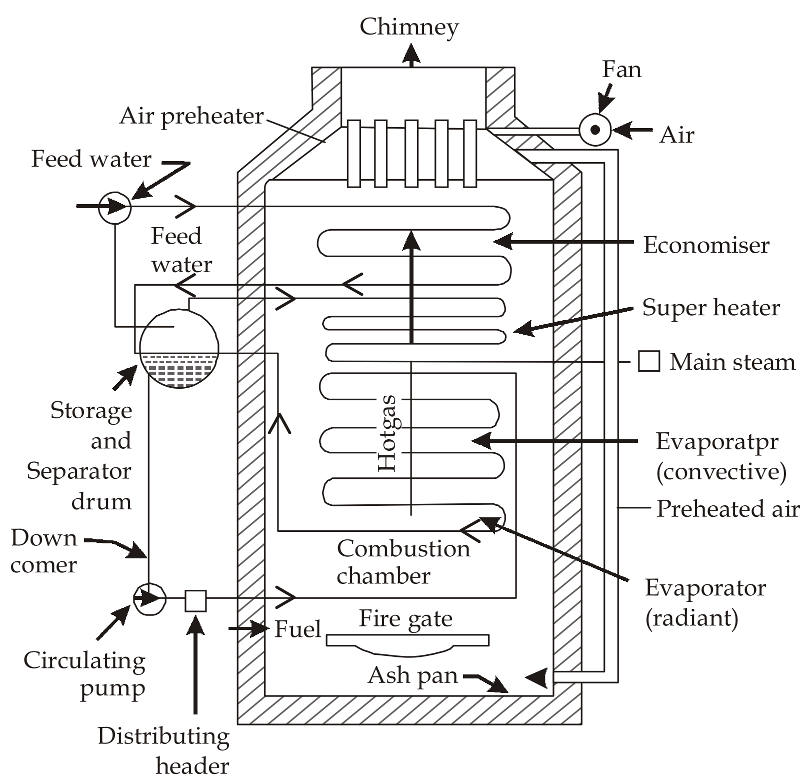



**Figure :** La Mont Boiler

The centrifugal pump delivers the water to the headers at a pressure of 2.5 bar above the drum pressure. The distribution headers distribute the water through the nozzle into the evaporator.

The steam separated in the boiler is further passed through the super-heater.

Secure a uniform flow of feed water through each of the parallel boiler circuits a choke is fitted entrance to each circuit.

These boilers have been built to generate 45 to 50 tonnes of superheated steam at a pressure of 120 bar and temperature of 500°C.

**8. (b) Solution:**

The word gasification (or thermal gasification) implies converting solid fuel into a gaseous fuel by thermochemical method without leaving any solid carbonaceous residue.

Gasification involves partial combustion (oxidation in restricted quantity of air/oxidant) and reduction operations of biomass. In a typical combustion process, generally the oxygen is surplus, while in a gasification process, the fuel is surplus. The combustion products, mainly carbon dioxide, water vapour, nitrogen, carbon monoxide and hydrogen pass through the glowing layer of charcoal for the reduction process to occur. During this stage, both carbon dioxide and water vapour, oxidize the char to form CO, $H_2$ and $CH_4$. The following are the typical reactions, which occur during gasification.

$$C + O_2 \rightarrow CO_2 \qquad \text{(Combustion, 240 kJ/mole)}$$

The moisture available in the biomass is converted to steam and generally no extra moisture is required. Thus the product of combustion of pyrolysis gases results in $CO_2$ and $H_2O$ and $H_2O$ (steam), which further react with char:

$$C + O_2 \rightarrow 2CO \quad \text{(Boudouard reaction, – 164.9 kJ/mole)}$$

$$C + H_2O \rightarrow CO + H_2 \quad \text{(Water gas reaction – 122.6 kJ/mole)}$$

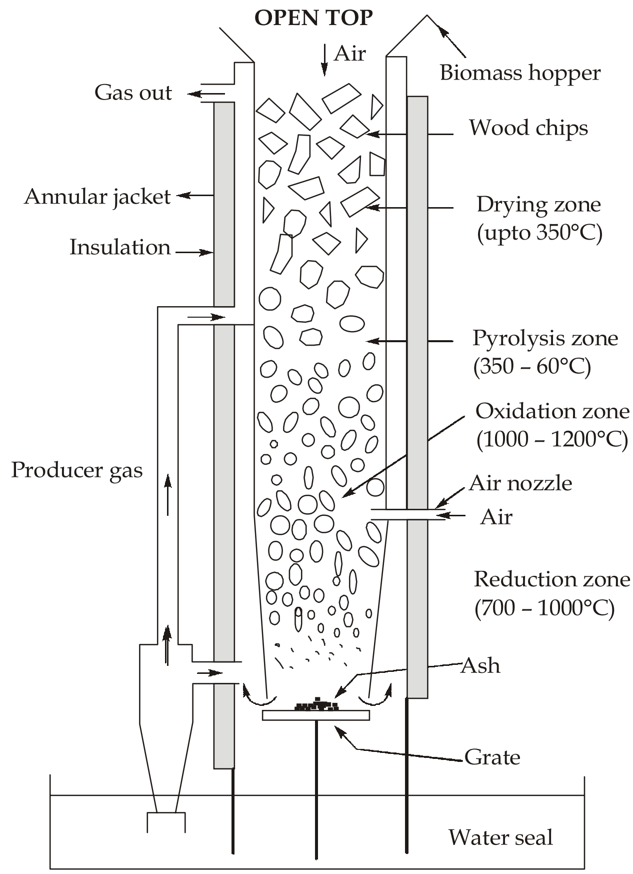
$$CO + H_2O \rightarrow CO_2 + H_2 \quad \text{(Water shift reaction, 40.2 kJ/mole)}$$

$$C + 2H_2 \rightarrow CH_4 \quad \text{(Methane reaction, 83.3 kJ/mole)}$$

The output gas is known as producer gas, a mixture of $H_2$(15 – 20%), CO(10 – 20%), $CH_4$(1 – 5%), $CO_2$(9 – 12%) and $N_2$(45 – 55%).

**Downdraft type gasifier :** The downdraft type is best suited for a variety of biomass. Its design forces the raw products to pass through a high-temperature zone so that most of the unburnt pyrolysis products (especially tars) can be cracked into gaseous hydrocarbons, thus producing a relatively clean gas.

In steady-state operation, heat from the combustion zone, near the air nozzle is transferred upwards by radiation, conduction and convection causing wood chips to pyrolyse and lose 70-80% of their weight. These pyrolysed gases burn with air to form CO, $CO_2$, $H_2$ and $H_2O$, thereby raising the temperature to 1000-1200°C. The product gases from the combustion zone further undergo reduction reaction with char to generate combustible products like CO, $H_2$ and $CH_4$. Generally about 40-70% air is drawn through the open top depending on the pressure drop conditions due to the size of wood chips and gas-flow rate. This flow of air opposite to the flame front helps in maintaining homogeneous air/gas flow across the bed. Combining the open top with the air nozzle towards the bottom of the reactor helps in stabilizing the combustion zone by consuming the uncovered char left and also by preventing the movement of the flame front to the top.



**Downdraft biomass gasification plant**

As a consequence, the high-temperature zone spreads above the air nozzle by radiation and conduction, aided by air flow from the top. The tar thus is eliminated in the best possible way by creating a high-temperature oxidizing atmosphere in the reactor itself. The gas produced is withdrawn from an exit at the bottom and reintroduced in the annular jacket for heat recovery. The hot gas which enters the annular jacket around 500°C, transfers some heat to the wood chips inside, improving the thermal efficiency of the system in addition to drying the wood in this zone. The inner wall temperature reaches more than 350°C after a few hours of operation. This aspect enables the use of wood chips with moisture content as high as 25%. The regenerative heating due to the transfer of heat from hot gas to the biomass moving downwards also increases its residence time in the high temperature zone. This leads to better tar cracking

8. **(c) (i) Solution:**

**Wear Debris Analysis:** Wear debris analysis is a useful and cost effective mechanical condition monitoring technique. However, compared with the complementary technique of vibration analysis, little information is available to the engineers that describes the technical details of wear debris analysis. When a machine wears, material from the contacting surfaces enters the lubricant. The forces, material and geometry of the wear site all influence the size, shape and quality of final wear product.

**Advantages of wear debris analysis:**

(i) There is no large capital investment risked in evaluating the wear debris analysis.

(ii) No requirement of a trained specialist.

(iii) It can also be useful and to identifying lubrication problem.

Wear debris analysis is a useful tool that can be used as part of an oil condition monitoring program to gain valuable insight into how machinary is operating, helping to plan for maintenance, reduce downtime, and optimise the lifespan of assets.

It is necessary to schedule debris sampling for wear debris analysis to know the type of wear, severity of wear rate on the equipment within a given time. After wear debris analysis necessary action has to the taken to minimize the failure of the system.

8. **(c) (ii) Solution:**

The failure density function of the electronic component is given by

$$f(t) = \begin{cases} 0.003e^{-0.003t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

So, the reliability of the component,

$$R(t) = \int_t^{\infty} f(t)dt$$

$$R(t) = \int_t^{\infty} 0.003 \cdot e^{-0.003t} \cdot dt$$

$$= \left[\frac{-0.003 \times e^{-0.003t}}{0.003}\right]_t^{\infty} = \left[e^{-0.003t}\right]_{\infty}^{t}$$

$$R(t) = e^{-0.003t}$$

The reliability of the component at 350 hours is

$$R(350) = e^{-0.003 \times 350} = 0.3499$$

$$R(350) = 34.99\%$$

Now, mean time to failure (MTTF) = $\int_0^\infty R(t)dt$

$$\text{MTTF} = \int_0^\infty e^{-0.003t} dt = \left[-\frac{e^{-0.003t}}{0.003}\right]_0^\infty = \left[\frac{1000}{3} \times e^{-0.003t}\right]_\infty^0$$

$$\text{MTTF} = \frac{1000}{3}[1 - 0] = 333.33 \text{ hours}$$

❍❍❍❍