MADE EASY

MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Electrical Engineering Test No : 8

Section A : Electromagnetic Theory + Control Systems + Communications Systems

Q.1 (a) Solution:

Relation in cylindrical and spherical coordinate

$$\rho = r\sin\theta$$

$$\Rightarrow \quad V(\vec{r}) = \frac{1}{r^2} + \frac{3\cos\phi}{r} + 3r\sin 2\theta \qquad \text{(spherical coordinates)}$$

(i)
$$\vec{E} = -\vec{\nabla} V$$

$$= -\left\{\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right\}$$

$$= -\left\{\left(\frac{-2}{r^3} - \frac{3\cos\phi}{r^2} + 3\sin 2\theta\right)\hat{a}_r + \frac{1}{r}(6r\cos 2\theta)\hat{a}_\theta + \frac{1}{r\sin\theta}\left(\frac{-3\sin\phi}{r}\right)\hat{a}_\phi\right\}$$

$$= \left(\frac{2}{r^3} + \frac{3\cos\phi}{r^2} - 3\sin 2\theta\right)\hat{a}_r - 6\cos 2\theta\,\hat{a}_\theta + \frac{3}{r^2}\operatorname{cosec}\theta\sin\phi\,\hat{a}_\phi$$

$$\vec{D} = \varepsilon_o \vec{E}$$

(ε_o : Permittivity of free space)

$$= \varepsilon_o\left(\frac{2}{r^3} + \frac{3\cos\phi}{r^2} - 3\sin 2\theta\right)\hat{a}_r - 6\varepsilon_o\cos 2\theta\,\hat{a}_\theta + \frac{3\varepsilon_o \operatorname{cosec}\theta\sin\phi}{r^2}\hat{a}_\phi \ \text{C/m}^2$$

(ii) Using Gauss's law

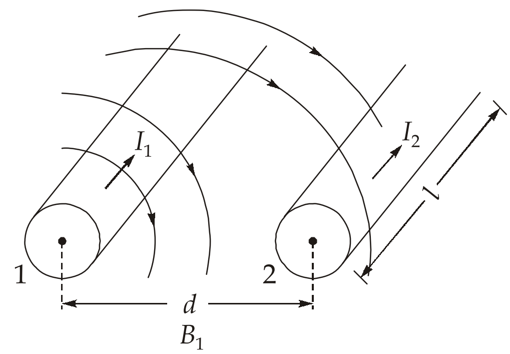
$$\oint \vec{D} \cdot d\vec{s} = q_{\text{enclosed}} = \oint \rho_s \cdot ds$$

$$\Rightarrow \quad \rho_s = D_r$$

$$= \varepsilon_o \left(\frac{2}{r^3} + \frac{3\cos\phi}{r^2} - 3\sin 2\theta \right) \text{C/m}^2$$

Q.1 (b) Solution:

(i) Consider two straight, long parallel lines situated in space as shown below,



Conductor 1 produces a field around and its value is B_1. At the location of conductor 2 its value is given by

$$B_1 = \mu_0 \frac{I_1}{2\pi d}$$

While conductor 2 is carrying a current of I_2 and is situated in the field with flux density B_1.

$$\overrightarrow{dF} = I\, d\vec{l} \times \vec{B}$$

We can write for length l,

$$F_2 = I_2 \vec{l} \times \vec{B}_1$$

$$F_2 = \mu_0 \frac{I_1 I_2 l}{2\pi d}$$

$$F_2 = \frac{4\pi \times 10^{-7} \times 6 \times 6 \times 4}{2\pi \times 5 \times 10^{-3}} = 5.76 \times 10^{-3} \text{ N}$$

According to right hand thumb rule flux between the conductors is nullified when same direction of current is flowing in conductors, so there will be force of attraction between conductors.

(ii) If direction of I_2 is reversed then the flux between the conductor is aiding so force of repulsion is between the conductors of same magnitude.

$$F = 5.76 \times 10^{-3} \text{ N}$$

Q.1 (c) Solution:

Given, $$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$$

$$\angle\phi = \angle G(s) = -90° - \tan^{-1}(0.2\omega) - \tan^{-1}(0.05\,\omega)$$

At ω_{pc} : $$\angle G(s) = -180°$$

$$-90° - \tan^{-1}(0.2\omega) - \tan^{-1}(0.05\omega) = -180°$$

$$\tan^{-1}\left[\frac{0.2\omega + 0.05\omega}{1-(0.2\omega)(0.05\omega)}\right] = 90°$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{0.25\omega}{1-0.01\omega^2}\right] = 90°$$

$$\Rightarrow \quad 1 - 0.01\,\omega^2 = 0$$

$$\Rightarrow \quad \omega = 10 \text{ rad/sec}$$

Given $$20\log(\text{GM}) = 40 \text{ dB}$$

$\therefore$ At ω_{pc} : $$|G(s)|_{s=j\omega} = \frac{K}{\omega\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.05\omega)^2}}$$

$$|G(s)|_{\omega_{pc}} = \frac{K}{10\sqrt{1+2^2}\sqrt{1+0.5^2}} = \frac{K}{25}$$

$\because$ $$20\log_{10}(\text{GM}) = 40$$

$$\log_{10}(\text{GM}) = 2$$

$$\Rightarrow \quad \text{GM} = 100 = \frac{1}{M}$$

$$\Rightarrow \quad \frac{25}{K} = 100$$

$$\Rightarrow \quad K = 0.25$$

Q.1 (d) Solution:

For sinusoidal modulation, current relation between carrier and modulated signal is given by,

$$\frac{I_t}{I_c} = \sqrt{1+\frac{m^2}{2}} \quad \text{...(i)}$$

Where, I_t = modulated current

I_c = carrier current

m = modulation index

According to question, $I_t = 11$ A, when $m = 0.4$

∴ From equation (i),

$$I_c = \frac{I_t}{\sqrt{1+\frac{m^2}{2}}} = \frac{11}{\sqrt{1+\frac{0.4^2}{2}}} = 10.585 \text{ A}$$

Now, simultaneous modulation by another audio sine wave:

∴ Again from equation (i),

$$m_t = \sqrt{2\left(\left(\frac{I_t}{I_c}\right)^2 - 1\right)}$$

$$m_t = \sqrt{2\left(\left(\frac{12}{10.585}\right)^2 - 1\right)} = 0.7553$$

Modulation index due to second wave,

$$m_2 = \sqrt{m_t^2 - m^2} = \sqrt{(0.7569)^2 - (0.4)^2} = 0.641$$

Modulation index due to second wave is 0.641.

Q.1 (e) Solution:

At point P :

Carrier frequency, $f_C = 4 \times 10^6 \times 10 = 40$ MHz

$\Delta f = 4 \times 10^3 \times 20 = 80$ kHz

Modulation index after the multiplier,

$$m_{fp} = 4 \times 5 = 20$$

Minimum frequency, f_{min} = 40 MHz – 80 kHz

= 39.92 MHz

Maximum frequency, f_{max} = 40 MHz + 80 kHz

= 40.08 MHz

At point Q : we have

f_C = 40 MHz + 10 MHz

= 50 MHz

$f_{max} = 40.08 + 10 = 50.08$ MHz

$f_{min} = 39.92 + 10 = 49.92$ MHz

$\therefore$ Frequency deviation, $\Delta f = f_{max} - f_C$

$= (50.08 - 50)$ MHz

$= 80$ kHz

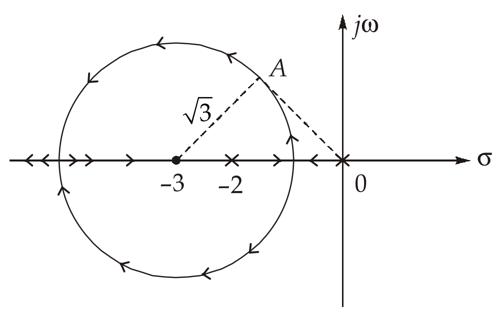
As there is no changing in frequency deviation due to mixing, the modulation index will remain same (i.e. $m_f = 20$)

Q.2 (a) Solution:

Given, $$G(s)H(s) = \frac{K(s+3)}{s(s+2)}$$

Poles occur at $s = 0$, $s = -2$

zero occur at $s = -3$



Breakaway point, $$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+3)}{s(s+2)} = 0$$

$$s^2 + (2 + K)s + 3K = 0$$

$$K = \frac{-(s^2+2s)}{s+3}$$

$$\frac{dK}{ds} = -\left[\frac{(s+3)(2s+2)-(s^2+2s)}{(s+3)^2}\right] = 0$$

$$s^2 + 6s + 6 = 0$$

$$s = \frac{-6 \pm \sqrt{36-24}}{2} = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$s = -1.27, -4.73$$

Number of asymptotes $= 1$

Angle of asymptotes : $$\theta_A = \frac{(2K+1)180°}{P-Z} = \frac{(2\times 0+1)}{1} \times 180° = 180°$$

By using angle criteria, and put $s = \alpha + j\omega$

$$\angle(s+3) - \angle s - \angle(s+2) = 180°$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\omega}{\alpha+3}\right) - \tan^{-1}\left(\frac{\omega}{\alpha}\right)\right] = \tan\left[180° + \tan^{-1}\left(\frac{\omega}{\alpha+2}\right)\right]$$

$$\Rightarrow \frac{\frac{\omega}{\alpha+3} - \frac{\omega}{\alpha}}{1 + \frac{\omega^2}{\alpha(\alpha+3)}} = \frac{0 + \frac{\omega}{\alpha+2}}{1 - 0 \times \frac{\omega}{\alpha+2}}$$

$$-3(\alpha + 2) = \alpha^2 + 3\alpha + \omega^2$$

$$\Rightarrow \alpha^2 + 6\alpha + \omega^2 + 6 = 0$$

$$\Rightarrow (\alpha + 3)^2 + (\omega)^2 = (\sqrt{3})^2$$

This equation represents circle of radius $\sqrt{3}$ with centre (–3, 0)

For underdamped conditions:

$$s = -1.27$$

and $$s = -4.73$$

$$K_{min} G(s)\big|_{s=-1.27} = -1$$

$$K_{min}\left(\frac{-1.27+3}{-1.27(-1.27+2)}\right) = -1$$

$$K_{min} = 0.536$$

and $$K_{max} G(s)\big|_{s=-4.73} = -1$$

$$K_{max}\left(\frac{(-4.73+3)}{(-4.73)(-4.73+2)}\right) = -1$$

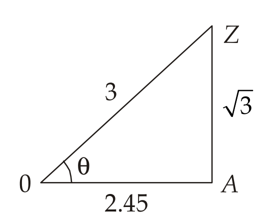
$$K_{max} = 7.46$$

The range of K: $0.536 < K < 7.46$

For minimum value of damping ratio:

$$\xi_{min} = \cos\theta$$

$$= \frac{2.45}{3} = 0.8167$$



Q.2 (b) Solution:

(i) The transfer function of the given system is

$$\frac{C(s)}{R(s)} = \frac{10}{s^2+2s+10}$$

Here, $H(s) = 1 \text{ and } G(s) = \frac{10}{s(s+2)};$

Therefore, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

The characteristic equation is

$$s^2 + 2s + 10 = 0$$

Comparing with characteristic equation of second-order unity feedback system,

we get, $\omega_n = \sqrt{10} = 3.16 \text{ rad/sec}$

and $2\xi\omega_n = 2$

or $\xi = \frac{1}{\omega_n} = \frac{1}{3.16} = 0.32$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.16\sqrt{1-0.32} = 3 \text{ rad/sec}$$

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-0.32^2}}{0.32} = 71.34°$$

Now the output response is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \beta)$$

Substituting the values in the expression, we get

$$c(t) = 1 - \frac{e^{-0.32\times3.16t}}{\sqrt{1-0.32^2}}\sin(3t + 71.34°)$$

or $c(t) = 1 - 1.05e^{-t}\sin(3t + 71.34°)$

(ii) The natural frequency (ω_n) and damping ratio (ξ) have been calculated in part (i) above as

$$\omega_n = 3.16 \text{ rad/sec}$$

(iii) Peak overshoot,

$$\%M_p = 100\times e^{-\frac{\xi}{\sqrt{1-\xi^2}}\pi} = 100\times e^{-\frac{0.32}{\sqrt{1-0.32^2}}\pi} = 34.63\%$$

Peak time, $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{3.16\sqrt{1-0.32^2}} = 1.04 \text{ sec}$

(iv) Stead-state error,

The input is 1 + 4t, In Laplace transform for

$$R(s) = \frac{1}{s} + \frac{4}{s^2} = \frac{s+4}{s^2}$$

Now, $$E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{1}{1+\dfrac{10}{s(s+2)}} \times \frac{s+4}{s^2} = \frac{s(s+4)(s+2)}{s^2(s^2+2s+10)}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \frac{s \times s(s+4)(s+2)}{s^2(s^2+2s+10)} = \frac{8}{10} = 0.8$$

Now, $$G(s) = \frac{10}{s(s+2)}$$

It is given that poles are added at $\pm j\sqrt{3}$. This means $G(s)$ will have another denominator term equal to $(s^2 + 3)$. Therefore,

$$G(s) = \frac{10}{s(s+2)(s^2+3)}$$

The characteristic equation is 1 + GH = 0;

i.e. $$1 + \frac{10}{s(s+2)(s^2+3)} \times 1 = 0$$

or $$s(s + 2)(s^2 + 3) + 10 = 0$$

or $$s^4 + 2s^3 + 3s^2 + 6s + 10 = 0$$

Now, we have to find the absolute stability. This we will ascertain by Routh Hurwitz criterion.

s^4	1	3	10
s^3	2	6	0
s^2	0	10	0

A zero has appeared in the first column of s^2 row,

Therefore, put $s = \dfrac{1}{z}$

Putting $s = \frac{1}{z}$ in the characteristic equation, we get

$$10z^4 + 6z^3 + 3z^2 + 2z + 1 = 0$$

Developing the Routh's array, we get

z^4	10	3	1
z^3	6	2	0
z^2	$-\frac{1}{3}$	1	0
z^1	20	0	0
z^0	1	0	0

Sign change ($z^3 \to z^2$), Sign change ($z^2 \to z^1$)

There are two sign changes in the first column of the Routh's array. Hence, there are two roots lying on the right hand side of s- plane. Hence, the closed- loop system unstable.

Q.2 (c) Solution:

- The open-loop transfer function of the uncompensated system is,

$$G_f(s) = \frac{10}{s(s+1)}$$

Finding the phase margin of uncompensated system :

- The gain crossover frequency of uncompensated system is,

$$\frac{10}{\omega_{gc}\sqrt{1+\omega_{gc}^2}} = 1$$

$$\omega_{gc}^4 + \omega_{gc}^2 - 100 = 0$$

$$\omega_{gc}^2 = 9.51 \quad \text{(taking only positive value)}$$

$$\omega_{gc} = 3.1 \text{ rad/sec}$$

- The phase of the system at $\omega = \omega_{gc}$ is,

$$\phi_{gc} = -90° - \tan^{-1}(\omega_{gc}) = -90° - \tan^{-1}(3.1) = -162°$$

- The phase margin of the uncompensated system is,

$$(PM)_{uncompensated} = 180° - 162° = 18°$$

To design the required compensator:

$$(PM)_{uncompensated} = 18°$$

$$(PM)_{overall} \geq 43°$$

- So, the compensator to be designed should be a lead-compensator and the phase lead to be provided at new gain crossover frequency is,

$$\phi_m = 43° - 18° + \varepsilon$$

ε = margin of safety and it can be taken as 5° for this problem, as the gain of the uncompensated system is rolling-off with a slope of –40 dB/decade nearer to its gain crossover frequency.

So, $\phi_m = 43° - 18° + 5° = 30°$

- The general form of the transfer function of lead compensator to be designed can be given as,

$$G_c(s) = \frac{K_c(1+s\tau)}{(1+s\alpha\tau)} \quad \text{...(i)}$$

- The value of K_c can be determined by using desired K_v as follows:

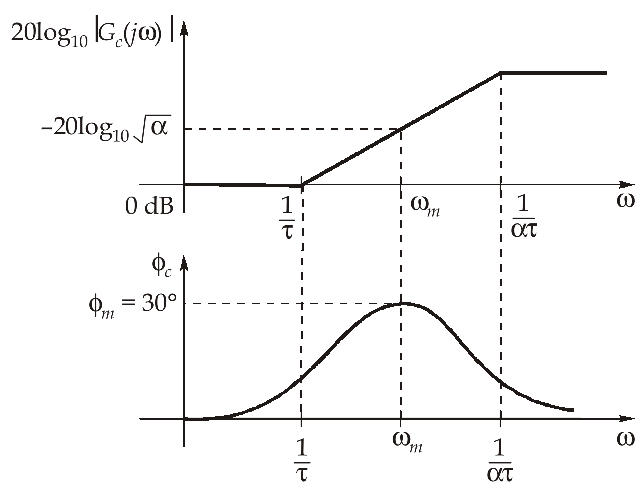
$$K_v = \lim_{s\to 0} sG_f(s)G_c(s) = \lim_{s\to 0} \frac{10K_c(1+s\tau)s}{s(s+1)(1+s\alpha\tau)}$$

$$10K_c = 10 \Rightarrow K_c = 1$$

- The constant "α" can be given as,

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1-0.5}{1+0.5} = \frac{1}{3}$$

- The magnitude and phase bode plots of the lead compensator block can be given as follows:



$$\omega_m = \sqrt{\left(\frac{1}{\tau}\right)\left(\frac{1}{\alpha\tau}\right)} = \frac{1}{\sqrt{\alpha}\,\tau}$$

$$\tau = \frac{1}{\omega_m \sqrt{\alpha}}$$

- The value of ω_m should be selected at the new gain crossover frequency. So, at $\omega = \omega_m$ the gain of overall open-loop transfer function should be unity and it can be determined as follows:

$$|G_f(j\omega)||G_c(j\omega)| = 1$$

$$|G_c(j\omega)|_{\omega=\omega_m} = \frac{1}{\sqrt{\alpha}}$$

So,
$$|G_f(j\omega)||_{\omega=\omega_m} = \sqrt{\alpha}$$

$$\frac{10}{\omega_m\sqrt{(1+\omega_m^2)}} = \sqrt{\alpha} = \frac{1}{\sqrt{3}}$$

$$\omega_m^4 + \omega_m^2 - 300 = 0$$

$$\omega_m^2 = 16.83 \qquad \text{(considering only positive value)}$$

$$\omega_m = 4.1 \text{ rad/sec}$$

$$\tau = \frac{1}{\omega_m\sqrt{\alpha}} = 0.42$$

$$\alpha\tau = 0.14$$

- By substituting the value of τ, $\alpha\tau$ and K_c in equation (i), we get the transfer function of the desired compensator, which is as follows:

$$G_c(s) = \frac{(1+0.42s)}{(1+0.14s)}$$

- The open-loop transfer function of the overall system (or) compensated system is,

$$G(s) = G_f(s)\,G_c(s) = \frac{10(1+0.42s)}{s(1+s)(1+0.14s)}$$

- The gain crossover frequency of the overall system is $\omega_{gc} = \omega_m = 4.1$ rad/sec
- The phase margin of the overall system can be calculated as,

$$\phi_{gc} = -90° - \tan^{-1}(\omega_{gc}) - \tan^{-1}(0.14\,\omega_{gc}) + \tan^{-1}(0.42\omega_{gc})$$

$$= -136.3°$$

$$(\text{PM})_{\text{compensated}} = 180° - 136.3° = 43.7°$$

- The phase margin of uncompensated system is 18°. So, the designed compensator improved the phase margin of the system by 25.7°.

Q.3 (a) Solution:

(i) Carrier amplitude is 10, the power is

$$P = \frac{(10)^2}{2} = 50$$

(ii) Frequency deviation is obtained as

$$\omega = \frac{d}{dt}(\theta(t))$$

$$= \omega_c + 15000 \cos 3000t + (20000\pi) \cos (2000\pi t)$$

Carrier deviation is 15000 cos 3000t + 20000π cos (2000πt)

Maximum carrier deviation,

$$\frac{\Delta\omega}{2\pi} = \Delta f = \frac{15000 + 20000\pi}{2\pi} = 12387.32 \text{ Hz}$$

(iii)

$$\beta = \frac{\Delta f}{B} = \frac{12387.32}{(2000\pi / 2\pi)} = 12.387$$

∵ B = Signal bandwidth = highest frequency in $m(t)$

$$B = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}$$

(iv)

$$\theta(t) = \omega t + 5 \sin (3000t) + 10 \sin (2000\pi t)$$

Phase deviation is maximum value of angle inside parenthesis, is given by Δϕ = 15 rad

(v)

$$B_{FM} = 2(\Delta f + B)$$

$$= 2(12387.32 + 1000)$$

$$= 26774.64 \text{ Hz}$$

Q.3 (b) (i) Solution:

- The transfer function of the given circuit is,

$$\frac{V_0(s)}{V_i(s)} = \frac{R_4}{R_3}\left(\frac{R_2 + \frac{1}{sC_2}}{R_1 \parallel \frac{1}{sC_1}}\right) = \frac{R_4}{R_3}\left(\frac{R_2 + \frac{1}{sC_2}}{\left(\frac{R_1}{1 + sR_1C_1}\right)}\right)$$

$$= \frac{R_4}{R_3R_1}\left[(1+sR_1C_1)\left(R_2+\frac{1}{sC_2}\right)\right]$$

$$= \frac{R_4}{R_3R_1}\left[\left(R_2+\frac{R_1C_1}{C_2}\right)+sR_1R_2C_1+\frac{1}{sC_2}\right]$$

$$= \left[\frac{R_4}{R_3R_1}\left(R_2+\frac{R_1C_1}{C_2}\right)\right]+\left(\frac{R_4R_2C_1}{R_3}\right)s+\left(\frac{R_4}{R_3R_1C_2}\right)\cdot\frac{1}{s}$$

- So, the given circuit can be used as a PID controller. The standard transfer function of a PID controller can be given by,

$$G_c(s) = K_P + K_D s + \frac{K_I}{s}$$

- By comparing the transfer function of the given controller circuit with that of the standard PID controller, we get,

$$K_P = \frac{R_4}{R_3R_1}\left(R_2+\frac{R_1C_1}{C_2}\right)$$

$$K_D = \frac{R_4R_2R_1}{R_3}$$

$$K_I = \frac{R_4}{R_3R_1C_2}$$

Q.3 (b) (ii) Solution:

1.
$$\dot{x}(t) = \begin{bmatrix}0 & 1\\0 & -3\end{bmatrix}x(t)+\begin{bmatrix}1\\0\end{bmatrix}u(t) \quad \text{...(i)}$$

$$\dot{x}(t) = Ax(t)+Bu(t) \quad \text{...(ii)}$$

Comparing equation (i) and (ii), we get

$$A = \text{System matrix} = \begin{bmatrix}0 & 1\\0 & -3\end{bmatrix} \text{ and } B = \begin{bmatrix}1\\0\end{bmatrix}$$

$$[sI-A] = \begin{bmatrix}s & 0\\0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\0 & -3\end{bmatrix} = \begin{bmatrix}s & -1\\s & s+3\end{bmatrix}$$

$$[sI-A]^{-1} = \frac{\begin{bmatrix}s+3 & 1\\0 & s\end{bmatrix}}{s(s+3)} = \begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+3)}\\0 & \frac{1}{s+3}\end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right) \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\text{State transition matrix} = \mathcal{L}^{-1}[(sI - A)^{-1}]$$

$$\phi(t) = \begin{bmatrix} 1 & \frac{1}{3}(1-e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$$

2. ZIR (Zero Input Response) $= \phi(t) \times x(0)$

$$= \begin{bmatrix} 1 & \frac{1}{3}(1-e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1-e^{-3t} \\ 3\,e^{-3t} \end{bmatrix} = \begin{bmatrix} -e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

ZSR (Zero State Response) $= \mathcal{L}^{-1}[(sI - A)^{-1}\,BU(s)]$

$$= \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{s}\right\}$$

$$= \mathcal{L}^{-1}\begin{bmatrix} 1/s^2 \\ 0 \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix}$$

∴ Complete response

$$= ZIR + ZSR = \begin{bmatrix} -e^{-3t} \\ 3e^{-3t} \end{bmatrix} + \begin{bmatrix} t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} t - e^{-3t} \\ 3e^{-3t} \end{bmatrix} u(t)$$

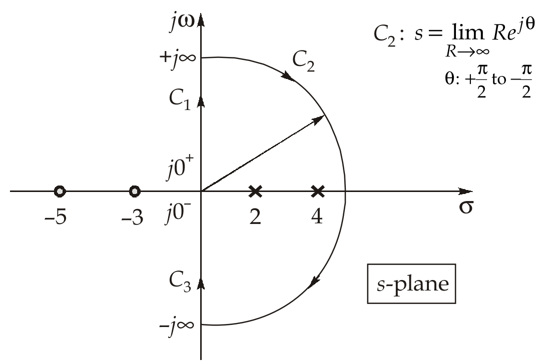
Q.3 (c) Solution:

- The open-loop transfer function of the given system is,

$$L(s) = G(s)H(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)} \quad \text{...(i)}$$

(i) Analysis using Nyquist stability criterion:

- The number of open-loop poles in the right-half of s-plane is, $P = 2$.
- The Nyquist plot of the given system can be obtained by mapping the contour which encloses the entire right-half of s-plane on to the $L(s) = G(s)H(s)$ plane. This contour which encloses the right-half of s-plane is called as the Nyquist contour, which is shown below.



Segment C_1 of the Nyquist contour can be mapped on to the $L(s)$ plane as follows:

- For this segment, $s = j\omega$ and ω is varying from 0^+ to ∞.
- For the purpose of mapping C_1, the open-loop transfer function of the given system can be modified by replacing "s" with "$j\omega$".

$$L(j\omega) = \frac{K(3+j\omega)(5+j\omega)}{(-2+j\omega)(-4+j\omega)}$$

At $\omega = 0^+$, $\quad L(j\omega) = \frac{15K}{8}\angle 0°$

At $\omega = \infty$, $\quad L(j\omega) = K\angle 0°$

At $\omega = 1$, $\quad L(j\omega) = 1.75K\angle 70.35°$

- The intersection points of Nyquist plot with 180° line can be investigated as follows:

$$\tan^{-1}\left(\frac{\omega}{3}\right)+\tan^{-1}\left(\frac{\omega}{5}\right)-180°+\tan^{-1}\left(\frac{\omega}{2}\right)-180°+\tan^{-1}\left(\frac{\omega}{4}\right) = -180°$$

$$\tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right)+\tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right)=180°$$

$$8\omega(8-\omega^2)+6\omega(15-\omega^2) = 0$$

$$\omega(64-8\omega^2+90-6\omega^2) = 0$$

$$14\omega_{pc}^2 - 154 = 0$$

$$\omega_{pc}^2 = 11$$

$$\omega_{pc} = \sqrt{11} \text{ rad/sec} = 3.316 \text{ rad/sec}$$

$$|L(j\omega)|\Big|_{\omega = \omega_{pc} = \sqrt{11}} = \frac{4K}{3} = 1.33K$$

- The intersection points of Nyquist plot with 90° line can be investigated as follows:

$$\tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) + \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right) = 90°$$

$$\tan^{-1}\left[\frac{8\omega(8-\omega^2)+6\omega(15-\omega^2)}{(15-\omega^2)(8-\omega^2)-48\omega^2}\right] = 90°$$

$$120 - 71\omega_{90}^2 + \omega_{90}^4 = 0$$

$$\omega_{90}^2 = \frac{71 \pm \sqrt{(71)^2 - 480}}{2} = 1.73,\ 69.3$$

Valid value ω_{90} will be less than ω_{pc}.

So, $\omega_{90} = \sqrt{1.73} = 1.316 \text{ rad/sec}$

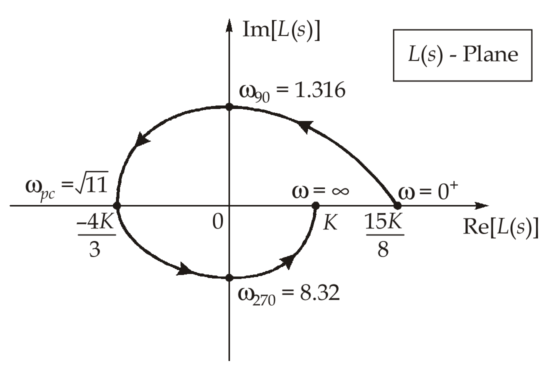
- The intersection points of the Nyquist plot with 270° line can be investigated as follows:

$$\tan^{-1}\left(\frac{8\omega}{15-\omega^2}\right) + \tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right) = 270°$$

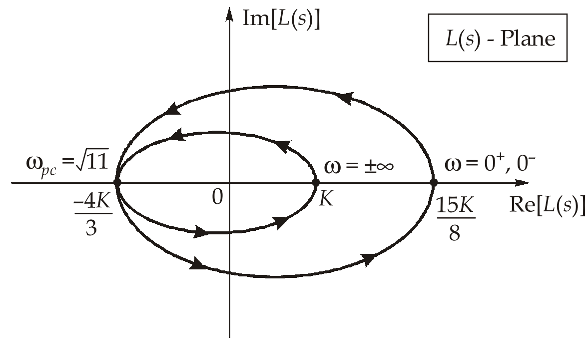
$$120 - 71\omega_{270}^2 + \omega_{270}^4 = 0$$

$$\omega_{270} = \sqrt{69.3} = 8.32 \text{ rad/sec} \qquad (\because \omega_{270} > \omega_{pc})$$

- Nyquist plot corresponding to the segment C_1 of the Nyquist contour is as follows:



Segment C_3 of the Nyquist contour can be mapped by taking the mirror image (about real axis) of the Nyquist plot corresponding to segment C_1 as shown below:

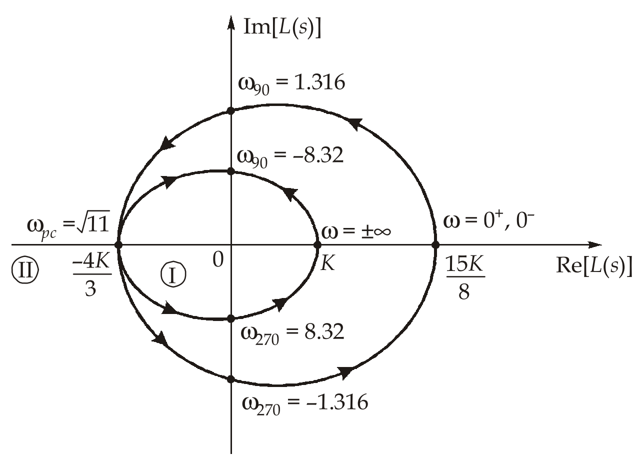


Segment C_2 of the Nyquist contour can be mapped on to the $L(s)$-plane as follows:

- Along the segment C_2, "s" can be written as $\lim\limits_{R\to\infty} Re^{j\theta}$ where θ varies from $+\frac{\pi}{2}$ to $-\frac{\pi}{2}$.

$$L(s) = \lim_{R\to\infty} \frac{K(3+Re^{j\theta})(5+Re^{j\theta})}{(-2+Re^{j\theta})(-4+Re^{j\theta})} = K\angle 0°$$

- So, the segment C_2 of the Nyquist contour can be mapped on to the $L(s)$-plane as a point $K\angle 0°$.
- The overall Nyquist plot of the given system can be drawn as follows:



Nyquist stability criterion:

- To determine the stability of the closed-loop system, the number of encirclements of the point $(-1 + j0)$ by the Nyquist plot should be determined, which will be equal to,

$$N_{(-1+j0)} = P - Z$$

where, P = Number of open-loop poles in the right-half of s-plane

Z = Number of closed-loop poles in the right-half of s-plane

- The closed-loop system will be stable, when $Z = 0$.
- When $Z = 0$, $N_{(-1+j0)} = P - 0 = P = 2$

- When the point $(-1 + j0)$ lies in the region-I of the Nyquist plot,

$$N_{(-1 + j0)} = 2 \Rightarrow \text{system will be stable}$$

When the point $(-1 + j0)$ lies in the region-II of the Nyquist plot,

$$N_{(-1 + j0)} = 0 \Rightarrow \text{system will be stable}$$

- So, to make the closed-loop system stable, the Nyquist plot should encircle the point $(-1 + j0)$, which is possible when $\frac{4K}{3} > 1$.
- Hence, the system will be stable for,

$$\frac{4K}{3} > 1$$

$$K > \frac{3}{4} = 0.75$$

(ii) To verify the result using Routh stability criterion:

- The characteristic equation of the given system is,

$(s^2 - 6s + 8) + K(s^2 + 8s + 15) = 0$

$(1 + K)s^2 + (8K - 6)s + (15K + 8) = 0$

- Using the Routh table, the system will be stable, when the following conditions are satisfied.

s^2	$(1+K)$	$(15K+8)$
s^1	$(8K-6)$	0
s^0	$(15K+8)$	

$(1 + K) > 0 \Rightarrow K > -1$ ...(i)

$(8K - 6) > 0 \Rightarrow K > 0.75$ ...(ii)

$(15K + 8) > 0 \Rightarrow K > -0.53$ ...(iii)

- From conditions (i), (ii) and (iii), we can conclude that the given closed-loop system will be stable for $K > 0.75$.
- The range of K obtained is same in both the methods.

Q.4 (a) Solution:

Given planes $z = 0$ and $z = 0.1$ m are perfect conductors. A voltage difference of 2 V is maintained such that $\vec{E} = |\vec{E}|\hat{a}_z$

(i) Electric field intensity, $\vec{E} = \frac{\Delta V}{\Delta Z}\hat{a}_z = \frac{2}{0.1}\hat{a}_z$

$$\vec{E} = 20\hat{a}_z \text{ V/m}$$

The current density, $\vec{J} = \sigma\vec{E}$

$$\vec{J} = 1000\,e^{-100\rho} \times 20\hat{a}_z$$

$$\vec{J} = 20000\,e^{-100\rho}\hat{a}_z \text{ A/m}^2$$

(ii) The total current in a cylinder of radius ρ is,

$$I = \int_s \vec{J}\cdot\vec{dS} = \int_0^{2\pi}\int_0^{\rho} 20000 e^{-100\rho}\rho d\rho d\phi$$

$$= 20000 \times 2\pi \times \int_0^{\rho} e^{-100\rho}\rho d\rho$$

$$I = 4\pi\times 10^4\left[\frac{-\rho e^{-100\rho}}{100} - \frac{e^{-100\rho}}{10000}\right]_0^{\rho}$$

$$= 4\pi\times 10^4\left[\frac{-\rho e^{-100\rho}}{100} - \frac{e^{-100\rho}}{10000} + \frac{1}{10000}\right]$$

$$I = 4\pi\left\{-100\rho e^{-100\rho} - e^{-100\rho} + 1\right\} A$$

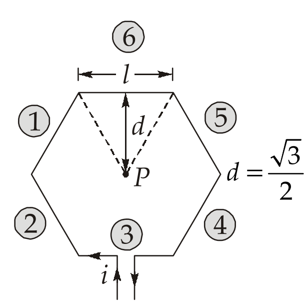
(iii) Applying Ampere's circuit law, taking a circular radial path of radius ρ, we obtain

$$\oint \vec{H}\cdot d\vec{l} = I$$

$$2\pi\rho\vec{H} = 4\pi\left\{-100\rho e^{-100\rho} - e^{-100\rho} + 1\right\}\hat{a}_\phi$$

$$\vec{H} = \frac{2}{\rho}\left\{1 - 100\rho e^{-100\rho} - e^{-100\rho}\right\}\hat{a}_\phi \text{ A/m}$$

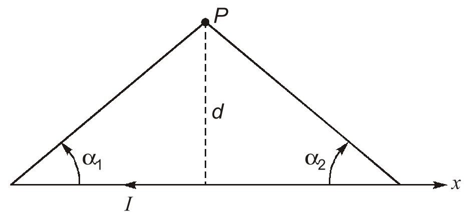
Q.4 (b) (i) Solution:



Here, B at point P is

$$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

We know for each segment of hexagon



Magnetic field intensity due to elemental length dx,

$$\overrightarrow{dH} = \frac{I\,dx}{4\pi r^2}\vec{i} \times \hat{u}_r$$

where, $i \times \hat{u}_r = (\sin\theta)\hat{k}$ .....Perpendicular to plane of paper

$$H = \int dH = \int \frac{I\sin\theta}{4\pi r^2}(dx)\hat{k} \text{ and } B = \mu_0 H$$

So, $$B_p = \frac{\mu_0 I}{4\pi d}(\cos\alpha_1 + \cos\alpha_2)\hat{k}$$

$$d = \frac{\sqrt{3}}{2}l \quad \text{... where } l = 1 \text{ m}$$

and $$\alpha_1 = \alpha_2 = 60°$$

$$|B_p| = \frac{\mu_0 I}{4\pi \frac{\sqrt{3}}{2}}(\cos 60° + \cos 60°)$$

For all six segments,

$$6 \times |B_p| = \frac{4\pi \times 10^{-7} \times 6 \times 1}{4\pi \times \frac{\sqrt{3}}{2}}(1)$$

$$= \frac{12}{\sqrt{3}} \times 10^{-7} = \frac{1.20}{\sqrt{3}} \times 10^{-6}\,T \Rightarrow B = 0.69\ \mu\text{T}$$

Q.4 (b) (ii) Solution:

From the given Bode plot, it is clear that corner frequencies,

$$\omega_{c_1} = \frac{1}{3} \text{ and } \omega_{c_2} = 1$$

∴ Transfer function of given system is given by

$$T(s) = \frac{C(s)}{R(s)} = \left[\frac{1+3s}{1+s}\right] = \left[\frac{1+Ts}{1+\alpha Ts}\right]$$

Here, $T = 3$ and $\alpha T = 1$ or, $\alpha = \frac{1}{3}$

As $\alpha < 1$, therefore the transfer function $T(s)$ represents a lead compensator having $\alpha = \frac{1}{3}$

∴ Maximum phase shift,

$$\phi_m = \tan^{-1}\left[\frac{1-\alpha}{2\sqrt{\alpha}}\right] = \tan^{-1}\left[\frac{1-\frac{1}{3}}{\frac{2}{\sqrt{3}}}\right]$$

$$= \tan^{-1}\left[\frac{2}{3}\times\frac{\sqrt{3}}{2}\right] = \tan^{-1}\left[\frac{1}{\sqrt{3}}\right] = 30°$$

∴ $\phi_m = 30°$

Also,

$$T(j\omega) = \frac{1+j3\omega}{1+j\omega}$$

∴ $$|T(j\omega)| = \frac{\sqrt{1+9\omega^2}}{\sqrt{1+\omega^2}}$$

∴ Gain G_m in dB = 20 log $|T(j\omega)|$

$$= 20\log_{10}\left[\frac{\sqrt{1+9\omega_m^2}}{1+\omega_m^2}\right]$$

Now, $$\omega_m = \frac{1}{T\sqrt{\alpha}} = \frac{1}{\sqrt{3}}$$

∴ $$G_m = 20\log_{10}\left[\sqrt{\frac{1+9\times\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}}\right]$$

$$= 20\log_{10}\left[\sqrt{\frac{1+3}{1+\frac{1}{3}}}\right] = 20\log_{10}\sqrt{3}$$

or, $GM = 4.77$ dB

Q.4 (c) Solution:

(i) Side band frequencies are given by,

$$f = f_c \pm f_m$$

Where, f_c = carrier frequency

f_m = modulating frequency

The upper and lower side frequencies are the sum and difference of frequencies.

$$f_{upper} = 500 + 10 = 510 \text{ kHz}$$

$$f_{lower} = 500 - 10 = 490 \text{ kHz}$$

(ii) Modulation index (m)

$$m = \frac{E_{max} - E_c}{E_c}$$

Where, $E_{max} - E_c$ = peak change in the amplitude of the output waveform

E_c = peak amplitude of carrier signal

$$m = \frac{7.5}{20} = 0.375$$

Percentage modulation (M),

$$M = m \times 100 = 37.5\%$$

(iii) The peak amplitude of the side band frequencies is

$$E_{upper} = \frac{mE_c}{2} = \frac{0.375 \times 20}{2} = 3.75 \ V_p$$

$$E_{lower} = \frac{mE_c}{2} = 3.75 \ V_p$$

(iv) Maximum and minimum amplitudes of the envelope of the output wave are,

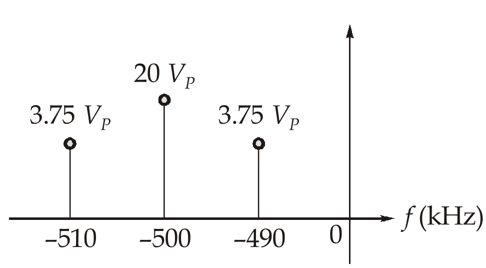
$$V_{max} = E_c(1 + m) = 20(1 + 0.375) \ V_p = 27.5 \ V_p$$

$$V_{min} = E_c(1 - m) = 20(1 - 0.375) \ V_p = 12.5 \ V_p$$

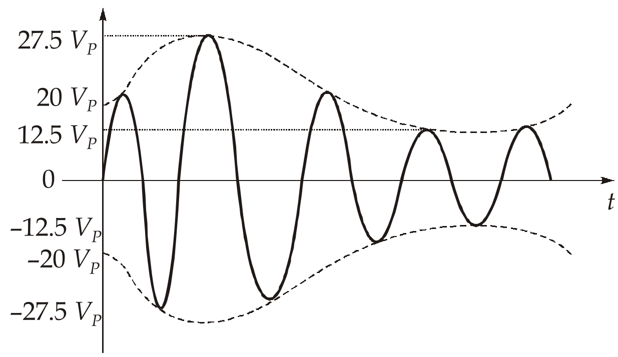
(v) The expression for the modulated wave

$V_{am}(t) = 20 \sin (3.14 \times 10^6)t - 3.75 \cos (3.20 \times 10^6)t + 3.75 \cos (3.08 \times 10^6)t$

Output spectrum:



(vi) AM envelope



Section B : Computer Fundamentals-1 + Electrical & Electronic Measurements-1 + Power Electronics & Drives-2 + Engineering Mathematics-2

Q.5 (a) Solution:

- In demand paging, pages of data are only brought into the main memory when a program accesses them.
- When a context switch occurs, the operating system does not copy any of the old program's pages out to the disk or any of the new program's pages into the main memory. Instead, it just begins executing the new program and fetches that program's pages as they are referenced.
- Swapping is a related technique that uses magnetic media to store the state of programs that are not currently running on the processor. In a system that uses swapping, the operating system treats all of the a program's data as an atomic unit and moves all of the data into or out of the main memory at one time. When the operating system on a computer that uses swapping select a program to run on the processor, it loads all of the program's data into the main memory, evicting other programs from the main memory if necessary.
- If all of the programs being executed on a computer fit into the main memory (counting both their instructions and data), both demand paging and swapping allow the computer to operate in a multiprogrammed mode without having to fetch data from disk. Swapping systems have the advantages that, one a program has been fetched from disk, all of the program's data is mapped in the main memory. This makes the execution time of the program more predictable, since page faults never occur during a program's use of the CPU.
- Demand paging systems have the advantages that they only fetch the pages of data that a program actually uses from the disk. If a program only needs to reference a fraction of its data during each timeslice of execution, this can significantly reduce the amount of time spent copying data to and from the disk. Also, systems that use

swapping typically cannot use their magnetic storage to allow a single program to reference more data than fits in the main memory, because all of a program's data must be swapped into or out of the main memory as a unit.

Q.5 (b) Solution:

Equation of ellipse is,

$$4x^2 + 9y^2 = 9$$

$$\frac{x^2}{9/4} + \frac{y^2}{1} = 1$$

i.e.

$$\frac{x^2}{\left(\frac{3}{2}\right)^2} + \frac{y^2}{1} = 1$$

∴ Major axis is $\frac{3}{2}$, Minor axis is 1

The ellipse meets the x-axis at $\pm\frac{3}{2}$ and the y-axis at ±1.

Given,

$$f(z) = \frac{z \sec z}{1 - z^2} = \frac{z}{(1+z)(1-z)\cos z}$$

The poles are the solutions of $(1 + z)(1 - z)\cos z = 0$

i.e. $z = -1$,

$z = 1$ are simple poles

and $z = (2n+1)\frac{\pi}{2}$

Out of these poles $z = \pm 1$ lies inside the ellipse

$z = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ lies outside the ellipse

$$[\text{Res } f(z)]_{z=1} = \lim_{z\to 1}(z-1)\frac{z}{(1+z)(1-z)\cos z}$$

$$= \lim_{z\to 1}\frac{-z}{(1+z)\cos z} = \frac{-1}{2\cos 1}$$

$$[\text{Res } f(z)]_{z=-1} = \lim_{z\to -1}(z+1)\frac{z}{(1+z)(1-z)\cos z}$$

$$= \lim_{z\to -1}\frac{z}{(1-z)\cos z} = \frac{-1}{2\cos 1}$$

∴

$$\int_c \frac{z \sec z}{1 - z^2}\,dz = 2\pi i \text{ [sum of the residues]}$$

$$= 2\pi i\left[\frac{-1}{2\cos 1} - \frac{1}{2\cos 1}\right] = -2\pi i[\sec 1]$$

Q.5 (c) Solution:

CISC (Complex Instruction Set Computer)	RISC (Reduced Instruction Set Computer)
1. They have large instruction set, with instruction formats of different lengths.	1. They have compact instruction set, with instruction format of same length.
2. Instructions perform both elementary and complex operation.	2. Instructions perform elementary operation.
3. Control unit is microprogrammed.	3. Control unit is simple and hard wired.
4. CISC is not pipelined or less pipelined.	4. RISC is pipelined in nature.
5. CISC has single register set.	5. RISC has multiple register set.
6. Emphasis is on hardware in case of CISC.	6. Emphasis is on software in case of RISC.
7. It includes multi-clock complex instructions.	7. It includes single clock reduced instructions only.
8. Small code sizes, high cycles per second is used in CISC.	8. Low cycles per second, large code sizes is used in RISC.
9. Transistors used for storing complex instruction in CISC architecture.	9. In RISC, more transistors are used for memory registers.

Q.5 (d) Solution:

Hence the output is a square wave, value of fundamental voltage is

$$V_{01} = \frac{4V_s}{\pi\sqrt{2}} = \frac{4\times 230}{\pi\sqrt{2}} = 207.0727 \text{ V}$$

Magnitude of load impedance at fundamental frequency is,

$$|Z_1| = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi\times 50\times 0.03)^2}$$

$$|Z_1| = 13.7414\ \Omega$$

$$I_{01} = \frac{V_{01}}{Z_{01}} = \frac{207.0727}{13.7414} = 15.0692 \text{ V}$$

$$V_{03} = \frac{4V_s}{n\pi\sqrt{2}} = \frac{4\times 230}{3\pi\sqrt{2}} = 69.0242 \text{ V}$$

$$|Z_3| = \sqrt{10^2 + (3\times 2\pi\times 50\times 0.03)^2} = 29.9906\ \Omega$$

$$I_{03} = \frac{69.0242}{29.9906} = 2.3015 \text{ A}$$

Similarly,
$$I_{05} = \frac{920}{5\times\pi\times\sqrt{2}}\times\frac{1}{\sqrt{10^2+(5\times 2\pi\times 50\times 0.03)^2}} = 0.8597 \text{ A}$$

$$I_{07} = \frac{920}{7\times\pi\times\sqrt{2}}\times\frac{1}{\sqrt{10^2+(7\times 2\pi\times 50\times 0.03)^2}} = 0.4433 \text{ A}$$

Rms value of resultant load current,

$$I_{0,\text{ rms}} = \sqrt{I_{01}^2 + I_{03}^2 + I_{05}^2 + I_{07}^2}$$

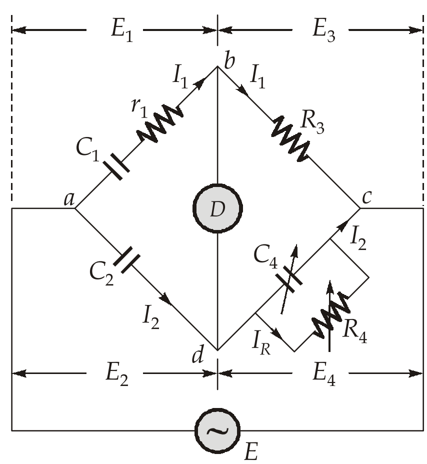
$$I_{0,\text{ rms}} = \sqrt{(15.0692)^2 + (2.3015)^2 + (0.8597)^2 + (0.4433)^2}$$

$$= 15.27 \text{ A}$$

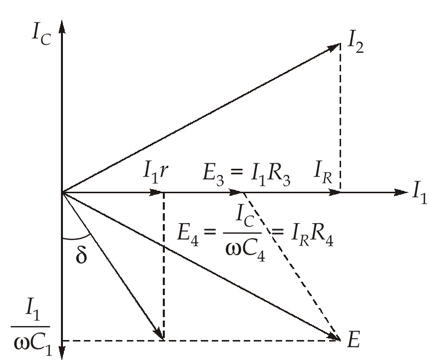
$$\text{Power delivered to load} = I_{0,\text{rms}}^2 R = (15.27)^2\times 10 = 2331.729 \text{ W}$$

Q.5 (e) Solution:

Schering bridge :



Phasor diagram,



Let, C_1 = Capacitor to be determined

r_1 = Series resistance representing loss in capacitor C_1

C_2 = Standard capacitor

R_3 = Non-inductive resistance

C_4 = A variable capacitor

R_4 = Variable non-inductive resistance in parallel with C_4

At balance,

$$\left(r_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega R_4 C_4}\right) = \frac{R_3}{j\omega C_2}$$

$$\left(r_1 + \frac{1}{j\omega C_1}\right)R_4 = \frac{R_3}{j\omega C_2}(1 + j\omega R_4 C_4)$$

$$r_1 R_4 - \frac{jR_4}{\omega C_1} = \frac{-jR_3}{\omega C_2} + \frac{R_3 R_4 C_4}{C_2}$$

Equating real imaginary part we obtain,

$$r_1 = \frac{C_4}{C_2} \cdot R_3$$

$$C_1 = \frac{R_4}{R_3} \cdot C_2$$

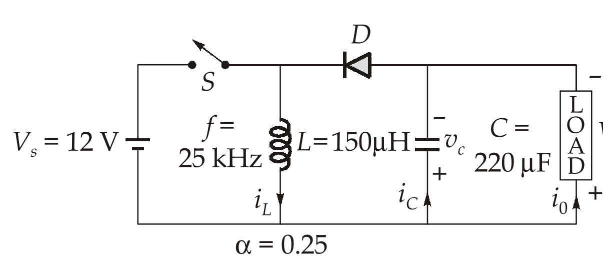
Two independent balance equation are obtained if C_4 and R_4 are chosen as variable elements

$$\text{Dissipation factor, } D = \tan\delta = \omega C_1 r_1 = \left(\omega \cdot \frac{R_4}{R_3}\right) C_2 \cdot \frac{C_4}{C_2} \cdot R_3$$

$\because$ $$D = \omega R_4 C_4$$

Q.6 (a) Solution:

The circuit diagram of Buck boost converter is,



(i) Peak to peak output ripple voltage is,

$$\Delta V_C = \frac{I_0\alpha}{fC} = \frac{1.25 \times 0.25}{25 \times 10^3 \times 220 \times 10^{-6}} = 56.8 \text{ mV}$$

(ii) The peak current through the switch,

$$I_P = \frac{I_s}{\alpha} + \frac{\Delta I}{2} \quad \text{...(i)}$$

$$\Delta I = \frac{V_s\alpha}{fL} = \frac{12 \times 0.25}{25 \times 10^3 \times 150 \times 10^{-6}}$$

$$= 0.8 \text{ A}$$

I_S = Average current through source

$$V_S I_S = V_0 I_0$$

$$I_S = \frac{V_0 I_0}{V_S} = \left(\frac{\alpha}{1-\alpha}\right) I_0 \qquad \left[\because V_0 = \left(\frac{\alpha}{1-\alpha}\right) V_s\right]$$

$$= \left(\frac{0.25}{0.75}\right) \times 1.25 = 0.4167 \text{ A}$$

∴ From equation (i),

$$I_{P\,\text{switch}} = \frac{0.4167}{0.25} + \frac{0.8}{2} = 2.067 \text{ A}$$

(iii) The critical value of inductor $= L_C = \dfrac{(1-\alpha)^2 R}{2f}$

Where R is the load resistance,

$$R = \frac{V_0}{I_0} = \frac{\left(\frac{\alpha}{1-\alpha}\right) V_s}{I_0}$$

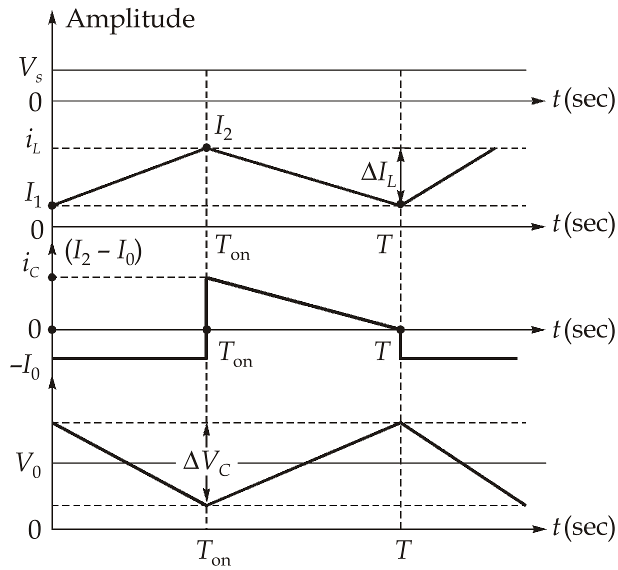
$$= \frac{\left(\frac{0.25}{0.75}\right) \times 12}{1.25} = 3.2 \ \Omega$$

∴
$$L_C = \frac{(1-0.25)^2 3.2}{2 \times 25 \times 10^3} = 36 \ \mu\text{H}$$

Critical value of capacitor $= \dfrac{\alpha}{2fR} = \dfrac{0.25}{2 \times 25 \times 10^3 \times 3.2}$

$$C_C = 1.56 \ \mu\text{F}$$

(iv) The waveforms of i_L, i_C and V_C are,



Q.6 (b) Solution:

In a three phase three wire system we require 3 measuring instruments but if we make the common points of the pressure coils coincide with one of the lines, then we will require only $n - 1 = 2$ measuring instruments.

Instantaneous power consumed by load

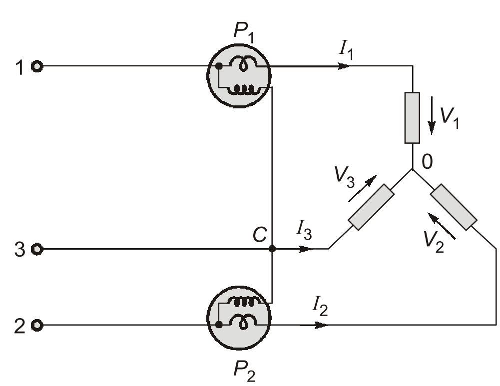
$$= V_1 I_1 + V_2 I_2 + V_3 I_3$$

Star (Wye) connection: Instantaneous reading of P_1 wattmeter,

$$P_1 = V_{13} I_1 = I_1(V_1 - V_3)$$

Instantaneous reading of P_2 wattmeter,

$$P_2 = V_{23} I_2 = I_2(V_2 - V_3)$$



Sum of instantaneous readings of two wattmeters

$$= P_1 + P_2$$
$$= I_1(V_1 - V_3) + I_2(V_2 - V_3)$$
$$= V_1 I_1 + V_2 I_2 - V_3(I_1 + I_2)$$

By Kirchhoff's law,

$$I_1 + I_2 + I_3 = 0$$

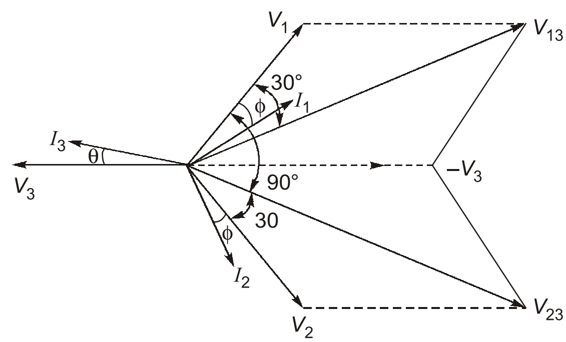
or $$I_3 = -(I_1 + I_2)$$

$\therefore$ Sum of instantaneous readings of two wattmeters

$$= V_1 I_1 + V_2 I_2 + V_3 I_3$$

Therefore, the sum of the two wattmeter reading is equal to the power consumed by the load.

Let V_1, V_2, V_3 be the rms values of the phase voltage and I_1, I_2, I_3 be the rms value of phase currents.



The load is balanced therefore,

Phase voltages, $v_1 = v_2 = v_3 = V$ (say)

Line voltages, $V_{13} = V_{23} = V_{12} = \sqrt{3}\,V$

Phase currents, $I_1 = I_2 = I_3 = I$ (say)

Line currents, $I_1 = I_2 = I_3 = I$

Power factor $= \cos\phi$

The phase current lag the corresponding phasor voltages by an angle ϕ.

The current through wattmeter P_1 is I_1 and voltage across its pressure coil is V_{13}. I_1 leads V_{13} by an angle $(30° - \phi)$.

$\therefore$ Reading of P_1 wattmeter,

$$P_1 = V_{13} I_1 \cos(30° - \phi) = \sqrt{3}\,VI\cos(30° - \phi)$$

The current through wattmeter P_2 is I_2 and voltage across its pressure coil is V_{23}. I_2 lags V_{23} by an angle $(30 + \phi)$.

$\therefore$ Reading of P_2 wattmeter,

$$P_2 = V_{23} I_2 \cos(30° + \phi) = \sqrt{3}\,VI\cos(30° + \phi)$$

Sum of reading of two wattmeters:

$$P_1 + P_2 = \sqrt{3}\,VI\,[\cos(30° - \phi) + \cos(30° + \phi)]$$
$$= 3\,VI\cos\phi$$

This is the total power consumed by load.

$\therefore$ Total power consumed by load, $P = P_1 + P_2$

Difference of readings of two wattmeters

$$P_1 - P_2 = \sqrt{3}\,VI\,[\cos(30° - \phi) - \cos(30° + \phi)] = \sqrt{3}\,VI\sin\phi$$

$$\therefore \quad \frac{P_1 - P_2}{P_1 + P_2} = \frac{\sqrt{3}\,VI\sin\phi}{3\,VI\cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

or $$\phi = \tan^{-1}\sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

Power factor, $$\cos\phi = \cos\left(\tan^{-1}\sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)\right)$$

Q.6 (c) Solution:

(i) Here in this case line commutated inverter means the battery is supplying power to source, since battery is supplying total power which is desired to be transferred i.e. 5000 W and $I_{0,rms}^2 R$ loss so, according to energy balance equation.

$$E_b I_0 = 5000 + I_{0,rms}^2 R$$

$$I_{0'rms} \simeq I_0 \text{ because of large inductor}$$

$$500\, I_0 = 5000 + I_0^2\, 12.4$$

$12.4 I_0^2 - 500 I_0 + 5000 = 0$ is a quadratic equation, the roots of this equation are I_0 = 21.96 A or 18.35 A. Considering the lowest value of load current,

$$I_0 = 18.35 \text{ A}$$

$$V_0 = I_0 R - E_b$$

$$\frac{3V_{mL}}{\pi}\cos\alpha = I_0 R - E_b$$

$$\frac{3\times\sqrt{2}\times 440}{\pi}\cos\alpha = -500 + (18.35)\,(12.4)$$

$$\alpha = \cos^{-1}\left(\frac{-272.46\times\pi}{3\times\sqrt{2}\times 440}\right) = 117.2920°$$

(a) Input power factor $= \dfrac{I_{s_1}}{I_s}\times\cos\alpha = \dfrac{3}{\pi}\cos(117.2920°)$

$= 0.4378$ lagging

(b) Rms value of fundamental ac current

$$I_{s_1} = \frac{3}{\pi}I_s = \frac{3}{\pi}\left(I_0\sqrt{\frac{2}{3}}\right) = \frac{\sqrt{6}}{\pi}I_0 = \frac{\sqrt{6}}{\pi}\times 18.35 = 14.307 \text{ A}$$

$= 14.307$ A

(c) Efficiency of energy transferred

$$= \frac{5000}{E_b I_0} = \frac{5000}{500\times 18.35}\times 100$$

$$\eta = 54.49\%$$

(ii) The maximum usable firing angle is obtained by making load current equal to zero,

$$\frac{3V_{mL}}{\pi}\cos\alpha = -E + I_0 R$$

$$\frac{3\times\sqrt{2}\times 440}{\pi}\cos\alpha = -500$$

$$\alpha = \cos^{-1}\left[\frac{-500}{594.208}\right]$$

$$\alpha = 147.29°$$

(iii) Rms current rating of SCR is,

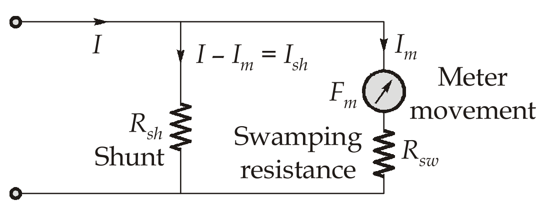
$$I_{T_{rms}} = I_0\sqrt{\frac{1}{3}} = 18.35\sqrt{\frac{1}{3}} = 10.6 \text{ A}$$

SCR voltage rating = Maximum value of source voltage

$$= V_m = 440\sqrt{2} = 623 \text{ V}$$

Q.7 (a) (i) Solution:

Case-I:



Multiplying-factor of shunt, $m = \frac{I}{I_m} = \frac{100}{1} = 100$

∴ Resistance of shunt, $R_{\text{sh}} = \frac{R_m}{(m-1)} = \frac{25}{(100-1)} = 0.2525\ \Omega$

Instrument resistance for 10° C rise in temperature,

$$R_{mt} = 25\,(1 + 10 \times 0.004) = 26\ \Omega$$

Shunt resistance for 10° C rise in temperature

$$R_{sht} = 0.2525(1 + 10 \times 0.00015) = 0.2529\ \Omega$$

Current I_{mt} through the meter for 100 mA in the main circuit for 10° C rise in temperature,

$$I_{mt} = 100\times\frac{0.2529}{26+0.2529} = 0.963 \text{ mA}$$

Normal meter current = 1 mA

∴ Error due to rise in temperature

$$= (0.963 - 1) \times 100 = -3.7\%$$

Case-II:

Total resistance in the meter circuit

$$R_m + R_{sw} = 25 + 75 = 100\ \Omega$$

$$\text{Shunt resistance, } R_{sh} = \frac{R_m}{(m-1)} = \frac{100}{(100-1)} = 1.01\ \Omega$$

Resistance of the instrument circuit for 10° C rise in temperature,

$$R_{mt} = 25(1 + 10 \times 0.004) + 75(1 + 10 \times 0.00015) = 101.11\ \Omega$$

Shunt resistance for 10° C rise in temperature,

$$R_{sh} = 1.01(1 + 10 \times 0.00015) = 1.0115\ \Omega$$

Instrument current for 100 mA in the main circuit for 10° C rise in temperature

$$I_{mt} = 100 \times \frac{1.0115}{101.11 + 1.0115} = 0.9905\text{ mA}$$

$$\therefore \quad \text{Error} = (0.9905 - 1) \times 100 = -0.95\%$$

The improvement in the error from 3.7% to about 1% has been obtained by the use of the additional series swamping resistance of 3 times as compared to the meter resistance. We could obtain better correction by increasing the ratio of the swamping resistance as compared to the meter resistance. But by increasing this ratio, the p.d. across the meter circuit would also be increased. Thus the disadvantage of using swamping resistors is a reduction in the full scale sensitivity as a higher voltage across the instrument is necessary to sustain the full scale current.

Q.7 (a) (ii) Solution:

Total resistance of the circuit when the meter is converted to a 250 V voltmeter

$$R = \frac{250}{(100 \times 10^{-3})} = 2500\ \Omega$$

Resistance of the series resistor

$$R_s = 2500 - 320 = 2180\ \Omega$$

When a voltage of 250 V is applied the total circuit resistance becomes

$$2180 + 369 = 2549\ \Omega$$

$$\therefore \quad \text{Voltage reading} = \left(\frac{2500}{2549}\right) \times 250 = 245.2\text{ V}$$

$$\text{Hence error due to self heating} = \frac{245.2 - 250}{250} \times 100 = -1.92\%$$

When a voltage of 125 V is applied the increase in temperature and consequently the change in resistance is 1/4 of the value with 250 V applied continuously,

$\therefore$ Change in resistance of coil $= \dfrac{(369-320)}{4} = 12.25\,\Omega$

Hence the resistance of voltmeter circuit,

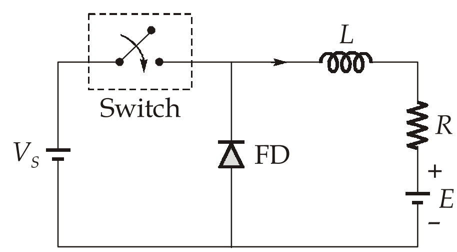
$$= 2500 + 12.25 = 2512.25\ \Omega$$

$\therefore$ Voltmeter reading $= \left(\dfrac{2500}{2512.25}\right)\times 125 = 124.4\text{ V}$

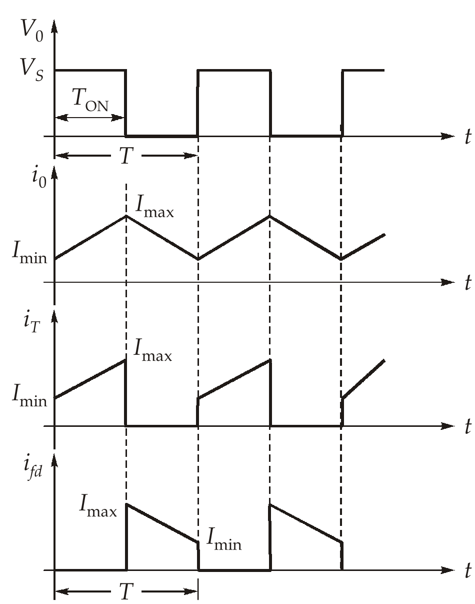
Hence, error $= \dfrac{124.4-125}{125}\times 100 = -0.48\%$

Q.7 (b) Solution:

Circuit diagram:



For an A-type chopper, waveforms are drawn for load current I_0, input (or thyristor) current i_T, freewheeling diode current i_{fd}.



When chopper is ON, voltage equation for chopper circuit

$$i_T R + L\frac{di_T}{dt} + E = V_S$$

$$R \cdot i_T dt + L\frac{di_T}{dt} \cdot dt = (V_s - E) \cdot dt$$

Taking average on both sides

$$R\frac{1}{T}\int_0^{T_{on}} i_T \cdot dt + \frac{L}{T}\int_0^{T_{on}} di_T = \frac{(V_s - E)}{T}\int_0^{T_{on}} dt$$

$$R \cdot I_{TAV} + \frac{L}{T}\int_{I_{min}}^{I_{max}} di_T = (V_s - E)\frac{T_{on}}{T}$$

$$R \cdot I_{TAV} + \frac{L}{T}[I_{max} - I_{min}] = (V_s - E)\alpha$$

$$I_{TAV} = \frac{\alpha(V_s - E)}{R} - \frac{L}{RT}(I_{max} - I_{min})$$

When the freewheeling diode is conducting, load voltage is zero. The voltage equation

$$Ri_{fd} + L\frac{di_{fd}}{dt} + E = 0$$

$$R \cdot \frac{1}{T}\int_{T_{on}}^{T} i_{Fd} \cdot dt + \frac{L}{T}\int_{T_{on}}^{T} \frac{di_{fd}}{dt} \cdot dt + \frac{E}{T}\int_{T_{on}}^{T} dt = 0$$

$$R \cdot i_{Fd} + \frac{L}{T}\int_{I_{max}}^{I_{min}} di_{fd} = \frac{-E(T - T_{on})}{T}$$

$$R \cdot i_{Fd} + \frac{L}{T}(I_{min} - I_{max}) = -E(1 - \alpha)$$

$$i_{Fd} = \frac{L(I_{max} - I_{min})}{TR} - \frac{E(1-\alpha)}{R}$$

Now, average load current over a complete cycle can be obtained by adding I_{TAV} and i_{Fd} from the above equations

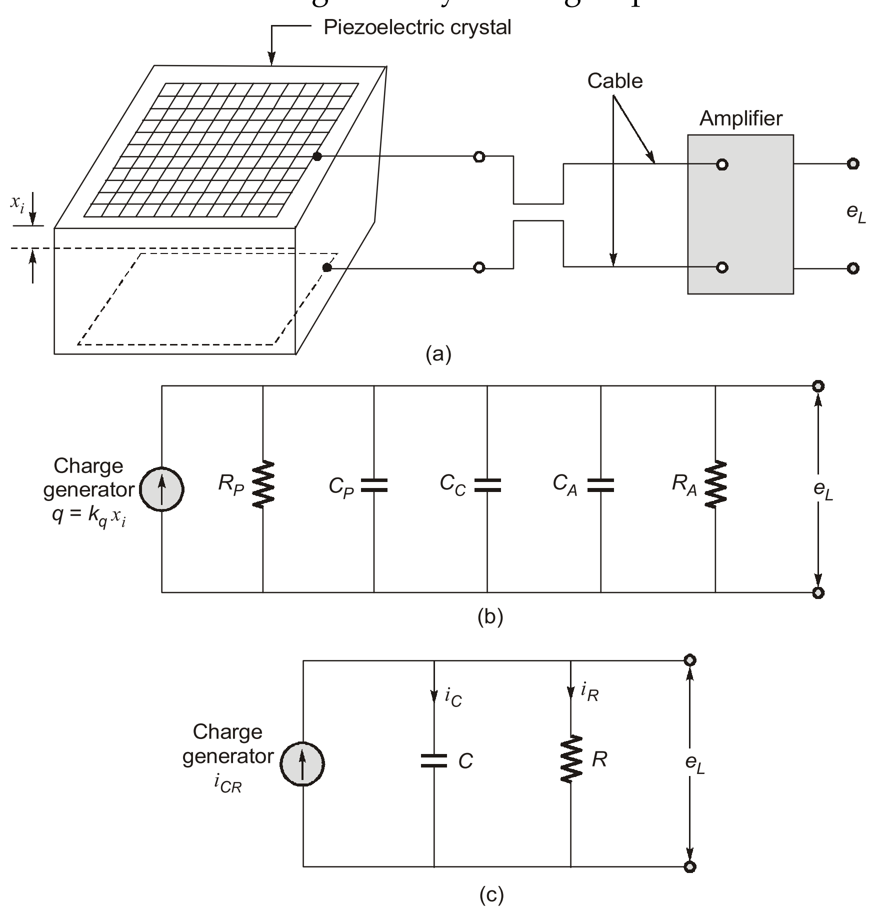
$$I_{av} = \frac{\alpha(V_s - E)}{R} - \frac{L}{RT}(I_{max} - I_{min}) + \frac{L}{RT}(I_{max} - I_{min}) - E\frac{(1-\alpha)}{R}$$

$$\frac{\alpha V_s - E}{R} = \frac{V_0 - E}{R}$$

Q.7 (c) Solution:

A piezo-electric material is one in which an electric potential appears across surfaces of a crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. The effect is reversible, i.e. conversely, if a varying potential is applied to the proper axis of the crystal, it will change the dimensions of the crystal thereby deforming it. This effect is known as piezo-electric effect. Elements exhibiting piezo-electric properties are called as electro-resistive elements. The materials that exhibit a significant and useful piezo-electric effect are divided into two categories: (a) Natural group and (b) Synthetic group.

Quartz and Rochelle salt belong to natural group while materials like lithium sulphate, ethylene diamine tartarate belong to the synthetic group.



(a) (b) (c)

Consider the transducer, the connecting cable and the amplifier as a unit. The impedance of the transducer is very high and hence an amplifier with a high input impedance has to be used in order to avoid loading errors.

Charge produced, $q = K_q x_i$ coulomb

where, K_q = sensitivity; C/m

and x_i = displacement; m

Figure (b) shows the equivalent circuit:

R_P = Leakage resistance of transducer; Ω
C_P = Capacitance of transducer; F
C_C = Capacitance of cable; F
C_A = Capacitance of amplifier; F
R_A = Resistance of amplifier; Ω

The charge generator is converted into a constant current generator as shown in figure (c). The capacitance connected across the current generator is C where:

$$C = C_P + C_C + C_A$$

Resistance, $$R = \frac{R_A R_P}{R_A + R_P}$$

Since the leakage resistance of transducer is very large (of the order of 0.1×10^{12} Ω) and therefore,

$$R = R_A$$

Converting the charge generator into a current generator

$$i_{CR} = \frac{dq}{dt} = K_q\left(\frac{dx_i}{dt}\right)$$

where i_{CR} is the current of the constant current generator.

Now, $$i_{CR} = i_C + i_R$$

∴ Output voltage at load $e_L = e_C = \frac{1}{C}\int i_C\, dt = \frac{1}{C}\int (i_{CR} - i_R)dt$

or $$\frac{d(e_L)}{dt} = \frac{1}{C}(i_{CR} - i_R)$$

or $$C\frac{d(e_L)}{dt} = i_{CR} - i_R = K_q\frac{d(x_i)}{dt} - \frac{e_L}{R}$$

or $$RC\frac{d(e_L)}{dt} + e_L = K_q R\frac{d(x_i)}{dt}$$

$$\tau\frac{d(e_L)}{dt} + e_L = K\tau\frac{d(x_i)}{dt}$$

where, K = sensitivity $= \frac{K_q}{C}$ V/m

Taking Laplace transform:

$$(\tau s + 1)\, E_L(s) = K\tau s\, X_i(s)$$

$\therefore$ Transfer function:

$$\frac{E_L(s)}{X_i(s)} = \frac{K\tau s}{1+\tau s}$$

Sinusoidal transfer function on keeping $s = j\omega$:

$$\frac{E_L}{X_i}(j\omega) = \frac{j\omega K\tau}{1+j\omega\tau}$$

Q.8 (a) (i) Solution:

Given equation is

$$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y = \sin(\log x)$$

$$(x^2D^2+4xD+2)y = \sin(\log x) \quad \text{...(i)}$$

Put,

$$x = e^z \text{ (or) } z = \log x$$

$$xD = D' \quad \text{...(ii)}$$

$$x^2D^2 = D'(D'-1) \quad \text{...(iii)}$$

Where D' denotes $\frac{d}{dz}$

Sub (ii) and (iii) in (i) we get,

$$(D'(D'-1)+4D'+2)y = \sin z$$

i.e.

$$(D'^2-D'+4D'+2)y = \sin z$$

$$(D'^2+3D'+2)y = \sin z \quad \text{...(iv)}$$

The A.E is

$$m^2+3m+2 = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

Complementary function, C.F. : $Ae^{-z}+Be^{-2z}$

$$\text{Particular integral, P.I.} = \frac{1}{D'^2+3D'+2}\sin z = \frac{1}{-1+3D'+2}\sin z$$

$$= \frac{1}{3D'+1}\sin z = \frac{3D'-1}{9D'^2-1}\sin z$$

$$= \frac{(3D'-1)\sin z}{9(-1)-1} \text{ [Replace } D'^2 \text{ by } -1]$$

$$= \frac{3D'(\sin z)-\sin z}{-10} = \frac{3\cos z-\sin z}{-10}$$

∴ The solution of equation is,

$$y = \text{C.F.} + \text{P.I.}$$

$$y = Ae^{-z} + Be^{-2z} + \frac{3\cos z - \sin z}{-10}$$

Sub, $z = \log x$

or, $x = e^z$,

we get $y = Ae^{-\log x} + Be^{-2\log x} - \frac{3\cos(\log x) - \sin(\log x)}{10}$

$$y = Ax^{-1} + Bx^{-2} - \frac{3\cos(\log x) - \sin(\log x)}{10}$$

$$y = \frac{A}{x} + \frac{B}{x^2} - \frac{3\cos(\log x) - \sin(\log x)}{10}$$

Q.8 (a) (ii) Solution:

$$f(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \le x \le 4 \\ 0, & x > 4 \end{cases}$$

If $f(x)$ is a probability density function, then $\int_{-\infty}^{\infty} f(x)dx = 1$

Here, $\int_2^4 \frac{1}{18}(2x+3)dx = \frac{1}{18}\left[x^2 + 3x\right]_2^4$

$$= \frac{1}{18}(16 + 12 - 4 - 6) = 1$$

$f(x) > 0$ for $2 \le x \le 4$

Hence, the given function is a probability density function.

Q.8 (b) Solution:

$$N_P = 1, \qquad N_S = 98,$$

$$\text{turn ratio, } n = 98$$

$$\text{Nominal ratio, } K_n = \frac{500}{5} = 100$$

Magnetizing current component is in phase while the loss component is in quadrature with the flux

$\therefore$ magnetizing mmf = 8 A,

mmf equivalent to loss = 10 A

$$I_m = \frac{\text{Magnetizing mmf}}{\text{Primary winding turns}} = \frac{8}{1} = 8\,\text{A}$$

$$\text{Loss current, } I_e = \frac{\text{Loss mmf}}{\text{Primary winding turns}} = \frac{10}{1} = 10\,\text{A}$$

Output VA = 15 VA

Impedance of secondary load burden

$$= \frac{VA}{I_s^2} = \frac{15}{(5)^2} = 0.6\,\Omega$$

$\therefore$ It is given that external burden is purely resistive

Resistance of external burden = 0.6 Ω,

Resistance total secondary burden= 0.6 + 0.35 Ω = 0.95 Ω

Reactance of total secondary burden = 0 + 0.3 = 0.3 Ω

δ = secondary phase angle

$$= \tan^{-1}\left(\frac{0.3}{0.95}\right) = 17.52^\circ$$

$\cos\delta$ = 0.95,

$\sin\delta$ = 0.3

$$\text{Actual ratio, } R = n + \frac{I_e\cos\delta + I_m\sin\delta}{I_s}$$

$$= 98 + \frac{10\times 0.95 + 8\times 0.3}{5} = 100.38$$

$$\text{Ratio error} = \frac{100 - 100.38}{100.38}\times 100 = -0.38\%$$

$$\text{Phase angle, } \theta = \frac{180}{\pi}\left[\frac{I_m\cos\delta - I_e\sin\delta}{nI_s}\right]$$

$$= \frac{180}{\pi}\left[\frac{8\times 0.95 - 10\times 0.3}{98\times 5}\right] = 0.537^\circ$$

Q.8 (c) Solution:

Inductive reactance of pressure coil $= 2\pi f L$

$= 2\pi \times 100 \times 10 \times 10^{-3} = 6.28\ \Omega$

β = phase angle of P.C. circuit

Resistance of pressure coil circuit $= 2000\ \Omega$

$$\tan \beta = \frac{6.25}{2000} = 3.14 \times 10^{-3} \text{ radian} = 0.1799°$$

For an inductive load:

Reading of wattmeter $\propto \cos \beta \cos (\phi - \beta)$

True power $\alpha \cos \phi$

$$\therefore \quad \text{True power} = \frac{\cos\phi}{\cos(\beta)\cos(\phi-\beta)} \times \text{reading of wattmeter}$$

$$\text{True power} = I^2 R = (4.5)^2 \times Z \cos \phi$$

Where, R = load resistance

$$\text{Impedance of load, } Z = \frac{240}{4.5} = 53.3\ \Omega$$

$$\text{True power} = (4.5)^2 \times 53.3 \cos \phi$$

$$\because \quad \text{Reading of wattmeter} = 23\text{ W}$$

$$\Rightarrow \quad (4.5)^2 \times (53.3) \cos \phi = \frac{\cos\phi}{\cos\beta\cos(\phi-\beta)} \times 23$$

$$\Rightarrow \quad \cos \beta \,.\, \cos (\phi - \beta) = \frac{23}{(4.5)^2 \times 53.3} = 0.0213$$

$$\cos (0.1799°) \times \cos(\phi - 0.1799°) = 0.0213$$

$$\Rightarrow \quad \phi = 88.96°$$

$$\% \text{ error} = (\tan \phi \times \tan \beta) \times 100$$

$$= (55.08) \times 0.0314 \times 100$$

$$= 17.3\%$$

❍❍❍❍