249/300

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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Civil Engineering

Test-5 : Section A : Flow of Fluids, Hydraulic Machines and Hydro Power (All Topics)

Section B : Design of Concrete and Masonry Structures-1

+ Strength of Materials-2 [Part syllabus]

Name : ....

Roll No :

Test Centres	Student's Signature
Delhi ✓ Bhopal ☐ Jaipur ☐ Pune ☐ Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	40
Q.2	56
Q.3	
Q.4	48
Section-B	
Q.5	48
Q.6	57
Q.7	
Q.8	
Total Marks Obtained	249/300

Signature of Evaluator ....D.......... Cross Checked by ....

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IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Flow of Fluids, Hydraulic Machines and Hydro Power

Q.1 (a) A three-dimensional flow field is defined by the velocity vector

$$\vec{V} = (3x^2 + 2y)\hat{i} + (-4xy + 2y^2 + 2zy)\hat{j} + \left(-\frac{5}{2}z^2 + 3xz - 4y^2z\right)\hat{k}$$

Based on this velocity field, determine the total acceleration and angular velocity vectors of a fluid particle at the point (1, 1, 1) at $t = 4$ sec. Assume the flow is steady.

[12 marks]

$u = 3x^2 + 2y$

$v = (-4xy + 2y^2 + 2zy)$

$w = \left(-\frac{5}{2}z^2 + 3xz - 4y^2z\right)$

~~check for continuity~~

$\cancel{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 6x + (-4x + 4y + 2z) + (-5z + 3x - 4y^2)}$

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$a_x = (3x^2+2y) \times (6x) + (-4xy + 2y^2 + 2zy) \times 2 + 0 + 0$

a_x @ (1, 1, 1) $\boxed{a_x = 30}$ — 1

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$

$= (3x^2+2y) \times (-4y) + (-4xy + 2y^2 + 2zy) \times (-4x + 4y + 2z)$

$+ \left(-\frac{5}{2}z^2 + 3xz - 4y^2z\right) \times (2y) + 0$

$a_y \rightarrow$ @ (1, 1, 1) $\longrightarrow$ $\boxed{a_y = -27}$

$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \cancel{\frac{\partial w}{\partial t}}^{\,0}$

$= (3x^2+2y) \times (3z) + (-4xy + 2y^2 + 2zy) \times (-8yz)$

$+ \left(-\frac{5}{2}z^2 + 3xz - 4y^2z\right) \times (-5z + 3x - 4y^2)$

$\boxed{a_z = 36}$

$\therefore \boxed{a = 30\hat{i} - 27\hat{j} + 36\hat{k}}$

$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}\left(-8zy \cancel{\phi} - 2y\right)$

$\omega_x = -5$ — (1) @ (1,1,1)

$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}\left(0 - 3z\right)$

$\omega_y = -1.5$ — (2) @ (1,1,1)

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(-4y - 2\right) = -3$ — (3)

$\therefore \vec{\omega} = -5\hat{i} - 1.5\hat{j} - 3\hat{k}$.

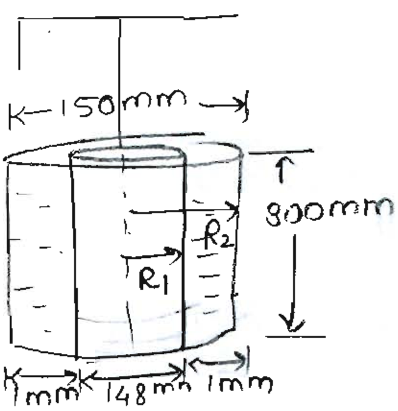
12

Q.1 (b) Explain the effect of pressure gradient on boundary layer separation and describe the various methods used to prevent the separation of the boundary layer.

[12 marks]

Q.1 (c) Two coaxial cylinders of height 300 mm have a liquid filled in the space between them. The outer cylinder has diameter of 150 mm, while the inner cylinder has diameter of 148 mm. When the outer cylinder rotates at 120 rpm, it is found that a torque of 1.5 N-m is exerted on the inner cylinder. Determine the viscosity of the liquid.

[12 marks]



$$\omega_{outer} = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi \times 120}{60} = 4\pi$$

$$v_{outer} = \omega \times r = \omega \times R_2$$

$$v_{outer} = 4\pi \times \frac{0.15}{2} = 0.9424$$

$$\text{Torque} = F \times r$$
$$= \tau A \times r$$

$$15 = \mu \times \frac{du}{dy} \times A \times R_1 = \mu \times \frac{(0.9424 - 0)}{10^{-3}} \times \pi \times D_1 \times L \times R_1$$

$$15 = \mu \times \frac{0.9424}{10^{-3}} \times \pi \times 0.148 \times 0.3 \times \frac{0.148}{2}$$

$$\boxed{\mu = 1.542}\ \text{Pa-s}$$

4

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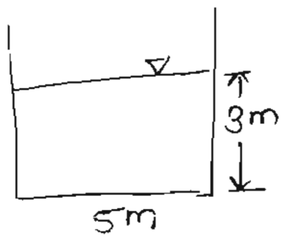
Q.1 (d) A rectangular channel has 5.0 m width and 3.0 m water depth. If the bed slope of the channel is 1 in 1200, find (i) minimum width of the throat, (ii) maximum height of the hump without changing the water level at the entrance. Take, Manning coefficient (n) = 0.02.

[12 marks]



By manning's eq^n

$$Q = \frac{A}{n} R^{2/3} S_o^{1/2} \qquad (n = 0.02 \text{ given})$$

$A = 5 \times 3 = 15\, m^2$

$P = 5 + 3 + 3 = 11\, m$

$R = A/P = \frac{15}{11}$

$$Q = \frac{15}{0.02} \times \left(\frac{15}{11}\right)^{2/3} \left(\frac{1}{1200}\right)^{1/2} \qquad \left(S_o = \frac{1}{1200}\right)$$

$Q = 26.623\ m^3/sec.$

(i) For minimum width

$E_1 = E_c \longrightarrow$

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{A_1^2 \times 2g} = 3 + \frac{26.623^2}{15^2 \times 2 \times 9.81}$$

$E_1 = 3.16056$

$E_1 = E_c \Rightarrow 3.16056 = \frac{3}{2} y_c$

$$\left\{ y_c = \left(\frac{q^2}{g}\right)^{1/3} \ \& \ q = \frac{Q}{B} \right.$$

$$3.16056 = \frac{3}{2}\left(\frac{Q^2}{B_{min}^2 \times 9.81}\right)^{1/3}$$

$$\boxed{B_{min} = 2.779 \simeq 2.78\, m}$$

(ii) $(\Delta z)_{max} = E_1 - E_c$

E_c @ $y = 3m \longrightarrow$ $E_c = \frac{3}{2} \times \left(\frac{q^2}{g}\right)^{1/3}$

$E_c = \frac{3}{2}\left(\frac{26.623^2}{5^2 \times 9.81}\right)^{1/3}$

$E_c = 2.1366\ m$

$\therefore (\Delta z)_{max} = E_1 - E_c$

$= 3.16056 - 2.1366$

$\boxed{\Delta z_{max} = 1.024\ m}$

12

Q.1 (e) Pelton wheel is designed to revolve at a speed of 210 r.p.m. and develops 6200 kW of power while working under a head of 250 m. The overall efficiency of the turbine is 85%. Determine the unit speed, unit discharge, and unit power for this turbine. Furthermore, calculate the predicted speed, discharge, and power if the turbine were to operate under a reduced head of 160 m.

[12 marks]

$N = 210\,rpm$

$P = 6200\,kW$

$H = 250\,m$

$\eta_o = 85\%$

(i) unit speed → $u \propto \frac{\pi D N}{60} \propto \sqrt{H}$

$N \propto \sqrt{H}$

$$N_u = \frac{N}{\sqrt{H}} = \frac{210}{\sqrt{250}} = 13.2815$$

unitspeed $\boxed{N_u = 13.2815}$

(ii) discharge → $Q \propto Au \propto A\sqrt{H}$

$$Q_u = \frac{Q}{\sqrt{H}} \Rightarrow$$

For Q; $\eta_o = \frac{S.P}{W.P} = \frac{6200\,kW}{W.P} = 0.8$

Water power $= 7750\,kW = \rho g Q H$

$7750 \times 10^3 = 10^3 \times 9.81 \times Q \times 250$

$$\boxed{Q = 3.16\ m^3/sec}$$

$$\therefore Q_u = \frac{Q}{\sqrt{H}} = \frac{3.16}{\sqrt{250}} = 0.1998$$

$\boxed{Q_u = 0.1998}$ → unit discharge

Power = $\rho g Q H \longrightarrow$ but $Q \propto \sqrt{H}$

$\therefore \; P \propto H \times \sqrt{H}$

$P \propto H^{3/2}$

$\therefore \; P_u = \dfrac{P}{H^{3/2}} = \dfrac{6200}{250^{3/2}} = 1.5684$

$\boxed{P_u = 1.5684}$

for H = 160 m

$N_u = \dfrac{N_1}{\sqrt{H_1}} = \dfrac{N_2}{\sqrt{H_2}}$

$\dfrac{210}{\sqrt{250}} = \dfrac{N_2}{\sqrt{160}}$

$\boxed{N_2 = 168 \text{ rpm}}$

$Q_u = \dfrac{Q_1}{\sqrt{H_1}} = \dfrac{Q_2}{\sqrt{H_2}}$

$\dfrac{3.16}{\sqrt{250}} = \dfrac{Q_2}{\sqrt{160}}$

$\boxed{Q_2 = 2.528 \text{ m}^3/\text{sec}}$

$P_u = \dfrac{P_1}{H^{3/2}} = \dfrac{P_2}{H^{3/2}}$

$\therefore \; \dfrac{6200}{250^{3/2}} = \dfrac{P_2}{160^{3/2}}$

$\boxed{P_2 = 3174.4 \text{ kW}}$

12

Q.2 (a) The discharge through an orifice depends on the diameter D of the orifice, head H over the orifice, density ρ of the liquid, viscosity μ of the liquid, and acceleration g due to gravity. Using dimensional analysis, obtain an expression for the discharge.

$$Q = D^{2.5} g^{0.5} \phi\left(\frac{H}{D}, \frac{\mu}{\rho D^{1.5} g^{0.5}}\right)$$

[20 marks]

$Q \propto f(D, H, \rho, \mu, g)$

Total no. of variables = 6

No. of dimensions involved = 3 $\{M, L, T\}$

$\therefore$ No. of π terms = 6 − 3 = 3.

Dimensions are,

$Q = [L^3T^{-1}]$, $H = [L]$, $D = [L]$, $\rho = [ML^{-3}]$

$\mu = [ML^{-1}T^{-1}]$ $\quad g = [LT^{-2}]$

let, repeating set of variables be, $[D, g, \rho]$

$\therefore \pi_1 = Q D^a g^b \rho^c$

$M^0L^0T^0 = [L^3T^{-1}][L]^a[LT^{-2}]^b[ML^{-3}]^c$

$c = 0$, $\quad 0 = 3 + a + b$, $\quad 0 = -1 - 2b$

$a + b = -3$ $\quad b = -0.5$

$\therefore a = -2.5$, $b = -0.5$, $c = 0$.

$\therefore \pi_1 = \dfrac{Q}{D^{2.5} \times g^{0.5}}$ — (1)

$\pi_2 = H D^d g^e \rho^f$

$M^0L^0T^0 = [L][L]^d[LT^{-2}]^e[ML^{-3}]^f$

$0 = f$, $\quad 0 = e$, $\quad 0 = 1 + d$

$d = -1$

$\therefore \pi_2 = \dfrac{H}{D}$ — (2)

$\pi_3 = \mu \, D^a g^b \rho^c$

$M^0 L^0 T^0 = [ML^{-1}T^{-1}] \, [L]^a \, [LT^{-2}]^b \, [ML^{-3}]^c$

$0 = 1 + c$, $\quad 0 = -1 + a + b - 3c$, $\quad 0 = -1 - 2b$

$c = -1$, $\quad -2 = a + b$, $\quad b = -0.5$

$a = -1.5$

$\therefore \pi_3 = \dfrac{\mu}{\rho D^{1.5} g^{0.5}}$

π_1, π_2, π_3 are $\dfrac{Q}{D^{2.5} \times g^{0.5}}, \dfrac{H}{D}, \dfrac{\mu}{\rho D^{1.5} g^{0.5}}$

$\pi_1 = f(\pi_2, \pi_3)$

$$\frac{Q}{D^{2.5} \times g^{0.5}} = \phi\left(\frac{H}{D}, \frac{\mu}{\rho D^{1.5} g^{0.5}}\right)$$

$$Q = D^{2.5} \times g^{0.5} \, \phi\left(\frac{H}{D}, \frac{\mu}{\rho D^{1.5} g^{0.5}}\right)$$

$\therefore$ Hence proved

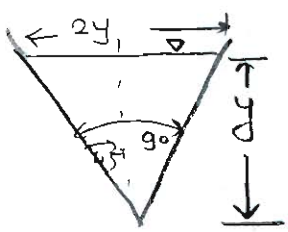
2

Q.2 (b) (i) A hydraulic jump takes place in a horizontal, frictionless triangular channel with a bottom angle of 90°. Find the discharge if the pre-jump and post-jump depths are 5 cm and 15 cm respectively.

(ii) Find the convective acceleration at the middle of a pipe which converges uniformly from 0.6 m diameter to 0.3 m diameter over 3 m length. The rate of flow is 40 lit/s. If the rate of flow changes uniformly from 40 lit/s to 80 lit/s in 40 seconds, find the total acceleration at the middle of the pipe at 20th second.

[8 + 12 = 20 marks]

(i)



$\longrightarrow$ $A = \frac{1}{2} \times 2y \times y$

$= y^2$

$\bar{y} = y/3$ (from top)

In HJ, specific force remains constant

$$\frac{P_1 + M_1}{\gamma} = \frac{P_2 + M_2}{\gamma}$$

$$\frac{\gamma A\bar{y} + \rho Q v}{\gamma} = \text{const}$$

$$\left(A\bar{y} + \frac{Q^2}{Ag}\right)_1 = \left(A\bar{y} + \frac{Q^2}{Ag}\right)_2$$

$$y_1^2 \times \frac{y_1}{3} + \frac{Q^2}{y_1^2 \times g} = \frac{y_2^2 \times y_2}{3} + \frac{Q^2}{y_2^2 \times g}.$$

$$\frac{y_1^3 - y_2^3}{3} = \frac{Q^2}{g}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right) \quad \text{—— (1)}$$

$y_1 = 5cm, \; y_2 = 15cm$

$$\therefore \frac{0.05^3 - 0.15^3}{3} = \frac{Q^2}{9.81}\left(\frac{1}{0.15^2} - \frac{1}{0.05^2}\right)$$

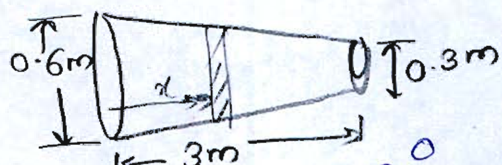
$$Q = 5.4671 \times 10^{-3} \; m^3/sec$$

$$\boxed{Q = 5.4671 \; l/sec}$$

Q

(ii)

$$Q = 40 \times 10^{-3} \; m^3/sec$$



$$(a_x)_{convective} = u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}}^{0} + \cancel{w\frac{\partial u}{\partial z}}^{0}$$

$$a_x = u \times \frac{\partial u}{\partial x} \quad ; \quad u = \frac{Q}{A} = \frac{4Q}{\pi D_x^2}$$

$$\& \; D_x = 0.6 - \frac{(0.6-0.3)}{3} \times x$$

$$= 0.6 - 0.1x$$

$$\therefore u = \frac{4Q}{\pi (0.6-0.1x)^2} = \frac{4Q}{\pi}(0.6-0.1x)^{-2}$$

$$\frac{\partial u}{\partial x} = \frac{4Q \times (-2) * (-0.1)}{\pi (0.6-0.1x)^3} = \frac{0.8Q}{\pi(0.6-0.1x)^3}$$

$$\therefore a_x = u\frac{\partial u}{\partial x} = \frac{4Q\,\cancel{(0.6-0.1x)} \times 0.8Q}{\pi (0.6-0.1x)^5}$$

$$\boxed{a_x = 0.0883 \; m/sec^2} \rightarrow con$$

$(\because Q = 40 \times 10^{-3}$
@ $x = 1.5$ m.

for temporal accl^n.

$$a_t = \frac{\partial u}{\partial t} = \cancel{(80-40)}$$

$$a_t = \frac{u_2 - u_1}{\Delta t} = \frac{0.503 - 0.2515}{40}$$

$$a_t = 6.2877 \times 10^{-3} \ m/sec^2$$

$$u_1 = \frac{40 \times 10^{-3}}{\frac{\pi}{4} \times 0.45^2}$$

$\left\{ \begin{array}{l} D_x = 0.45 \\ @ \ x = 1.5 \end{array} \right.$

$$u_1 = 0.2515$$

$$u_2 = \frac{80 \times 10^{-3}}{\frac{\pi}{4} \times 0.45^2}$$

$$u_2 = 0.503$$

& ~~a_c~~

at 20th sec, $Q = 60$ lit/sec

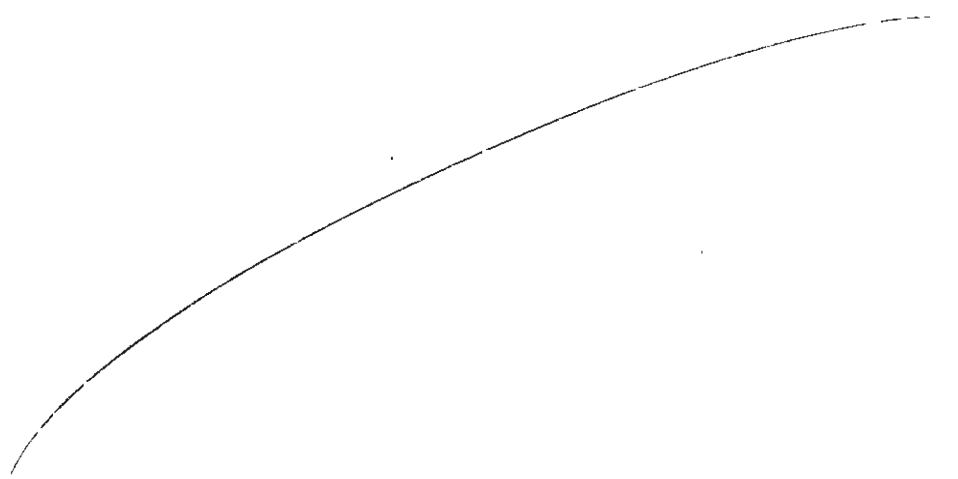
$$\therefore a_x = \frac{0.32 Q^2}{\pi (0.6 - 0.15x)^5}$$

@ $x = 1.5$ m & $Q = 60$ lit/sec

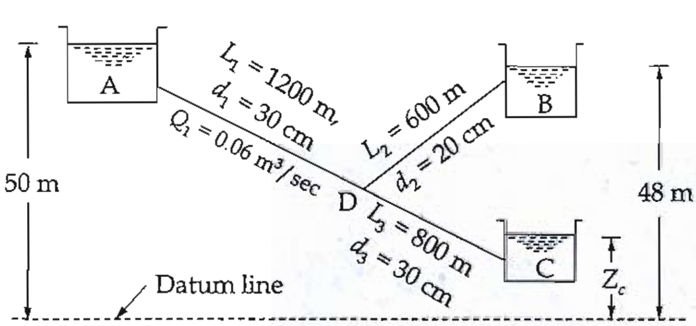
$$a_x = \cancel{0.0198} \ 0.1987 \ m/sec^2$$

$$\therefore a_{total} = a_x + a_t = 0.205 \ m/sec^2$$

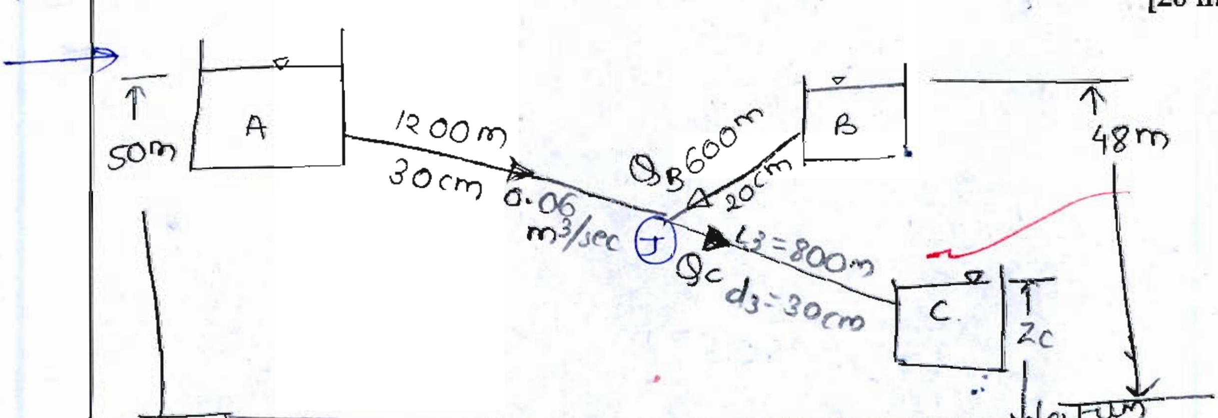
8



Q.2 (c) Three reservoirs *A*, *B*, and *C* are connected by a pipe system. Reservoir *A* has a water level height of 50 m and reservoir *B* has a water level height of 48 m. The pipes are connected at a common junction *D*. Pipe *AD* has a length of 1200 m and a diameter of 300 mm; pipe *DB* has a length of 600 m and a diameter of 200 mm; and pipe *DC* has a length of 800 m and a diameter of 300 mm. The rate of flow from reservoir *A* is 60 litres/s. Find the discharge into or from the reservoirs *B* and *C* and find the height of water level in the reservoir C. Take friction factor $f = 0.024$ for all pipes.



[20 marks]



Let the direction of flow be as shown.

Applying Bernoulli's principle betn A & j

$$\frac{\cancel{P_A}}{\cancel{\rho g}}^{0} + \frac{\cancel{v_A^2}}{\cancel{2g}}^{0} + z_A = \frac{P_j}{\rho g} + \frac{\cancel{v_j^2}}{\cancel{2g}}^{0} + z_j + h_L$$

$$50 = \frac{P_j}{\rho g} + z_j + \frac{8Q^2}{\pi^2 g}\frac{fL}{D^5}$$

$$\frac{P_j}{\rho g} + z_j = 50 - \frac{8\times(0.06)^2}{\pi^2\times 9.81}\times\frac{0.024\times1200}{0.3^5}$$

$$= \cancel{46.67}\ 46.4746\,m \quad \text{---} (1)$$

Applying Bernoulli's betn B & j

$$\frac{\cancel{P_B}}{\cancel{\rho g}}^{0} + \frac{\cancel{v_B^2}}{\cancel{2g}}^{0} + z_B = \frac{P_j}{\rho g} + \frac{\cancel{v_j^2}}{\cancel{2g}}^{0} + z_j + h_L$$

$$48 = \frac{P_j}{\rho g} + z_j + \frac{8\times Q_B^2}{\pi^2\times 9.81}\times\frac{0.024\times600}{0.2^5}$$

$\left\{ \text{from eq}^n \ (1) \quad \frac{P_j}{\rho g} + z_j = 46.4746 \right.$

$\therefore Q_B = 0.02025 \ m^3/sec$

$$\boxed{Q_B = 20.254 \ l/sec}$$

$$\therefore \boxed{Q_C = 60 + Q_B = 80.254 \ l/sec}$$

Applying Bernoullis betn j & C

$$\frac{\cancel{P_j}}{\cancel{\rho g}} + \frac{\cancel{v_j^2}}{\cancel{2g}}^{0} + z_j = \frac{\cancel{P_c}}{\cancel{\rho g}}^{0} + \frac{\cancel{v_c^2}}{\cancel{2g}}^{0} + z_c + h_L$$

$$\frac{P_j}{\rho g} + z_j = 46.4746 = z_c + \frac{8\times Q_c^2}{\pi^2 g}\times\frac{0.024\times800}{0.3^5}$$

$$\therefore \boxed{z_c = 42.27\,m}$$

20

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Q.3 (a) (i) A spillway model is to be built to a geometrically similar scale of 1/40 across a flume of 50 cm width. The prototype is 20 m high and the maximum head on it is expected to be 2 m.

1. What height of model and what head on the model should be used?
2. If the flow over the model at a particular head is 10 litres/s, what flow per metre length of the prototype is expected?
3. If the negative pressure in the model is 150 mm, what is the negative pressure in the prototype? Is it practicable?

(ii) A venturi meter is installed in a 300 mm diameter horizontal pipeline. The throat pipe ratio is 1/3. Water flows through the installation. The pressure in the pipeline is 13.783 N/cm^2 (gauge) and vacuum in the throat is 37.5 cm of mercury. Neglecting head loss in the venturi meter, determine the rate of flow in the pipeline. Take S.G of $Hg = 13.6$

[12 + 8 = 20 marks]

Q.3 (b) A cylindrical vessel of diameter 15 cm and depth 40 cm is completely filled with water and is open at the top. The vessel is rotated about its vertical axis at a speed of 600 r.p.m. Determine the quantity of liquid left in the vessel and the pressure force acting on the bottom of the vessel in kN. Neglect viscosity and assume steady rotation.

[20 marks]

Q.3 (c) A Francis turbine operates under a net head of 8 m and is designed to produce 147.15 kW of power with an overall efficiency of 70%. The turbine runs at a speed of 200 r.p.m., and the peripheral velocity at the inlet is given by $0.30\sqrt{2gH}$ while the radial velocity of flow at the inlet is $0.96\sqrt{2gH}$. It is observed that the hydraulic losses within the turbine amount to 20% of the total available energy. Assuming the discharge at the outlet is radial, determine the guide blade angle, the wheel vane angle at the inlet, the diameter of the wheel at the inlet and the width of the wheel at the inlet.

[20 marks]

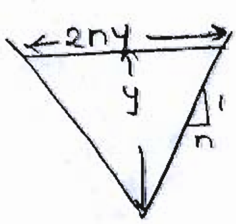
Q.4 (a) **(i)** Prove that the most efficient triangular cross-section channel is half of a square with its diagonal horizontal

(ii) Show that in a triangular channel, the Froude numbers F_1 and F_2 corresponding to alternate depths y_1 and y_2 respectively are related as

$$\left(\frac{F_1}{F_2}\right)^2 = \left(\frac{4+F_1^2}{4+F_2^2}\right)^5$$

[10 + 10 = 20 marks]

(i)



let side slopes be 1V : n H

$\therefore A = \frac{1}{2} \times y \times 2ny = ny^2$

$P = 2y\sqrt{1+n^2}$

For most efficient channel $\longrightarrow$ Q is max^m but, ~~en~~ implies P_{min}

for type I channel Q_{max^m}

$\therefore$ For most efficient channel P is min^m.

$A = ny^2 \longrightarrow y = \sqrt{\frac{A}{n}}$

& $P = 2y\sqrt{1+n^2}$

$P = 2\sqrt{A} \times \sqrt{\frac{1+n^2}{n}} \longrightarrow P^2 = 4A\left(\frac{1+n^2}{n}\right)$

for P_{min^m} $\frac{dP^2}{dn} = 0$

$\frac{dP^2}{dn} = \cancel{2\sqrt{A}}\ 4A * \left(\frac{2n \times n - (1+n^2) \times 1}{n^2}\right) = 0$

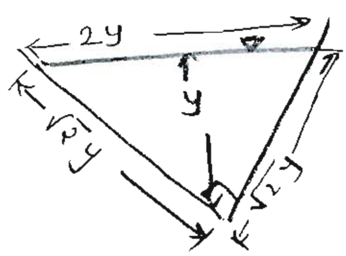
$2n^2 = 1+n^2$

$n^2 = 1$

$n = \pm 1 \longrightarrow$ ~~as~~ $n = -1$ not possible

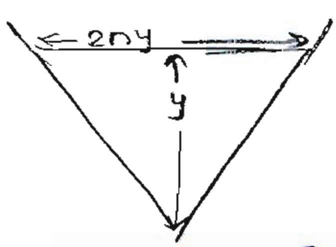
$\therefore n = 1$

for $n=1$ channel, angle at vertex $= 90°$ & it resembeles square with it's diagonal horizontal.



9

(ii)



alternate depths have same specific energy.

$A = \frac{1}{2} \times y \times 2ny = ny^2$

For most efficient channel, $n = 1$

$\therefore A = y^2$

$E_1 = E_2$

$y_1 + \frac{Q^2}{A_1^2 \times 2g} = y_2 + \frac{Q^2}{A_2^2 \times 2g}$

$y_1 + \frac{Q^2}{y_1^4 \times 2g} = y_2 + \frac{Q^2}{2g \times y_2^4}$

$(y_1 - y_2) = \frac{Q^2}{2g}\left(\frac{1}{y_2^4} - \frac{1}{y_1^4}\right) = \frac{Q^2 (y_1^4 - y_2^4)}{2g\ y_1^4 \times y_2^4}$

$\cancel{\frac{Q^2}{2g} = \frac{2y_1^2 y_2^2}{(y_1+y_2)}}$ — (1) $\frac{Q^2}{g} = \frac{y_1^4 \times y_2^4 (y_1 - y_2)}{(y_1^2 - y_2^2)(y_1^2+y_2^2)} = \frac{y_1^4 y_2^4}{(y_1^2+y_2^2)(y_1+y_2)}$

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$$F_1^2 = \frac{Q^2 T}{gA^3} = \frac{Q^2 \times 2y_1}{g \times (y_1^2)^3} = \frac{2Q^2 y_1}{g\, y_1^6} = \frac{2Q^2}{g y_1^5} \quad -(2)$$

$$F_2^2 = \frac{2Q^2}{g y_2^5} \quad -(3)$$

$$\text{LHS} \Rightarrow \left(\frac{F_1}{F_2}\right)^2 = \frac{F_1^2}{F_2^2} = \frac{y_2^5}{y_1^5} \quad -(3) \; \{\text{from } (2) \,\&\, (3)\}$$

$$\text{RHS} = \left(\frac{4+F_1^2}{4+F_2^2}\right)^5 = \left(\frac{4+\frac{2Q^2}{gy_1^5}}{4+\frac{2Q^2}{gy_2^5}}\right)^5$$

$$= \left(\frac{4+\frac{2\times y_1^4 y_2^4 \times 2}{(y_1+y_2)y_1^5(y_1^2+y_2^2)}}{4+\frac{2\times 2\times y_1^2 y_2^2}{(y_1+y_2)\,y_2^5(y_1^2+y_2^2)}}\right)^5$$

$$= \left(\frac{1+\frac{y_2^2}{(y_1+y_2)y_1^3}}{1+\frac{y_1^2}{(y_1+y_2)y_2^3}}\right)^5 = \left(\frac{(y_1+y_2)y_1^3 + y_2^2 y_1 y_2}{(y_1+y_2)y_2^3 + y_1^2 y_2 y_1}\right)$$

$$= \left(\frac{y_2}{y_1}\right)^5$$

10

LHS = RHS

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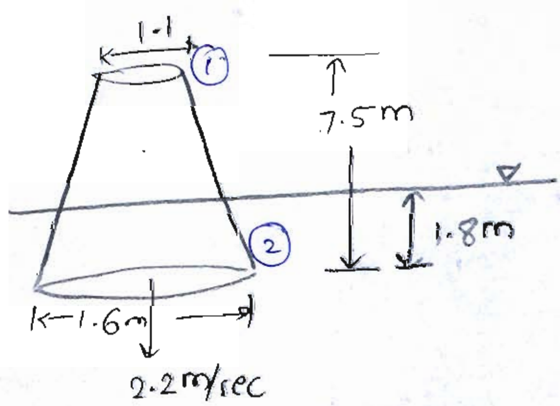
Q.4 (b) **(i)** A conical draft-tube with inlet and outlet diameters of 1.1 m and 1.6 m, respectively, discharges water at the outlet with a velocity of 2.2 m/s. The total vertical length of the draft-tube is 7.5 m, and 1.8 m of this length remains immersed in the tailrace water. If the atmospheric pressure head is 10.3 m of water and the loss of head due to friction within the draft-tube is equal to 0.25 times the velocity head at the outlet of the tube, determine:

(a) The pressure head at the inlet.

(b) The efficiency of the draft-tube.

(ii) A river has a width of 45 m, a depth of 3.5 m and a mean velocity of 1.5 m/s. A weir is to be constructed on the river floor to produce an afflux of 1.2 m. Determine the height of the weir required. Assume the coefficient of discharge of the weir as 0.92.

[12 + 8 = 20 marks]



~~By Bernoulli's~~

applying continuity

$A_1 v_1 = A_2 v_2$

$\frac{\pi}{4} \times 1.1^2 \times v_1 = \frac{\pi}{4} \times 1.6^2 \times 2.2$

$v_1 = 4.6545 \text{ m/sec}$

applying Bernoulli's bet^n ① & ②

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_{LS}$$

$\frac{P_2}{\rho g} = \frac{P_{atm}}{\rho g} + 1.8m = 10.3 + 1.8 = 12.1 \text{ m}$

$$\frac{P_1}{\rho g} + \frac{4.6545^2}{2 \times 9.81} + 7.5 = 12.1 + \frac{2.2^2}{2 \times 9.81} + 0 + \frac{0.25 \times 2.2^2}{2 \times 9.81}$$

$$\boxed{\frac{P_1}{\rho g} = 4.729 \text{ m}}$$

$$\eta = \frac{\frac{v_1^2}{2g} - \frac{v_2^2}{2g} - h_s}{\frac{v_1^2}{2g}} = \frac{\frac{4.6545^2}{2 \times 9.81} - \frac{2.2^2}{2 \times 9.81} - \frac{0.25 \times 2.2^2}{9.81 \times 2}}{\frac{4.6545^2}{2 \times 9.81}}$$

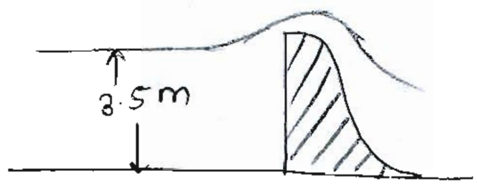
$\simeq 0.7207$

$\boxed{\eta = 72.074\ \%}$

6

(ii)

Initially,



$B = 45\,m$

$y = 3.5\,m$

$v = 1.5\ m/sec$

$\therefore Q = A \times v = B y \times v$

$Q = 236.25\ m^3/sec$

$C_d = 0.92$

Now $y = 3.5 + 1.2 = 4.7\,m$

$\therefore V_a = \frac{Q}{A} = \frac{236.25}{4.7 \times 45} = 1.117\ m/sec$

$\therefore H_a = \frac{V_a^2}{2g} = 0.0636\ m$ —(1)

Q = discharge through rectangular weir:

$= \frac{2}{3} C_d L \sqrt{2g}\left[(H + H_a)^{3/2} - (H_a)^{3/2}\right]$ {H = height of weir.

$236.25 = \frac{2}{3} \times 0.92 \times 45 \sqrt{2 \times 9.81}\left((H + 0.0636)^{3/2} - (0.0636)^{3/2}\right)$

~~$H = 1.6236\,m$~~ $H = 1.4964\,m$

~~$\simeq 1.62\,m$~~

$\boxed{H \simeq 1.5\,m}$

5

Q.4 (c) The outer diameter of an impeller of a centrifugal pump is 500 mm and the outlet width is 40 mm. The pump runs at 900 r.p.m. and works against a total head of 20 m. The vane angle at the outlet is 30° and the manometric efficiency is 80%. Determine the velocity of flow at the outlet, the absolute velocity of water leaving the vane, the angle made by the absolute velocity at the outlet with the direction of motion, and the discharge.

[20 marks]

$D_2 = 500\ mm$

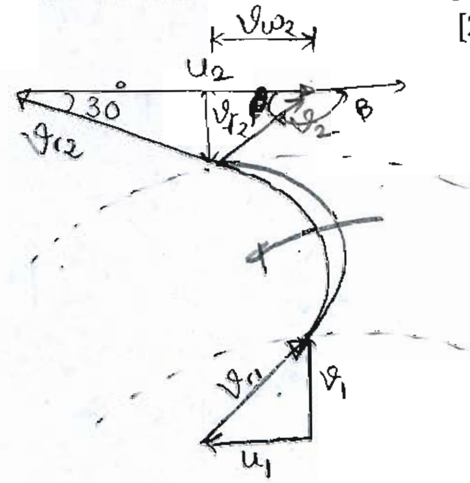
$B_2 = 400\ mm$

$N = 900\ rpm$

$H_m = 20m.$

$\phi = 30^\circ$

$\eta_m = 80\%$



$V_{f_2} = ?$

$V_2 = ?$

$\beta = ?$

$Q = ?$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 900}{60} = 23.562\ m/sec.$$

$$W.P = \rho g Q H = 10^3 \times 9.81 \times Q \times 20 = 196.2Q\ kW$$

$$\eta_m = \frac{W.P}{I.P} \rightarrow I.P = \frac{W.P}{0.8} = \frac{196.2Q}{0.8}\ kW$$

$$I.P = 245.25\,Q\ kW$$

$$245.25\cancel{Q} \times 10^3 = (V_{w_2} u_2)\,\dot{m} = \rho\cancel{Q}(V_{w_2} u_2)$$

$$245.25 \times \cancel{10^3} = \cancel{10^3} \times V_{w_2} \times 23.562$$

$$V_{w_2} = 10.4087\ m/sec.$$

$$\tan 30^\circ = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow \tan 30^\circ (23.562 - 10.4087) = V_{f2}$$

$$\boxed{V_{f2} = 7.594\ m/sec}$$

$$v_2 = \sqrt{v_{w_2}^2 + v_{f_2}^2} = \sqrt{7.594^2 + 10.4087^2}$$

$$\boxed{v_2 = 12.8845 \text{ m/sec}}$$

$$\tan(180-\beta) = \frac{v_{f_2}}{v_{w_2}} = \frac{7.594}{10.4087}$$

$$\boxed{\beta = 143.886^\circ}$$

$$Q = v_{f_2} \times \pi D_2 B_2 = 7.594 \times \pi \times 0.5 \times 0.04$$

$$\boxed{Q = 0.4771 \text{ m}^3/\text{sec}}$$

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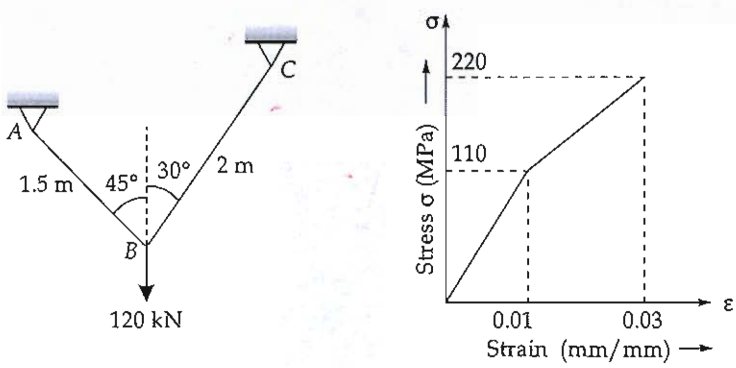
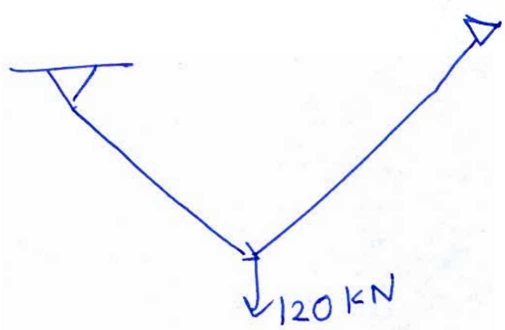
Section B : Design of Concrete and Masonry Structures-1 + Strength of Materials-2

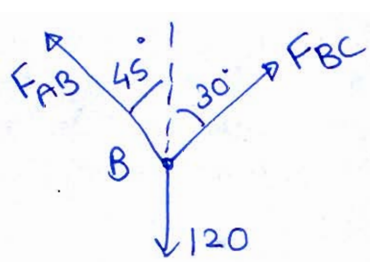
Q.5 (a) Two wires *AB* and *BC* have original length 1.5 m and 2 m and diameter of 20 mm and 35 mm respectively. These wires are made of a material with the stress strain behavior as shown in figure.



Determine the elongation of wires *AB* and *BC* after 120 kN load is applied.

[12 marks]





$F_{AB} \sin 45° = F_{BC} \sin 30°$

$\frac{F_{AB}}{\sqrt{2}} = \frac{F_{BC}}{2}$

$\sqrt{2}\, F_{AB} = F_{BC}$ —— (1)

$F_{AB} \cos 45° + F_{BC} \cos 30° = 120$

$\frac{F_{AB}}{\sqrt{2}} + \frac{F_{BC}\sqrt{3}}{2} = 120$ —— (2)

from (1) & (2)

$F_{AB} = 62.116$ kN, $F_{BC} = 87.846$ kN

$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \cancel{0.1977}\ 197.7\ N/mm^2$

$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = 91.305\ N/mm^2$

$(\text{Strain})_{AB} = 0.025949 \longrightarrow \Delta_{AB} = 38.9235\,mm$

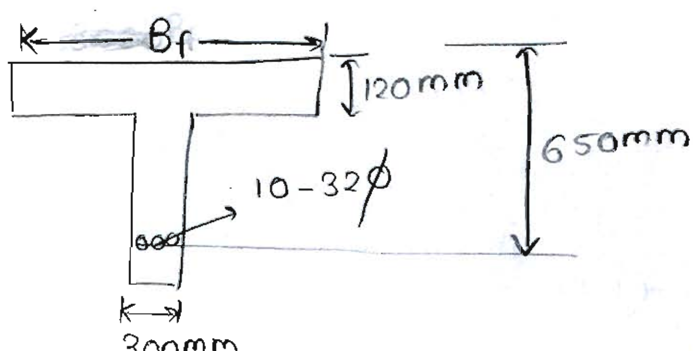
$(\text{Strain})_{BC} = 8.300 \times 10^{-3} \longrightarrow \Delta_{BC} = 16.6\,mm$

$\therefore$ Elongations of AB & BC are 38.9235, 16.6 mm

12

Q.5 (b) Determine the ultimate moment of resistance for a reinforced concrete T-beam with the following specifications: Clear span 8.5 m, web width b_w = 300 mm, flange thickness D_f = 120 mm, effective depth d = 650 mm, center-to-center spacing of beams = 3.5 m, tension reinforcement 10 bars of 32 mm diameter is provided, materials M-25 concrete and Fe-500 steel.

[12 marks]



M25/Fe500

$\left\{ \begin{array}{l} d_f = 120\,mm \\ A_{st} = 8042.48\,mm^2 \end{array} \right.$

$L_o = L_{eff} = L_c + d = 8.5 + 0.65 = 9.15\,m$ {simply supported span.

$B_f = \dfrac{L_o}{6} + b_w + 6d_f \longrightarrow$ for multiple T beams.

$= \dfrac{9.15 \times 10^3}{6} + 300 + 6 \times 120$

$= 2545\,mm < 3500\,mm$ (OK) $\{B = 3.5\,m\}$

$=$

let the NA depth, $x_u < d_f$

$C = T$

$0.36 f_{ck} B_f x_u = 0.87 f_y A_{st}$

$0.36 \times 25 \times 2545 \times x_u = 0.87 \times 500 \times 8042.48$

$x_u = 152.738 > 120 mm$

Assuming, $x_u > d_f$ & $\frac{3}{7} x_u < d_f$

$\therefore y_f = 0.15 x_u + 0.65 d_f$

$= 0.15 x_u + 0.65 \times 120$

$= 0.15 x_u + 78$

$C = T$

$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) \times y_f = 0.87 f_y A_{st}$

$0.36 \times 25 \times 300 \times x_u + 0.45 \times 25 (2545 - 300) \times (0.15 x_u + 78)$

$= 0.87 \times 500 \times 8042.48$

$x_u = 235.57 > d_f = 120 mm$

$\frac{3}{7} x_u = 100.959 < d_f = 120 mm$

$\therefore x_u = 235.57 mm$

$\therefore MOR = 0.36 f_{ck} b_w x_u \times (d - 0.42 x_u)$

$+ 0.45 f_{ck} (B_f - b_w) \times y_f \times (d - \frac{y_f}{2})$

$= 0.36 \times 25 \times 300 \times 235.57 \times (650 - 0.42 \times 235.57)$

$+ 0.45 \times 25 (2545 - 300) \times (0.15 \times 235.57 + 78)$

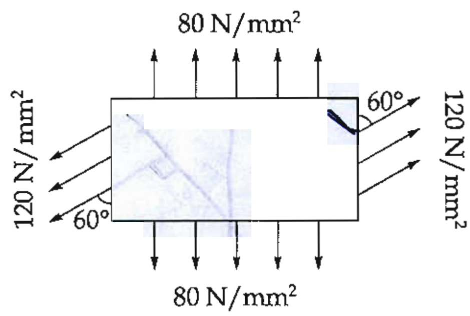
$\times (650 - \frac{y_f}{2}) / 10^6$

$= 350.496 + 1698.372$

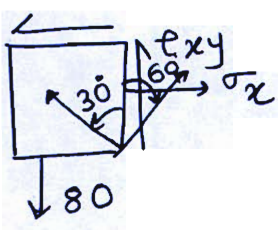
$\boxed{MOR = 2048.867 kNm}$

9

Q.5 (c) A point in a strained material is subjected to the stresses as shown in figure. Locate the principal planes, evaluate the principal stresses. Show the principal stresses on the suitable element



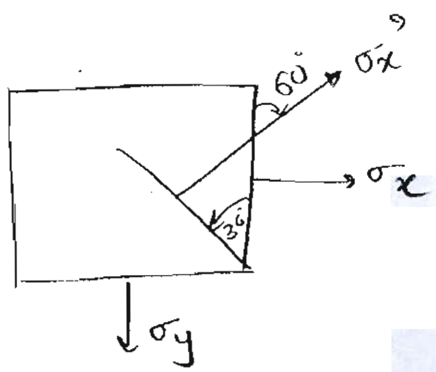
[12 marks]



$\tau_{xy} = 0$, $\sigma_y = 80$

$\cancel{\theta = 60°}$

σ_x' $\theta = 30° \longrightarrow \sigma_x' = 120$

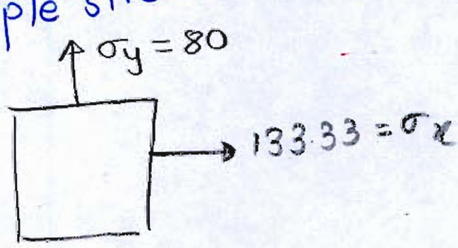


$$\sigma_x' = 120 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \cancel{\tau_{xy}}\sin 2\theta \quad @\ \theta = 30°$$

$$120 = \left(\frac{\sigma_x + 80}{2}\right) + \left(\frac{\sigma_x - 80}{2}\right)\cos 60°$$

$\sigma_x = 133.333$ N/mm²

$\therefore$ principle stresses $\Rightarrow \sigma_x, \sigma_y \Rightarrow 80, 133.33$ N/mm² @ $\theta = 90°, 0°$



3

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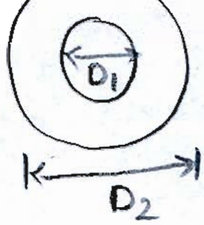
Q.5 (d) The maximum allowable shear stress in a hollow shaft of external diameter equal to twice the internal diameter is 80 N/mm^2. Determine the diameter of the shaft if it is subjected to a torque of 13.5×10^6 Nmm and a bending moment of 10.125×10^6 Nmm.

[12 marks]

$(\tau_{max}) = 80\ N/mm^2$



~~D hollow = 2~~

$D_2 = 2D_1$

$T = 13.5 \times 10^6$ Nmm

$M = 10.125 \times 10^6$ Nmm

By combined bending & Torsion,

$T_{eq} = \sqrt{T^2 + M^2} = \sqrt{(13.5 \times 10^6)^2 + (10.125 \times 10^6)^2}$

$T_{eq} = 16.875 \times 10^6$ Nmm

$\dfrac{\tau_{max}}{r_{max}} = \dfrac{T}{J}$

$\dfrac{2 \times 80}{D_2} = \dfrac{16.875 \times 10^6}{\frac{\pi}{32}(D_2^4 - D_1^4)}$

$\dfrac{\cancel{2}80}{\cancel{2}D_1} = \dfrac{16.875 \times 10^6 \times 32}{\pi((2D_1)^4 - (D_1)^4)} \rightarrow \dfrac{80}{D_1} = \dfrac{16.875 \times 10^6 \times 32}{\pi \times 15 D_1^4}$

12

$D_1 = 52.322$ mm

$\boxed{D_2 = 2D_1 = 104.644\ mm}$

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Q.5 (e) Determine the shrinkage deflection at the free end of a reinforced concrete cantilever beam with a effective span of 4.0 m. The beam has a rectangular cross-section of 350 mm × 600 mm (overall) and is reinforced with 4 bars of 25 mm ϕ at the top with an effective cover of 50 mm. The beam is subjected to an environment where the ultimate shrinkage strain of concrete (ε_{cs}) is 0.0005. Use M-25 grade concrete and Fe-500 grade steel.

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ANNEX C

(*Clauses* 22.3.2, 23.2.1 *and* 42.1)

CALCULATION OF DEFLECTION

C-1 TOTAL DEFLECTION

C-1.1 The total deflection shall be taken as the sum of the short-term deflection determined in accordance with C-2 and the long-term deflection, in accordance with C-3 and C-4.

C-2 SHORT-TERM DEFLECTION

C-2.1 The short-term deflection may be calculated by the usual methods for elastic deflections using the short-term modulus of elasticity of concrete, E_c and an effective moment of inertia I_{eff} given by the following equation:

$$I_{eff} = \frac{I_r}{1.2 - \dfrac{M_r}{M}\dfrac{z}{d}\left(1 - \dfrac{x}{d}\right)\dfrac{b_w}{b}}; \text{ but}$$

$$I_r \le I_{eff} \le I_{gr}$$

where

I_r = moment of inertia of the cracked section,

M_r = cracking moment, equal to $\dfrac{f_{cr} I_{gr}}{y_t}$ where f_{cr} is the modulus of rupture of concrete, I_{gr} is the moment of inertia of the gross section about the centroidal axis, neglecting the reinforcement, and y_t is the distance from centroidal axis of gross section, neglecting the reinforcement, to extreme fibre in tension,

M = maximum moment under service loads,

z = lever arm,

x = depth of neutral axis,

d = effective depth,

b_w = breadth of web, and

b = breadth of compression face.

For continuous beams, deflection shall be calculated using the values of I_r, I_{gr} and M_r modified by the following equation:

$$X_e = k_1\left[\frac{X_1 + X_2}{2}\right] + (1 - k_1)X_o$$

where

X_e = modified value of X,

X_1, X_2 = values of X at the supports,

X_o = value of X at mid span,

k_1 = coefficient given in Table 25, and

X = value of I_r, I_{gr} or M_r as appropriate.

C-3 DEFLECTION DUE TO SHRINKAGE

C-3.1 The deflection due to shrinkage a_{cs} may be computed from the following equation:

$$a_{cs} = k_3 \Psi_{cs} l^2$$

where

k_3 is a constant depending upon the support conditions,

0.5 for cantilevers,

0.125 for simply supported members,

0.086 for members continuous at one end, and

0.063 for fully continuous members.

Ψ_{cs} is shrinkage curvature equal to $k_4 \dfrac{\varepsilon_{cs}}{D}$

where ε_{cs} is the ultimate shrinkage strain of concrete (*see* 6.2.4).

$$k_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}} \le 1.0 \text{ for } 0.25 \le P_t - P_c < 1.0$$

$$= 0.65 \times \frac{P_t - P_c}{\sqrt{P_t}} \le 1.0 \text{ for } P_t - P_c \ge 1.0$$

Table 25 Values of Coefficient, k_1

(*Clause* C-2.1)

k_2	0.5 or less	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
k_1	0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97	1.0

NOTE — k_2 is given by

$$k_2 = \frac{M_1 + M_2}{M_{F1} + M_{F2}}$$

where

M_1, M_2 = support moments, and

M_{F1}, M_{F2} = fixed end moments.

[12 marks]

→ Shrinkage deflection

$\delta = k_3 \psi l_e^2$ $\qquad \{ l_{eff} = 4\,m \}$

$k_3 = 0.5 \rightarrow$ for cantilever.

$\psi_{cs} = \dfrac{k_4 \varepsilon_{cs}}{D}$

$\cancel{\delta = 0}$

$\cancel{k_4}\ \% P_t = \dfrac{4 \times \frac{\pi}{4} \times 25^2 \times 100}{350 \times 550}$ $\qquad \left\{ \begin{aligned} d &= D - \varepsilon_c \\ &= 600 - 50 \\ &= 550\,mm \end{aligned} \right.$

$= 1.0199\%$

$\% P_c = 0.$

$\therefore\ k_4$ @ $\% P_t - P_c = 1.0199\%$

$k_4 = 0.65 \times \dfrac{P_t - 0}{\sqrt{P_t}} = 0.65 \times \sqrt{P_t} = 0.6564 < 1 \rightarrow ok$

$\psi_{cs} = \dfrac{0.6564 \times (0.0005)}{600}$ $\qquad \{ \varepsilon_{cs} = 0.0005 \}$

$\psi_{cs} = 5.47055 \times 10^{-7}.$

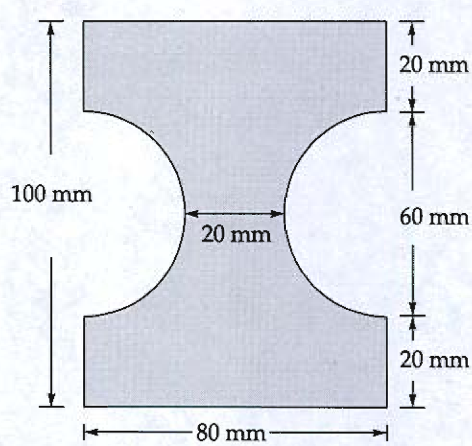
$\delta = 0.5 \times 5.47055 \times 10^{-7} \times (4000)^2$

$\boxed{\delta = 4.376\,mm}$

12

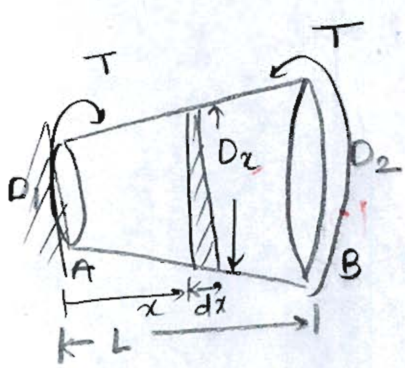
Q.6 (a) **(i)** A tapered solid circular shaft of length L, has cross-section with its diameter varying from D_1 at one end to D_2 at the other. Determine the expression for angle of rotation when the shaft is subjected to a pair of equal and opposite torque T applied at its ends. G is modulus of rigidity.

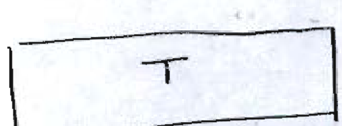
(ii) A steel section as shown in figure is subjected to a shear force of 20 kN. Determine the shear stress at the important points and sketch the shear stress distribution diagram.



[10 + 10 = 20 marks]

(i)





torque diagram

$$\theta_{B/A} = \frac{TL}{GJ}$$

but, J is varying?
Take a strip of width dx at sect^n x distance from A.

$$\theta = \int_0^L \frac{T\,dx}{G J_x} \quad ; \quad J_x = \frac{\pi D_x^4}{32}$$

$$\& \quad D_x = D_1 + \frac{(D_2 - D_1)}{L} \times x$$

$$= D_1 + Kx$$

$$\left\{ K = \frac{(D_2 - D_1)}{L} \right.$$

$$\theta = \int_0^L \frac{T\,dx}{G \frac{\pi}{32} D_x^4}$$

$$\theta = \frac{32T}{G\pi} \int_0^L \frac{dx}{(D_1 + Kx)^4}$$

$$\theta = \frac{32T}{G\pi} \left[\frac{1}{(D_1 + Kx)^3 (-3) \times K} \right]_0^L$$

$$\theta = \frac{-32T}{3KG\pi} \left[\frac{1}{D_2^3} - \frac{1}{D_1^3} \right]$$

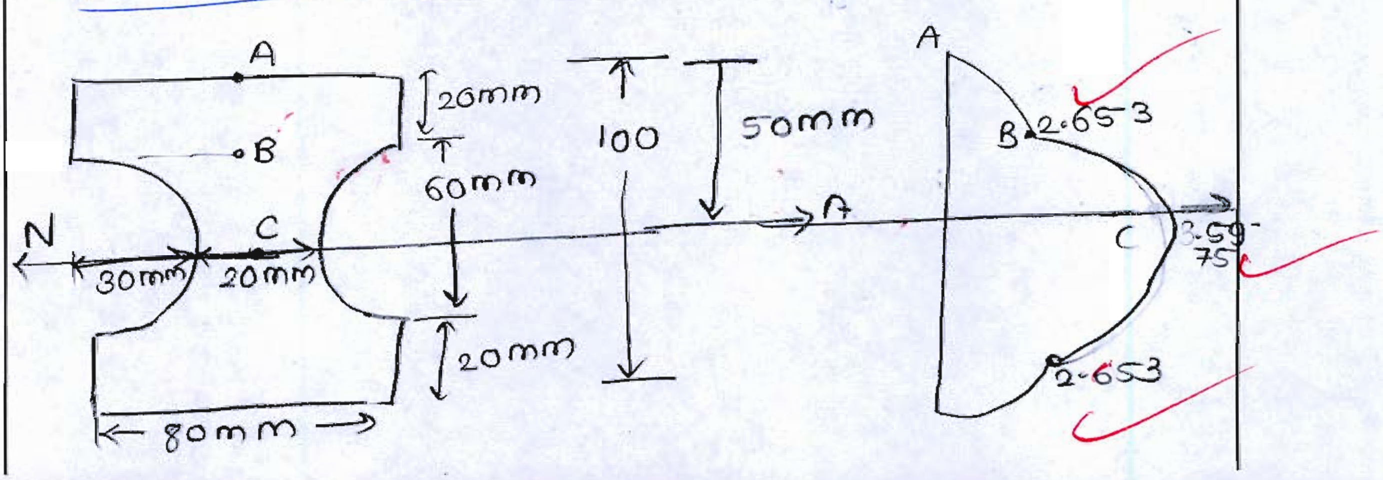
$$\theta = \frac{32T}{3KG\pi} \left[\frac{1}{D_1^3} - \frac{1}{D_2^3} \right] = \frac{32T(D_2^3 - D_1^3)}{3KG\pi D_2^3 D_1^3}$$

$$\theta = \frac{32T(D_2^3 - D_1^3)}{3 \times \frac{D_2 - D_1}{L} G\pi (D_1 D_2)^3} = \frac{32TL(D_1^2 + D_2^2 + D_1 D_2)}{3G\pi D_2^3 D_1^3}$$

$$\boxed{\theta = \frac{32TL(D_1^2 + D_2^2 + D_1 D_2)}{3G\pi D_2^3 D_1^3}}$$

10

(ii)



NA ~~lies~~ passes through C.G

∴ By symmetry depth of N.A = 50mm (from Top)

$I_{NA} = I_{\square} - I_{O}$

$= \frac{80\times100^3}{12} - \frac{\pi\times d^4}{64}\times\frac{1}{\cancel{2}}\times\cancel{2}$

$= \frac{80\times100^3}{12} - \frac{\pi\times60^4}{64}$

$= 60.305\times10^5\ mm^4$

$\tau_A = \frac{VQ}{It}$ {but, $Q = A\bar{y}$ }

$\tau_A = 0,$

$\tau_B = \frac{VQ}{It} = \frac{20\times10^3\times(20\times80\times40)}{60.305\times10^5\times80}$ ∵ V = 20kN

$\tau_B = 2.653\ N/mm^2$

$(\tau_C)_{middle} = \frac{VQ}{It} = \frac{20\times10^3\times2586.2833\times31.705}{60.305\times10^5\times20}$

$\tau_C = 13.5975\ N/mm^2$

$A_{\cancel{g}c} = 80\times100 - \frac{\pi\times60^2}{4}$

$= \frac{5172.567}{2}\ mm^2$

$\bar{y}_c = \frac{Ay_1 - A_2y_2}{A_1 - A_2}$

$\bar{y}_c = \frac{80\times50\times25 - \frac{\pi\times60^2}{4\ \ 2}\times\frac{4\times30}{3\pi}}{80\times50 - \frac{\pi}{4}\times\frac{60^2}{2}}$

$\bar{y}_c = 31.705\ mm$

Because of symmetry of section about NA shear stress diagram will also become symmetrical.

10

Q.6 (b) Design a short circular column with helical reinforcement to support a factored axial load of 1800 kN. The column is effectively held in position at both ends and restrained against rotation at one end, with an unsupported length of 3.5 m. Use M25 grade concrete and Fe415 grade steel. Perform all necessary design checks including diameter of longitudinal bars, diameter of helical reinforcement, and pitch of helix in accordance with IS 456:2000. Assume a clear cover of 40 mm.

[20 marks]

$P_u = 1800\,kN$

$\lambda < 12 \rightarrow$ short column

$K = 0.8$

$l_0 = 3.5\,m$

M25/Fe415

$c_c = 40\,mm$

let dia of column be D.

$\therefore\ P_u = 1.05\,[0.4 f_{ck} A_c + 0.67 f_y A_s]$ —— (1)

$\lambda = \frac{l_{eff}}{D} = \frac{0.8\times3500}{D} = 6 < 12$ $\left\{ \text{Let's assume } \lambda = 6 \right\}$

$D > 466.67$

$D \cancel{> 466.67}\ \cancel{350}$

Let, $D \cancel{\simeq 500\,mm}$. $D = 500\,mm$

$e_{min} = \left(\frac{l_0}{500} + \frac{D}{30}\right)$ or $20\,mm$ $\}_{max^m}$

$= \frac{3500}{500} + \frac{3500}{30}$ or $20\,mm$

$= 23.67\,mm\ (20 < 0.05\times D = 25\,mm$

& No external moments, $\lambda < 12$.

$\therefore\ P_u = 1.05\left[0.4\times25\times\left(\frac{\pi}{4}\times500^2 - A_s\right) + 0.67\times415\times A_s\right]$

$1800\times10^3 = 1.05\,[10\,(62500\pi - A_s) + (278.05 A_s)$

$A_s \rightarrow$ is coming -ve.

$\therefore$ let's provide min^m r/f $\Rightarrow \frac{0.8}{100}\times\frac{\pi}{4}\times500^2$

$= 1570.8\,mm^2 \rightarrow$ 8 bars of 16mm ϕ

Calculat^n of pitch

$$0.36\frac{f_{ck}}{f_y}\left(\frac{A_g}{A_c}-1\right) \leq \frac{V_h}{V_c}$$

$$\left\{ \begin{aligned} D_c &= D - 2E.C \\ &= 500 - 2\times40 \\ &= 420\ mm \end{aligned} \right.$$

$$\frac{0.36\times25}{415}\left(\frac{\pi/4\times500^2}{\pi/4\times420^2}-1\right) \leq \frac{\frac{\pi}{4}\phi_h^2\times\pi D_h\times\left(\frac{1000}{p}\right)}{\frac{\pi}{4}\times420^2\times1000}$$

let's $\phi_h = 6mm,$

$p \rightarrow$ pitch

$D_h = D_c - \phi_h = 420-6$
$= 416\ mm.$

$\therefore\ p = 29.47\ mm$

$p \ngtr 75\ mm.$

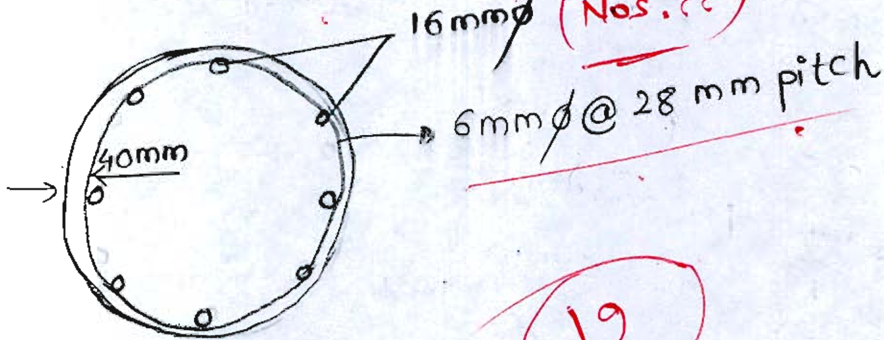
$\nless 25\ mm$

$\ngtr \frac{D_c}{6} = 70\ mm$

$\nless 3\phi_h = 18\ mm$

$\therefore$ provide 6mm ϕ stirrup @ pitch of 28mm

(Nos.??)



19

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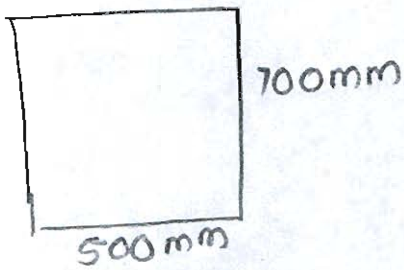
Q.6 (c) Design a section of a rectangular beam 500 mm wide and 700 mm deep subjected to a bending moment of 200 kNm, twisting moment of 15 kNm and a shear force of 150 kN (all are factored). Use M20 mix and Fe415 grade steel. Provide 35 mm effective cover and 0.25% tension steel. Design shear strength of for M20 concrete τ_c (N/mm^2) with percentage tension steel in as given below:

P_t	τ_c (N/mm^2)
0.25	0.36
0.50	0.48
0.75	0.56

[20 marks]



$M_u = 200$ kNm

$T_u = 15$ kNm

$V_u = 150$ kN

M20/Fe415

$d_c = 35$ mm

$A_{st} = 0.25\%$

$B = 500$ mm

$D = 700$ mm

$d = 665$ mm

(i) $V_{ue} = V_u + \frac{1.6\,T_u}{B}$

$= 150 + \frac{1.6 \times 15}{0.5} = 198$ kN

$\tau_v = \frac{198 \times 10^3}{500 \times 665} = \frac{V_{ue}}{Bd} = 0.5954 < (\tau_{cmax} = 2.8$ N/mm^2)

(ii) $M_{Tu} = \frac{T_u}{1.7}\left(1 + \frac{D}{B}\right) = \frac{15}{1.7}\left(1 + \frac{0.7}{0.5}\right)$

$M_{Tu} = 21.176 < M_u$

$\therefore M_{ue_1} = M_u + M_{Tu} = 221.176$ kNm

(No need to design for M_{ue_2})

~~$d_{req} = \sqrt{\quad}$~~

$M_{ulim} = Q B d^2/10^6 = 0.138 \times f_{ck}\, B d^2$ (for Fe415)

$= 0.138 \times 20 \times 500 \times 665^2/10^6$

$= 610.2705$ kNm

$M_{ue_1} < M_{ulim} \rightarrow$ Under r/f sect^n

$\therefore A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_{ue1}}{f_{ck} B d^2}}\right) B d$

$= \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 221.176 \times 10^6}{20 \times 500 \times 665^2}}\right) \times 500 \times 665$

$= 981.806\ mm^2 > A_{stmin}$, $\% P_t = \frac{0.25}{100} \times 500 \times 700$

$= 875\ mm^2$

$A_{stmin} \rightarrow \frac{A_{stmin}}{Bd} = \frac{0.85}{f_y}$ (∴ provide $981.81\ mm^2$)

$A_{stmin} = 681.024$

∴ provide 5 bars of 16 mm ϕ.

Shear r/f design ; $\% P_t = \frac{5 \times \frac{\pi}{4} \times 16^2 \times 100}{500 \times 665}$

$V_s = (\tau_v - \tau_c) \times B \times d$ $= 0.302\%$

$= (0.5954 - 0.385) \times \frac{500 \times 665}{10^3}$

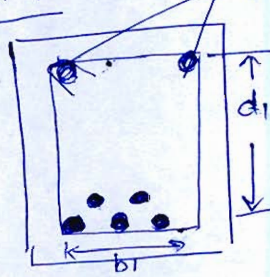
$= 69.958\ kN$

τ_c @ 0.302%

$\tau_c = 0.385$

$S_v = \frac{0.87 f_y A_{sv} d_1}{\left(\frac{V_u}{2.5} + \frac{T_u}{B_1}\right)} = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2 \times 630}{\left(\frac{150 \times 10^3}{2.5} + \frac{15 \times 10^6}{430}\right)}$

$= 241\ mm$



$S_v = \frac{0.87 f_y A_{sv} d}{V_s} = 345\ mm$

let's $\phi_s = 8\ mm$

2legged 8mm ϕ stirrup.

Refer Fig ??

18

$S_{v_{min}} = \frac{0.87 f_y A_{sv}}{0.4 B} = 181.48\ mm$

$S_v \not>$ $x_1 = 454$

$\frac{x_1 + y_1}{4} = 277\ mm$

300

∴ provide ~~8t~~ 2L-8ϕ stirrup @ 180 mm c/c.

$d_1 = d - \varepsilon_c$

$d_1 = 630\ mm$

$b_1 = 500 - 2 \times 35$

$= 430\ mm$

$x_1 = b_1 + \phi_{main} + \phi_{stirrup}$

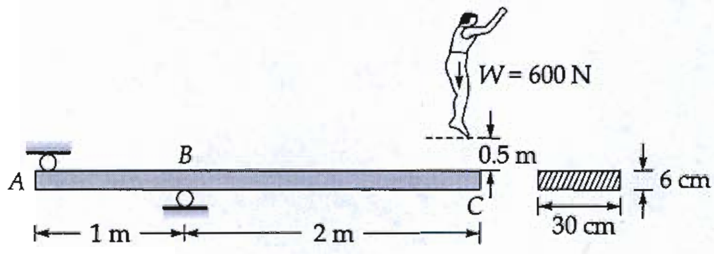
$= 430 + 16 + 8$

$= 454\ mm$

$y_1 = d_1 + \phi_{main} + \phi_{stirrup}$

$= 654\ mm$

Q.7 (a) A man weighing 600 N jumps from a height of 0.5 m on a diving board of dimensions 30 cm × 6 cm supported as shown in figure. Find the maximum stress produced in the board.



Take, $E = 10$ GPa.

[20 marks]

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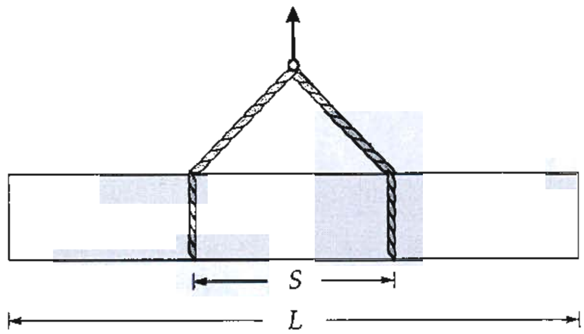
Q.7 (b) A fiberglass pipe is lifted by a sling, as shown in the figure.



The outer diameter of the pipe is 180 mm, its thickness is 6 mm, and its weight density is 20 kN/m^3. The length of pipe is L =18 m and the distance between lifting points is S = 5 m.

(i) Determine the maximum bending stress in the pipe due to its own weight.

(ii) Find spacing S between lift points which will minimize the bending stress. What will be minimum bending stress?

(iii) What spacing S will lead to maximum bending stress? What is that stress?

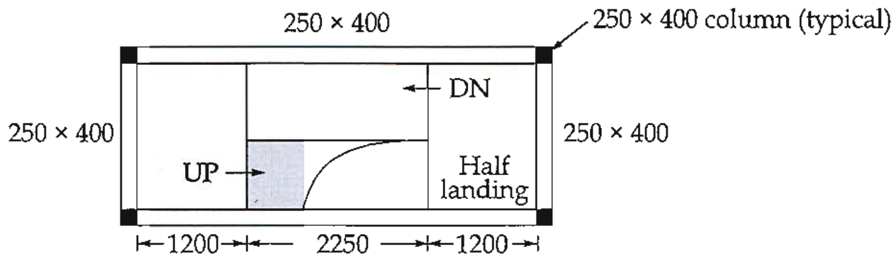
[20 marks]

Q.7 (c) A shaft *ABC* of 500 mm length and 40 mm external diameter is bored for a part of its length *AB*, to a 20 mm diameter and for the remaining length *BC*, to a 30 mm diameter bore. If the shear stress is not to exceed 80 N/mm^2, find the maximum power, the shaft can transmit at a speed of 200 rpm. Take, shear modulus is 80 GPa.

If the angle of twist in the length of 20 mm diameter bore is equal to that in the length of 30 mm diameter bore, find the lengths of the shaft that has been bored to 20 mm and 30 mm diameter. Also determine the total angle of twist.

[20 marks]

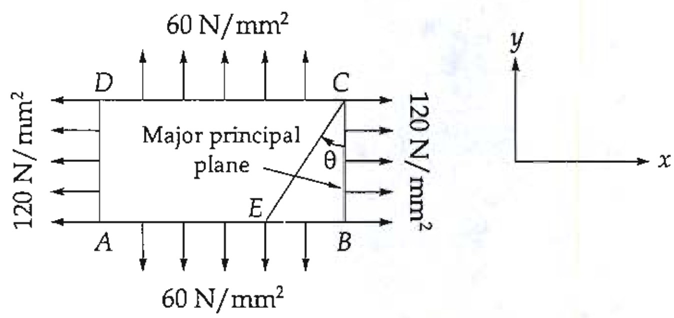
Q.8 (a) Details of a dog-legged stairs for a building is shown in the figure below. The floor to floor height is 3.0 m. The live load may be taken as 2.5 kN/m^2. Thickness of the stair case slab is 150 mm. The rise and tread are 150 mm and 250 mm respectively. Design and detail the typical flight. Use M-25 grade concrete and Fe500 grade of steel. Use limit state method.



M_u/bd^2	2.5	2.6	2.7	2.8	2.9	3.0	3.1
P_t	0.65	0.695	0.727	0.76	0.794	0.826	0.863

[20 marks]

Q.8 (b) (i) At a point in a strained material the principal tensile stresses across two perpendicular planes are 120 N/mm^2 and 60 N/mm^2. Determine normal stress, shear stress and the resultant stress on a plane inclined at 20° in clockwise direction with the major principal plane. Determine also the obliquity. What will be the intensity of stress in x-direction, which acting alone will produce the same maximum strain if Poisson's ratio = 0.25



(ii) A solid circular shaft of 10 cm diameter and of length 4 m is transmitting 75 kW power at 150 rpm. Determine:

(a) The maximum shear stress induced in the shaft and

(b) Strain energy stored in the shaft. Take $G = 8 \times 10^4$ N/mm^2

[12 + 8 = 20 marks]

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Q.8 (c) A hollow mild steel shaft having 100 mm external diameter and 50 mm internal diameter is subjected to a twisting moment of 13.2 kN-m and a bending moment of 4.125 kN-m. Calculate the principal stresses and find the direct normal stress which, acting alone, would produce the same (i) By maximum elastic strain energy theory, (ii) By maximum elastic shear strain energy theory, as that produced by the principal stresses acting together. Take Poisson's ratio as 0.25.

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work