MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 7

Section A : Advanced Electronics + Electronic Measurements and Instrumentation

Q.1 (a) Solution:

Given,

4-dial decade has been set at 4639 Ω. Thus,

decade *a* of 10 × 1000 Ω ±0.1% is set at 4,

∴ resistance of '*a*': 4 × 1000 Ω = 4000 Ω

error in decade '*a*' = 0.1% of 4000 Ω = $\frac{0.1}{100} \times 4000 = 4\ \Omega$

∴ Resistance at decade '*a*' is 4000 ± 4 Ω

decade *b* of 10 × 100 Ω ± 0.1% is set at 6,

∴ resistance of '*b*': 6 × 100 Ω = 600 Ω

error in decade '*b*' = 0.1% of 600 = $\frac{0.1}{100} \times 600 = 0.6\ \Omega$

∴ Resistance at decade '*b*' is = 600 ± 0.6 Ω

decade '*c*' of 10 × 10 Ω ± 0.5% is set at 3,

∴ Resistance of '*c*' = 3 × 10 = 30 Ω

error in decade '*c*' = 0.5% of (30) = $\frac{0.5}{100} \times 30 = 0.15\ \Omega$

∴ Resistance at decade '*c*' is 30 ± 0.15 Ω

decade '*d*' of 10 × 1 Ω ± 1.0% is set at 9,

∴ resistance of '*d*' = 9 × 1 = 9 Ω

error in decade 'd' = 1.0% of (9) = $\frac{1}{100} \times 9 = 0.09\ \Omega$

$\therefore$ Resistance at decade 'd' is $9 \pm 0.09\ \Omega$

$\therefore$ Total resistance of 4-dial decade set is,

$$R = (4000 \pm 4\ \Omega) + (600 \pm 0.6\ \Omega) + (30 \pm 0.15\ \Omega) + (9 \pm 0.09\ \Omega)$$

$$R = 4639 \pm 4.84\ \Omega$$

$\therefore$ Limiting Error, $\in_r = \frac{4.84}{4639} \times 100 = 0.104\%$

Maximum resistance = 4639 + 4.84 = 4643.84 Ω

Minimum resistance = 4639 – 4.84 = 4634.16 Ω

Thus, the resistance value varies from 4634.16 Ω to 4643.84 Ω.

Q.1 (b) Solution:

From the given state diagram, the state table can be obtained as below:

State Table :

Present State	Next State		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	g	f	0	1
g	a	f	0	1

Two states are considered to be equivalent if and only if for every input sequence the circuit produces the same output sequence irrespective of which one of the two states is the starting state. Thus, state 'g' and 'e' are equivalent states. So, we replace state 'g' with state 'e'.

Thus, the reduced state table can be drawn as :

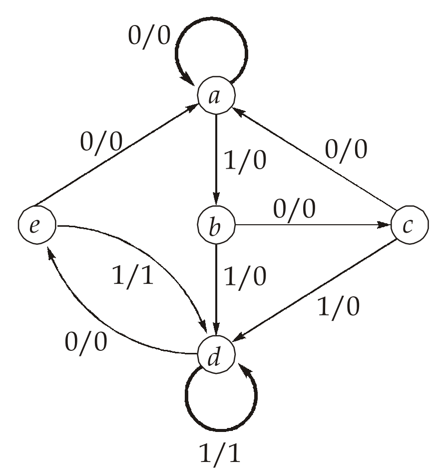
Reduced the State Table :

Present State	Next State		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	e	f	0	1

From above state table, it can be seen that state d and f are equivalent, so replacing state f by state d, we get the reduced state table as below:

Present State	Next State		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	d	0	1
e	a	d	0	1

Now, the reduced state diagram can be drawn with the help of reduced state table as below:



Q.1 (c) Solution:

The flux linking with the moving coil is given by $N(A \cos \theta) \times B$

where $A = \frac{\pi}{4}d^2$ = area of moving coil

d = diameter of moving coil

θ = angle between axes of fixed and moving coils

B = flux density

N = number of turns of moving coil.

$\therefore$ Mutul inductance, $M = \dfrac{\left(\frac{\pi}{4}\right)d^2 NB\cos\theta}{I}$

where I is the current through the fixed or field coil.

$$\frac{dM}{d\theta} = \frac{\pi d^2 NB}{4I}\sin\theta$$

Deflection torque, $$T_d = \frac{V}{R_p} I \cos\phi \frac{dM}{d\theta}$$

$$= \frac{V}{R_p} I \cos\phi \frac{\pi d^2 NB}{4I} \sin\theta = \frac{\pi d^2 NBV}{4R_p} \cos\phi \sin\theta$$

$$= \frac{\pi(2.5\times10^{-2})^2 \times 500 \times 1.1\times10^{-3} \times 100 \times 0.7}{4\times2000} \sin\theta$$

$\therefore$ $T_d = 9.45 \times 10^{-6} \sin\theta$ Nm

(i) Given, $\theta = 45°$

$\therefore$ $T_d = 9.45 \times 10^{-6} \times \sin 45° = 6.69 \times 10^{-6}$ Nm

(ii) Given, $\theta = 90°$

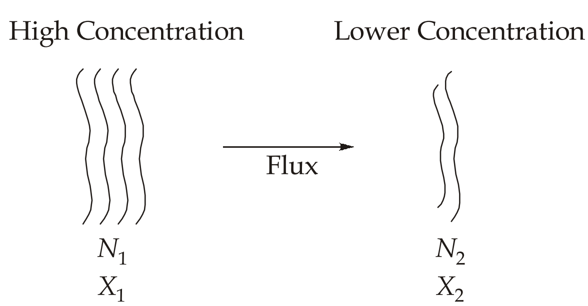
$\therefore$ $T_d = 9.45 \times 10^{-6} \times \sin 90° = 9.45 \times 10^{-6}$ Nm

Q.1 (d) Solution:

(i) Diffusion : Diffusion is the process of introduction of controlled amount of dopant atoms into the semiconductor.

Diffusion alters the type of conductivity of semiconductor. Commonly used diffusion methods are : diffusion from a chemical source, diffusion from a doped oxide source and diffusion and annealing from an ion implanted layer.

In the diffusion process, the dopant atoms move from region of higher concentration to lower concentration.



$N_1 \rightarrow$ higher concentration of atoms at a distance X_1

$N_2 \rightarrow$ lower concentration of atoms at a distance X_2

Here, $N_1 > N_2$

$X_1 < X_2$

Concentration gradient is given by

$$\frac{\partial N}{\partial X} = \frac{N_2 - N_1}{X_2 - X_1}$$

Example : Room Freshener

Process of diffusion is governed by Fick's law. This theory is based on the analogy between material transfer in a solution and heat transfer by conduction.

According to Fick's law, particles diffuse from a higher concentration to a lower concentration and flux density is proportional to the concentration gradient.

Fick's Ist Law : The diffusion flux is defined as the number of dopant atoms passing through a unit area in a unit time and is given by :

$$\boxed{J = -D\frac{\partial N}{\partial x}} \quad ...(1)$$

where, J = Diffusion flux

D = Diffusion constant

$\frac{\partial N}{\partial x}$ = Concentration gradient

Diffusion is temperature dependent and can be described by the Arrhenius expression:

$$D = D_0 e^{-E_A/KT}$$

where, D = Diffusion coefficient

D_0 = Maximum diffusion constant, independent of temperature

E_A = Activation energy

K = Boltzmann constant = 8.6×10^{-5} eV/Kelvin

T = Temperature in Kelvin

Fick's IInd Law : According to law of conservation of matter, i.e., under the condition that no materials are formed or consumed in the host semiconductor, the continuity equation is written as:

$$\boxed{\frac{\partial J}{\partial x} = -\frac{\partial N}{\partial t}} \quad ...(2)$$

where $N \rightarrow$ Number of dopant atoms/volume

and $\nabla \cdot \bar{J} = -\frac{\partial \rho_V}{\partial t}$

where ρ_V = Charge density

Now, from equation (1) and (2),

$$J = -D\frac{\partial N}{\partial x}$$

$$\frac{\partial J}{\partial x} = -D\frac{\partial^2 N}{\partial x^2} = \frac{-\partial N}{\partial t}$$

$$\boxed{D\frac{\partial^2 N}{\partial x^2} = \frac{\partial N}{\partial t}}$$

Thus, Concentration is a function of both space and time.

Q.1 (e) Solution:

Given,

$$\text{Resistance, } R_1 = 3980\ \Omega \text{ at } T_1 = 0°C = 273\text{ K}$$

$$\text{Resistance, } R_2 = 794\ \Omega \text{ at } T_2 = 50°C = 323\text{ K}$$

The relation between resistance and temperature is given as,

$$R_T = aR_0\, e^{b/T}$$

at T_1: $$R_1 = 3980\ \Omega$$

$$\therefore \quad 3980 = aR_0 e^{\frac{b}{273}} \quad \text{...(i)}$$

at T_2: $$R_2 = 794\ \Omega$$

$$794 = aR_0 e^{\frac{b}{323}} \quad \text{...(ii)}$$

Dividing equation (i) by equation (ii),

$$\frac{3980}{794} = e^{b\left(\frac{1}{273} - \frac{1}{323}\right)}$$

$$5.0126 = e^{b(0.0036\ -\ 0.0030)}$$

$$\ln(5.0126) = b(0.000567)$$

$$\therefore \quad b = \frac{1.612}{0.000567} = 2843\text{ K}$$

From equation (i),

$$3980 = aR_0 e^{\frac{2843}{273}}$$

$$aR_0 = 3980e^{-10.414}$$

$$\therefore \quad aR_0 = 0.1195$$

given, $$R_0 = 1\ \Omega$$

$$\therefore \quad a = 0.1195$$

Q.2 (a) Solution:

(i) Given data :

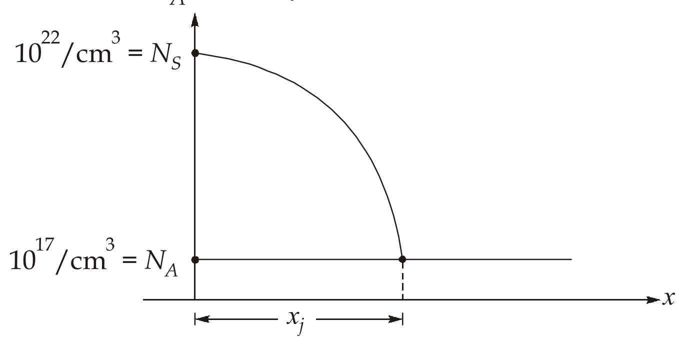
Diffusion coefficient, $D = 10^{-14}\ \text{cm}^2/\text{sec}$;

Pre-deposition time, $t = 1\ \text{hr.} = 3600\ \text{sec}$

Solid solubility, $N_S = 10^{22}/\text{cm}^3$

Sample doping concentration,

$$N_A = 10^{17}/\text{cm}^3$$



As we know, the total dose i.e., total number of atoms per unit area is given by S

$$S = 2N_S\sqrt{\frac{Dt}{\pi}}$$

$$= 2\times 10^{22}\sqrt{\frac{10^{-14}\times 3600}{3.14}}$$

$$\boxed{S = 6.77\times 10^{16}/\text{cm}^2}$$

For pre-deposition, $N(X, t) = N_S erfc\dfrac{X}{2\sqrt{Dt}}$

At the junction depth X_j, the dopant concentration equals the background concentration i.e.

$$N_A = N_S erfc\frac{X_j}{2\sqrt{Dt}}$$

Junction depth X_j is given by

$$X_j = 2\sqrt{Dt}\ erfc^{-1}\left(\frac{N_A}{N_S}\right)$$

$$= 2\sqrt{10^{-14}\times 3600}\ erfc^{-1}\left(\frac{10^{17}}{10^{22}}\right)$$

$$= 1.2 \times 10^{-5} \text{ erfc}^{-1} (10^{-5})$$

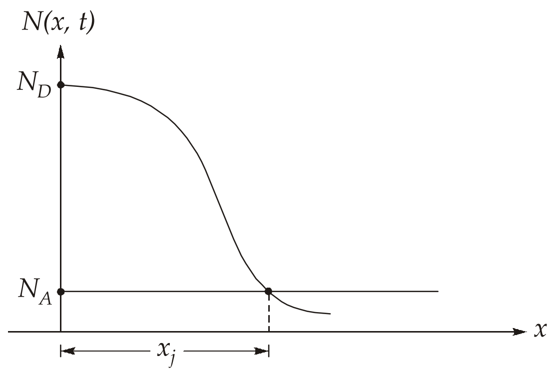
$$= 1.2 \times 10^{-5} \times 3.12$$

$$X_j = 3.74 \times 10^{-5} \text{ cm}$$

$$\boxed{X_j = 0.3704 \ \mu\text{m}}$$

(ii) For drive in profile, the concentration profile is Gaussian given as:

$$N(X, t) = \frac{S}{\sqrt{\pi D t}} e^{\frac{-X^2}{4Dt}}$$



At the junction,

$$X = X_j$$

and

$$N(X, t) = N_A$$

Thus,

$$N_A = \frac{S}{\sqrt{\pi D t}} e^{-X_j^2 / 4Dt}$$

$$\ln\left(\frac{N_A \sqrt{\pi D t}}{S}\right) = \frac{-X_j^2}{4Dt}$$

$$4Dt \ln\left(\frac{S}{N_A \sqrt{\pi D t}}\right) = X_j^2$$

$$X_j = \sqrt{4Dt \ln\left(\frac{S}{N_A \sqrt{\pi D t}}\right)}$$

Given,

$$S = 6.77 \times 10^{16} \text{ cm}^{-2}$$

$$N_A = 10^{17} \text{ cm}^{-3}$$

$$D = 10^{-14} \text{ cm}^2/\text{sec};$$

$$\text{Drive-in time, } t = 7200 \text{ sec} = 2 \text{ hours}$$

Now, substituting the above formula in above formula of X_j,

$$X_j = \sqrt{4\times10^{-14}\times7200\ln\left(\frac{6.77\times10^{16}}{10^{17}\sqrt{\pi\times10^{-14}\times7200}}\right)}$$

$$X_j = 5.55 \times 10^{-5} \text{ cm}$$

$$\boxed{X_j = 0.55\ \mu\text{m}}$$

Surface concentration $N(X, t)$ at $X = 0$

$$N(X,t)\big|_{X=0} = \frac{S}{\sqrt{\pi Dt}} = \frac{6.77\times10^{16}}{\sqrt{\pi\times10^{-14}\times7200}}$$

$$\boxed{N(X,t)\big|_{X=0} = 4.5\times10^{21}/\text{cm}^3}$$

Q.2 (b) Solution:

The state diagram is already given. The states are already assigned.

Based on this state assignment, the state table is obtained as below.

Present State		Next State		Output	
y_1	y_2	$X = 0$	$X = 1$	$X = 0$	$X = 1$
0	0	0 1	0 0	0 0	0 0
0	1	1 1	1 1	0 1	0 1
1	1	1 0	1 0	1 1	1 1
1	0	1 0	0 0	1 0	1 0

Two state variables are required. Two state variables can have a maximum of four states. So, there are no invalid states.

Now, by selecting the memory elements as S-R flip flops and writing the excitation table.

Excitation Table :

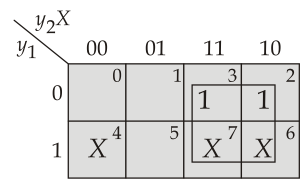
Present State		Input	Next State		Required Excitations				Output	
y_1	y_2	X	y_1^+	y_2^+	S_1	R_1	S_2	R_2	z_1	z_2
0	0	0	0	1	0	X	1	0	0	0
0	0	1	0	0	0	X	0	X	0	0
0	1	0	1	1	1	0	X	0	0	1
0	1	1	1	1	1	0	X	0	0	1
1	1	0	1	0	X	0	0	1	1	1
1	1	1	1	0	X	0	0	1	1	1
1	0	0	1	0	X	0	0	X	1	0
1	0	1	0	0	0	1	0	X	1	0

Now, drawing the K-maps, simplifying them the minimal expressions for S_1, R_1, S_2, R_2 and Z_1, Z_2 in terms of y_1, y_2 are obtained as shown below.

From the excitation we can see that $z_1 = y_1$ and $z_2 = y_2$.

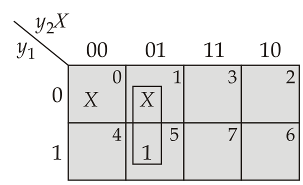
K-Maps :

For S_1 :



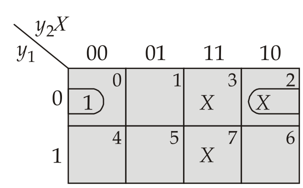
$S_1 = y_2$

For R_1 :



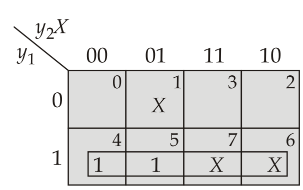
$R_1 = \bar{y}_2 X$

For S_2 :



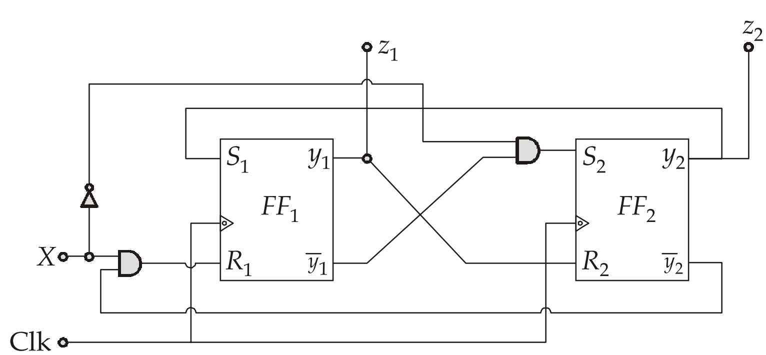
$S_2 = \bar{y}_1 \bar{X}$

For R_2 :



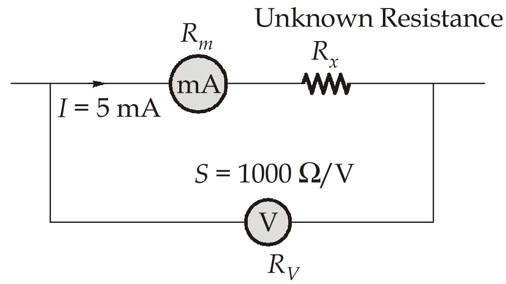
$R_2 = y_1$

The logic circuit using S-R flip flops based on these minimal expressions is shown in figure below.



Q.2 (c) Solution:

(i) The measurement setup is as given below,



Here, R_m is the resistance of milliammeter and R_V is the resistance of voltmeter.

1. The apparent resistance is the resistance calculated directly from the instrument readings (Voltage/Current) without accounting for the internal resistance of the meters. Apparent resistance of the unknown resistance:

$$R_T = \frac{V_T}{I_T} = \frac{100}{5\times10^{-3}} = 20\text{ k}\Omega$$

Neglecting the resistance of milliammeter and voltmeter, the value of unknown resistor is

$$R_a = 20\text{ k}\Omega$$

2. Actual resistance of the unknown resistance is R_x,

Resistance of voltmeter,

$$R_V = S \times V_{\text{scale}} = 1000 \times 150 = 150\text{ k}\Omega$$

$$\because \quad R_T = \frac{R_x \cdot R_V}{R_x + R_V} = 20\times10^3$$

$$\frac{R_x\times150\times10^3}{R_x+150\times10^3} = 20 \times 10^3$$

$$R_x \times 150 \times 10^3 = 20 \times 10^3 (R_x + 150 \times 10^3)$$

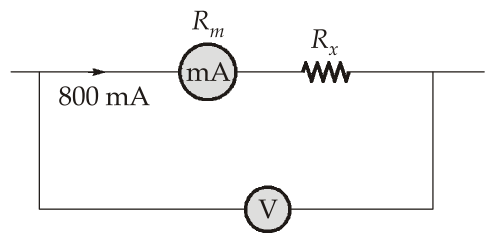
$$7.5R_x = R_x + 150 \times 10^3$$

$$R_x = 23.077\text{ k}\Omega$$

3. Percentage error, $$\%\in_r = \frac{\text{Measured value} - \text{True value}}{\text{True value}}\times100$$

$$\%\in_r = \frac{20-23.077}{23.077}\times100 = -13.33\%$$

(ii) When milliammeter reads 800 mA and the voltmeter reads 40 V on its 150 V scale:



S = 1000 V/Ω
Voltmeter reading = 40 V
R_V = 1000 × 150 = 150 kΩ

1. Apparent resistance:

$$\text{Total Resistance, } R_T = \frac{V_T}{I_T} = \frac{40}{800} \times 1000 = 50\ \Omega$$

As millimeter is assumed to be ideal, then apparent resistance R_a = 50 Ω

2. Actual resistance of the unknown resistor,

$$R_T = R_x \parallel R_V$$

$$R_T = \frac{R_x \cdot R_V}{R_x + R_V} = 50$$

$$50 = \frac{R_x \times 150000}{R_x + 150000}$$

$$50(R_x + 150000) = R_x \times 150000$$

$$R_x + 150000 = 3000R_x$$

$$2999R_x = 150000$$

$$R_x \approx 50.016\ \Omega$$

3. Percentage error,

$$\%\in_r = \frac{M \cdot V - T \cdot V}{T \cdot V} \times 100$$

$$= \frac{50 - 50.016}{50.016} \times 100 = -0.03\%$$

(iii) The error present in the above calculation is due to the resistance offered by the voltmeter and that effect is termed as loading error.

Loading error:

One of the most common errors committed by beginners, is the improper use of an instrument for measurement work. For example, a well calibrated voltmeter may give a misleading voltage reading when connected across a high resistance circuit. The same voltmeter, when connected in a low resistance circuit, may give a more dependable reading. These examples illustrate that the voltmeter has a loading effect on the circuit, altering the actual circuit conditions by the measurement process.

To minimize the loading effect (where the voltmeter draws current away from the circuit, leading to an inaccurately low voltage reading), it must be ensured that the voltmeter's internal resistance is significantly higher than the resistance being measured.

Q.3 (a) Solution:

Given:
$$I = 4\theta^n$$
$$\text{Sping constant } C = 0.16 \text{ N-m/rad}$$
$$L = 10 \text{ mH at } \theta = 0$$

(i) The deflecting torque (T_d) and controlling torque (T_c) in a moving iron instrument is given by,

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}, T_c = C\theta$$

At equilibrium: $T_d = T_c$ i.e.,

$$\frac{1}{2} I^2 \frac{dL}{d\theta} = C\theta$$

Substituting $I = 4\theta^n$

$$I^2 = 16\theta^{2n}, \text{ we get}$$

$$\frac{1}{2} \cdot 16\theta^{2n} \frac{dL}{d\theta} = C\theta$$

$$8\theta^{2n} \frac{dL}{d\theta} = C\theta$$

Rearrange:
$$\frac{dL}{d\theta} = \frac{C}{8} \theta^{1-2n}$$

Integrate:
$$L = \frac{C}{8} \int \theta^{1-2n} d\theta = \frac{C}{8} \cdot \frac{\theta^{2-2n}}{2-2n} + K$$

Substitute $C = 0.16$:

$$L = \frac{0.16}{8(2-2n)} \theta^{2-2n} + K$$

$$L = \frac{0.02}{2(1-n)}\theta^{2-2n} + K$$

Given: L = 10 mH at θ = 0. Thus, we get

$$K = 10 \text{ mH} = 0.01$$

Thus, the expression for self-inductance is obtained as,

$$L(\theta) = \frac{0.02}{2(1-n)}\theta^{2-2n} + 0.01$$

(ii) For n = 0.75 and L = 60 mH

$$L = \frac{0.02}{2(0-0.75)}\theta^{0.5} + 0.01 = \frac{0.02}{0.5}\theta^{0.5} + 10 = 0.04\theta^{0.5} + 0.01$$

$$0.06 = 0.04\theta^{0.5} + 0.01$$

$$0.05 = 0.04\theta^{0.5}$$

$$\theta^{0.5} = \frac{0.05}{0.04} = 1.25$$

$$\theta = (1.25)^2 = 1.5625 \text{ rad}$$

Meter current: $$I = 4\theta^{0.75}$$

$$I = 4(1.5625)^{0.75} = 4 \times 1.3975 \approx 5.59 \text{ A}$$

Q.3 (b) Solution:

(i) Silicon Manufacturing Process :

Raw material for silicon is quartzite. Quartzite is a rock of pure SiO_2.

Manufacturing of Metallurgical Silicon :

SiO_2 is heated to a temperature of 1900°C and carbon is added to it.

$$SiO_2 + C \xrightarrow{1900°C} Si + CO_2$$

- Silicon obtained is called Metallurgical Si and it has the purity of 98%.
- This cannot be used for electronic purposes.

Manufacturing of Electronic Grade Silicon :

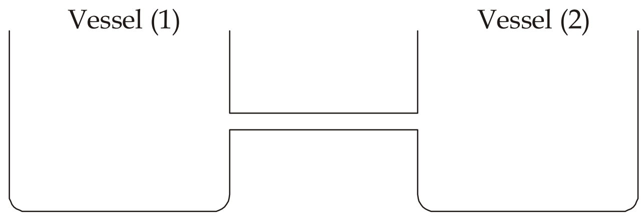
- It is a two step process :

  Step 1 :

$$Si + 3HCl \longrightarrow SiHCl_3 + H_2$$

- Powdered metallurgical Si is mixed to HCl at elevated temperature (1000°C) to form Trichlorosilane ($SiHCl_3$) along with light impurities.

- Trichloro Silane is liquified and purified using fractional distillation.



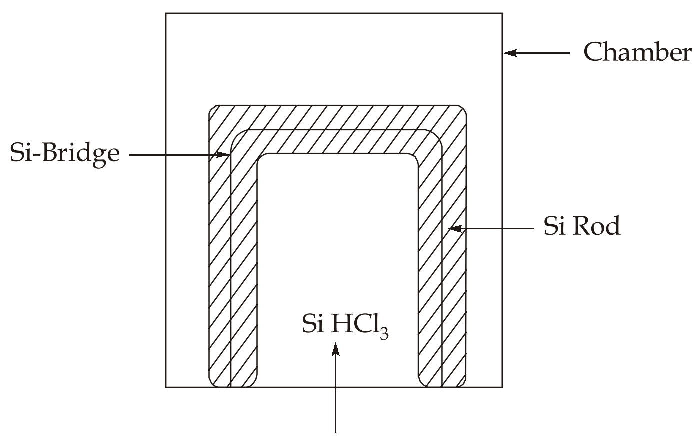
Assume *A* has lesser boiling point as compared to *B*.

When heated upto temperature of boiling point of *A*. *A* will get evaporated and can be collected in vessel (2). This method is known as fractional distillation, used for separation of liquids.

$SiHCl_3$ has a low boiling point and distillation is used to purify the $SiHCl_3$ from the impurities.

Step 2 :

Manufacturing of Si from $SiHCl_3$



$$H_2 + SiHCl_3 \xrightarrow{1100°C} Si + 3HCl$$

- Trichloro Silane is mixed with H_2 to form Si and HCl.
- The reaction is surface catalysed. The Si atoms released can be deposited as an epitaxial layer.
- The Si obtained in this process is called as electronic grade Silicon or Poly Crystalline Silicon. It has a purity of 99.99%.

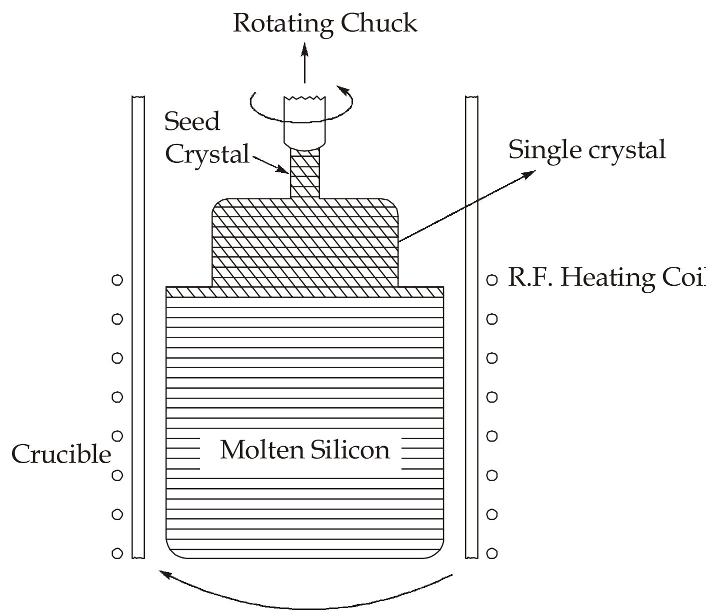
(ii) Czochralski Method (CZ Method)

The CZ method is used to convert polycrystalline Si to single crystalline Si. To initiate this process, seed crystal is required.

Seed crystal is a small piece of single crystalline silicon.

Process of CZ Method :

The schematic of this growth process is shown in figure.



The various components of the process are :

1. Furnace
2. Crystal Pulling Mechanism
3. Ambient Control – Atmosphere
4. Control System

The starting material for CZ process is electronic grade silicon, which is melted in the furnace. To minimize contamination, the crucible is made of SiO_2 or SiN_x. The drawback is that at the high temperature, the inner liner of the crucible also starts melting and has to be replaced periodically.

The furnace is heated above 1500°C, since Si melting point is 1412°C. A small seed crystal, with the desired orientation of the final wafer, is dipped in the molten Si and slowly withdrawn by crystal pulling mechanism. The seed crystal is also rotated while it is being pulled; to ensure uniformity across the surface. The furnace is rotated in the direction opposite to the crystal puller.

The molten Si sticks to the seed crystal and starts to solidify with the same orientation as the seed crystal. Thus, a single crystal ingot is obtained. To create doped crystals, the dopant material is added to the Si melt so that it can be incorporated in the growing crystal.

The process control, i.e., speed of withdrawal and the speed of rotation of the crystal puller is crucial to obtain a good quality single crystal. There is a feedback system that control this process.

The final solidified Si obtained is the single crystal ingot.

Q.3 (c) Solution:

(i) 1. The ±1 Count Error

- When an electronic counter makes a measurement, a ±1 count ambiguity can exist in the least significant digit. This is often referred to as quantization error. This ambiguity can occur because of the non-coherence between the internal clock frequency and the input signal.

2. The Time Base Error

- Any error resulting from the difference between the actual time base oscillator frequency and its nominal frequency is directly translated into a measurement error.
- The difference is the cumulative effect of all the individual time base oscillator errors and may be expressed as dimensionless factor such as so many parts per million.

3. Trigger Error

- Trigger error is a random error caused by noise on the input signal and noise from the input channels of the counter.
- In period and time interval measurements, the input signal(s) control the opening and closing of the counter's gate.
- This causes the main gate to be open for an incorrect period of time.
- This results in a random timing error for period and time interval measurements.

4. Frequency Measurement Error

- The accuracy of an electronic counter is dependent on the mode of operation.
- The total frequency measurement error may be defined as the sum of its ±1 count error and its total time base error.
- The relative frequency measurement error due to ±1 count ambiguity is

$\Delta f/f = (\pm 1)/f_{in}$

where f_{in} is the input signal frequency.

(ii) Given, count displayed = 1133

Pulses of f_2 are counted during 100 cycles of f_1

$$f_1 = 33 \text{ kHz}$$

Let T = time for 100 cycles of f_1

$$T = \frac{100}{f_1} = \frac{100}{33000} = \frac{1}{330} \text{sec}$$

The display shows the total number of pulses of frequency f_2 that occurred during this time T. Thus,

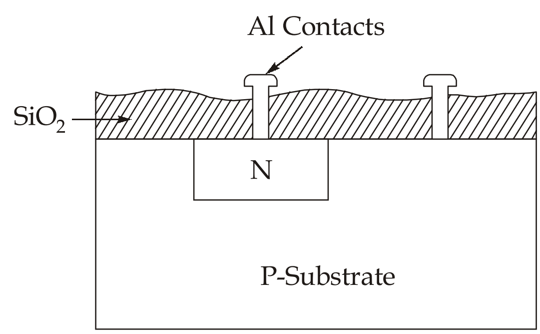
$$\text{Count} = f_2 \times T$$

$$1133 = f_2 \times \frac{1}{330}$$

$$\therefore \quad f_2 = 1133 \times 330 = 373890 \text{ Hz} = 373.89 \text{ kHz}$$

Q.4 (a) Solution:

A complete structure of PN junction diode is shown as



- SiO_2 is made by using dry oxidation or wet oxidation.

  Dry oxidation :

$$Si + O_2 \xrightarrow{900°C\text{-}1300°C} SiO_2$$

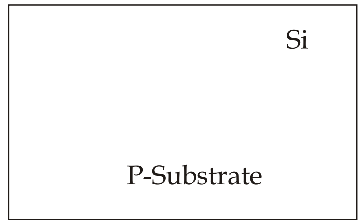
  Wet oxidation :

$$Si + 2H_2O \xrightarrow{900°C\text{-}1300°C} SiO_2 + 2H_2$$

- N-type inside P-type is achieved through diffusion or ion implantation.
- All contacts are formed by physical vapour deposition or sputtering.

The following series of steps are involved in the fabrication process of PN junction diode:

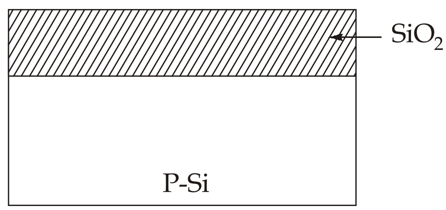
(1) Take a P-type substrate



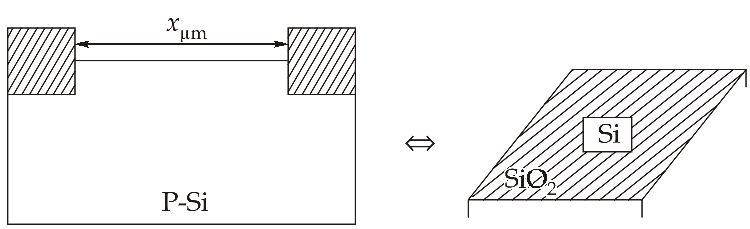
(2) Grow Fox (Field Oxide)

It is a low quality oxide used for insulation purpose.

$$Si + 2H_2O \longrightarrow SiO_2 + 2H_2$$

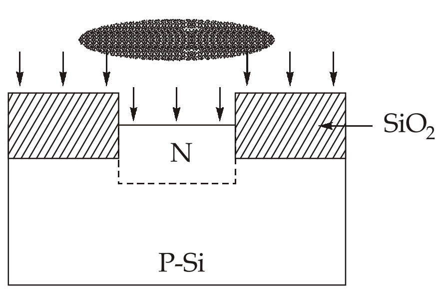


(3) Open Window using Photolithography and Etching



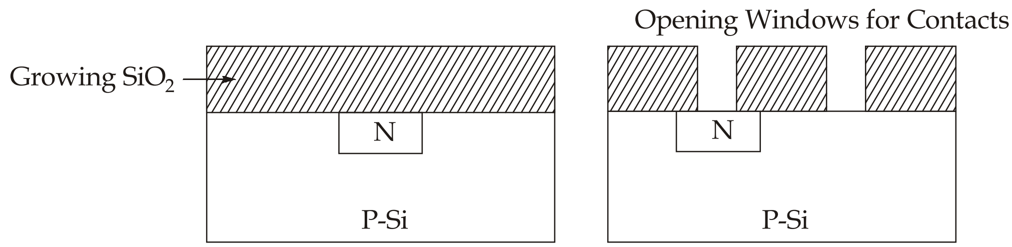
The window dimensions are decided by photolithography. A light-sensitive polymer called photoresist is spun onto the SiO_2 layer. The photomask with the desired diode pattern is placed over the wafer, which is then exposed to UV light. The exposed photoresist is dissolved leaving a pattern that exposes the underlying SiO_2.

(4) Doping the p-type substrate using diffusion or ion implantation.



SiO_2 acts as mask against dopant.

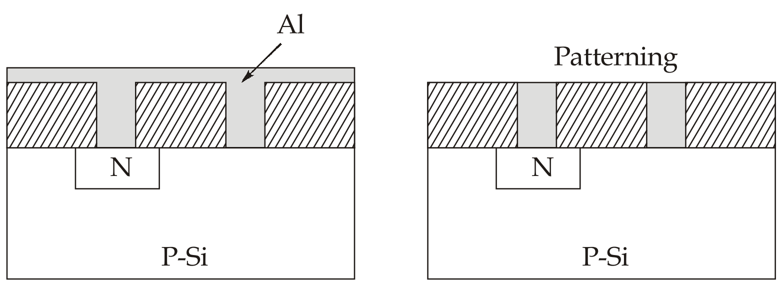
(5) Again grow SiO_2 and open windows for contacts.



Another round of photolithography and etching ensures the metal only remains at the specific contact points for the Anode (p-side) and Cathode (n-side).

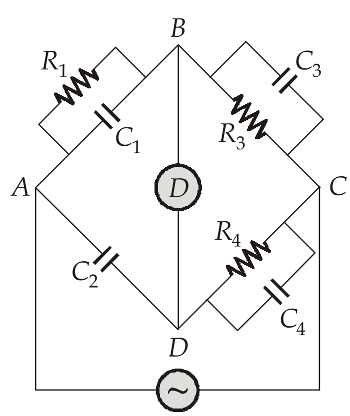
(6) Metallization and patterning.

A thin layer of metal (usually Aluminum) is deposited over the entire wafer using physical vapour deposition or sputtering, and patterned to form the ohmic contacts.



Q.4 (b) Solution:

The given Schering Bridge circuit can be drawn as below,



Given, Without the specimen between electrodes, balance is obtained with

$$C_3 = C_4 = 120 \text{ pF}, C_2 = 150 \text{ pF}, R_3 = R_4 = 5000 \ \Omega$$

With specimen inserted, these values becomes

$$C_3 = 200 \text{ pF}, C_4 = 1000 \text{ pF}, C_2 = 900 \text{ pF}, R_3 = R_4 = 5000 \ \Omega$$

$$\omega = 5000 \text{ rad/sec}$$

For balance: $$Y_1Y_4 = Y_2Y_3$$

$$\left(\frac{1}{R_1}+j\omega C_1\right)\left(\frac{1}{R_4}+j\omega C_4\right) = (j\omega C_2)\left(\frac{1}{R_3}+j\omega C_3\right)$$

$$\frac{1}{R_1R_4}+\frac{j\omega C_4}{R_1}+\frac{j\omega C_1}{R_4}-\omega^2C_1C_4 = \frac{j\omega C_2}{R_3}-\omega^2C_2C_3$$

Equating real and imaginary parts, we have,

$$\frac{1}{R_1R_4}-\omega^2C_1C_4 = -\omega^2C_2C_3 \quad ...(1)$$

and
$$\frac{\omega C_4}{R_1}+\frac{\omega C_1}{R_4} = \frac{\omega C_2}{R_3}$$

$$\frac{C_4}{R_1}+\frac{C_1}{R_4} = \frac{C_2}{R_3} \quad ...(2)$$

From equation (1)
$$\omega^2C_1C_4 = \frac{1}{R_1R_4}+\omega^2C_2C_3$$

$$C_1 = \frac{1}{\omega^2R_1R_4C_4}+\frac{\omega^2C_2C_3}{\omega^2C_4}$$

$$C_1 = \frac{1}{\omega^2R_1R_4C_4}+\frac{C_2C_3}{C_4} \quad ...(3)$$

From equation (2),
$$\frac{C_4}{R_1} = \frac{C_2}{R_3}-\frac{C_1}{R_4} = \frac{C_2R_4-C_1R_3}{R_3R_4}$$

$$R_1 = \frac{R_3R_4C_4}{(C_2R_4-C_1R_3)}$$

From equation (3),
$$C_1 = \frac{(C_2R_4-C_1R_3)}{\omega^2R_3R_4C_4R_4C_4}+\frac{C_2C_3}{C_4}$$

$$C_1 = \frac{\left(\frac{C_2R_4-C_1R_3}{R_3}\right)+C_2C_3\omega^2R_4^2C_4}{\omega^2R_4^2C_4^2}$$

$$C_1\left(\omega^2R_4^2C_4^2\right) = \frac{C_2R_4}{R_3}-C_1+C_2C_3\omega^2R_4^2C_4$$

$$(1+\omega^2R_4^2C_4^2)C_1 = \frac{C_2R_4}{R_3}+C_2C_3C_4\omega^2R_4^2$$

$$C_1 = \frac{\frac{C_2R_4}{R_3} + C_2C_3C_4\omega^2R_4^2}{\left(1+\omega^2R_4^2C_4^2\right)}$$

Now $\quad C_2C_3C_4\omega^2R_4^2 <<< \frac{C_2R_4}{R_3}$

and $\quad \omega^2R_4^2C_4^2 <<< 1$

Hence, $\quad C_1 = C_2\frac{R_4}{R_3}$

When the capacitor C_1 is without specimen dielectric, let its capacitance be C_0

$\therefore \quad C_0 = \frac{R_4}{R_3}C_2 = \frac{5000}{5000}\times 150$

$$C_0 = 150 \text{ pF}$$

When the capacitor C_1 is with specimen dielectric, let the capacitance be C_s.

$$C_s = \frac{C_2R_4}{R_3} = \frac{900\times 5000}{5000}$$

$$C_s = 900 \text{ pF}$$

Now, $\quad C_0 = \frac{\epsilon_0 A}{d}$

and $\quad C_s = \frac{\epsilon_0\epsilon_r A}{d}$

Hence, relative permittivity of specimen

$$\epsilon_r = \frac{C_s}{C_0} = \frac{900}{150} = 6$$

Q.4 (c) Solution:

(i)

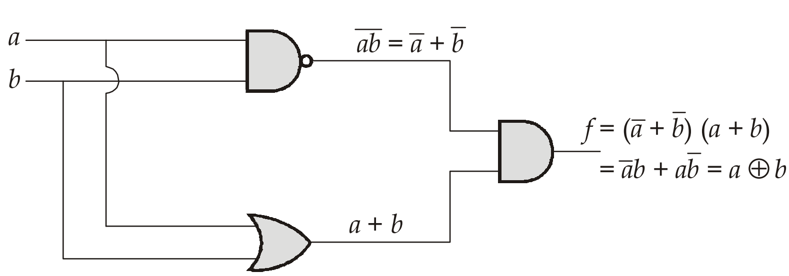


Fig. 1: Faultless circuit

For faultless circuit: $\quad f = a \oplus b$

Considering fault at P only, and obtaining the output expression (f_P).

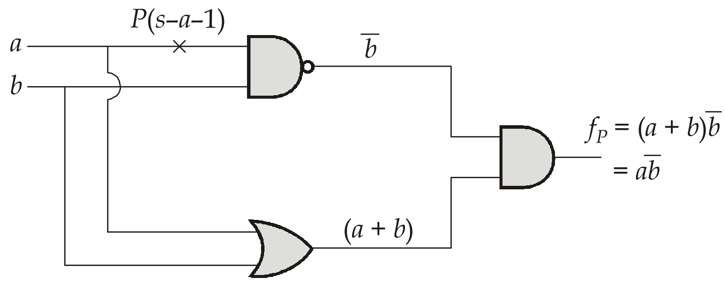


Fig. 2: Faulty circuit (considering fault at P)

Using EX-OR method approach to calculate the test vector.

a	b	Faultless circuit $f = a \oplus b$	Faulty circuit (Fault at P) $f_P = a\overline{b}$	$f \oplus f_P$
0	0	0	0	0
0	1	1	0	1
1	0	1	1	0
1	1	0	0	0

Hence 01 is the test vector for fault P.

Considering fault at Q only and obtaining the output expression (f_Q):

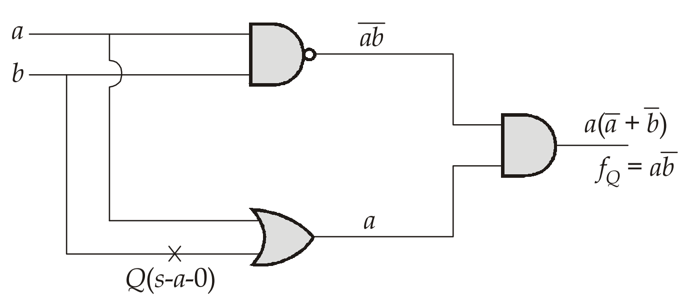


Fig. 3: Faulty circuit (considering fault at Q)

a	b	Faultless circuit $f = a \oplus b$	Faulty circuit (Fault at Q) $f_Q = a\overline{b}$	$f \oplus f_Q$
0	0	0	0	0
0	1	1	0	1
1	0	1	1	0
1	1	0	0	0

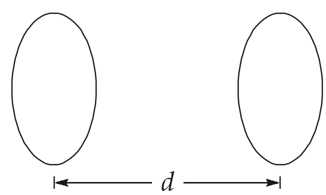
Hence, 01 is the test vector for fault Q.

(ii) If the faulty output functions are identical, no input test pattern can distinguish between the two faults. Thus, two faults are considered indistinguishable (or equivalent) if they produce the same faulty output function $f_P(a, b) = f_Q(a, b)$ for all possible input combinations (a, b). Since the output expression is same for fault at P and fault at Q, it means that the fault behaviour is same. Hence, fault P and Q are indistinguishable.

i.e, $$f_P(a, b) = f_Q(a, b)$$

Section B : Electromagnetics-1 + Basic Electrical Engineering-1
Computer Organization and Architecture-2 + Materials Science-2

Q.5 (a) Solution:



$$\text{Radius of the disc, } r = 0.1 \text{ m}$$

$$d = 0.05 \text{ m}$$

$$V = 50 \cos 10^4 t$$

⇒ Electric field between the plates,

$$E = \frac{V}{d} = \frac{50 \cos 10^4 t}{0.05} = 1000 \cos 10^4 t \text{ V/m}$$

According to the Maxwell's equation, the displacement current density is given by

$$J_d(t) = \frac{\partial D(t)}{\partial t} = \varepsilon_o \frac{\partial E(t)}{dt}$$

$$= -\varepsilon_o \times 1000 \times 10^4 \sin 10^4 t$$

$$= -8.85 \times 10^{-12} \times 10^7 \sin 10^4 t$$

$$= -88.5 \sin 10^4 t \ \mu\text{A}$$

$$\text{Displacement current, } I_d = J_d \times A$$

$$= -88.5 \sin 10^4 t \ \mu\text{A} \times \pi r^2$$

$$= -88.5 \sin 10^4 t \ \mu\text{A} \times \pi \times (0.1)^2$$

$$I_d = -2.78 \sin 10^4 t \ \mu\text{A}$$

RMS value of the displacement current,

$$(I_d)_{\text{rms}} = \frac{(I_d)_{\max}}{\sqrt{2}} = 1.96 \ \mu\text{A} \quad \text{...(1)}$$

$$\text{Capacitance, } C = \frac{\varepsilon_0 A}{d} = \frac{1}{36\pi}\times 10^{-9}\times\frac{\pi\times(0.1)^2}{0.05}$$

$$= \frac{10^{-11}}{1.8}\text{F}$$

In circuit theory, the current through a capacitor is given by

$$I_C = C\frac{dV}{dt} = -\frac{10^{-11}}{1.8}\times 50\times 10^4 \sin 10^4 t$$

$$= -2.78 \sin 10^4 t \ \mu\text{A}$$

$$(I_c)_{rms} = 1.96\ \mu\text{A} \quad ...(2)$$

From (1) and (2), $(I_c)_{rms} = (I_d)_{rms}$

Thus, the RMS value of the displacement current calculated via Maxwell's equations is identical to the RMS value of the conduction current calculated using the standard capacitor voltage equation.

Q.5 (b) Solution:

The total core loss is the sum of hysteresis loss (P_h) and eddy current loss (P_e):

Given,

At 400 V, 50 Hz,

Total core loss $\Rightarrow P_e + P_h = 2800$ W ...(1)

At 200 V, 25 Hz,

Total core loss $\Rightarrow P_e' + P_h' = 1000$ W ...(2)

We know,

Eddy current loss, $P_e \propto f^2$ $\left(\because \left(\frac{V}{f}\right) \text{ is constant}\right)$

$$\frac{P_e}{P_e'} = \left(\frac{f_1}{f_2}\right)^2$$

$$P_e' = \left(\frac{f_2}{f_1}\right)^2 P_e = \left(\frac{25}{50}\right)^2 P_e$$

$$P_e' = \frac{P_e}{4}$$

Also hysteresis loss $P_h \propto (f)$ $\left[\because \left(\frac{V}{f}\right) \text{ is constant}\right]$

$$\frac{P_h'}{P_h} = \frac{f_2}{f_1}$$

$$P_h' = P_h \times \left(\frac{25}{50}\right)$$

$$P_h' = \frac{P_h}{2}$$

From eqn (2),

$$P_e' + P_h' = 1000$$

$$\frac{P_e}{4} + \frac{P_h}{2} = 1000$$

$$P_e + 2P_h = 4000 \quad ...(3)$$

Solving equation (1) and (3), we get

$$P_e = 1600 \text{ W}, \quad P_h = 1200 \text{ W}$$

Now at 800 V, 25 Hz:

$\left(\frac{V}{f}\right) \neq$ constant compared to case (1)

Eddy current loss $P_e \propto B_m^2 f^2$. Since $B_m \propto V/f$.

$$\therefore \quad P_e \propto V^2 \quad ...(4)$$

Hysteresis loss $P_h \propto B^n f B_m^2 f$ ($\because$ Steinmetz exponent, $n = 2$)

$$P_h \propto \left(\frac{V}{f}\right)^2 \times f$$

$$P_h \propto \frac{V^2}{f} \quad ...(5)$$

From equation (4),

$$\frac{P_e}{P_e''} = \frac{V_1^2}{V_3^2}$$

$$P_e'' = P_e\left(\frac{V_3}{V_1}\right)^2 = 1600\left(\frac{800}{400}\right)^2$$

$P_e'' = 6400$ W $\rightarrow$ Eddy current losses at 800 V, 25 Hz.

Now, from equation (5),

$$\frac{P_h}{P_h''} = \left(\frac{V_1}{V_3}\right)^2 \times \left(\frac{f_3}{f_1}\right)$$

$$\frac{1200}{P_h''} = \left(\frac{400}{800}\right)^2 \times \left(\frac{25}{50}\right)$$

$$P_h'' = 9600 \text{ W} \quad \rightarrow \quad \text{Hysteresis losses at 800 V, 25 Hz.}$$

Q.5 (c) Solution:

(i) In pipelining, we take cycle time as maximum stage delay plus the register delay if there is any. Here, there is no register delay.

So, t_p = max(8 μsec, 24 μsec) = 24 μs

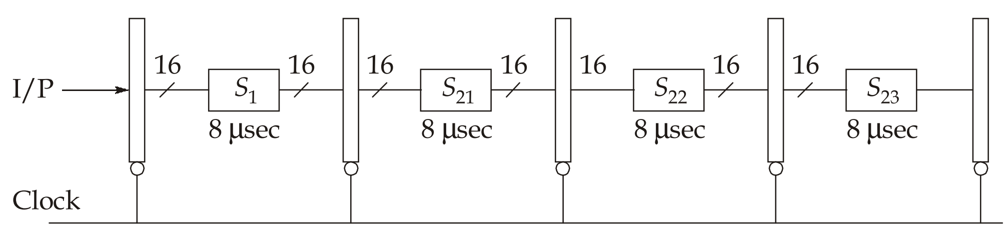
Now, Latency = No. of stages in pipeline × Cycle time of pipeline

= 2 × 24 μsec = 48 μsec

Thus, throughput $= \dfrac{1}{\text{Cycle time of pipeline}}$

$= \dfrac{1}{24\,\mu\,\text{sec}}$ = 0.0416 instructions/μsec

(ii) When stage 2 is split up in three equal substages each with stage delay of 8 μsec, let these substages are S_{21}, S_{22}, S_{23} respectively.



Now, t_p = 8 μsec; where t_p = cycle time of pipeline

Thus, Latency = No. of stages in pipeline × t_p = 4 × 8 μsec = 32 μsec

throughput $= \dfrac{1}{t_p} = \dfrac{1}{8\,\mu\,\text{sec}}$ = 0.125 instructions/μsec

Q.5 (d) Solution:

(i) Hysteresis loss per cycle = Area of B-H curve

= Area of parallelogram

= Base × Height

$= 400 \times 10^3 \text{ A/m} \times 2 \text{ Wb/m}^2 = 8 \times 10^5 \text{ J/m}^3$

(ii) $(BH)_{max}$ is derived from the demagnetizing portion (or) second quadrant of the *B-H* curve in the given figure.

The equation of demagnetizing curve is

$$B = \frac{1}{200}H + 1$$

$$BH = \frac{1}{200}H^2 + H$$

For $(BH)_{max}$, differentiating w.r.t. H,

$$\frac{d}{dH}(BH) = \frac{2}{200}H + 1 = 0$$

$$H_{max} = -100 \text{ kA/m}$$

$$B_{max} = -\frac{100}{200} + 1 = 0.5 \text{ Wb/m}^2$$

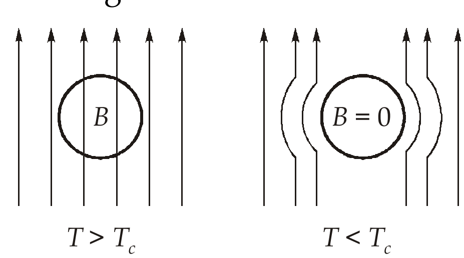
$\therefore$ $(BH)_{max} = 100 \times 10^3 \times 0.5 = 50 \text{ kJ/m}^3$

Q.5 (e) Solution:

(i) Meissner Effect:

Superconductors not only exhibits zero resistance but also spontaneously expel all magnetic flux when cooled below its critical temperature (T_c), that is they are also perfect diamagnets. This is called as Meissner effect.

Meissner effect is not a consequence of zero resistance and Lenz's law. The flux is expelled as the superconductor is cooled in constant magnetic field. There is no time rate of change of the magnetic induction. Lenz's law does not apply. Perfect diamagnetism is an independent property of superconductors and shows that superconductivity involves a change of thermodynamic state, not just a spectacular change in electrical resistance. Figure below shows the illustration of Meissner effect:



It has been observed that when a long superconductor is cooled in a longitudinal magnetic field from above the transition temperature, the lines of induction are pushed out. Then inside the specimen, $B = 0$. We know from magnetic properties of materials that $B = \mu_0(H + M)$, for $B = 0$, $H = -M$, consequently since $\chi_m = \frac{M}{H}$, we may state that magnetic susceptibility of superconductor is negative, this is referred to as perfect diamagnetism. This phenomenon is called Meissner effect. From the Maxwell's equation,

$$\nabla \times \vec{E} = \frac{-\partial B}{\partial t}$$

and from ohm's law, $J = \sigma E$ (or) $E = \rho J$

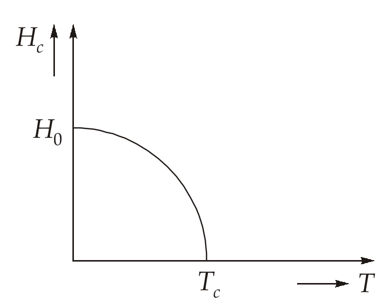
with $\rho = 0$, $E = 0$, so $\frac{\partial B}{\partial t} = 0$, but this is not so because the flux exclusion from normal to superconducting state takes place. Thus, a perfect diamagnetism and zero resistivity are two independent essential properties of superconducting state.

(ii) Silsbee's Rule:

The Silsbee rule states that a superconductor transitions to a normal, resistive state when the self-induced magnetic field produced by its own current exceeds the material's critical magnetic field (H_c). The critical value of magnetic field for destruction of superconductivity, H_c is a function of temperature. At $T = T_c$, $H_c = 0$. With only small deviations, the critical field H_c varies with temperature according to parabolic law as,

$$H_c = H_0\left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

H_0 is the critical field at absolute zero and T_c is the transition temperature. For any particular superconductor, the shape of variation of H_c with temperature is shown in figure below.



The magnetic field which causes a superconductor to transition to normal state from superconducting state is not necessarily an external applied field, it may arise as a result of electric current flow in the conductor.

In a long superconductor wire of radius r, the superconductivity may be destroyed when a current I exceeds the critical current value I_C, which at the surface of wire will produce a critical field H_C, given by $I_C = 2\pi r H_c$.

(iii) Frequency effect:

Superconductivity is observed for d.c. and upto radio frequencies. It is not observed for higher frequencies. For a superconductor, the resistance is zero only when the current is steady or varies slowly. When the current fluctuates or alternates, small absorption of energy roughly proportional to rate of alternation occurs. When the frequency of alternation rises above 10 MHz, appreciable resistance arises, and at infrared frequencies (10^{13} Hz) the resistivity is same in the normal and superconducting states, and is independent of temperature.

Q.6 (a) Solution:

(i) Given, Main memory size = 4 MB; Block size = 64 B, Tag bits = 10

4-Way Set Associative Cache is used

$$\text{Physical Address space} = 4 \text{ MB}$$
$$= 2^2 \times 2^{20} = 2^{22} \text{ Bytes}$$

Since the memory is byte-addressable, physical address is of 22 bits.

$$\text{Block size} = 64 \text{ B} = 2^6 \text{ Bytes}$$

Thus: $\text{Block offset} = 6 \text{ bit}$

Physical address split

22 bits

Tag	Set Index	Block offset
10 bits	6 bits	6 bits

(ii) Number of main memory blocks $= \dfrac{\text{Physical address size}}{\text{Block size}}$

$$= \frac{2^{22}}{2^6} = 2^{16} = 65{,}536$$

(iii) The cache is 4-way set associative, and

$$\text{No. of set index bits} = 22 - 10 - 6 = 6$$
$$\text{No. of sets inside cache} = 2^6$$

∴ No. of lines inside cache = Number of sets × Associativity

$= 2^6 \times 4 = 2^8$

Thus, Cache size = (No. of lines) × (Block size)

$= 2^8 \times 2^6\ B = 2^{14}\ B$

Cache size = 16 kB

(iv) The tag directory stores the tag for every line in the cache. Thus,

Tag Directory size = (No. of lines in cache) × Tag bits

$= 10 \times 2^8$ bits

= 2560 bits = 320 bytes

(v) In *k*-way set associative mapping, main memory is divided into blocks, and the cache is divided into *v* sets, with each set containing *k* lines. A main memory block *j* is mapped to a specific set calculated by the formula *j* (mod *v*), but it can be placed in any of the *k* lines available within that set. Thus, in a *k*-way set-associative cache, when a memory address is accessed, the tags of all k lines in a specific set must be compared simultaneously to check for a cache hit. So, we need *k* comparators to know the line number and *k* : 1 MUX to select one of the *k* cache lines. Hence,

No. of comparators needed = 4

Q.6 (b) Solution:

(i)

Carbon Dots	Quantum Dots
1. Carbon dots are small carbon nanoparticles having some form of surface passivation.	1. Quantum dots are small semiconductor particles that exhibit 3 dimensional quantum confinement, having optical and electronic properties that differ from large particles according to quantum mechanics.
2. Top-down and bottom-up methods are used for production of carbon dots.	2. Colloidal synthesis, plasma synthesis, fabrication, electrochemical assembly can be used for production of Quantum dots.
3. The properties of carbon dots solely depends on their structures and compositions.	3. The properties of quantum dots are intermediate to those of bulk semiconductors and discrete atoms.
4. Carbon dots are used in bioimaging, sensing, drug delivery, catalysis, optronics etc.	4. Quantum dots are used in LEDs, single photon sources, quantum computing etc.

(ii) Top-Down Technique: In top-down technique, generally a bulk material is taken and machined to modify into the desired shape and product. Examples of this type of technique are the manufacturing of integrated circuits using a sequence of steps such as crystal growth, lithography, etching, ion implantation, etc. For nanomaterial synthesis, ball-milling is an important top-down approach, where macrocrystalline structures are broken down to nanocrystalline structures, but original integrity of the material is retained. Sometimes this method is used to prepare nanostructured metal oxides by chemical reaction between two constituents during crushing. The crystallites are allowed to react with each other by the supply of kinetic energy during milling process to form the required nanostructured oxide.

Bottom-Up Technique: Bottom-up technique is used to build something from basic materials, for example, assembling materials from the atoms/molecules up, and, therefore very important for nano-fabrication. Unlike lithographic technique of top-down approach, which is extensively used in silicon industry, this bottom-up nonlithographic approach of nanomaterial synthesis is not completely proven in manufacturing yet, but has great potential to become important alternative to lithographic process. Examples of bottom-up technique are self-assembly of nanomaterials, solgel technology, electrodeposition, physical and chemical vapour deposition (PVD, CVD), epitaxial growth, laser ablation, etc.

Q.6 (c) Solution:

Given: $\mu_r = 4$, $\varepsilon_r = 9$ and $f = 10$ MHz $= 10^7$ Hz

$\Rightarrow$ $$\omega = 2\pi f = 2\pi \times 10^7 \text{ rad/sec}$$

(i) For a lossless medium, $$\beta = \omega\sqrt{\mu\epsilon} = 2\pi\times10^7\times\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r} = 2\pi\times10^7\times\frac{6}{3\times10^8}$$

$\Rightarrow$ $$\beta = \frac{2\pi}{5} \text{ rad/m}$$

$\Rightarrow$ $$\lambda = \frac{2\pi}{\beta} = 5 \text{ m}$$

We have, $$v_p = \text{Phase velocity} = f\lambda$$

$\Rightarrow$ $$v_p = 5\times10^7 \text{ m/sec}$$

$$\eta = \text{Intrinsic impedance} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}}$$

$$= \frac{2}{3}\times377 = 251.33\ \Omega$$

(ii) At t = 60 ns and $P(0.6, 0.6, 0.6)$, it is given that

$$E_{x0} = 400 \text{ V/m}$$
$$E_{y0} = E_{z0} = 0$$

The wave is propagating in $\hat{a}_y$ direction. Thus,

$$E(t) = E_0 e^{j(\omega t - \beta y)} \hat{a}_x$$

At t = 60 ns and $P(0.6, 0.6, 0.6)$,

we have

$$400 = E_0 e^{j\left(2\pi\times10^7\times60\times10^{-9} - \frac{2\pi}{5}\times0.6\right)} \hat{a}_x$$

$$E_0 = 400e^{-j3.0159} \text{ V/m}$$

Hence, $$E(t) = 400e^{j(\omega t-\beta y)}e^{-j3.0159}\hat{a}_x \text{ V/m}$$

(iii) $$H(t) = \frac{-400e^{-j3.0159}}{\eta} e^{j(\omega t-\beta y)}\hat{a}_z \text{ A/m} \quad \left[\because \hat{a}_E\times\hat{a}_H = \hat{a}_k = \hat{a}_y\right]$$

$$H(t) = \frac{-400e^{-j3.0159}}{251.33} e^{j(\omega t-\beta y)}\hat{a}_z$$

$$H(t) = -1.59e^{-j3.0159}e^{j(\omega t-\beta y)}\hat{a}_z \text{ A/m}$$

where ω, β and η have been calculated in part (i).

Q.7 (a) Solution:

(i) Given, $$\frac{\tau_{st}}{\tau_{max}} = \frac{\tau_{fl}}{2\tau_{fl}} = \frac{1}{2}$$

Also, $$\frac{\tau_{st}}{\tau_{max}} = \frac{2s_M}{1+s_M^2} = \frac{1}{2}$$ [At starting, slip s = 1]

∴ $$s_M^2 - 4s_M + 1 = 0$$

$$s_M = \frac{4\pm\sqrt{16-4}}{2} = 3.732, 0.268$$

Since the slip must be between 0 and 1, thus

$$s_M = 0.268$$

(ii) Let the slip at full load be 's'. We have,

$$\frac{\tau_{fl}}{\tau_{max}} = \frac{2s\,s_M}{s^2 + s_M^2}$$

$$\frac{1}{2} = \frac{2s\,s_M}{s^2 + s_M^2}$$

$$s^2 - 4s\,s_M + s_M^2 = 0$$

$$s = \frac{4s_M \pm \sqrt{16s_M^2 - 4s_M^2}}{2} = 0.268s_M, 3{,}732s_M$$

Since the slip must be between 0 and 1, thus

$$s = 0.268s_M = 0.268 \times 0.268 = 0.0718$$

(iii) The rotor current I_2 at any slip s is given by

$$I_{2fl} = \frac{E_{2s}}{Z_{2s}} = \frac{sE_{20}}{\sqrt{R_2^2 + (sX_{20})^2}}$$

where E_{20} is the rotor induced EM per phase at standstill, R_2 is the rotor resistance per phase and X_2 is the rotor reactance per phase at standstill.

At starting, $s = 1$

$$I_{2st} = \frac{E_{20}}{\sqrt{R_2^2 + X_{20}^2}}$$

$$\therefore \quad \frac{I_{2st}}{I_{2fl}} = \frac{E_{20}}{\sqrt{R_2^2 + X_{20}^2}} \times \frac{\sqrt{R_2^2 + (sX_{20})^2}}{sE_{20}}$$

$$\left(\frac{I_{2st}}{I_{2fl}}\right)^2 = \frac{R_2^2 + (sX_{20})^2}{s^2(R_2^2 + X_{20}^2)} = \frac{X_{20}^2\left(\dfrac{R_2^2}{X_{20}^2} + s^2\right)}{X_{20}^2\left(\dfrac{R_2^2}{X_{20}^2} + 1\right)s^2}$$

Since, $s_M = \dfrac{R_2}{X_{20}}$ is the slip at which maximum torque occurs.

$$\left(\frac{I_{2st}}{I_{2fl}}\right)^2 = \frac{s_M^2 + s^2}{s^2(1 + s_M^2)} = \frac{(0.268)^2 + (0.0718)^2}{(0.0718)^2(1 + 0.268^2)} = 13.9316$$

$$\frac{I_{2st}}{I_{2fl}} = \sqrt{13.9316} = 3.7325$$

The starting rotor current is 3.727 p.u. of the full-load rotor current.

Q.7 (b) Solution:

(i) For an EM wave,

Attenuation constant, $$\alpha = \omega\sqrt{\frac{\mu\in}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}-1\right]}$$

Given : $\omega = 10^{10}\pi$ rad/sec, $\mu_r = 1, \in_r = 80, \sigma = 4$ S/m

$$\alpha = \frac{10^{10}\pi}{3\times10^8}\sqrt{\frac{80}{2}\left(\sqrt{1+\left(\frac{4}{10^{10}\pi\times80\times8.854\times10^{-12}}\right)^2}-1\right)}$$

$$= \frac{100\pi}{3}\sqrt{\frac{80}{2}\times0.016}$$

$\therefore$ $$\alpha = 83.8 \text{ Np/m}$$

- Intrinsic impedance,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\in}} = \sqrt{\frac{j10^{10}\pi\times4\pi\times10^{-7}}{4+j10^{10}\pi\times80\times8.85\times10^{-12}}}$$

$$\eta = 41.8\angle5.1°$$

We have $$\frac{\alpha}{\beta} = \tan\theta_\eta = \tan 5.1° \Rightarrow \beta = 938.96 \text{ rad/m}$$

- Wavelength, $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{938.96} = 6.7\times10^{-3} \text{ m}$$

- Skin depth, $$\delta = \frac{1}{\alpha} = 11.9\times10^{-3} \text{ m}$$

(ii) The amplitude of $\vec{H}$ decays with distance as

$$H(y) = H_o e^{-\alpha y}$$

$$0.01 = 0.1e^{-83.8y}$$

$\Rightarrow$ $$83.8y = -\ln 0.1$$

$\Rightarrow$ $$y = 27.4 \text{ mm}$$

(iii) We can write, $$\vec{H}(y,t) = 0.1e^{-\alpha y}\sin\left(10^{10}\pi t-\frac{\pi}{3}-\beta y\right)\hat{a}_x \text{ A/m}$$

At $y = 0.5$ m $$\vec{H}(0.5m,t) = 0.1e^{-41.9y}\sin\left(10^{10}\pi t-\frac{\pi}{3}-938.96y\right)\hat{a}_x \text{ A/m}$$

Similarly,
$$E(y, t) = E_o e^{-\alpha y} \sin\left(10^{10}\pi t - \frac{\pi}{3} - 938.96y\right)\hat{a}_z \text{ V/m}$$

$$\left[\because \hat{a}_E \times \hat{a}_H = \hat{a}_k = \hat{a}_y\right]$$

where,
$$E_o = H_o|\eta| = 4.18$$

$$E(y, t) = 4.18e^{-41.9y} \sin\left(10^{10}\pi t - \frac{\pi}{3} - 938.96y\right)\hat{a}_z \text{ V/m}$$

Q.7 (c) Solution:

(i) The magnetic field produced by the solenoid

$$H = \frac{NI}{l} = \frac{1000\times 2.5}{0.25} = 10000 \text{ A/m}$$

Increase in magnetic induction when placed in oxygen

$$= 1.04 \times 10^{-8} \text{ Wb/m}^2 = \mu_o M$$

This increase is due to magnetization (M)

$$M = \frac{1.04\times 10^{-8}}{4\pi\times 10^{-7}} = 8.276\times 10^{-3} \text{ A/m}$$

The magnetic susceptibility of oxygen is given as

$$\chi_m = \frac{M}{H} = \frac{8.276\times 10^{-3}}{10000} = 8.276\times 10^{-7}$$

(ii) A nanoparticle of Si can be made by laser evaporation of a Si substrate in the region of a helium gas pulse. The beam of neutral clusters is photolyzed by a UV laser producing ionized clusters whose mass to charge ratio is then measured in a mass spectrometer. The most striking property of nanoparticles made of semiconducting elements is the pronounced changes in their optical properties compared to those of bulk material. There is a significant shift in the optical absorption spectra toward the blue (shorter wavelength) as the particle size is reduced.

In a bulk semiconductor a bound electron hole pair, called an exciton can be produced by a photon having an energy greater than that of the bandgap of material. The photon excites an electron from the filled band to the unfilled band above. The separation between the hole and the electron is many lattice parameters.

The existence of the exciton has strong influence on the electronic properties of the semiconductor and its optical absorption. The exciton can be modeled as a hydrogen like atom and has energy levels with relative spacings analogous to the energy levels of the hydrogen atom but lower actual energies. Light induced transitions between these hydrogen like energy levels produce a series of optical absorptions.

When the size of the nanoparticle becomes smaller than or comparable to the radius of the orbit of the electron-hole pair, there are two situations called weak-confinement and the strong confinement regimes. In the weak regime, the particle radius is larger than the radius of the electron-hole pair, but the range of motion of the exciton is limited, which causes the blue shift of the absorption spectrum. When the radius of the particle is smaller than the orbital radius of the electron-hole pair, the motion of the electron and the hole become independent and the exciton does not exist. The hole and electron have their own set of energy levels. Here there is also a blue shift and the emergence of a new set of absorption lines.

Q.8 (a) Solution:

(i) We can write,

$$\vec{E_s} = \frac{100}{r}\sin\theta\, e^{-j\beta r}\hat{a}_\theta \text{ V/m}$$

$$\vec{H_s} = \frac{0.265}{r}\sin\theta e^{-j\beta r}\hat{a}_\phi \text{ A/m}$$

$$\vec{H_s^*} = \frac{0.265}{r}\sin\theta\, e^{j\beta r}\hat{a}_\phi \text{ A/m}$$

The average power crossing per unit area is given by

$$P_{\text{avg}} = \frac{1}{2}\text{Re}\left\{\vec{E_s}\times\vec{H_s^*}\right\}$$

$$= \frac{1}{2}\text{Re}\left\{\frac{100}{r}\sin\theta e^{-j\beta r}\hat{a}_\theta \times \frac{0.265}{r}\sin\theta e^{-j\beta r}\hat{a}_\phi\right\}$$

$$\vec{P_{avg}} = \frac{1}{2}\frac{100(0.265)}{r^2}\sin^2\theta\,\hat{a}_r\left(\frac{\text{Watt}}{\text{m}^2}\right)$$

The average power crossing the hemispherical shell is

$$\vec{P} = \iint \vec{P_{avg}}\cdot\vec{ds}$$

$$= \iint \frac{1}{2}\times\frac{26.5}{r^2}\sin^2\theta\hat{a}_r\cdot r^2\sin\theta\, d\theta\, d\phi\hat{a}_r$$

$$\vec{P} = \frac{26.5}{2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\sin^3\theta\cdot d\theta\, d\phi$$

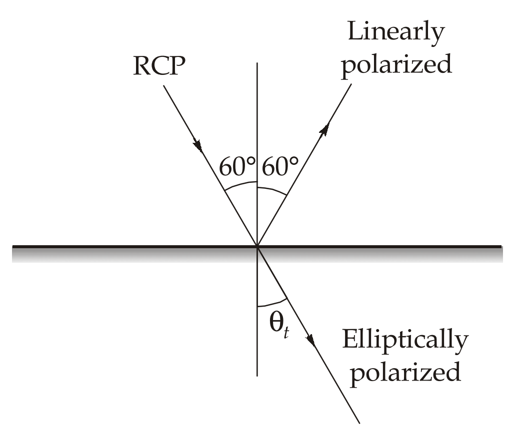
$$= 13.25\int_{\phi=0}^{2\pi} d\phi * \frac{2}{3}$$

$$\left[\because \int_0^{\pi/2} \sin^n x\,dx = \begin{cases} \frac{(n-1)}{n}\times\frac{(n-3)}{(n-2)}\times\ldots\frac{1}{2}\times\frac{\pi}{2}; \text{ if } n = 2,4,6,\ldots \\ \frac{(n-1)}{n}\times\frac{(n-3)}{(n-2)}\times\ldots\frac{2}{3}; \quad \text{if } n = 3,5,7,\ldots \end{cases}\right]$$

$$\vec{P} = 13.25\times 2\pi\times\frac{2}{3} = 55.501 \text{ Watt}$$

(ii) Both circularly and elliptically polarized waves are composed of two orthogonal components (s and p) with a specific phase difference. When light is incident at the Brewster angle (θ_B), the reflection coefficient for the p-polarized component is zero. Thus, 100% of the p-polarized light is transmitted into the second medium. However, a significant portion of the s-polarized light is also transmitted (only a fraction of s-light is reflected). If incident angle is Brewster angle and incident wave is circularly polarised or Elliptically polarised, then reflected wave is linearly polarized and transmitted wave is elliptically polarized.

From question



Given incident wave is circularly polarised and reflected wave is linearly polarized.
So, $\theta_i = \theta_B$ (i.e., Brewster angle)

Hence, $\theta_B = 60°$

$$\tan\theta_B = \sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}$$

$$\tan^2 60° = \frac{\epsilon_{r_2}}{\epsilon_{r_1}}$$

$$(\sqrt{3})^2 = \frac{\epsilon_0\epsilon_{r_2}}{\epsilon_0 * 1}$$

$$\epsilon_{r_2} = 3$$

Q.8 (b) Solution:

We have,
$$\eta = \frac{x \cdot \text{kVA rating} \cos\theta}{x \cdot \text{kVA rating} \cos\theta + P_i + x^2 P_{cufl}}$$

Given: $\eta = \frac{98}{100}$ for $x = 1$ and $x = \frac{1}{2}$ and unity power factor

For $x = 1$ and $\cos\theta = 1$

$$\frac{98}{100} = \frac{1 \times 200 \times 10^3}{200 \times 10^3 + P_i + P_{cufl}}$$

$$P_i + P_{cufl} = 4081.63 \text{ Watt} \qquad \text{...(i)}$$

For $x = \frac{1}{2}$ and $\cos\theta = 1$

$$\frac{98}{100} = \frac{\frac{1}{2} \times 200 \times 1000}{\frac{1}{2} \times 200 \times 1000 + P_i + \frac{P_{cufl}}{4}}$$

$$P_i + \frac{P_{cufl}}{4} = 2040.82 \text{ Watt} \qquad \text{...(ii)}$$

On solving (i) and (ii), we get

$$P_i = 1360.55 \text{ W}$$

$$P_{cufl} = 2721.08 \text{ Watt}$$

We can write, Iron loss, $P_i = V_0 I_0 \cos\theta_0 = 1360.55$ W

$$384 \times I_\mu = 1360.55 \qquad [\because I_\mu = I_0 \cos\phi]$$

$$I_\mu = 3.54 \text{ A}$$

Full load Copper loss, $P_{cufl} = 2721.08 = I^2_{rated} R_{02}$

$$\left[\frac{200 \times 10^3}{384}\right]^2 R_{02} = 2721.08$$

$$R_{02} = 0.01\ \Omega$$

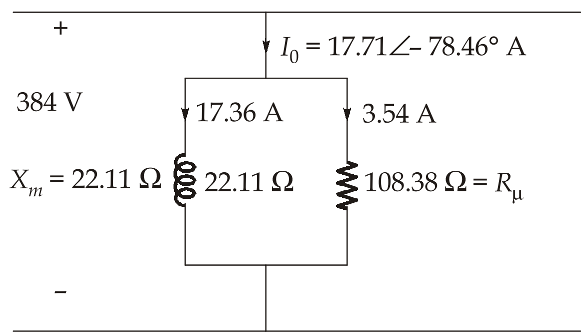
$$R_\mu = \frac{V_2}{I_\mu} = \frac{384}{3.54} = 108.38\ \Omega$$

No load power factor, $\cos\theta = 0.2 \Rightarrow \theta = 78.46°$

$$I_m = I_0 \sin\theta_0$$

$$= \frac{1360.55}{384 \times 0.2} \times 0.98$$

$$I_m = 17.36 \text{ A}$$

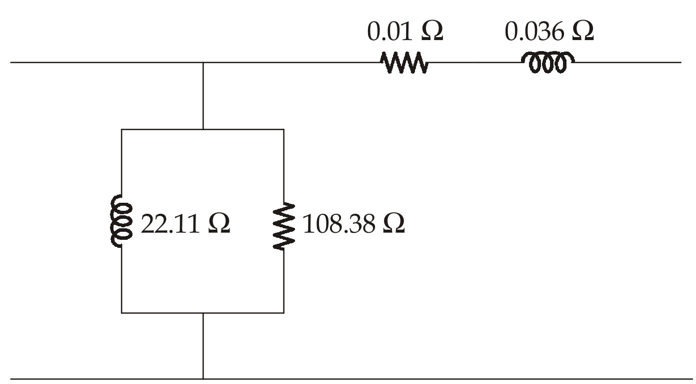


$$X_m = \frac{V_2}{I_m} = \frac{384}{17.36} = 22.11\ \Omega$$

For lagging load,
$$V.R. = \frac{I_2(R_{02}\cos\phi + X_{02}\sin\phi)}{V_2}$$

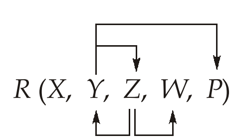
$$0.04 = \frac{200 \times 10^3 (0.01 \times 0.8 + X_{02} \times 0.6)}{384 \times 384}$$

$$X_{02} = 0.036\ \Omega$$



Q.8 (c) Solution:

(i) Since, X does not appear in any FD, it is an independent attribute and must be part of any candidate key. Thus,



$$F = \{Y \rightarrow Z, Z \rightarrow Y, Z \rightarrow W, Y \rightarrow P\}$$

X is the prime attribute.

$$(XY)^+ = (XYZWP) \Rightarrow R$$

Hence, XY is candidate key of relation R.

$$(XZ)^+ = (XYZWP) \Rightarrow R$$

Hence, XZ is candidate key of relation R.

$$(XW)^+ = XW \neq R$$

Hence, XW is not candidate key of relation R.

$$(XP)^+ = XP \neq R$$

Hence, XP is not candidate key of relation R.

Thus, the candidate keys for the relation R are: $\{XY, XZ\}$

(ii) Closure of FDs.

1. $$AB \rightarrow C$$
$$(AB)^+ = ABCDEF$$

2. $$C \rightarrow A$$
$$(C)^+ = AC$$

3. $$BC \rightarrow D$$
$$(BC)^+ = ABCDEF$$

4. $$ACD \rightarrow B$$
$$(ACD)^+ = ABCDEF$$

5. $$BE \rightarrow C$$
$$(BE)^+ = BCEDFA = ABCDEF$$

6. $$CE \rightarrow FA$$
$$(CE)^+ = ACEFBD = ABCDEF$$

7. $$CF \rightarrow BD$$
$$(CF)^+ = BCDFEA \Rightarrow ABCDEF$$

8. $$D \rightarrow EF$$
$$(D)^+ = DEF$$

❍❍❍❍