



MADE EASY
Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026
Mains Test Series

Electrical Engineering
Test No : 7

Section A : Computer Fundamentals + Electrical & Electronic Measurements

Q.1 (a) Solution:

Main memory is divided in page size = 16 bytes

We know that, page size = frame size

So in virtual address least 4 bit represent page size and remaining bits represents page number.

So, page number request are:

$$0 = \underbrace{00}_{\text{page}} \underbrace{0000}_{\text{frame}} \text{ i.e. Page number} = 0$$

$$4 = \underbrace{00}_{\text{page}} \underbrace{0100}_{\text{frame}} \text{ i.e. Page number} = 0$$

$$8 = \underbrace{00}_{\text{page}} \underbrace{1000}_{\text{frame}} \text{ i.e. Page number} = 0$$

$$20 = \underbrace{01}_{\text{page}} \underbrace{0100}_{\text{frame}} \text{ i.e. Page number} = 1$$

$$24 = \underbrace{01}_{\text{page}} \underbrace{1000}_{\text{frame}} \text{ i.e. Page number} = 1$$

$$36 = \underbrace{10}_{\text{page}} \underbrace{0100}_{\text{frame}} \text{ i.e. Page number} = 2$$

$$44 = \underbrace{10}_{\text{page}} \underbrace{1100}_{\text{frame}} \text{ i.e. Page number} = 2$$

$$12 = \underbrace{00}_{\text{page}} \underbrace{1100}_{\text{frame}} \text{ i.e. Page number} = 0$$

$$68 = \underbrace{100}_{\text{page}} \underbrace{0100}_{\text{frame}} \text{ i.e. Page number} = 4$$

$$72 = \underbrace{100}_{\text{page}} \underbrace{1000}_{\text{frame}} \text{ i.e. Page number} = 4$$

$$80 = \underbrace{101}_{\text{page}} \underbrace{0000}_{\text{frame}} \text{ i.e. Page number} = 5$$

$$84 = \underbrace{101}_{\text{page}} \underbrace{0100}_{\text{frame}} \text{ i.e. Page number} = 5$$

$$28 = \underbrace{1}_{\text{page}} \underbrace{1100}_{\text{frame}} \text{ i.e. Page number} = 1$$

$$32 = \underbrace{010}_{0000} \text{ i.e. Page number} = 2$$

$$88 = \underbrace{101}_{1000} \text{ i.e. Page number} = 5$$

$$92 = \underbrace{101}_{1100} \text{ i.e. Page number} = 5$$

Request are:

0	0	0	1	1	4	2	0	4	4	5	5	1	2	5	5
						2	2	2	2	2	2	1	1	1	1
					4	4	4	4	4	4	4	4	4	4	4
			1	1	1	1	1	1	1	5	5	5	5	5	5
0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2
F	H	H	F	H	F	F	H	H	H	F	H	F	F	H	H

So, at the end pages 1, 2, 4, 5 are in memory and 7 page fault are present.

Q.1 (b) Solution:

For derivation of the balance condition of the bridge. At balance,

$$\left(R_x + \frac{1}{j\omega C_x} \right) \left(\frac{R_1}{1 + j\omega C_1 R_1} \right) = \frac{1}{j\omega C_s} \cdot R_2$$

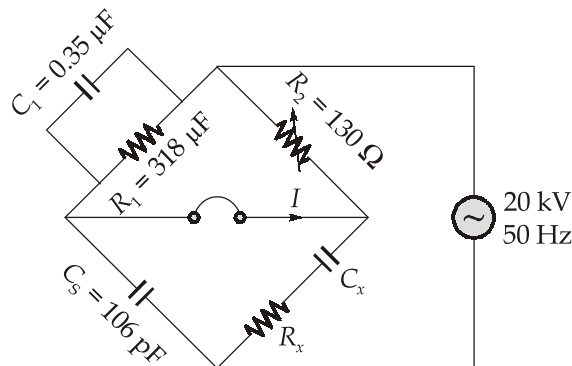
or
$$\left(R_x + \frac{1}{j\omega C_x} \right) R_1 = \frac{R_2}{j\omega C_s} (1 + j\omega C_1 R_1)$$

or
$$R_1 R_x - \frac{jR_1}{\omega C_x} = -j \frac{R_2}{\omega C_s} + \frac{C_1 R_1 R_2}{C_s}$$

Equating the real and imaginary terms we get,

$$R_x = \frac{C_1}{C_s} \cdot R_2; C_x = \frac{C_s}{R_2} \cdot R_1$$

Series resistance of the capacitor,



$$R_x = \frac{C_1}{C_s} R_2 = \frac{0.35 \times 10^{-6}}{106 \times 10^{-12}} \times 130$$

$$= 0.42924 \times 10^6 \Omega$$

3. Mantissa

∴ Normal Mantissa = 1.M = 1.1101101

Data + 1.1101101 × 2⁻³ {± M × B^{±e}}

mantissa align to right upto 3 times

$$\begin{array}{c} \downarrow \\ +0.0011101101 \\ \downarrow \\ 0.228 \\ \downarrow \\ 2.28 \times 10^{-1} \end{array}$$

Q.1 (d) Solution:



To extend the range of moving iron voltmeter add a series resistance of high value in the voltmeter circuit

Let the resistance of voltmeter be R_v and inductance be L

Let, the external series resistance to increase the range be R_s

Voltmeter current,

$$I = \frac{V}{\sqrt{(R_s + R_v)^2 + \omega^2 L^2}}$$

$$= \frac{300}{\sqrt{(2 \times 10^3 + R_s)^2 + (2\pi \times 50 \times 0.6)^2}}$$

Voltage drop across instrument,

$$v = I \sqrt{R_v^2 + (\omega L)^2}$$

$$v = \frac{V}{\sqrt{(R_s + R_v)^2 + \omega^2 L^2}} \cdot \sqrt{R_v^2 + (\omega L)^2}$$

$$\frac{V}{v} = \frac{\sqrt{(R_s + R_v)^2 + (\omega L)^2}}{\sqrt{R_v^2 + (\omega L)^2}}$$

$$\frac{300}{100} = \frac{\sqrt{(2000 + R_s)^2 + (2\pi \times 50 \times 0.6)^2}}{\sqrt{(2000)^2 + (2\pi \times 50 \times 0.6)^2}}$$

Solving, $9[(2000)^2 + (60\pi)^2] = (2000 + R_s)^2 + (60\pi)^2$

$$6023.64 = 2000 + R_s$$

$$R_s = 4023.64 \Omega$$

The multiplier resistance is shunted by a capacitor in order to compensate for the frequency errors introduced due to inductance of the operating coil.

$$C = 0.41 \times \frac{L}{R_s^2} = 0.41 \times \frac{0.6}{(4023.64)^2} = 15.19 \text{ nF}$$

Q.1 (e) Solution:

Computer architecture is a set of rules and method that describe the functionality organization and implementation of computer system. It is description of capabilities and programming model of a computer but not a particular implementation.

Reduced Instruction Set Computer architecture (RISC) has following properties:

1. RISC has simpler instruction, hence simpler instruction decoding is involved in process.
2. Instructions size is less than size of one word.
3. Instruction take single clock cycle to be executed.
4. RISC has more number of general purpose registers.
5. RISC has simple addressing models.
6. RISC has less data type involved.
7. It has hardwired unit of programming.
8. RISC processors are highly pipelined.
9. Execution time in RISC is very less.
10. RISC has no requirement of external memory for calculations.
11. RISC has applications in video processing telecommunication and image processing.

Q.2 (a) Solution:

When a file is used then the stored information in the file must be accessed and read into the memory of a computer system. Various mechanism are provided to access a file from the operating system.

1. Sequential access
2. Direct Access
3. Index Access

Sequential Access: It is the simplest access mechanism, in which informations stored in a file are accessed in an order such that one record is processed after the other. For example editors and compilers usually access files in this manner.

Direct Access: It is an alternative method for accessing a file, which is based on the disk model of a file, since disk allows random access to any block or record of a file. For this method, a file is viewed as a numbered sequence of blocks or records which are read/written in an arbitrary manner, i.e. there is no restriction on the order of reading or writing. It is well suited for Database management System.

Index Access: In this method an index is created which contains a key field and pointers to the various block. To find an entry in the file for a key value , we first search the index and then use the pointer to directly access a file and find the desired entry. With large files , the index file itself may become too large to be keep in memory. One solution is to create an index for the index file. The primary index file would contain pointers to secondary index files, which would point to the actual data items.

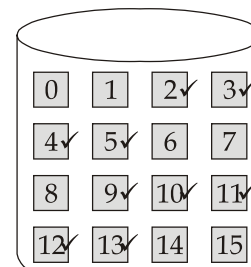
File Allocation Method:

1. Contiguous Allocation
2. Linked List Allocation
3. I-Nodes

Contiguous allocation: It requires each file to occupy a set of contiguous addresses on a disk. It store each file as a contiguous run of disk blocks. Thus on a disk with 1-KB blocks, a 50-KB file would be allocated 50 consecutive blocks. Both sequential and direct access is supported by the contiguous allocation method. Contiguous disk space allocation has two significant advantages.

- (i) First, it is simple to implement because keeping track of where a file's blocks are is reduced to remembering two numbers: the disk address of the first block and the number of blocks in the file. Given the number of the first block, the number of any other block can be found by a simple addition.
- (ii) Second, the read performance is excellent because the entire file can be read from the disk in a single operation. Only one seek is needed (to the first block). After that, no more seeks or rotational delays are needed so data come in at the full bandwidth of the disk. Thus contiguous allocation is simple to implement and has high performance. Unfortunately, contiguous allocation also has a major drawback: in time, the disk becomes fragmented, consisting of files and holes. It needs compaction to avoid this.

File	Starting disk block address	Size
abc.doc	2	4
xyz.doc	9	5

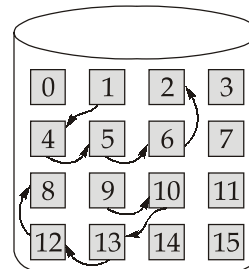


Linked List Allocation:

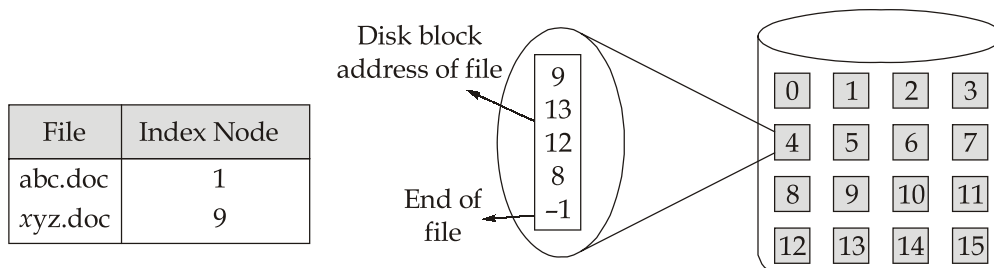
keep each file as a linked list of disk blocks as shown in the fig. The first word of each block is used as a pointer to the next one. The rest of the block is for data. Unlike contiguous

allocation, every disk block can be used in this method. No space is lost to disk fragmentation. The major problem with linked allocation is that it can be used only for sequential access files. To find the *i*th block of a file, we must start at the beginning of that file, and follow the pointers until we get the *i*th block. It is inefficient to support direct access capability for linked allocation of files. Another problem of linked list allocation is reliability. Since the files are linked together with the pointer scattered all over the disk. Consider what will happen if a pointer is lost or damaged.

File	Starting disk block address	Size
abc.doc	1	2
xyz.doc	9	8

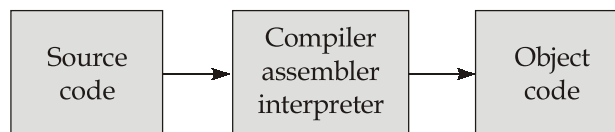


Indexed allocation (I-Nodes): It solves the external fragmentation and size declaration problems of contiguous allocation. In this allocation all pointers are brought together into one location called Index block. Each file has its own index block, which is an array of disk-block addresses. The *i*th entry in the index block points to the *i*th block of the file. The directory contains the address of the index block.



Q.2 (b) (i) Solution:

Translator software is used to convert a program written in high-level language and assembly language to a form that the computer can understand. Translator software converts a program written in assembly language, and high-level language to a machine-level language program. The translated program is called the object code.



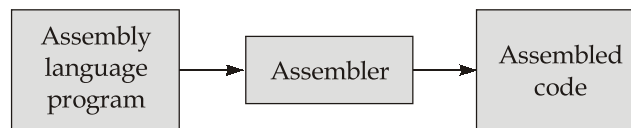
(Block diagram of translator software)

There are three different kinds of translator software:

- Assembler
- Compiler and
- Interpreter

Assembler converts a program written in assembly language to machine language. Compiler and interpreter convert a program written in high-level language to machine language.

Assembler: Assembly language is also referred to as a symbolic representation of the machine code. Assembler is a software that converts a program written in assembly language into machine code. There is usually a one-to-one correspondence between simple assembly statements and machine language instructions. The machine language is dependent on the processor architecture, though computers are generally able to carry out the same functionality in different wave. Thus the corresponding assembly language program also differ for different computer architectures.



Compiler: A program written in a high level language that the computer can understand, i.e., binary form. Compiler is the software that translates the program written in a high-level language to machine language. The program written in high-level language as referred to as the source code and compiled program is referred as the object code. The object code is the executable code, which can run as a stand-alone code. It does not require the compiler to be present during execution. Each programming language has its own compiler. Some languages that use a compiler are C++, COBOL, Pascal and FORTRAN. The compilation process generally involves two parts: breaking down the source code into small pieces and creating an intermediate representation, and constructing the object code for the intermediate representation. The compiler also reports syntax errors, if any, in the source code.

Interpreter: The purpose of interpreter is similar to that of a compiler. The interpreter is used to convert the high-level language program into computer understandable form. However, the interpreter functions in a different way than a compiler. Interpreter performs line-by-line execution of the source code during program execution. Interpreter reads the source code line-by-line, converts it into machine understandable form, execute the line, and then proceeds to the next line. Some languages that use an interpreter are BASIC and Python.

Difference between a Compiler and an Interpreter: Compiler and interpreter are used to convert a program written in high-level language to machine language, however, they

work differently. The key differences between a compiler and an interpreter are as follows:

- Interpreter looks at a source code line-by-line. Compiler looks at the entire source code.
- Interpreter converts a line into machine executable form, executes the line, and proceeds with the next line. Compiler converts the entire source code into object code and creates the object code. The object code is then executed by the user.
- For a given source code, once is compiled, the object code is created. This object code can be executed multiple number of times by the user. However, interpreter executes line-by-line, so executing the program using an interpreter means that during each execution, the source code is first interpreted and then executed.
- During execution of an object code, the compiler is not required. However, for interpretation, both interpreter and the source code is required during execution (because source code is interpreted during execution).
- Since interpreter interprets line-by-line, the interpreted code runs slower than the compiled code.

Q.2 (b) (ii) Solution:

ROM stands for read only memory and it does not lose its content when the power is switched-off. The features of ROM are described as follows:

- ROM, as the name implies, has only read capability and no write capability. After the information is stored in ROM, it is permanent and cannot be corrected.
- ROM comes programmed by the manufacturer. It stores standard processing programs that permanently reside in the computer. ROM stores the data needed for the start up of the computer. The instructions that are required for initializing the devices attached to a computer are stored in ROM.
- The ROM memory chip stores the Basic Input Output System (BIOS). BIOS provides the processor with the information required to boot the system. It provides the system with the settings and resources that are available on the system. BIOS is a permanent part of the computer. It does not load from disk but instead is stored in a ROM memory chip. The program code in the BIOS differs from ordinary software since it acts as an integral part of the computer. When the computer is turned-on, the BIOS does the following things:
 - Power on Self Test (POST) is a program that runs automatically when the system is booted. BIOS performs the power on self test. It checks that the major hardware components are working properly.
 - BIOS setup program, which is a built-in utility in BIOS, lets the user set the many functions that control how the computer works. BIOS displays the system settings

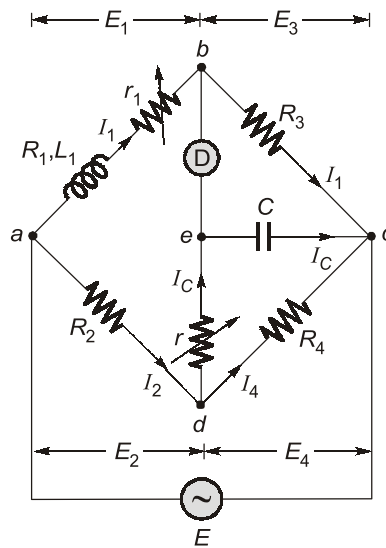
and finds the bootable devices. It loads the interrupt handlers and device drivers. It also initializes the registers.

- Bootstrap Loader is a program whose purpose is to start the computer software for operation when the power is turned-on. It loads the operating system into RAM and launches. It generally, seeks the operating system on the hard disk. The bootstrap loader resides in the ROM. The BIOS initiates the bootstrap sequence.
 - ◆ Different kinds of ROMs has been classified below. They have evolved from the fixed read only memory to the ones that can be programmed and re-programmed. They vary in the number of re-writes and the method used for the re-writing. Programmable ROM (PROM), Erasable Programmable ROM (EPROM) and Electrically Erasable Programmable ROM (EEPROM) are some of the ROMs. All the different kinds of ROM retain their content when the power is turned-off.
 - ◆ **PROM** can be programmed with a special tool, but after it has been programmed the contents cannot be changed. PROM memories have thousands of fuses (or diodes). High voltage (12 V) is applied to the fuses to be burnt. The burnt fuses correspond to 0 and the others to 1.
 - ◆ **EPROM** can be programmed in a similar way as PROM, but it can be erased by exposing it to ultra violet light and re-programmed. EPROM chips have to be removed from the computer for re-writing.
 - ◆ **EEPROM** memories can be erased by electric charge and re-programmed. EEPROM chips do not have to be removed from the computer for re-writing.

Q.2 (c) Solution:

- (i) **Anderson's Bridge:** This bridge, in fact, is a modification of the Maxwell's inductance-capacitance bridge. In this method, the self-inductance is measured in terms of a standard capacitor. This method is applicable for precise measurement of self-inductance over a very wide range of values.

Let, L_1 = self-inductance to be measured
 R_1 = resistance of self-inductor
 r_1 = resistance connected in series with self-inductor
 and r, R_2, R_3, R_4 = known non-inductive resistances
 C = fixed standard capacitor



At balance,

$$I_1 = I_3 \text{ and } I_2 = I_c + I_4$$

Now,

$$I_1 R_3 = I_c \times \frac{1}{j\omega C}$$

∴

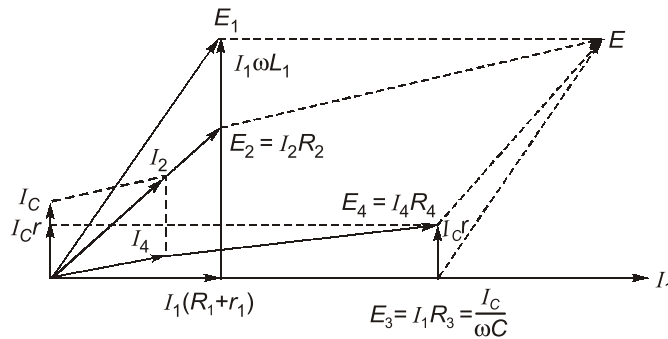
$$I_c = I_1 j\omega C R_3$$

Writing the other balance equations,

$$I_1 (r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_c r$$

and

$$I_c \left(r + \frac{1}{j\omega C} \right) = (I_2 - I_c) R_4$$



Substituting the value of I_c in the above equations,

$$I_1 (r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_1 j\omega C R_3 r$$

or
$$I_1 (r_1 + R_1 + j\omega L_1 - j\omega C R_3 r) = I_2 R_2 \quad \dots(i)$$

and
$$j\omega C R_3 I_1 \left(r + \frac{1}{j\omega C} \right) = (I_2 - I_1 j\omega C R_3) R_4$$

or
$$I_1 (j\omega C R_3 r + j\omega C R_3 R_4 + R_3) = I_2 R_4 \quad \dots(ii)$$

From equations (i) and (ii),

$$I_1(r_1 + R_1 + j\omega L_1 - j\omega C R_3 r) = I_1 \left(\frac{R_2 R_3}{R_4} + \frac{j\omega C R_2 R_3 r}{R_4} + j\omega C R_3 R_2 \right)$$

Equating the real and the imaginary parts:

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

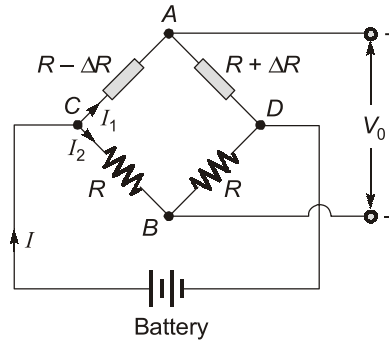
and

$$L_1 = C \frac{R_3}{R_4} [r (R_4 + R_2) + R_2 R_4]$$

- (ii) Strain gauge 1 is in compression mode, hence resistance of strain gauge 1 after application of strain = $R - \Delta R$

Similarly resistance of strain gauge 2 after application of strain = $R + \Delta R$

Redraw the circuit



$$V_0 = V_A - V_B \quad \dots(i)$$

$$V_0 = I_1(R + \Delta R) - I_2(R) \quad \dots(ii)$$

Total resistance in arm CAD = Total resistance in arm CBD = $2R$

$$I_1 = I_2 = \frac{I}{2}$$

Putting these values in equation (ii), we get

$$V_0 = \frac{I}{2}(R + \Delta R) - \frac{I}{2}(R)$$

$$V_0 = \frac{I}{2}(\Delta R) \quad \dots(iii)$$

1. For strain gauge, change in resistance ΔR due to application of strain (ϵ) is given by

$$\begin{aligned} \Delta R &= R \times G_f \times \epsilon \\ &= 120 \times 2 \times 150 \times 10^{-6} \Omega \\ &= 36 \times 10^{-3} \Omega \end{aligned}$$

From equation (iii), $V_0 = \frac{100}{2} \times 36 \times 10^{-3} \text{ mV} = 1.8 \text{ mV}$

2. Sensitivity : It is defined as ratio of output voltage to unit microstrain
Output voltage for unit micro strain,

$$V_0 = \frac{I}{2}(\Delta R) = \frac{I}{2} \times R \times G_f \times \epsilon$$

Here,

$$\epsilon = 1 \times 10^{-6}$$

$$V_0 = \frac{100}{2} \times 120 \times 2 \times 1 \times 10^{-6} = 12 \text{ mV}$$

$$S = 0.012 \mu\text{V}/\mu\text{S}$$

3. Resolution: It is defined as the minimum input quantity that can be measured accurately.

Output of Galvanometer per scale division = 1 mV

For 1/10th of division can be read with accuracy.

The minimum output voltage that can read by the Galvanometer =

$$1 \text{ mV} \times \frac{1}{10} = 0.1 \text{ mV}$$

Here minimum strain that can be measured by the circuit is equal to strain at which output voltage = 0.1 mV

$$V_0 = \frac{I}{2}(\Delta R)$$

$$V_0 = \frac{I}{2}(R \times G_f \times \epsilon)$$

$$\epsilon = \frac{V_0 \times 2}{I \times R \times G_f}$$

$$\begin{aligned} \epsilon &= \frac{0.1 \times 10^{-3} \times 2}{100 \times 10^{-3} \times 120 \times 2} = 8.33 \times 10^{-6} \\ &= 8.33 \text{ microstrain} \end{aligned}$$

Q.3 (a) Solution:

(i) Advantages of LVDT are:

- Linearity
- Infinite resolution
- High output
- High sensitivity
- Ruggedness

- Less friction
- Low hysteresis
- Low power consumption

Disadvantages of LVDT:

- Large displacements are required for appreciable differential output.
- Limited dynamic response
- Temperature affects on the performance of transducer.

$$(ii) \quad \text{Sensitivity LVDT} = \frac{\text{Output voltage}}{\text{Displacement}} = \frac{2 \times 10^{-3}}{0.5}$$

$$= 4 \times 10^{-3} \text{ V/mm} = 4 \text{ mV/mm}$$

$$\text{Sensitivity instrument} = (\text{Amplification factor}) \times (\text{Sensitivity of LVDT})$$

$$= (4 \times 10^{-3}) \times (250) = 1 \text{ V/mm}$$

$$1 \text{ scale division} = \frac{5}{100} = 50 \text{ mV}$$

Minimum voltage that can be read on voltmeter

$$= \frac{1}{5} \times 50 = 1 \text{ mV}$$

$$\therefore \text{Resolution of instrument} = 1 \times \frac{1}{1000} = 1 \times 10^{-3} \text{ mm}$$

Q.3 (b) (i) Solution:

Demand paging:

- Demand paging is used with virtual memory concept.
- Main memory is divided into equal size blocks known as frames.
- Virtual (user) program is divided into 'pages', where page size is equal to frame size.
- Whenever CPU generates read/write request for a word, that requested word belongs to one of the page of a program.
- If demanded page is present in one of the frame of main memory then it is known as 'page hit'.
- If demanded page is not available then this condition is known as 'page fault' and page fault service routine invoked.
- Page fault service routine loads demanded page into main memory and updates the page table.

Q.3 (b) (ii) Solution:

When computer moved into mainstream use, personal computers were connected together through LANs to control server. These control servers were much more powerful than the PCs so any large data processing took place on these servers. Since the PCs have become much more powerful and they are able to handle the data processing locally rather than on central servers; because of this, PC to PC or peer-to-peer computing can now occur when individual computers bypass control servers to connect and collaborate directly with each other.

There are three peer-to-peer computing models:

- (i) Multiple peer relationship:** PCs are connected to each other through servers, and files can be shared and collected from anyone else on that same network. One key problem in this is, it can lead to major breaches in security and intellectual property issues.
- (ii) Distributed peer relationship:** A group of computers are connected together to combine their computing and processing abilities to search the internet or solve very complex problems requiring massive process crunching.
- (iii) Collaborative peer relationship:** A small group of people agree to collaborate through a common interface, such as online gaming chat rooms, instant messaging core-learning environment.

Q.3 (c) (i) Solution:

Given :

$$r_p = 96 \Omega, r_s = 0.88 \Omega, x_p = 67.2 \Omega, x_{1e} = 115 \Omega$$

Now,

$$n = \frac{E_p}{E_s} = \frac{1000}{100} = 10$$

1. Phase angle error can be given as :

$$\theta = \frac{\frac{I_s}{n}(x_{1e} \cos \Delta - r_{1e} \sin \Delta) + I_e x_p - I_m r_p}{n V_s}$$

At no-load,

$$I_s = 0 \text{ A}$$

$$\theta = \frac{I_e x_p - I_m r_p}{n V_s} \quad \dots(i)$$

$$\cos \phi = 0.4, I_o = 0.03 \text{ A}, I_e = I_o \cos \phi_o = 0.012 \text{ A}$$

$$I_m = I_o \sin \phi = 0.02749 \text{ A}$$

Now from eqn. (i)

$$\begin{aligned} \theta &= \frac{0.012 \times 67.2 - 0.02749 \times 96}{10 \times 100} \text{ rad} \\ &= -1.8326 \times 10^{-3} \text{ rad} = -0.105^\circ = -6.3' \end{aligned}$$

2. At unity pf, $\cos \Delta = 1, \sin \Delta = 0$

$$\therefore \theta = \frac{\frac{I_s}{n} x_{1e} \cos \delta + I_e x_p - I_m r_p}{n V_s}$$

$$0 = \frac{\frac{I_s}{10} \times 115 + 0.012 \times 67.2 - 0.02749 \times 96}{10 \times 100}$$

$$\therefore I_s = 0.1593 \text{ A}$$

$$\therefore \text{Load in VA} = V_s I_s = 100 \times 0.1593 = 15.93 \text{ VA}$$

Q.3 (c) (ii) Solution:

An analog data acquisition system typically consists of some or all of the following elements:

1. Transducers for translating physical parameters into electrical signals.
2. Signal conditioners for amplifying, modifying, or selecting certain portions of these signals in order to bring them to a presentable form so that it is acceptable to the output device.
3. Visual display devices for continuous monitoring of the input signals. These devices may include single- or multichannel oscilloscopes, storage oscilloscopes, panel meters, numerical displays, and so on.
4. Graphic recording instruments for obtaining permanent records of the input data. These instruments include stylus-and-ink recorders to provide continuous records on paper charts, optical recording systems such as mirror galvanometer recorders, and ultraviolet recorders.
5. Magnetic tape instrumentation for acquiring input data, preserving their original electrical form, and reproducing them at a later date for more detailed analysis.

Q.4 (a) (i) Solution:

Input Output Interface : The method that is used to transfer information between internal storage and external I/O devices is known as I/O interface. The CPU is interfaced using special communication links by the peripherals connected to any computer system. These communication links are used to resolve the differences between CPU and peripheral. There exists special hardware components between CPU and peripherals to supervise and synchronize all the input and output transfers that are called interface units.

Data transfer to and from the peripherals may be done in any of the three possible ways

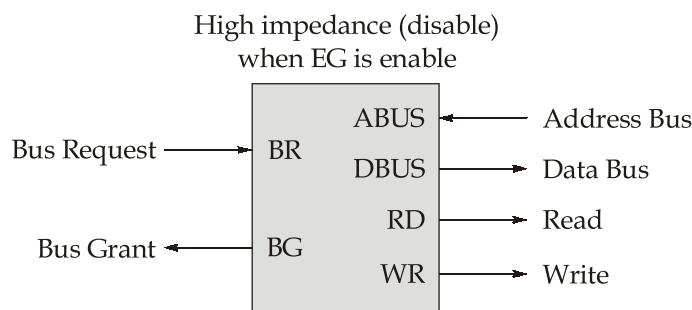
1. Programmed I/O.
2. Interrupt- initiated I/O.
3. Direct memory access(DMA).

Programmed I/O: It is due to the result of the I/O instructions that are written in the computer program. Each data item transfer is initiated by an instruction in the program. Usually the transfer is from a CPU register and memory. In this case it requires constant monitoring by the CPU of the peripheral devices.

Example of Programmed I/O: In this case, the I/O device does not have direct access to the memory unit. A transfer from I/O device to memory requires the execution of several instructions by the CPU, including an input instruction to transfer the data from device to the CPU and store instruction to transfer the data from CPU to memory. In programmed I/O, the CPU stays in the program loop until the I/O unit indicates that it is ready for data transfer. This is a time consuming process since it needlessly keeps the CPU busy.

Interrupt- initiated I/O: Since in the above case we saw the CPU is kept busy unnecessarily. This situation can very well be avoided by using an interrupt driven method for data transfer. By using interrupt facility and special commands to inform the interface to issue an interrupt request signal whenever data is available from any device. In the meantime the CPU can proceed for any other program execution. The interface meanwhile keeps monitoring the device. Whenever it is determined that the device is ready for data transfer it initiates an interrupt request signal to the computer. Upon detection of an external interrupt signal the CPU stops momentarily the task that it was already performing, branches to the service program to process the I/O transfer, and then return to the task it was originally performing.

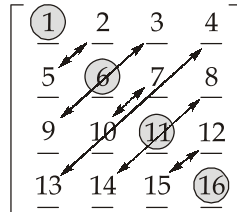
Direct Memory Access: The data transfer between a fast storage media such as magnetic disk and memory unit is limited by the speed of the CPU. Thus we can allow the peripherals directly communicate with each other using the memory buses, removing the intervention of the CPU. This type of data transfer technique is known as DMA or direct memory access. During DMA the CPU is idle and it has no control over the memory buses. The DMA controller takes over the buses to manage the transfer directly between the I/O devices and the memory unit.



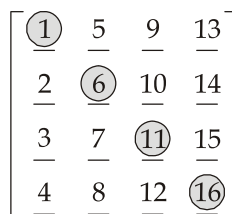
Bus Request: It is used by the DMA controller to request the CPU to relinquish the control of the buses.

Bus Grant: It is activated by the CPU to Inform the external DMA controller that the buses are in high impedance state and the requesting DMA can take control of the buses. Once the DMA has taken the control of the buses it transfers the data. This transfer can take place in many ways.

Q.4 (a) (ii) Solution:

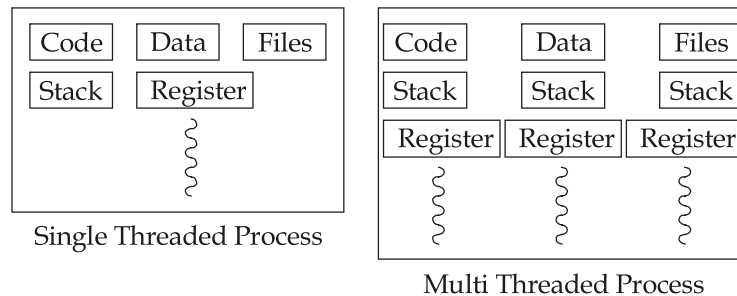


```
#include <stdio.h>
#define ROW 4
#define COL 4
int M[ROW][COL] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16};
main(){
    int i, j, t;
    for (i = 0; i < 4; ++i){
        for(j = i; j < 4; ++j){
            t = M[j][i];
            M[i][j] = M[j][i];
            M[j][i] = t;
        }
    }
    for (i = 0; i < 4; ++i){
        for (j = 0; j < 4; ++j){
            printf ("%d", M[i][j]);
        }
    }
}
```



Q.4 (b) (i) Solution:

Multithreading refers to ability of an operating system to support multiple threads of execution within a single process. Multithreading gives the benefit of separation of resource ownership (Address space and used files I/O) and execution. Resources like code data, files and memory will be shared among all threads within process but stacks and registers can not be shared and every new thread will have its own stack and register as shown below.

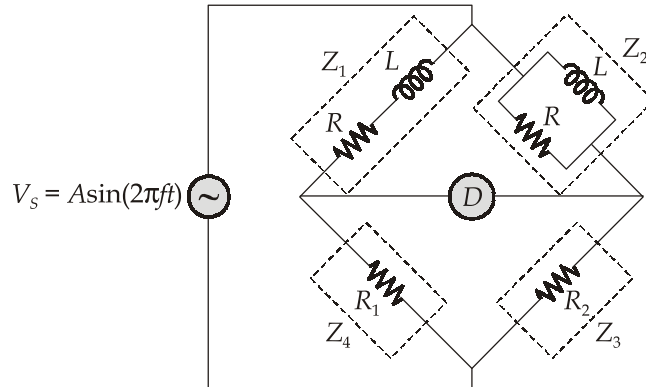
**Advantages of Multi Threading :**

- It performs foreground and background work in parallel.
- It increases responsiveness of system.
- It allows synchronous processing, i.e., it separates the execution of independent tasks.
- It increases the execution speed, i.e., it overlaps CPU execution time and I/O wait time.
- Concurrency can be achieved by multi threading.
- Reduces context switch time.
- Enhances the throughput.

Comparison of Multi-threading and Multi-tasking :

- Multi-threading is faster compared to multi-tasking.
- In multi-threading, creating a thread takes less time than termination of a process in multi-tasking.
- Switching between threads takes less time compared to switching between processes.
- In multi-threading, threads of same process can share memory, I/O and files so communication between the threads can be done at user level.

Q.4 (b) (ii) Solution:



At balance condition, we have

$$Z_1 Z_3 = Z_2 Z_4$$

Where,

$$Z_1 = R + j\omega L$$

$$Z_2 = \frac{R(j\omega L)}{R + j\omega L}$$

$$Z_3 = R_2$$

and

$$Z_4 = R_1$$

$$\therefore (R + j\omega L)R_2 = \frac{j\omega RL}{R + j\omega L} \times R_1$$

$$(R^2)(R + j\omega L)^2 = j\omega RL(R_1)$$

$$R_2 \left[(R^2 - \omega^2 L^2) + j2\omega LR \right] = j\omega RL(R_1)$$

Equating real part on both sides, we get

$$R^2 - \omega^2 L^2 = 0$$

$$\omega = \frac{R}{L}$$

$$f = \frac{1}{2\pi} \left(\frac{R}{L} \right)$$

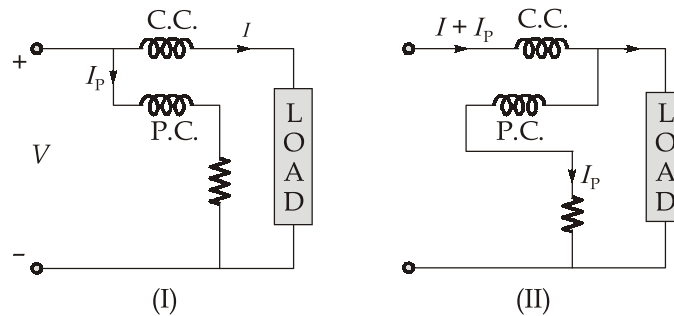
Substituting the given values $R = 300 \Omega$ and $L = 30 \text{ mH}$

$$\text{We get, } f = \frac{1}{2\pi} \left(\frac{300}{30 \times 10^{-3}} \right) = \frac{10}{2\pi} \times 10^3 = 1.59 \text{ kHz}$$

Q.4 (c) Solution:

Sources of errors in electro-dynamometer type wattmeter are:

- 1. Pressure coil inductance:** In ideal dynamometer type wattmeter the current in pressure coil is in phase with applied voltage. But the pressure coil does have inductance which causes the current to lag behind the voltage applied. It is because of this, wattmeter will read high with lagging power factor load and will read low with leading power factor load.
- 2. Pressure coil capacitance:** The capacitance is mainly due to inter turn capacitance of the series resistance. The effect of capacitance is opposite to that of inductance.
- 3. Mutual inductance between pressure coil and current coil** is also a source of error. These errors are significant for high frequency measurements.
- 4. Eddy current error:** Eddy currents are induced in the solid metal parts and within the thick conductors by A.C. field produced by current coil. The field by Eddy current will affect the field by main current in current coil.
- 5. Error due to connection:** Two possible connections



In connection (I) power measured by instrument = Power consumed + Power loss in current coil

In connection (II) power measured by instrument = Power consumed by load + Power loss in pressure coil

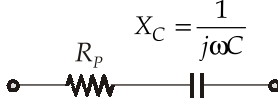
For smaller load current, connection (I) is preferable whereas for larger current connection (II) is preferable.

- 6. Temperature error:** Change in room temperature will change the resistance of pressure coil and stiffness of springs providing controlling torque.
- 7. Stray magnetic field error:** These wattmeters has relatively weaker field and therefore are affected by stray magnetic field.
- 8. Error due to vibration of moving system:** Torque on the moving system varies with frequency which is twice to that of voltage. If some part of moving system have a natural frequency which is in resonance with the frequency of torque pulsation, the moving system would vibrate.

Power measured $P_1 \propto VI \cos \phi$... (i)

$\cos \phi$ is power factor of load.

Consider pressure coil with a capacitor,

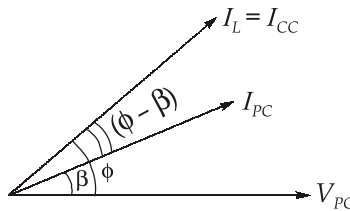


$$R_p + \frac{1}{j\omega C} = R_p - \frac{j}{\omega C}$$

Given, $\frac{1}{\omega C} = R_p$... (ii)

The current in pressure coil will lead the voltage by an angle β

$$\beta = \tan^{-1} \left(-\frac{1}{\omega R_p C} \right) \quad \left[\because R_p = \frac{1}{\omega C} \right]$$



From equation (ii), $\beta = \tan^{-1} (-1) = -45^\circ$
 $\beta = -45^\circ$

Wattmeter reading in this case

$$P_2 \propto VI \cos \beta \cos (\phi - \beta) \quad \dots \text{(iii)}$$

From equation (i) and (iii),

$$\frac{P_2}{P_1} = \frac{\cos \beta \cos (\phi - \beta)}{\cos \phi} = \frac{\cos \beta (\cos \phi \cos \beta + \sin \phi \sin \beta)}{\cos \phi}$$

$$\frac{P_2}{P_1} = \cos \beta (\cos \beta + \tan \phi \sin \beta)$$

$$\beta = -45^\circ$$

$$\frac{P_2}{P_1} = \frac{1}{2} - \frac{1}{2} \tan \phi$$

$$\Rightarrow \tan \phi = \frac{P_1 - 2P_2}{P_1}$$

$$\tan \phi = 1 - \frac{2P_2}{P_1}$$

**Section B : Power Electronics & Drives-1 + Engineering Mathematics-1
+ B.E.E.-2 + Analog Electronics-2 + Electrical Materials-2**

Q.5 (a) Solution:

Given :

$$A = \begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$$

Conjugate,

$$\bar{A} = \begin{bmatrix} 2 & 3-4i \\ 3+4i & 2 \end{bmatrix}$$

$$\bar{A}^T = \begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix} = A$$

Thus, A is Hermitian matrix.

The characteristic equation for A is

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 3+4i \\ 3-4i & 2-\lambda \end{vmatrix} = 0$$

$$(2 - \lambda)^2 - (3 + 4i)(3 - 4i) = 0$$

$$4 + \lambda^2 - 4\lambda - (9 + 16) = 0$$

i.e., $\lambda^2 - 4\lambda - 21 = 0$

$$(\lambda + 3)(\lambda - 7) = 0$$

Eigen values of A , Hermitian matrix are real $-3, 7$.

For $\lambda = -3$

$$\begin{bmatrix} 5 & 3+4i \\ 3-4i & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$x_1 = -\left(\frac{3+4i}{5}\right)x_2$$

The eigen vector corresponding to $\lambda = -3$ is $X_1 = \begin{bmatrix} -3-4i \\ 5 \end{bmatrix}$.

For $\lambda = 7$

$$\begin{bmatrix} -5 & 3+4i \\ 3-4i & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$x_1 = \frac{3+4i}{5}x_2$$

The eigen vector corresponding to $\lambda = 7$ is

$$X_1 = \begin{bmatrix} 3 + 4i \\ 5 \end{bmatrix}$$

Q.5 (b) Solution:

Nano materials show huge change in properties when materials are brought to nano scale. The two reason which prominently effect the properties, when materials are brought to nano scale are:

- Increase in surface area to volume ratio.
- Quantum confinement effect.

Properties which distinguish nano materials from normal materials are :

- Improved electrical conductivity in ceramics and magnetic nano composites, increased electric resistance in metals.
- Increased magnetic coercivity upto a critical gain size, super paramagnetic behavior.
- Improved hardness and toughness of metals and alloys, ductility and super plasticity of ceramic.
- High catalytic efficiency through higher surface to volume ratio.
- Spectral shift of optical absorption and fluorescence properties, increased quantum efficiency of semiconductor crystal.

Bucky ball is a sphere of around 1 nm diameter which is formed by 60 carbon atoms which are covalently bonded to three adjacent carbon atoms. It is also known as buckminster fluorene.

Applications of buckminster fluorene:

- They have hardness greater than diamond and hence used within armors.
- They are also used in various polymers to make them stronger.
- Modified Bucky balls are also developed as antioxidants for use by humans.
- Functionalized Bucky balls are developed for target drug delivery where Bucky balls encases a minute dose of particular drug.

Q.5 (c) Solution:

(i) The rms value is defined as

$$I_r = \left[\frac{1}{T} \int_0^{t_1} (I_m \sin \omega_s t)^2 dt + \frac{1}{T} \int_{t_2}^{t_3} I_a^2 dt \right]^{1/2}$$

$$I_r = (I_{r1}^2 + I_{r2}^2)^{1/2} \quad \dots(i)$$

where

$$\omega_s = 2\pi f_s = 31415.93 \text{ rad/sec}$$

$$t_1 = \frac{\pi}{\omega_s} = 100 \mu\text{s}, T = \frac{1}{f}$$

$$\begin{aligned} I_{r1} &= \left[\frac{1}{T} \int_0^{t_1} (I_m \sin \omega_s t)^2 dt \right]^{1/2} = I_m \sqrt{\frac{f \cdot t_1}{2}} \\ &= 450 \times \sqrt{\frac{250 \times 100 \times 10^{-6}}{2}} \\ &= 50.31 \text{ A} \end{aligned} \quad \dots(\text{ii})$$

$$\begin{aligned} I_{r2} &= \left[\frac{1}{T} \int_{t_2}^{t_3} I_a^2 dt \right]^{1/2} = I_a \sqrt{f(t_3 - t_2)} \\ &= 150 \sqrt{250 \times (150) \times 10^{-6}} \\ &= 29.05 \text{ A} \end{aligned} \quad \dots(\text{iii})$$

Now putting eqn. (ii) and eqn. (iii) in eqn. (i),

$$I_r = (50.31^2 + 29.05^2)^{1/2}$$

$$I_r = 58.09 \text{ A}$$

(ii) The average current is found from

$$\begin{aligned} I_{av} &= \left[\frac{1}{T} \int_0^{t_1} (I_m \sin \omega_s t) dt + \frac{1}{T} \int_{t_2}^{t_3} I_a dt \right] \\ I_{av} &= I_{d1} + I_{d2} \end{aligned} \quad \dots(\text{iv})$$

$$I_{d1} = \frac{1}{T} \int_0^{t_1} (I_m \sin \omega_s t) \cdot dt = \frac{I_m f}{\pi f_s} = \frac{450 \times 250}{\pi \times 5000} = 7.162 \text{ A}$$

$$\begin{aligned} I_{d2} &= \frac{1}{T} \int_{t_2}^{t_3} I_a dt = I_a f (t_3 - t_2) \\ &= 150 \times 250 \times 150 \times 10^{-6} = 5.62 \text{ A} \end{aligned}$$

Now, average passing current diode is I_{av} :

$$I_{av} = (7.162 + 5.62)$$

$$I_{av} = 12.782 \text{ A}$$

Q.5 (d) Solution:

First draw is from Urn A, second draw is from Urn B and third draw is again from Urn A.

Let events A_R and A_B be drawing red ball and black ball, respectively from bag A.

Also let events B_R and B_B be drawing red ball and black ball respectively, from bag B.

Case-I:

$$A_R \rightarrow B_R \rightarrow A_R$$

Required probability,

$$\begin{aligned} P(A_R \cap B_R \cap A_R) &= P(A_R)P\left(\frac{B_R}{A_R}\right)P\left(\frac{A_R}{(A_R \cap B_R)}\right) \\ &= \frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} = \frac{18}{110} \end{aligned}$$

Case-II:

$$A_R \rightarrow B_B \rightarrow A_R$$

Required probability,

$$\begin{aligned} P(A_R \cap B_B \cap A_R) &= P(A_R)P\left(\frac{B_B}{A_R}\right)P\left(\frac{A_R}{(A_R \cap B_B)}\right) \\ &= \frac{6}{10} \times \frac{6}{11} \times \frac{6}{10} = \frac{18}{110} \end{aligned}$$

Case III:

$$A_B \rightarrow B_R \rightarrow A_R$$

Required probability,

$$\begin{aligned} P(A_B \cap B_R \cap A_R) &= P(A_B)P\left(\frac{B_R}{A_B}\right)P\left(\frac{A_R}{(A_B \cap B_R)}\right) \\ &= \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} = \frac{56}{550} \end{aligned}$$

Case IV:

$$A_B \rightarrow B_B \rightarrow A_R$$

Required probability,

$$P(A_B \cap B_B \cap A_R) = P(A_B)P\left(\frac{B_B}{A_B}\right)P\left(\frac{A_R}{(A_B \cap B_B)}\right)$$

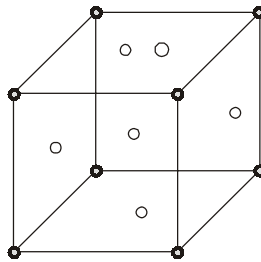
$$= \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10} = \frac{84}{550}$$

∴ Total probability = $\frac{18}{110} + \frac{18}{110} + \frac{56}{550} + \frac{84}{550}$

$$= \frac{90 + 90 + 56 + 84}{550} = \frac{320}{550} = \frac{32}{55}$$

Q.5 (e) Solution:

FCC unit cell structure :



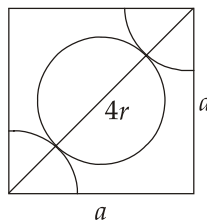
In this structure, the eight corners of the cube are occupied by eight atoms and six atoms occupy the centres of six faces of cube. Metals that crystallize in FCC structure are nickel, aluminium, copper, silver, gold, platinum, lead and iron.

Number of atoms in the unit cell of FCC structure :

$$\begin{aligned} \text{Total number of atoms} &= \frac{1}{8} \times (8 \text{ corner atom}) + (6 \text{ atoms at faces}) \times \frac{1}{2} \\ &= 4 \text{ atoms} \end{aligned}$$

Therefore, the unit cell of FCC structure contains 4 atoms.

Atomic packing factor of FCC :



Now,

$$a^2 + a^2 = 16r^2$$

$$r = \frac{a}{2\sqrt{2}}$$

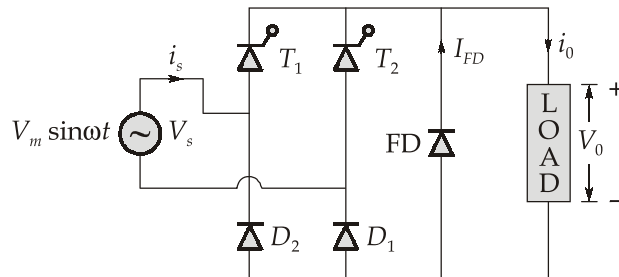
$$\text{Atomic packing factor} = \frac{\text{Volume of atoms in the unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{4 \times \frac{4}{3} \pi r^3}{a^3} = \frac{4 \times \frac{4}{3} \pi \times \frac{a^3}{16\sqrt{2}}}{a^3}$$

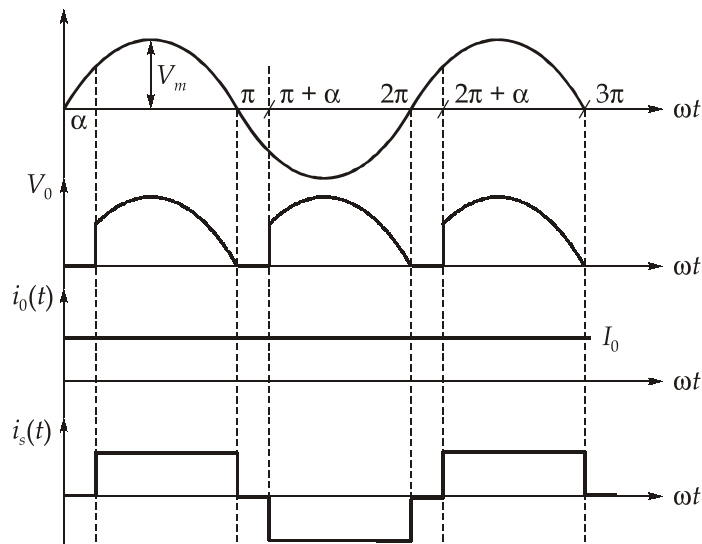
$$\text{APF} = \frac{\pi}{3\sqrt{2}} = 0.74$$

Q.6 (a) Solution:

1- ϕ semi-converter circuit is shown in figure,



- Load current (I_0) is assumed to be continuous and ripple free.
- The various waveform V_s, V_0, I_0, I_s are shown below,



The instantaneous supply current $i_s(t)$ can be expressed in Fourier series as

$$i_s(t) = a_0 + \sum_{n=1,2,3,\dots}^{\infty} (a_n \cos n \omega t + b_n \sin n \omega t)$$

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} i_s(t) d(\omega t) = \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} I_0 d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_0 d(\omega t) \right] = 0$$

$$\begin{aligned}
 a_n &= \frac{1}{\pi} \left[\int_{\alpha}^{\pi} I_0 \cos n\omega t \cdot d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_0 \cos n\omega t \cdot d(\omega t) \right] \\
 &= \frac{I_0}{n\pi} \left[\sin n\omega t \Big|_{\alpha}^{\pi} - \sin n\omega t \Big|_{\pi+\alpha}^{2\pi} \right] \\
 &= \begin{cases} \frac{-2I_0}{\pi} \sin n\alpha & \text{for } n = 1, 3, 5, \dots \\ 0 & \text{for } n = 2, 4, 6, \dots \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 b_n &= \frac{1}{\pi} \left[\int_2^{\pi} (I_0 \sin n\omega t) d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_0 \sin n\omega t d(\omega t) \right] \\
 &= \begin{cases} \frac{2I_0}{n\pi} (1 + \cos n\alpha) & \text{for } n = 1, 3, 5, \dots \\ 0 & \text{for } n = 2, 4, 6, \dots \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 c_n &= \sqrt{a_n^2 + b_n^2} \\
 &= \left[\left(\frac{-2I_0}{n\pi} \sin n\alpha \right)^2 + \left(\frac{2I_0}{n\pi} (1 + \cos n\alpha) \right)^2 \right]^{1/2} \\
 &= \frac{2\sqrt{2}}{n\pi} I_0 (1 + \cos n\alpha)^{1/2}
 \end{aligned}$$

As we know, $1 + \cos 2\theta = 2 \cos^2 \theta$

$$(1 + \cos n\alpha)^{1/2} = 2 \cos n \frac{\alpha}{2}$$

$$\begin{aligned}
 \theta_n &= \tan^{-1} \left(\frac{a_n}{b_n} \right) = \tan^{-1} \left(-\frac{\sin n\alpha}{1 + \cos n\alpha} \right) \\
 &= \tan^{-1} \left(\frac{-2 \sin \frac{n\alpha}{2} \cdot \cos \frac{n\alpha}{2}}{2 \cos^2 \frac{n\alpha}{2}} \right) = \frac{-n\alpha}{2}
 \end{aligned}$$

$$c_n = \frac{2\sqrt{2}}{n\pi} I_0 \left[2 \cos^2 \frac{n\alpha}{2} \right]^{1/2} = \frac{4I_0}{n\pi} \cos \left(\frac{n\alpha}{2} \right)$$

$$i_s(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi} \cos \frac{n\alpha}{2} \sin \left(n\omega t - \frac{n\alpha}{2} \right)$$

Rms value of n^{th} harmonic input current,

$$I_{sn} = \frac{4I_0}{\sqrt{2n\pi}} \cos \frac{n\alpha}{2} = \frac{2\sqrt{2}}{n\pi} I_0 \cos \frac{n\alpha}{2}$$

Hence, rms value of fundamental current,

$$I_{s1} = \frac{2\sqrt{2}}{n\pi} I_0 \cos \frac{\alpha}{2} \quad \text{for } n = 1$$

Rms value of total input current,

$$I_s = \left[\frac{I_0^2(\pi - \alpha)}{\pi} \right]^{1/2} = I_0 \sqrt{\frac{\pi - \alpha}{\pi}}$$

Current distortion factor, (CDF) = $\frac{I_{s1}}{I_s} = \frac{\frac{2\sqrt{2}}{\pi} I_0 \cos \frac{\alpha}{2}}{I_0 \sqrt{\frac{\pi - \alpha}{\pi}}} = \frac{2\sqrt{2} \cos \frac{\alpha}{2}}{\sqrt{\pi(\pi - \alpha)}}$

Harmonic factor, $\text{HF} = \sqrt{\frac{1}{(\text{CDF})^2} - 1} = \left[\frac{\pi(\pi - \alpha)}{4(1 + \cos \alpha)} - 1 \right]^{1/2}$

Hence, harmonic factor of input current for $\alpha = \frac{\pi}{2}$

$$= \left[\frac{\pi \left(\pi - \frac{\pi}{2} \right)}{4 \left(1 + \cos \frac{\pi}{2} \right)} - 1 \right]^{1/2} = 0.4834$$

Q.6 (b) Solution:

(i) Substitute $x = e^z$, the differential equation becomes

$$D(D - 1)y - 2y = e^{-z} + e^{2z}$$

$$(D^2 - D - 2)y = e^{-z} + e^{2z}$$

The homogenous part of differential equation is

$$(D - 2)(D + 1)y = 0$$

$$D = 2, -1$$

Thus, complimentary function is

$$y_{\text{complimentary function}} = C_1 e^{2z} + C_2 e^{-z}$$

The particular integral is

$$y_{\text{Particular integral}} = \frac{e^{-z} + e^{2z}}{(D-2)(D+1)}$$

$$= \frac{-ze^{-z}}{3} + \frac{z}{3}e^{2z}$$

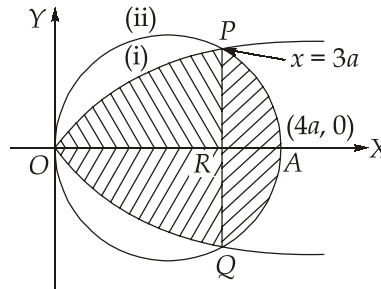
Complete solution is given by

$$y = y_{\text{Complimentary function}} + y_{\text{Particular integral}}$$

$$= \left(C_1x^2 + \frac{C_2}{x} \right) + \frac{\log x}{3} \left(x^2 - \frac{1}{x} \right)$$

(ii) Given parabola is $y^2 = ax$ and the circle is

$$x^2 + y^2 = 4ax$$



Both these curves are symmetrical about x -axis.

Solving (i) and (ii) for x , we have

$$x^2 + ax = 4ax$$

$$x(x - 3a) = 0$$

$$x = 0, 3a$$

Thus, the two curves intersect at the points where $x = 0$ and $x = 3a$.

Also, (ii) meets the x -axis at $A(4a, 0)$.

Area common to (i) and (ii), i.e., the shaded area

$$= 2[\text{Area ORP} + \text{Area PRA}]$$

$$= 2 \left[\int_0^{3a} y dx \text{ from (i)} + \int_{3a}^{4a} y dx \text{ from (ii)} \right]$$

$$= 2 \left[\int_0^{3a} \sqrt{ax} dx + \int_{3a}^{4a} \sqrt{(4ax - x^2)} dx \right]$$

$$\begin{aligned}
&= 2\sqrt{a} \left| \frac{x^{3/2}}{\frac{3}{2}} \right|_0^{3a} + 2 \int_{3a}^{4a} \sqrt{[4a^2 - (x-2a)^2]} dx \\
&= \frac{4\sqrt{a}}{3} (3a)^{3/2} + 2 \left[\frac{1}{2} (x-2a) \sqrt{4a^2 - (x-2a)^2} + \frac{4a^2}{2} \sin^{-1} \frac{x-2a}{2a} \right]_{3a}^{4a} \\
&= 4\sqrt{3}a^2 + 2 \left[\left\{ 0 - \frac{1}{2} a\sqrt{3}a \right\} + 2a^2 \left\{ \frac{\pi}{2} - \frac{\pi}{6} \right\} \right] \\
&= 4\sqrt{3}a^2 - \sqrt{3}a^2 + \frac{4}{3}\pi a^2 \\
&= \left(3\sqrt{3} + \frac{4}{3}\pi \right) a^2
\end{aligned}$$

Q.6 (c) Solution:

$$\begin{aligned}
\text{(i)} \quad |A - \lambda I| &= \begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} \\
&= (3-\lambda)(2-\lambda)(5-\lambda)
\end{aligned}$$

Hence the characteristic equation of matrix A is given by

$$\begin{aligned}
|A - \lambda I| &= 0 \\
(3-\lambda)(2-\lambda)(5-\lambda) &= 0
\end{aligned}$$

$$\therefore \lambda = 2, 3, 5$$

Thus the eigen values of matrix A are 2, 3, 5.

The eigen vectors of the matrix A corresponding to the eigen value λ is given by the non-zero solution of the equation $(A - \lambda I)X = 0$.

$$\begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When $\lambda = 2$, the corresponding eigen vector is given by

$$\begin{bmatrix} 3-2 & 1 & 4 \\ 0 & 2-2 & 6 \\ 0 & 0 & 5-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + 4x_3 = 0$$

$$\frac{x_1}{6-0} = \frac{x_2}{0-6} = \frac{x_3}{0-0} = k$$

$$\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{0} = k$$

$$x_1 = k, x_2 = -k, x_3 = 0$$

Hence, $X_1 = \begin{bmatrix} k \\ -k \\ 0 \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ can be taken as an eigen vector of A corresponding to the eigen values $\lambda = 2$.

When $\lambda = 3$,

$$\begin{bmatrix} 3-3 & 1 & 4 \\ 0 & 2-3 & 6 \\ 0 & 0 & 5-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 + x_2 + 4x_3 = 0$$

$$0x_1 - x_2 + 6x_3 = 0$$

$$\frac{x_1}{6+4} = \frac{x_2}{0-0} = \frac{x_3}{0-0}$$

$$\frac{x_1}{10} = \frac{x_2}{0} = \frac{x_3}{0} = \frac{k}{10}$$

Hence, $X_2 = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix} = k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ can be taken as an eigen vector of A corresponding to the eigen value $\lambda = 3$.

When $\lambda = 5$.

Again, when $\lambda = 5$, substituting in (1), the corresponding eigen vector is given by

$$\begin{bmatrix} 3-5 & 1 & 4 \\ 0 & 2-5 & 6 \\ 0 & 0 & 5-5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 + 4x_3 = 0$$

$$-3x_2 + 6x_3 = 0$$

$$\frac{x_1}{6+12} = \frac{x_2}{0+12} = \frac{x_3}{6-0}$$

$$\frac{x_1}{18} = \frac{x_2}{12} = \frac{x_3}{6}$$

$$\frac{x_1}{3} = \frac{x_2}{2} = \frac{x_3}{1} = k$$

$$x_1 = 3k, x_2 = 2k, x_3 = k$$

Hence, $X_3 = \begin{bmatrix} 3k \\ 2k \\ k \end{bmatrix} = k \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ can be taken as an eigen vector of A corresponding to the eigen value $\lambda = 5$.

(ii) Given $N = 6.5 \times 10^{25}$ atoms/m³

Corresponding temperature = 300 K

$$\text{Susceptibility, } \chi_m = \frac{\mu_0 N M^2}{3KT} \quad \dots(i)$$

The magnetic moment of each atom

$$\begin{aligned} M &= n \left(\frac{eh}{4\lambda m} \right) \\ &= \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} \\ &= 9.24 \times 10^{-24} \text{ Am}^2 \end{aligned}$$

From equation, we get

$$\begin{aligned} \chi_m &= \frac{(4\pi \times 10^7) \times 6.5 \times 10^{25} \times (9.24 \times 10^{-24})^2}{3 \times 1.38 \times 10^{-23} \times 300} \\ &= 5.612 \times 10^{-7} \end{aligned}$$

Q.7 (a) (i) Solution:

The value of gain of feedback network, β can be obtained using the basic feedback relation,

$$A_{FB} = 100 \text{ and } A = 200$$

We have,
$$100 = \frac{200}{1 + 200 \times \beta}$$

$\Rightarrow 1 + 200 \beta = 2$

or
$$\beta = \frac{1}{200}$$

Further, the value of β is CE amplifier circuit is given as

$$\beta = \frac{R_E}{R_C}$$

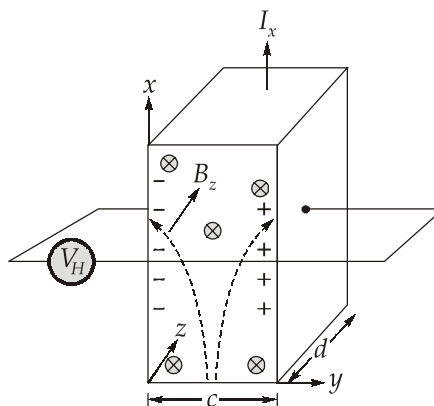
or
$$R_E = \beta R_C = \frac{6 \times 1000}{200} = 30 \Omega$$

So,
$$R_E = 30 \Omega$$

Q.7 (a) (ii) Solution:

Hall effect is observed when a potential difference (Hall voltage) is generated across an electric material, that is transverse to an electric current in the material and to an applied magnetic field perpendicular to current.

In given specimen when magnetic field is imposed in positive z-direction, the resulting force brought to bear on the charge carrier will cause them to be deflected in the y-direction (for holes in right direction in specimen and for electrons to the left in specimen).



Where

V_H = Hall voltage

B_z = Magnetic field

I_x = Current (x-direction)

$$V_H = \frac{R_H I_x B_z}{d}$$

R_H is Hall coefficient, n is number of charge carrier.

$$R_H = \frac{1}{n|e|}$$

Electron mobility, $\mu_e = \frac{\sigma}{n|e|}$

or $\mu_e = |R_H|\sigma$

Q.7 (b) (i) Solution:

$$E_{dc} = \frac{3\sqrt{3}}{\pi} E_{\text{mph}} \cdot \cos \alpha - \frac{3\omega L_s}{\pi} I_d$$

Given, $E_{dc} = 360 \text{ V},$

$$E_{\text{mph}} = 400 \times \left(\frac{\sqrt{2}}{\sqrt{3}} \right)$$

$$\alpha = \frac{\pi}{4},$$

$$I_d = 10 \text{ A}$$

$$\therefore 360 = \frac{3\sqrt{3}}{\pi} \times 400 \left(\frac{\sqrt{2}}{\sqrt{3}} \right) \cos \left(\frac{\pi}{4} \right) - \frac{3 \times 2\pi \times 50 \times L_s \times 10}{\pi}$$

$$360 = 381.17 - 3000L_s$$

$$L_s = 7.3 \text{ mH}$$

Source inductance per phase will be 7.3 mH

$$E_{dc} = \frac{3\sqrt{3}}{\pi} E_{\text{mph}} \cos(\alpha + \mu) + \frac{3\omega L_s}{\pi} I_d$$

$$360 = \frac{3\sqrt{3}}{\pi} 400 \left(\frac{\sqrt{2}}{\sqrt{3}} \right) \cos(\alpha + \mu) + \frac{3 \times 2\pi \times 50 \times 7 \times 10^{-7} \times 10}{\pi}$$

$$\therefore \cos(\alpha + \mu) = 0.63$$

$$\alpha + \mu = 50.95$$

$$\mu = 50.95 - 45^\circ$$

$$\mu \approx 6^\circ$$

Q.7 (b) Solution:

(ii) Given $\phi = 2.4 \times 10^{-5}$ Wb and $A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$

The magnetic flux density is given by

$$B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2 \text{ NA}^{-1} \text{ m}^{-1}$$

The permeability is given by

$$\mu = \frac{B}{H}$$

$$\mu = \frac{1.2}{500} = 2.4 \times 10^{-3} \text{ N/m}$$

The susceptibility is given by

$$\chi_m = \frac{\mu}{\mu_0} - 1 = \frac{2.4 \times 10^{-3}}{4 \times 3.14 \times 10^{-7}} - 1$$

$$= 1910 - 1 = 1909$$

Q.7 (c) (i) Solution:

The dc output voltage of the bridge,

$$V_0 = \frac{3V_{mL-L}}{\pi} \cos \alpha = \frac{3\sqrt{2}(4160)}{\pi} \cos 120^\circ = -2809 \text{ V}$$

The average output current,

$$I_0 = \frac{V_0 + V_{dc}}{R} = \frac{-2809 + 3000}{2} = 95.5 \text{ A}$$

The power absorbed by the bridge and transferred back to the ac system is

$$P_{ac} = -I_0 V_0 = (-95.5)(-2809) = 268.3 \text{ kW}$$

Power supplied by the dc source is

$$P_{dc} = I_0 V_{dc} = (95.5)(3000) = 286.5 \text{ kW}$$

Power absorbed by the resistance is

$$P_R = I_{\text{rms}}^2 R = (95.5)^2 (2) = 18.2 \text{ kW}$$

Variation in load current is due to the ac terms in the Fourier series. The load current amplitudes for each of the ac terms is

$$I_n = \frac{V_n}{Z_n}$$

Since the decreasing amplitude of the voltage terms and the increasing magnitude of the impedance both contribute to diminishing ac currents as n increase the peak to peak current variation will be estimated from the first ac term.

$$\text{For } n = 6, \quad V_6 = 0.28(4160\sqrt{2}) = 1650 \text{ V}$$

The peak to peak variation of 10 percent corresponding to a zero to peak amplitude $0.05(95.5) = 4.8 \text{ A}$

$$Z_6 = \frac{V_6}{I_6} = \frac{1650}{4.8} = 343 \Omega$$

$$L \approx \frac{Z_6}{6\omega_0} = \frac{343}{6(377)} = 0.15 \text{ H}$$

Q.7 (c) (ii) Solution:

We know that

$$H_C(T) = H_C(0) \left[1 - \left(\frac{T}{T_C} \right)^2 \right] \quad \dots(i)$$

Given,

$$\text{At } T = 14 \text{ K, } H_C(T) = 0.176 \text{ T}$$

and

$$\text{at } T = 13 \text{ K, } H_C(T) = 0.528 \text{ T}$$

$$0.176 = H_C(0) \left[1 - \left(\frac{14}{T_C} \right)^2 \right] \quad \dots(ii)$$

and

$$0.528 = H_C(0) \left[1 - \left(\frac{13}{T_C} \right)^2 \right] \quad \dots(iii)$$

Dividing equation (iii) by equation (ii), we get

$$\frac{1 - \left(\frac{13}{T_C} \right)^2}{1 - \left(\frac{14}{T_C} \right)^2} = \frac{0.528}{0.176} = 3$$

$$1 - \frac{169}{T_C^2} = 3 - \frac{588}{T_C^2}$$

$$\frac{419}{T_C^2} = 2$$

$$T_C = \left(\frac{419}{2} \right)^{1/2} = 14.5 \text{ K}$$

Substituting this value of T_C in equation (ii),

$$\begin{aligned} 0.176 &= H_C(0) \left[1 - \left(\frac{14}{14.5} \right) \right]^2 \\ &= H_C(0) [1 - (0.9655)^2] \\ H_C(0) &= \frac{0.176}{1 - 0.932} = 2.588 \text{ T} \end{aligned}$$

For $T = 4.2 \text{ K}$

$$\begin{aligned} H_C(T) &= 2.588 \left[1 - \left(\frac{4.2}{14.5} \right)^2 \right] = 2.58[1 - 0.0839] \\ &= 2.37 \text{ T} \end{aligned}$$

Q.8 (a) Solution:

Given :

$$V_l = 400 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$R = 10 \ \Omega, E = 350 \text{ V}, L \rightarrow \infty$$

(i) For $\alpha = 30^\circ$:

The average output voltage of rectifier is given as :

$$V_0 = \frac{3}{\pi} V_{ml} [\cos(60^\circ + \alpha) - \cos(120^\circ + \alpha)] \quad (\text{for } \alpha \leq 60^\circ)$$

$$V_0 = \frac{3}{\pi} \times 400\sqrt{2} [\cos 90^\circ - \cos 150^\circ]$$

$$V_0 = 467.81 \text{ Volt}$$

$$\therefore \text{Load current, } I_0 = \frac{V_0 - E}{R} = \frac{467.81 - 350}{10} = 11.78 \text{ A}$$

As the load current is constant and ripple free, therefore output power can be calculated as

$$P = EI_0 + I_0^2 R$$

$$P = 350 \times 11.78 + (11.78)^2 \times 10 = 5510.684 \text{ Watts}$$

Since each phase of supply is conducting for 240° in every full cycle of supply, hence RMS value of supply current,

$$I_{sr} = I_o \times \sqrt{\frac{240^\circ}{360^\circ}} = 11.78 \sqrt{\frac{2}{3}} = 9.6183 \text{ A}$$

Input power factor,

$$\cos \phi = \frac{P_o}{\sqrt{3}V_L \cdot I_{sr}} = \frac{5510.684}{\sqrt{3} \times 400 \times 9.6183} = 0.8269 \text{ lag}$$

(ii) For firing advance angle of 60° ,

$$\alpha = 180^\circ - 60^\circ = 120^\circ$$

Therefore,

$$V_o = \frac{3}{\pi} V_{ml} \cos \alpha = \frac{3}{\pi} \times 400 \sqrt{2} \cos(120^\circ)$$

$$V_o = -270.10 \text{ V}$$

As V_o is negative, this converter is operating as line commutated inverter. The polarity of load emf E must therefore be reversed.

Now,

$$V_o = -E + I_o R$$

$$-270.10 = -350 + I_o R$$

\therefore Load current,

$$I_o = \frac{350 - 270.10}{10} = 7.99 \text{ A}$$

RMS value of load current,

$$I_{or} = I_o = 7.99 \text{ A}$$

Power delivered by the battery to the ac source through the line commutated inverter,

$$P_o = EI_o - I_{or}^2 R$$

$$= 350 \times 7.99 - (7.99)^2 \times 10$$

$$P_o = 2158 \text{ Watts}$$

RMS value of source current,

$$I_{sr} = I_o \sqrt{\frac{2}{3}}$$

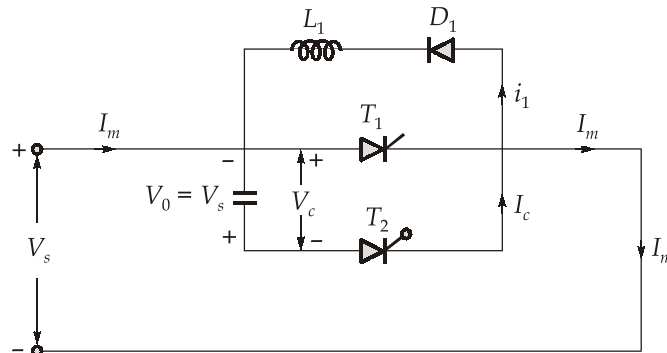
$$I_{sr} = 7.99 \sqrt{\frac{2}{3}} = 6.52 \text{ A}$$

Input power factor = $\frac{P_o}{\sqrt{3}V_L I_{sr}}$

$$\cos \phi = \frac{2158}{\sqrt{3} \times 400 \times 6.52} = 0.477 \text{ lagging}$$

Q.8 (b) (i) Solution:

The equivalent circuit during the commutation period is shown in figure below :



Now, from the circuit

$$i_c = i_1 + I_m$$

$$v_c = \frac{1}{C} \int i_c dt + v_c(t=0)$$

$$= -L_1 \frac{di_1}{dt} = -L_1 \frac{di_c}{dt}$$

The initial conditions $i_{c(t=0)} = I_m$ and $v_{c(t=0)} = -V_o = -V_s$.

The solutions of these equations yields the capacitor current as

$$i_c = V_o \sqrt{\frac{C}{L_1}} \cdot \sin \omega_1 t + I_m \cos \omega_1 t$$

The voltage across the capacitor is expressed as

$$V_c(t) = I_m \sqrt{\frac{L_1}{C}} \cdot \sin \omega_1 t - V_o \cos \omega_1 t$$

where,

$$\omega_1 = \frac{1}{\sqrt{L_1 C}}$$

The available turn-off times or circuit turn-off times is obtained from the condition $V_c(t = t_q) = 0$ and is solved as

$$t_q = \sqrt{CL_1} \cdot \tan^{-1} \left(\frac{V_o}{I_m} \sqrt{\frac{C}{L_1}} \right)$$

Now for $C = 20 \mu\text{F}$, $L_1 = 25 \mu\text{H}$, $V_o = 200 \text{ V}$ and $I_m = 50 \text{ A}$.

$$\Rightarrow t_q = 29.03 \mu\text{s}$$

and for $C = 20 \mu\text{F}$, $L_1 = 25 \mu\text{H}$, $V_o = 200 \text{ V}$ and $I_m = 200 \text{ A}$

$$\Rightarrow t_q = 16.32 \mu\text{s}$$

Hence, as the load current increases from 50 A to 200 A, the turn off decreases from 29.03 μs to 16.32 μs . The use of extra diode makes the turn-off time loss dependent on the load.

Q.8 (b) (ii) Solution:

The differential gain of instrument amplifier can be given as

$$A_d = 1 + \frac{2R_2}{R_1}$$

For minimum differential voltage gain

$$A_d = A_{d,\min} = 5$$

$$R_1 = R_{1,\max} = 50 \text{ k}\Omega$$

$\therefore A_d$ will be minimum only when R_1 will be maximum.

Substituting $A_d = 5$, $R_1 = 50 \text{ k}\Omega$

$$5 = 1 + \frac{2R_2}{50}$$

$$\Rightarrow R_2 = 100 \text{ k}\Omega$$

For maximum differential voltage gain

$$A_d = A_{d,\max} = 200$$

Substitute $A_d = 200$, $R_2 = 100 \text{ k}\Omega$ in eqn. (1), we have

$$200 = 1 + \frac{2 \times 100}{R_1}$$

$$\Rightarrow R_1 = \frac{200 \text{ k}\Omega}{199} \simeq 1 \text{ k}\Omega$$

For maximum A_d , R_1 will have minimum value. Therefore,

$$R_{1(\min)} = 1 \text{ k}\Omega$$

Thus, $R_1 = 1$ to $50 \text{ k}\Omega$ potentiometer

$$R_2 = 100 \text{ k}\Omega$$

$$R_3 = R_4 = \text{say, } 15 \text{ k}\Omega \text{ each}$$

Q.8 (c) Solution:

The main advantage of a clapp oscillator is its high frequency stability. This is because the additional capacitance C_3 in the feedback network is not influenced by any parameter such as changes in supply voltage or change in temperature or changes in the transistor parameters and the frequency of oscillations is dependent on C_3 as the approximate frequency of oscillations is given by

$$f = \frac{1}{2\pi\sqrt{L_3 C_3}}$$

Given that,

$$C_1 = C_2 = 150 \text{ pF} = 150 \times 10^{-12}$$

$$L_3 = 50 \text{ }\mu\text{H} = 50 \times 10^{-6}$$

$$C_S = 10 \text{ pF} = 10 \times 10^{-12}$$

Frequency of oscillations for a colpitt's oscillator

$$f = \frac{1}{2\pi\sqrt{CL_3}}$$

Where,

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{150 \times 150}{300} = 75 \text{ pF}$$

∴

$$f = \frac{1}{2\pi\sqrt{75 \times 10^{-12} \times 50 \times 10^{-6}}} = 2.598$$

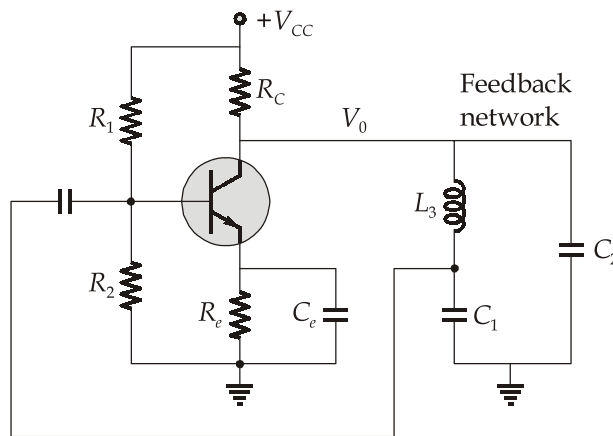
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_S} = \frac{1}{150} + \frac{1}{150} + \frac{1}{10}$$

$$C_{eq} = 8.82 \text{ pF}$$

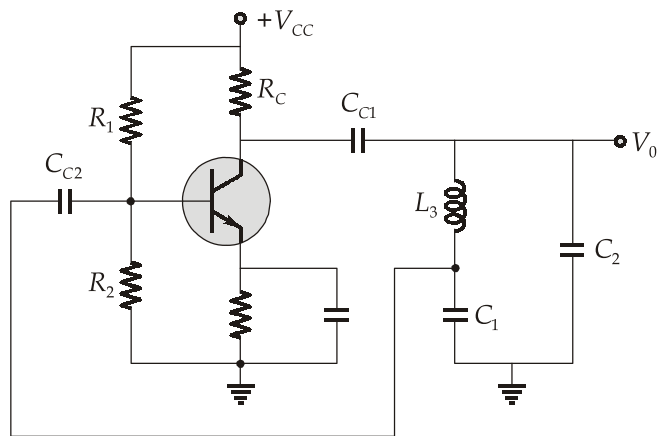
Frequency of oscillations for a clapp oscillator

$$f = \frac{1}{2\pi\sqrt{C_{eq} L_3}} ;$$

$$f = \frac{1}{2\pi\sqrt{8.82 \times 10^{-12} \times 50 \times 10^{-6}}} = 7.58 \text{ MHz}$$



Colpitt's oscillator



Clapp oscillator

