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Detailed Solutions

ESE-2026
Mains Test Series

Mechanical Engineering
Test No : 7

Section A : Heat Transfer + Power Plant
Section B : Renewable Sources of Energy-1 + Industrial & Maintenance Engineering-1
Production Engineering and Material Science-2

Section A : Heat Transfer + Power Plant

1. (a) Solution:

Desirable characteristics of working fluid in vapour-power cycle.

(i) High critical temperature

(ii) Large enthalpy of vaporization.

(iii) Saturation pressure at the temperature of heat rejection should be above the atmospheric pressure so as to avoid the necessity of maintaining vacuum in the condenser.

(iv) The specific heat of liquid should be small so that little heat transfer is required to raise the liquid state to boiling.

(v) Saturated vapour-line of T-s diagram should be steep very close to turbine expansion process so that excessive moisture does not appear during expansion.

(vi) The freezing-point of the fluid should be below room-temperature.

(vii) Working fluid be chemically-stable, non-corrosive & non-toxic.

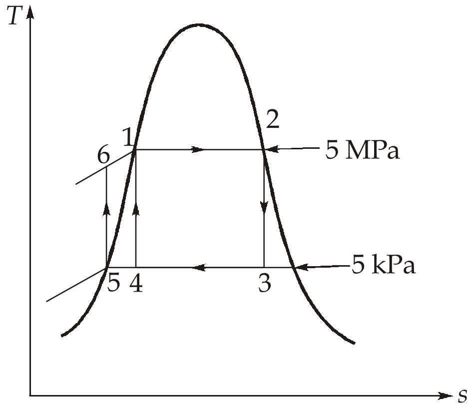
Important Properties of Coal for Power plant Application

1. **Swelling Index:** Qualitative evaluation method to determine the extent of caking of a coal is called swelling index. High value of swelling-index indicates that the coal expands in volume during combustion.

2. **Grindability:** It is measured by standard-grindability-index and is inversely proportional to power-required to grind the coal to a specific particle size. Grindability of standard coal is defined as 100, and the coal selected for the use in power plant is 50.
3. **Weatherability :** It is measure of how well coal can be stockpiled for long period of time without crumbling to pieces.
4. **Sulphur Content :** Sulhur content in coal is combustible and generates same energy by its oxidation to SO_2 is most major source of atmospheric pollution. The operating cost of sulphur removal equipment needed to be considered while selecting coal.
5. **Calorific value or Heating value :** Calorific-Value is the property of fundamental importance. It is the heat transferred when the products of complete combustion of a sample of coal (or other fuel) are cooled to the initial temperature of air and fuel. It is determined in a standard test in a bomb-calorimeter.
6. **Ash softening Temperature:** The temperature at which ash-softens and becomes plastic. This is somewhat below the melting point of ash. The design of the steam-generator greatly depends on ash-softening temperature of the coal.

1. **(b) Solution:**

Carnot cycle and Rankine cycle on same T-s coordinate are shown below:



Carnot cycle: 1-2-3-4-1

Rankine cycle: 1-2-3-5-6-1

As process 2-3 is isentropic, So $s_2 = s_3$

and $s_3 = s_{f,\,5kPa} + x_3 \times s_{fg,\,5kPa} = s_2 = s_{g,\,5MPa}$

$x_3 = 0.694$

Hence enthalpy at 3, $h_3 = h_{f,\,5kPa} + x_3 \times h_{fg,\,5kPa}$

$h_3 = 1819.85 \text{ kJ/kg}$

Enthalpy at 2, $h_2 = h_{g,\,5\text{MPa}} = 2794.3$ kJ/kg

Process 1-4 is isentropic, so $s_1 = s_4$

$$s_1 = 2.92 = 0.4764 + x_4 \times (8.3951 - 0.4764)$$

$$x_4 = 0.31$$

Enthalpy at 4, $h_4 = 137.82 + [0.31 \times (2561.5 - 137.82)]$

$$h_4 = 889.16 \text{ kJ/kg}$$

Enthalpy at 1, $h_1 = h_{f,\text{ at }5\text{MPa}}$

$$h_1 = 1154.23 \text{ kJ/kg}$$

Carnot cycle (1-2-3-4-1) efficiency:

$$\eta_{\text{Carnot}} = \frac{\text{Net work}}{\text{Heat added}} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

$$\eta_{\text{Carnot}} = \frac{[(2794.3 - 1819.85) - (1154.23 - 889.16)]}{(2794.3 - 1154.23)} = 0.4325$$

$\eta_{\text{Carnot}} = 43.25\%$ **Ans. (a)**

In Rankine cycle, 1-2-3-5-6-1

Pump work, $h_6 - h_5 = v_{f,5}(p_6 - p_5) = 0.001005\,(5000 - 5) = 5.02$ kJ/kg

$$h_5 = h_{f\text{ at }5\text{kPa}} = 137.82 \text{ kJ/kg}$$

Hence, $h_6 = 137.82 + 5.02 = 142.84$ kJ/kg

$$h_6 = 142.84 \text{ kJ/kg}$$

Net work in Rankine cycle $= (h_2 - h_3) - (h_6 - h_5)$

$= 974.45 - 5.02 = 969.43$ kJ/kg

Heat added $= h_2 - h_6 = 2794.3 - 142.84$

$= 2651.46$ kJ/kg

Rankine cycle efficiency $= \dfrac{969.43}{2651.46}$

$\eta_{\text{Rankine}} = 0.3656$ or 36.56% **Ans. (b)**

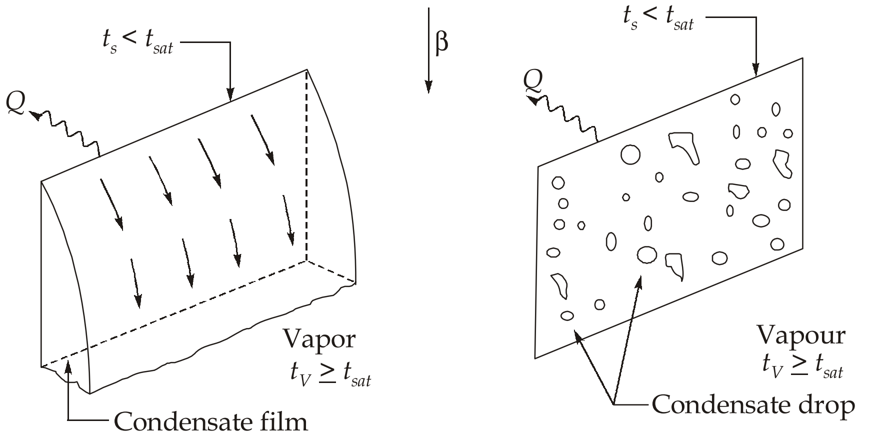
1. **(c) Solution:**

Depending upon the behaviour of condensate upon the cooled surface, the condensation process has been categorised into the following distinct modes:

(i) **Film condensation :** The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface. The liquid flows down the cooling surface under the action of gravity and the layer continuously grows in thickness because of newly condensing vapours. The continuous film offers thermal resistance and restricts further transfer of heat between the vapour and the surface.

Film condensation usually occurs when a vapour, relatively free from impurities, is allowed to condense on a clear surface.

(ii) **Dropwise condensation :** The liquid condensate collects in droplets and does not wet the solid cooling surface. The droplets develop in cracks and pits on the surface grow in size, break away from the surface, knock off other droplets and eventually run off the surface without forming a film. A part of the condensation surface is directly exposed to the vapour without an insulating film of condensate liquid. Evidently, there is no film barrier to heat flow and higher heat transfer rates are experienced. Heat transfer fluxes of the order of 750 kW/m^2 have been obtained with dropwise condensation.



(a) Film condensate

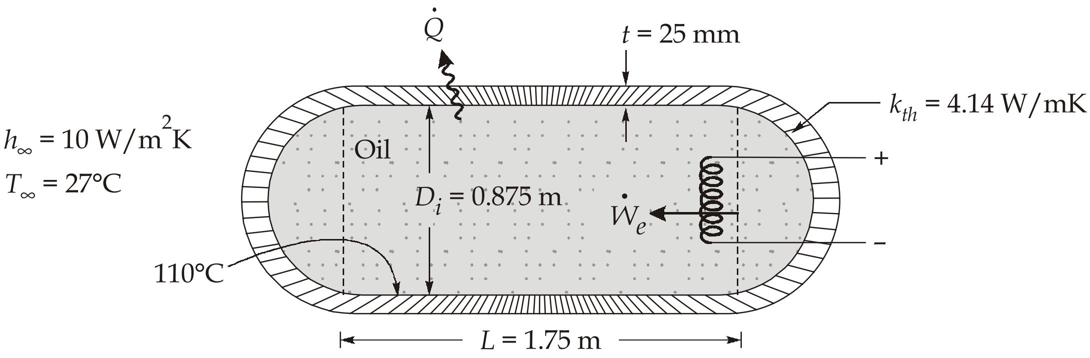
(b) Dropwise condensation

Film and dropwise condensation on a surface

Dropwise condensation has been observed to occur either on highly polished surfaces, or on surfaces contaminated with impurities like fatty acids and organic compounds. Dropwise condensation gives coefficient of heat transfer generally five to ten times larger than with film condensation. Because of potential performance gain, dropwise condensation is provoked artificially by surface coatings called promoters that inhibit wetting. Silicons, teflons and an assortment of waxes and fatty acids are often used for this purpose. These substances are either applied to the heat transfer surface or introduced into the vapour. However, the phenomenon is highly unstable as these coatings gradually lose their effectiveness due to oxidation, fouling or outright removal and the surfaces become wetted when exposed to condensing vapour over an extended length of time. Consequently film condensation is generally encountered in industrial applications and is usually planned for condenser design calculations.

1. (d) Solution:

Given data :



Hot oil storage tank

$$R_i = \frac{D_i}{2} = \frac{0.875}{2} = 0.4375 \text{ m}$$

$$R_o = R_i + t = 0.4375 + 0.025 = 0.4625 \text{ m}$$

$$R_o - R_i = t = 0.025 \text{ m}$$

Assumptions:

1. Steady-state radial conduction.
2. Radiation effects are neglected.
3. Variation of thermal conductivity with temperature is neglected.

For steady state heat transfer

$$\left(\frac{dE}{dt}\right)_{cv} = 0$$

$$\Rightarrow \qquad \dot{E}_{in} = \dot{E}_{out}$$

$$\Rightarrow \qquad \dot{W}_e = \dot{Q}$$

where; $\dot{W}_e$ is the electrical power consumed to maintain the given situation.

$$\dot{Q} = \dot{Q}_{cyl} + \dot{Q}_{sph}$$

$$R_{th,cyl} = \frac{\ln\left(\frac{R_o}{R_i}\right)}{2\pi k_{th}.L};\; R_{th,sph} = \frac{R_o - R_i}{4\pi k_{th} R_o R_i}$$

$$\dot{Q}_{cyl} = \frac{\Delta T}{R_{th,cyl} + \frac{1}{h(A_0)_{cyl}}}$$

$$\dot{Q}_{cyl} = \frac{(110 - 27)}{\frac{\ln\left(\frac{0.4625}{0.4375}\right)}{2\times\pi\times 4.14\times 1.75} + \frac{1}{10\times 2\times\pi\times 0.4625\times 1.75}}$$

$$\dot{Q}_{cyl} = 3974.208 \text{ Watt}$$

$$\dot{Q}_{sph} = \frac{\Delta T}{R_{th,sph} + \frac{1}{h(A_0)_{sph}}}$$

$$= \frac{110 - 27}{\frac{0.4625 - 0.4375}{4\times\pi\times 4.14\times 0.4625\times 0.4375} + \frac{1}{10\times 4\times\pi\times (0.4625)^2}}$$

$$= 2097.183 \text{ Watt}$$

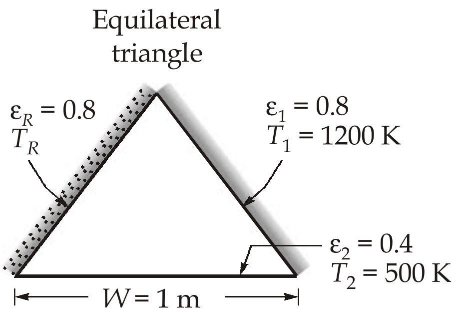
$$\dot{Q} = \dot{Q}_{cyl} + \dot{Q}_{sph} = 3794.208 + 2097.183$$

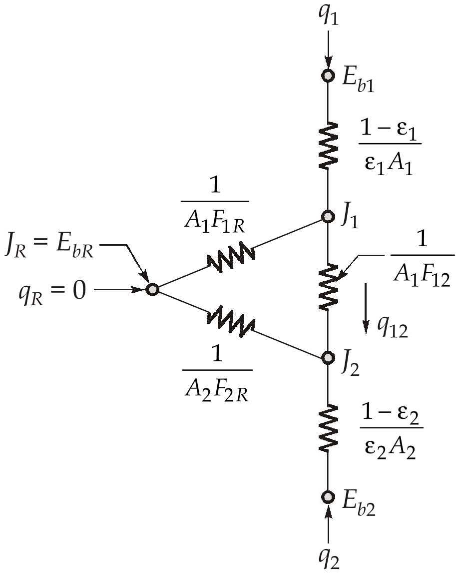
$$= 6071.39 \text{ Watt or } 6.07 \text{ kW}$$

1. (e) Solution:

Assumptions:

1. Steady-state conditions exist.
2. All surfaces are opaque, diffuse, gray, and of uniform radiosity.
3. Convection effects are negligible.
4. Surface R is reradiating.
5. End effects are negligible.





The rate at which energy must be supplied to the heated surface is

$$q_1 = \frac{E_{b_1} - E_{b_2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12} + \left[\left(\frac{1}{A_1F_{1R}}\right) + \left(\frac{1}{A_2F_{2R}}\right)\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

From symmetry, $F_{12} = F_{1R} = F_{2R} = 0.5$

Also, $A_1 = A_2 = W \cdot L$, where L is the duct length.

The heat transfer to the duct per unit length as

$$q_1' = \frac{q_1}{L} = \frac{5.67 \times 10^{-8} \times \left(1200^4 - 500^4\right)}{\frac{1-0.8}{0.8 \times 1} + \frac{1}{1 \times 0.5 + (2+2)^{-1}} + \frac{1-0.4}{0.4 \times 1}}$$

$$q_1' = 36.982 \text{ kW/m}$$ **Ans.**

Applying the surface energy balance, to surface 1 and 2, and recognizing from the network that $q_2' = -q_1'$,

$$J_1 = E_{b_1} - \frac{1-\varepsilon_1}{\varepsilon_1 W} q_1'$$

$$= 5.67 \times 10^{-8} (1200)^4 - \frac{1-0.8}{0.8 \times 1} \times 36982 = 108327.62 \text{ W/m}^2$$

$$J_2 = E_{b_2} - \frac{1-\varepsilon_2}{\varepsilon_2 W} q_2'$$

$$= 5.67 \times 10^{-8} (500)^4 - \frac{1-0.4}{0.4 \times 1} \times (-36982) = 59016.75 \text{ W/m}^2$$

From the energy balance for the reradiating surface,

We get, $$\frac{108327.62 - J_R}{\frac{1}{W \times L \times 0.5}} - \frac{J_R - 59016.75}{\frac{1}{W \times L \times 0.5}} = 0$$

Hence, the radiosity of the reradiating surface is

$$J_R = 83672.19 \text{ W/m}^2$$

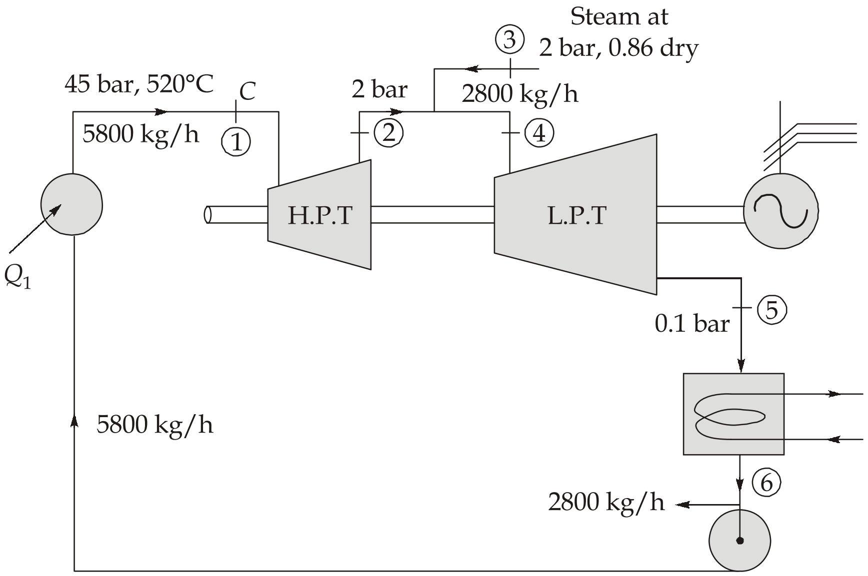
Also, $J_R = E_{bR} = \sigma T_R^4$ for the reradiating surface, its temperature is

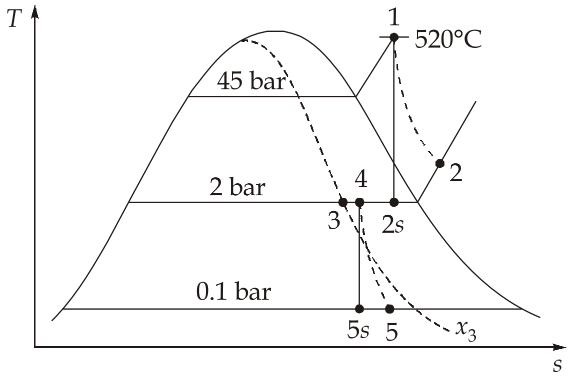
$$T_R = \left(\frac{J_R}{\sigma}\right)^{1/4} = \left(\frac{83672.19}{5.67 \times 10^{-8}}\right)^{1/4} = 1102.2 \text{ K}$$

2. (a) Solution:

Given data : $T_1 = 520°\text{C}$; $P_1 = 45$ bar; $P_2 = 2$ bar; $P_5 = 0.1$ bar

$$\dot{m}_1 = 5800 \text{ kg/h}; m_4 = (2800 + 5800) \text{ kg/h} = 8600 \text{ kg/h}$$





From steam table, at 45 bar

$$s_1 = 7.0912 \text{ kJ/kgK}$$

$$h_1 = 3486.5 \text{ kJ/kg}$$

At 2 bar,

$$s_{fg} = 5.5967 \text{ kJ/kgK}$$

$$s_f = 1.5302 \text{ kJ/kgK}$$

$$h_{fg} = 2201.5 \text{ kJ/kg}$$

$$h_f = 504.7 \text{ kJ/kg}$$

At 0.1 bar,

$$s_{fg} = 7.4996 \text{ kJ/kgK}$$

$$s_f = 0.6492 \text{ kJ/kgK}$$

$$h_{fg} = 2392.1 \text{ kJ/kg}$$

$$h_f = 191.81 \text{ kJ/kg}$$

For 1 - 2s,

$$s_1 = s_{2s} = s_f + x_2 s_{fg}$$

$$7.0912 = 1.5302 + x_2 \times 5.5967$$

$$x_2 = 0.9936$$

$$h_{2s} = h_f + x_2 h_{fg}$$

$$h_{2s} = 504.7 + 0.9936 \times 2201.5$$

$$= 2692.15 \text{ kJ/kg}$$

$$h_1 - h_2 = \eta_{HPT} \times (h_1 - h_{2s})$$

$$h_1 - h_2 = 0.85 \times (3486.5 - 2692.15)$$

$$h_1 - h_2 = 675.19 \text{ kJ/kg}$$

$$h_2 = 3486.5 - 675.23$$

$$= 2811.308 \text{ kJ/kg}$$

Now,

$$h_3 = h_f + x_3 h_{fg}$$

$$= 504.7 + 0.86 \times 2201.5$$

$$= 2397.99 \text{ kJ/kg}$$

For mixing, $m_2h_2 + m_3h_3 = (m_2 + m_3)h_4$

$$\frac{5800 \times 2811.308 + 2800 \times 2397.99}{5800 + 2800} = h_4$$

$$h_4 = 2676.739 \text{ kJ/kg} = h_f + x_4 h_{fg}$$

$$2676.739 = 504.7 + x_4 \times 2201.5$$

$$x_4 = 0.9866$$

$$s_4 = 1.5302 + 0.9866 \times 5.5967$$

$$= 7.0519 \text{ kJ/kgK}$$

$$s_4 = s_{5s} = s_f + x_5\, s_{fg}$$

$$7.0519 = 0.6492 + x_5 \times 7.4996$$

$$x_5 = 0.8537$$

$$h_{5s} = h_f + x_5\, h_{fg}$$

$$= 191.81 + 0.8537 \times 2392.1$$

$$= 2234.07 \text{ kJ/kg}$$

$$h_4 - h_5 = \eta_{\text{LPT}} \times (h_4 - h_{5s})$$

$$h_4 - h_5 = 0.80 \times (2676.739 - 2234.07)$$

$$h_4 - h_5 = 354.129 \text{ kJ/kg}$$

$$h_5 = 2322.61 \text{ kJ/kg}$$

$$h_6 = 191.81 \text{ kJ/kg}$$

Power output $= \dot{m}_1(h_1 - h_2) + \dot{m}_4(h_4 - h_5)$

$$\dot{W} = \frac{5800}{3600} \times (675.19) + \frac{8600}{3600} \times (354.129)$$

$$\dot{W} = 1933.78 \text{ kW}$$ **Ans.**

Heat added, $\dot{Q}_1 = \dot{m}_1 \times (h_1 - h_6)$

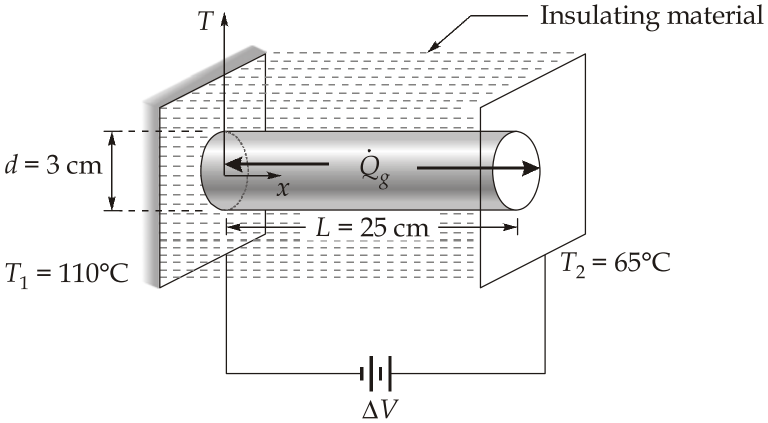
$$\dot{Q}_1 = \frac{5800}{3600} \times (3486.5 - 191.81) = 5308.11 \text{ kW}$$

Thermal efficiency, $\eta_{\text{cycle}} = \dfrac{\dot{W}}{\dot{Q}_1} = \dfrac{1933.78}{5308.11}$

$$\eta_{\text{cycle}} = 0.3643 \text{ or } 36.43\%$$ **Ans.**

2. (b) Solution:

Given data : $\dot{Q}_g = 0.17$ W; $k_{th} = 40$ W/m°C



Assumptions:

1. Perfect thermal insulation.
2. Steady state, one dimensional heat conduction.
3. Uniform heat generation.

Heat generated per unit volume

$$\dot{q}_g = \frac{\dot{Q}_g}{\text{Volume}} = \frac{0.17}{\pi\left(\frac{0.03}{2}\right)^2 \times 0.25} = 962.00321 \text{ W/m}^3$$

Heat conduction equation for rod.

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_g}{k} = 0$$

On integrating,

$$\frac{dT}{dx} + \frac{\dot{q}_g}{k}x = c_1 \qquad \text{...(i)}$$

Again integrating,

$$T + \frac{\dot{q}_g}{k}\left(\frac{x^2}{2}\right) = c_1x + c_2 \qquad \text{(General temperature profile)} \qquad \text{...(ii)}$$

Using Boundary conditions

(i) At $x = 0$, $T = T_1 = 110°C$

$$\Rightarrow \qquad c_2 = T_1 = 110°\text{C}$$

(ii) At x = L, $T = T_2 = 65°\text{C}$

$$\Rightarrow \qquad T_2 + \frac{\dot{q}_g}{k}\left(\frac{L^2}{2}\right) = c_1 L + T_1$$

$$\Rightarrow \qquad c_1 = \left(\frac{T_2 - T_1}{L}\right) + \frac{\dot{q}_g L}{2k} = -176.9937 \text{ °C/m}$$

Now, temperature distribution along the rod.

$$T + \frac{\dot{q}_g}{k}\left(\frac{x^2}{2}\right) = \left[\left(\frac{T_2 - T_1}{L}\right) + \frac{\dot{q}_g L}{2k}\right] x + T_1$$

Maximum temperature occurs where $\frac{dT}{dx} = 0$

$$\frac{dT}{dx} = c_1 - \frac{\dot{q}_g}{k} \cdot x; \qquad \text{from (i)}$$

$$c_1 = \frac{\dot{q}_g}{k} \cdot x$$

At $x = \frac{c_1 k}{\dot{q}_g}; T = T_{\max}$

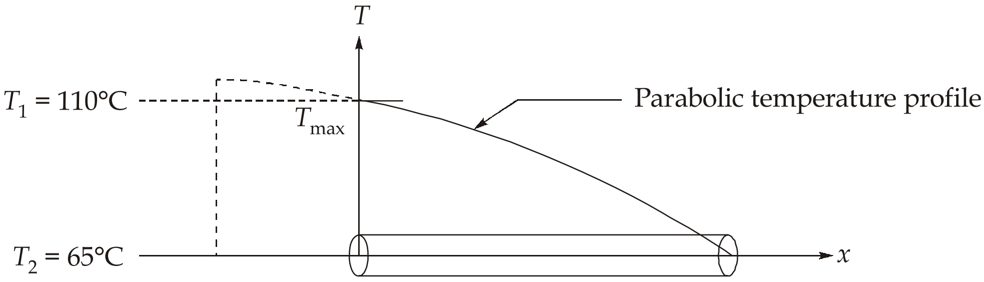
At $x = \left(\frac{T_2 - T_1}{L}\right)\frac{k}{\dot{q}_g} + \frac{L}{2}$

$$x = \left(\frac{65 - 110}{0.25}\right)\frac{40}{962} + \frac{0.25}{2}$$

$$= -7.359 \text{ m} \notin [0 \text{ m}, \ 0.25 \text{ m}]$$

Hence, at $x = 0$ m

$$T = T_{\max} = 110°\text{C}$$ **Ans.**



(ii)

On putting the values of c_1 and c_2 in equation (i)

$$\frac{dT}{dx} = -176.9937 - (24.05)x$$

$$\text{Heat flux} = \frac{\dot{Q}}{A} = q''$$

$$q''\Big|_{x=0\text{m}} = \frac{\dot{Q}\Big|_{x=0\text{m}}}{A} = -k\frac{dT}{dx}\Big|_{x=0}$$

$$= -40(-176.9937) = 7079.748 \text{ W/m}^2$$

$$q''\Big|_{x=0.25\text{m}} = \frac{\dot{Q}\Big|_{x=0\text{m}}}{A} = -k\frac{dT}{dx}\Big|_{x=0.25}$$

$$= -40(-176.9937 - 24.05(0.25)) = 7320.248 \text{ W/m}^2$$ **Ans.**

$$\dot{Q}_{gen} = 0.17 \text{ W}$$

$$Q_{x\,=\,0\text{ m}} = q''_{x=0\text{m}} \times A = 5.0044 \text{ W}$$

$$Q_{x\,=\,0.25\text{ m}} = q''_{x=0.25\text{m}} \times A = 5.1744 \text{ W}$$

Checking from energy balance

$$Q_2 = Q_1 + \dot{Q}_g$$

$$\dot{Q}_g = (Q_2 - Q_1) = 5.1744 - 5.0044 = 0.17 \text{ W}$$

which comes out same as provided in problem.

2. (c) Solution:

Let, us consider 100 kg of fuel oil, let x moles of oxygen are supplied for combustion.

$$\frac{84}{12}C + \frac{10}{2}H_2 + \frac{3.2}{32}S + \frac{1.6}{32}O_2 + xO_2 + 3.76xN_2 \rightarrow bCO_2 + dSO_2 + eO_2 + fN_2 + gH_2O$$

Equating coefficients,

Carbon : $\frac{84}{12} = b \quad \Rightarrow \quad b = 7$

Hydrogen : $$\frac{10}{2} = g \Rightarrow g = 5$$

Oxygen : $$\frac{1.6}{32} + x = b + d + e + \frac{g}{2}$$

Sulphur : $$\frac{3.2}{32} = d \Rightarrow d = 0.1$$

Nitrogen : $$3.76x = f$$

Volume analysis,

$$\frac{b+d}{b+d+e+f} = 0.1572, \quad \frac{e}{b+d+e+f} = 0.01$$

On dividing $$\frac{b+d}{e} = 15.72$$

$$\frac{7+0.1}{15.72} = e$$

$$e = 0.452$$

Substituting in O_2-balance

$$0.05 + x = 7 + 0.1 + 0.452 + 2.5$$

$$x = 10.002$$

Mass of oxygen supplied = $10.002 \times 32 = 320.06$ kg

∴ Mass of air supplied for 100 kg fuel = $\frac{320.06}{0.232} = 1379.58$ kg

∴ Mass of air supplied per kg fuel (m_A) = 13.79 kg

$$m_{\text{th}} = 11.5C + 34.5\left(H - \frac{O}{8}\right) + 4.3S$$

$$= 11.5 \times 0.84 + 34.5\left(0.1 - \frac{0.016}{8}\right) + 4.3 \times 0.032$$

$$= 9.66 + 3.38 + 0.14$$

$$= 13.18 \text{ kg}$$

Percentage excess air $$= \frac{m_A - m_{th}}{m_{th}} \times 100$$

$$= \frac{13.79 - 13.18}{13.18} \times 100 = 4.63\%$$

Mass of dry flue gas formed for 100 kg fuel = $bCO_2 + dSO_2 + eO_2 + fN_2$

$$= 7 \times 44 + 0.1 \times 64 + 0.452 \times 32 + 3.76 + 10.002 \times 28$$

$$= 1381.87 \text{ kg}$$

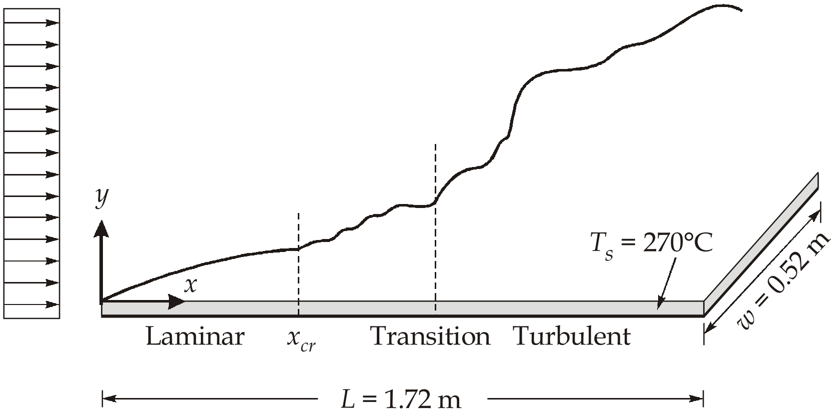
Mass of dry flue gas formed per kg fuel = 13.82 kg

Mass of water vapour formed per kg fuel,

$$g \times H_2O = \frac{5 \times 18}{100} = 0.9 \text{ kg}$$

3. (a) Solution:

Given data : $u_\infty = 50$ m/s; $T_\infty = 27°\text{C}$



Thermophysical properties of air

$\rho = 0.8$ kg/m^3; $\mu = 24 \times 10^{-6}$ kg/ms; $c_p = 1000$ J/kg°C; $k = 35 \times 10^{-3}$ W/m°C

Assumptions:

1. Thermophysical properties are independent of temperature.
2. Effects of radiation are neglected.
3. $Re_{,cr} = 5 \times 10^5$

(i)

$$Pr = \frac{\mu c_p}{k} = \frac{(24 \times 10^{-6}) \times 1000}{35 \times 10^{-3}} \simeq 0.6857 \in (0.6, 60)$$

$$Re_{,L} = \frac{\rho u_\infty L}{\mu} = \frac{0.8 \times 50 \times 1.72}{24 \times 10^{-6}} \simeq 2.8667 \times 10^6$$

$$Re_{,L} = 28.667 \times 10^5 > Re_{,cr} = 5 \times 10^5$$

lies between [5×10^5 and 1000×10^5] it implies that the flow is combined laminar and turbulent or transition flow.

Average combined Nusselt number $(\overline{Nu})$

$$(\overline{Nu}) = \left(0.037\,\text{Re}_L^{0.8} - 871\right)(\text{Pr})^{1/3};$$

$$(\overline{Nu}) = \left(0.037(2.8667\times10^6)^{0.8} - 871\right)(0.6857)^{1/3}$$

$$(\overline{Nu}) = 4012.5285$$

$$\frac{\bar{h}L_c}{k} = 4012.5285$$

$$\bar{h} = \frac{(4012.5285)\times35\times10^{-3}}{1.72} \simeq 81.6503\ \text{W/m}^2\,°\text{C}$$

Heat transferred from one side of the plate.

$$\dot{Q} = \bar{h}\cdot A\,(\Delta T)$$

$$= (81.6503)(0.52\times1.72)(270-27)$$

$$= 17745.8085\ \text{W}$$

$$= 17.7458\ \text{kW}$$

Ans.

Average combined skin friction or drag coefficient,

$$\overline{c_f} = \frac{0.074}{\text{Re}_{,L}^{0.2}} - \frac{1742}{\text{Re}_{,L}}$$

$$= \frac{0.074}{(28.667\times10^5)^{1/5}} - \frac{1742}{(28.667\times10^5)}$$

$$= 3.174\times10^{-3}$$

Drag force per side of the plate is

$$F_D = \left[\overline{c_f}\left(\frac{1}{2}\rho u_\infty^2\right)\right](\text{Plate area})$$

$$= \left(3.174\times10^{-3}\right)\left(\frac{1}{2}\times0.8\times(50)^2\right)(0.52\times1.72)$$

$$F_D = 2.839\ \text{N}$$

Ans.

(ii) For laminar portion

We know,
$$x_{cr} = \frac{\text{Re}_{,cr}}{\left(\frac{\rho u_\infty}{\mu}\right)} = \frac{5\times10^5}{\left(\frac{0.8\times50}{(24\times10^{-6})}\right)} = \frac{3}{10}\ \text{m}$$

$$x_{cr} = 0.3 \text{ m}$$

Also; we know for laminar flow over flat plate

$$\overline{Nu} = 0.664\,\text{Re}_{,cr}^{0.5}\,\text{Pr}^{1/3} = 414.0293$$

$$\overline{c_f} = \frac{1.328}{\text{Re}_{,cr}^{1/2}} = 1.8781 \times 10^{-3}$$

So;

$$\bar{h} = \frac{\overline{Nu} \cdot k}{x_{cr}} = 48.3034 \text{ W/m}^2\text{°C}$$

$$\dot{Q}_{\text{Laminar}} = \bar{h}_{\text{laminar}} \cdot A_{\text{laminar}} \cdot \Delta T$$

$$= 48.3034 \times (0.52 \times 0.3) \times (270 - 27)$$

$$= 1831.0859 \text{ W}$$

$$F_{D,\text{laminar}} = \left[\overline{c_f}_{\text{ laminar}} \cdot \left(\frac{1}{2}\rho u_\infty^2\right)\right] A_{\text{laminar}}$$

$$= 1.8781 \times 10^{-3} \times \left(\frac{1}{2} \times 0.8 \times 50^2\right) \times 0.52 \times 0.3$$

$$= 0.29298 \text{ N}$$

3. (b) Solution:

Given data :

Inlet, $T_1 = 500°\text{C};\ P_1 = 45 \text{ bar}$

Exit, $P_2 = 0.10 \text{ bar}$

From stable table,

Inlet, $h_1 = 3440.4 \text{ kJ/kg},\ s_1 = 7.0323 \text{ kJ/kgK}$

Exit,

at 0.10 bar $s_f = 0.6492 \text{ kJ/kgK}$

$$s_g = 8.1488 \text{ kJ/kgK}$$

$$h_f = 191.81 \text{ kJ/kg}$$

$$h_g = 2583.9 \text{ kJ/kg}$$

$\because$ Expansion is isentropic

$$s_1 = s_2 = s_f + x_2\,(s_g - s_f)$$

$$7.0323 = 0.6492 + x_2 \times (8.1488 - 0.6492)$$

$$x_2 = 0.8511$$

$$h_2 = h_f + x_2\,(h_g - h_f)$$

$$h_2 = 191.81 + 0.8511 \times (2583.9 - 191.81)$$

$$h_2 = 2227.717 \text{ kJ/kg}$$

$$\Delta h = h_1 - h_2$$

$$= 3440.4 - 2227.17$$

$$= 1212.68 \text{ kJ/kg}$$

(i) Single impulse stage,

$$\frac{V_1^2}{2000} = \Delta h$$

$$\frac{V_1^2}{2000} = 1212.68$$

$$\Rightarrow \quad V_1 = 1557.358 \text{ m/s}$$

$$\frac{u}{V_1} = \frac{\cos\alpha}{2} = \frac{\cos 16}{2}$$

$$u = 748.51 \text{ m/s} = \frac{\pi D_m N}{60}$$

$$748.51 = \frac{\pi D_m \times 3000}{60}$$

$$D_m = 4.765 \text{ m}$$ **Ans.**

(ii) Single 50% reaction stage,

$$\frac{V_1^2}{2000} = \frac{\Delta h_{stage}}{2} = \frac{1212.68}{2}$$

$$V_1 = 1101.21 \text{ m/s}$$

$$\frac{u}{V_1} = \cos\alpha$$

$$u = 1101.21 \times \cos 16 = 1058.55 \text{ m/s}$$

$$u = \frac{\pi D_m N}{60} = \frac{\pi \times D_m \times 3000}{60} = 1058.55$$

$$D_m = 6.738 \text{ m}$$ **Ans.**

(iii) Four pressure stage,

$$\frac{V_1^2}{2000} = \frac{\Delta h}{4}$$

$$\frac{V_1^2}{2000} = \frac{1212.68}{4}$$

$$\Rightarrow \quad V_1 = 778.67 \text{ m/s}$$

$$\frac{u}{V_1} = \frac{\cos\alpha}{2} = \frac{\cos 16}{2}$$

$$u = 374.25 \text{ m/s} = \frac{\pi D_m \times 3000}{60}$$

$$D_m = 2.382 \text{ m}$$ **Ans.**

(iv) One two row curtis stage

$$\frac{V_1^2}{2000} = \Delta h$$

$$\Rightarrow \quad V_1 = 1557.358 \text{ m/s}$$

$$\frac{u}{V_1} = \frac{\cos\alpha}{4} = \frac{\cos 16}{4}$$

$$u = 1557.358 \times \frac{\cos 16}{4} = 374.257 \text{ m/s} = \frac{\pi D_m \times 3000}{60}$$

$$D_m = 2.382 \text{ m}$$ **Ans.**

(v) Four 50% reaction stage

$$(\Delta h)_{\text{stage}} = \frac{\Delta h}{4} = \frac{1212.68}{4} = 303.17 \text{ kJ/kg}$$

$$\frac{V_1^2}{2000} = \frac{(\Delta h)_{stage}}{2} = \frac{303.17}{2}$$

$$V_1 = 550.608 \text{ m/s}$$

$$\frac{u}{V_1} = \cos\alpha = \cos 16$$

$$\Rightarrow \quad u = 529.27 \text{ m/s}$$

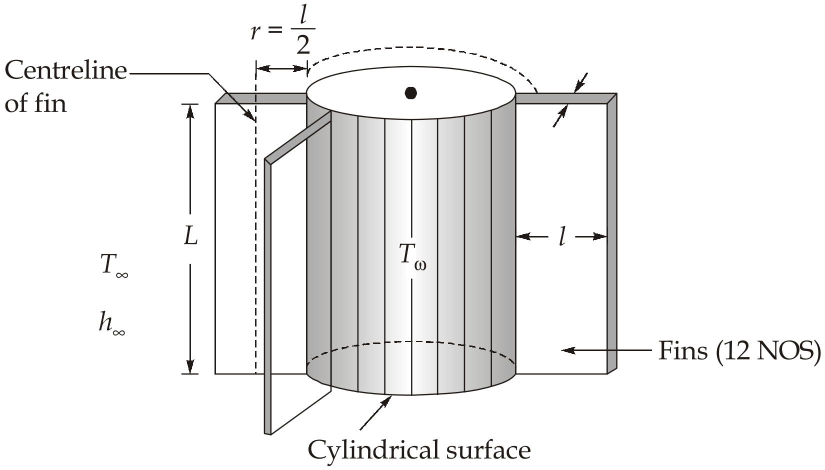
$$u = \frac{\pi D_m \times 3000}{60} = 529.27$$

$$D_m = 3.369 \text{ m}$$ **Ans.**

3. (c) Solution:

Given data : $k_{th} = 73.8$ W/mK; $t = 0.73$ mm; $l = 24.8$ mm; $L = 1.1$ m; $T_\infty = 40°\text{C}$; $h_\infty = 19 \text{ W/m}^2\text{K}$; $T_w = 150°\text{C}$

$$\theta_0 = T_w - T_\infty = 110°\text{C}$$



Assumptions:

(i) Fins are rectangular of uniform cross-section area.

(ii) Thermophysical properties are independent of temperature variation.

(iii) One-dimensional radial conduction.

(iv) Effects of radiation heat transfer are neglected.

(i) For single fin:

$$\text{Perimeter, }(P) = 2(L + t) = 2\left(1.1 + \frac{0.73}{1000}\right) = 2.20146 \text{ m}$$

$$\text{Fin cross-section }(A) = t \times L = 8.03 \times 10^{-4} \text{ m}^2$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{19 \times 2.20146}{73.8 \times 8.03 \times 10^{-4}}} = 26.567 \text{ m}^{-1}$$

$$ml = 0.6588$$

$$\tanh(ml) = 0.5776$$

$$\dot{Q}_f = mkA\theta_0 \tanh(ml)$$

$$= 26.567 \times 73.8 \times 8.03 \times 10^{-4} \times 110 \times 0.5776$$

$$= 100.031 \text{ W}$$

Heat transfer from 12 fins:

$$12 \times \dot{Q}_f = 1200.37 \text{ W}$$

Heat transfer from unfinned portion of the surface:

$$\dot{Q}_{uf} = hA\theta_0$$

$$= 19 \times [\pi \times 0.05 \times 1.1 - 12 \times 8.03 \times 10^{-4} \times L](150 - 40)$$

$$= 340.98 \text{ W}$$

Total heat transfer from the cylindrical surface:

$$\dot{Q}_{Total} = 12 \times \dot{Q}_f + \dot{Q}_{uf}$$

$$= 1541.356 \text{ W}$$ **Ans.**

If the cylinder were without fins:

$$\dot{Q}_{WF} = hA\theta_0 = 19 \times (\pi \times 0.05 \times 1.1)(150 - 40)$$

$$= 361.126 \text{ W}$$ **Ans.**

% increase in rate of heat transfer due to fins:

$$= \frac{\dot{Q}_{with\ fins} - \dot{Q}_{without\ fins}}{\dot{Q}_{without\ fins}} \times 100\%$$

$$= \frac{1541.356 - 361.126}{361.126} \times 100 = 326.82\%$$ **Ans.**

(ii) Temperature profile of fin

$$\frac{\theta}{\theta_0} = \frac{\cos h\, m(l - x)}{\cos h\, ml}$$

$$\frac{T_c - 40}{150 - 40} = \frac{\cosh\left[(26.567) \times \left(\frac{0.0248}{2}\right)\right]}{\cos h(0.6588)}$$

$$\frac{T_c - 40}{110} \simeq 0.861$$

$$T_c = 134.71°\text{C}$$ **Ans.**

$$\text{Fin efficiency, } (\eta_{fin}) = \frac{\tan h(ml)}{ml} = 0.8767 \text{ or } 87.67\%$$ **Ans.**

$$\text{Fin effectiveness } (\in_{fin}) = \frac{Q_{fin}}{Q_{without}} = \frac{1541.356}{361.126} = 4.268$$ **Ans.**

4. (a) Solution:

Large amounts of air are needed for combustion of the fuel. The gaseous combustion products in huge quantity have also to be removed continuously from the boiler furnace. To produce the required flow of either air or combustion gas, a pressure differential is needed. The term "draught" or "draft" is used to define the static pressure in the furnace, in the various ducts, and the stack. The function of the draught system is basically two-fold:

1. To supply to the furnace the required quantity of air for complete combustion of fuel.
2. To remove the gaseous products of combustion from the furnace and throw these through chimney or stack to the atmosphere.

Given : $h = 20$ mm; $H = 35$ m; $T_a = 300$ K; $T_g = 650$ K

(i)

Draught in mm of water column,

$$h = 353 \times H\left[\frac{1}{T_a} - \frac{m+1}{m} \times \frac{1}{T_g}\right]$$

$$20 = 353 \times 35\left[\frac{1}{300} - \frac{m+1}{m} \times \frac{1}{650}\right]$$

$\Rightarrow$ $m = 8.736$ kg air/kg coal **Ans.**

(ii)

Let H_g be the height of the hot gas column producing the draught. Then

$$H_g = H\left[\frac{m}{m+1} \times \frac{T_g}{T_a} - 1\right]$$

$$= 35 \times \left[\frac{8.736}{8.736+1} \times \frac{650}{300} - 1\right] = 33.04 \text{ m}$$ **Ans.**

(iii)

As, $$\rho_g = 353 \times \left(\frac{m+1}{m}\right) \times \frac{1}{T_g}$$

$$= 353 \times \left(\frac{8.736+1}{8.736}\right) \times \frac{1}{650} = 0.605 \text{ kg/m}^3$$

Mass flow of flue gas = Mass of coal + Mass of air

$$= \frac{2052 \times (1+8.736)}{3600} = 5.55 \text{ kg/s}$$

$$\text{Flue gas velocity, } V_g = \sqrt{2gH_g} = \sqrt{2\times 9.81\times(0.1\times 33.04)} = 8.05 \text{ m/s}$$

$$\therefore \quad \text{Volume flow of flue gas} = \frac{5.55}{0.605} = 9.17 \text{ m}^3/\text{s}$$

$$\text{Volume of flow of flue gas} = \frac{\pi}{4}\times d^2 \times V_g$$

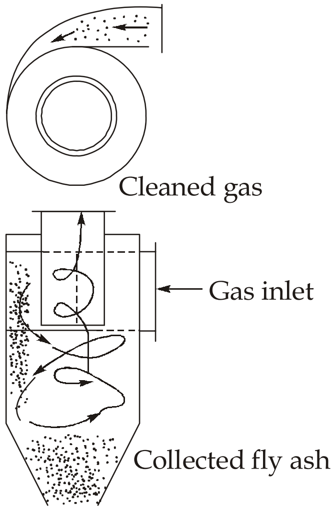
$$\Rightarrow \quad 9.17 = \frac{\pi}{4}\times d^2 \times 8.05$$

$$\Rightarrow \quad d = 1.204 \text{ m}$$

Ans.

4. **(b) (i) Solution:**

Cyclone separators: The cyclone is a separating chamber wherein high-speed gas rotation is generated for the purpose of "centrifuging" the particles from the carrying gases. Usually, there is an outer downward flowing vertex which turn into an inward flowing vertex. Involute inlets and sufficient velocity head pressure are used to produce the vortices. As multiple, small diameter vortices with high pressure drops appear to have high cleaning efficiency, that type is now being exploited. Skimming cyclones shave off the dust at the periphery of the vortex along with a small portion of the gas flow. This concentrated flow is then led to a secondary chamber for final separation.



Cyclone separator

The factors which affect the performance of this collector are gas volume, particulate loading, inlet velocity, temperature, diameter to height ratio of cyclone and dust characteristics.

The advantages and disadvantages of cyclone collectors are given below:

Advantages:

1. Rugged in construction.
2. Maintenance costs are relatively low.
3. Efficiency increases with increase in load.
4. Easy to remove bigger size particles.

Disadvantages:

1. Requires more power than other collectors.

2. Incapable to remove dust and ash particles which remain in suspension with still air.
3. Less flexible (in terms of volume handled).
4. High pressure loss comparatively.
5. Requires considerable head room and must be placed outside the boiler room.
6. As the fineness of the dust particle increases its collection efficiency decreases.

4. (b) (ii) Solution:

Given data : $\rho = 850\ \text{kg/m}^3$; $k = 0.14\ \text{W/m°C}$; $C_p = 2000\ \text{J/kgK}$; $\nu = 5.1 \times 10^{-6}\ \text{m}^2/\text{s}$

$L = 3.97$ m; $d = 19.3$ mm; $T_i = 30°\text{C}$; $T_o = 39°\text{C}$; $\dot{m} = 178.7$ kg/hr

Assumptions:

(k, ρ, ν, c ....) are constant with temperature variation.

Velocity of fluid flowing through the tube

$$V = \frac{\dot{m}}{\rho A} = \frac{\left(\dfrac{178.7}{3600}\right)}{850 \times \dfrac{\pi}{4} \times (0.0193)^2} \simeq 0.1996\ \text{m/s}$$

$$\text{Re} = \frac{\rho V d}{\mu} = \frac{Vd}{\nu} = \frac{0.1996 \times 0.0193}{5.1 \times 10^{-6}} = 755.41527$$

$$\text{Pr} = \frac{\mu C_p}{k} = \frac{\rho \nu C_p}{k} = \frac{850 \times 5.1 \times 10^{-6} \times 2000}{0.14} \simeq 61.9286$$

Let,

$$y = \left(\frac{d}{L} \text{Re} \cdot \text{Pr}\right) = \frac{0.0193}{3.97} \times 755.41527 \times 61.9286 \simeq 227.4278$$

∴

$$\text{Nu} = 3.65 + \frac{0.67(y)}{1 + 0.04(y)^{0.67}} = 64.1756$$

$$\frac{hL_c}{k} = 64.1756\ ; \qquad (\text{where } L_c = d)$$

$$h = \frac{64.1756 \times 0.14}{0.0193} = 465.522\ \text{W/m°C}$$ **Ans.**

$$\text{Heat gained by viscous fluid} = \dot{m}C_p \cdot \Delta T = \frac{178.7}{3600} \times 2000(39 - 30) = 893.5\ \text{Watts}$$

Mean temperature of viscous fluid,

$$T_m = \frac{39 + 30}{2} = 34.5°\text{C}$$

Convective heat flow from tube wall to viscous-fluid = $h(\pi dL)(\Delta T)$

$$= (465.522)\pi \times 0.0193 \times 3.97\left(T_w - 34.5\right)$$

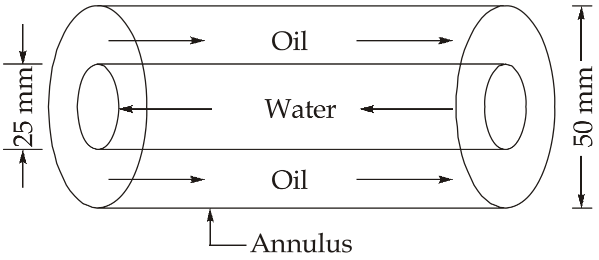
$$\Rightarrow \quad 893.5 = 122.0567\ (T_w - 34.5)$$

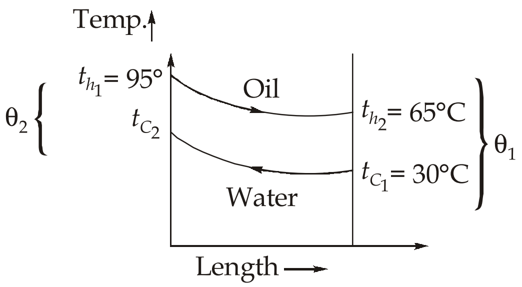
$$\Rightarrow \quad T_w = 42.474°\text{C}$$ **Ans.**

4. **(c) Solution:**

Given : d_i = 25 mm = 0.025; d_o = 50 mm = 0.05 m; $\dot{m}_w = \dot{m}_c = 0.2$ kg/s ;

$\dot{m}_{oil} = \dot{m}_h = 0.15$ kg/s; t_{h_1} = 95°C; t_{h_2} = 65°C; t_{c_1} = 30°C





The rate of heat transfer is given by,

$$Q = \dot{m}_h c_{ph}\left(t_{h_1} - t_{h_2}\right) = \dot{m}_c c_{pc}\left(t_{c_2} - t_{c_1}\right)$$

$$0.15 \times 2131 \times (95 - 65) = 0.2 \times 4174\left(t_{c_2} - 30\right)$$

$$\therefore \quad t_{c_2} = 41.48°\text{C}$$

LMTD is given by,

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln\left(\frac{\theta_2}{\theta_1}\right)} = \frac{(95 - 41.48) - (65 - 30)}{\ln\left(\frac{95 - 41.48}{65 - 30}\right)} = 43.6°\text{C}$$

Reynolds number for water,

$$Re = \frac{4\dot{m}_w}{\pi d_i \mu} = \frac{4 \times 0.2}{\pi \times 0.025 \times 725 \times 10^{-6}} = 1.4 \times 10^4$$

∴ Turbulent flow,

$$\therefore \quad \text{Nu} = \frac{h_i d_i}{k} = 0.023 \times \text{Re}^{0.8} \times \text{Pr}^{0.4}$$

$$= 0.023 \times (1.4 \times 104)^{0.8} \times (4.85)^{0.4}$$

$$\frac{h_i d_i}{k} = 89.72$$

$\therefore$ $$h_i = \frac{89.72 \times 0.625}{0.025} = 2243\ \text{W/m}^2\text{°C}$$

For Annulus part, $$D_h = d_o - d_i = 0.025\ \text{m}$$

$\therefore$ $$\text{Re} = \frac{\rho \cdot V \cdot D_h}{\mu} = \frac{\rho(d_o - d_i)}{\mu} \times \frac{\dot{m}_h}{\frac{\pi}{4}(d_o^2 - d_i^2) \cdot \rho}$$

or $$\text{Re} = \frac{4\dot{m}_h}{\pi(d_o + d_i)\,\mu} = \frac{4 \times 0.15}{\pi \times (0.075) \times 0.0325}$$

$$\text{Re} = 78.35 \qquad \text{(Laminar flow)}$$

Assuming uniform temperature along the inner surface of the annulus,

$$\text{Nu} = \frac{h_0 D_h}{k} = 3.65$$

$$h_0 = \frac{3.65 \times 0.138}{0.025} = 20.148\ \text{W/m}^2\text{°C}$$

The overall heat transfer coefficient,

$$\frac{1}{u} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{u} = \frac{1}{2243} + \frac{1}{20.148} = 19.96\ \text{W/m}^2\text{°C}$$

Also, $$Q = \dot{m}_h c_{ph}\left(t_{h_1} - t_{h_2}\right)$$

$$= uA\theta_m = u \cdot (\pi d_i L) \times \theta_m$$

$\therefore$ $$L = \frac{0.15 \times 2131 \times (95 - 65)}{19.96 \times \pi \times 0.025 \times 43.6}$$

$\therefore$ $$L = 140.3\ \text{m}$$

Section B : Renewable Sources of Energy-1 + Industrial & Maintenance Engineering-1 Production Engineering and Material Science-2

5. (a) Solution:

Given data : $\alpha = 5°$; $t_1 = 0.2$ mm; $w = 5$ mm; $\tau_s = 350$ MPa; $\mu = 0.5$, $\beta = \tan^{-1}(\mu) = 26.57°$

We know; from Merchant's Analysis:

Merchant's condition for minimum power consumption

$$2\phi + \beta - \alpha = 90°$$

$$2\phi + \tan^{-1}(\mu) - 5 = 90°$$

$$\phi = \frac{95 - \tan^{-1}(0.5)}{2} = 34.2175°$$

$$\phi = 34.2175°$$

$$\text{Shear force, } F_s = \frac{\tau_s \cdot w\, t_1}{\sin\phi} = \frac{350 \times 5 \times 0.2}{\sin(34.2175°)} = 622.40 \text{ N}$$

$$F_s = R \cdot \cos(\beta - \alpha + \phi)$$

$$R = \frac{F_s}{\cos(\beta - \alpha + \phi)}$$

$$R = \frac{622.4041}{\cos(\tan^{-1}(0.5) - 5 + 34.2175)}$$

$$R = 1106.8204 \text{ N}$$

$$F_c = R \cdot \cos(\beta - \alpha)$$

$$= 1106.8204 \times \cos 21.565°$$

$$= 1029.343 \text{ N}$$ **Ans.**

$$F_T = R \cdot \sin(\beta - \alpha)$$

$$= 1106.8204 \times \sin 21.565°$$

$$= 406.81999 \text{ N}$$ **Ans.**

5. (b) Solution :

Total variable costs as percentage of sales = 32.8 + 28.4 + 12.6 + 4.1 + 1.1 = 79 percent

$$\text{P/V ratio} = \frac{\text{Contribution}}{\text{Sales}} = \frac{100 - 79}{100} = 21 \text{ percent}$$

$$\text{Total fixed costs} = 315000$$

(i) Break-even sales volume

$$\text{BEP} = \frac{\text{Fixed cost}}{\text{P/V ratio}} = \frac{315000}{0.21} = 1500000$$

(ii) Profit at budgeted sales volume,

$$\text{Profit} = \text{Sales} \times \text{P/V ratio} - \text{Fixed cost}$$

$$= 1850000 \times 0.21 - 315000$$
$$= 388500 - 315000$$
$$= 73500$$

(iii) Profit when actual sales drop by 10 percent

$$\text{Actual sales} = 1850000 - 185000$$
$$= 1665000$$
$$\text{Profit} = \text{Sales} \times \text{P/V ratio} - \text{Fixed cost}$$
$$= 1665000 \times 0.21 - 315000$$
$$= 34650$$

Profit when actual sales are increased by 5 percent

$$\text{Actual sales} = 1850000 + 92500 = 1942500$$
$$\text{Profit} = 1942500 \times 0.21 - 315000$$
$$= 407925 - 315000 = 92925$$

5. (c) Solution :

$n = 120$; $t = 1$ hr; $T = 15000$ hours

The probability of failure, $P_{f_1} = \frac{n \cdot t}{T} = \frac{120 \times 1}{15000} = 8 \times 10^{-3}$ **Ans. (i)**

The group redundancy case,
The probability of failure if the elements are grouped as a set

$$P_{f_2} = N \cdot n \cdot \left(\frac{t}{T}\right)^2$$
$$= 120 \times 10 \times \left(\frac{1}{15000}\right)^2$$ [where N =10 elements]
$$= 5.33 \times 10^{-6}$$ **Ans. (ii)**

The reliability improvement factor,

$$\text{RIF} = \frac{\text{Probability of failure before improvement}}{\text{Probability of failure after improvement}}$$
$$= \frac{P_{f_1}}{P_{f_2}} = \frac{8 \times 10^{-3}}{5.33 \times 10^{-6}} = 1500.94$$ **Ans.(iii)**

5. (d) Solution:

W_{Ag} is the weight percent of Silver = 80%
W_{Cu} is the weight percent of Copper = 20%

C_{Ag} is the atom percent of silver

C_{Cu} is the atom percent of Copper

$$\therefore \quad C_{\text{Ag}} = \frac{W_{Ag} \times A_{Cu}}{W_{Ag} \times A_{Cu} + W_{Cu} \times A_{Ag}} \times 100$$

$$C_{\text{Ag}} = \frac{80 \times 63.55}{80 \times 63.55 + 20 \times 107.87} \times 100$$

$$\therefore \quad C_{\text{Ag}} = 70.2\%$$ **Ans.**

$$C_{\text{Cu}} = \frac{W_{Cu} \times A_{Ag}}{W_{Ag} \times A_{Cu} + W_{Cu} \times A_{Ag}} \times 100$$

$$= \frac{20 \times 107.87}{80 \times 63.55 + 20 \times 107.87} \times 100 = 29.8\%$$

5. (e) Solution:

Hydraulic energy required per day to lift water = mgh

$$= 10^3 \times 8 \times 9.81 \times 45$$

$$= 3531600 \text{ Joules}$$

or

$$\text{Power} = \frac{3531600}{3600} \text{Wh}$$

$$P = 981 \text{ Wh}$$

$$\therefore \quad \text{Required output in 5 hours, } P_0 = \frac{981}{5} = 196.2 \text{ W}$$

$$\text{Thus, actual output required, } (P_0)_{\text{act}} = \frac{196.2}{0.3} = 654 \text{ W}$$

$$\text{Required panel power considering losses} = \frac{654}{0.8 \times 0.85} = 961.76 \text{ W}$$

$$\therefore \quad \text{Power obtained from one PV module} = V_P \cdot I_P$$

$$= 20 \times 8$$

$$= 160 \text{ W}$$

$$\therefore \quad \text{Required number of PV module, } n = \frac{961.76}{160} = 6.011$$

$$n = 6 \text{ modules required}$$ **Ans.**

6. (a) Solution:

Given: $\beta = 30°$, $\phi = 28.7°$, $L = 77.2°$, $n = 319$ (for 15th November), Standard time = 1:30 PM

Now, from cooper relationship,

$$\text{Declination angle, } \delta = 23.45 \times \sin\left[\frac{360}{365}(284 + 319)\right]$$

$$= -19.147° \qquad \text{...Ans (i)}$$

$$\text{Solar time, ST} = 13 \text{ hr } 30 \text{ min} - 4\,(81.73 - 77.2)$$

$$\text{ST} = 13.198 \text{ hrs.}$$

$$\therefore \quad \text{Hour angle, } \omega = [12{:}00 - \text{ST}] \times 15 \text{ degrees}$$

$$\omega = [12 - 13.198] \times 15$$

$$\omega = -17.97° \qquad \text{...Ans (ii)}$$

Zenith angle, θ_z

$$\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$$

$$\cos\theta_z = \sin(28.7°)\sin(-19.147°) + \cos(28.7°) \times \cos(-19.147°)\cos(-17.97°)$$

or, $$\cos\theta_z = 0.63$$

or, $$\theta_z = 50.89° \qquad \text{...Ans (iii)}$$

Angle of incidence, θ_i $$\cos\theta_i = \sin(\phi - \beta)\sin\delta + \cos(\phi - \beta)\cos\delta\cos\omega$$

or $\cos\theta_i = \sin(28.7° - 30°)\sin(-19.147°) + \cos(28.7° - 30)\cos(-19.147)\cos(-17.97°)$

or, $$\cos\theta_i = 0.905$$

$\therefore$ $$\theta_i = 25.17° \qquad \text{...Ans (iv)}$$

Now, number of day light hours

$$t_d = \frac{2}{15} \times \omega_{SR}, \qquad \text{where } \omega_{SR} = \text{Hour angle at sunrise}$$

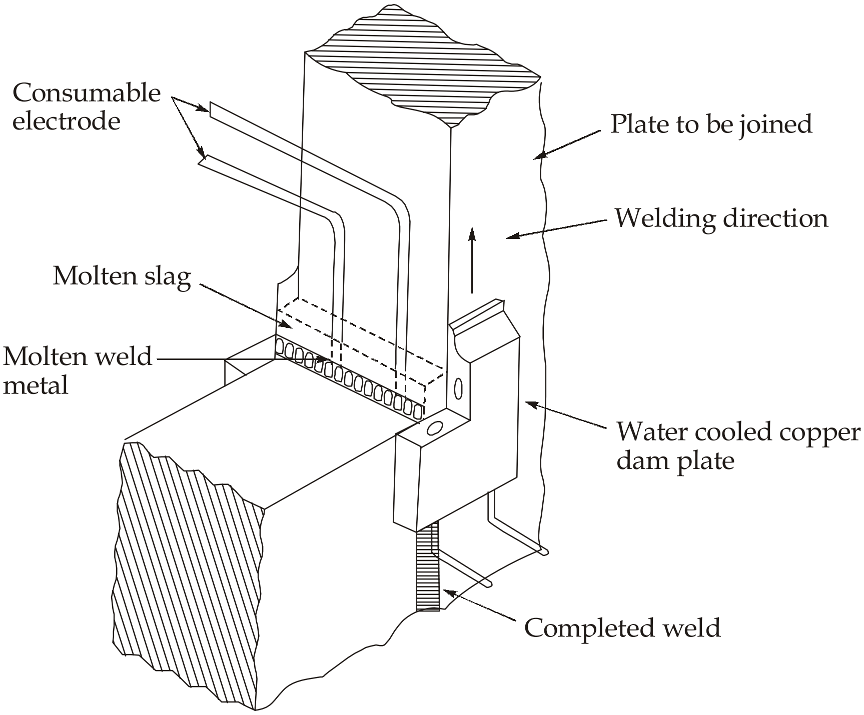
$$\therefore \quad t_d = \frac{2}{15}\cos^{-1}(-\tan(\phi - \beta)\tan\delta)$$

$$t_d = \frac{2}{15}\cos^{-1}(-\tan(28.7° - 30°)\tan(-19.147°))$$

$$t_d = 12.06 \text{ hours} \qquad \text{...Ans (v)}$$

6. (b) (i) Solution

The electro slag welding (ESW) process is developed essentially to weld very large plates without any edge preparation. This is essentially a single pass process using a consumable electrode for filling the gap between the two heavy plates. The heat required for melting the plates and the electrode is obtained initially by means of an arc so that flux will form the molten slag. Once the molten slag is formed, the arc is extinguished and the heat of welding is obtained by the resistance heating of the slag itself. The typical electro slag welding setup is shown in figure below. For the effective welding it is necessary to maintain a continuous slag pool and therefore the best way to maintain it, is to weld vertically. The slag pool is contained in the groove with the help of two water cooled copper dam plates which move along with the weld, as shown. The size and type of electrodes chosen depending on the width of the joint. In figure, two electrodes for feeding through the feed rollers (not shown) into the weld zone, are shown.



Electro slag welding process

The flux required to maintain a satisfactory amount of slag is fairly small, of the order of 0.2 to 0.3 kg per metre of weld length irrespective of the plate thickness. Thus, the heat utilised for melting the slag is much less. Most of the heat supplied in electro slag welding thus, melts the joint. By this process, a plate of thickness, 200 mm, can easily be welded in a single pass. Because of the vertical welding, any gas present easily bubbles out through the slag and, therefore, better welds can be produced. The heating and cooling of the edge is more gradual. The slag floating at the top would be preheating the joint before the actual melting by the heat liberated from the electrode. Whatever be the thickness of the plate no edge preparation is required. Electro slag welding is useful for welding very thick plates.

6. (b) (ii) Solution

$V = 90$ mm^3; $I = 20000$ A; $E = 15$ J/mm^3; $Q_{\text{lost}} = 750$ J; $R = 0.0002$ ohms

Total energy supplied during process = 90 × 15 + 750 = 2100 J

Also, $$i^2Rt = 2100\text{ J}$$

$$\therefore \quad t = \frac{2100}{(20000)^2 \times 0.0002} = 0.02625\,\text{sec} \qquad \textbf{Ans. (i)}$$

$$\text{Thermal efficiency} = \frac{\text{Heat required}}{\text{Heat supplied}} \times 100$$

$$\therefore \quad \eta = \frac{90 \times 15}{2100} \times 100 = 64.28\% \qquad \textbf{Ans. (ii)}$$

6. (c) (i) Solution:

1. **Chemical action theory or direct corrosion:** The direct corrosion is the simplest corrosion produced by means of a chemical attack and it includes oxidation in which the oxygen of the atmosphere combines with all or a part of the surface of material. The chemical reactions involved are as follows:

$$\text{Fe} + \text{O} + 2\text{CO}_2 + \text{H}_2\text{O} \rightarrow \text{Fe(HCO)}_3)_2 \qquad ...(i)$$

$$2\text{Fe(HCO)}_3)_2 + \text{O} \rightarrow 2\text{Fe(OH)CO}_3 + 2\text{CO}_2 + \text{H}_2\text{O} \qquad ...(ii)$$

$$\text{Fe(OH)CO}_3 + \text{H}_2\text{O} \rightarrow \text{Fe(OH)}_3 + \text{CO}_2 \qquad ...(iii)$$

The combined action of oxygen, carbon dioxide and moisture on iron results into soluble ferrous bicarbonate $\text{Fe(HCO}_3)_2$ as shown by reaction (i).

This ferrous bicarbonate is then oxidized to basic ferric carbonate 2Fe(OH)CO_3 as shown by reaction (ii).

This basic ferric carbonate is converted into hydrated ferric oxide and carbon dioxide is liberated as shown by reaction (iii).

The above theory of corrosion is supported by the following two observations:

- The analysis of rust shows small amounts of ferrous bicarbonate, ferric carbonate and hydrated ferric oxide.
- If carbon dioxide is excluded by immersing iron into a solution of sodium hydroxide or lime water, the intensity of rusting is considerably decreased.

The chemical reaction can also take place at the interface between a liquid and a metal. It may take one of the following three forms:

- The atoms from the liquid may diffuse into the surface of the solid metal and cause serious changes in the mechanical properties of the metal.

- The liquid atmosphere may dissolve the surface of the parent metal by a simple solution action.
- The liquid may react with the parent metal surface to form a compound similar to the oxide films.

The surface with direct corrosion has an etched or worn away appearance and its common examples are as follows:

- acid pickling or chemical bath used to clean the metal surfaces;
- corrosion of copper flashing or sheet metal in the atmosphere;
- reactions of dry chlorine, hydrogen, etc.;
- rusting of iron and steel;
- tarnishing of silverware; etc.

2. **Electrolytic theory or electro-chemical corrosion:** This is the commonly accepted theory of corrosion. According to this theory, the corrosion takes place due to chemical reaction in combination with electrolysis. It takes place at or near room temperature when the metal comes into contact with moisture or with aqueous solutions of salts, acids or bases. For electro-chemical corrosion to occur, the following conditions should be satisfied simultaneously:

   - There should be an electrolyte.
   - The current should be passing through the circuit.
   - There should be a difference of potential between a metal and its surrounding or between different parts of the same metal.
   - The circuit must be closed.

   In electro-chemical corrosion, the cathodic and anodic regions of the metal surface are involved. The metal surface from which current leaves the electrolyte and returns to the metal is called the cathode. The cathodic area does not corrode and it remains unchanged by the corrosion attack. The cathodes and anodes may be separate and independent units. The different areas on the same piece of metal may also represent the anode and cathode. The rate of corrosion will depend on the intensity of current between the anodic and cathodic sites and the nature of electrolyte. The multi-phase metals and alloys corrode at higher rates than pure metals.

3. **Galvanic action theory:** The galvanic corrosion occurs when two dissimilar metals are in electrical contact with each other and are exposed to an electrolyte. For instance, a less noble metal like zinc will dissolve and form the anode whereas the

more noble metal such as copper will act as the cathode. Thus the anode metal is made to corrode or dissolve continuously by the galvanic action. It is therefore necessary to observe that the direct contact between dissimilar metals is avoided in the fabrication work to prevent the corrosion of the anodic metal.

4. **High-temperature oxidation:** The rusting of ferrous alloys at high temperatures forms scales and oxides. It indicates the high-temperature dry corrosion. The other form of the high-temperature corrosion occurs when the liquid metals flow through other metals. The corrosion is due to the tendency of the solid to dissolve in the liquid metal up to the solubility limit at the given temperature. The liquid-metal attack may take any of the following three forms:
   - simple solution of the solid metal, or
   - formation of chemical compound, or
   - selective extraction of one of the component metals in a solid alloy.

6. (c) (ii) Solution:

In actual practice, the specific terms are used to denote a particular type of corrosion. The meanings of such terms are as follows:

- **Atmospheric corrosion:** The rain water and humid air act as electrolytes and they are mainly responsible for this type of corrosion. It is very frequent on ferrous materials.
- **Corrosion fatigue:** The combined action of corrosion and repeated stresses result in the corrosion fatigue. It is most common to the environments which cause pitting on the surface of the material.
- **Erosion corrosion:** The combined effect of the basic corrosion mechanism on a metallic surface and mechanical abrasion produce erosion corrosion and it is associated with the formation of cavities in the metal by fast moving liquids.
- **Fretting corrosion:** A fret indicates a hole, worn spot or path made by abrasion or erosion. This type of corrosion occurs when contact areas between the surfaces of any two materials are subjected to vibrational stresses. It is common in bolted and riveted joints, clamped surfaces, machine slides, etc.
- **Intergranular corrosion:** This type of corrosion occurs when a pronounced

difference in reactivity exists between grain boundaries and the remainder of the alloy. It is observed in defective welding and heat treatment of stainless steels, copper and aluminium alloys.

- **Pitting corrosion:** It is a localized type of corrosion and it is recognized by the presence of holes or pits. The shapes of pits vary widely. But generally, they are hemispherical with electropolished inner surfaces. The pitting is observed in aluminium, steel, copper and nickel alloys.
- **Selective corrosion:** When electro-chemical corrosion encourages preferential corrosion of one of the component metals, it is known as the selective corrosion. It is common in brass pipes and for brass articles containing more than 15% of zinc.
- **Stress corrosion:** The combined effect of mechanical stress and a corrosive environment produce this type of corrosion on the metal. The stress may either be applied or residual. The stress corrosion is usually confined to a local area which ultimately gives rise to small cracks and finally results in the failure of the material in service.
- **Uniform corrosion:** When the whole surface of the metal is corroded uniformly, it is known as the uniform corrosion and it is observed in metals like aluminium, lead and zinc.

7. (a) Solution:

Nanomaterials: Defined as materials with atleast one dimension between 1 and 100 nm.

These materials possess unique mechanical, electrical, thermal and chemical properties that are different from normal- engineering materials.

Some of the important types of nanomaterials used in engineering application are

1. Carbon based Nanomaterials :

- This include Graphene, Carbon Nanotubes (CNTs), fullerenes and carbon nanofibres.
- Primarily recognised for their extreme strength, light weight and high electrical conductivity.

- Graphene is a single- layer of carbon atoms almost 300 times stronger than steel, making it ideal for reinforcing composite materials.
- Carbon nanotubes which are cylindrical rolled sheets of graphe and are widely used in structural engineering.

2. **Metal and Metal-Oxide Nanomaterials:**
   - It include nanosilver, nanogold and metal oxides like Titanium dioxide (TiO_2), Zinc oxide (ZnO) and Iron oxide, favoured for their catalytic, magnetic and antimicrobial properties.
   - TiO_2 nanoparticles are used in self cleaning surfaces, photocatalysts and as UV blocking pigments in protective coatings.
   - Nano silver particles are integrated into textiles, air filters, and water purification systems because of their strong antimicrobial properties.
   - Quantum dots which are semiconductor nanocrystals are used in LED displays for improved color resolution and as a high efficiency components in PV cells.
3. **Ceramic and Inorganic Nanomaterials:**
   - Nanoceramics, such as Al_2O_3, zirconia (ZrO_2) f SiO_2 are used to improve toughness and scratch resistance of materials.
   - Nano-silica is extensively studied in structural engineering to densify the cementitious matrix, reducing porosity and permeability which enhances durability against sulphate attacks and chloride penetration.
4. **Polymeric composites Nanomaterials :**
   - Nanocomposites combine nanoparticles with polymers, ceramics or metals to achieve superior properties that conventional bulk materials cannot provide.
   - Polymer nanocomposites, often reinforced with clay nanoparticles as nanofibres to exhibit improved tensile strength.

Nanomaterials behave differently than bulk - materials because they exist in a transition - zone between individual atoms and macroscopic - solids . when a material is reduced to nanoscale its physical, chemical and other thermophysical of chemical properties drastically change due to two primary factors.

1. Increase relative surface area.
2. Quantum confinement - effect

Comparison between bulk engineering material and nanomaterials:

Feature	Bulk Engineering Materials	Nano Materials
1. Size Constraint	Macro-scale (> 100 nm) in all dimensions.	Nano-scale (< 100 nm) in atleast one dimension
2. Surface to Volume Ratio	Low (hence; bulk properties dominate)	Extremely High; (hence; surface atoms dictate behaviour)
3. Mechanical Strength	Limited by internal defects or dislocations.	Approaching theoretical ideal strength due to fewer defects.
4. Ductility and hardness	"Hall-Pech effect" (Smaller grains ⇒ harder)	"Inverse Hall-Pech" may occur. Eg. Some ceramics become harder.
5. Melting Point	Constant for given material	Size dependent, i.e. decreases as particle size shrinks.
6. Chemical Reactivity	Low/Moderate (e.g Bulk Gold in Inert)	Extremely High, often used as high efficiency catalyst.
7. Optical properties	Stable colour & transparency	Tunable; changes band on size.
8. Visibility	Visible to naked eye	Invisible to naked eye
9. Examples	Steel beams, Aluminium blocks, Concrete.	Carbon Nanotubes, Graphene, Quantum dots

In Nanotechnology "Top down" and "Bottom up" approach refers to two fundamental philosophies used to manufacture materials at the nano-scale.

(1) **The Top- down approach:** The top- down approach starts with a large (bulk) piece of material and removes parts of it until desired nanostructure is achieved.

Key characteristics

- Large scale objects are reduced to nano scale
- Involve high material waste
- Example: Photolithography, etching, laser- ablation.

(2) **The Bottom- up approach :** The bottom up approach builds nanostructures atom-by- atom or molecule- by- molecule until desired nanocomponent is produced.

Key characteristics

- Atomic scale to nano scale
- Often relies on chemical self-assembly
- Very little material wastage
- Capable of creating much smaller and more complex structures.

7. (b) Solution:

A loss in a solar cell refers to loss of photon energy (partial or full) which, due to some reason, is not able to deliver an electron out of a solar cell. This loss could be due to the fundamental reason, reason (limited by material properties) or it could be due to the technological reason (limited by cell processing capabilities). There are several ways in which photon energy loss could occur.

Loss of low energy photons: The photons having energy less than the band gap energy do not get absorbed in the material and, therefore, do not contribute to the generation of electron- pairs. This is referred as transmission loss, and is almost equal to 23% for a single junction solar cell.

Loss due to excess energy of photons: In an ideal case only, photon of energy equal to the band gap energy is required to excite an electron from valence band to conduction. When the photon energy E is higher than the band gap energy E_g, the excess energy = $E - E$, is given off as a heat to the material. This loss is referred as the thermalization loss. For a single junction solar cell, this is equal to about 33%.

Voltage loss: The voltage corresponding to the band gap of a material is obtained by dividing the band gap (potential energy) by charge, i.e., E_g/q. This is referred as the band gap voltage. The actual voltage obtained from a solar cell is V_{oc}. This happens due to the unavoidable intrinsic Auger recombination. The ratio of $V_o/E_g/q$ lies in the range of 0.65 to 0.72.

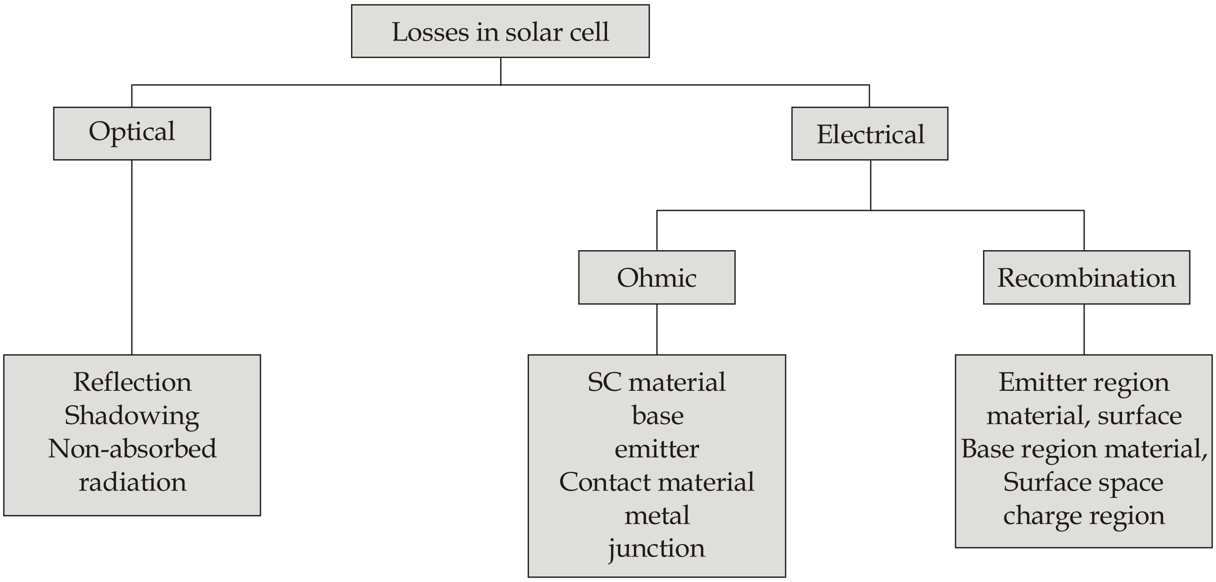
Fill factor loss: The I-V curve of ideal solar cell is square (i.e., $FF = 1$), but in reality, the cell I-V curve is given by the exponential behaviour. In the best case, the FF could be 0.89. This type of loss arises from the parasitic resistance (series and shunt resistance) of the cell.

The losses due to the technological reasons are as follows:

Loss by reflection: A part of incident photons is reflected from the cell surface. The reflection can be minimized by using anti-reflective coating and surface texturing.

Loss due to incomplete absorption: It refers to the loss of photons which have enough energy (i.e., $> E_g$) to get absorbed in the solar cell, but do not get absorbed in the cell due to limited solar cell thickness. The incomplete absorption is becoming important in the current scenario as the thickness of the cell (both wafer based and thin film) is being reduced in order to save the active material for cost reduction purpose. This type of loss can be minimized by having appropriate light trapping schemes.

Loss due to metal coverage: In wafer-based solar cell, the contact to the front side of the cell (from where light enters) is made in the form of finger and busbar. This metal contact shadows some light which can be up to 10%. Several approaches have been adopted to minimize this loss which include one-side contacted cell, buried-contact solar cell or transparent contact as used in thin film solar cells.



Categorization of loss mechanisms in solar cell arising from technological limitations

Recombination losses: Not all the generated electron-hole pairs contribute to the solar cell current and voltage due to recombination. The recombination could occur in the bulk of material or at the surfaces. This type of recombination can be minimized by appropriate surface and bulk passivation techniques.

The optical loss is referred to as the loss of photons which may result in the generation of electron-hole pairs. And the electrical loss is referred to as the loss of photons, which are absorbed in solar cell, but do not contribute to the cell output power due to either recombination or ohmic losses.

The optical losses can be reduced by the following design:

- Putting anti-reflection coating on the solar cell surface;
- Texturing front surface to reduce the reflection;
- Minimizing the front metal contact coverage area to reduce contact shading; and
- Making solar cell thicker to increase absorption of low energy photons.

7. (c) Solution:

(i)

Here we have

$D = 3000$ units, $C_0 =$ ₹15 per order, $C_h =$ ₹5 per unit per year

$\therefore$ EOQ $$Q^* = \sqrt{\frac{2C_0D}{C_h}} = \sqrt{\frac{2\times 15\times 3000}{5}}$$

$$Q^* = 134.164 \text{ units}$$ **Ans.**

Optimal number of orders, $n = \frac{D}{Q^*} = \frac{3000}{134.164} = 22.36$ **Ans.**

(ii) We are given:

D = 3000 units; C_0 = 250 per production run, C_h = ₹5 per unit per year; P = 4800 units

$\therefore$ EOQ, $$Q_1^* = \sqrt{\frac{2C_0D}{C_h}} \times \sqrt{\frac{P}{P-D}}$$

$$Q_1^* = \sqrt{\frac{2\times 250\times 3000}{5}} \times \sqrt{\frac{4800}{4800-3000}}$$

$$Q_1^* = 894.427 \text{ units}$$ **Ans.**

Average duration of the production run,

$$t = \frac{Q_1^*}{D} = \frac{894.427}{3000} = 0.298 \text{ year}$$ **Ans.**

(iii) When item is purchased from outside

$$\text{T.C}_1 = DC + \frac{D}{Q^*}\times C_0 + \frac{Q^*}{2}\times C_h$$

$$= 3000\times 32 + \frac{3000}{134.164}\times 15 + \frac{134.164}{2}\times 5$$

$$= ₹96670.82/-$$

when item is produced internally,

$$C' = 0.80 \times 32 = ₹25.6/-$$

$$\therefore \quad T.C_2 = DC' + \frac{D}{Q_1^*}\times C_0 + \frac{Q_1^*}{2}\times\frac{P-D}{P}\times C_h$$

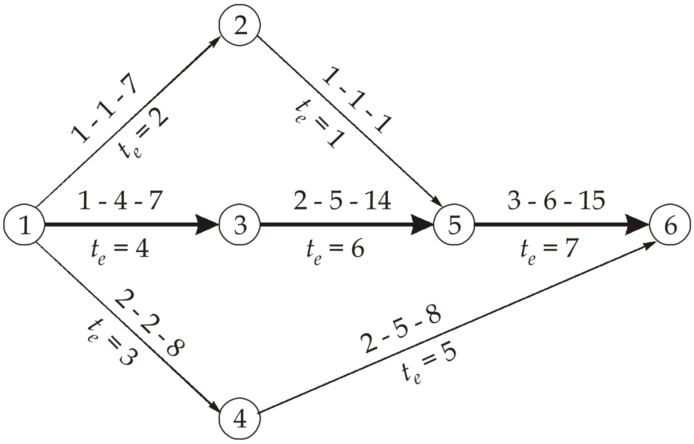
$$= 3000\times 25.6 + \frac{3000}{894.427}\times 250 + \frac{894.427}{2}\times\frac{4800-3000}{4800}\times 5$$

$$\therefore \quad T.C_2 = ₹78477.05/-$$

$\because$ $T.C_2 < T.C_1$, hence the company should manufacture the product internally.

8. (a) Solution:

The network diagram for the given data is drawn in figure below:



For determining the expected project length, the expected activity times need to be calculated. The same, along with the variances, are computed below.

Activity	t_0	t_m	t_p	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
1 - 2	1	1	7	2	1
1 - 3	1	4	7	4	1
1 - 4	2	2	8	3	1
2 - 5	1	1	1	1	0
3 - 5	2	5	14	6	4
4 - 6	2	5	8	5	1
5 - 6	3	6	15	7	4

Since 1 - 3 - 5 - 6 has the longest duration, it is the critical path of the Network.

$\therefore$ $T_s = 17$ weeks **Ans. (i)**

Now, Variance, $\sigma^2 = 1 + 4 + 4 = 9$

$\therefore$ $\sigma = 3$ weeks

(ii) When the project due date is 18 weeks

$$\therefore \quad z = \frac{T_S - T_E}{\sigma} = \frac{18 - 17}{3} = 0.333$$

For which P = 0.6304 or 63.04%.

∴ The probability of meeting the due date is 63.04% and the probability of not meeting the due date is 36.96%. **Ans.**

(iii) If $T_s = 21$ weeks

∴ $z = \dfrac{21-17}{3} = 1.333$

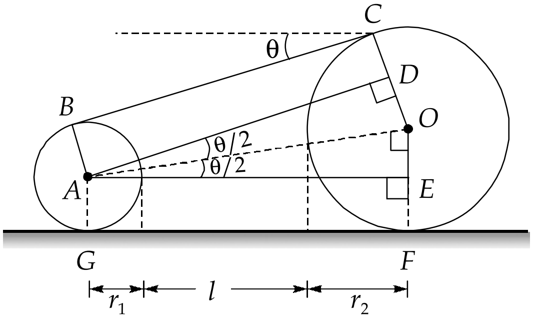
For which, $P = 0.9087$ or 90.87% **Ans.**

The probability that the project will be completed on schedule is 90.87%.

8. (b) Solution:

(i)

Given : $r_1 = 9$ mm, $r_2 = 15$ mm, $l = 23.87$ mm



Here, line $BC \parallel AD$ & $AB = CD$

Now, $DO = OC - CD = OC - AB = r_2 - r_1 = 15 - 9 = 6$ mm

and line $AE \parallel GF$ & $AG = EF$

So, $OE = OF - EF = OF - AG = r_2 - r_1 = 6$ mm

$$AE = r_1 + l + r_2 = 9 + 23.87 + 15 = 47.87 \text{ mm}$$

In ΔOAE,

$$\tan\frac{\theta}{2} = \frac{OE}{AE} = \frac{6}{47.87}$$

$$\Rightarrow \quad \frac{\theta}{2} = \tan^{-1}\left(\frac{6}{47.87}\right)$$

$$\Rightarrow \quad \theta = 2(7.144) = 14.29°$$

(ii)

Tolerance sink : A design engineer keeps one section of the part blank (without tolerance) so that production engineer can dump all the tolerance on that section which becomes most inaccurate dimension of the part, known as the tolerance sink.

Tolerance for the sink is cumulative sum of all the tolerances and only like minded tolerances can be added i.e. either equally bilateral or equally unilateral.

Now,
$$\begin{aligned} P &= 30 \pm 0.04 \text{ mm} \\ Q &= 13 \pm 0.03 \text{ mm} \\ R &= 12 \pm 0.07 \text{ mm} \\ \text{Basic size of } W &= P - (Q + R) \\ &= 30 - (13 + 12) = 5 \text{ mm} \\ \text{Tolerance} &= \text{Cumulative sum of tolerances} \\ &= 0.04 + 0.03 + 0.07 \\ &= 0.14 \end{aligned}$$

So,
$$W = 5 \pm 0.14 \text{ mm}$$

(iii)

Given, $h = d$, $a = 0.10$, $b = 0.03$ and $c = 1.00$

$$\begin{aligned} \text{Volume of casting} &= 300 \times 300 \times 100 = 9 \times 10^6 \text{ mm}^3 \\ \text{Surface area of casting} &= 2\,(300 \times 300) + 4\,(300 \times 100) \\ &= 18 \times 10^4 + 12 \times 10^4 \\ &= 30 \times 10^4 \text{ mm}^2 \end{aligned}$$

$$\text{Volume of riser} = \frac{\pi}{4}d^2 \times h = \frac{\pi}{4}d^3 \qquad [\because d = h]$$

$$\text{Surface area of riser} = \left(\frac{\pi}{4}d^2\right)2 + \pi dh = \frac{3}{2}\pi d^2 \qquad [\because d = h]$$

$$\text{For freezing ratio, } X = \frac{\left(\frac{V}{A}\right)_{riser}}{\left(\frac{V}{A}\right)_{casting}} = \frac{\left(\dfrac{\frac{\pi}{4}d^3}{\frac{3}{2}\pi d^2}\right)}{\left(\dfrac{9 \times 10^6}{30 \times 10^4}\right)}$$

$$X = \frac{\left(\frac{d}{6}\right)}{30} = \frac{d}{180} \qquad \text{...(i)}$$

From Caine's curve,

$$X = \frac{a}{Y-b} + c \qquad \text{...(ii)}$$

where,

$$Y = \frac{V_{riser}}{V_{casting}} = \frac{\frac{\pi}{4}d^3}{9 \times 10^6} \qquad \text{...(iii)}$$

Now, putting values of a, b, c, X and Y in equation (ii), we get

$$\frac{d}{180} = \frac{0.10}{\frac{\left(\frac{\pi}{4}d^3\right)}{9 \times 10^6} - 0.03} + 1$$

$$\Rightarrow \quad \frac{d}{180} = \frac{0.10}{\left(\frac{\pi d^3}{36 \times 10^6} - 0.03\right)} + 1$$

$$\Rightarrow \quad \frac{d-180}{180} = \frac{0.10}{(8.72 \times 10^{-8})d^3 - 0.03}$$

$$\Rightarrow \quad d^4(8.72 \times 10^{-8}) - 180 \times 8.72 \times 10^{-8} d^3 - 0.03d + 180(0.03) = 18$$

$$\Rightarrow \quad d^4 - 180d^3 - \left(\frac{0.03}{8.72 \times 10^{-8}}\right)d + \frac{180 \times 0.03}{8.72 \times 10^{-8}} - \frac{18}{8.72 \times 10^{-8}} = 0$$

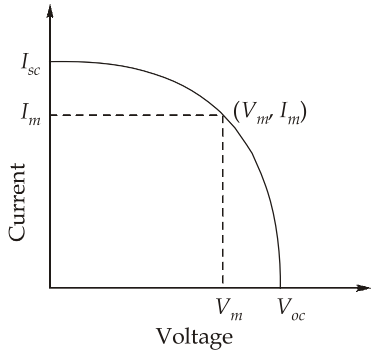
$$\Rightarrow d^4 - 180d^3 - 344036.7d - 144495412.8 = 0$$

$$\Rightarrow \quad d = 204.9689$$

8. **(c) Solution**

Current-Voltage characteristic of a solar cell: Electrical characteristics of a solar cell are expressed by the current-voltage curves plotted under a given illumination and temperature conditions as shown in figure.

The important points on the curve are short circuit current I_{sc} and open circuit voltage V_{oc}. Maximum useful power of the cell is represented by the rectangle with the largest area. When the cell yields maximum power, the current and voltage are represented by the symbols I_m and V_m respectively. Leakage across the cell increases with temperature which reduces voltage and maximum power.



Fill factor: Fill factor indicates the quality of a cell. It is defined as the ratio of the peak power to the product of open-circuit voltage and short-circuit current.

$$\text{Fill factor, } FF = \frac{V_m \times I_m}{V_{oc} \times I_{sc}}$$

An ideal cell will have a fill factor of unity. In order to maximize the fill factor, the ratio of the photo current to reverse saturation current should be maximized while minimizing internal series resistance and maximizing the shunt resistance. For a commercial silicon cell, its value is in the range of 0.5 to 0.83.

Maximum efficiency of a solar cell: It is defined as the ratio of maximum electric power output to the incident solar radiation.

$$\eta_{\max} = \frac{I_m \times V_m}{I_s \times A_c} = \frac{(FF) \times V_{oc} \times I_{sc}}{I_S \times A_C}$$

where, I_S = Incident solar flux
A_C = Cell's area
FF = Fill factor
I_{sc} = Short circuit current
V_{oc} = Open circuit voltage

Major losses in photovoltaic cell: Photovoltaic cells have low efficiency around 15%, i.e., only about 1/6th of the sunlight striking the cell generates electricity. The low efficiency is due to the following major losses:

i. When photons of light energy from the sun strike the cell, some of them (30%) are reflected (since reflectance from semiconductors is high). However, reflectance can be reduced to nearly 3% by surface texturing and anti-reflection coating.

ii. Photons of quantum energy ($h\nu < E_g$), cannot contribute to photoelectric current production (where h is the Planck's constant and ν is the frequency). This energy is converted into thermal energy and lost.

iii. Excess energy of active photons ($h\nu > E_g$) given to the electrons beyond the required amount to cross the band gap cannot be recovered as useful electric power. It appears as heat (about 33%) and lost.

iv. Photovoltaic cells are exposed directly to the sun. As the temperature rises, leakage across the cell increases. Consequently there is reduction in power output relative to input of solar energy. For silicon, the output decreases by 0.5% per °C.

v. Incident active photons produce electron-hole pairs with high quantum efficiency. Better cell design is required to ensure 95% absorption.

vi. Generated electric current flows out of the top surface by a mesh of metal contacts provided to reduce series resistance losses. These contacts cover a definite area which reduces the active surface and provides an obstacle to incident solar radiation.

vii. For higher efficiency, the p-n junction should be located near to the top surface (within 0.15 µm).

viii. The semiconductor with optimum band gap should be used for maximum efficiency.

Remedy or Methods of increasing efficiency of photovoltaic cell: The band gap of a semiconductor is required to match the solar spectrum and for obtaining a high efficiency, the band gap range is from 1.1 to 1.4 eV. Cells need to have absorptance so as to absorb the maximum number of photons in solar spectrum. This can be achieved by using a series of solar cells with varying band-gaps planned in a multi-layer structure. The correct approach is to use filters and split the solar spectrum into several spectral regions. Then, focusing is required for spectral beam towards a particular cell that carries a matching band gap and spectral response. With this concept, tandem solar cells with efficiency of 28% have been produced.

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