



MADE EASY

Leading Institute for ESE, GATE & PSUs

ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Mechanical Engineering

Test-4 : Section A : Theory of Machines [All Topics]

Section B : Fluid Mechanics & Turbo Machinery-1 [Part Syllabus]

Strength of Materials & Mechanics-2 [Part Syllabus]

Name :

Roll No :

Test Centres	Student's Signature
Delhi <input checked="" type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input type="checkbox"/> Pune <input type="checkbox"/> Hyderabad <input type="checkbox"/>	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	27
Q.2	53
Q.3	-
Q.4	40
Section-B	
Q.5	05
Q.6	-
Q.7	-
Q.8	21
Total Marks Obtained	146

Signature of Evaluator

Cross Checked by

Censor

Keep up this consistent effort

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

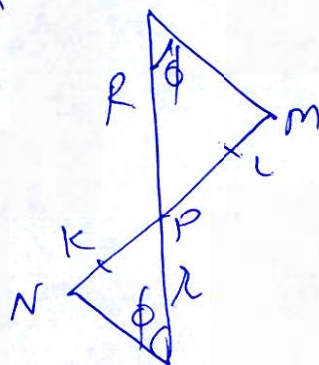
1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Theory of Machines

- Q.1 (a) For a pair of involute spur gear, module is 10 mm, pressure angle (α) is 18° . The number of teeth on pinion and gear are 20 and 40 respectively. The addendum on each gear is such that the path of approach and path of recess on each side is 60% of the maximum possible length. Determine the addendum for the pinion and the gear and the length of arc of contact.

$$r = \frac{mr}{2} = \frac{10 \times 20}{2} = 100 \text{ mm}$$

$$R = \frac{mT}{2} = \frac{10 \times 40}{2} = 200 \text{ mm}$$



[12 marks]

Given $KP = 0.6 \times NP = 0.6 \times r \sin \phi$

Similarly $PL = 0.6 \times MP = 0.6 \times R \sin \phi$

$$\therefore KP = 0.6 \times 0.1 \times \sin(18^\circ) = \sqrt{R_a^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$0.6 \times \frac{100}{\text{mm}} \times \sin(18^\circ) = \sqrt{R_a^2 - 200^2 \cos^2(18^\circ)} - 200 \times \sin(18^\circ)$$

$$R_a = \frac{206.4838}{\text{m}}$$

And $PL = 0.6 \times 200 \times \sin(18^\circ) = \sqrt{R_a^2 - 100^2 \cos^2(18^\circ)} - 100 \sin(18^\circ)$

$$R_a = \frac{116.9054}{\text{mm}}$$

Addendum of Gear = $R_a - R = 206.4838 - 200$

Gear add. = $\frac{6.4838 \text{ mm}}{\text{Answer}}$

Addendum of Pinion = $R_a - r = 116.9054 - 100$

Pinion add. = $\frac{16.9054 \text{ mm}}{\text{Answer}}$

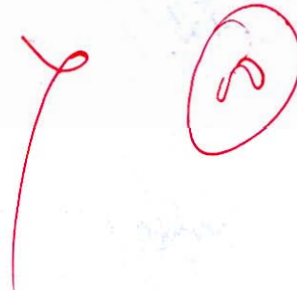
$$\text{Arc of Contact} = \frac{\text{Path of cut}}{\cos \phi} = \frac{KL + PL}{\cos \phi} = \frac{0.6 \times 80 \text{ mm} (18^\circ) \left[\frac{20}{100+20} \right]}{\cos(18^\circ)}$$

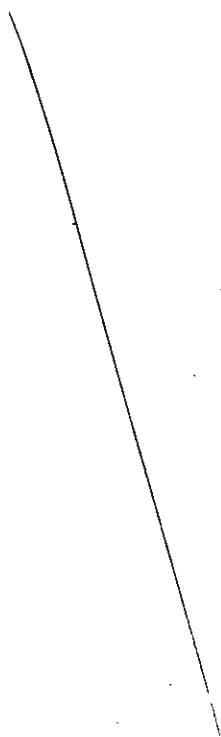
$$\text{Arc of Contact} = \underline{58.4855 \text{ mm}} \quad \text{Answer}$$

(12)

- Q.1 (b) A punching machine is required to punch 6 holes per minute of 40 mm diameter in 30 mm thick plate. The ultimate shear strength of plate material is 225 MPa. The punch has a stroke of 90 mm. Find the power of motor required if mean speed of flywheel is 15 m/s. If coefficient of fluctuation of energy is 3%, then determine the mass of flywheel. [Assume rim type flywheel]

[12 marks]

Power $P =$ 

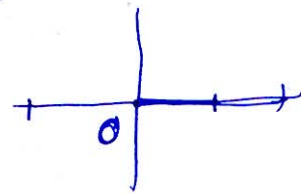
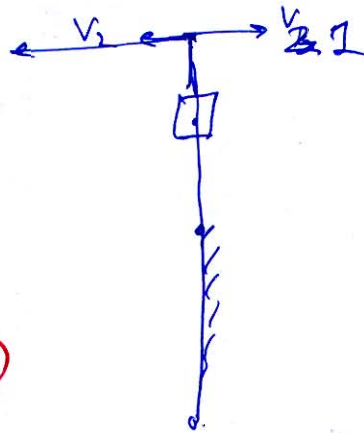


- Q.1 (c) In a quick-return motion of the crank and slotted lever type, the ratio of maximum velocities during return and forward motion is 2. If the stroke length is 280 mm, find
- the length of the slotted lever;
 - ratio of times of cutting and return strokes;
 - maximum cutting speed during the cutting stroke, if crank rotates at 45 rpm.

[12 marks]

$$\frac{v_2}{v_1} = 2$$

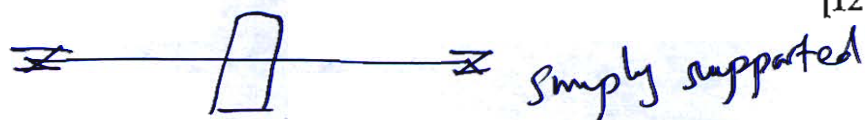
$$\frac{(\omega/cr) \times r_{cr}}{(\omega/cr)_i \times r_{cr}} = 2$$





- Q.1 (d) A rotor has a mass 15 kg and is mounted midway on a 25 mm diameter horizontal shaft supported at the ends by two bearings. The bearings are 1.2 apart. The shaft rotates at 2100 rpm. If the centre of mass of rotor is 0.12 mm away from the geometric centre of rotor due of a certain manufacturing defect, find the amplitude of the steady-state vibration and the dynamic force transmitted to the bearings. [Take $E = 220$ GPa]

[12 marks]



$$m = 15 \text{ kg}$$

$$d = 0.025 \text{ m}$$

$$L = 1.2 \text{ m}$$

$$e = 0.12 \text{ mm} = 12 \times 10^{-5} \text{ m}$$

$$N = 2100 \text{ rpm} \Rightarrow \omega = \frac{2\pi \times 2100}{60}$$

$$= 219.9115 \text{ rad/s}$$

assumy $\zeta = 0$ (No damping)

$$A = \frac{e}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\Delta_{st} = \frac{WL^3}{48EI} = \frac{15 \times 9.81 \times 1.5^3}{48 \times 220 \times 10^9 \times \frac{\pi}{64} \times (0.025)^4}$$

$$= 2.4527 \times 10^{-3} \text{ m}$$

$$\omega_n = \sqrt{\frac{g}{\Delta_{st}}} = \frac{63.2430}{\text{rad/s}}$$

So,

$$A = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} = \frac{12 \times 10^{-5}}{\left(\frac{63.2430}{219.9115}\right)^2 - 1}$$

$$A = 1.3082 \times 10^{-4} \text{ m}$$

$$A = 0.1308 \text{ mm} \quad / \text{ Answer}$$

Dynamic force Transmitted to bearing

$$F_{dy} = S A \quad S = m \omega_n^2$$

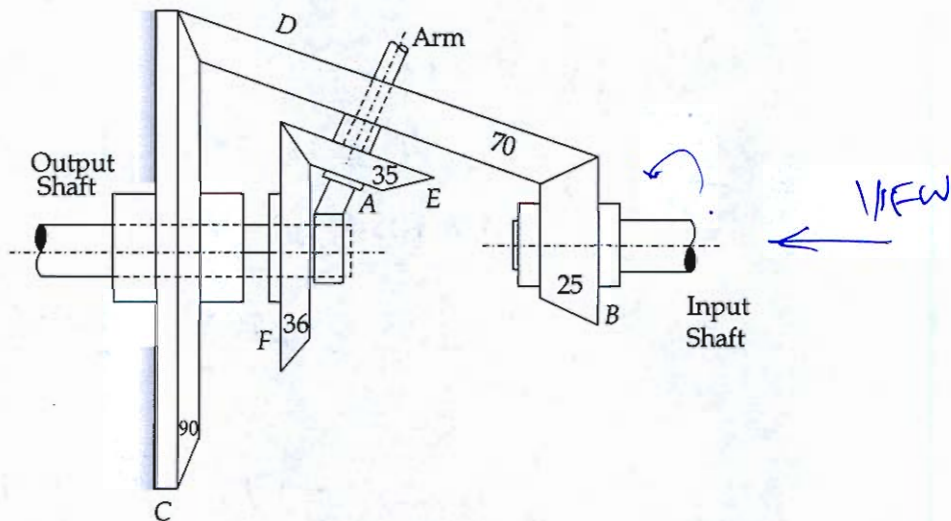
$$= m \omega_n^2 \times A$$

$$= 15 \times (63.2430)^2 \times 1.3082 \times 10^{-4}$$

$$F_{dy} = 7.8486 \text{ N} \quad / \text{ Answer}$$

Q.1 (e) In a gear train, as shown in the figure, gear B is connected to the input shaft. The arm A carrying the compound wheels D and E, turns freely on the output shaft. If the input speed is 1100 rpm counter-clockwise, when seen from the right, determine the speed of the output shaft under the following conditions:

1. When the gear C is fixed
2. When gear C rotates at 10 rpm clockwise



[12 marks]

Arm	B 25	D/E 70 35	C (90)	F (36)
0	+x	$\pm x \times \frac{25}{70}$	$(-)\frac{25}{90}x$	$(+)\frac{25 \times 35}{70 \times 36}x$
y	y+x	$y \pm x \times \frac{25}{70}$	$y - \frac{25}{90}x$	$y - \frac{25 \times 35}{70 \times 36}x$

(i) Also given $y - \frac{25}{90}x = 0$

$$y = \frac{25}{90}x \quad \text{--- (1)}$$

and $y + x = 1100 \text{ rpm (C.C.W.)}$

$$\left(\frac{25}{90} + 1\right)x = 1100$$

$$x = \underline{860.8696 \text{ rpm (C.C.W.)}}$$

$$y = \frac{25}{90} \times x = \underline{239.1304 \text{ rpm}}$$

$$N_f = y - \frac{25}{70} \times \frac{35}{36} x = (-) 59.7827 \text{ rpm}$$

$$N_f = \underline{59.7827 \text{ rpm (C.W.)}}$$

$$(ii) N_c = \cancel{110} - 10 \text{ rpm} = y - \frac{25}{90} x$$

$\omega \rightarrow -ve$
 $ccw \rightarrow +ve$

$$(c.w) \quad y + x = 1100$$

$$\left(\frac{25}{90} + 1\right)x - 10 = 1100$$

$$x = \underline{868.6957 \text{ rpm}}$$

$$y = -10 + \frac{25}{90} \times (868.6957) = \underline{231.3849 \text{ rpm}}$$

$$N_f = y - \frac{25}{70} \times \frac{35}{36} x = (-) 70.3261 \text{ rpm}$$

$$N_f = \underline{70.3261 \text{ rpm (C.W.)}}$$

12

Q.2 (a) The following data relate to the connecting rod of a reciprocating engine:

Mass = 60 kg

Distance between bearing centres = 1000 mm

Diameter of big end bearing = 110 mm

Diameter of small end bearing = 90 mm

Time of oscillation when the connecting rod is suspended from
big end = 1.8 sec

small end = 1.95 sec

Determine the

- (i) Radius of gyration of the rod about an axis through centre of mass perpendicular to the plane of oscillation.
- (ii) Moment of inertia of the rod about the same axis.
- (iii) Dynamically equivalent system of the connecting rod comprising two masses, one at the small end-bearing centre.

[20 marks]

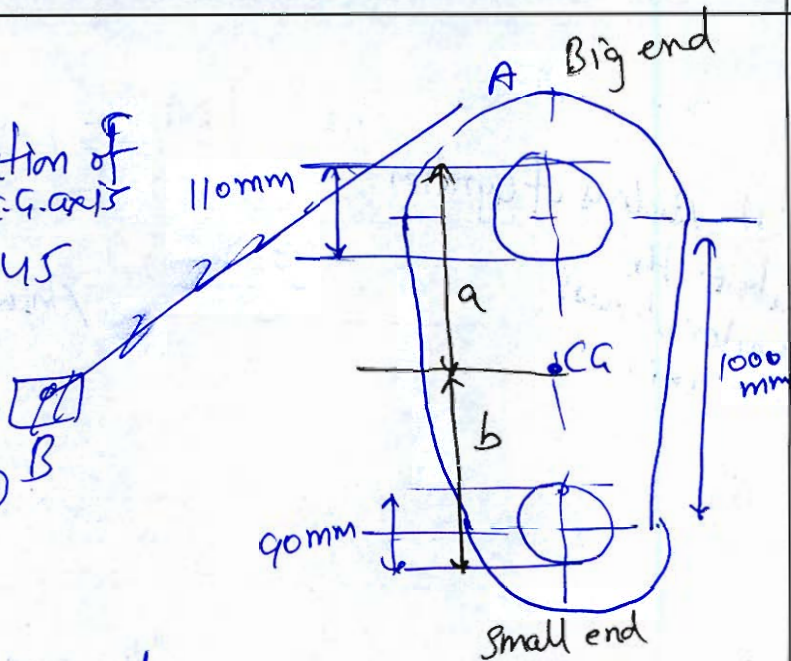
$$m_{cr} = 60 \text{ kg}$$

$k = 0.15 \text{ m}$
 $k \rightarrow$ radius of gyration of
 e.r about its C.G. axis

$$a + b = 1000 + 55 + 45$$

$$a + b = 1100 \text{ mm}$$

$$a + b = 1.1 \text{ m} \quad \text{--- (1)}$$



First Oscillation about Big end

$$T = 2\pi \sqrt{\frac{le}{g}} = 2\pi \sqrt{\frac{a + \frac{k^2}{a}}{g}} = 1.8 \text{ s}$$

$$a + \frac{k^2}{a} = \left(\frac{1.8}{2\pi}\right)^2 \times g \quad \text{--- (2)}$$

Oscillation about small end

$$T = 2\pi \sqrt{\frac{le}{g}} = 2\pi \sqrt{\frac{b + \frac{k^2}{b}}{g}} = 1.95 \text{ s}$$

$$b + \frac{k^2}{b} = \left(\frac{1.95}{2\pi}\right)^2 g$$

$$(1.1 - a) + \frac{k^2}{(1.1 - a)} = \left(\frac{1.95}{2\pi}\right)^2 g$$

from eqn (2)

$$(1.1 - a) + \frac{\left(\left(\frac{1.8}{2\pi}\right)^2 g - a\right)a}{(1.1 - a)} = \left(\frac{1.95}{2\pi}\right)^2 g$$

by solving

$$a = 0.3792 \text{ m}$$

and from eqn (1)

$$b = 0.7208 \text{ m}$$

From eqn - ②

(i) Radius of Gyration about the center of mass axis

$$k^2 = \left[\frac{(1.8)^2}{2\pi} \times 9.81 - 0.3792 \right] \times 0.3792$$

$$k = 0.4019 \text{ m} \quad \text{Answer}$$

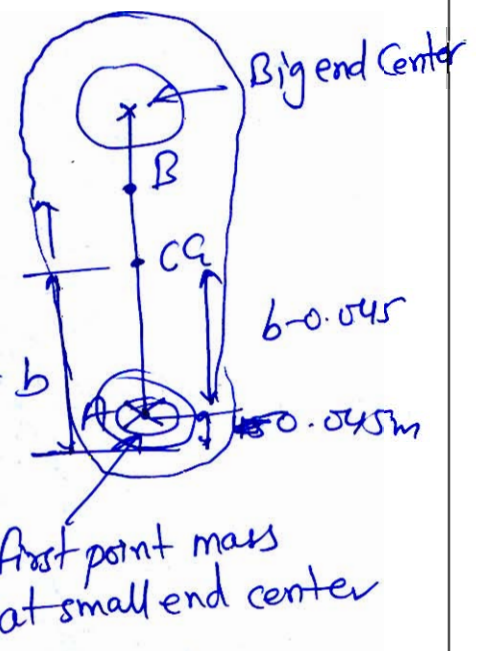
(ii) $I_{C.G.} = m_{C.R.} \cdot k^2 = 60 \times (0.4019)^2 \text{ kg-m}^2$

$$I_{C.G.} = \text{---} \text{ kg-m}^2 \quad \text{Answer}$$

(iii) Point mass at small end center

$$m_A = \frac{m_{C.R.} \times \left(\frac{k^2}{b-0.045} \right)}{\left[(b-0.045) + \frac{k^2}{b-0.045} \right]}$$

$$m_A = \frac{60 \times (0.4019)^2}{(0.7208 - 0.045) + (0.4019)^2} = 0.2390 \text{ m}$$



$$OB = 0.2390 \text{ m}$$

$$AO = 0.6758 \text{ m}$$

Distance of point masses from C.G.

Answer

$$m_A = \frac{60 \times 0.2390}{0.2390 + 0.6758} = 15.6756 \text{ kg}$$

$$m_B = 44.3244 \text{ kg}$$

$$m_A = 15.6756 \text{ kg}$$

Point masses of equivalent dynamic system.



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- Q.2 (b) The lengths of the ball and sleeve arms of the bell crank lever of Hartnell governor are 150 mm and 130 mm respectively. The mass of each governor ball is 6 kg. The fulcrum of the bell-crank lever is at a distance of 170 mm. At the mean speed of the governor which is 280 rpm, the ball arms are vertical and sleeve arms are horizontal. The sleeve moves up by 14 mm for an increase in speed of 5%. Neglecting friction, determine the
- spring stiffness.
 - minimum equilibrium speed when sleeve moves by 28 mm.
 - sensitiveness of the governor.
 - spring stiffness for the governor to be isochronous at the mean speed.

[20 marks]

$f = 0$ (friction zero)

$a = 0.150 \text{ m}$

$b = 0.130 \text{ m}$

$m = 6 \text{ kg}$

$N = 280 \text{ rpm}$
mean

$\omega_{\text{mean}} = \frac{2\pi \times 280}{60} = 29.3215 \text{ rad/s} = \omega_1$

$\omega_2 = 1.05 \times \omega_1 = 30.7876 \text{ rad/s}$

~~$\omega_1 = 0.95 \times \omega_1 = 27.8554 \text{ rad/s}$~~

~~Sleeve movement = 0.014 m~~

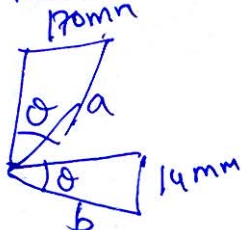
(i) Stiffness

$$S = 2 \times \frac{(F_2 - F_3)}{(r_2 - r_3)} \times \left(\frac{a}{b}\right)^2$$

When sleeve moves 14mm up

When sleeve moves by 28mm

from Geometry

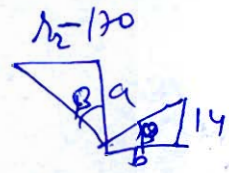


$$\frac{14}{b} = \frac{(170 - r_1)}{a}$$

$$\frac{14 \times 150}{130} = 170 - r_1$$

$$r_1 = 170 - \frac{14 \times 150}{130}$$

$$r_1 = 153.8462 \text{ mm}$$



$$\frac{r_2 - 170}{a} = \frac{14}{b}$$

$$r_2 = 170 + \frac{14 \times 150}{130}$$

$$r_2 = 186.538 \text{ mm}$$

$$F_2 = m r_2 \omega_2^2 = 6 \times 0.1862 \times (30.7876)^2 = \underline{1058.9674 \text{ N}}$$

$$F_1 = m r_1 \omega_1^2 = 6 \times 0.1538 \times (27.4024)^2 = \underline{694.9232 \text{ N}}$$

$$F_3 = m r_3 \omega_3^2 = 6 \times 0.170 \times (29.8245)^2 = \underline{876.9454 \text{ N}}$$

So, stiffness

$$s = 2 \times \frac{(1058.9674 - 876.9454)}{(0.1862 - \frac{0.1538}{0.170})} \times \left(\frac{0.15}{0.13} \right)$$

Stiffness
of
spring

$$s = \underline{29918.1624 \text{ N/m}} \quad \text{Answer}$$

(ii) Minimum equilibrium speed when sleeve moves by
20mm

$$r_1 = 0.1538 \quad (\text{by geometry})$$

$$F_2 = s = 2 \times \frac{(F_2 - F_1)}{(r_2 - r_1)} \times \left(\frac{g}{b} \right)^2$$

$$29918.1624 = 2 \times \left[\frac{1058.9674 - F_1}{0.1862 - 0.1538} \right] \times \left(\frac{0.15}{0.13} \right)^2$$

by solving $F_1 = \underline{694.9232 \text{ N}}$

$$m r_1 \omega_1^2 = \underline{694.9232 \text{ N}}$$

$$\omega_1 = \sqrt{\frac{694.9232}{6 \times 0.1538}} = \underline{27.4024 \text{ rad/s}}$$

Minimum
speed

$$N_1 = \underline{261.6736 \text{ rpm}} \quad \text{Answer}$$

(iii) Sensitiveness = $\frac{N_{\max} - N_{\min}}{N_{\text{mean}}}$

$$N_{\max} = 1.05 \times 280 \text{ rpm} \quad N_{\text{mean}} = 280 \text{ rpm} \quad N_{\min} = \underline{261.6736 \text{ rpm}}$$

$$\text{So sensitivity} = \frac{1.05 \times 280 - 261.6736}{280}$$

$$\text{sensitivity} = \boxed{0.1155} \text{ Answer}$$

(iv) Stiffness of spring to be isochronous at mean speed

$$\frac{\lambda_3}{\lambda_1} = \frac{m_3 + F_3}{m_1 + F_1}$$

$$\frac{F_3}{F_1} = \frac{\lambda_3}{\lambda_1} = \frac{170}{153.8}$$

$$F_3 - F_1 = S \cdot \Delta \lambda_{1-3}$$

$$S = \frac{F_3 - F_1}{\Delta \lambda_{1-3}}$$

$$m \lambda_3 \omega_3^2 \times a = F_3 \times b$$

$$F_3 = m \lambda_3 \omega_3^2 \times \frac{a}{b}$$

$$= 6 \times 0.17 \times (29.3215)^2 \times \frac{15}{13}$$

$$F_3 = 1011.86 \text{ N}$$

$$F_1 = m \lambda_1 \omega_3^2 \times \frac{a}{b}$$

isochron

$$= 6 \times 0.1538 \times (29.3215)^2 \times \frac{15}{13}$$

$$= 915.4357 \text{ N}$$

$$\text{So Stiffness for isochronous} = \frac{F_3 - F_1}{\Delta \lambda_{1-3}} = \frac{1011.86 - 915.4357}{0.074}$$

$$\text{at mean speed for isochronous } S = \boxed{6887.45 \text{ N/m}} \text{ Answer}$$

- Q.2 (c) A machine supported symmetrically on four springs has a mass of 100 kg. The mass of the reciprocating parts is 2.5 kg which moves through a vertical stroke of 110 mm with simple harmonic motion. Neglecting damping, determine the combined stiffness of the springs so that the force transmitted to the foundation is $\frac{1}{20^{\text{th}}}$ of the impressed force. The machine crankshaft rotates at 900 rpm. If under actual working conditions, the damping reduces the amplitudes of successive vibrations by 25%. Find the
- Force transmitted to the foundation at 900 rpm.
 - Force transmitted to the foundation at resonance.
 - Amplitude of vibrations at resonance.

[20 marks]

$$m = 100 \text{ kg}$$

$$m_{\text{rec}} = 2.5 \text{ kg}$$

$$x = 0.11 \text{ m}$$

$$\text{when } z = 0 \quad e = \frac{F_T}{F_0} = \frac{1}{20}$$

$$N = 900 \text{ rpm} \quad \omega = \frac{2\pi \times 900}{60} \text{ rad/s} = 94.2478 \text{ rad/s}$$

$$e_{(z=0)} = \frac{\sqrt{1+(0)}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 0}} = \frac{1}{\pm \left(1 - \frac{\omega^2}{\omega_n^2}\right)} = \frac{1}{20}$$

$$\pm \left(1 - \frac{\omega^2}{\omega_n^2}\right) = 20$$

By +ve sign

$$+ \left(1 - \frac{\omega^2}{\omega_n^2}\right) = 20$$

$$\frac{\omega^2}{\omega_n^2} = -19$$

↑
not possible

-ve sign

$$- \left(1 - \frac{\omega^2}{\omega_n^2}\right) = 20$$

$$\frac{\omega^2}{\omega_n^2} = 21$$

$$\frac{\omega}{\omega_n} = \sqrt{21} = 4.5826$$

$$\# \text{ Given } \frac{x_1}{x_0} = 0.75 = e^{-\zeta \omega_n t_0}$$

$$\omega_n = \frac{\omega}{\sqrt{21}} = 20.5664 \text{ rad/s}$$

$$\Rightarrow \delta = \ln(0.75)$$

$$\frac{1}{\sqrt{1-\xi^2}} = \ln(0.75)$$

$$\xi = \ln(0.75) \sqrt{1-\xi^2}$$

$$4\xi^2 = (\ln(0.75))^2 (1-\xi^2)$$

$$\xi^2 [4 + (\ln(0.75))^2] = (\ln(0.75))^2$$

$$\xi = \frac{\ln(0.75)}{\sqrt{4 + (\ln(0.75))^2}}$$

$$\xi = 0.0457$$

(1)

$$\xi_{\xi=0.0457} = \frac{\sqrt{1 + \left(\frac{2370}{\omega_n}\right)^2}}{\sqrt{\left(-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2370}{\omega_n}\right)^2}} = \frac{\sqrt{1 + (2 \times 0.0457 \times 45826)^2}}{\sqrt{(1 - 45826)^2 + (2 \times 0.0457 \times 45826)^2}}$$

$$\xi = 0.0542 = \frac{F_T}{F_0} = \frac{F_T}{m_{\text{rec}} \times \omega_{\text{res}}^2}$$

$$F_T = 0.0542 \times 2.5 \times 0.055 \times [94.2478]^2$$

$$F_T = 66.1979 \text{ N} \quad \text{Answer}$$

(ii) at Resonance $\omega = \omega_n = 20.5664 \text{ rad/s}$

$$F_0 = m_{\text{rec}} \omega_n^2 = 2.5 \times 0.055 \times (20.5664)^2$$

$$= 58.1593 \text{ N}$$

$$\xi = \frac{\sqrt{1 + (2\xi)^2}}{2\xi} = \frac{\sqrt{1 + (2 \times 0.0457)^2}}{2 \times 0.0457}$$

$$\xi = 10.9865$$

$$F_T = \xi \times F_0 = 10.9865 \times 58.1593$$

$$\text{at Resonance } F_T = 638.9671 \text{ N} \quad \text{Answer}$$

~~600~~
$$S = m \omega_n^2 = 100 \times (20.5664)^2 = \underline{42297.6809 \frac{N}{m}}$$

at Resonance; Amplitude of vibration

$$A = \frac{F_0/s}{2\xi}$$

$$= \frac{58.1593}{2 \times 0.0457}$$

$$A = 1.5044 \times 10^{-2} \text{ m}$$

$$A = 15.044 \text{ mm} \quad \text{Answer}$$

20

- Q.3 (a) The mass of each ball of a proell governor is 8 kg and the load on the sleeve is 90 kg. Each of the arms is 350 mm long. The upper arms are pivoted on the axis of rotation whereas the lower arms are pivoted to links of 45 mm from the axis of rotation. The extensions of the lower arms to which the balls are attached are 105 mm long and are parallel to the governor axis at the minimum radius. Determine the equilibrium speeds corresponding to extreme radii of 185 mm and 245 mm. [Neglecting friction]

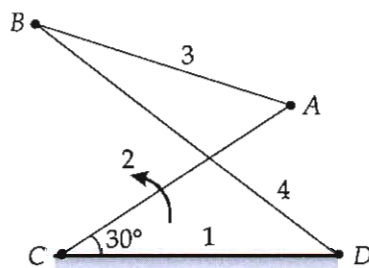
[20 marks]







- Q.3 (b) Locate all the instantaneous centers for the crossed four bar mechanism as shown in figure.



The dimensions of various links are : $CD = 65$ mm, $CA = 60$ mm, $DB = 80$ mm and $AB = 55$ mm. Find the angular velocities of the links AB and DB , if the crank CA rotates at 100 rpm in the anticlockwise direction.

[20 marks]







Q.3 (c) The following data relate to a shaft held in long bearings.

Length of shaft = 1.5 m

Diameter of shaft = 15 mm

Mass of rotor at midpoint = 17 kg

Eccentricity of centre of mass of rotor from centre of rotor = 0.4 mm

Modulus of elasticity of shaft material = 220 GPa

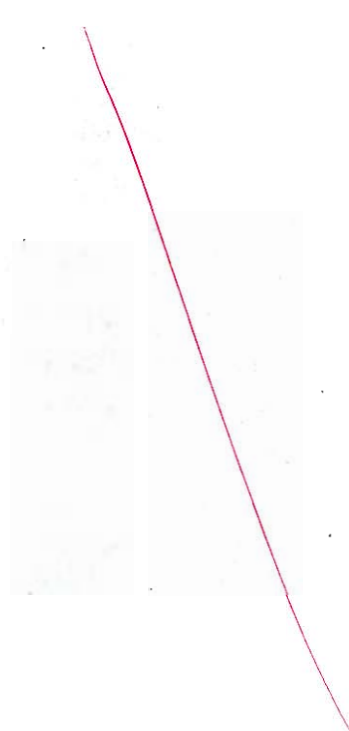
Permissible stress in shaft material = $70 \times 10^6 \text{ N/mm}^2$

Determine the critical speed of shaft and the range over which it is unsafe to run the shaft.

Assume the shaft to be massless.

[20 marks]

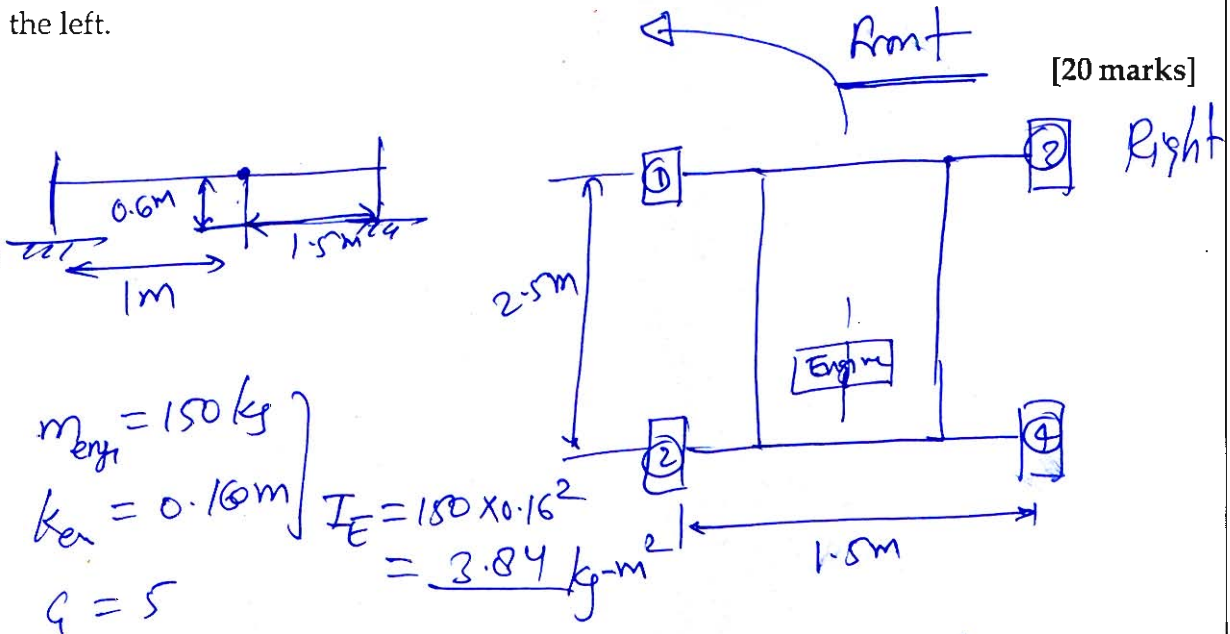






Q.4 (a) A 2 tonne racing car has a wheel base of 2.5 m and a track of 1.5 m. The centre of mass of the car lies at 0.6 m above ground and 1.5 m from the rear axle. The equivalent mass of engine parts is 150 kg with radius of gyration of 160 mm. The back axle ratio is 5. The engine shaft and flywheel rotates clockwise when viewed from the front. Each wheel has a diameter of 0.7 m and a moment of inertia of 0.6 kgm². Determine the load distribution on the wheels when the car is rounding a curve of 110 m radius at a speed of 75 km/h to the left.

[20 marks]



$m_{\text{eng}} = 150 \text{ kg}$

$k_{\text{en}} = 0.16 \text{ m}$

$G = 5$

$I_E = 150 \times 0.16^2 = 3.84 \text{ kg-m}^2$

$R_w = 0.35 \text{ m}$
 $I_w = 0.6 \text{ kg-m}^2$
 $\omega_w = \frac{V}{R_w} = 59.5237 \text{ rad/s}$

$R = 110 \text{ m}$
 $V = 75 \text{ kmph} = 75 \times \frac{5}{18} = 20.8333 \text{ m/s}$
 $\omega_p = \frac{V}{R} = 0.1894 \text{ rad/s}$

$\omega_E = 5\omega_w = 297.6185 \text{ rad/s}$ (CW from front)

(i) Due to Weight
 on front wheels = $\frac{Mg \times \frac{5}{2.55}}{2.55} = \frac{3}{5} Mg$

on each front wheel = $\frac{3}{10} Mg$

on rear wheels = $\frac{2}{5} Mg$

on each rear wheel = $\frac{1}{5} Mg$

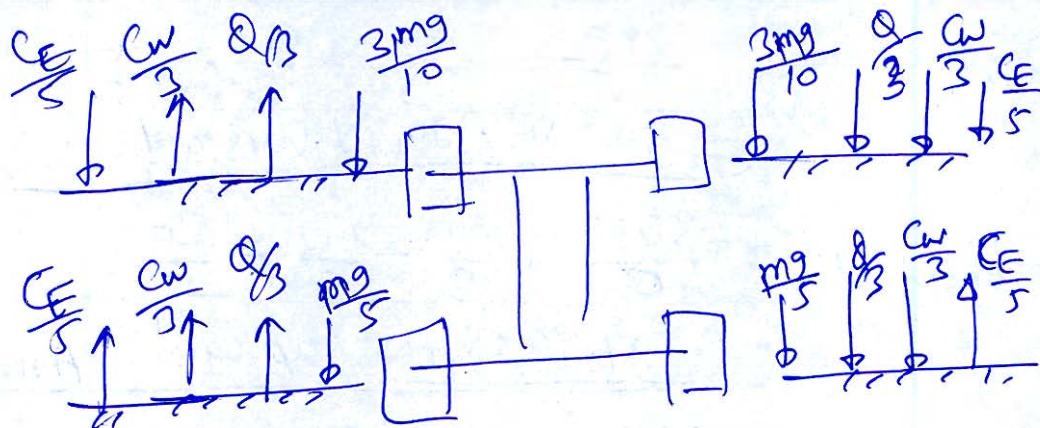
$Mg = 2 \times 10^3 \times 9.81 = 19620 \text{ N}$

(ii) Centrifugal effect

$Q = \frac{Mk^2}{R} \times 0.6 = \frac{2 \times 10^3 \times (20.8333)^2}{110} \times 0.6$

0

$Q = 4734.8333 \text{ N-m}$



centrifugal effect on ~~each~~ ^{each} wheel.

$$= \frac{Q}{2 \times 1.5} = \frac{Q}{3}$$

(iii) Cyrosopic Couple

↳ of wheels $\rightarrow C_w = 4 \times [I_w \omega_w \cdot \omega_p]$

$$= 4 \times 0.6 \times 59.5237 \times 0.1894$$

$$= 27.0571 \text{ N-m}$$

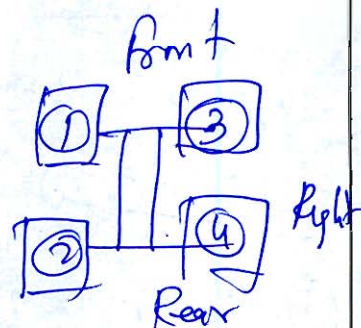
on each wheel $\frac{C_w}{2 \times 1.5} = \frac{C_w}{3}$

↳ on-engine $C_E = I_E \omega_E \times \omega_p = 3.84 \times 297.6185 \times 0.1894$

$$= 216.4567 \text{ N-m}$$

on each wheel $\frac{C_E}{2 \times 2.5} = \frac{C_E}{5}$

So load distribution on wheels



Wheel-1 $= \frac{3mg}{10} - \frac{Q}{3} - \frac{C_w}{3} + \frac{C_E}{5}$

$$= \frac{3}{10} \times 19620 - \frac{4734.8833}{3} - \frac{27.0571}{3} + \frac{216.4567}{5}$$

$$= 4341.9945 \text{ N (upward)} \quad \text{Answer}$$

$$\text{wheel-2} = \frac{mg}{5} - \frac{Q}{3} - \frac{C_w}{3} - \frac{C_E}{5}$$

$$= 2293.4119 \text{ N (upward)} \quad \text{Answer}$$

$$\text{wheel-3} = \frac{3mg}{10} + \frac{Q}{3} + \frac{C_w}{3} + \frac{C_E}{5}$$

$$= 7516.5881 \text{ N (upward)} \quad \text{Answer}$$

$$\text{wheel-4} = \frac{8mg}{5} + \frac{Q}{3} + \frac{C_w}{3} - \frac{C_E}{5}$$

$$= 5468.0055 \text{ N (upward)} \quad \text{Answer}$$

20

Q.4 (b) The torque delivered by a two stroke engine is represented by

$$T = (1200 + 350 \sin 2\theta - 600 \cos 2\theta) \text{ Nm};$$

where θ is the angle turned by the crank from the inner dead centre. The engine speed is 280 rpm. The mass of flywheel is 400 kg and radius of gyration 350 mm. Determine the

- (i) Power developed
- (ii) Total percentage fluctuation of speed
- (iii) Angular acceleration of flywheel when the crank has rotated an angle of 70° from the inner dead centre.
- (iv) Maximum angular acceleration and retardation of the flywheel.

[20 marks]

$$N = 280 \text{ rpm} \quad m = 400 \text{ kg} \quad k = 0.350 \text{ m}$$

$$\text{2-stroke engine} \quad I_{\text{fly}} = I = 400 \times 0.35^2 = 49 \text{ kg-m}^2$$

(2π)

(i) Power Developed per cycle

$$\text{work/cycle} = \int_0^{2\pi} T d\theta = \int_0^{2\pi} [1200 + 350 \sin(2\theta) - 600 \cos(2\theta)] d\theta$$

$$\text{work/cycle} = 7539.8224 \text{ J/cycle}$$

$$\text{Mean Torque } T_{\text{mean}} = \frac{\text{Work/cycle}}{T_{\text{period}}} = \frac{7539.8224}{2\pi}$$

$$= 1200 \text{ N-m}$$

$$\text{Power Developed } P = T_{\text{mean}} \times \omega_{\text{mean}} = 1200 \times \frac{2\pi \times 200}{60}$$

$$P = 35185.8377 \text{ W}$$

$$P = 35.1858 \text{ kW} \quad \text{Answer}$$

(ii)

$$T = T_{\text{mean}}$$

$$1200 + 350 \sin(2\theta) = 600 \cos 2\theta = 1200$$

$$350 \sin 2\theta = 600 \cos 2\theta$$

$$\tan 2\theta = \frac{600}{350}$$

$$2\theta = \tan^{-1}\left(\frac{60}{35}\right)$$

$$2\theta = 59.7436^\circ, 239.7436^\circ, 419.7436^\circ$$

$$\theta = \frac{0}{\theta_1}, \frac{29.8718^\circ}{\theta_2}, \frac{119.8718^\circ}{\theta_3}, \frac{209.8718^\circ}{\theta_4}, \frac{299.8718^\circ}{\theta_5}$$

$$\theta_6 = 360$$

$$\Delta E_1 = \int_{\theta_1}^{\theta_2} (T - T_{\text{mean}}) d\theta = 172.311 \text{ J} \quad -$$

$$\Delta E_2 = \int_{\theta_2}^{\theta_3} (T - T_{\text{mean}}) d\theta = 694.6222 \text{ J} \quad +$$

$$\Delta E_3 = \int_{\theta_3}^{\theta_4} (T - T_{\text{mean}}) d\theta = 694.6222 \text{ J} \quad -$$

$$\Delta E_4 = \int_{\theta_4}^{\theta_5} (T - T_{\text{mean}}) d\theta = 694.6222 \text{ J} \quad +$$

$$\Delta E_5 = \int_{\theta_5}^{\theta_6} (T - T_{\text{mean}}) d\theta = 522.3112 \text{ J}$$

$$1200 \text{ a} \quad \text{b} \quad \text{c} \quad \text{d} \quad \text{e} \quad \text{f}$$

$$\Delta E_{\max} = I \omega^2 \alpha_s$$

$$C_s = \frac{\Delta E_{\max}}{I \omega^2} = \frac{694.6222}{49 \times \left(\frac{2\pi \times 2880}{60}\right)^2}$$

$$C_s = 0.0165 = \frac{N_{\max} - N_{\min}}{N_{\text{mean}}}$$

$$C_s = 1.65\% \text{ / Answer}$$

% of fluctuation of speed

(iii)
$$T_{\theta=90^\circ} = 1200 + 350 \times \sin(140^\circ) - 600 \cos(140^\circ)$$

$$= 1884.60 \text{ N-m}$$

$$(T - T_{\text{mean}}) = I \times \alpha_{\text{fly}}$$

$$\alpha_{\text{fly}} = \frac{T - T_{\text{mean}}}{I} = \frac{1884.60 - 1200}{49}$$

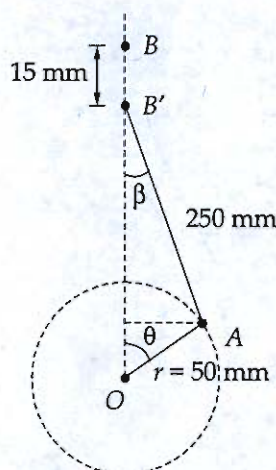
$$\alpha_{\text{fly}} = 13.9714 \text{ rad/s}^2 \text{ / Answer}$$

25

(iv)
$$T_{\text{max}} \text{ at } \alpha_{\text{max}}$$

$$T - T_{\text{mean}}$$

- Q.4 (c) The crank and the connecting rod of a vertical single cylinder gas engine as shown in the figure running at 1900 rpm are 50 mm and 250 mm respectively. The diameter of the piston is 88 mm and the mass of the reciprocating parts is 1.5 kg. At a point during the power stroke when the piston has moved 15 mm from the top dead centre position, the pressure on the piston is 750 kPa.

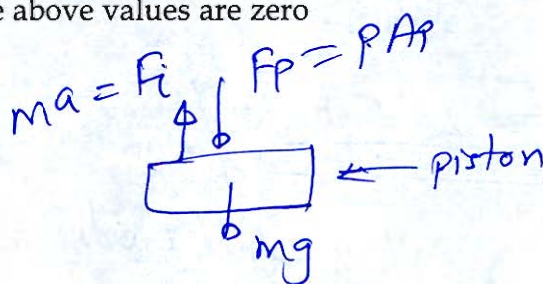


Determine the

- Net force on the piston
- Thrust in the connecting rod
- Thrust on the sides of the cylinder walls
- Engine speed at which the above values are zero

[20 marks]

Friction = 0
N = 1900 rpm



$$x = r \left[(1 - \cos \theta) + (n^2 - \sqrt{n^2 - \sin^2 \theta}) \right]$$

$$0.015 = 0.050 \times \left[1 - \cos \theta + \left[5^2 - \sqrt{5^2 - \sin^2 \theta} \right] \right]$$

[∵ n = 5]

By solving

$$\theta = 45.573^\circ$$

$$a = r \omega^2 \left[\cos \theta + \frac{\cos 2\theta}{n} \right]$$

$$= 0.05 \times \left(\frac{2\pi \times 1900}{60} \right)^2 \left[\cos(45.573) + \frac{\cos(2 \times 45.573)}{5} \right]$$

$$a = 1377.665 \text{ m/s}^2$$

$$A_p = \frac{\pi}{4} d_p^2 = \frac{\pi}{4} \times (0.088)^2 \text{ m}^2$$

(i) Net force on piston

$$F_p = F_{\text{gas}} - F_i + mg$$

$$= 750 \times 10^3 \times \frac{\pi}{4} \times (0.088)^2 - 1.5 \times (377.665)^2 + 1.5 \times 9.81$$

$$F_{\text{piston net}} = 2509.81 \text{ N} \quad \text{Answer}$$

(ii) Thrust on connecting rod

$$F_c = \frac{F_{\text{piston}}}{\cos \beta} \quad \beta = \sin^{-1} \left(\frac{sm}{n} \right)$$

$$\beta = 8.2116^\circ$$

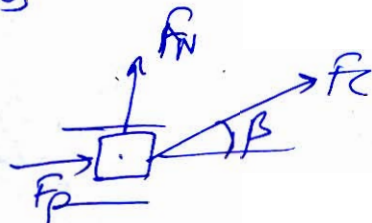
$$= \frac{2509.81}{\cos(8.2116^\circ)}$$

$$F_c = 2535.81 \text{ N} \quad \text{Answer}$$

(iii) Thrust on side of cylinder wall

$$F_N = F_c \sin \beta$$

$$= 2535.81 \times \sin(8.2116^\circ)$$



$$F_N = 362.188 \text{ N} \quad \text{Answer}$$

Side Thrust on cylinder wall

$$(iv) F_p = 0 = F_{\text{gas}} - F_i + mg$$

$$F_i = F_{\text{gas}} + mg$$

$$15 \times 0.05 \times \omega^2 \left(\cos 45.873^\circ + \frac{\cos(45.873^\circ \times 2)}{5} \right) = 750 \times 10^3 \times \frac{\pi}{4} \times 0.088^2 + 1.5 \times 9.81$$

$$\omega = \underline{296.0891 \text{ rad/s}}$$

$$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 296.0891}{2\pi}$$

Speed
at
which

$$N = \underline{2827.443 \text{ rpm}}$$

Answer

F_{ptw} , F_c , F_N
will be zero.

**Section B : Fluid Mechanics & Turbo Machinery-1 +
Strength of Materials & Mechanics-2**

Q.5 (a) Empirical pressure-density relation for water is given by

$$\frac{P}{P_0} = 3000 \left[\left(\frac{\rho}{\rho_0} \right)^7 - 1 \right]; P > P_0 = 100 \text{ kPa}$$

where; P_0 and ρ_0 are atmospheric-pressure and density of water at atmospheric conditions respectively.

Determine the isothermal bulk-modulus of elasticity and compressibility of water at 1.1 atm and 100 atm.

[12 marks]



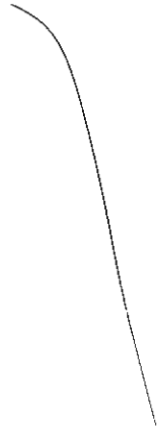
Q.5 (b) A jet of water having a velocity of 30 m/s impinges on a series of vanes moving with a velocity of 14 m/s. The jet makes an angle of 25° to the direction of motion of vanes entering and leaves at an angle of 125° . Draw the velocity triangles at inlet and outlet and find

1. the angles of vanes tip so that water enters and leaves without shock.
2. the work done per unit weight of water entering and leaves without shock.
3. the efficiency

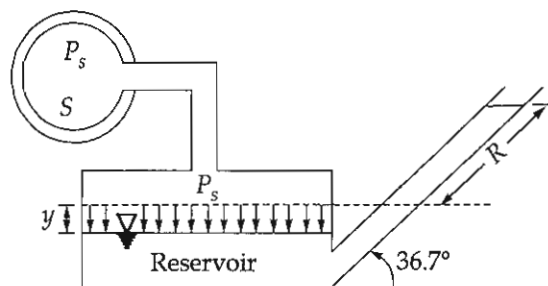
[12 marks]







- Q.5 (c) An inclined tube manometer measures the gauge pressure P_s of a system 'S' as shown in figure. The reservoir and tube-diameters of the manometer are 55 mm and 4 mm respectively. The inclination angle of tube is 36.7° .



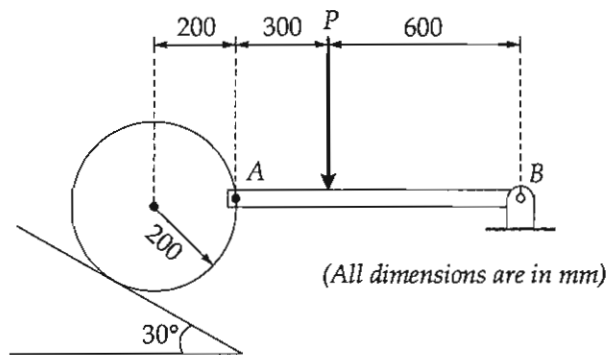
What will be the percentage error in measuring P_s , if the reservoir deflection is neglected?

[12 marks]





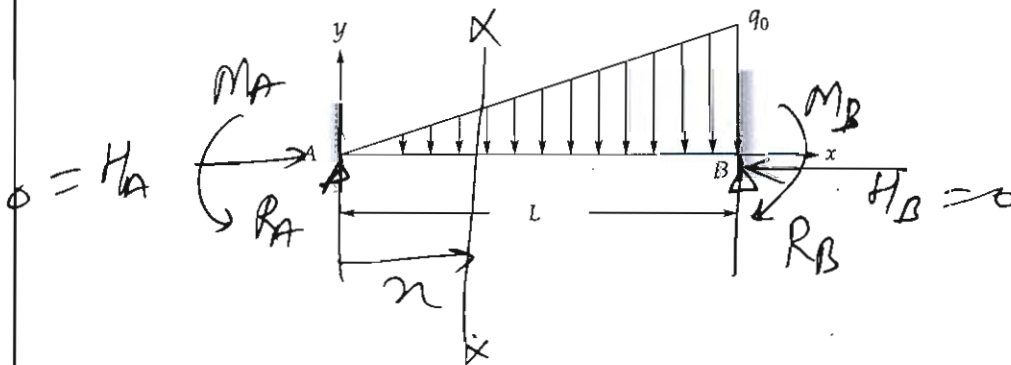
- Q.5 (d) A 40 kg disc rests on an inclined surface for which $\mu_s = 0.3$ as shown in the figure. Determine the maximum vertical force P that may be applied to link AB without causing the disc to slip at C .



[12 marks]



- Q.5 (e) A fixed-end beam of length L is loaded by triangularly distributed load of maximum intensity q_0 at B as shown in the figure. Find reactions at A and B and also the equation of deflection curve.



[12 marks]

$$R_A + R_B = \frac{1}{2} \times q_0 \times L$$

$$EI \frac{d^2y}{dx^2} = M$$

$$EI \frac{d^3y}{dx^3} = \frac{dM}{dx} = SF$$

$$EI \frac{d^4y}{dx^4} = -w_x \quad w_x \text{ ~~downward~~ upward +ve}$$

$$EI \frac{d^4y}{dx^4} = \Rightarrow 90 \times \frac{x}{L} \quad \cdot 90 \times \frac{x}{L}$$

$$EI \frac{d^3y}{dx^3} = -90 \frac{x^2}{2L} + C_1$$

$$EI \frac{d^2y}{dx^2} = -90 \frac{x^3}{6L} + C_1x + C_2$$

$$EI \frac{dy}{dx} = -90 \frac{x^4}{24L} + \frac{C_1x^2}{2} + C_2x + C_3$$

$$\text{at } x=0 \frac{dy}{dx} = 0$$

$$C_3 = 0$$

$$\text{at } x=L \frac{dy}{dx} = 0$$

$$0 = -\frac{90 \times L^4}{24 \times 24} + \frac{C_1 L^2}{2} + \frac{C_2 L}{2} \quad \text{--- (1)}$$

$$EI(y) = -\frac{90x^5}{120L} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_4$$

$$x=0 \quad y=0 \Rightarrow C_4 = 0$$

$$x=L \quad y=0$$

$$0 = -\frac{90 \times L^5}{120L} + \frac{C_1 L^3}{6} + \frac{C_2 L^2}{2}$$

$$C_4 \left[\frac{1}{6} - \frac{1}{4} \right] - 90 \left[\frac{1}{120} - \frac{1}{40} \right] \Rightarrow C_1 = (+) 90 \times L \times (0.15)$$

$$C_2 = \frac{90}{24} L^2 - (0.159L) \times \frac{L}{2} = 90 L^2 (-0.0333)$$

So,

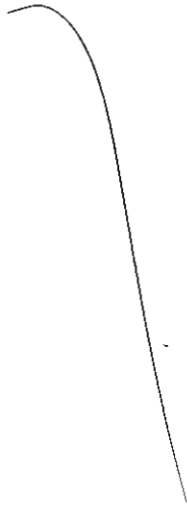
$$EI(y) = -\frac{90}{120} \frac{x^5}{L} + C_1 + 0.1590 \frac{L \times x^3}{6}$$

$$- \frac{0.0333}{2} 90 L^2 \cdot x^2$$

Equation of
deflection

- Q.6 (a) (i) What are metacentric height and metacentric radius? Explain their significance in the stability of floating bodies. Also state the conditions for rotational and translational stability of floating bodies.
- (ii) A solid hemisphere of density ' ρ ' and radius ' R ' floats with its plane base immersed in a liquid of density ' ρ_l ' ($\rho_l > \rho$). Show that the equilibrium is stable and find its metacentric height in terms of ρ , ρ_l and R . Also determine the metacentric height if the radius of hemisphere is 1.5 m and specific gravity of the material of hemisphere is 0.7.

[10 + 10 = 20 marks]

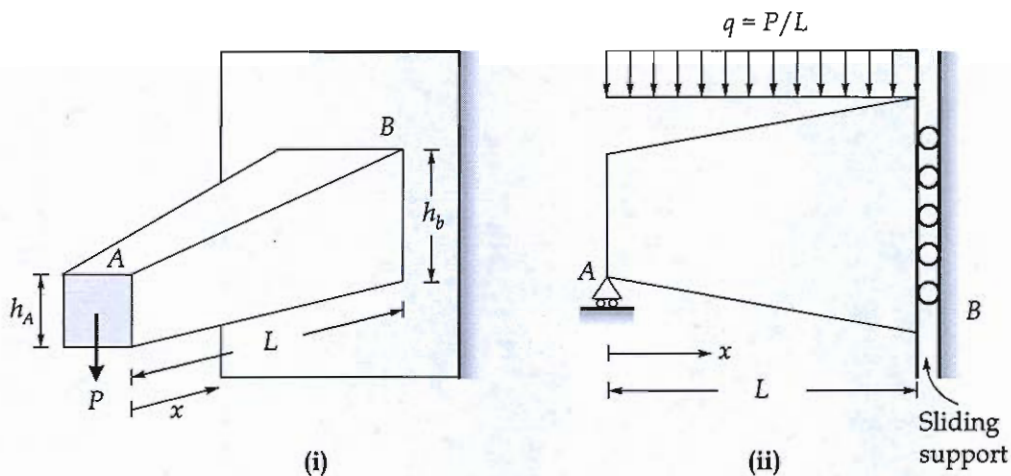




Q.6 (b) A tapered cantilever beam AB of length L has square cross section and supports a concentrated load P at the free end as shown in the figure (i). The width and height of beam vary linearly from h_A at the free end to h_B at the fixed end.

Determine the distance x from the free end A to the cross section of maximum bending stress if $h_B = 3h_A$.

- (i) What is the magnitude σ_{\max} of the maximum bending stress? What is the ratio of the maximum stress to the largest stress σ_B at the support?
- (ii) Repeat part (i) if load P is now applied as uniform load of intensity $q = \frac{P}{L}$ over the entire beam, A is restrained by a roller support and B is a sliding support [See figure (ii)]



[20 marks]





- Q.6 (c) A centrifugal pump has dimensionless specific speed 0.075 rev . The blades are forward facing on the impeller and the outlet angle is 118° to the tangent, with an impeller passage width at the outlet being equal to the one-tenth of the diameter. The pump is to be used to raise water a vertical through distance of 35 m at flow rate of $0.05 \text{ m}^3/\text{s}$. The suction and delivery pipes are each of 150 mm diameter and have a combined length of 40 m with a friction factor of 0.005 . Other losses at the pipe entry, exit, bends etc. are three times the velocity head in the pipes. If the blades occupy 6% of the circumferential area and the hydraulic efficiency (neglecting slip) is 75% . What will be the diameter of the pump impeller?

[20 marks]

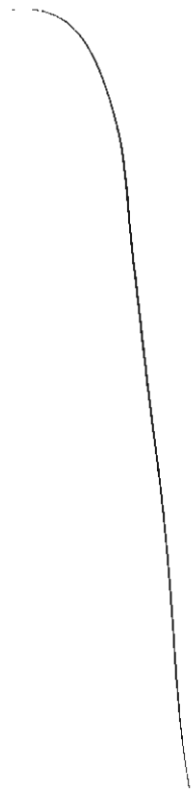


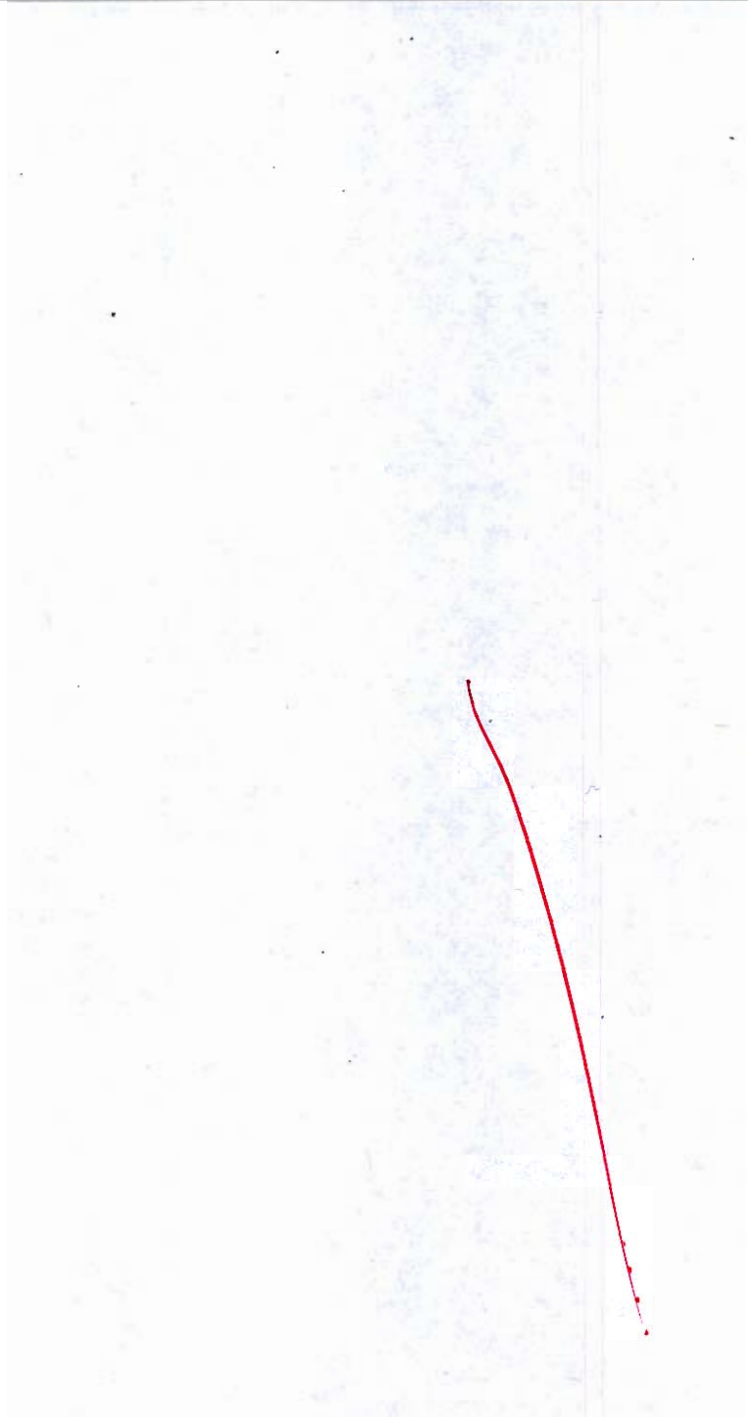


- Q.7 (a) An inward flow vertical shaft reaction turbine runs at a speed of 375 rpm under an available head of 60 m above the atmospheric pressure. The external diameter of the runner is 1.5 m and the dimensionless specific speed based on the power transferred to the runner is 0.14 rev. Water enters the turbine without shock with a flow velocity of 9 m/s and leaves the runner without whirl with an absolute velocity of 7 m/s. The discharge velocity of water at tailrace is 2 m/s. The mean height of runner entry plane is 2 m above the tailrace level while the entrance to the draft tube is 1.7 m above the tailrace level. At the entrance to the runner, the static pressure head is 35 m above the atmospheric pressure, while at the exit from the runner the static pressure head is 2.2 m below the atmospheric pressure. Assuming a hydraulic efficiency of 95%. Calculate
- Runner blade entry angle.
 - Head loss in the guide vanes, in the runner and in the draft tube.

[20 marks]







Q.7 (b) (i) The tangential component of velocity of incompressible fluid in 2-D flow is

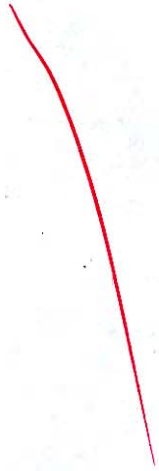
$$v_{\theta} = -\frac{c \sin \theta}{r^2}$$

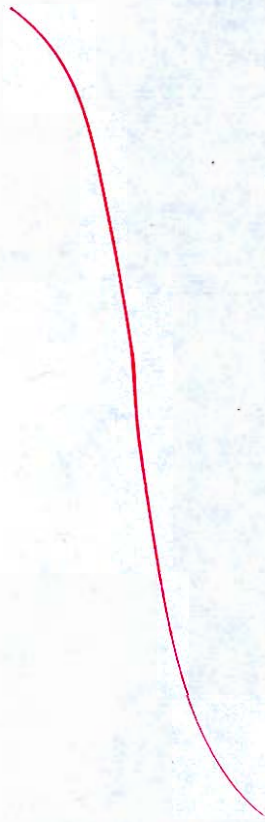
where c is a constant

1. Using continuity equation, determine the expression for radial velocity v_r .
2. Find the magnitude and direction of resultant velocity

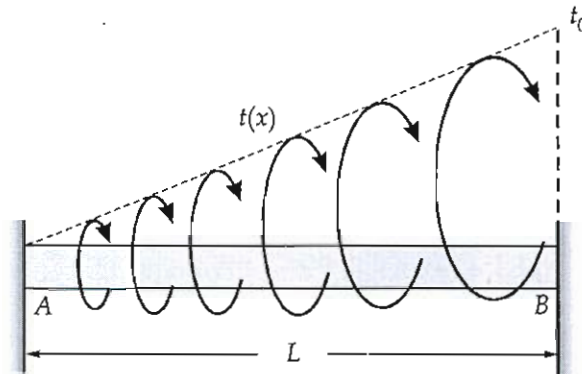
(ii) If the velocity field is given by $u = (16y - 8x)$, $v = (8y - 7x)$, find the circulation around the closed curve defined by $x = 4$, $y = 2$, $x = 8$, $y = 8$.

[10 + 10 = 20 marks]

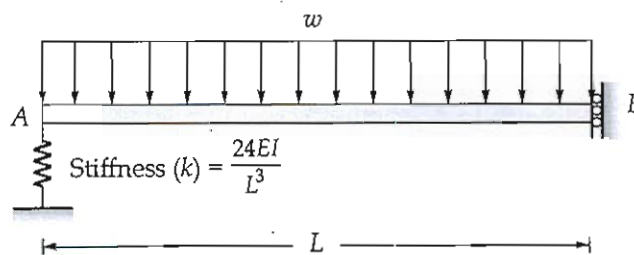




- Q.7 (c) (i) A circular bar AB of length L is fixed against rotation at the ends and loaded by a distributed torque $t(x)$ that varies linearly in intensity from zero at the end A to t_0 at end B .

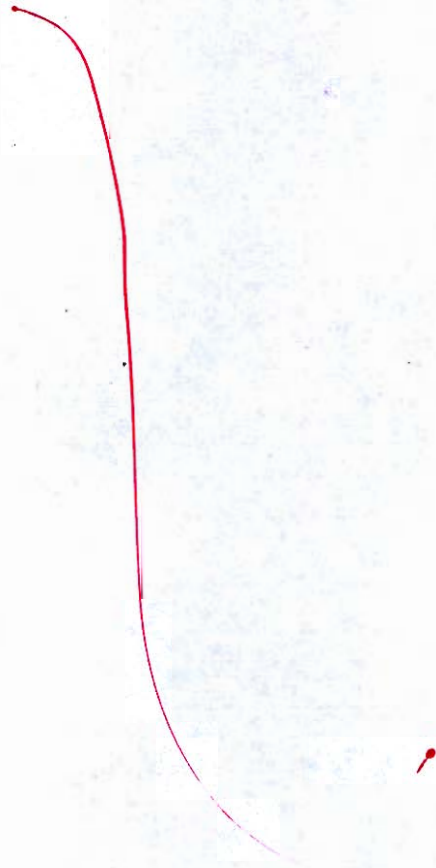


1. Obtain formulas for the fixed-end torques T_A and T_B .
 2. Find an expression for the angle of twist $\phi(x)$. What is ϕ_{\max} and where does it occur along the bar?
- (ii) The beam shown in the figure has a spring support at A and a guided support at B . The guided support permits vertical movement but no rotation. Determine the expression for deflection at B due to the uniform load of intensity w using the second-order differential equation of the deflection curve. The beam is prismatic with constant flexural rigidity EI .



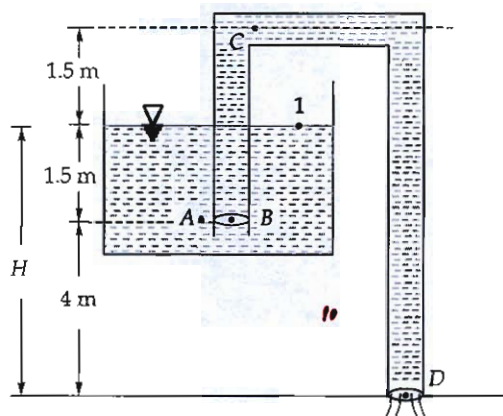
[10 + 10 = 20 marks]





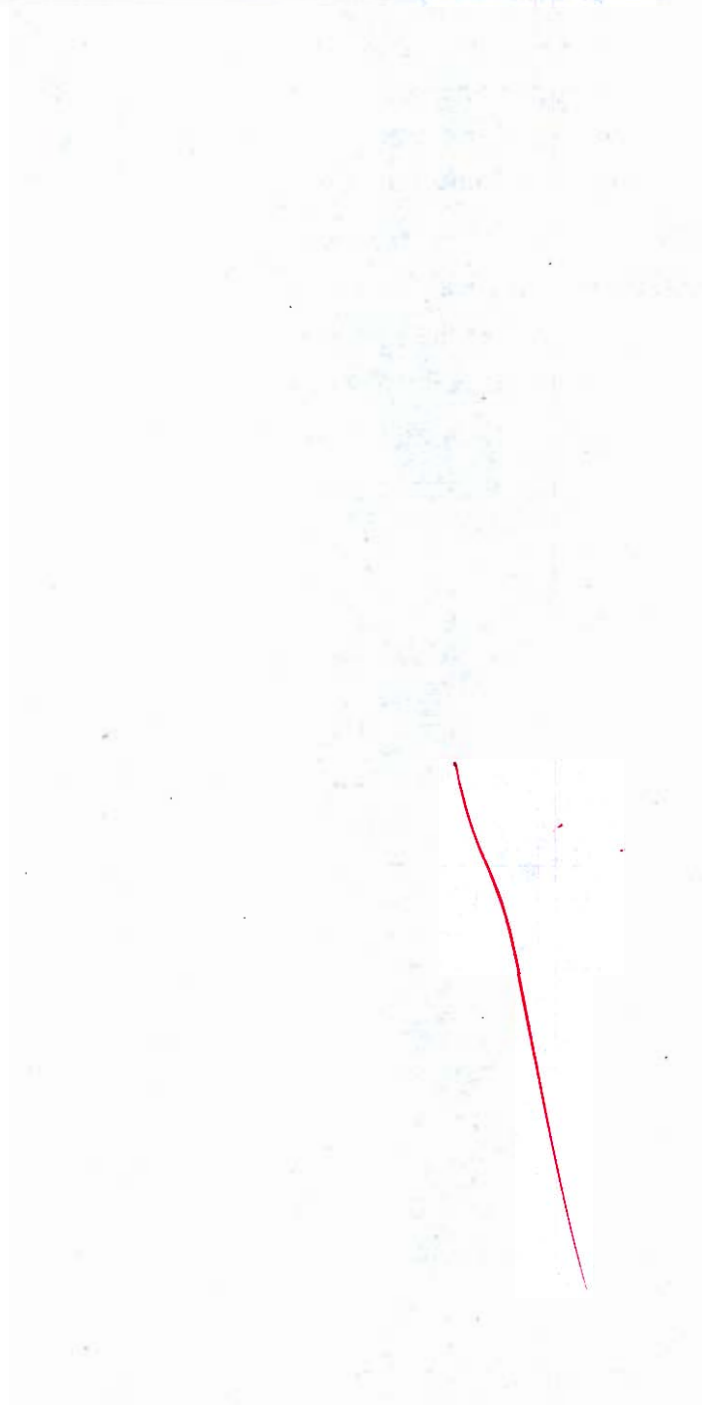


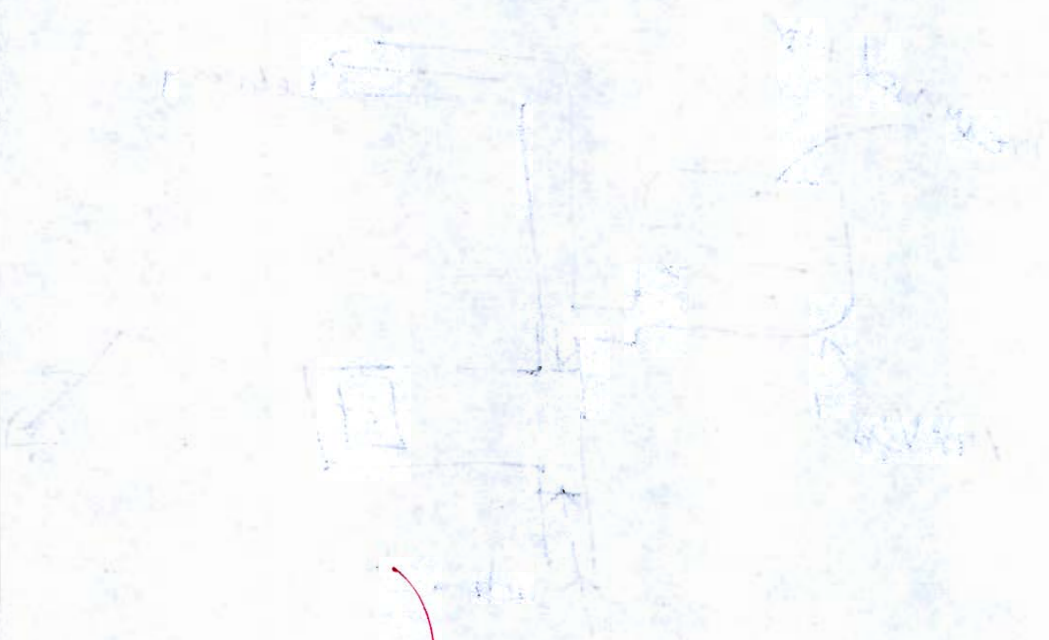
- Q.8 (a) A tube is used as a siphon to discharge an oil of specific gravity 0.8 from a large open vessel into a drain at atmospheric pressure as shown in figure. The vapour pressure of liquid at the working temperature is 30 kPa and atmospheric pressure is 101 kPa. Neglecting the effect of friction. Determine the :
- Pressure at points 'A' and 'B'.
 - Velocity of oil through the siphon.
 - Pressure at the highest point 'C' and maximum height of point 'C' that can be accommodated above the level in the vessel.
 - The maximum vertical depth of right limb of the siphon.



[20 marks]

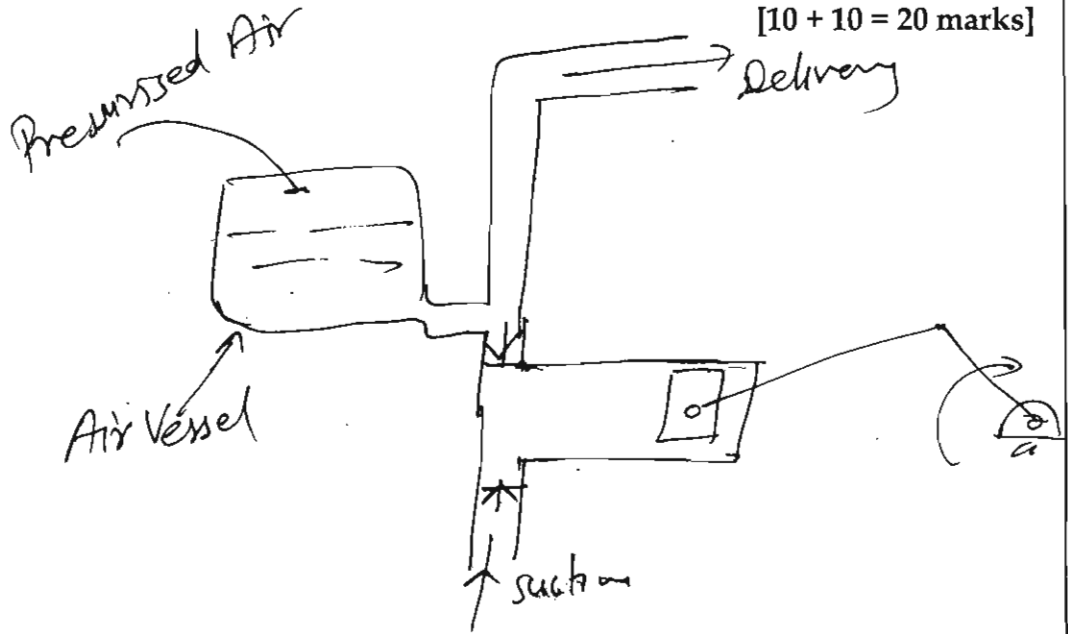






[Faint, illegible handwritten text, possibly bleed-through from the reverse side of the page.]

- Q.8 (b) (i) What is an Air-vessel and Why it is used in reciprocating pump system? Explain its working principle with a neat sketch and discuss the advantages of installing it on the suction and delivery sides of the pump.
- (ii) Derive the expressions of accelerating heads during suction and delivery strokes of a reciprocating pump. Assuming single cylinder and single acting pump. [10 + 10 = 20 marks]



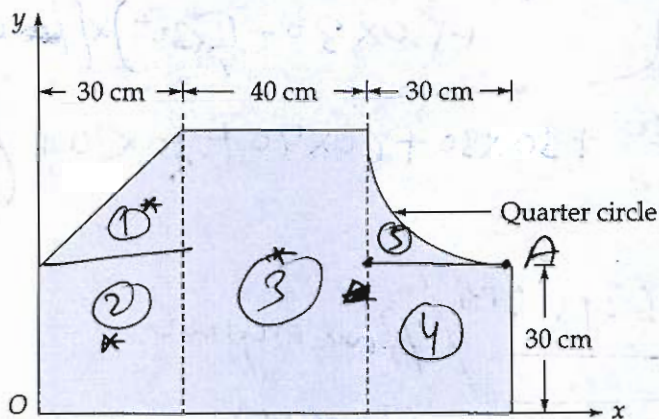
Air vessels are used to maintain constant discharge. When ~~pressure~~ ^{discharge} at the delivery pipe is less than the air vessel supplies the excess water since it has pressurised water filled.

3

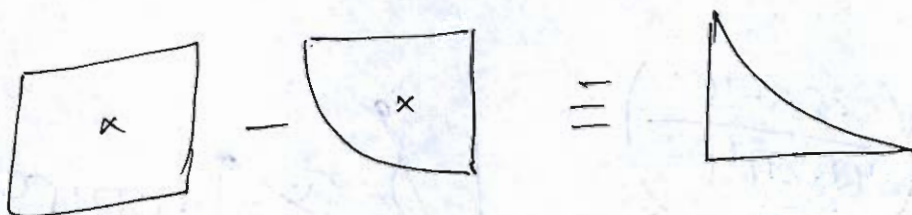




Q.8 (c) Locate the centroid of the composite area shown below and write its coordinates about point O in x-y reference frame. What would be the coordinate of centroid if the shape is rotated about point O clockwise by 37°?



[20 marks]



local reference A

$$\bar{x}_s = \frac{30 \times 30 \times 80 \times 30 \times 15 - \frac{\pi \times 30^2}{4} \times \frac{4 \times 30}{3\pi}}{30 \times 30 - \frac{\pi \times 30^2}{4}}$$

$$\bar{x}_s = 23.299$$

$$\bar{X} = \frac{\frac{1}{2} \times 30 \times 30 \times \frac{2}{3} \times 30 + 30 \times 30 \times 15 + 40 \times 60 \times 50 + 30 \times 30 \times 85 + \left[30 \times 30 - \frac{\pi \times 30^2}{4} \right] \times (100 - 23.299)}{\frac{1}{2} \times 30 \times 30 + 30 \times 30 + 40 \times 60 + 30 \times 30 + \left[30 \times 30 - \frac{\pi \times 30^2}{4} \right]}$$

$$\bar{X} = \frac{\frac{1}{2} \times 30 \times 30 + 30 \times 30 + 40 \times 60 + 30 \times 30 + \left[30 \times 30 - \frac{\pi \times 30^2}{4} \right]}{\dots}$$

$\bar{X} = 48.2774 \text{ cm}$ from origin - O

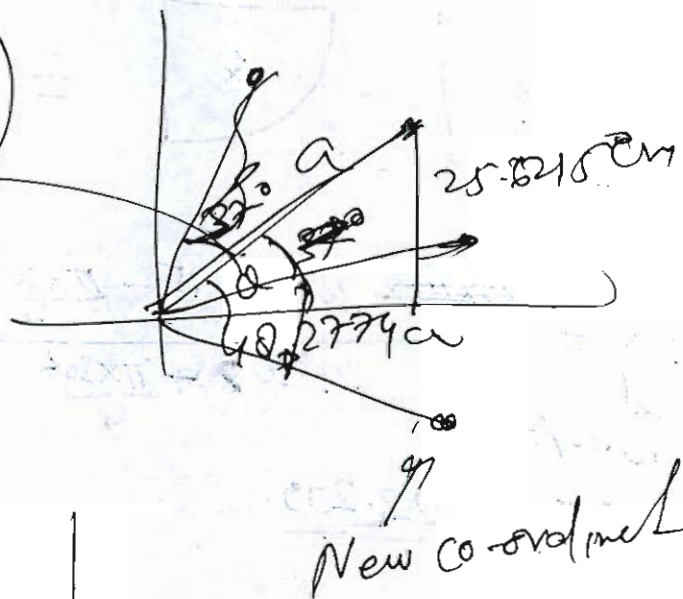
Answer

$$\bar{y} = \frac{\frac{1}{2} \times 30 \times 30 \times (\cancel{30} + \cancel{30} + 40) + 30 \times 30 \times 15 + 60 \times 40 \times 30 + 30 \times 30 \times 15 + (30 \times 30 - \frac{\pi}{4} \times 30^2) \times (60 - 23.299)}{\frac{1}{2} \times 30 \times 30 + 30 \times 30 + 60 \times 40 + 30 \times 30 + (30 \times 30 - \frac{\pi}{4} \times 30^2)}$$

$\bar{y} = 25.6215 \text{ cm}$ from origin
Answer

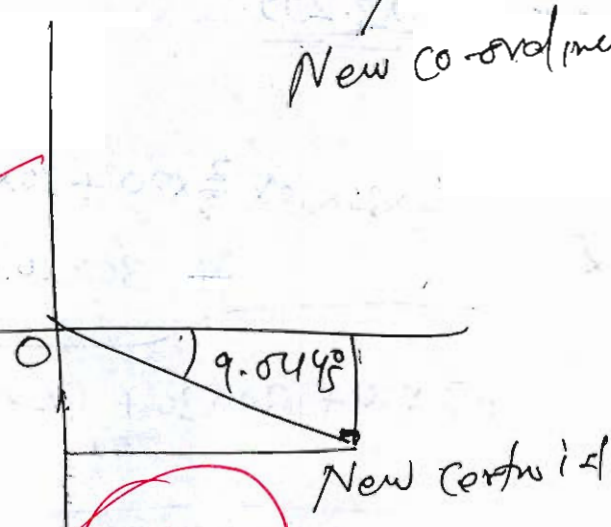
$$\theta = \tan^{-1} \left(\frac{25.6215}{48.2774} \right) = 27.953^\circ$$

$$a = 54.685^\circ$$



$\bar{x}_{new} = 53.9754 \text{ cm}$

$\bar{y}_{new} = -8.5918 \text{ cm}$



18

oooo

Space for Rough Work

Space for Rough Work
