



**MADE EASY**  
Leading Institute for ESE, GATE & PSUs

# ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

## Civil Engineering

Test-3 : Section A : Strength of Materials [All Topics]

Section B : Highway Engineering-1 + Surveying and Geology-1 + Geo-technical & Foundation Engineering - 2 + Environmental Engineering - 2 [Part syllabus]

Name : .....

Roll No :

Test Centres	Student's Signature
Delhi <input checked="" type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input type="checkbox"/> Pune <input type="checkbox"/> Hyderabad <input type="checkbox"/>	

### Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

### FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	55
Q.2	49
Q.3	43
Q.4	
Section-B	
Q.5	39
Q.6	
Q.7	
Q.8	39
<b>Total Marks Obtained</b>	<b>225</b>

Signature of Evaluator

Cross Checked by

## IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

### DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

### DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

→ Very good Attempt.

→ Include units and proper notation

→ Use keywords in theory.

→ Represent answer neat and well structured

→ UPSC does not ask for answer, it also require step-by-step solution along with good representation.

## Section A : Strength of Materials [All Topics]

- (a) A cylindrical specimen of 22 mm diameter is subjected to uniaxial tension over a gauge length of 120 mm. Under an applied axial load of 55.725 kN, the specimen undergoes an elongation of 0.0765 mm. Subsequently, a torsion test is performed on another specimen of identical material and diameter, wherein an applied torque of 130.150 kN-m produces an angular twist of 0.018 radians over a length of 250 mm. Determine:

- (i) Poission ratio of the material.  
 (ii) Shear modulus.  
 (iii) Bulk modulus.  
 (iv) Young's modulus of elasticity.

[12 marks]

Given,  $d = 22 \text{ mm}$   
 $l_g = 120 \text{ mm}$ ,  $P = 55.725 \text{ kN}$ ,  $\Delta l = 0.0765 \text{ mm}$ .

for Torque,

$$T = 130.150 \text{ kNm}, \theta = 0.018 \text{ rad.}, l = 250 \text{ mm}, d = 22 \text{ mm}$$

we know

$$\Delta l = \frac{P l}{A E}$$

$$\Rightarrow 0.0765 = \frac{(55.725 \times 10^3) \times 120}{\frac{\pi}{4} \times 22^2 \times E}$$

$$\Rightarrow E = 229950.65 \text{ N/mm}^2$$

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also,

$$\theta = \frac{T L}{G J}$$

$$0.018 = \frac{(130.150) \times 10^6 \times 250}{G \times \frac{\pi}{32} \times (22)^4}$$

$$\Rightarrow G = 78599730.75 \text{ (unit)?}$$

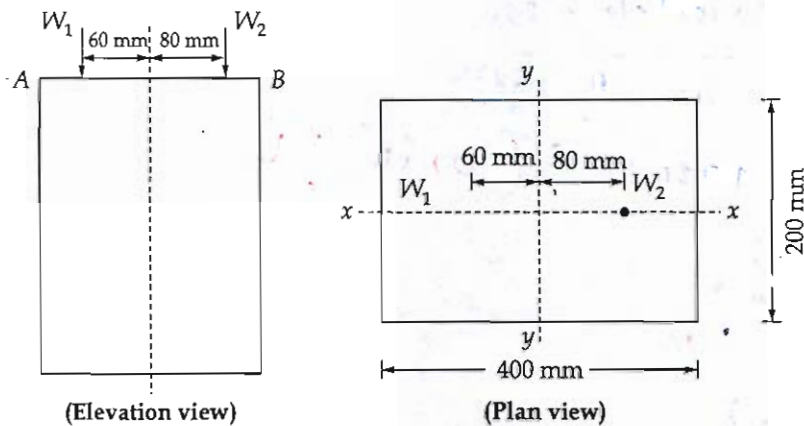
$$\therefore E = 2G(1+\mu)$$

$$\Rightarrow \mu = -0.99$$

$$\& E = 3K(1-2\mu)$$

$$\Rightarrow K = 25575.01 \text{ N/mm}^2$$

- Q.1 (b) A short wooden pillar is rectangular in section  $400 \text{ mm} \times 200 \text{ mm}$ . It carries at the top, two point loads  $W_1$  and  $W_2$  in vertical plane as shown in figure below. If the stress is throughout compressive and extreme stress on the side in which  $W_1$  acts i.e. at A is four times the extreme intensity on the other side i.e. at B, then compute the value of  $W_1$  if  $W_2 = 50 \text{ kN}$ .



[12 marks]

Sim.  $\epsilon_A = 452$  (1)

$w_2 = 50 \text{ kN}$ ,  $w_1 = ?$

$I_{yy} = \frac{200 \times 400^3}{12} = 16.67 \times 10^4 \text{ cm}^4$

$\epsilon_A = \frac{w_1}{A} + \frac{w_2}{A} + \left( \frac{w_1 \times 60}{I} \right) \times 200 - \left( \frac{w_2 \times 80}{I} \right) \times 200$

$\epsilon_B = \frac{w_1}{A} + \frac{w_2}{A} + \left( \frac{w_2 \times 80}{I} \right) \times 200 - \left( \frac{w_1 \times 60}{I} \right) \times 200$

also  $\epsilon_A = 452$

$\therefore \left( \frac{w_1 + w_2}{A} \right) + \frac{200}{I} [w_1 \times 60 - w_2 \times 80] = 4 \left( \frac{w_1 + w_2}{A} \right) + \frac{4 \times 200}{I} [w_2 \times 80 - w_1 \times 60]$

$w_1 \left[ \frac{200 \times 60}{I} + \frac{1}{A} - \frac{4}{A} + \frac{800 \times 60}{I} \right] = \left[ \frac{3}{A} + \frac{800 \times 80}{I} + \frac{200 \times 80}{I} \right] \times 50$

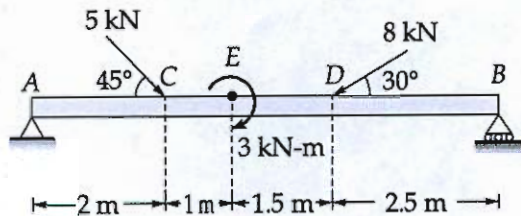
$w_1 \times (1.975 \times 10^{-5}) = 5.625 \times 10^{-3}$

$\therefore w_1 = 300 \text{ kN}$

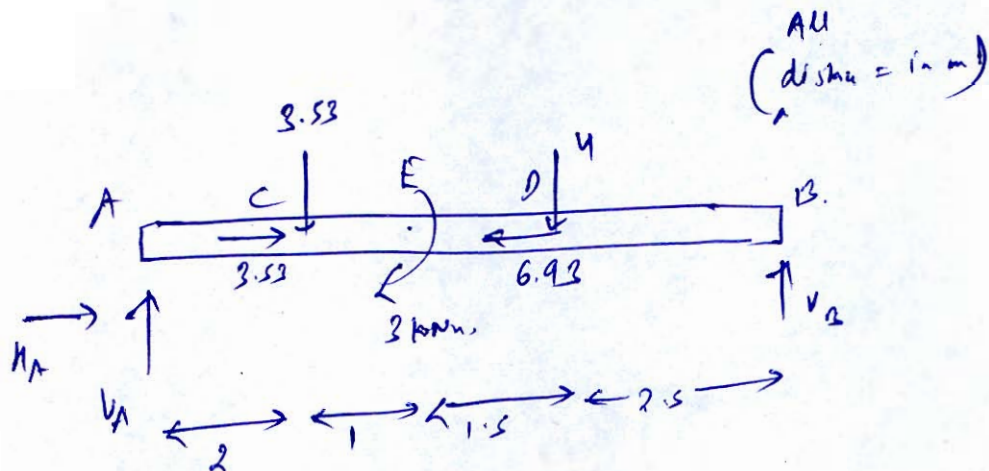
12

*[Faint, illegible handwritten text, likely bleed-through from the reverse side of the page]*

1 (c) A beam is loaded as shown in the figure. Find the reactions at the supports. Draw the BMD and SFD.



[12 marks]



$\therefore H_A + 3.53 - 6.93 = 0$

$H_A = 3.4 \text{ kN}$

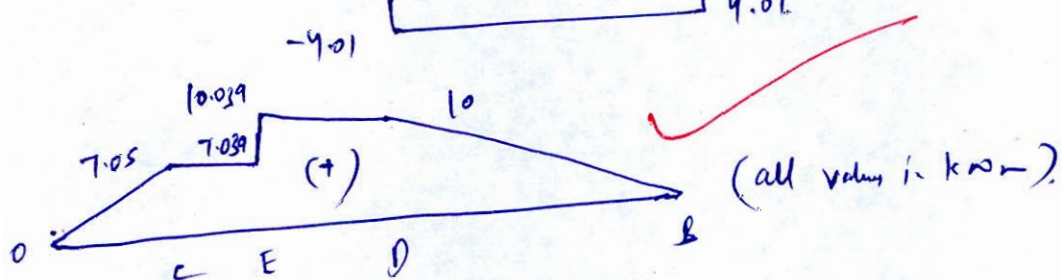
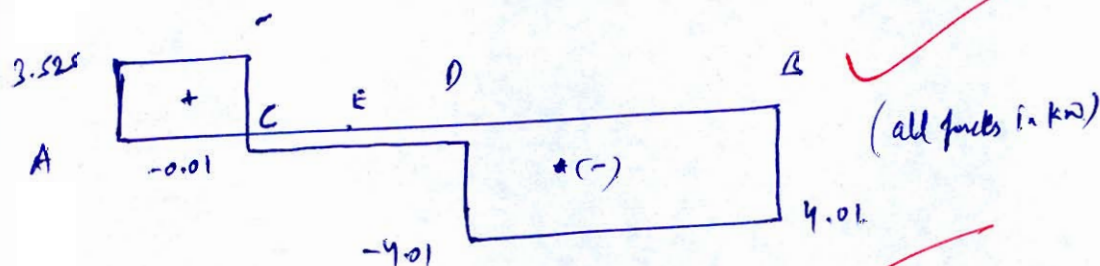
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$\sum M_B = 0$

$\therefore V_A \times 7 - 3.53 \times 5 + 3 - 4 \times 2.5 = 0$

$\therefore V_A = 3.525 \text{ kN}$

$\therefore V_B = 3.53 + 4 - V_A = 4.01 \text{ kN} \approx 4 \text{ kN}$



write the steps of drawing SFD & BMD.



101

201 101 101 101

1 (d) A solid rectangular beam of width 200 mm and depth 300 mm is subjected to a transverse shear force of 100 kN.

Determine:

- (i) Average shear stress in the section.
- (ii) Maximum shear stress developed.
- (iii) Shear stress at a point located 125 mm from the top.

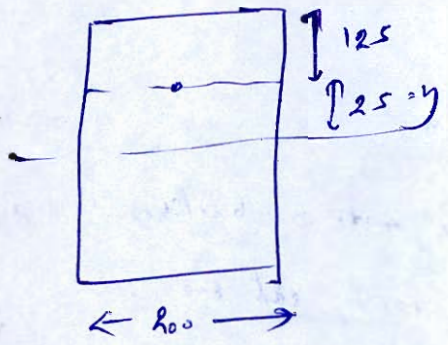
[3 + 3 + 6 = 12 marks]

Soln. Given,  $b = 200\text{ mm}$ ,  $D = 300\text{ mm}$ ,  $V = 100\text{ kN}$ .

i) Avg. Shear stress,  $\tau_{avg} = \frac{V}{A} = \frac{100 \times 10^3}{200 \times 300} = 1.67\text{ N/mm}^2$

ii)  $\tau_{max} = \frac{3}{2} \tau_{avg} = 2.5\text{ N/mm}^2$

iii)  $\tau = \frac{VA\bar{y}}{Ib} = \frac{3}{2} \frac{V}{bh^3} (h^2 - y^2)$



$\tau = \frac{3}{2} \times \frac{100 \times 10^3 \times (300^2 - 4 \times 25^2)}{200 \times (300)^3}$   
 $= 2.43\text{ N/mm}^2$

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Q.1 (e) State the assumptions of Euler's theory of columns. Derive the Euler crippling load for a strut hinged at both the end supports.

[4 + 8 = 12 marks]

Sol<sup>n</sup>.

Assumptions:

- i) failure mode is buckling of column.
- ii) load acting at the centre.
- iii) column is long slender column.
- iv) Material is homogeneous & isotropic.



$$M_x = -ly.$$

$$M_x + ly = 0.$$

$$EI \frac{d^2y}{dx^2} + ly = 0$$

$$\frac{d^2y}{dx^2} + \frac{l}{EI} y = 0$$

$$\left( l^2 + \frac{l}{EI} \right) y = 0 \quad \rightarrow \quad \text{i.e. } (l^2 + m^2) y = 0$$

$$\left( m^2 = \frac{l}{EI} \right)$$

$\therefore$  The sol<sup>n</sup> of the differential eq<sup>n</sup>.

$$y = A \cos mx + B \sin mx \quad \textcircled{0}$$

at  $x=0, y=0$

$$\therefore 0 = A \Rightarrow A=0 \quad (1)$$

lets in (2)

$$\therefore y = B \sin mx$$

and at  $x=L, y=0$

$$\therefore 0 = B \sin(mx)$$

$B \neq 0$ , otherwise  $y=0$  for every  $x$ .

$$\therefore \sin(mx) = 0$$

$$mx = n\pi$$

$$\sqrt{\frac{P}{EI}} \left(\frac{L}{2}\right) = n\pi$$

$$\therefore P = \frac{n^2 \pi^2 EI}{L^2}$$

$\therefore$  for  $x=2$ , i.e.

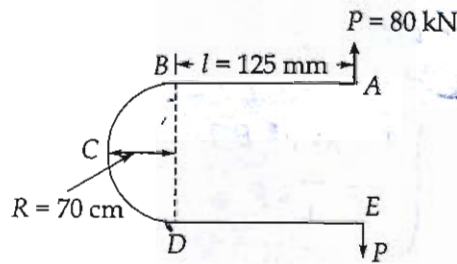
$$P = \frac{n^2 \pi^2 EI}{L^2}$$

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Q.2(a)

A circular bar of diameter 10 mm is bent into a U-shaped configuration consisting of a curved portion of radius 70 cm and two straight limbs, each of length 125 mm, as shown in Fig. Equal loads of 80 kN are applied at the free ends A and E. Determine the relative displacement between points A and E.

Take:  $E = 200 \text{ GPa}$ ,  $I = 45 \times 10^6 \text{ mm}^4$ .



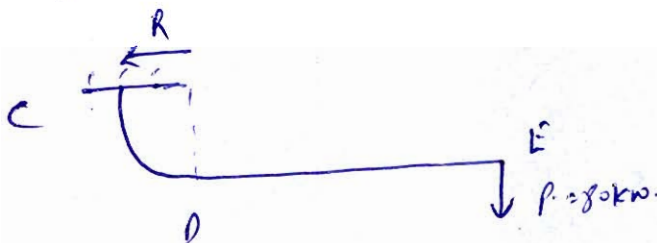
[20 marks]

Sol<sup>n</sup>.

Given  $E = 200 \text{ GPa}$ ,  $I = 45 \times 10^6 \text{ mm}^4$ .

To find relative displacement b/w A & E.

$\therefore$  Symmetric  $\therefore$



$\therefore$  relative displacement =  $2 \times$  displacement from half of the total portion.

for portion DE,

$$M_x = -Px \quad \& \quad \frac{\partial M_x}{\partial P} = -x.$$

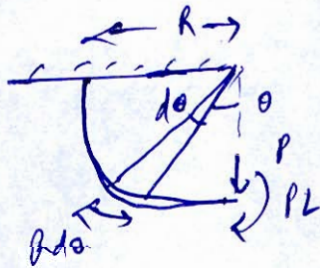


∴ for DE,

$$\delta = \frac{\partial U}{\partial P} = \int_0^L \frac{M_x \cdot \frac{\partial M_x}{\partial P} dx}{EI}$$

$$= \int_0^L \frac{(-Px)(-x)}{EI} dx = \frac{P}{EI} \left. \frac{x^3}{3} \right|_0^L = \frac{PL^3}{3EI} \text{ (1)}$$

for portion DC,



$$M_\theta = -PR \sin\theta - PL, \quad \frac{\partial M_\theta}{\partial P} = -R \sin\theta - L$$

$$\therefore \delta = \frac{\partial U}{\partial P} = \int_0^{\pi/2} \frac{(-PR \sin\theta - PL)(-R \sin\theta - L)(R d\theta)}{EI}$$

$$= \int_0^{\pi/2} \frac{PR(R \sin\theta + L)^2 d\theta}{EI}$$

$$= \frac{PR}{EI} \int_0^{\pi/2} (R^2 \sin^2\theta + L^2 + 2R \sin\theta L) d\theta$$

$$= \frac{PR}{EI} \int_0^{\pi/2} (R^2 \sin^2\theta + L^2 + 2R \sin\theta L) d\theta$$

$$= \frac{PR}{EI} \left[ \frac{R^2}{2} \int_0^{\pi/2} (1 - \cos 2\theta) d\theta + \frac{L^2}{2} \theta + \frac{2RL}{2} (-\cos\theta) \right]_0^{\pi/2}$$

$$= \frac{PR}{EI} \left[ \frac{R^2}{2} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/2} + \frac{L^2}{2} \left( \frac{\pi}{2} \right) - \frac{2RL}{2} [0 - 1] \right]$$

$$= \frac{PR}{EI} \left[ \frac{R^2}{2} \left[ \frac{\pi}{2} \right] + \frac{L^2}{2} \frac{\pi}{2} + \frac{2RL}{2} \right]$$

$$= \frac{(80 \times 10^3) \times (700)}{2 \times 200 \times 10^9} \left[ \frac{(700)^2 \times \pi}{2} + \frac{(125)^2 \times \pi}{2} + 2 \times 700 \times 125 \right]$$

∴ δ = 2.636 mm.

$$\therefore \text{total deflection} = \left[ \frac{Pl^3}{3EI} + 3.636 \right] \times 2$$

$$= \left( \frac{(80 \times 10^3) \times (125)^3}{3 \times 200 \times 10^3 \times 454106} + 3.636 \right) \times 2$$

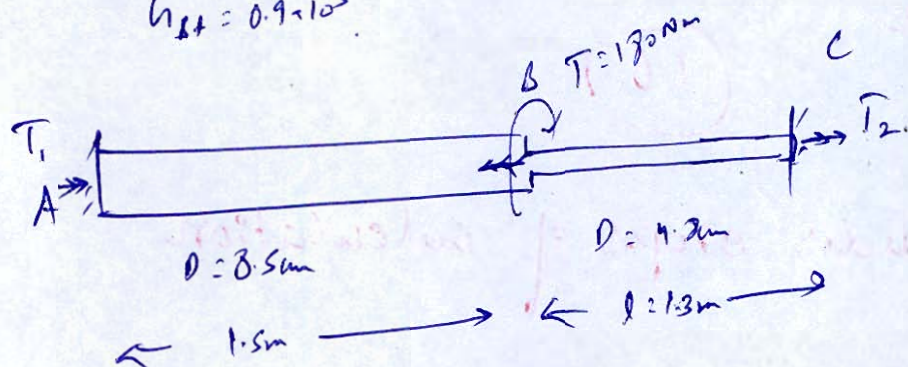
$$= 7.273 \text{ mm}$$

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- 2(b) Two solid circular shafts, AB (aluminium) and BC (steel), are rigidly connected at point B and fixed to rigid supports at points A and C respectively. Shaft AB has a diameter of 8.5 cm and a length of 1.5 m, while shaft BC has a diameter of 4.8 cm and a length of 1.3 m. A torque of 180 N-m is applied at the junction B. Determine the maximum shear stress developed in each shaft. Also, calculate the angle of twist at the junction B. Take: the modulus of rigidity,  $G_{al} = 0.3 \times 10^5 \text{ N/mm}^2$  and  $G_{st} = 0.9 \times 10^5 \text{ N/mm}^2$ .

**[20 marks]**

Given,  $G_{Al} = 0.3 \times 10^5$   
 $G_{St} = 0.9 \times 10^5$



$$T_1 + T_2 = 180 \quad \text{--- (1)}$$

total angle twist = 0 from A to C.

$$\theta_{AB} + \theta_{BC} = 0$$

$$\frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} = 0$$

$$\frac{T_1 L_1}{G_1 J_1} = - \frac{T_2 L_2}{G_2 J_2}$$

$$T_1 \times 1.5 = T_2 \times 1.2$$

$$\frac{(0.3 \times 10^5) \times \pi \times (8.5)^4}{32} \times 1.5 = \frac{(0.9 \times 10^5) \times \pi \times (4.8)^4}{32} \times 1.2$$

$$T_1 = 2.84 T_2$$

$\therefore$  using in (1)  $\rightarrow$  we get,

$$3.84 T_2 = 180$$

$$\Rightarrow T_2 = 46.86 \text{ Nm}$$

$$T_1 = 133.13 \text{ Nm}$$

$$\therefore (\tau_{max})_{Al} = \frac{16 T_1}{\pi D_1^3} = \frac{16 \times 133.13 \times 10^3}{\pi \times (85)^3} = 1.10 \text{ N/mm}^2$$

$$(\tau_{max})_{St} = \frac{16 T_2}{\pi D_2^3} = \frac{16 \times 46.86 \times 10^3}{\pi \times (48)^3} = 2.16 \text{ N/mm}^2$$

angle of Twist at B,

$$\begin{aligned} \theta_{AB} &= \theta_B - \theta_A = \frac{T_1 L_1}{G_1 J_1} \\ &= \frac{(133.13 \times 10^3) \times (1.5 \times 10^3)}{(0.3 \times 10^{-5}) \times \frac{\pi}{32} \times (85)^4} \\ &= 1.298 \times 10^{-2} \text{ radians} \end{aligned}$$

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→ includes steps of calculation

2(c) A short tubular specimen having an internal diameter of 3.5 cm and an external diameter of 5 cm fails in compression under a load of 230 kN. When a 3 m long specimen of the same tube is tested as a strut with both ends fixed, the failure load is found to be 1,80,000 N. Assuming that the crushing stress in Rankine's formula is obtained from the first test, determine the value of the Rankine constant ( $\alpha$ ).

[20 marks]

Given

$$D_i = 3.5 \text{ cm}$$

$$D_o = 5 \text{ cm}$$

$$P_c = 230 \text{ kN}$$

$$\Rightarrow F_c = \frac{P_c}{A} = \frac{230 \times 10^3}{\frac{\pi}{4} \times (50^2 - 35^2)} = 229.67 \text{ N/mm}^2$$

Specimen:

$$L = 3 \text{ m}, L_{eff} = (0.5) \times 3 = 1.5 \text{ m}$$

$$\text{failure load} = 180 \text{ kN}$$

We know,

$$\text{failure load} = \frac{\sigma_c A}{1 + \alpha d^2}$$

$$(180 \times 10^3) = \frac{(229.67) \times \frac{\pi}{4} (50^2 - 35^2)}{1 + \alpha d^2} \quad \text{--- (1)}$$

$$d = \frac{L_{eff}}{n_{min}} = \frac{1500}{\sqrt{\frac{F_{min}}{A}}} = \frac{1500}{\sqrt{\frac{16744 \times \frac{\pi}{4} (50^2 - 35^2)}{180000}}}$$

using i- (1)

$$d = 98.31$$

$$180 \times 10^3 = (230 \times 10^3)$$
$$It \propto (98.31)^2$$

$$\Rightarrow \alpha = 2.37 \times 10^{-5}$$

(15)

→ Represent answer in well structured manner.

→

3(a) For a delta strain rosette mounted on an aluminium specimen, the following strain readings are recorded:

$$\epsilon_{0^\circ} = -100 \mu, \epsilon_{60^\circ} = +700 \mu, \epsilon_{120^\circ} = -600 \mu$$

Determine the principal strains, the corresponding principal stresses.

Take, Young's modulus for aluminium,  $E = 0.8 \times 10^5 \text{ N/mm}^2$ , Poisson's ratio for aluminium,  $\mu = 0.32$ .

[20 marks]

Given.

$$\epsilon_0 = -100 \mu$$

$$\epsilon_{60} = 700 \mu, \epsilon_{120} = -600 \mu$$

$$\therefore \epsilon_{\theta} = \epsilon_0 \cos^2 \theta + \epsilon_{90} \sin^2 \theta + \gamma_{xy} \cos \theta \sin \theta$$

$$700 = (-100) \cos^2 60 + \epsilon_{90} \sin^2 60 + \gamma_{xy} \cos 60 \sin 60$$

$$725 = \epsilon_{90} (0.75) + 0.433 \gamma_{xy} \quad \text{--- (1)}$$

$$\epsilon_{120} = \epsilon_0 \cos^2 120 + \epsilon_{90} \sin^2 120 + \gamma_{xy} \cos 120 \sin 120$$

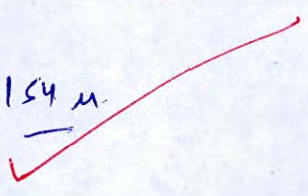
$$-600 = (-100) (0.25) + 0.75 \epsilon_{90} - 0.433 \gamma_{xy}$$

$$-575 = 0.75 \epsilon_{90} - 0.433 \gamma_{xy} \quad \text{--- (2)}$$

(1) + (2)

we get  $\Rightarrow \epsilon_{90} = 100 \mu$

$\therefore$  using in (1) we get  $\gamma_{xy} = 1501.154 \mu$



∴ Principal strains

$$\begin{aligned} \epsilon_{p_1/p_2} &= \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \\ &= \left(\frac{-100 + 100}{2}\right) \pm \sqrt{\left(\frac{-100 - 100}{2}\right)^2 + \left(\frac{150 \cdot 154}{2}\right)^2} \end{aligned}$$

$$\epsilon_{p_1/p_2} = \pm 757.21 \mu$$

∴ Principal stress,

$$\begin{aligned} \sigma_{p_1} &= \frac{E}{1-\mu^2} (\epsilon_{p_1} + \mu \epsilon_{p_2}) \\ &= \frac{0.8 \times 10^5}{1 - 0.32^2} \left[ 757.21 + (0.32)(-757.21) \right] \end{aligned}$$

$$\sigma_{p_1} = 45.89 \text{ N/mm}^2$$

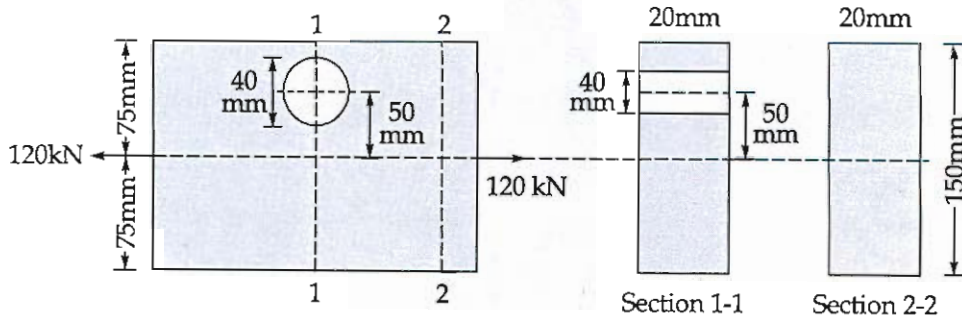
$$\sigma_{p_2} = \frac{0.8 \times 10^5}{1 - 0.32^2} \left[ -757.21 + (0.32)(757.21) \right]$$

$$= -45.89 \text{ N/mm}^2$$

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Q.3 (b) A 150 mm × 20 mm steel plate is subjected to a pull of 120 kN along its longitudinal centroidal axis. A hole of 40 mm diameter is drilled through the plate whose centre is 50 mm from the original longitudinal axis of the bar as shown in figure. Determine the extreme stress induced at section 1-1 and 2-2.



[20 marks]

Soln.

To find extreme stress,

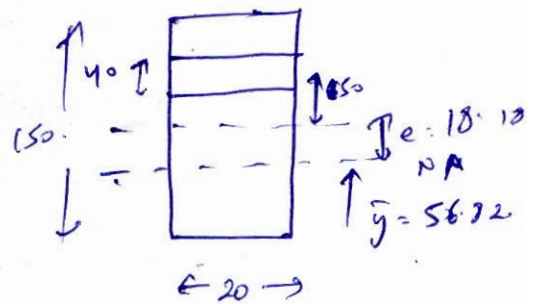
Section 1-1

~~$I = \frac{A^3}{12}$~~

$\bar{y}$  from bottom,

$$\bar{y} = \frac{150 \times 20 \times 75 - (40 \times 20) \times (125)}{150 \times 20 - 40 \times 20}$$

$$\bar{y} = 56.82 \text{ mm}$$



Moment of Inertia, about NA.

$$I = \frac{(150)^3 \times 20}{12} + (150 \times 20) \times (75 - 56.82)^2 - \left[ \frac{20 \times 40^3}{12} + 40 \times 20 \times [50 + (75 - 56.82)]^2 \right]$$

$$= 6616537.2 - 3725476.537$$

$$= 2791060.613$$

∴ Stress at bottom,

(+ve = tensile  
-ve = comp.)

$$\sigma = \frac{P}{A} - \left( \frac{P e}{I} \right) \bar{y}$$

$$= \frac{120 \times 10^3}{(150 \times 20 - 40 \times 20)} - \frac{(120 \times 10^3) \times 18.18 \times 56.82}{2791060.613}$$

$$= 54.54 - 44.41 = 10.13 \text{ N/mm}^2 \quad \left( \begin{matrix} +ve \\ T. \end{matrix} \right)$$

Stress at top,

$$\sigma_t = \frac{P}{A} + \left( \frac{Pe}{I} \right) (75 + 18.18)$$

$$= \frac{120 \times 10^3}{(150 \times 20 - 40 \times 20)} + \frac{(120 \times 10^3 \times 18.18)}{2791060.613} \times (75 + 18.18)$$

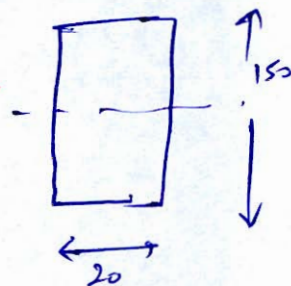
$$= 54.54 + 72.83$$

$$= 127.37 \text{ N/mm}^2 \quad \begin{matrix} (+ve) \\ \text{(Tensile)} \end{matrix}$$

for steel (2-2)

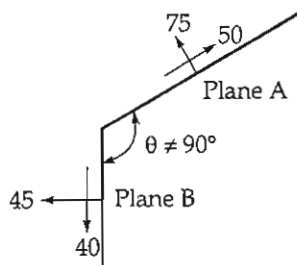
at bottom

$$\sigma_b = \sigma_{bottom} = \frac{P}{A} = \frac{120 \times 10^3}{150 \times 20} = 40 \text{ N/mm}^2 \quad \begin{matrix} (+ve) \\ \text{(Tensile)} \end{matrix}$$



18

- Q.3(c)
- (i) A circular shaft  $ABC$ , having a total length of 60 cm and an external diameter of 45 mm, is hollow for its entire length. Over a portion  $AB$ , the shaft has an internal diameter of 18 mm, while over the remaining portion  $BC$ , the internal diameter is 35 mm. If the maximum permissible shear stress in the shaft is limited to  $75 \text{ N/mm}^2$ . Determine the maximum power that can be transmitted by the shaft when it rotates at a speed of 250 rpm. Further, if the angle of twist in the portion having 18 mm bore is equal to that in the portion having a 35 mm bore, determine the lengths of the shaft portions bored to 18 mm and 35 mm diameters.
- (ii) At a point in a material subjected to two dimensional stresses, the stresses on a certain plane are  $75 \text{ N/mm}^2$  (tension) and  $50 \text{ N/mm}^2$  (shear) and on another plane the stresses are  $45 \text{ N/mm}^2$  (tension) and  $40 \text{ N/mm}^2$  (shear) as shown in figure. Find the principal stresses.



(All values are in  $\text{N/mm}^2$ )

[10 + 10 = 20 marks]

i). Given, total length = 60m.

$$r_e = 45 \text{ mm.}$$

$$(D_i)_{AB} = 18 \text{ mm.}, \quad (D_i)_{BC} = 35 \text{ mm.}$$

$$(\tau_{max})_{\text{shear}} = 75 \text{ N/mm}^2, \quad N = 250 \text{ rpm.}$$

Time for AB,

$$\tau_{max} = \frac{16T}{\pi r^3 (1 - k^4)}$$

$$75 = \frac{16 \times T}{\pi \times (45)^3 \left(1 - \left(\frac{18}{45}\right)^4\right)}$$

$$\Rightarrow T = 1.31 \text{ kNm.}$$

Time for BC,

$$75 = \frac{16 \times T}{\pi \times 45^3 \left(1 - \left(\frac{35}{45}\right)^4\right)}$$

$$\Rightarrow T = 0.85 \text{ kNm.}$$

$\therefore$  Torque that can be applied = min of above  $T = 0.85 \text{ kNm.}$

$$\therefore P_{max} \text{ transmitted: } T_{\omega} = \frac{(0.85 \times 10^3) \times 250}{60}$$

$$P_{max} = 3541.67 \text{ W.}$$

also, given  $\theta_{AB} = \theta_{BC}$ .

$$\frac{T_1 L_1}{G J_1} = \frac{T_2 L_2}{G J_2}$$

$$\frac{L_1}{(45^4 - 18^4)} = \frac{L_2}{(45^4 - 35^4)} \Rightarrow L_1 = 1.536 L_2.$$

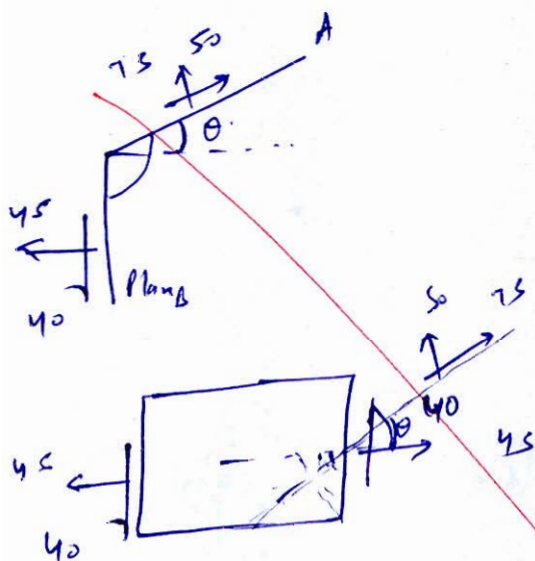
$$d_1 = 1.536 d_2$$

$$2 d_1 + d_2 = 60 \text{ mm}$$

$$\therefore d_2 = 23.65 \text{ cm} \rightarrow \text{i.e. } (d_i = 35 \text{ mm})$$

$$d_1 = \underline{36.34 \text{ cm}} \rightarrow \text{i.e. } (18 \text{ mm } d_i)$$

ii)



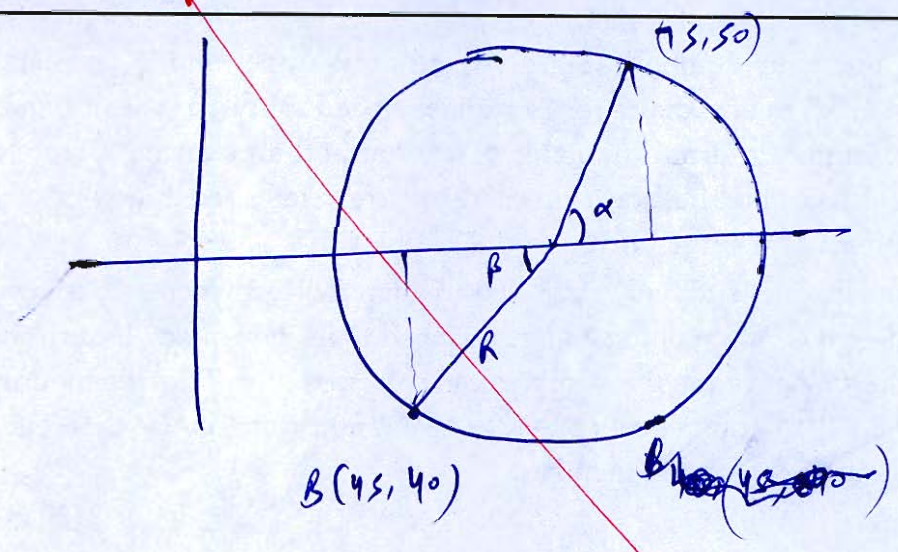
$$\therefore \text{Angle made by Plane A with vertical} = -(90^\circ - \theta) = \theta - 90^\circ$$

$$\therefore \sigma_\theta = \left( \frac{\sigma_x + \sigma_y}{2} \right) + \left( \frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\therefore 75 = \left( \frac{45 + 40}{2} \right) + \left( \frac{45 - 40}{2} \right) \cos 2(\theta - 90^\circ) + \tau_{xy} \sin 2(\theta - 90^\circ)$$

$$75 = 0.5(45 + 40) + (45 - 40) \cdot 0.5(-\cos 2\theta) - 40 \sin 2\theta$$

$$75 = 0.5(45 + 40) - 0.5(45 - 40) \cos 2\theta - 40 \sin 2\theta \quad \text{--- (1)}$$



~~Ans:  $r = 45$~~   
 $R \sin \beta = 40$

$R \sin \alpha = 50$

~~Eqn~~  $R \cos \alpha + R \cos \beta = 30$

Q.4(a) A vertical compound tie member is rigidly fixed at its upper end. It consists of a steel rod of length 3.5 m and diameter 24 mm, enclosed within a brass tube of the same length having an internal diameter of 25 mm and an external diameter of 40 mm. The steel rod and the brass tube are securely fastened together at both ends so that they act as a single unit.

The compound member is suddenly subjected to a tensile load when a 25 kN weight falls through a height of 3 mm onto a flange attached to its lower end. Determine the maximum stresses developed in the steel rod and the brass tube. Take the modulus of elasticity as:  $E_s = 2 \times 10^5 \text{ N/mm}^2$  (steel),  $E_b = 1.0 \times 10^5 \text{ N/mm}^2$  (brass). Also determine the instantaneous elongation of the tie member.

[20 marks]



- Q.4(b) A bolt is subjected simultaneously to an axial tensile load of 750 kg and a transverse shear load of 400 kg. Using appropriate theories of failure, determine the required diameter of the bolt based on:
- Assume a factor of safety of 2.5.
- (a) Maximum principal stress theory.
  - (b) Maximum shear stress theory.
  - (c) Maximum shear strain energy theory.
  - (d) Strain energy theory.

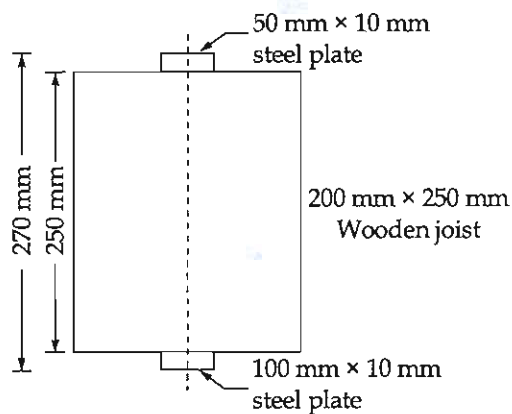
The material properties of the bolt are: Yield strength =  $3010 \text{ kg/cm}^2$ ,  $\mu = 0.33$ .

[20 marks]

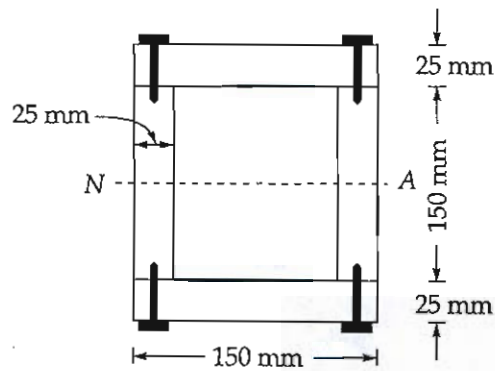




- 4 (c) (i) Find the moment of resistance of the flitched beam section as shown in figure below if the stress in steel and wood are not to exceed  $150 \text{ N/mm}^2$  and  $7.5 \text{ N/mm}^2$  respectively. Take  $E_s = 2 \times 10^5 \text{ N/mm}^2$  and  $E_w = 1 \times 10^4 \text{ N/mm}^2$ .



- (ii) The box beam as shown in figure is made up of four  $150 \text{ mm} \times 25 \text{ mm}$  wooden planks connected by screws. Each screw can safely transmit a shear force of  $1250 \text{ N}$ . Estimate the minimum necessary spacing of screws along the length of the beam if the maximum shear force transmitted by the cross-section is  $5000 \text{ N}$ . Sketch corresponding shear stress distribution across the section.



[10 + 10 = 20 marks]





**Section B : Highway Engineering-1 + Surveying and Geology-1 + Geo-technical & Foundation  
Engineering - 2 + Environmental Engineering - 2**

- 5(a) A rectangular raft foundation measuring 3.0 m × 4.5 m is to be constructed at a depth of 1.6 m below the ground surface. The soil supporting the foundation has a unit weight of 1.65 t/m<sup>3</sup>, a cohesion of 1.2 t/m<sup>2</sup>, and an angle of internal friction of 21.9°. Assuming shear failure, determine the maximum safe load that the footing can carry using a factor of safety of 2.5. Use Terzaghi's bearing capacity theory. Relevant bearing capacity factors corresponding to the angle of internal friction are provided in the table.

$\phi'$	$N_c$	$N_q$	$N_\gamma$
0°	5.7	1.0	0
5°	6.7	1.4	0.2
10°	8.0	1.9	0.5
15°	9.7	2.7	0.9
20°	11.8	3.9	1.7
25°	13.7	5.9	2.80

[12 marks]

Soln.

Given,

raft size  $23 \times 4.5 \text{ m}$ 

$$q_g = 1.6 \text{ m}^2, \quad \gamma = 1.65 \text{ t/m}^3, \quad c = 1.2 \text{ t/m}^2, \quad \phi = 21.9^\circ$$

$$FOS = 2.5$$

$$N_c, N_q, N_{\gamma} \text{ for } \phi = 21.9^\circ \text{ are,}$$

$$N_c = 11.8 + \frac{(13.7 - 11.8)}{5} (1.9) = 12.522$$

$$N_q = 3.9 + \frac{(2)}{5} \times 1.9 = 4.66$$

$$N_{\gamma} = 1.7 + \frac{(2.8 - 1.7)}{5} \times 1.9 = 2.118$$

local shear case

$$\tan \phi_m = \frac{2}{3} \tan \phi$$

As per IS 800:

$$q_u = \left(1 + 0.3 \frac{b}{L}\right) c N_c + q N_q + 0.5 \left(1 - 0.2 \frac{b}{L}\right) \times B \gamma N_{\gamma}$$

$$= \left(1 + 0.3 \times \frac{3}{4.5}\right) \times 1.2 \times 12.522 + (1.65 \times 1.6) \times 4.66 + 0.5 \times \left(1 - 0.2 \times \frac{3}{4.5}\right) \times 3 \times 1.65 \times 2.118$$

$$q_u = 30.33 + 4.54 = 34.87 \text{ t/m}^2$$

$$q_s = \frac{q_u - q}{F}$$

$$q_s = \frac{34.87 - 1.65 \times 1.6 + 1.65 \times 1.6}{2.5} = 15.53 \text{ t/m}^2$$

0

5 (b) Results of two plate load tests corresponding to a settlement of 25.4 mm are given. A circular plate of diameter 0.30 m requires a load of 40 kN, while a circular plate of diameter 0.75 m requires a load of 150 kN. A square column foundation is to be designed to carry a load of 1200 kN with the same allowable settlement of 25.4 mm. Determine the size of the square foundation.

[12 marks]

Sol<sup>n</sup>. Circular Plate 1, | Plate 2,  
 $D = 0.3m, P = 40kN,$  |  $D = 0.75m, P = 150kN$

We know

$$Q = A_m + P_m$$

$$\therefore 40 = \frac{\pi}{4} \times 0.3^2 \times m + \pi \times 0.3 \times n \rightarrow \text{Plate 1}$$

$$40 = 0.07m + 0.94n \quad \text{--- (1)}$$

for Plate 2

$$150 = \frac{\pi}{4} \times 0.75^2 \times m + \pi \times 0.75 \times n$$

$$150 = 0.44m + 2.36n \quad \text{--- (2)}$$

$$[40 = 0.07m + 0.94n] \times 0.44 \rightarrow 17.6 = 0.0308m + 0.4136n$$

$$150 = 0.0308m + 0.1652n$$

$$(150 = 0.44m + 2.36n) \times 0.07 \rightarrow$$

$$\therefore 7.1 = 0.2156m + 0.1652n$$

$$\Rightarrow n = 28.53$$

$$\therefore m = 187.6$$

$$m = 137.6, n = 23.58$$

$$\therefore S = Am + Pn \quad \text{for } S_n$$

$$1200 = (b^2)137.6 + (4b)23.58$$

$$\Rightarrow 137.6b^2 + 94.32b - 1200 = 0$$

$$\Rightarrow b = 8.24 \text{ m}$$

12

1.5 (c) (i) A rectangular sewer having width as 1.5 times of its depth is hydraulically equivalent to a circular sewer. Find the relation between the width of the rectangular sewer and the diameter of the circular sewer.

(ii) In a test conducted for determining the relative stability at  $20^{\circ}\text{C}$ , the period of incubation was found to be 11 days. Calculate the percent of relative stability ( $S$ ).

[8 + 4 = 12 marks]

sol<sup>n</sup> i)  
Given  $B = 1.5d$   $\rightarrow$  for rect.  
A rect & circ are hydraulically equivalent.  
we know, for hydraulic equivalent  
 $Z = \text{const.}$

$$ZRS = \text{const.}$$

$$\sqrt{R_1 S_1} = \sqrt{R_2 S_2}$$

$$h_1 = h_2$$

(assume full depth flow)

$$\therefore \frac{(B) \times D}{2(B+D)} = \frac{y}{4}$$

( $y = \text{dia. of circular sewer}$ )

$$\frac{(1.5D) \times D}{2 \times 2.5D} = \frac{y}{4}$$

$$\Rightarrow y = 1.2D$$

10

ii) we know,  
relative stability,  $S = 100 \left( 1 - (0.793)^{t_{20}} \right)$   
 $= 100 \left( 1 - (0.793)^{11} \right)$   
 $= 92.20\%$

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5(d) The maximum increase in temperature after the construction of a cement concrete (CC) pavement is expected to be 24°C. If the expansion joint gap is 2.0 cm, determine the required spacing between the expansion and contraction joints using the following data: the coefficient of thermal expansion ( $\alpha$ ) is  $12 \times 10^{-6}$  per °C, the unit weight of concrete ( $w$ ) is 2400 kg/m<sup>3</sup>, the allowable tensile stress ( $\sigma_{at}$ ) is 0.9 kg/cm<sup>2</sup>, and the coefficient of interface friction ( $f$ ) is 1.5.

[12 marks]

01<sup>m</sup>.

Given,  $\Delta T = 24^\circ C$ .  
 gap = 2cm. ,  $\alpha = 12 \times 10^{-6} / ^\circ C$  ,  $\gamma_w = 2400 \text{ kg/m}^3$  ,  $\sigma_{at} = 0.9 \text{ kg/cm}^2$  ,  
 $f = 1.5$ .

for expansion joint,

$$l \alpha \Delta T = \frac{\text{gap}}{2}$$

$$l \times (12 \times 10^{-6}) \times 24 = \frac{0.02}{2}$$

$$l = 34.72 \text{ m} < 140 \text{ m} \text{ (OK)}$$

10

for contraction joint,

$$f \alpha \left( \frac{L}{2} B h \right) \times \alpha = \frac{\sigma_{at}}{B h}$$

$$L = \frac{2 \sigma_{at}}{f \alpha} = \frac{2 \times 0.9 \times 10^4}{2400 \times 1.5} = 5 \text{ m} > 4.5 \text{ m}$$

$\therefore$  Spacing should be limited to 4.5m.

- Q.5 (e) (i) Differentiate between Plane Surveying and Geodetic Surveying.  
(ii) What are the principles of surveying? Explain "working from whole to part."

[6 + 6 = 12 marks]

i). Plane Surveying

Geodetic Surveying

1. Earth's curvature is not taken into account

Effect of earth's curvature is considered.

2. done when surveying to be done in small area.

when survey to done is large area.

3

3. Plumb lines in case of plane survey are  $\perp$  to each other.

Plumb lines in this case are intersecting in nature

4

Whole to part:

In whole to part, large ~~scale~~ measurements are taken with higher degree of precision and smaller measurements are taken with lower degree of precision. This is done in order to localize the error & doesn't let the effect of error in the other case.

Principles of Surveying:

- i) working from whole to part
- ii)  $\odot$  At least <sup>one</sup> angle and side may be measured for subsequent measuring & surveying.
- iii) Surveying with triangles.

- Q.6(a) Explain the different type of Geosynthetics with their primary properties. Do mention atleast one field application of each. What are the key design considerations while selecting them for design?

[20 marks]





- 6 (b) (i) What is meant by "face left" and "face right" of a theodolite? What instrumental errors are eliminated by taking observations on both face left and face right?
- (ii) A valley curve of a State Highway is formed by a descending gradient of 1 in 20 meeting an ascending gradient of 1 in 30. Design the length of the valley curve to fulfill both the comfort condition and the headlight sight distance requirement for a design speed of 80 kmph. Assume the allowable rate of change of centrifugal acceleration,  $C = 0.60 \text{ m/sec}^3$ . Suggest the most suitable shape of the valley curve. Consider reaction time,  $t = 2.5 \text{ sec}$ , and coefficient of longitudinal friction,  $f = 0.35$ .
- [8 + 12 = 20 marks]





- Q.6 (c) (i) An observer standing on the deck of a ship just sees a lighthouse. The top of the lighthouse is 64 m above sea level, and the height of the observer's eye is 16 m above sea level. Find the distance of the observer from the lighthouse.
- (ii) Design the size of an oxidation pond for a residential community of 1,200 persons contributing sewage at 150 litres/capita/day. The 5-day BOD of the sewage is 250 mg/L.
- Design Constraints:**  
Organic loading rate in the pond = 250 kg/ha/day.  
Length-to-Width ratio ( $L: B$ ) = 2:1  
Assume suitable water depth and calculate the detention time.
- (iii) Write about the common defects to be observed in design life of a rigid pavement.

[4 + 8 + 8 = 20 marks]







Q.7(a)

Determine the Safe load capacity of a driven concrete pile with a diameter of 500 mm and a total length of 18 m. The pile penetrates through a multi-layered soil profile consisting of three distinct strata:

**Layer 1 (Top):** Soft clay extending from the ground surface to a depth of 6 m. The undrained cohesion is  $30 \text{ kN/m}^2$  and the adhesion factor is 0.8. Unit weight is  $18.5 \text{ kN/m}^3$ .

**Layer 2 (Middle):** Medium dense sand from 6 m to 14 m depth. The angle of internal friction ( $\phi$ ) is  $32^\circ$ , the unit weight is  $18.5 \text{ kN/m}^3$ , the earth pressure coefficient ( $k$ ) is 1.2, and the wall friction angle ( $\delta$ ) is  $0.75\phi$ .

**Layer 3 (Bottom):** Stiff clay from 14 m to 18 m. The undrained cohesion is  $100 \text{ kN/m}^2$ , the adhesion factor is 0.45, and the bearing capacity factor ( $N_c$ ) is 9.

Assume the water table is at great depths and consider the arching effect (critical depth) for the calculations in the sandy layer. For medium dense sand, assume a critical depth ( $L_c$ ) equal to  $15 \times$  diameter.

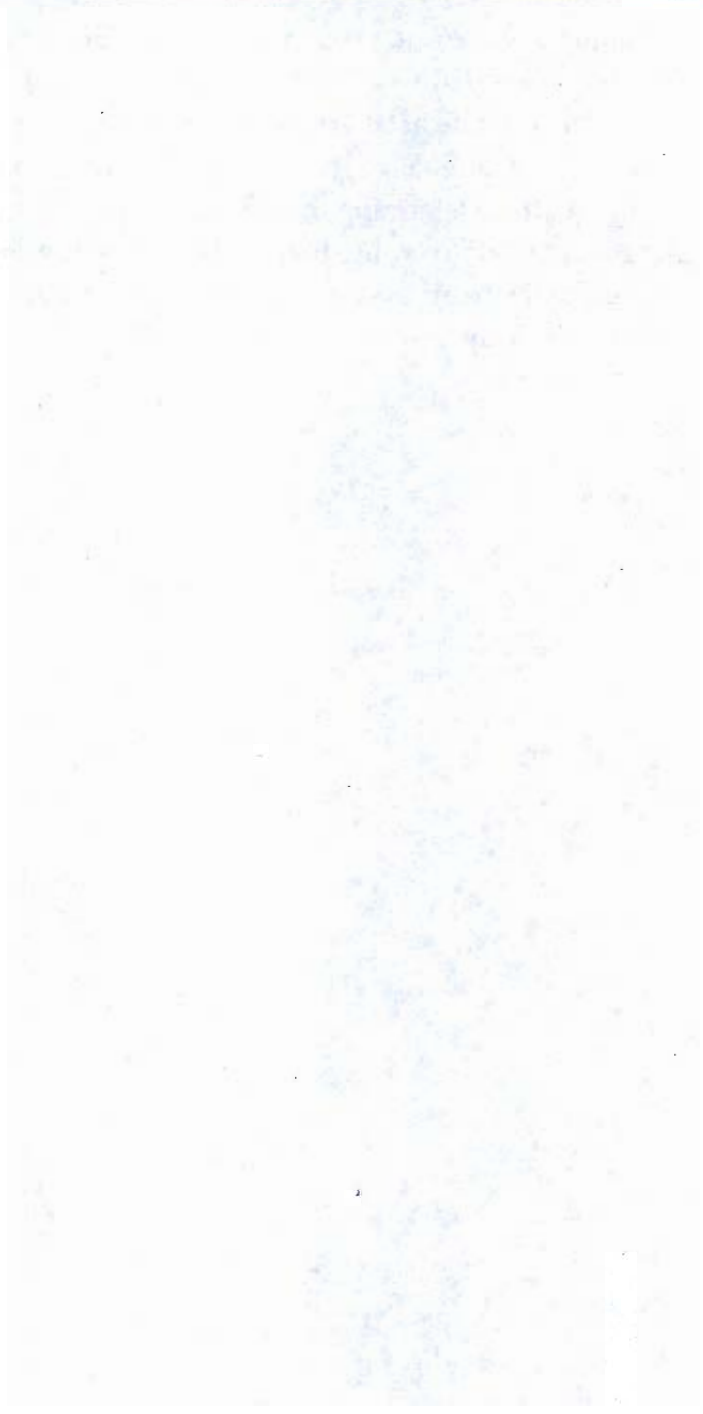
[20 marks]





- 7(b) You are tasked with designing a wastewater treatment facility for a city that generates 1500 L/s of sewage with a 5-day BOD ( $20^{\circ}\text{C}$ ) of 200 mg/L. The sewage is to be discharged into a stream with a minimum flow rate of 6000 L/s, where the existing BOD is 1 mg/L and the dissolved oxygen (DO) is at 90% of its 9.17 mg/L saturation level. Both the sewage and the stream are at  $20^{\circ}\text{C}$ . Determine the minimum percentage reduction in BOD required at your treatment plant to ensure that the DO concentration in the stream never falls below 4.5 mg/L downstream. Assume de-oxygenation and re-oxygenation coefficients of 0.1 and 0.3 respectively.

[20 marks]



- (c) (i) A four-sided closed traverse PQRS has been surveyed with the following lengths and Whole Circle Bearings (WCB). Compute the corrected latitudes and departures using Bowditch rule. Also calculate the angle of closure and its direction.

Line	Length (m)	WCB
PQ	150.00	45°00'
QR	125.00	130°00'
RS	160.00	220°30'
SP	132.00	315°00'

- (ii) Explain briefly any four test to be performed on bitumen before using pavement construction.

[12 + 8 = 20 marks]





- Q.8 (a) (i) A fill 12 m high is to be constructed with a factor of safety of 1.25. The soil has cohesion  $c = 20 \text{ kN/m}^2$ , angle of internal friction  $\phi = 15^\circ$ , and unit weight  $\gamma = 17.0 \text{ kN/m}^3$ . Determine the required inclination of the filling.

Stability number	Mobilized friction angle ( $\phi_m$ ) = $12^\circ$	
	Inclination $i = 30^\circ$	Inclination $i = 45^\circ$
$S_n$	0.063	0.098

- (ii) Write down the ways of improving stability of slopes.

[12 + 8 = 20 marks]

Given,  $H = 12\text{m}$ ,  $FOS = 1.25$   
 $C = 20\text{ kN/m}^2$ ,  $\phi = 15^\circ$ ,  $\gamma = 17\text{ kN/m}^3$ .

we know,

$$S_n = \frac{C}{\gamma H_c}$$

$$S_n = \frac{20}{17 \times 12 \times 1.25}$$

$$\Rightarrow S_n = 0.0784$$

$\therefore$  interpolation from the table.

$$f = 30^\circ + \frac{45-30}{(0.093-0.063)} (0.0784-0.063)$$

$$i = 36.6^\circ$$

(12)

Ways of improving slope stability:

1. Proper compaction during slope building, which in turn lead to minimum settlement.
2. Weep holes to be given to drain excess water.
3. Use of ~~geomembrane~~ geotextiles for reinforcement & acting as a separator. two layers.
4. Use of chemical grouting.
5. Use of lime stabilization for cohesive soils.
6. Avoid high height of slope.

(4)

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- Q.8 (b) To determine the difference in elevation between two points A and B located on opposite sides of a river, reciprocal levelling was carried out using a tilting level. Observations were taken with the instrument set up near point A and near point B. When the instrument was near point A, the staff readings observed were 1.455 m on point A and 2.685 m on point B. When the instrument was near point B, the corresponding staff readings were 0.925 m on point A and 2.045 m on point B. The horizontal distance between points A and B is 800 m, and the reduced level (RL) of point A is known to be 120.500 m. Based on these observations, determine
- Reduced level of point B.
  - Total combined error due to curvature and refraction.
  - Collimation error of the instrument.

[20 marks]

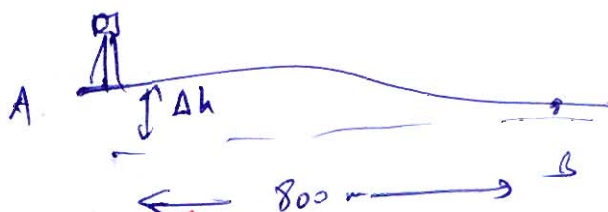
Soln.

Inst. at A  $h_A = 1.455$  ,  $h_B = 2.685$

Inst. at B  $h'_A = 0.925$  ,  $h'_B = 2.045$

We know,  
 exact ht. diff =  $\frac{(h_B - h_A) + (h'_B - h'_A)}{2}$   
 $= 1.175m$

$\therefore RL_B = RL_A + h_A - 1.175$   
 $= 120.73m$   
 after correction  $RL_B = 120.73$



$RL_B = RL_A - \Delta h = 119.325m$

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Measured ht. at B, when inst. at A =  $h_B = 2.685m$

True ht. at B =  $h_A + \Delta h = 2.63m$

total error =  $MV - TV$   
 $= 2.685 - 2.63 = 0.055m$  (1)

error due to curvature & refraction =  $+0.0673 (d^2)$   
 $= 0.043m$  (2)

$\therefore$  error due to collimation = total error - error due to refraction & curvature  
 $= 0.055 - 0.043 = 0.012m$   
 (i.e. inclined axis upwards)





- (c) You are evaluating an existing Activated Sludge facility. The wastewater flow to the plant is  $12,000 \text{ m}^3/\text{day}$ , and the aeration tank volume is  $3,000 \text{ m}^3$ . The influent and effluent  $\text{BOD}_5$  concentrations are  $250 \text{ mg/L}$  and  $25 \text{ mg/L}$ , respectively. The mixed liquor suspended solids (MLSS) concentration in the aeration tank is  $3,500 \text{ mg/L}$ , while the return sludge concentration is  $10,000 \text{ mg/L}$ . The effluent suspended solids concentration is  $15 \text{ mg/L}$ . The kinetic parameters include a yield coefficient of  $0.5 \text{ kg VSS per kg of BOD removed}$ , a decay coefficient of  $0.06 \text{ day}^{-1}$ , and an MLVSS-to-MLSS ratio of  $0.8$ .
- Determine:
- Total biomass in Kg to be maintained in aeration tank in steady state condition.
  - Total gross biomass in Kg per day created from the consumed BOD.
  - Biomass lost due to natural cell death and respiration in Kg per day.
  - Net growth of biomass in kg per day.
  - Its operational sludge age ( $\theta_c$ ).
  - Biomass loss in effluent kg per day and biomass to be wasted in kg per day.
  - Daily sludge wasting rate ( $Q_w$ ).
  - Recycle ratio ( $R$ ).

[20 marks]

$Q_{\text{wss}} = 12000 \text{ m}^3/\text{d}, V = 3000 \text{ m}^3.$   
 $y_i = 250 \text{ mg/l}, y_e = 25 \text{ mg/l}.$   
 $\text{MLSS} = 3500 \text{ mg/l}, X_u = 10,000 \text{ mg/l}.$   
 $S_{Se} = 15 \text{ mg/l}, y = 0.5, k = 0.06 \text{ d}^{-1}, \frac{\text{MLVSS}}{\text{MLSS}} = 0.8$

$\therefore \text{MLVSS} = 2800 \text{ mg/l}.$

- Total biomass =  $VX = 8400 \text{ kg}$
- Total biomass created per day =  $Qy_i = 3000 \text{ kg/d}$

v) Operational sludge age,

$$\frac{1}{\theta_c} + k_{ER} = \frac{Q(y_i - y_e) Y}{V X}$$

$$\frac{1}{\theta_c} + 0.06 = \frac{(12000)(225) \times 0.5}{3000 \times 2800}$$

$\Rightarrow \theta_c = 9.93 \text{ days}$

vii)  $R = \frac{X}{X_u - X} = 0.389$

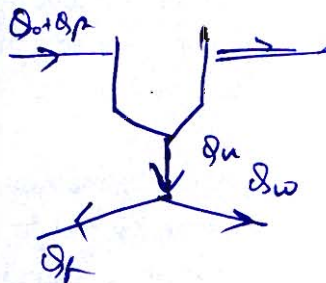
5

$$\therefore h = \frac{x_2}{x_1 - x} ; \frac{dR}{d_0}$$

$$\therefore dR = 4667.$$

$$\therefore dR =$$

$$d_e = d_0 - d_w$$







**Space for Rough Work**

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Space for Rough Work

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