MADE EASY

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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-2 : Digital Circuits + Microprocessors and Microcontroller [All topics]
Network Theory-1 + Signals and Systems-1 [Part Syllabus]

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☐	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	17
Q.2	
Q.3	
Q.4	25
Section-B	
Q.5	22
Q.6	57
Q.7	
Q.8	27
Total Marks Obtained	148

Signature of Evaluator .... Cross Checked by ....

1/4/26

○ Keep it up --

○ Revise network

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

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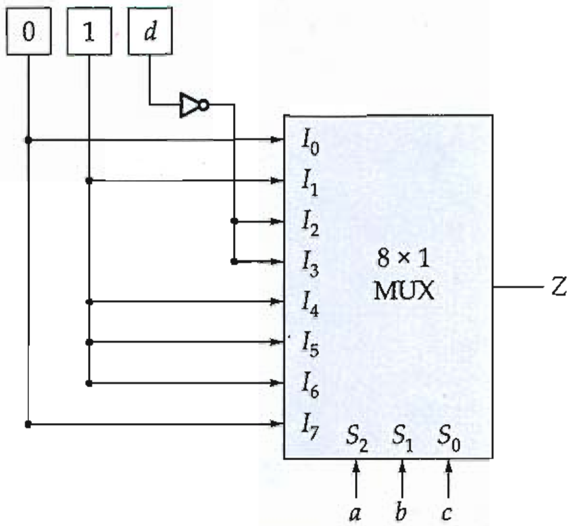
Section A : Digital Circuits + Microprocessors & Microcontroller

1. (a) Consider the combinational circuit shown below has four inputs (a, b, c, d) and one output Z.



For the above given combinational circuit,

(i) construct the truth table.

(ii) write the minimized logic expression for output $Z = f(a, b, c, d)$.

[12 marks]

Ans:- (ii) The expression for Z is,

$$Z = \bar{a}\bar{b}\bar{c}(0) + \bar{a}\bar{b}c(1) + \bar{a}b\bar{c}(\bar{d}) + \bar{a}bc(\bar{d}) + a\bar{b}\bar{c}(1) + a\bar{b}c(1) + ab\bar{c}(1) + abc(0)$$

$$\therefore Z = \bar{a}\bar{b}c + \bar{a}b\bar{c}\bar{d} + \bar{a}bc\bar{d} + a\bar{b}\bar{c} + a\bar{b}c + ab\bar{c}$$

K-map.

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$			1	1
$\bar{A}B$	1			1
AB	1	1		
$A\bar{B}$	1	1	1	1

$$\therefore Z = A\bar{C} + B\bar{C}\bar{D} + A\bar{B} + \bar{B}C + \bar{A}C\bar{D}$$

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1 (b) **(i)** Convert n-bit base-3 number to n-bit base-9 number.

(ii) Convert $(211101222211122)_3$ to $()_9$ using the result obtained in part (i).

[6 + 6 marks]

Sol: (i) let the n bit be for example is 1010.

$(1010)_3$

Step1:- Multiple by 3 at the digits place to get the decimal number.

$$1\times3^3+0\times3^2+1\times3^1+0\times3^0=30$$

Step2:- Divide by 9 to get base in 9.

9	30	
	3	3

$\therefore (1010)_3 \longleftrightarrow (30)_{10} \longleftrightarrow (33)_9$

0

(ii) $(211101222211122)_3$

$$2\times3^{14}+1\times3^{13}+1\times3^{12}+1\times3^{11}+0\times3^{10}+1\times3^9+2\times3^8+2\times3^7+2\times3^6+2\times3^5+1\times3^4+1\times3^3+1\times3^2+2\times3^1+2\times3^0$$

5

$= (11908097)_{10} \longleftrightarrow (84785322)_9$

9	11908097	8
9	1323121	4
9	147013	7
9	16334	8
9	1814	5
9	201	3
9	22	2
	2	2

$\leftrightarrow (84785322)_9$

0

Q.1 (c) Write an assembly language program using 8086 microprocessor to add two 16-bit hexadecimal numbers stored in memory at addresses 0200 H and 0202 H. Store the result in memory.

[12 marks]

soln: In 8086, microproper, the program for addition is,

```
LHLD 0200H    : load the content at
                address 0200H at HL.
MOV A, H
MOV B, L      : MOV the content of lower
                bit (L) at B and content
                of higher bit at A.
                (H)
LHLD, 0202    : content of 0202 address
MOV C, H        is moved at HL
MOV D, L        register pair
Add C         : content of higher of higher
Add B, D        bit A and C are added
SHLD 0300 H   : content of lower bit B,
                and D are added.
              : Store the resultant at
                memory .0300 H.
```

0

8086 !!

1 (d) Using 8085 microprocessor, write an assembly language program to find the 9's complement of a decimal digit stored in memory at address location 2050 H.

[12 marks]

Q.1 (e) Consider the logic function given below:

$$F = ABCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}C + AB$$

(i) Simplify the above function using K-map.

(ii) Realize the logic function using only NAND gates.

(iii) Realize the logic function using only NOR gates. **[12 marks]**

Sol: (i) $F = ABCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}C + AB$

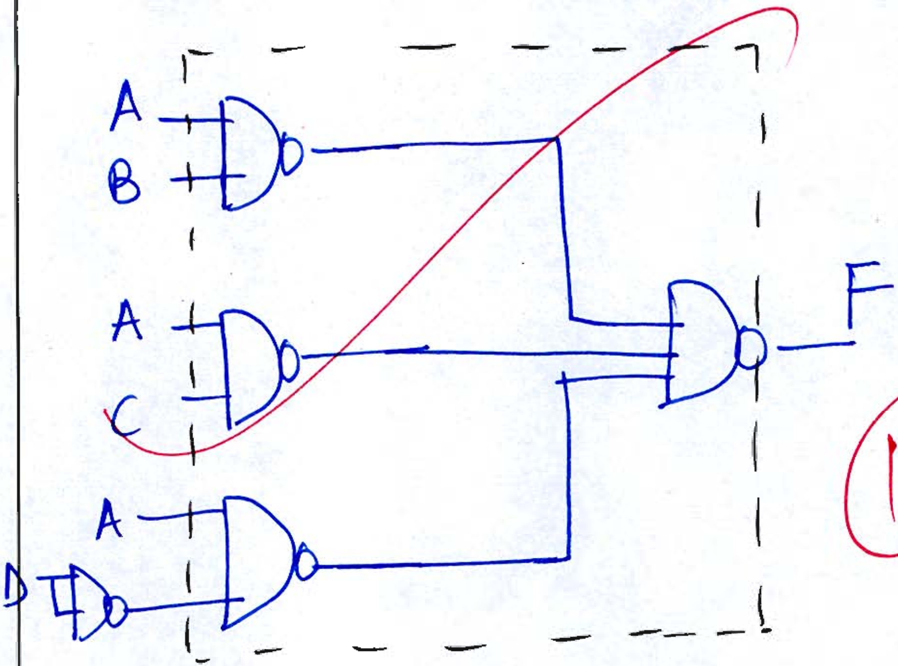
AB \ CD	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$				
$\overline{A}B$				
AB	1	1	1	1
$A\overline{B}$	1		1	1

$$F = AB + AC + A\overline{D}$$

(ii) logic function using NAND Gate: $(\overline{xy})$

$$\overline{\overline{F}} = \overline{\overline{AB + AC + A\overline{D}}}$$

$$F = \overline{\overline{AB}\cdot(\overline{AC})\cdot(\overline{A\overline{D}})}$$



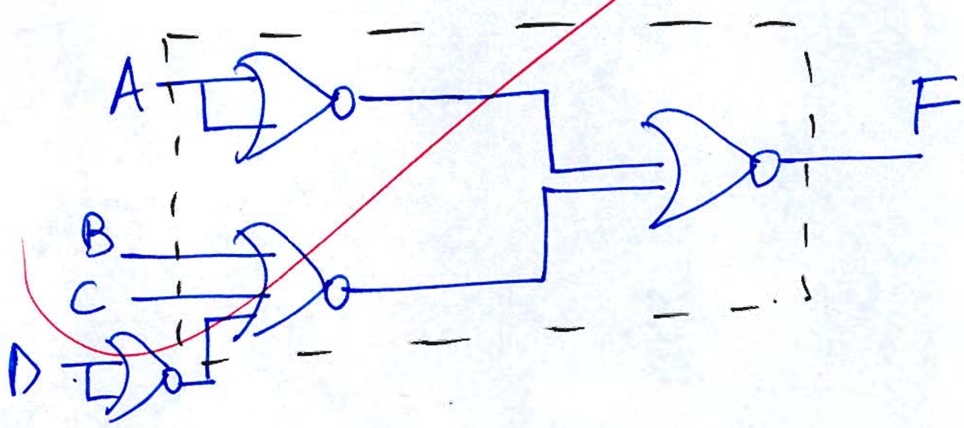
12 Good

(iii) logic function using NOR gates: $(\overline{x+y})$

$$F = A(B + C + \overline{D})$$

$$\overline{\overline{F}} = \overline{\overline{A(B+C+\overline{D})}} = \overline{\overline{A} + (\overline{B+C+\overline{D}})}$$

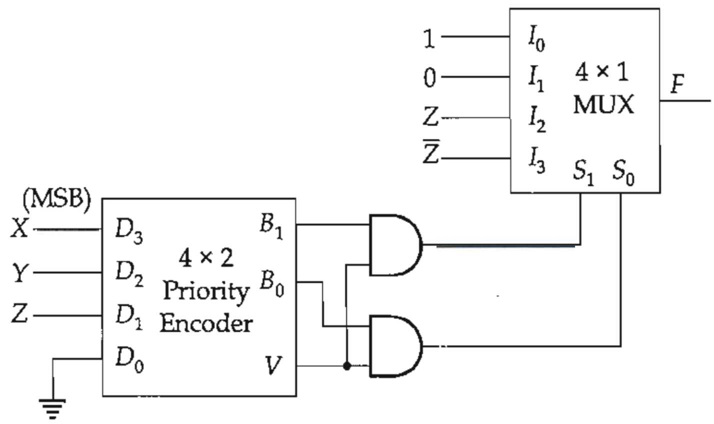
$$F = \overline{\overline{A} + (\overline{B+C+\overline{D}})} = \overline{\overline{A} + (\overline{B}\cdot\overline{C}\cdot\overline{\overline{D}})}$$



Q.2 (a) Write an assembly language program for an 8051 microcontroller to generate 100 kHz square wave (70% duty cycle) at Pin P2.0 by using Timer 1 operating in Mode 2. Assume that the microcontroller is operating with 12 MHz crystal oscillator.

[20 marks]

Q.2 (b) For the following combinational circuit, construct the truth table and obtain the simplified SOP expression of the output function (F).

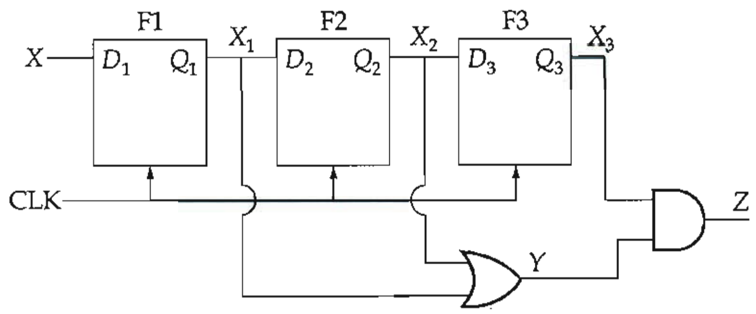


[20 marks]

Q.2 (c) Explain different transfer modes of an 8237 DMA controller in active cycle.

[20 marks]

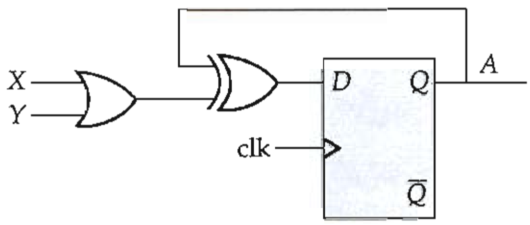
Q.3 (a) (i) A digital design is implemented by the circuit given below:



The design has D-type flip flops, F1, F2 and F3 driven by the clock 'CLK'. It has one input 'X' and one output 'Z'.

1. Find output logic expression for 'Z'.
2. Identify the functionality of the given circuit.

(ii) Analyze the logic circuit shown below and also draw the state diagram for the given circuit.



[10 + 10 marks]

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3 (b) Write an assembly language program using 8051 microcontroller (with clock frequency 12 MHz) for a simple traffic light control system using LEDs connected to port 2 of the microcontroller.

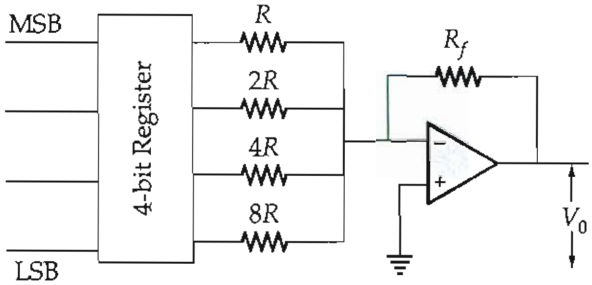
Assume LED connections

- P2.0 → Red LED (ON for 5 sec).
- P2.1 → Yellow LED (ON for 5 sec).
- P2.2 → Green LED (ON for 2 sec).

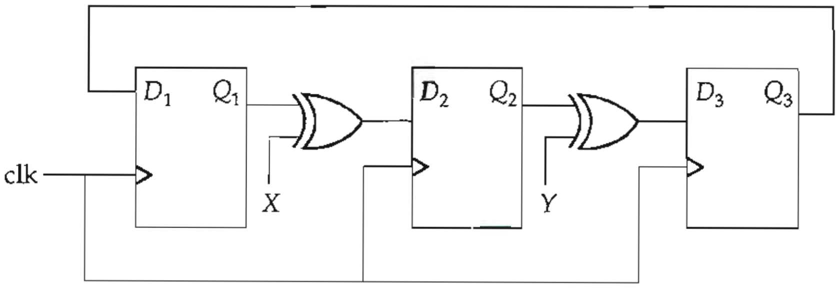
Assume this program cycles through Red-Green-Yellow lights with delays.

[20 marks]

Q.3 (c) **(i)** Calculate the output voltage for an input code word 0110 if a logic 1 is 10 V and logic 0 is 0 V. Assume $R = R_f = 1\ k\Omega$.



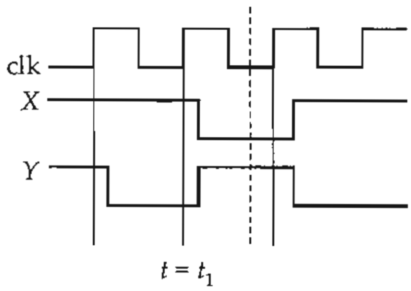
(ii) Consider the sequential circuit shown below:



1. Fill in the table for the next state values of the three flip-flops for the given current state of the flip-flops and the inputs X and Y. Assume setup and hold times are synchronized with flip-flop inputs.

Q_1	Q_2	Q_3	X	Y	Q_1^+	Q_2^+	Q_3^+
0	0	0	0	1			
1	1	0	1	1			
0	0	1	1	0			

2. For the timing diagram shown below, what is the value of Q_1, Q_2 and Q_3 at the time indicated by the dashed line in the figure if the value at $t = t_1$ for $Q_1Q_2Q_3 = 001$? (Assume the flip-flops are negative edge trigerred)



[10 + 10 marks]

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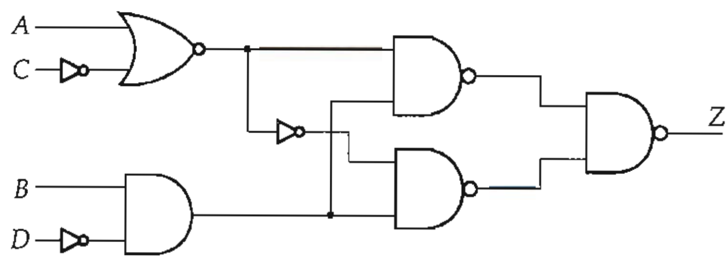
Q.4 (a) **(i)** Design an Excess-3 to BCD code converter (Use don't cares for unused codes).

(ii) The simplified logic expression for output in the circuit shown in below figure is



[15 + 5 marks]

Solu: (i) Excess 3 to BCD code.

	A	B	C	D	A'	B'	C'	D'	x	y	z	w
3.	0	0	0	0	0	0	1	1	0	0	0	0
4.	0	0	0	1	0	1	0	0	0	0	0	1
5.	0	0	1	0	0	1	0	1	0	0	1	0
6.	0	0	1	1	0	1	1	0	0	0	1	1
7.	0	1	0	0	0	1	1	1	0	1	0	0
8.	0	1	0	1	1	0	0	0	0	1	0	1
9.	0	1	1	0	1	0	0	1	0	1	1	0
10	0	1	1	1	1	0	1	0	0	1	1	1
11	1	0	0	0	1	0	1	1	1	0	0	0
12	1	0	0	1	1	1	0	0	1	0	0	1
13	1	0	1	0	1	1	0	1	X	X	X	X
14	1	0	1	1	1	1	1	0	X	X	X	X
15	1	1	0	0	1	1	1	1	X	X	X	X

1 1 0 1	X X X X	X X X X
1 1 1 0	X X X X	X X X X
1 1 1 1	X X X X	X X X X

For x

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0 3	2
$\bar{A}B$	0 4	0 5	7	6
AB	1 12	X 13	X 15	X 14
$A\bar{B}$	8	9	1 11	10

$x = AB + ACD$

for y

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$			1	
AB		X	X	X
$A\bar{B}$	1	1		1

$y = A\bar{B} + BCD + AC\bar{D}$

for z

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$		1		1
AB		X	X	X
$A\bar{B}$		1		1

$z = B\bar{C}D + A\bar{C}D + BC\bar{D} + AC\bar{D}$

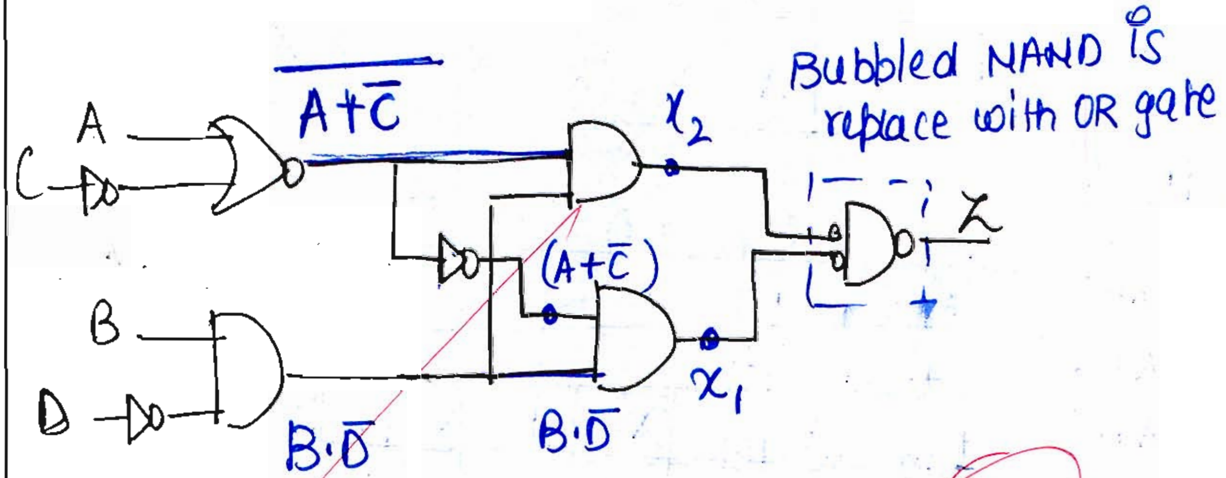
$\therefore z = B(C \oplus D) + A(C \oplus D)$

For w

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$	1			1
AB	1	X	X	X
$A\bar{B}$	1		0	1

$w = B\bar{C}\bar{D} + AB + BC\bar{D} + AC\bar{D}$

Q.4(a)(ii) $Z \Rightarrow$ expression from given circuit:



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$\therefore \; x_1 = (B\overline{D}) \cdot (A+\overline{C})$

$x_2 = (B\overline{D}) \cdot (\overline{A+\overline{C}})$

$\therefore \; Z \Rightarrow$ x_1, x_2 → $Z = x_1 + x_2$

$Z = B\overline{D}(A+\overline{C}) + B\overline{D}(\overline{A+\overline{C}})$

$\therefore \; Z = B\overline{D}\left[(A+\overline{C}) + (\overline{A+\overline{C}})\right]$

$\therefore \; x + \overline{x} = 1$

$\therefore$ $\boxed{Z = B\overline{D}}$

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4 (b) Implement the following functions using single (3 × 6 × 4) programmable logic array (PLA) with programmable output polarity feature.

$$F_1(A, B, C) = \Sigma m\,(1, 2, 4, 6)$$
$$F_2(A, B, C) = \Sigma m\,(0, 1, 6, 7)$$
$$F_3(A, B, C) = \Sigma m\,(2, 6)$$
$$F_4(A, B, C) = \Sigma m\,(1, 2, 3, 5, 7)$$

[20 marks]

Sol:

~~$F_1(A, B, C, D) = \Sigma m(1, 2, 4, 6)$~~

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 (0)	1 (1)	0 (3)	1 (2)
$\bar{A}B$	1 (4)	0 (5)	0 (7)	1 (6)
AB	0 (12)	0 (13)	0 (15)	0 (14)
$A\bar{B}$	0 (8)	0 (9)	0 (11)	0 (10)

~~writing compliment of F_1: $= \bar{C}D + \bar{A} + BD + \bar{A}\bar{B}C\bar{D}$~~

~~$F_2(A, B, C$~~

$F_1(A, B, C) = \Sigma m(1, 2, 4, 6)$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	(0)	1 (1)	(3)	1 (2)
A	1 (4)	(5)	(7)	1 (6)

$F_1 = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1 (0)	1 (1)	(3)	(2)
A	(4)	(5)	1 (7)	1 (6)

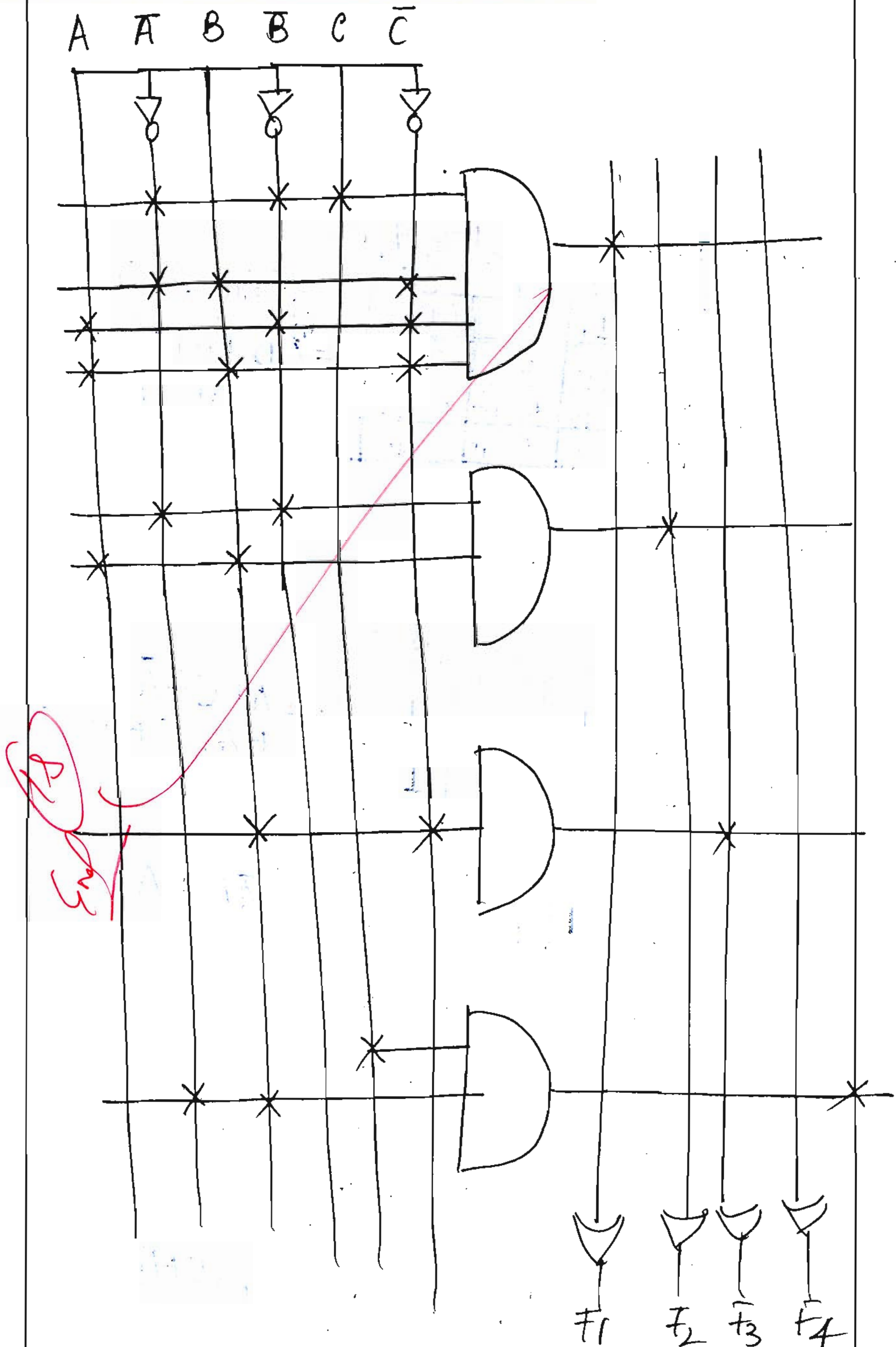
$F_2 = \bar{A}\bar{B} + AB$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	(0)	(1)	(3)	1 (2)
A	(4)	(5)	(7)	1 (6)

$F_3 = B\bar{C}$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	(0)	1 (1)	1 (3)	1 (2)
A	(4)	1 (5)	1 (7)	(6)

$F4 = C + \bar{A}B$


A
$\bar{A}$
B
$\bar{B}$
C
$\bar{C}$
F1
F2
F3
F4

.4 (c) (i) Describe memory segmentation in 8086 microprocessor with the help of block diagram.

(ii) What are the different addressing modes of 8051 microcontroller?

[10 + 10 marks]

Section B : Network Theory-1 + Signals and Systems-1

Q.5 (a) A triangular wave shown in figure (a) is applied as an input to a series RL circuit shown in figure (b). Find the current $i(t)$.

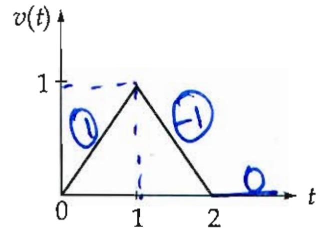


Fig. (a)

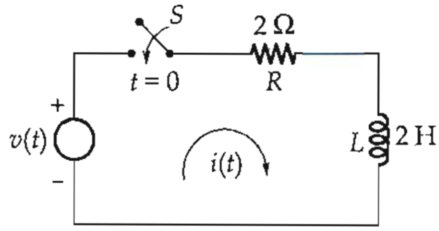


Fig. (b)

[12 marks]

Solu: The expression for v(t) is

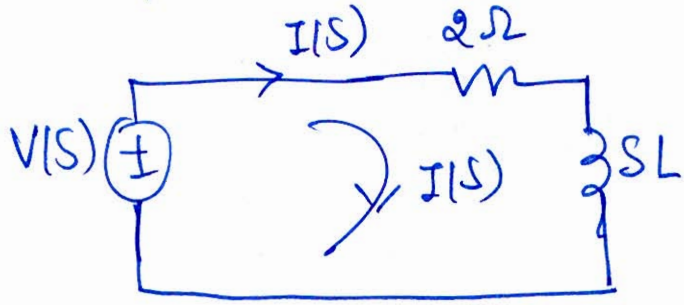
$v(t) = r(t) - 2r(t-1) + 1\, r(t-2)$

on taking laplace transform;

$V(S) = \frac{1}{S^2} - \frac{2}{S^2} e^{-S} + \frac{e^{-2S}}{S^2}$

Redraw the circuit in s domain;

$i_L(0^-) = 0$



on applying KVL in given circuit,

$V(S) - (2 + S \times 2) I(S) = 0$

$\therefore \frac{V(S)}{(2+2S)} = I(S)$

$I(S) = \frac{1}{2(S+1)} \left[\frac{1}{S^2} - \frac{2}{S^2} e^{-S} + \frac{e^{-2S}}{S^2} \right]$

$$\Rightarrow I(s) = \frac{1}{2(s+1)(s^2)} - \frac{2e^{-s}}{s^2(s+1)} + \frac{e^{-2s}}{2s^2(s+1)}$$

$$I(s) = \frac{e^{-2s} - 1 + 2e^{-s}}{2s^2(s+1)}$$

$$\boxed{I(s) = \frac{(e^{-s}-1)^2}{2s^2(s+1)}}$$

8

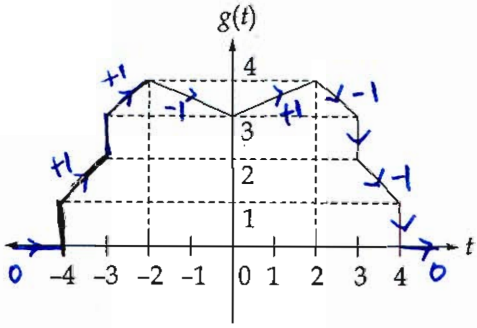
incomplete soln

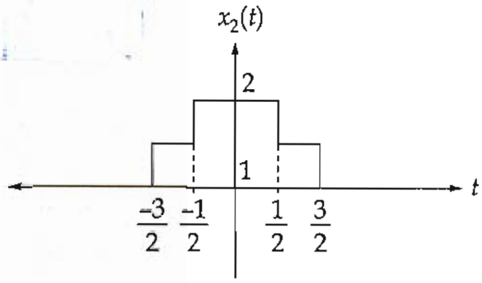
Do not write in this ma

Q.5 (b) **(i)** Find whether the following system is static, linear, time-invariant, causal, invertible.

$$y(t) = \int_{t}^{t+1} x(\tau - \alpha)d\tau \text{ where, '}\alpha\text{' is a constant.}$$

(ii) The response of an LTI system to an input signal $x_1(t) = u(t+1) - u(t-1)$ is denoted as $g(t)$, as illustrated in the figure below. If a new input $x_2(t)$ is applied to the same system, resulting in an output $y(t)$. Determine the value of the output at $t = 0$.





[6 + 6 marks]

Soln: (i)

$$y(t) = \int_{t}^{t+1} x(\tau - \alpha)d\tau.$$

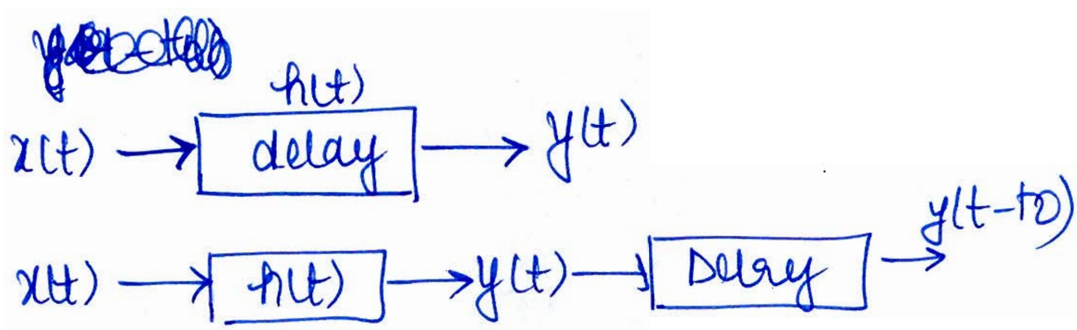
$$y(t) = [x'(\tau - \alpha)]_{t}^{t+1}$$

$$y(t) = x'(t+1-\alpha) - x'(t)$$

(4)

The given system is not static since its depends upon the future value of input.

For the same reason system is Non-causal.



$$y(t) = \int_{t}^{t+1} x(\tau - t_0 - \alpha)\, d\tau$$

$$y(t-t_0) = \int_{t-t_0}^{t-t_0+1} x(\tau-\alpha)\, d\tau .$$

system is time varient.

$$y_1(t) = \int_{t}^{t+1} x_1(\tau-\alpha) dt$$

$$y_2(t) = \int_{t}^{t+1} x_2(\tau-\alpha) dt$$

$$\therefore y_1(t) + y_2(t) = \int_{t}^{t+1} x_1(\tau-\alpha) dt + \int_{t}^{t+1} x_2(\tau-\alpha) dt$$

$$= \int_{t}^{t+1} [x_1(\tau-\alpha) + x_2(\tau-\alpha)] dt$$

system is linear.

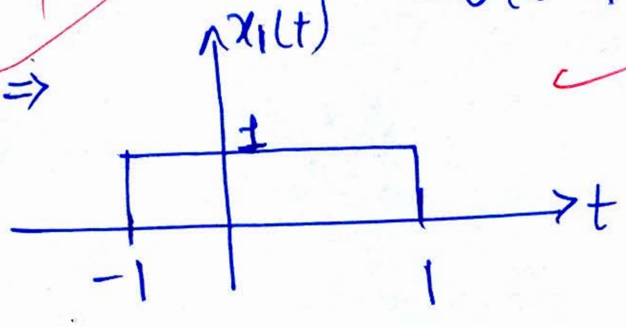
---

ii) $x_1(t) \rightarrow$ [$h(t)$] $\rightarrow g(t)$

$$x_1(t) = u(t+1) - u(t-1)$$

$$g(t) = u(t+4) + r(t+4) + u(t+3) + r(t+3) - 2r(t-2)$$
$$+ 2r(t) - 2r(t-2) - u(t-3) - r(t-3)$$
$$- u(t-4)$$

$x_1(t) \Rightarrow$



incomplete soln

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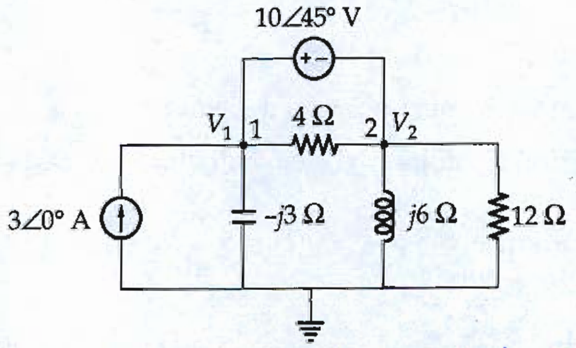
.5 (c) Compute V_1 and V_2 in the below circuit:



[12 marks]

Sol: By using supernode and nodal analysis:

$$3 = \frac{V_1}{(-3j)} + \frac{V_1 - V_2}{4} + \frac{V_2}{j6} + \frac{V_2}{12} \quad -(1)$$

By using supernode concept:

$$V_1 - 10\angle 45^\circ - V_2 = 0 \quad \text{———} (2)$$

$$V_1 - V_2 = 10\angle 45^\circ$$

Now,

$$3 = V_1\left[\frac{1}{-3j} + \frac{1}{4}\right] + V_2\left[-\frac{1}{4} + \frac{1}{j6} + \frac{1}{12}\right]$$

$$\therefore 3 = V_1(0.416\angle 53.13) + V_2(0.235\angle -135^\circ)$$

——— (3)

From (2), put in (3) $V_1 = 10\angle 45° + V_2$

$3 = V_1(0.416\angle 53.13) + V_2(0.235\angle -135°)$

$3 = (10\angle 45° + V_2)(0.416\angle 53.13) + V_2(0.235\angle -135°)$

$3 = (10\angle 45)(0.416\angle 53.13) + V_2[0.416\angle 53.13 + 0.235\angle -135°]$

$3 - (1.46\angle 98.13) = V_2(0.186\angle 63.40)$

$\therefore$ $V_2 = 18.90\angle -87.66$ V

$V_1 = 14.18\angle -56.44$ V

Q.5 (d) (i) What are eigen functions?

(ii) Express $\sin t + \cos 2t$ in terms of exponential eigen functions.

(iii) Using the eigen functions obtained above, calculate the response of the system having difference equation $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + y(t) = 5\frac{dx}{dt} + x(t)$ for the input $\sin t + \cos 2t$?

[12 marks]

Soln. (ii) $\sin t + \cos 2t$

$\sin t = \frac{e^{t} - e^{-t}}{2i}$

$\cos 2t = \frac{e^{2t} + e^{-2t}}{2}$

$\therefore \sin t + \cos 2t = \frac{e^{t} - e^{-t}}{2i} + \frac{e^{2t} + e^{-2t}}{2}$

$\sin t + \cos 2t = \frac{e^{t}}{2i} - \frac{e^{-t}}{2i} + \frac{e^{2t}}{2} + \frac{e^{-2t}}{2}$

(iii) Differential Equation:

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + y(t) = 5\frac{dx}{dt} + x(t)$$

on taking laplace transform:

$$s^2Y(s) + 6sY(s) + Y(s) = 5sX(s) + X(s)$$

$$\frac{Y(s)}{X(s)} = H(s) = \left[\frac{5s+1}{s^2+6s+1}\right]$$

$s = j\omega$

$$H(j\omega) = \frac{5j\omega+1}{-\omega^2+6j\omega+1} = \frac{1+5j\omega}{(1-\omega^2)+6j\omega}$$

$$\therefore |H(j\omega)| = \frac{\sqrt{1+(5\omega)^2}}{\sqrt{(1-\omega^2)^2+(6\omega)^2}}$$

for eigen value concept

$\omega = 1$ and $\omega = 2$.

$$\therefore y(t) = x(t)\,|H(j\omega_0)| \angle H(j\omega_0)$$

For $\omega = 1$

$$\therefore y_1(t) = \sin t \times \left(\frac{\sqrt{1+(5\times1)^2}}{\sqrt{(6\times1)^2}}\right) \angle \tan^{-1}\left(\frac{5\omega}{1}\right)_{\omega_0} - \tan^{-1}\left(\frac{6\omega}{1-\omega^2}\right)$$

2

$$y(t) = 0.849\sin(t + (78.69 - 90°))$$

$$\boxed{y_1(t) = 0.849\sin(t - 11.30°)}$$

↓ Mistake

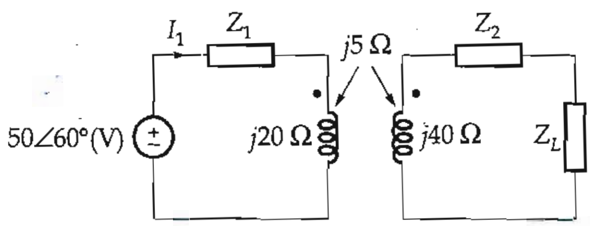
For $\omega = 2$

$y_2(t) = \cos 2t \left| \dfrac{\sqrt{1+(5\times2)^2}}{\sqrt{(1-2^2)^2+(6\times2)^2}} \right| \cos\left(2t + \tan^{-1}(10) - \tan^{-1}\left(\dfrac{6\times2}{1-2^2}\right)\right)$

$y_2(t) = (0.846)\cos(2t+84.35)$

∴ $y(t) = 0.849\sin(t-11.30^\circ) + 0.846\cos(2t+84.35^\circ)$

Q.5 (e) For a linear transformer shown in figure below, with $Z_1 = (60 - j100)\ \Omega$, $Z_2 = (30 + j40)\ \Omega$ and $Z_L = (80 - j60)\ \Omega$; find the input impedance and the current I_1.



[12 marks]

Soln:

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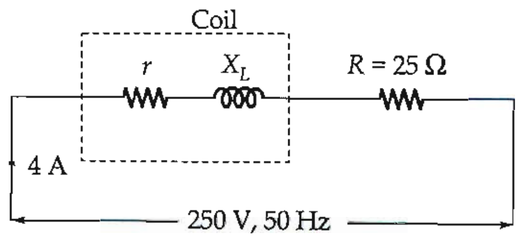
Q.6 (a) **(i)** Consider the discrete time signals $x[n] = a^n u[n]$, $0 < a < 1$ and $h[n] = u[n]$. Explain the graphical method for computing the discrete-time convolution. Illustrate the steps to obtain the analytical expression for $y[n]$ graphically. Sketch the output signal $y[n]$.

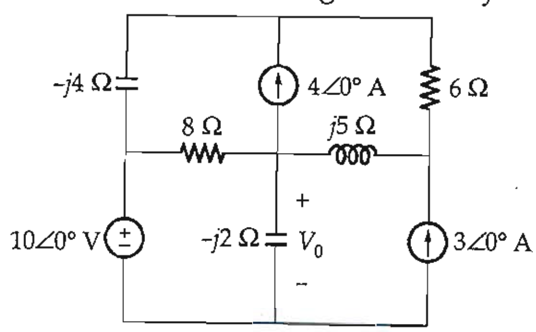
(ii) Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = \left(\frac{1}{2}\right)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = \frac{1}{2}$, determine the value of $g[1]$.

[12 + 8 marks]

Q.6 (b) (i) A resistance of 25 Ω is connected in series with a choke coil. The series combination when connected across a 250 V, 50 Hz supply, draw a current of 4-A which lags behind the voltage by 65°. Calculate: total power ($VA = W \pm jVAR$), Power consumed by resistance (R = 25 Ω), Power consumed by choke coil, and resistance and inductance of the coil.



(ii) Solve for V_0 in the below circuit using mesh analysis.



[10 + 10 marks]

Q.6 (c) (i) If $i_s = 2\cos 10t\ (A)$, find the total energy stored in the passive network shown in figure (a) at $t = 0$ for coefficient of coupling, $k = 0.6$ and terminals x and y left open-circuited.

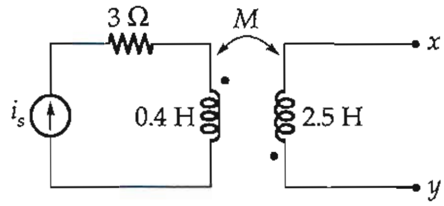


Fig. (a)

(ii) Determine the amount of energy stored after 0.5s, when the primary side of the circuit shown in figure (b) is connected to a dc source of 15 V and the secondary is short-circuited. Given: $L_1 = 2$ H, $L_2 = 3$ H and $M = 1$ H.

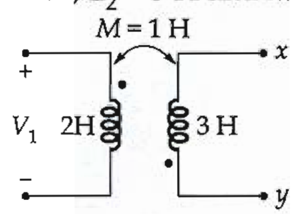


Fig. (b)

[5 + 15 marks]

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(i) Find the Inverse Fourier Transform $x(t)$ for the given frequency domain expression:

$$X(j\omega) = e^{-j5\omega/2}\left[\frac{\sin\left(\frac{\omega}{2}\right)+2\sin\left(\frac{3\omega}{2}\right)}{\omega}\right]$$

(ii) Evaluate the following integral involving the doublet function:

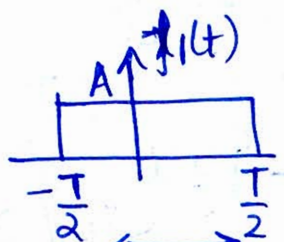
$$I = \int_{-\infty}^{2} \cos\frac{\pi}{2}t\,[\delta'(2t-1)+\delta(t-4)]\,dt$$

[12 + 8 marks]

(i)

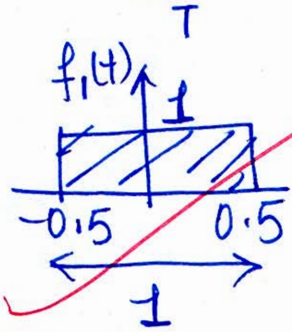
$$X(\omega) = \frac{e^{-j\frac{3\omega}{2}}\sin\left(\frac{\omega}{2}\right)}{\omega} + \frac{2\sin\left(\frac{3\omega}{2}\right)e^{-j\frac{5\omega}{2}}}{\omega}$$

$X_1(\omega)$ $X_2(\omega)$



$$\longleftrightarrow AT\,Sa\left(\omega\cdot\frac{T}{2}\right) = \frac{AT\sin\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega T}{2}\right)}$$

∴



$$\longleftrightarrow 1\cdot Sa\left(\omega\cdot\frac{1}{2}\right)$$

$$1\,\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}} = \frac{2\sin\left(\frac{\omega}{2}\right)}{\omega}$$

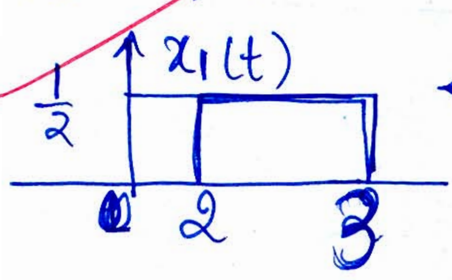
10

$$f_1(t) = f\left(t+\frac{3}{2}\right) \longleftrightarrow \frac{2\sin\left(\frac{\omega}{2}\right)}{\omega} = f_1(\omega)$$

using time shifting property

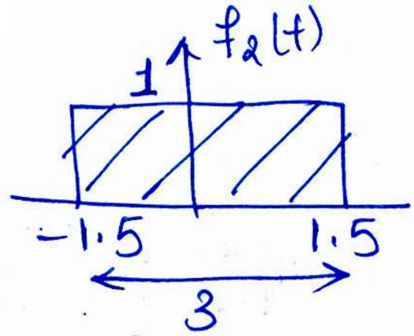
$$\therefore \frac{1}{2}f\left(t-\frac{3}{2}\right) \longleftrightarrow e^{-j\frac{3}{2}\omega}\cdot\frac{\sin\left(\frac{\omega}{2}\right)}{\omega} = F(\omega)$$

$$\therefore x_1(t) \longleftrightarrow X_1(\omega)$$



$$\longleftrightarrow e^{-j\frac{3}{2}\omega}\,\frac{\sin\left(\frac{\omega}{2}\right)}{\omega}$$

$$X_2(\omega) = 2e^{-j\frac{5\omega}{2}} \frac{\sin\left(\frac{3\omega}{2}\right)}{\omega}$$

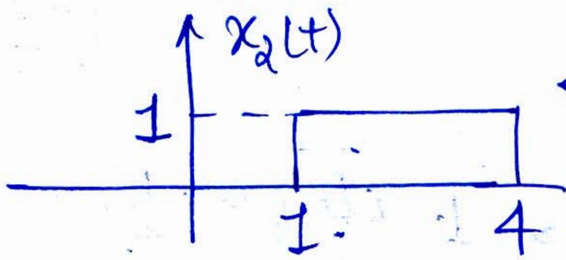


$$\longleftrightarrow 3\,Sa\left(\omega\cdot\frac{3}{2}\right)$$

$$3\frac{\sin\left(\frac{\omega 3}{2}\right)}{\frac{3\omega}{2}}$$

$$= 2\frac{\sin\left(\frac{3\omega}{2}\right)}{\omega}$$

$$f_2(t) = f\left(t-\frac{5}{2}\right)$$

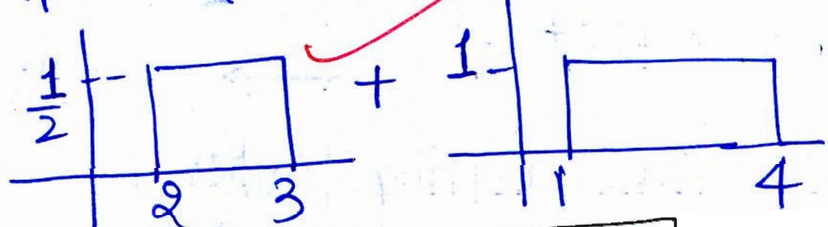


$$\longleftrightarrow 2e^{-j\frac{5}{2}\omega}\sin\left(\frac{3\omega}{2}\right)$$

∴ $X(\omega)$ can be written as:

$x(t) = x_1(t) + x_2(t)$

$x(t) =$



$$x(t) = \frac{1}{2}\,rec\left(\frac{t}{1}\right) + 1\cdot rec\left(\frac{t}{3}\right)$$

(ii) $I = \int_{-\infty}^{2} \cos\frac{\pi}{2}t \left[\delta'(2t-1) + \delta(t-4)\right]dt$

$I = \int_{-\infty}^{2} \cos\frac{\pi}{2}t \cdot \delta'(2t-1) + \cos\frac{\pi}{2}t\, \delta(t-4)\, dt$

$x(t)\,\delta'(t) \longleftrightarrow (-1)^{n} \left.\frac{d}{dt}x(t)\right|_{t=0}$ Doublet property

using property of impulse function.

~~$I = \int_{-\infty}^{2} (-\sin\frac{\pi}{2}(t)) \left[\frac{\pi}{2}\right]$~~ Time scaling property

also $\delta(at+b) \longleftrightarrow \frac{1}{|a|}\delta\left(t+\frac{b}{a}\right)$

$\therefore I = \frac{1}{2}\int_{-\infty}^{2} \cos\frac{\pi}{2}t\, \delta'\left(t-\frac{1}{2}\right) + \cos\frac{\pi}{2}t\, \delta(t-4)\, dt$

$I = \frac{1}{2} \times -\frac{\pi}{2} \sin\frac{\pi}{2}\left(\frac{1}{2}\right) \int_{-\infty}^{2} \delta'\left(t-\frac{1}{2}\right) dt$

$+ \int_{-\infty}^{2} \cos\frac{\pi}{2} \times 4\, \delta(t-4) = 0$

since $t = 4$, is outside the limit.

$\therefore I = \frac{1}{2}\left(-\frac{\pi}{2}\right)\sin\left(\frac{\pi}{4}\right)$ $\left[\int_{-\infty}^{2} \delta'\left(t-\frac{1}{2}\right) = 1\right.$

$I = +\frac{\pi}{4} \times \frac{1}{\sqrt{2}}$

$\frac{\pi}{4\sqrt{2}}$

(7)

Q.7 (b) **(i)** A continuous-time signal $x(t) = e^{2t}u(-t)$ is applied to the input of a continuous-time Linear Time-Invariant system. The system is characterized by its unit impulse response $h(t) = u(t-3)$. Determine the mathematical expression for the output $y(t)$ of the system using convolution integral. Sketch the resulting output signal $y(t)$ labelling all critical time instances and the steady state value.

(ii) The unit sample response of a LTI system is $h(n) = 3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^{n-1} u(n)$ and the output of the system is $y(n)$ when unit step $u(n)$ is applied at the input. Determine the steady state value of the output as $n \to \infty$.

[10 + 10 marks]

Sol: (i) $x(t) = e^{2t}u(-t)$

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

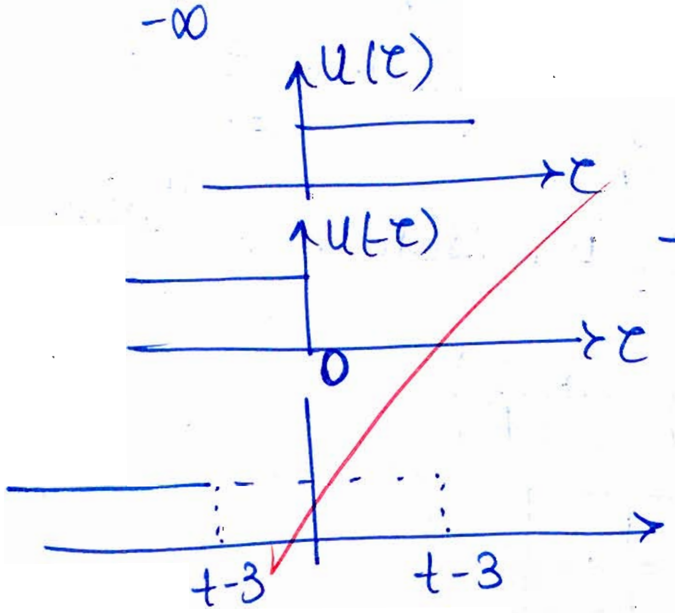
$h(t) = u(t-3)$

$y(t) = x(t) * h(t)$

$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$y(t) = \int_{-\infty}^{\infty} e^{2\tau} u(-\tau) \cdot u(t-\tau-3)\, d\tau$

NOW,



$-\infty < \tau < 0$

$t-\tau-3 = 0$

$t-3 = \tau$

$-\infty < \tau < t-3$

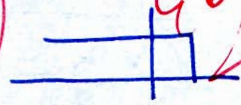
for $\tau < 0 \Rightarrow t-3 < 0 \Rightarrow t < 3$

$\therefore$  $\Leftrightarrow -\infty < \tau < t-3$

$$y(t) = \int_{-\infty}^{t-3} 1 \cdot e^{2\tau} d\tau = \left[\frac{e^{2\tau}}{2}\right]_{-\infty}^{t-3}$$

$$y(t) = \frac{1}{2}\left[e^{2(t-3)} - 0\right] = \frac{1}{2} e^{2(t-3)}$$

for $\tau > 0 \Rightarrow t-3 > 0 \Rightarrow t > 3$



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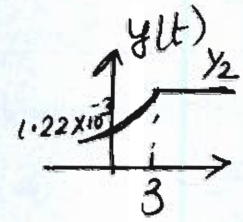
$$\therefore y(t) = \int_{-\infty}^{0} 1 \cdot e^{2\tau} d\tau = \frac{1}{2}\left[e^{2\tau}\right]_{-\infty}^{0} = \frac{1}{2}[1-0] = \frac{1}{2}$$

$$\therefore y(t) = \begin{cases} \frac{1}{2} e^{2(t-3)} & t < 3 \\ \frac{1}{2} & t > 3 \end{cases}$$



(ii) $h(n) = 3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^{n-1} u(n)$

10

$x(n) \rightarrow$ [$h(n)$] $\rightarrow y(n)$

$u(n)$

$$y(n) = \sum_{n=-\infty}^{n} 3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^{n-1} u(n)$$

$$y(n) = \sum_{n=0}^{n} 3\left(\frac{1}{2}\right)^n - \sum_{n=0}^{n} 2\left(\frac{1}{3}\right)^{n-1} \quad \left\{ \because y(n) = u(n) * h(n), \quad y(n) = \sum_{n=-\infty}^{n} h(n) \right.$$

$$y(n) = 3\left[1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots\right] - 2\left[\left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{3}\right)^0 + \left(\frac{1}{3}\right)^1 + \cdots\right]$$

$$y(n) = 3\left[1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\cdots\right] - 2\left(\frac{1}{3}\right)^{-1}\left[1+\left(\frac{1}{3}\right)^1+\left(\frac{1}{3}\right)^2+\cdots\right]$$

$$y(n) = \frac{3\left[1\left(1-\left(\frac{1}{2}\right)^{n+1}\right)\right]}{\left[\frac{1}{2}\right]} - 6\left[\frac{\left(1-\left(\frac{1}{3}\right)^{n+1}\right)}{\left(\frac{2}{3}\right)}\right]$$

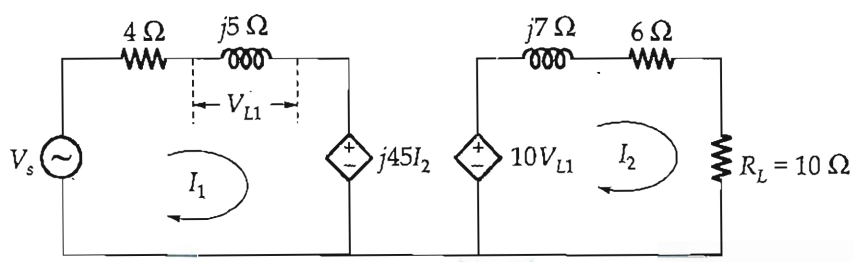
for $n \to \infty$

G.P. $= \frac{a(1-r^n)}{(1-r)}$

$$y(n) = 3\left(\frac{a}{1-r}\right) - 6\left(\frac{a}{1-r}\right)$$

$$y(n) = \frac{3\times 1}{\left(\frac{1}{2}\right)} - \frac{6\times 1}{\left(\frac{2}{3}\right)} = 6 - 6\times\frac{3}{2} = -3.$$

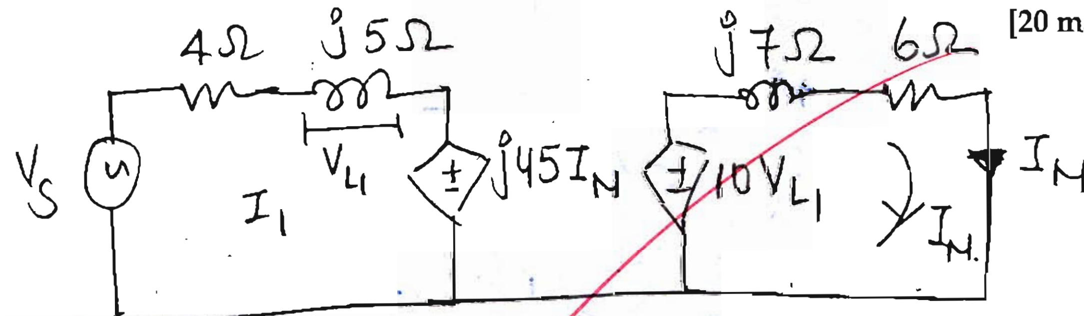
Q.7 (c) In the circuit shown below, if the source voltage $V_s = 110\angle 53.13°$V.



With the help of Norton and Maximum power transfer theorem, determine whether the maximum power is transferred to load for R_L = 10 Ω, and also calculate the power delivered to load for R_L = 10 Ω.

[20 marks]

Soln:



By KVL,
$$10V_{L1} - I_N(j7+6) = 0$$

$$\therefore \frac{10V_{L1}}{(6+j7)} = I_N \quad \text{--- (1)}$$

By using KVL₁

$$V_S - (4+j5)I_1 - j45 I_N = 0$$

$$\frac{V_S - j45 I_N}{(4+j5)} = I_1 \quad —(2)$$

and By ohm's law

$$V_{L1} = I_1 (j5) \quad —— (3)$$

from (2) equation

$$V_{L1} = \left[\frac{V_S - j45 I_N}{(4+j5)}\right](j5)$$

from (1) equation

$$I_N = \frac{10(5j)}{(6+j7)}\left[\frac{V_S - j45 I_N}{(4+j5)}\right]$$

$$\therefore I_N = 0.846\angle -10.73 (V_S) - j45 \times (0.846\angle -10.73) I_N$$

$$\therefore I_N [1 + (j45)(0.846\angle -10.73)] = [0.846\angle -10.73][110\angle 53.13]$$

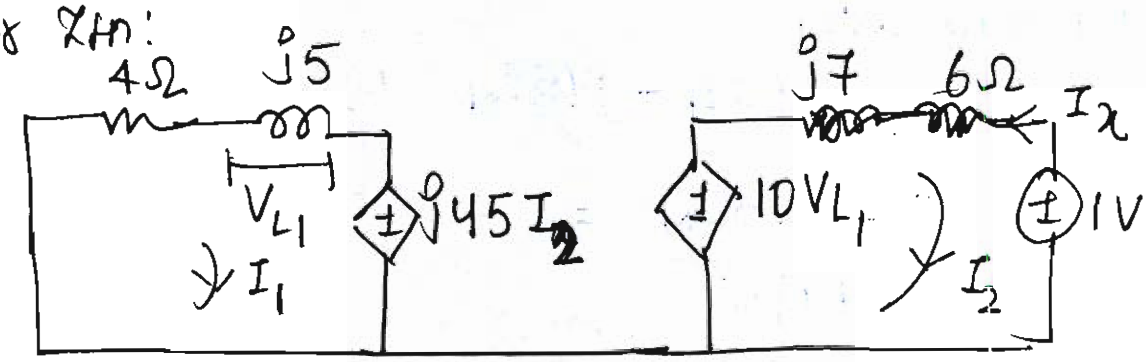
$$\therefore I_N = \frac{93.06\angle 42.4}{38.26\angle 77.79}$$

$$\boxed{I_N = 2.432\angle -35^\circ.39}$$

20

Good

For x_{th}:



where, $I_2 = -I_x$

$$-(4+j5)I_1 - j45(-I_x) = 0$$

$$-(4+j5)I_1 + j45 I_x = 0$$

$$-(4+j5)I_1 = -j45 I_x$$

$$\therefore \quad I_1 = \frac{j45 I_x}{(4+j5)}$$

for, $10V_{L1} - I_2(6+7j) - 1 = 0$

$\Rightarrow$ where, $V_{L1} = I_1(j5)$

$$\frac{10V_{L1} - 1}{(6+7j)} = I_2$$

$$\therefore \quad \frac{10V_{L1}}{(6+7j)} - \frac{1}{(6+7j)} = -I_x$$

and

$$\frac{10(j5)}{(6+7j)} \times \left[\frac{j45 I_x}{4+j5}\right] - \frac{1}{(6+7j)} = -I_x$$

$$\left[38.11\angle 79.26\, I_x\right] + I_x = \frac{1}{(6+7j)}$$

$$38.31\angle 77.79\, I_x = \frac{1}{(6+7j)}$$

$$I_x = 2.831\times 10^{-3}\angle -127.19$$

$\therefore Z_{th} = \frac{1}{I_x} = 353.22 \angle 127.19\ \Omega$

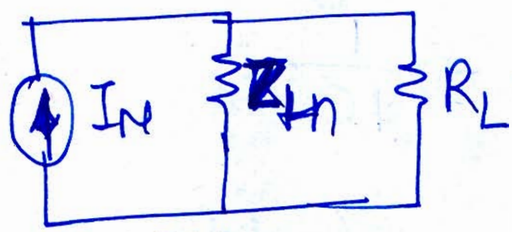
$V_{th} = I_{sc} \cdot Z_{th}$.

$V_{th} = (2.432 \angle -35.39)(353.22 \angle 127.19)$

$V_{th} = 859.04 \angle 91.80\ V$

Maximum power transferred to load:

$Power = \frac{V_{th}^2}{4R_L} = 18448.74\ Watt$



$\therefore I = \frac{V_{th}}{Z_{th}+R_L} = \frac{859.04 \angle 91.80}{353.22+10}$

$I = 2.365 \angle 91.8\ A$

$\therefore P_{delivered} = I^2 R_L$
$= 5.59 \times 10$
$= 55.93\ Watt$

Q.8 (a) (i) When connected to a 120 V (rms), 60 Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance required to be added in parallel to raise the pf to 0.95 while the load absorbs the same power.

(ii) A series-connected circuit has $R = 4\ \Omega$ and $L = 25$ mH.

1. Calculate the value of C that will produce a quality factor of 50.
2. Find half power frequencies ω_1, ω_2 and BW.
3. Determine the average power dissipated at $\omega = \omega_0, \omega_1, \omega_2$.

(Take $V_m = 100$ V.)

[10 + 10 marks]

Solu: (ii) 1.

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$50 = \frac{1}{4}\sqrt{\frac{25\times10^{-3}}{C}}$$

$$\therefore \frac{50\times4}{\sqrt{25\times10^{-3}}} = \frac{1}{\sqrt{C}}$$

$$\therefore \frac{25\times10^{-3}}{(50\times4)^2} = C$$

$$\therefore \boxed{C = 6.25\times10^{-7}\ F}$$

2. $\omega_1 = \pm\frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_0^2}$

$$\omega_1 = \pm\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\therefore \omega_1 = \frac{4}{2\times25\times10^{-3}} + \sqrt{\left(\frac{4}{2\times25\times10^{-3}}\right)^2 + \frac{1}{25\times10^{-3}\times6.25\times10^{-7}}}$$

$$\omega_1 = 8080.005\ \frac{rad}{sec}$$

$$\omega_2 = -\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right]$$

$$\omega_2 = -7920.005 \frac{rad}{sec}$$

$$BW = \frac{\omega_0}{Q}$$

$$BW = \frac{1}{\sqrt{LC}} \times \frac{1}{50}$$

$$BW = \frac{1}{50\sqrt{25\times10^{-3}\times 6.25\times10^{-7}}}$$

$$\boxed{BW = 160\ rad/sec}$$

To Good

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(i) An LTI system S is defined by its impulse response:

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}$$

The system is excited by an input signal $x(t) = \cos\left(6t + \frac{\pi}{2}\right)$. Using frequency-domain analysis, determine the mathematical expression for the output $y(t)$.

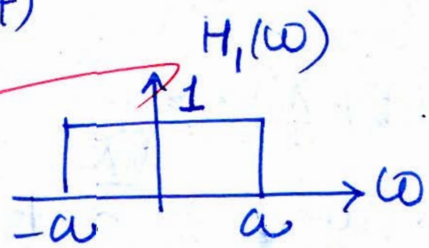
(ii) The impulse response of a causal discrete time LTI system is given by $h[n] = a^n u[n]$, where $a > 0$ and $u[n]$ is the unit step function. Derive the expression for the unit step response $s[n]$ of the system. It is given that the step response at $n = 2$ is $s[2] = 7$. Determine the value of the parameter a.

[10 + 10 marks]

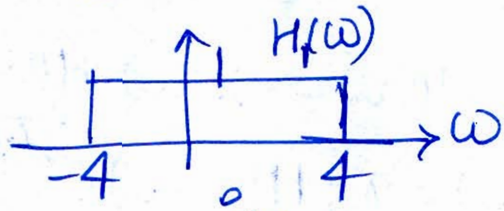
(i) $h(t) = \dfrac{\sin(4(t-1))}{\pi(t-1)}$

$y(t) = x(t) * h(t)$

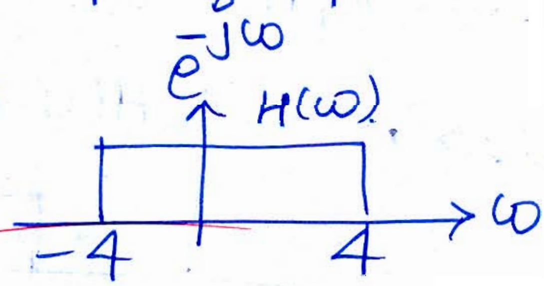
$h_1(t) = \dfrac{\sin at}{\pi t} \longleftrightarrow$



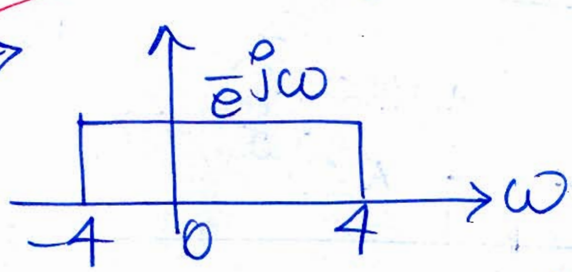
$\dfrac{\sin 4t}{\pi t} \longleftrightarrow$



$\dfrac{\sin 4(t-1)}{\pi(t-1)} \longleftrightarrow$



$\therefore$ $H(\omega) \Rightarrow$



$x(t) = \dfrac{e^{j6t} \cdot e^{j\frac{\pi}{2}}}{2} + \dfrac{e^{-j6t} \cdot e^{-j\frac{\pi}{2}}}{2}$

$\cos(6t + \frac{\pi}{2}) \leftrightarrow \pi[\delta(\omega - 6) + \delta(\omega + 6)]$

$\frac{1}{2} e^{j6t} \cdot e^{j\frac{\pi}{2}} \leftrightarrow ?$

$1 \leftrightarrow 2\pi\,\delta(\omega)$

$\frac{1}{2} \leftrightarrow \pi\,\delta(\omega)$

$\frac{1}{2} e^{j6t} \leftrightarrow \pi\,\delta(\omega - 6)$

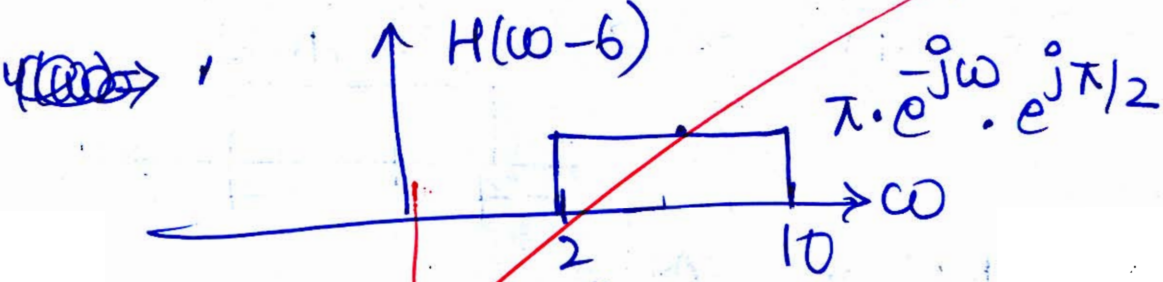
$\frac{1}{2} e^{j6t} \cdot \left(e^{j\frac{\pi}{2}}\right) \leftrightarrow [\pi\,\delta(\omega - 6)]\left(e^{j\frac{\pi}{2}}\right)$

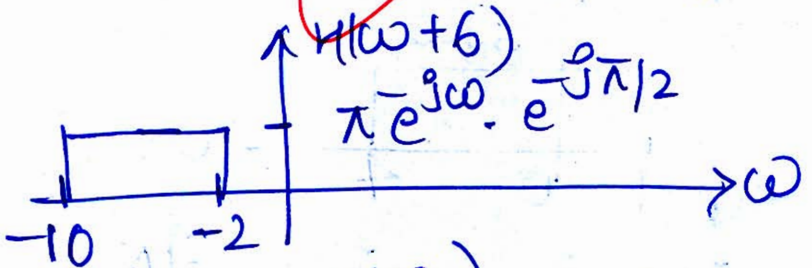
$\therefore \cos(6t + \frac{\pi}{2}) \leftrightarrow$

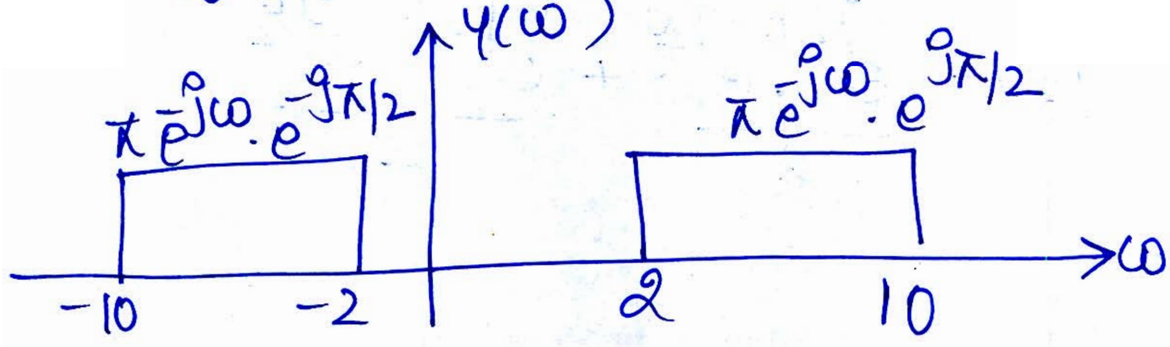
$\pi\left[\delta(\omega - 6)\, e^{j\frac{\pi}{2}} + \delta(\omega + 6)\, e^{-j\frac{\pi}{2}}\right] = X(\omega)$

$Y(\omega) = X(\omega)\, H(\omega)$

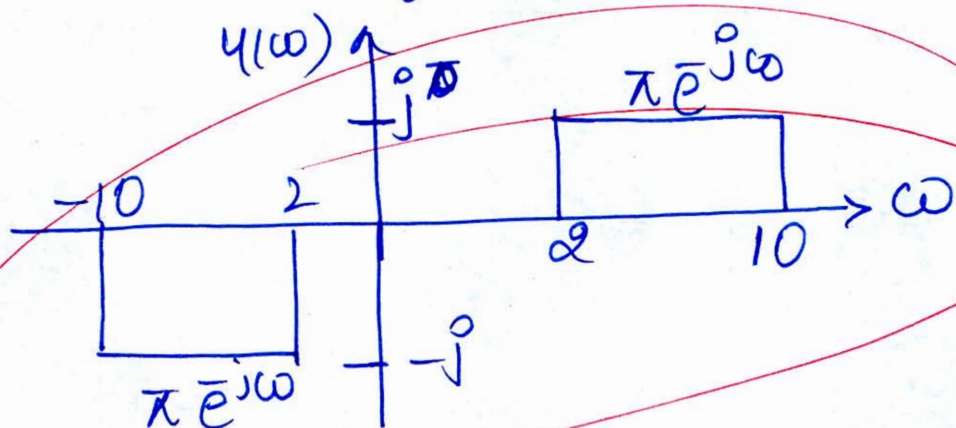
$\therefore Y(\omega) = \pi\left[H(\omega - 6)\, e^{j\pi/2} + H(\omega + 6)\, e^{-j\pi/2}\right]$







since $e^{j\pi/2} = +j$ and $e^{-j\pi/2} = -j$



given, $h(n) = a^n u(n)$ $\quad a>0$

$$s(n) = \sum_{n=-\infty}^{n} h(n)$$

since, unit step response is

$$s(n) = x(n) * h(n)$$

where, $x(n) = u(n)$

∴ $s(n) = u(n) * h(n)$

$$s(n) = \sum_{n=-\infty}^{n} h(n)$$

∴ $$s(n) = \sum_{n=-\infty}^{n} \alpha^n u(n)$$

$$s(n) = \sum_{n=0}^{n} \alpha^n = \alpha^0 + \alpha^1 + \dots + \alpha^n$$

$$s(n) = \frac{a(1-r^{n+1})}{(1-r)} = \frac{1(1-\alpha^{n+1})}{(1-\alpha)}$$

for $s(2) = 7 \Rightarrow 7 = \frac{1-\alpha^3}{1-\alpha} = \frac{(1-\alpha)(\alpha^2+\alpha+1)}{(1-\alpha)}$

$7 = \alpha^2 + \alpha + 1$

∴ $\alpha^2 + \alpha - 6 = 0$

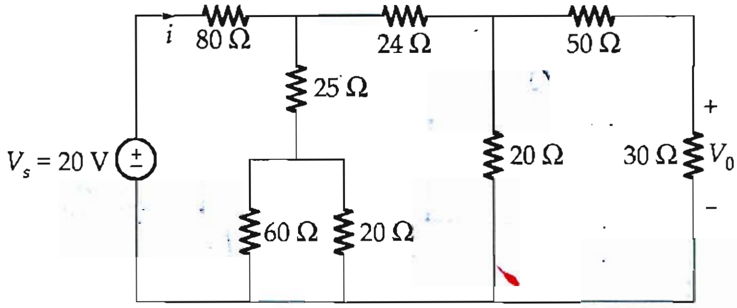
$\alpha = +2, \alpha = -3$

∵ $\alpha > 0$ ∴ $\alpha = 2$

on solving above equation for $s(2) = 7$

∴ $\alpha = 2$

Q.8 (c) **(i)** For the circuit shown in figure below:



Calculate:

1. Input current, i
2. Output voltage, V_0
3. Power Efficiency of the system if V_s and i is considered as input and P_0 is considered as output power.

(ii) Let $u(t)$ be the unit step function and $r(t) = tu(t)$ be the unit ramp function. Derive the expression for the convolution $z(t) = u(t+1) * r(t-2)$. Simplify the result and sketch the waveform of $z(t)$.

[12 + 8 marks]

Soln.

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Space for Rough Work

Space for Rough Work