• Try to avoid over writing

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MADE EASY

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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Digital Electronics + Microprocessors + Electrical Circuits-1 + Systems and Signal Processing-1

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Hyderabad ☐		

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	24
Q.2	
Q.3	31
Q.4	
Section-B	
Q.5	41
Q.6	46
Q.7	53
Q.8	
Total Marks Obtained	195

Signature of Evaluator: Sourabh Kumar

Cross Checked by: ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
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6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Digital Electronics + Microprocessors

(a) Derive a minimized POS expression for the given function and realize using NOR-gate only.

$$F(A, B, C, D) = \Sigma m(0, 1, 3, 4, 6, 9, 13, 14)$$

[12 marks]

$F(A,B,C,D) = \Pi M(2,5,7,8,10,11,12,15)$

AB \ CD	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	0	1	3	1 (2)
$A+\bar{B}$	4	1 (5)	1 (7)	6
$\bar{A}+\bar{B}$	1 (12)	13	1 (15)	14
$\bar{A}+B$	1 (8)	9	1 (11)	1 (10)

$F = (C+D+\bar{A})\cdot(A+\bar{B}+\bar{D})(\bar{A}+\bar{C}+\bar{D})\cdot(B+\bar{C}+D)$

$= (\bar{A}+C+D)(A+\bar{B}+\bar{D})(\bar{A}+\bar{C}+\bar{D})(B+\bar{C}+D)$

we know $\overline{A+B} = \bar{A}\cdot\bar{B}$

$= (\bar{A}+\overline{\bar{C}\bar{D}})(A+\overline{B\cdot D})(\overline{A\cdot CD})(\overline{\bar{B}\bar{D}}+\bar{C})$.

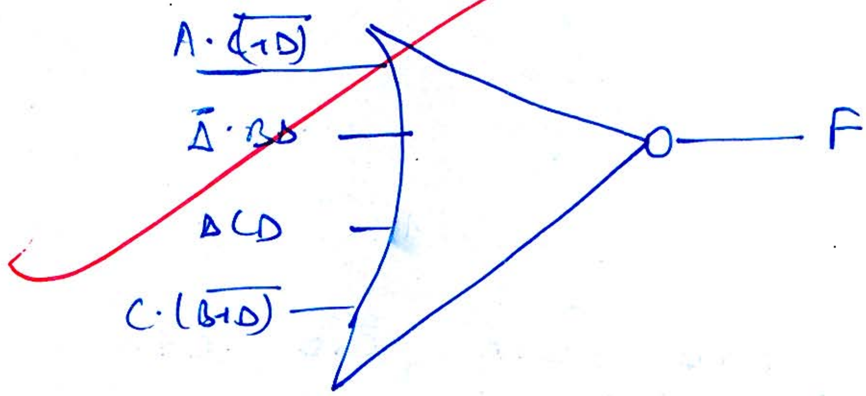
$\overline{A\cdot B} = \bar{A}+\bar{B}$

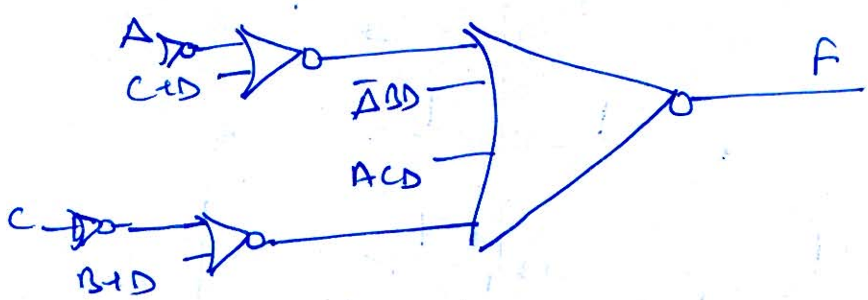
$= (\overline{A\cdot\bar{C}\bar{D}})\cdot(\overline{\bar{A}\cdot B\cdot D})\cdot(\overline{A\cdot C\cdot D})(\overline{C\cdot\bar{B}\bar{D}})$

$= \overline{A\cdot\bar{C}\bar{D} + \bar{A}\cdot BD + ACD + C\bar{B}\bar{D}}$

$= \overline{A\cdot(\overline{C+D}) + \bar{A}\cdot BD + ACD + C\cdot(\overline{B+D})}$

$$F = (\bar{A} + C + D)(A + \bar{B} + \bar{D})(\bar{A} + \bar{C} + \bar{D})(B + \bar{C} + D)$$





$$A \cdot \overline{C+D} = \overline{\overline{A \cdot \overline{C+D}}} = \overline{\bar{A} + (C+D)} =$$

8

Go Through the made easy solution

(b) Design an odd parity bit generator using four bit input.

[12 marks]

Odd parity bit

no of 1	Q_3	Q_2	Q_1	Q_0	Y
0	0	0	0	0	0
1	0	0	0	1	1
1	0	0	1	0	1
2	0	0	1	1	0
1	0	1	0	0	1
2	0	1	0	1	0
2	0	1	1	0	0
3	0	1	1	1	1
1	1	0	0	0	1
2	1	0	0	1	0
2	1	0	1	0	0
3	1	0	1	1	1
2	1	1	0	0	0
3	1	1	0	1	1
3	1	1	1	0	1
4	1	1	1	1	0

no of 1 = even $Y = 0$

no of 1 = odd $Y = 1$ (odd parity)

So $Y = \Sigma m(1, 2, 4, 7, 8, 11, 13, 14)$

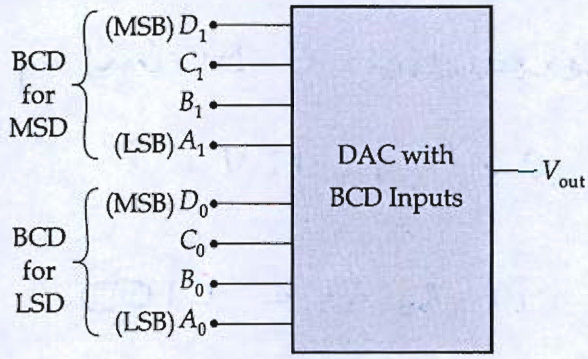
K-map

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_3\bar{Q}_2$	$_{0}$	$1\ _{1}$	$_{3}$	$1\ _{2}$
$\bar{Q}_3 Q_2$	$1\ _{4}$	$_{5}$	$1\ _{7}$	$_{6}$
$Q_3 Q_2$	$_{12}$	$1\ _{13}$	$_{15}$	$1\ _{14}$
$Q_3\bar{Q}_2$	$1\ _{8}$	$_{9}$	$1\ _{11}$	$_{10}$

$$Y = Q_3 \oplus Q_2 \oplus Q_1 \oplus Q_0$$

8

(c) A digital to analog converter using BCD input code is shown in the figure below.



If the weight of A_1 is 2 V, then **find:**

(i) Step size.

(ii) Full scale output voltage.

(iii) Percentage resolution.

(iv) V_{out} for $D_1C_1B_1A_1$ = 0110 and $D_0C_0B_0A_0$ = 0100. **[12 marks]**

i) Resol^n in terms of smallest voltage = step size.

If $A_1 = 2V$ weight i.e (0001)

For BCD Input Max^m weight $D_1C_1B_1A_1 = 1001$.

$\boxed{\text{step size} = 2V}$ ← Ans

ii) Full scale o/p voltage = Resol^n × Decimal equivalent

$= 2 \times 9 = 18V$.

iii) Percentage Resolution $= \dfrac{1}{\text{no. of step discrete}} \times 100$.

Total no of step = ~~100~~ / ~~9999~~ = 0000 to 9999

% resolution $= \dfrac{100}{10000} = 0.01\%$
↳ Ans

iv) V_{out} for $D_1 C_1 B_1 A_1 = 0110$

V_{out} = Resolution × Decimal eq. of 0110

$= 2 \times (6) = 12V$

V_{out} for $D_0 C_0 B_0 A_0 = 0100$

V_{out} = Resolution × Decimal equivalent of 0100

$= 2 \times (4) = 6$ Volt

↳ Ans

4

(d) List of the functional classification of 8085 instruction set. Give one example for each class.

[12 marks]

Functional classification 8085 instruction set

Program counter [PC] → address top of next stack

Stack pointer → Push [SP - 2] & POP [SP + 2]

Pair of Accumulator

BC pair, DE pair, HL pair.

There are 5 diff types of Instruction type

Register direct Addressing

Immediate Address

Register Indirect

Opcode fetch.

Memory Read

Memory Write

I/O read

I/O write

4

Incomplete solution

1 (e) A bar code scanner scans the boxes being shipped from the loading dock and record all the codes in computer memory; the end of the data is indicated by the byte 00. The code 10100011 (A3H) is assigned to 19″ television sets. Write a program to count the number of 19″ television sets that were shipped from the following data set: Data: FA, 67, A3, B8, A3, A3, FA, 00. Write comments in the program.

[12 marks]

Q.2 (a) Design a 3-bit counter that goes through the states 2, 4, 5, 7, 2, 4, ... using T-flip flops. Assume the unused states as don't cares. Check whether the designed counter is self starting or not and there by give the complete sequence diagram for the designed counter.

[20 marks]

2 (b) (i) Calculate the delay produced by the following subroutine program of an 8085 microprocessor, which is operating with a clock frequency of 2 MHz.

```
DELAY  :    MVI B, 02H
LOOP2  :    MVI C, FFH
LOOP1  :    DCR C
            JNZ LOOP1
            DCR B
            JNZ LOOP2
            RET
```

[14 marks]

Do not write in this margin

2 (b) (ii) Explain the features of the three sources of interrupts in the 8086 microprocessor.

[6 marks]

Q.2 (c) Draw the state diagram of a modulo-4 UP/DOWN counter. Design the circuit using JK flip flops.

[20 marks]

Do not write in this margin

Q.3 (a) (i) Design a synchronous counter using D-flip flop that counts in the following sequence: 6, 3, 5, 0, 2, 6, 3, 5, 0, 2, 6

[10 marks]

It is repeating $6 \rightarrow 3 \rightarrow 5 \rightarrow 0 \rightarrow 2$. = Used state

Total 5 sequence means 5 states, (Unused state 1, 7, 4)

we know $2^n \geq N \Rightarrow 2^n \geq 5$ so $n = 3$

no of flipflop required = 3.

	Present state			Next state			Flip flop I/D		
Dec	Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
0	0	0	0	0	1	0	0	1	0
(1)	0	0	1	X	X	X	X	X	X
(2)	0	1	0	1	1	0	1	1	0
(3)	0	1	1	1	0	1	1	0	1
(4)	1	0	0	X	X	X	X	X	X
(5)	1	0	1	0	0	0	0	0	0
(6)	1	1	0	0	1	1	0	1	1
(7)	1	1	1	X	X	X	X	X	X

K-map for D2

			Q_1Q_0	$Q_1\bar{Q_0}$
$\bar{Q_2}$	0	X 1	1 3	1 2
Q_2	X 4	5	X 7	6

$D_2 = \bar{Q_2} Q_1$

K-map for D1

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2$	1 (0)	X (1)	(3)	1 (2)
Q_2	X (4)	(5)	X (7)	1 (6)

$D_1 = \cancel{Q_1\bar{Q}_0 + \bar{Q}_1\bar{Q}_0}$

or

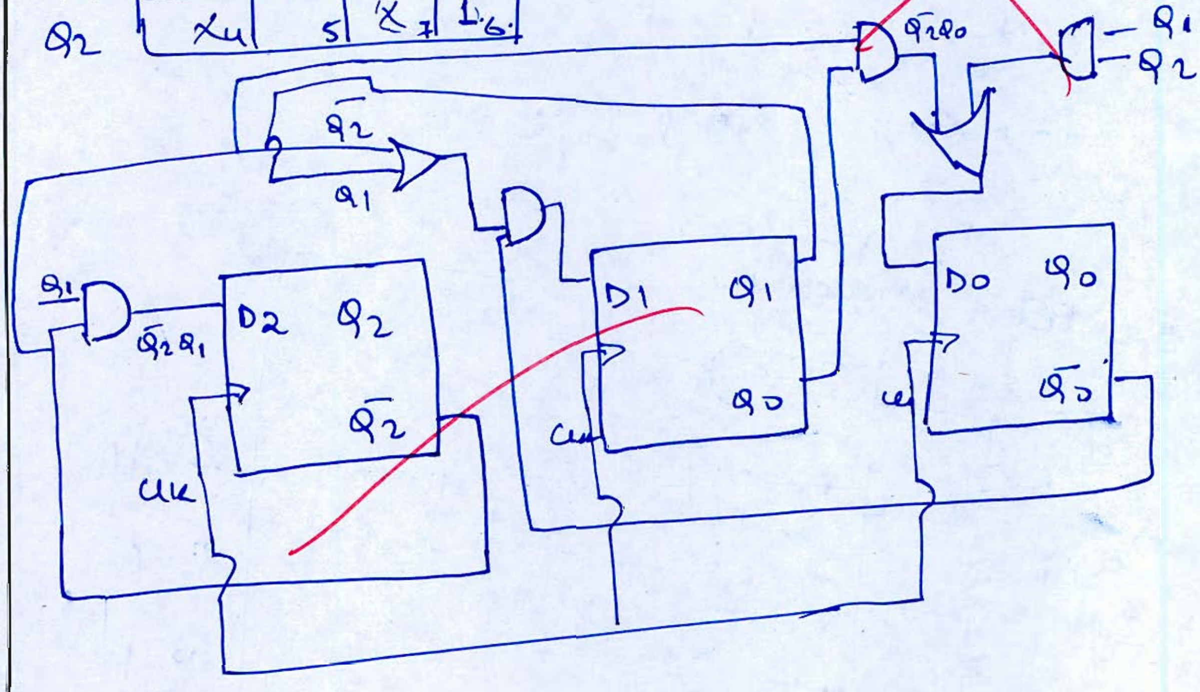
$\cancel{Q_1 + \bar{Q}_0 + \bar{Q}_2\bar{Q}_1}$.

$D_1 = Q_1\bar{Q}_0 + \bar{Q}_2\bar{Q}_0 = \bar{Q}_0(Q_1 + \bar{Q}_2)$.

K-map for D0

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2$	(0)	X (1)	1 (3)	(2)
Q_2	X (4)	(5)	X (7)	1 (6)

$D_0 = \bar{Q}_2 Q_0 + Q_2 Q_1$



7

Q.3 (a) (ii) Explain the working of 3-bit flash type ADC.

[10 marks]

3 bit Flash Type ADC is fastest ADC.

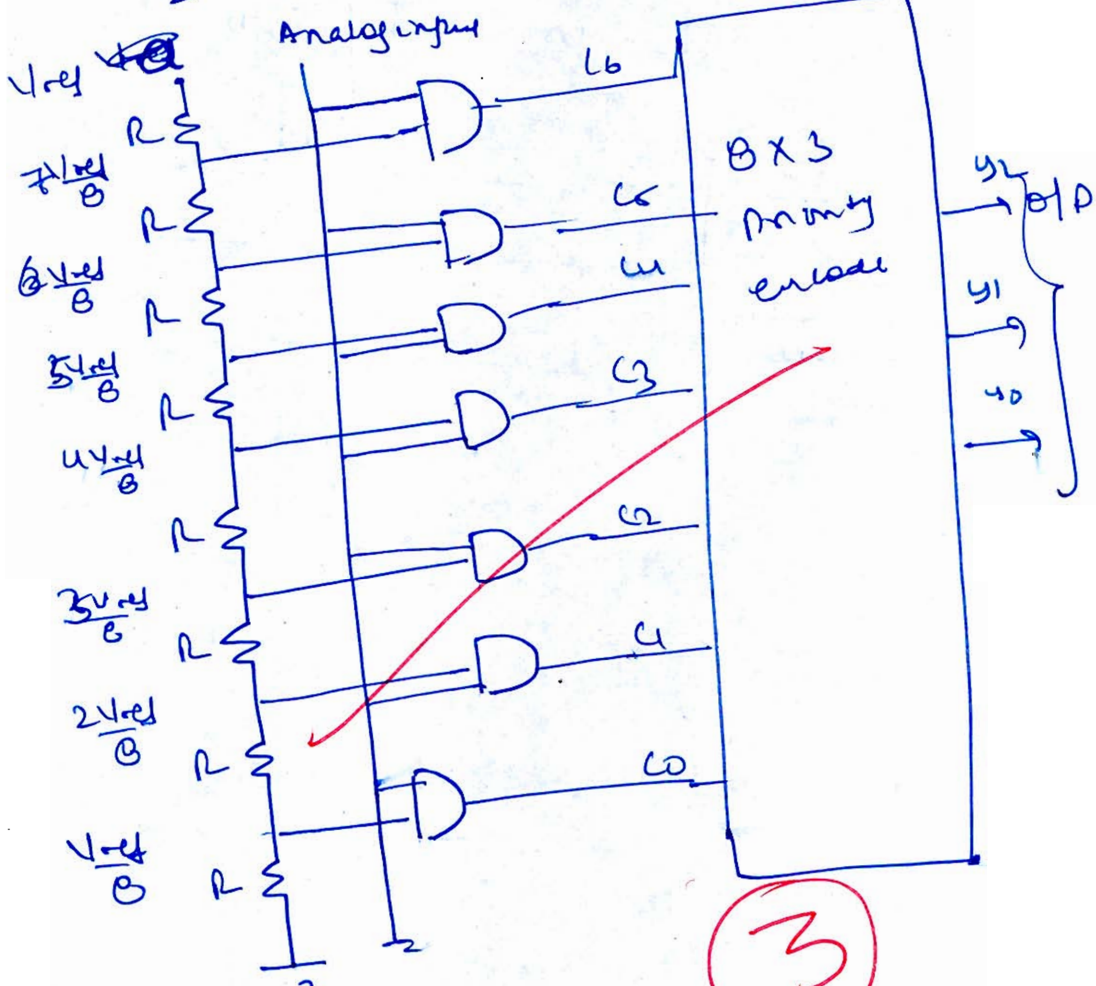
It requires 7 comparator, 8 resistor, 8X3 encoder

For N bit flash ADC

(2^N-1) comparator.

$(2^N \times N)$ priority encoder

2^N resistors



3

Binary i/p Analog	Digital o/p	O/P
	C_6 C_5 C_4 C_3 C_2 C_1 C_0	y_2 y_1 y_0
	0 0 0 0 0 0 1	
	0 0 0 0 0 1 0	
	0 0 0 0 1 0 0	
	0 0 0 1 0 0 0	
	0 0 1 0 0 0 0	
	0 1 0 0 0 0 0	
	1 0 0 0 0 0 0	

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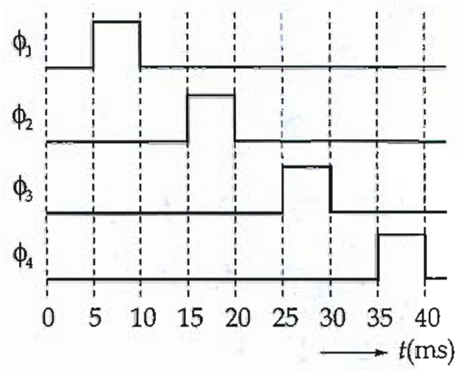
Q.3 (b) (i) Design a synchronous 3-bit binary up-counter using *D*-flip flops.

(ii) A stepper-motor drive circuit requires four periodic signal waveforms, each with a period of 40 ms, as shown below. By using the counter circuit obtained in part (i), design a circuit to generate the necessary signal waveforms for this stepper motor.



[10 + 10 marks]

i) Syncronous 3 bit Binary up counter using DFF

Present State			Next State			Flip flop I/P		
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0	D_2	D_1	D_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	0
0	1	0	0	1	1	0	1	1
0	1	1	1	0	0	1	0	0
1	0	0	1	0	1	1	0	1
1	0	1	1	1	0	1	1	0
1	1	0	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0

$D_2 = \Sigma m(3,4,5,6)$

$D_1 = \Sigma m(1,2,5,6)$

$D_0 = \Sigma m(0,2,4,6)$

· K-map D2

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	0	1	1 3	2
Q_2	1 4	1 5	7	1 6

$D_2 = \bar{Q}_2 Q_1 Q_0 + Q_2\bar{Q}_1 + Q_2\bar{Q}_0$

$= Q_2(\bar{Q}_1 + \bar{Q}_0) + \bar{Q}_2 Q_1 Q_0$

$= Q_2(\overline{Q_1 Q_0}) + \bar{Q}_2 Q_1 Q_0$

$= Q_2 \oplus Q_1 Q_0$

K-map D1

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$		1		1
Q_1		1		1

$D_1 = \bar{Q}_1 Q_0 + Q_1\bar{Q}_0$

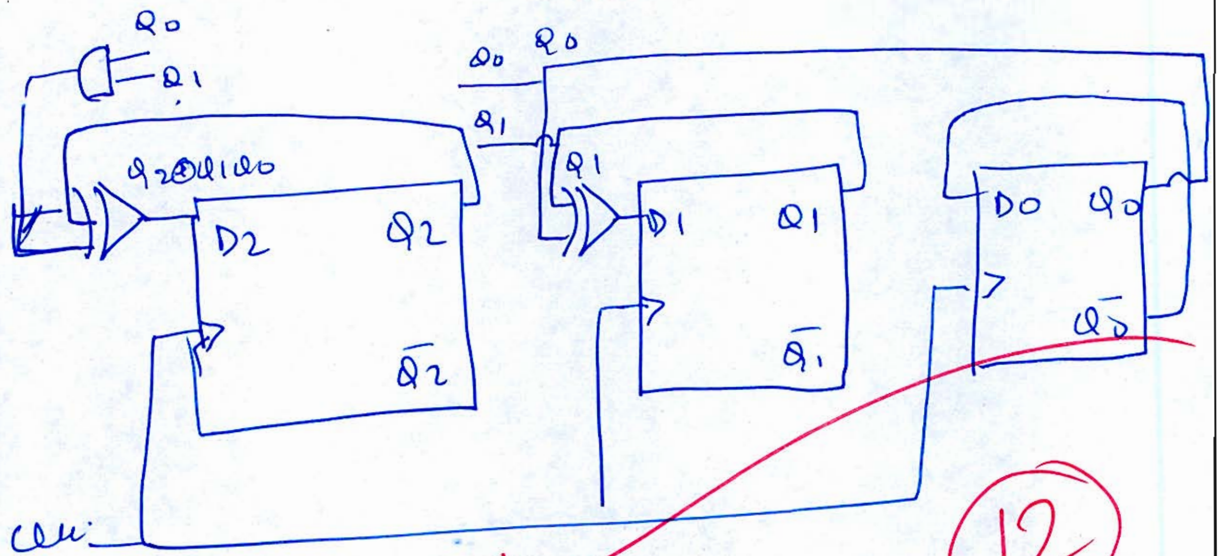
$= Q_1 \oplus Q_0$

K-map D0

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	1			1
Q_2	1			1

$D_0 = \bar{Q}_1\bar{Q}_0 + Q_1\bar{Q}_0$

$D_0 = \bar{Q}_0$



12

Incomplete solution

) $T = 40\,ms$ (Given)

$T = nT_{clk}$ So As part (1) total 8 state.

$T_{clk} = \frac{1}{8T}$

$= \frac{1}{320} = \frac{1000}{320} = 3.125\,sec$

.3 (c) (i) Write a short note on the flag register of 8086 microprocessor.

[14 marks]

i) 8086 Intel ~~8~~ flag register

S, Z, AC, P, CY, O, D, I, T.

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
X	X	X	O	D	I	T	S	Z	X	AC	X	P	X	CY

S → sign flag → If MSB = 0 no +ve
MSB = 1 no −ve

Z → zero flag   If All are 0 sign flag set
If not sign flag reset

AC → Auxiliary Carry flag   Carry Transfer lower nibble to upper nibble

P → Parity flag   If no of 1's even PF set
no of 1's odd PF reset

C → Carry flag   If CY = 1 carry flag set
CY = 0 carry flag reset

9

O → overflow flag

D → Directional flag

I → Interrupt flag   when Interrupt occur it is 1 otherwise 0.

T → Trap flag.   Trap is non maskable

3 (c) (ii) Write down the purpose of each bit in SIM (Set Interrupt Mask) Instruction. Give three different functions of SIM instruction.

[6 marks]

SIM → It is Mask enable of disable

SME

Q.4 (a) (i) Write the steps involved in DMA data transfer. Also, describe the functions of 8085 pins which are used in DMA data transfer.

[10 marks]

Q.4 (a) **(ii)** How can we generate a square wave with a variable bit rate, using microprocessor? Output should be available on a chosen port, using bit 'O'.

[10 marks]

4 (b) A set of five 16-bit readings of the power consumption of industrial control units is monitored by meters and stored at memory locations starting at 2050H. Each reading occupies two memory locations: the lower order byte is stored first, followed by the higher order byte. The corresponding maximum limits for each control unit are stored at memory locations starting at 2090H, also with the lower order byte stored first followed by the higher order byte. Write an 8085 assembly language program to subtract each reading from its specified maximum limit and store the difference at the same memory locations of the readings. Also provide a provision in the program to call the indicator subroutine if the reading is higher than its maximum limit and then continue checking the remaining reading.

[20 marks]

4 (c) **(i)** Given that $(292)_{10} = 1204$ in some number system, find the base of that system.

(ii) In the following series, the same integer is expressed in different number systems. Determine the missing number of the series : 10000, 121, 100, ? , 24, 22, 20.

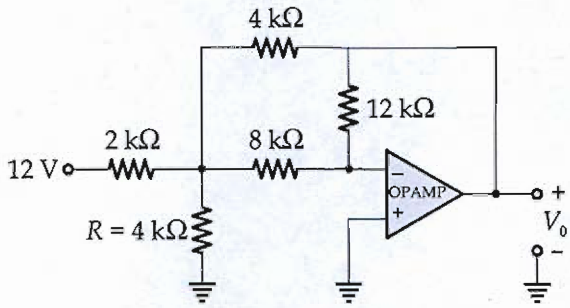
(iii) Add the binary numbers of 1101.101 and 111.011. Find its decimal equivalent.

(iv) Subtract 14 from 46 using 8-bit 2's complement arithmetic.

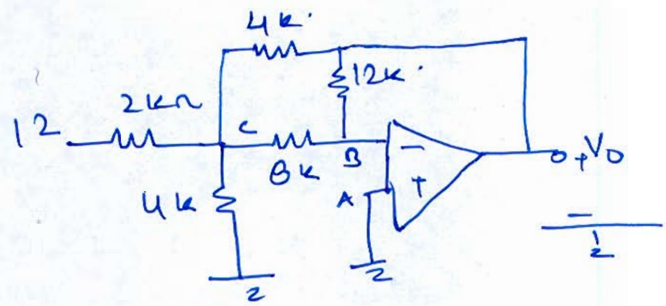
[5 × 4 marks]

Section B : Electrical Circuits-1 + Systems and Signal Processing-1

.5 (a) In the operational amplifier circuit shown in figure below, calculate the current in $R = 4\ \text{k}\Omega$ resistor, using Thevenin's theorem.



[12 marks]



By virtual short

$V_A = V_B = 0$

Apply KCL at node C

$$\frac{V_C - 12}{2} + \frac{V_C}{4} + \frac{V_C - V_B}{8} + \frac{V_C - V_0}{4} = 0$$

$$4V_C - 48 + 2V_C + V_C - 0 + 2V_C - 2V_0 = 0$$

$$9V_C - 2V_0 = 48 \quad \text{— (1)}$$

11

Apply KCL at node B.

$$\frac{V_B - V_C}{8} + \frac{V_B - V_0}{12} = 0$$

$$3V_B - 3V_C + 2V_B - 2V_0 = 0$$

As $V_B = 0$ $\quad 3V_C = -2V_0$

Good Approach

Apply in eqn(1)

$$9V_C - 2V_0 = 48 \Rightarrow 9V_C + 3V_C = 48$$

so $V_C = 4V$

$$\boxed{I_{4k\Omega} = \frac{V_C}{4} = \frac{4}{4} = 1\ \text{mA}} \leftarrow \text{Ans}$$

Q.5 (b) The input $x[n]$ and the impulse response $h[n]$ of a discrete time LTI system are given by:

$$x[n] = u[n]$$

$$h[n] = \alpha^n u[n]; \quad 0 < \alpha < 1$$

Compute the output $y[n]$ by method of convolution.

[12 marks]

Given:

$x(n) = u[n]$, $h[n] = \alpha^n u[n]$ $0<\alpha<1$

To find $y[n]$ by convolution method.

Soln: we know convolution formulae

$$y[n] = x[n] * h[n].$$

$$= \sum_{K=-\infty}^{\infty} x(K)\, h[n-K] = \sum_{K=-\infty}^{\infty} h(K)\, x[n-K].$$

$$= \sum_{K=-\infty}^{\infty} u[n] \cdot \alpha^{n-K} u[n-K] = \sum_{n=0}^{\infty} \alpha^{n-K} u[n-K]$$

$$= \sum_{K=-\infty}^{\infty} \alpha^K u[K]\, \cancel{*} u[n-K] = \sum_{K=0}^{\infty} \alpha^K u[n-K].$$

$$= \sum_{K=0}^{n} \alpha^K =$$

(It is summation of GP series where $\alpha<1$

$$= 1+\alpha+\alpha^2+ \text{--}\ \alpha^n$$

$$\boxed{y[n] = \frac{1(1-\alpha^{n+1})}{1-\alpha}}$$

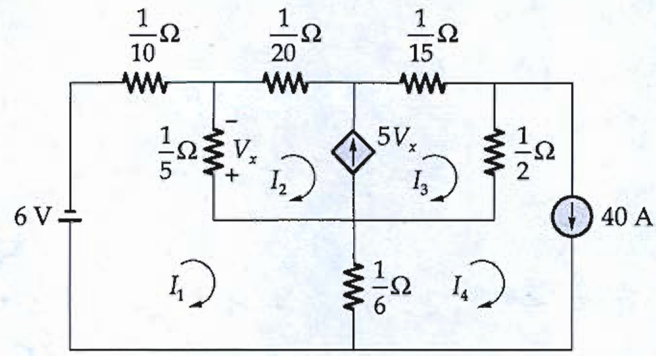
where $\alpha<1$.
$n \geq 0$

↳ Ans

11

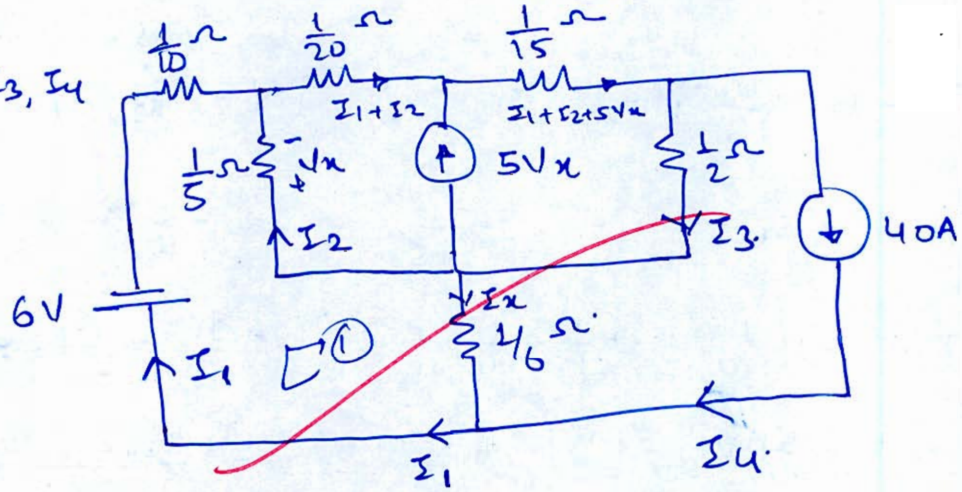
Good Approach

Q.5 (c) Find currents I_1, I_2, I_3 and I_4.



[12 marks]

To find:
I_1, I_2, I_3, I_4



Current in $\frac{1}{6}\Omega$ resistor $= I_x$

$\cancel{I_x + 5V_x}$ $\quad I_x + I_4 = I_1 \Rightarrow I_x = (I_4 - I_1)$

$I_x = (I_1 - I_4)$

Also $I_x + 5V_x + I_2 = I_3$.

& $I_2 = \frac{V_x}{1/5} = 5V_x$. So $I_x + 2I_2 = I_3$

So $I_x = I_3 - 2I_2 = I_1 - I_4 \Rightarrow I_3 + I_4 = I_1 + 2I_2$.

Also current in $\frac{1}{20}\Omega = I_1 + I_2$

$I_1 + I_2 + 5V_x = I_3 + 40$

$I_4 = 40A$

$I_2 = 5V_x$

$I_x = I_1 - I_4 = I_3 - 2I_2$

$I_1 + 2I_2 = I_3 + 40$

Apply KVL loop 1

$-6 - \frac{1}{10}I_1 + V_x - \frac{1}{6}I_x = 0 \Rightarrow \frac{1}{6}I_x + \frac{1}{10}I_1 + 6 = V_x$

$\frac{(I_1 - I_4)}{6} + \frac{I_1}{10} + 6 = \frac{I_2}{5}$

$$\frac{I_1}{6}+\frac{I_1}{10}-\frac{40}{6}+6=\frac{I_2}{5}$$

$$\frac{5I_1+3I_1}{30}-\frac{I_2}{5}=\frac{40}{6}-6$$

$$\frac{8I_1}{30}-\frac{I_2}{5}=\frac{4}{6} \Rightarrow \frac{8I_1-6I_2}{30}=\frac{20}{30}$$

$4I_1-3I_2=20$ — (1),

Drop above
$I_1+2I_2=I_3+40$
$I_4=40A$
$I_2=54x$
$I_x=I_1-I_4=I_3-2I_2$

Again Apply KVL in loop1

$$-6-\frac{1}{10}I_1-\frac{1}{20}(I_1+I_2)-\frac{1}{15}(I_1+I_2+54x)-\frac{1}{2}I_3-\frac{1}{6}I_x=0$$

$$-6-\frac{1}{10}I_1-\frac{1}{20}I_1-\frac{I_2}{20}-\frac{1}{15}(I_1+2I_2)-\frac{1}{2}I_3-\frac{1}{6}(I_1-40)=0$$

$$6+I_1\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{15}+\frac{1}{6}\right)+I_2\left(\frac{1}{20}+\frac{2}{15}\right)+\frac{1}{2}I_3=\frac{40}{6}$$

$$6+\frac{1}{10}I_1+\frac{1}{20}I_1+\frac{I_2}{20}+\frac{1}{15}I_1+\frac{2}{15}I_2+\frac{(I_1+2I_2-40)}{2}+\frac{1}{6}(I_1-40)=\frac{40}{6}$$

$$6+I_1\left(\frac{6+3+4+10}{60}\right)+I_2\left(\frac{3+8}{60}\right)+\frac{I_1}{2}+I_2-40=\frac{40}{6}$$

$$I_1\left(\frac{23}{60}+\frac{1}{2}\right)+I_2\left(\frac{11}{60}+1\right)=40+\frac{40}{6}-6=\frac{244}{6}$$

$$I_1(23+30)+I_2(60+11)=\frac{244}{6}\times 60$$

$53I_1+71I_2=2440$ — (2)

6

Wrong Value calculated

Solve eqn 1 & 2

$$53I_1+71\left(\frac{4I_1-20}{3}\right)=2440 \Rightarrow I_1\left(53+\frac{284}{3}\right)=2440+\frac{71\times 20}{3}$$

$$\frac{443}{3}I_1=\frac{8740}{3} \Rightarrow \boxed{I_1=6.576A}$$

$$\text{so } I_2=\frac{4I_1-20}{3}= \boxed{I_2=2.101A}$$

$$I_3=I_1+2I_2-40= \boxed{I_3=-29.22A}$$

$I_4=40A$

Ans

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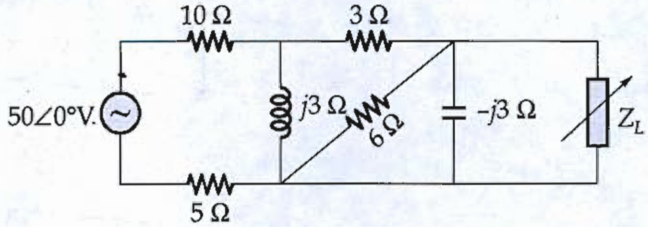
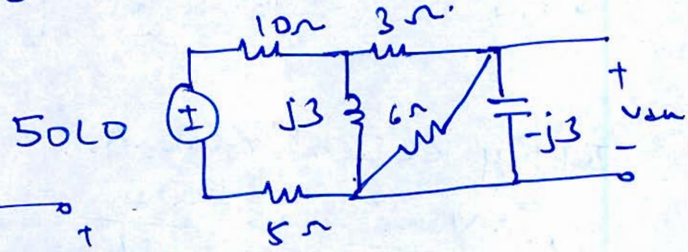
Q.5 (d) Find the impedance Z_L so that maximum power can be transferred to it in the network shown below. Also, find the maximum power delivered to load Z_L.

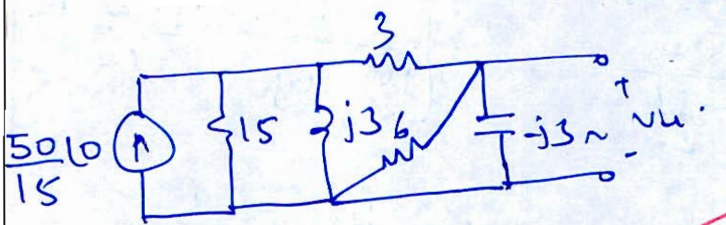


[12 marks]

For Max^m Power find V_{th} & R_{th}.

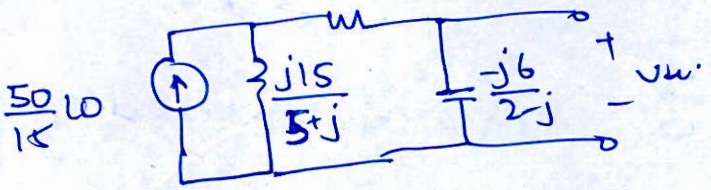
For V_{th}:





$6 \parallel -j3 = \frac{6 \times -j3}{6-j3} = \frac{-j6}{2-j}$

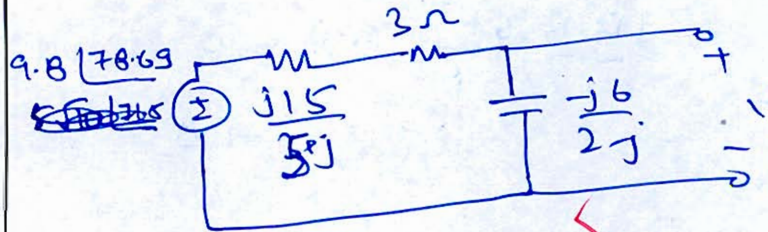
$15 \parallel j3 = \frac{15 \times j3}{15+j3} = \frac{j15}{5+j}$



$\frac{50}{15} \times \frac{j15}{j+5}$

$= \frac{j50}{j+5}$

$= 9.8\angle 78.69$

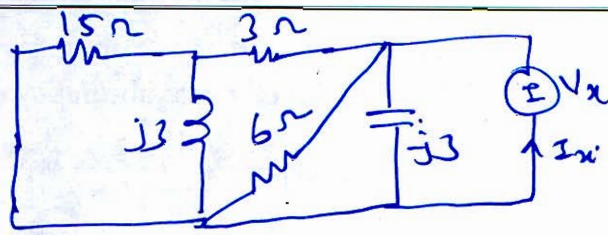


$V_{th} = (5+15i) \times \left[\frac{\frac{-j6}{2-j}}{3-\frac{j6}{2-j}+\frac{j15}{5+j}}\right] = 8.82+0.36i$

$= 8.836\angle 2.33$ volt

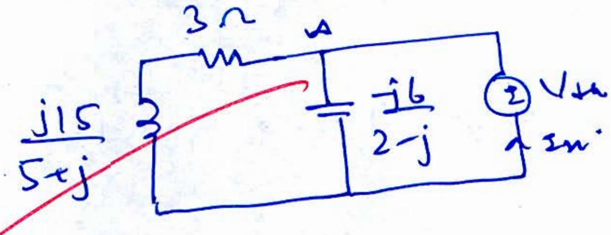
For R_{th}:

deactivate voltage source



$15 \parallel j3$

$= \frac{15 \times j3}{15 + j3} = \frac{j15}{5+j}$



Apply KCL at node A

$$I_x = \frac{V_{th}}{3 + \frac{j15}{5+j}} + \frac{V_{th}}{\frac{-j6}{2-j}} \Rightarrow V_{th}\left[\frac{5+j}{15+18j} + \frac{j-2}{j6}\right]$$

$$I_x = V_{th}\left[\frac{41}{122} + \frac{12}{61} i\right] \Rightarrow R_{th} = \frac{V_{th}}{I_x} = \frac{1}{\frac{41}{122} + \frac{12}{61} i}$$

$$Z_{th} = 2.216 - 1.297i$$

For Max^m Power $Z_L = Z_{th}^* = 2.216 + 1.927i$

↳ Ans

For Max^m Power delivered to load Z_L

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(8.836)^2}{4 \times 2.216} = 8.8 \text{ watt}$$

↳ Ans

4

Q.5 (e) Show that the resonant frequency ω_0 of an *R-L-C* series circuit is geometric mean of ω_1 and ω_2, the lower and upper half-power frequencies respectively.

[12 marks]

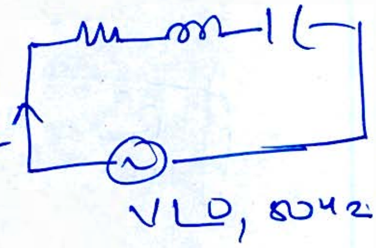
To proof: $\omega_0 = \sqrt{\omega_1 \omega_2}$.

we know

lower half power freq $= \omega_1$

upper half power freq $= \omega_2$.

for series R-L-C



ω_1 occur

when $P = \frac{P_{max}}{2}$

& $I = \frac{I_{max}}{\sqrt{2}}$.

$$\frac{V}{Z} = \frac{V}{\sqrt{2}R}.$$

$$Z = \sqrt{2}R$$

$$R + j\omega L - \frac{1}{j\omega C} = \sqrt{2}R.$$

$$R + \left(j\omega L - \frac{1}{j\omega C}\right) = \sqrt{2}R$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

$$R^2 + \omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} = 2R^2.$$

$$R^2 + \frac{2L}{C} + \omega^2 L^2 + \frac{1}{\omega^2 C^2} = 0$$

$$\frac{\omega^4 L^2 C^2 + 1}{\omega^2 C^2} + R^2 + \frac{2L}{C} = 0$$

So by solving ω we get

ω_1, ω_2.

$$\omega_1 = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}.$$

$$\omega_2 = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}.$$

$$\omega_2 - \omega_1 = \frac{R}{2L} - \left(-\frac{R}{2L}\right) = \frac{R}{L}.$$

$$\omega_1\omega_2 = \left(-\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\right)\left(\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\right)$$

$$= \left(\sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\right)^2 - \left(-\frac{R}{2L}\right)^2.$$

$$= \frac{R^2}{4L^2} + \frac{1}{LC} - \frac{R^2}{4L^2} = \frac{1}{LC}$$

Also resonant frequency $= \omega_0 = \frac{1}{\sqrt{LC}}$ rad/s.

$$\omega_1\omega_2 = \omega_0^2$$

$$\boxed{\omega_0 = \sqrt{\omega_1\omega_2} \text{ rad/s.}}$$

↳ Hence proved

9

Good Approach

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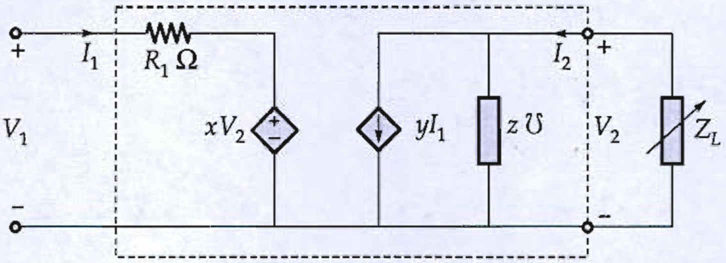
Q.6 (a) Consider a two port network shown in figure below,



If transmission parameters matrix of the network is $\begin{bmatrix} 10^{-2} & 10^{2} \\ 0 & 10^{-1} \end{bmatrix}$.

Then, calculate:

(i) parameters of the circuit: R_1, x, y and z.

(ii) the value of load impedance (Z_L), for maximum power transfer.

(iii) maximum power transfer to load for $V_1 = 0.1$ volt.

[20 marks]

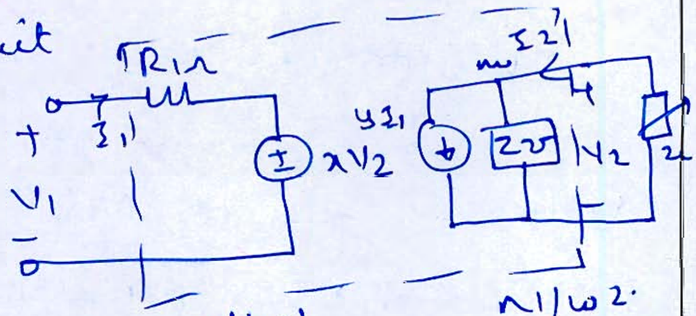
Given Tparameter $= \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{100} & 100 \\ 0 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$.

$V_1 = \frac{1}{100} V_2 - 100 I_2$

$I_1 = -\frac{I_2}{10}$

Also by Given circuit in N/w 1. Apply KVL.

$V_1 - I_1 R_1 - xV_2 = 0$



$V_1 = xV_2 + I_1 R_1$ —(1)

Also Apply KCL in N/w 2 at node m

$I_2 = yI_1 + V_2 z$. So $yI_1 = I_2 - V_2 z$.

$I_1 = \frac{I_2}{y} - \frac{z}{y} V_2$. —(2)

Put value of I_1 in eq^n(1)

$V_1 = xV_2 + R_1\left(\frac{I_2}{y} - \frac{z}{y} V_2\right) = V_2\left(x - \frac{zR_1}{y}\right) + \frac{R_1}{y} I_2$.

So $V_1 = \left(x - \frac{zR_1}{y}\right)V_2 + \frac{R_1}{y}(I_2)$ — (3)

lly eq^n 2 $I_1 = -\frac{z}{y}V_2 + \frac{1}{y}I_2$

Compare above both eq^n 3 & 2 with Given transmission parameter matrix.

$V_1 = \frac{V_2}{100} - 100 I_2.$

$I_1 = -\frac{I_2}{10}.$

So $\frac{1}{y} = -\frac{1}{10} \Rightarrow y = -10$

$\frac{R_1}{y} = -100 \Rightarrow R_1 = 1000 = 1k\Omega.$

$\frac{z}{y} = 0 \Rightarrow$ so $z = 0$

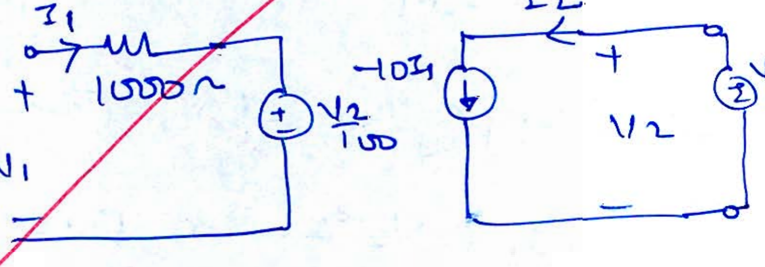
$x - z\frac{R_1}{y} = \frac{1}{100} \Rightarrow x = \frac{1}{100}$

i) So $R_1 = 1000\,\Omega$, $x = 0.01$, $y = -10$, $z = 0$
↳ Ans

ii) Value of Z_L for Max^m Power Transfer.

So Draw circuit apply R_1, x, y, z value

To find Z_L:
remove Z_L
&



$I_2 = -10 I_1$

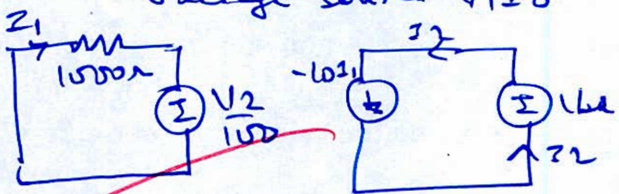
& $V_{th} = V_2.$ & $R_{th} = \frac{V_{th}}{I_2}.$

Apply KVL loop 1.

$V_1 - 1000 I_1 - \frac{V_2}{100} = 0$

↳ (A)

& For R_{th} deactivate voltage source $V_1 = 0$



$-1000 I_1 - \frac{V_2}{100} = 0$

$I_2 = -10 I_1$

$-1000 I_1 = \frac{V_2}{100}$

$-1000\left(\frac{I_2}{-10}\right) = \frac{V_{th}}{100} \Rightarrow 100 \times 100 = \frac{V_{th}}{I_2}$.

$\boxed{Z_L = \frac{V_{th}}{I_2} = 10\,k\Omega}$ ← Ans

iii) For $V_1 = 0.1V$ max^m Power Transfer

we know $P_{max} = \frac{V_{th}^2}{4R_{th}}$

See eq^n A) Above

$V_1 - 1000 I_1 = \frac{V_2}{100} \Rightarrow 0.1 - 1000 I_1 = \frac{V_{th}}{100}$.

Also for finding V_{th} open circuit load that means $I_2 = 0$ so $I_1 = 0$ as $I_2 = -10 I_1$

$+0.1 - 1000(0) = \frac{V_{th}}{100} \Rightarrow V_{th} = 0.1 \times 100 = 10V$.

Now $P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{10 \times 10}{4 \times 10000} = 2.5 mW$

$\boxed{P_{max} = 0.0025\,W = 2.5 \times 10^{-3}\,\text{watt} = 2.5\,mW}$ ↳ Ans

18

Good Approach

Q.6 (b) **(i)** The transfer function relating the input $x(t)$ to the output $y(t)$ of a system is given by $G(s) = \dfrac{1}{(s+3)}$. A unit-step input is applied to the system at time $t = 0$. Assuming that $y(0) = 3$, find the value of $y(t)$ at time $t = 1$.

[12 marks]

Given $G(s) = \dfrac{1}{s+3} = \dfrac{Y(s)}{X(s)}$.

unit step I/P $x(t) = u(t)$ at $t = 0$.

$y(0) = 3$

To find $y(t)|$ at $t=1$.

Ans As $\dfrac{Y(s)}{X(s)} = \dfrac{1}{s+3}$

$sY(s) + 3Y(s) = X(s)$

Apply ILT on above eqn.

$Y(s)[s+3] = X(s)$

$X(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$

5

$y(t) = x(t) \cdot g(t)$

$Y(s) = X(s) \cdot G(s)$

As $x(t) = u(t)$

$= \dfrac{1}{s} \cdot \dfrac{1}{s+3} = \dfrac{1}{3}\left(\dfrac{s+3-s}{s(s+3)}\right) = \dfrac{1}{3}\left(\dfrac{1}{s} - \dfrac{1}{s+3}\right)$

Apply ILT

$y(t) = \dfrac{1}{3}(1 - e^{-3t})\, u(t)$

Above is Forced Response

$y(t)_{total} = y_{natural\ resp} + y_{forced\ resp}$

$y(total) = y_0 + \dfrac{1}{3}(1 - e^{-3t})\, u(t)$.

y_0 means value of y at $t = 0$.

$$y_{total} = 3 + \frac{1}{3}(1 - e^{-3t})\,u(t)$$

$$y|_{at\ t=1} = 3 + \frac{1}{3}\left(1 - e^{-3(1)}\right]$$

$$= 3 + \frac{1}{3}(1 - e^{3})$$

$$\boxed{y|_{at\ t=1} = 3.316} \rightarrow \underline{Ans}$$

Q.6 (b) **(ii)** Consider the signal $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$, where t is in seconds. Find its fundamental time period.

(iii) For a periodic signal $v(t) = 30\sin 100t + 10\cos 300t + 6\sin\left(500t + \frac{\pi}{4}\right)$, find the fundamental frequency in rad/s.

[6 + 2 marks]

$f(t) = 1 + 2\cos\pi t + 3\sin\frac{2\pi}{3}t + 4\cos\left(\frac{\pi t}{2} + \frac{\pi}{4}\right)$

$\omega_1 = \pi$, $\omega_2 = \frac{2\pi}{3}$, $\omega_3 = \frac{\pi}{2}$.

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\pi} = 2$, $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{2\pi/3} = 3$, $T_3 = \frac{2\pi}{\omega_3} = \frac{2\pi}{\pi/2}$

$T_3 = 4$.

Fundamental Time period

$T = LCM(T_1, T_2, T_3) = LCM(2, 3, 4) = 12$.

But Above dc function also given i.e 1.

so $f(t) =$ (Sum of 1 + Periodic f^n)

1 is always continuous

& we know

$f(t) =$ Sum Aperiodic + Periodic

= Aperiodic

5

$HCF(\omega_1, \omega_2, \omega_3, 0) = HCF(1, \pi, \frac{2\pi}{3}, \frac{\pi}{2}) = 1$.

Fundamental time Period = $T = \frac{2\pi}{\omega} = 2\pi$ sec. Ans

(iii) $v(t) = 30\sin 100t + 10\cos 300t + 6\sin\left(500t + \frac{\pi}{4}\right)$.

$\omega_1 = 100$, $\omega_2 = 300$, $\omega_3 = 500$

Fundamental freq = $\omega_0 = HCF(\omega_1, \omega_2, \omega_3)$

$= HCF(100, 300, 500) = 100$

$\boxed{\omega_0 = 100 \text{ rad/s}}$ ← Ans

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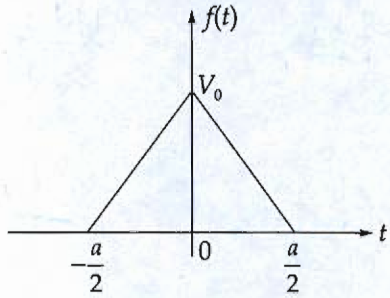
6 (c) (i) The figure shows a triangular pulse which is zero for all time except $-\frac{a}{2} \le t \le \frac{a}{2}$. For this pulse:

1. determine the Fourier transform,
2. sketch the continuous amplitude spectrum.

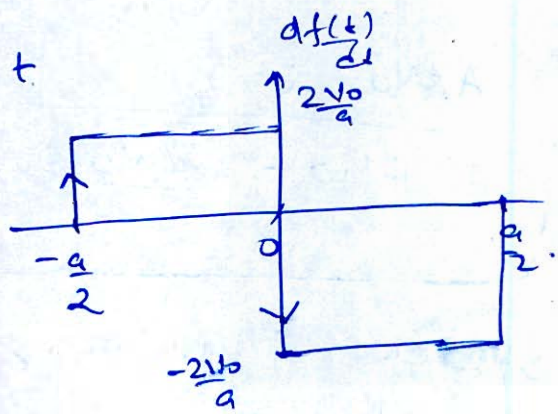


[12 marks]

i) To find Fourier transform

$f(t) = V_0 \, tri\left(\frac{t}{a/2}\right)$

differentiate $f(t)$ w.r.to t



slope $= \frac{V_0}{a/2} = \frac{2V_0}{a}$

Now

$\frac{df(t)}{dt} = \frac{2V_0}{a}\left[u\left(t+\frac{a}{2}\right)\right] - \frac{4V_0}{a}\left[u(t-0)\right] + \frac{2V_0}{a}\left[u\left(t-\frac{a}{2}\right)\right]$

Apply Fourier Transform

$j\omega F(\omega) = \frac{2V_0}{a}\left[\frac{e^{+s\frac{a}{2}}}{s}\right] - \frac{4V_0}{a} \times \frac{1}{s} + \frac{2V_0}{a}\frac{e^{-s\frac{a}{2}}}{s}$

Here $s = j\omega$

$j\omega F(\omega) = \frac{2V_0}{a}\frac{e^{j\omega\frac{a}{2}}}{j\omega} - \frac{4V_0}{aj\omega} + \frac{2V_0}{a}\frac{e^{-aj\frac{\omega}{2}}}{j\omega}$

$j\omega F(\omega) = \frac{2V_0}{aj\omega}\left[e^{j\omega\frac{a}{2}} + e^{-j\omega\frac{a}{2}}\right] - \frac{4V_0}{aj\omega}$

$= \frac{2V_0}{aj\omega}\left[2\cos\left(\frac{\omega a}{2}\right)\right] - \frac{4V_0}{aj\omega}$

$$F(\omega) = \frac{4V_0}{a(j\omega)^2}\cos\left(\frac{a\omega}{2}\right) - \frac{4V_0}{a(j\omega)^2}$$

$$= \frac{4V_0}{a\omega^2}\left[\frac{\cos\left(\frac{a\omega}{2}\right)}{-1} + 1\right] = \frac{4V_0}{a\omega^2}\left[1-\cos\left(\frac{a\omega}{2}\right)\right]$$

$$= \frac{4V_0}{a\omega^2} \times 2\sin^2\left(\frac{a\omega}{4}\right) = \frac{8V_0}{a\omega^2}\,\frac{\sin^2\left(\frac{a\omega}{4}\right)}{\left(\frac{a\omega}{4}\right)^2} \times \left(\frac{a\omega}{4}\right)^2$$

$$= \frac{8V_0}{a\omega^2} \times \frac{a^2\omega^2}{16}\left[\frac{\sin\left(\frac{a\omega}{4}\right)}{\left(\frac{a\omega}{4}\right)}\right]^2 = \frac{aV_0}{2} Sa^2\left(\frac{a\omega}{4}\right)$$

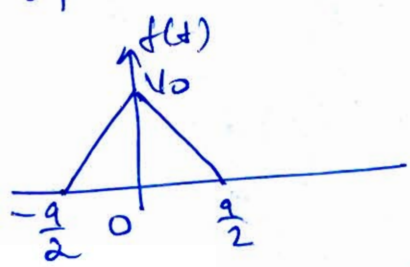
Also we know $A\,tri\left(\frac{t}{\tau}\right) \rightarrow A\tau\, Sa^2\left(\frac{\omega\tau}{2}\right)$

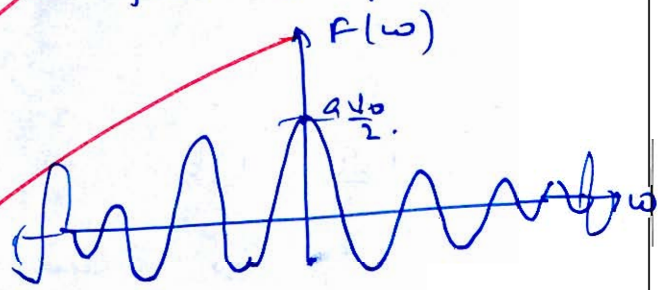
$$V_0\, tri\left(\frac{t}{a/2}\right) \rightarrow \frac{a}{2}V_0\, Sa^2\left(\frac{\omega a}{4}\right)$$

$A = V_0$, $\tau = \frac{a}{2}$

$$\boxed{\text{so } F(\omega) = \frac{aV_0}{2} Sa^2\left(\frac{a\omega}{4}\right)} \rightarrow \text{Ans}$$

(iv) Sketch continuous Amplitude spectrum





Good Approach

11

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6 (c) **(ii)** Using duality property show that the Fourier transform of $\left[\frac{1}{1+j2\pi t}\right]$ is equal to $e^f u(-f)$ where $u(t)$ is the unit step.

[8 marks]

ii) duality property states

if $x(t) \leftrightarrow X(\omega)$

so $\omega \to t$ then $t \to -\omega$

$X(t) \leftrightarrow 2\pi x(-\omega)$.

To prove: $\frac{1}{1+j2\pi t} \leftrightarrow e^{f} u(-f)$.

Sol^n we know Fourier Transform

$e^{-at} u(t) \longleftrightarrow \frac{1}{a+j\omega} = \frac{1}{a+j2\pi f}$.

Apply duality

$\frac{1}{a+jt} \longleftrightarrow 2\pi e^{+a\omega} u(-\omega)$.

If $a=1$

$\frac{1}{1+jt} \longleftrightarrow 2\pi e^{\omega} u(-\omega)$.

Same

$e^{-at} u(t) \longleftrightarrow \frac{1}{a+j2\pi f}$.

so $x(t) \to X(f)$.

then for duality $X(t) \to x(-f)$. $\left\{\begin{matrix} f \to t \\ t \to -f \end{matrix}\right\}$

$\frac{1}{a+j2\pi t} \longleftrightarrow e^{-a(-f)} u(-f)$

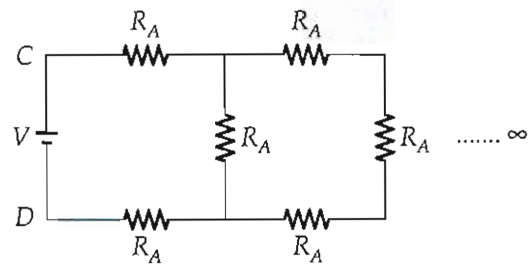
If $a=1$

$\boxed{\frac{1}{1+j2\pi t} \longleftrightarrow e^{f} u(-f)}$ → Hence proved

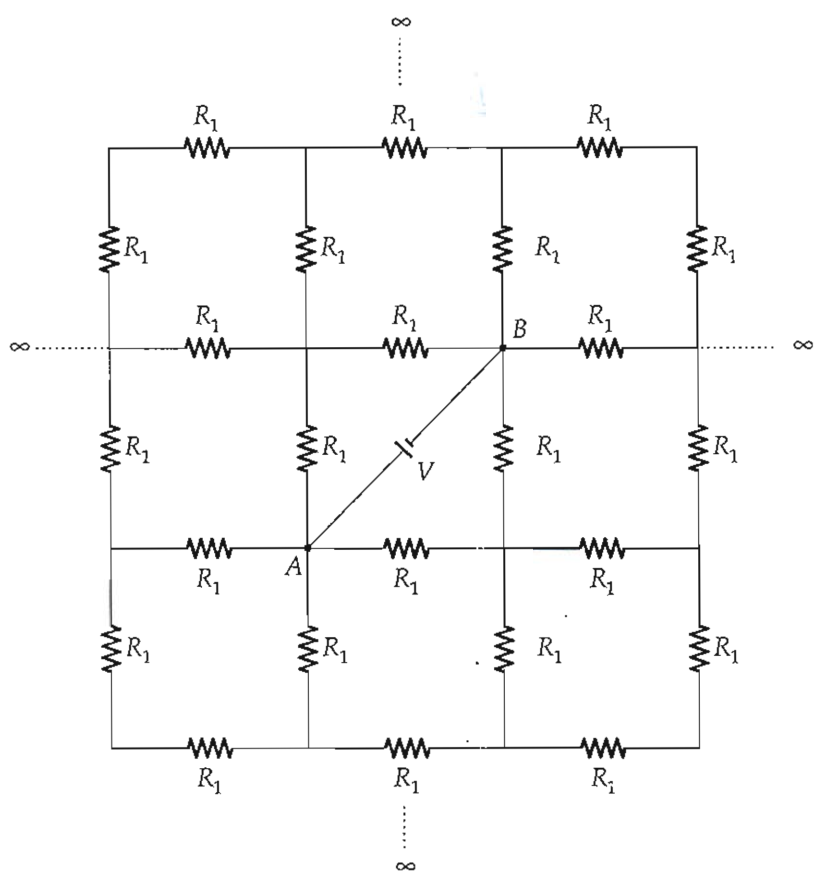
7

Good Approach

Q.7 (a) For the networks shown in figure below,



Network (i)

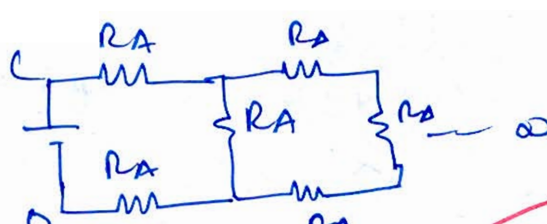


Network (ii)

On applying 10 V voltage source across terminal CD and AB respectively in network (i) and (ii), we get same power delivered by the source. Derive the relationship between the resistance present in network (i) and (ii). And also calculate the power delivered by the source for $R_A = 10\ \Omega$.

[20 marks]

N/w (i)



So $R_{CD} = R_A + (R_A \| R_{CD}) + R_A$.

$R_{CD} = 2R_A + \frac{R_A \cdot R_{CD}}{R_A + R_{CD}}$

$R_{CD}(R_A + R_{CD}) = 2R_A^2 + 3R_AR_{CD}$

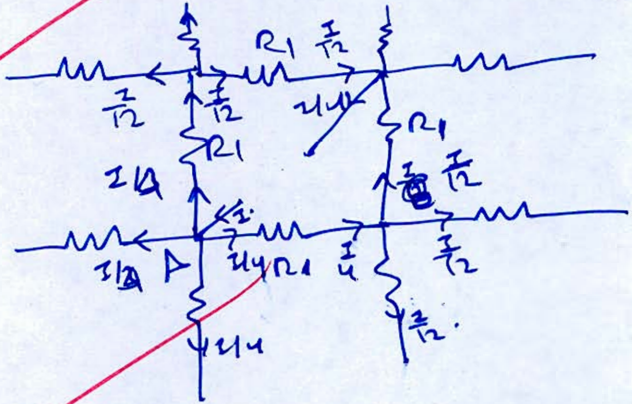
$R_{CD}^2 + R_AR_{CD} = 2R_A^2 + 3R_AR_{CD}$

$R_{CD}^2 - 2R_A R_{CD} - 2R_A^2 = 0$

$R_{CD} = \frac{2R_A \pm \sqrt{4R_A^2 - 4(-2R_A)^2}}{2} = \frac{2R_A \pm 2\sqrt{3}R_A}{2}$

$= (1 \pm \sqrt{3})R_A$

As R_{CD} cant be -ve so $\boxed{R_{CD} = (1+\sqrt{3})R_A\ \Omega}$



$V_B - \frac{I}{4}R_1 - \frac{I}{12}R_1 - V_A = 0 \Rightarrow V_{BA} = IR_1\left(\frac{1}{12} + \frac{1}{4}\right)$

$= IR_1 \frac{1+3}{12} = \frac{IR_1}{3}$

$R_{AB} = \frac{R_1}{3}$.

Power delivered N/w (1) & N/w 2 same.

$P_{CD} = P_{AB}$

$(I_{CD})^2 R_{CD} = (I_{AB})^2 R_{AB}$

$\frac{V_{CD}^2}{R_{CD}} = \frac{V_{AB}^2}{R_{AB}}$

So $R_{CD} = R_{AB}$ if $V_{CD} = V_{AB}$

$(1+\sqrt{3})R_A = \frac{R_1}{3}$.

$\boxed{R_1 = (3+3\sqrt{3})R_A\ \Omega}$ $\boxed{R_A = \frac{R_A}{3+3\sqrt{3}}}$

) Power delivered by source for $R_A = 10\Omega$

If $R_A = 10\Omega$

$R_{CD} = (1+\sqrt{3})R_A = 10(\sqrt{3}+1) = 27.32\Omega$

$$P = \frac{V_{CD}^2}{R_{CD}} = \frac{10 \times 10}{27.32}$$

$$\boxed{P = 3.66 \text{ watts}} \leftarrow \underline{\text{Ans}}$$

17

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Q.7 (b) Consider a signal $x(t)$ with Fourier transform $X(j\omega)$. Suppose we are given the following facts :

1. $x(t)$ is real and non-negative.
2. $F^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$, where A is independent of t.
3. $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi$

Determine a closed-form expression of $x(t)$.

[20 marks]

1 Given $x(t)$ Real & non-ve.

2) $F^{-1}(X(\omega) + j\omega X(\omega)] = Ae^{-2t}u(t)$

$F^{-1}(X(\omega) + F^{-1}[j\omega X(\omega)] = Ae^{-2t}u(t)$

$x(t) + \frac{dx(t)}{dt} = Ae^{-2t}u(t)$

Or $X(\omega)[1+j\omega] = F[Ae^{-2t}u(t)] = \frac{A}{2+j\omega}$

$e^{-at}u(t) \rightarrow \frac{1}{a+j\omega}$

$X(\omega) = \frac{A}{(2+j\omega)(1+j\omega)}$

Apply Partial fraction expansion

$X(\omega) = \frac{A}{(2+j\omega)(1+j\omega)} = \frac{A_0}{2+j\omega} + \frac{A_1}{1+j\omega}$

$A_0|_{j\omega=-2} = \frac{A}{1-2} = -A$

$A_1|_{j\omega=-1} = \frac{A}{2-1} = A$

$X(\omega) = \frac{-A}{2+j\omega} + \frac{A}{1+j\omega}$

By Applying Inverse Fourier transform

$x(t) = -A\cdot e^{-2t}u(t) + Ae^{-t}u(t)$

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3) $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$

we know by Parseval energy theorem

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$$

$$\text{So Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{2\pi}{2\pi} = 1.$$

$$\int_{-\infty}^{\infty} [-Ae^{-2t}u(t) + Ae^{-t}u(t)]^2 dt = 1.$$

$$\int_{-\infty}^{\infty} [A^2 e^{-4t}u(t) + A^2 e^{-2t}u(t) - 2A^2 e^{-3t}u(t)] dt = 1.$$

$$A^2\int_0^{\infty} e^{-4t} dt + A^2\int_0^{\infty} e^{-2t} dt - 2A^2\int_0^{\infty} e^{-3t} dt = 1.$$

$$A^2\left[\left[\frac{e^{-4t}}{-4}\right]_0^{\infty} + \left[\frac{e^{-2t}}{-2}\right]_0^{\infty} - 2\left|\frac{e^{-3t}}{-3}\right|_0^{\infty}\right] = 1.$$

$$A^2\left[-\frac{1}{4}(e^{-\infty}-1) + \left(-\frac{1}{2}\right)[e^{-\infty}-1] + \frac{2}{3}[e^{-\infty}-1]\right] = 1.$$

$$A^2\left[\frac{1}{4}+\frac{1}{2}-\frac{2}{3}\right] = 1 \Rightarrow A^2\left[\frac{3+6-8}{12}\right] = 1.$$

18

$A^2 = 12.$

$A = \pm\sqrt{12} = \pm 2\sqrt{3}.$

Good Approach

By point 1
So for $x(t)$ is real & non-negative so $A = 2\sqrt{3}$

Hence $x(t) = A[e^{-t} - e^{-2t}]u(t).$

$$\boxed{x(t) = 2\sqrt{3}[e^{-t} - e^{-2t}]u(t)} \rightarrow \text{Ans}$$

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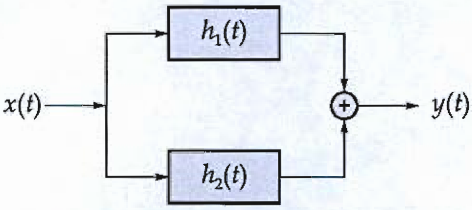
7 (c) (i) Consider the parallel combination of two LTI systems shown in the figure,



The impulse responses of the systems are

$h_1(t) = 2\delta(t+2) - 3\delta(t+1);$

$h_2(t) = \delta(t-2)$

If the input $x(t)$ is a unit step signal, then find the energy of $y(t)$.

[10 marks]

i) $h_{eq} = h_1(t) + h_2(t)$.

$= [2\delta(t+2) - 3\delta(t+1)] + \delta(t-2]$.

If $x(t) = u(t)$.

$y(t) = x(t) * [h_1(t) + h_2(t)] = x(t) * h_{eq}(t)$

$y(t) = x(t) * [2\delta(t+2) - 3\delta(t+1) + \delta(t-2)]$.

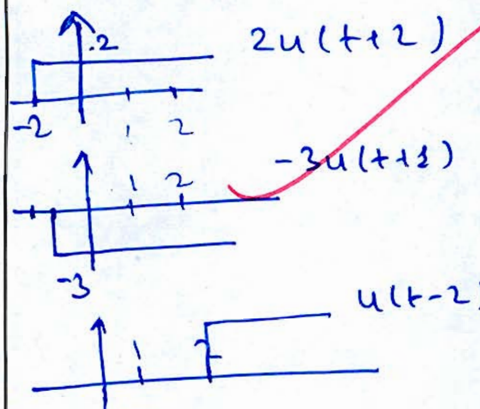
Now we know convolution property

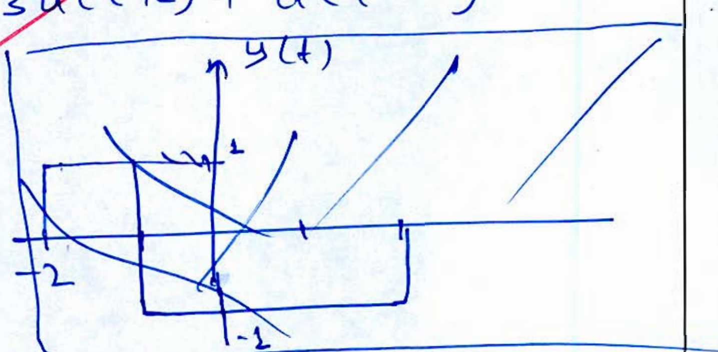
$x(t) * \delta(t-t_0) = x(t-t_0)$.

$y(t) = 2x(t+2) - 3x(t+1) + x(t-2)$.

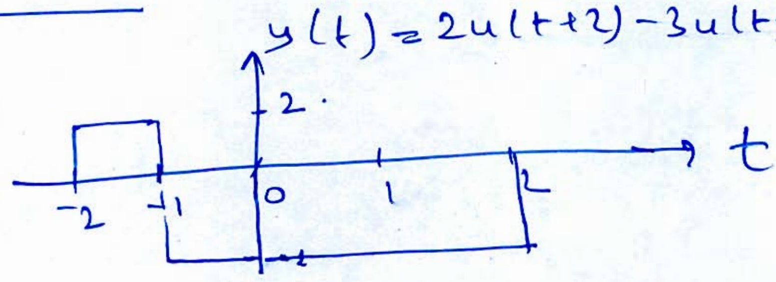
If $x(t) = u(t)$

so $y(t) = 2u(t+2) - 3u(t+1) + u(t-2)$.





$y(t) = 2u(t+2) - 3u(t+1) + u(t-2)$



Energy of $y(t) = \int_{-\infty}^{\infty} |y(t)|^2 dt$

$= \int_{-2}^{2} y(t)^2 dt.$

$= \int_{-2}^{-1} (2)^2 dt + \int_{-1}^{2} (-1)^2 dt = 4[-1-(-2)] + 1[2-(-1)]$

Energy $= 4[1] + 3 = 7$ Joules

↳ Ans

9

Good Approach

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7 (c) (ii) The exponential Fourier series representation of a continuous-time periodic signal $x(t)$ is defined as $x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$, where ω_o is the fundamental angular frequency of $x(t)$ and the coefficients of the series are a_k. The following information is given about $x(t)$ and a_k.

1. $x(t)$ is real and even, having a fundamental period of 6.
2. The average value of $x(t)$ is 2.
3. $a_k=\begin{cases}k, & 1\le k\le 3\\ 0, & k>3\end{cases}$

Find the average power of the signal $x(t)$.

[10 marks]

$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$x(t)\longleftrightarrow a_k$$

1) $x(t)$ real even $T=6$.

2) $\frac{1}{T}\int_{\langle T\rangle} x(t)\,dt=2$.

3) $a_k=\begin{cases}k & 1\le k\le 3\\ 0 & k>3\end{cases}$

To find: Avg Power of signal $x(t)$

$$P_{avg}=\frac{1}{T}\int_{\langle T\rangle}|x(t)|^2\,dt.$$

$$\omega_0=\frac{2\pi}{T}=\frac{2\pi}{6}=\frac{\pi}{3}.$$

$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\frac{\pi}{3}t}=a_1e^{j\frac{\pi}{3}t}+a_{-1}e^{-j\frac{\pi}{3}t}+a_2e^{j\frac{2\pi}{3}t}+a_{-2}e^{-j\frac{2\pi}{3}t}+a_3e^{j\frac{\pi}{3}3t}+a_{-3}e^{-j\frac{3\pi}{3}t}+a_0.$$

To find $a_0=\frac{1}{T}\int_0^T x(t)\,dt=2. \rightarrow$ Given

$$a_n=\frac{1}{T}\int_{\langle T\rangle} x(t)e^{-jn\omega_0 t}\,dt.$$

As $x(t)$ is real & even $x(t) = x(-t)$

So it means coefficient is also real & even

So $a_K = a_{-K}$.

Avg Power we know

$$P = \sum_{n=-\infty}^{\infty} |c_n|^2 = \sum_{K=-\infty}^{\infty} |a_K|^2 = \frac{1}{T}\int_{<T>} x^2(t)\,dt$$

$$= a_0^2 + 2(a_1^2 + a_2^2 + a_3^2).$$

As $a_0 = 2$ & $a_1 = 1$, $a_2 = 2$, $a_3 = 3$

$\because a_K = K$.

So Avg Power.

$$= 2^2 + 2(1^2 + 2^2 + 3^2)$$

$$= 4 + 2(1 + 4 + 9) = 32$$

$\boxed{P_{avg} = 32 \text{ watt}}$ ← Ans

9

Good Approach

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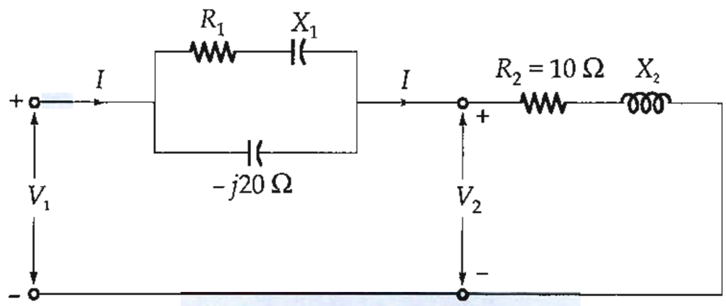
3 (a) In the circuit shown in the figure below, $|V_1| = 200$ V, $V_2 = 200\angle 0°$ V and $|I| = 12$ A. The total power absorbed by the circuit is 1.8 kW. Find R_1, X_1 and X_2.



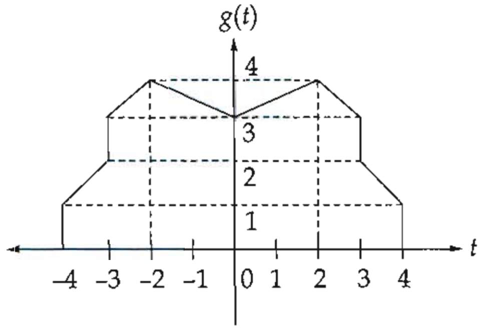
[20 marks]

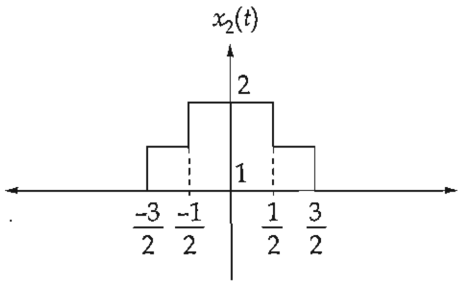
Q.8 (b) **(i)** Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by $y''(t) + y'(t) - 2y(t) = x(t)$.

1. Find the system function $H(s)$.
2. Determine the impulse response $h(t)$ for each of the following three cases : (1) The system is causal, (2) The system is stable, (3) The system is neither causal nor stable.

[13 marks]

Q.8 (b) (ii) The response of an LTI system to an input signal $x_1(t) = u(t+1) - u(t-1)$ is denoted as $g(t)$, as illustrated in the figure below. If a new input $x_2(t)$ is applied to the same system, resulting in an output $y(t)$. Determine the value of the output at $t = 0$.



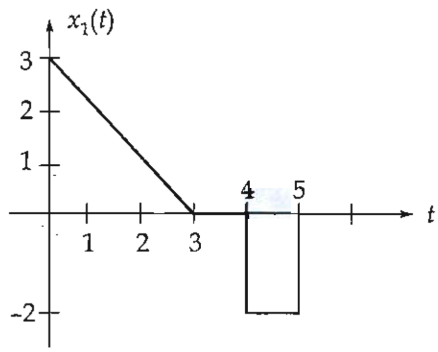


[7 marks]

3 (c) Consider a continuous-time LTI system with an impulse response $h(t)$ defined as a rectangular pulse of amplitude A and duration $T(0 < t < T)$. When the system is subjected to an input $x_1(t)$ given below, it produces an output $y_1(t)$. It is observed that $y_1(5) = 0$.

Furthermore, when the input is $x_2(t) = \sin\left(\frac{\pi t}{3}\right)u(t)$, the output $y_2(t)$ at $t = 9$ is equal to 9.

Determine the value of the product $A \times T$.



[20 marks]

OOOO

Space for Rough Work

1, π, $\frac{2\pi}{3}$ $\frac{\pi}{2}$

$\sin\left(\frac{\pi}{2L}\right)$

1, 3

$\frac{i}{3}$ 4 + 4 $\frac{9}{6}$

204 + 40