• Write all steps in detail

Try to avoid overwriting

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ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Digital Electronics + Microprocessors + Electrical Circuits-1 + Systems and Signal Processing-1

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	43
Q.2	48
Q.3	
Q.4	
Section-B	
Q.5	50
Q.6	46
Q.7	54
Q.8	
Total Marks Obtained	241

Signature of Evaluator: Saurabh kumar

Cross Checked by ....

IMPORTANT INSTRUCTIONS

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DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

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1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Digital Electronics + Microprocessors

Q.1 (a) Derive a minimized POS expression for the given function and realize using NOR-gate only.

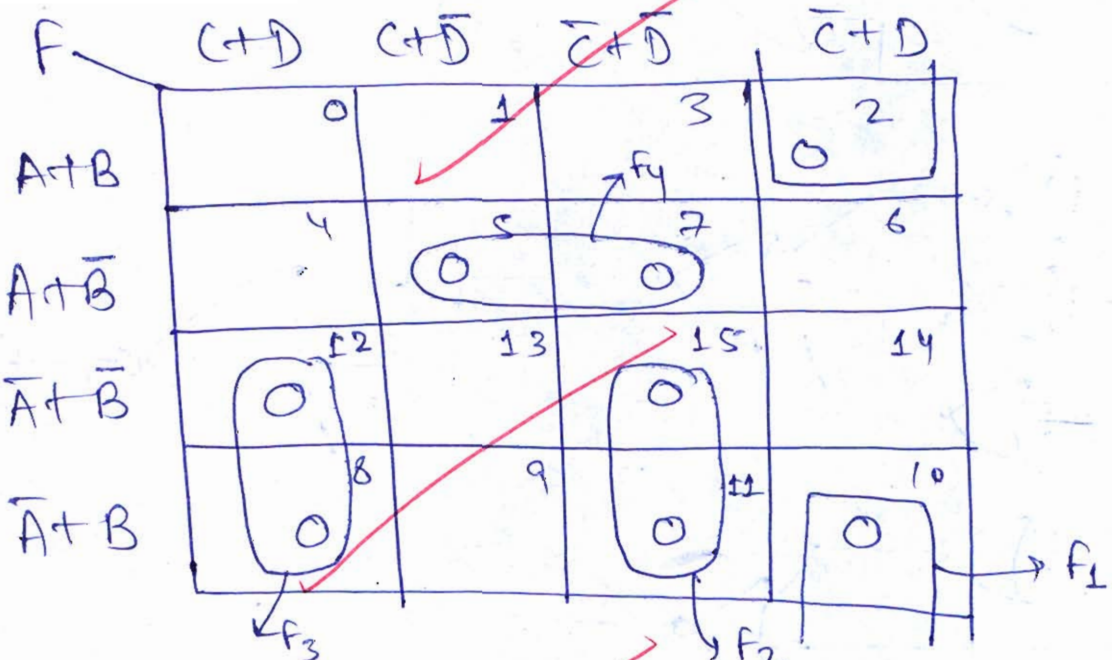
$$F(A, B, C, D) = \Sigma m(0, 1, 3, 4, 6, 9, 13, 14)$$

[12 marks]

Sol^n:-

$$F(A,B,C,D) = \Sigma m(0,1,3,4,6,9,13,14)$$

$$= \Pi M(2,5,7,8,10,11,12,15)$$



$$f_1 = (A+B+\bar{C}+D)(\bar{A}+B+\bar{C}+D) = (B+\bar{C}+D)$$

$$f_2 = \bar{A}+\bar{C}+\bar{D}$$

$$f_3 = \bar{A}+C+D$$

$$f_4 = A+\bar{B}+\bar{D}$$

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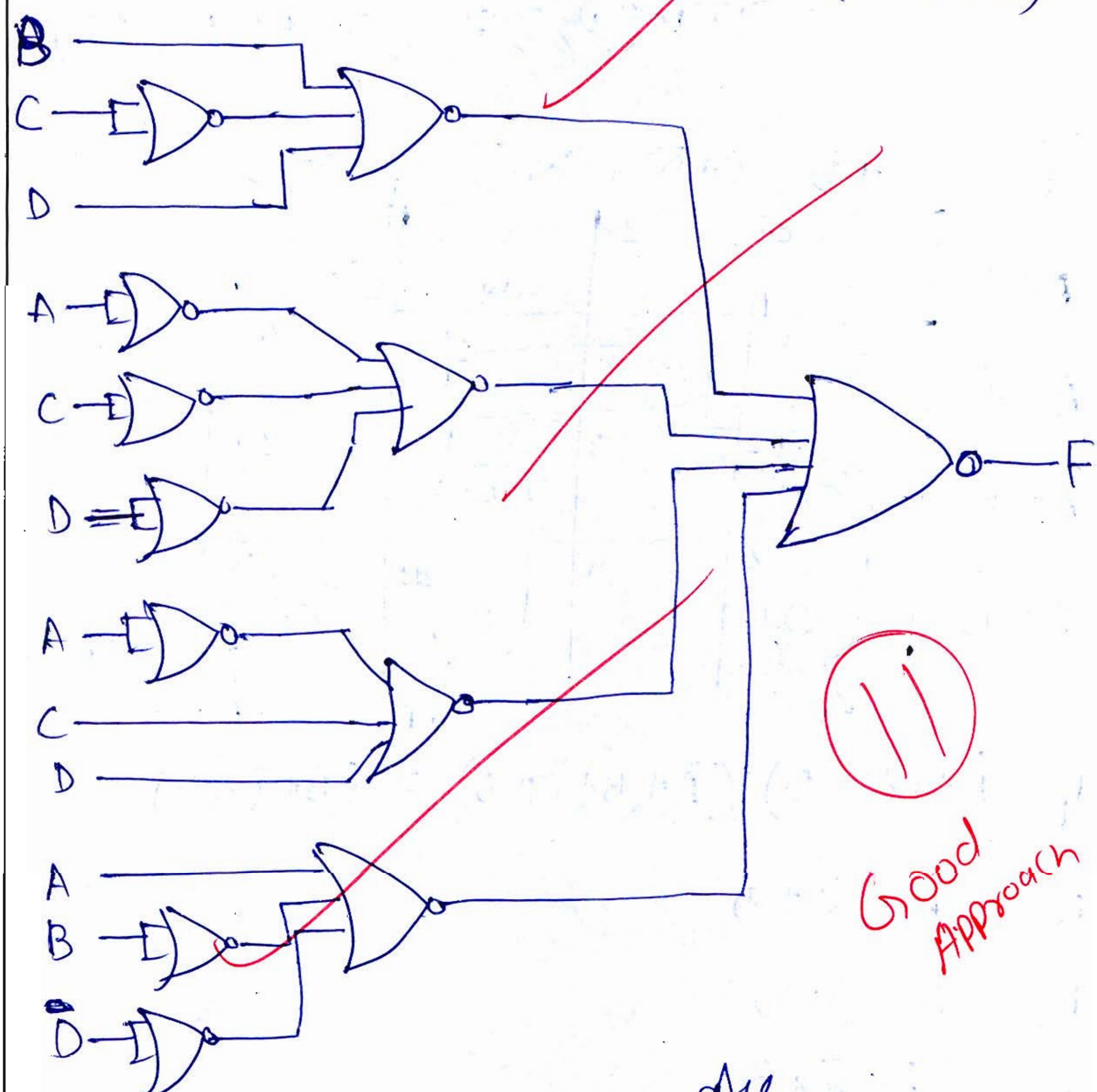
So, $\boxed{F = (B+\bar{C}+D)(\bar{A}+\bar{C}+\bar{D})(\bar{A}+C+D)(A+\bar{B}+\bar{D})}$

$$F = (B+\bar{C}+D)(\bar{A}+\bar{C}+\bar{D})(\bar{A}+C+D)(A+\bar{B}+\bar{D})$$

$$F = \overline{\overline{(B+\bar{C}+D)} + \overline{(\bar{A}+\bar{C}+\bar{D})} + \overline{(\bar{A}+C+D)} + \overline{(A+\bar{B}+\bar{D})}}$$



11

Good Approach

Ans

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.1 (b) Design an odd parity bit generator using four bit input.

[12 marks]

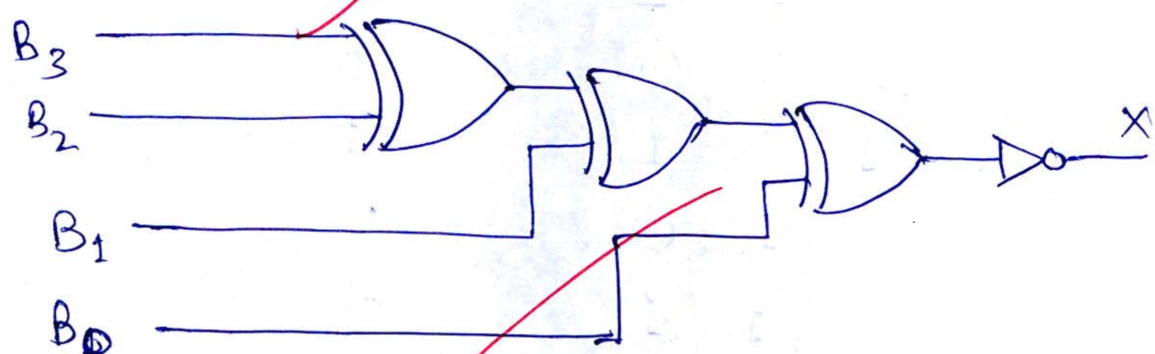
Solⁿ:-

Input				Parity Bit (X)
B_3	B_2	B_1	B_0	
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

X	$\bar{B}_1\bar{B}_0$	$\bar{B}_1 B_0$	$B_1 B_0$	$B_1\bar{B}_0$
$\bar{B}_3\bar{B}_2$	1	0	1	0
$\bar{B}_3 B_2$	0	1	0	1
$B_3 B_2$	1	0	1	0
$B_3\bar{B}_2$	0	1	0	1

Sol: $X = \bar{B_3}\bar{B_2}\bar{B_1}\bar{B_0} + \bar{B_3}\bar{B_2}B_1B_0 + \bar{B_3}B_2\bar{B_1}B_0 + \bar{B_3}B_2B_1\bar{B_0} + B_3B_2\bar{B_1}\bar{B_0} + B_3B_2B_1B_0 + B_3\bar{B_2}\bar{B_1}B_0 + B_3\bar{B_2}B_1\bar{B_0}$

$$\boxed{X = \overline{B_3 \oplus B_2 \oplus B_1 \oplus B_0}}$$

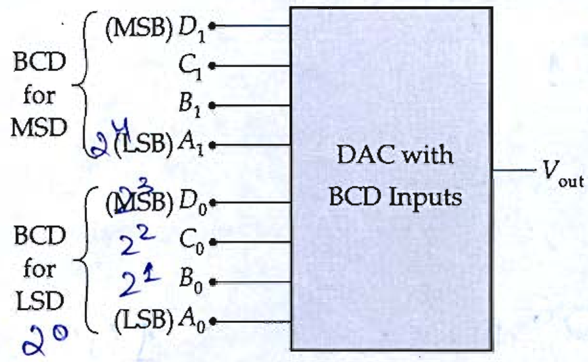


Odd parity bit generator

11

Good Approach

Q.1 (c) A digital to analog converter using BCD input code is shown in the figure below.



If the weight of A_1 is 2 V, then **find:**

(i) Step size.

(ii) Full scale output voltage.

(iii) Percentage resolution.

(iv) V_{out} for $D_1C_1B_1A_1$ = 0110 and $D_0C_0B_0A_0$ = 0100. **[12 marks]**

$n = 8$

Soln:

$$A_1 = 2^4 \times \frac{V_0}{2^n - 1} = 2V$$

$$\text{Step size} = \frac{V_0}{2^n - 1} = \frac{V_0}{255}$$

$$\frac{V_0}{255} \times 2^4 = 2 \Rightarrow V_0 = \frac{510}{2^4}$$

(ii) $\boxed{V_0 = 31.875\,V}$ Ans

full scale o/p voltage.

$$\text{Step size} = \frac{31.875}{255}$$

(i) $\boxed{\text{Step size} = \frac{1}{8}}$ Ans

(iii) % resolution

$$= \frac{1}{2^n - 1} \times 100\%$$

$\boxed{\%\ \text{Res.} = 0.392\%}$

iv) $V_{out} = \frac{V_0}{2^n - 1} \sum_i 2^i b_i$

For 0 1 1 0 0 1 0 0
$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$\sum_i 2^i b_i = 2^6 + 2^5 + 2^2 = 100$

So, $V_{out} = \frac{1}{8} \times 100$

$\boxed{V_{out} = 12.5V}$ Ans

11

Good Approach

Q.1 (d) List of the functional classification of 8085 instruction set. Give one example for each class.

[12 marks]

Sol. Classification of 8085 instruction set -

1.) Read / write instructions → These instructions are used to transfer to and from memory to microprocessor.
Ex:- MVI A, 05H → write 05 data from memory into accumulator.

2.) Input / output instructions → These instructions are used to transfer data to / from I/O device in microprocessor. Ex:- IN 03H
OUT 0BH

3.) Arithmetic & logical instruction → These instructions are used to perform arithmetic operations like addition, subtraction e.t.c. Ex:- ADD 03H
SBB 07H

Logical instruction are used to perform logical operations like AND, OR, XOR over the content of accumulator. Ex:- ORA, XRA e.t.c

4.) Serial I/P & O/P data txf. instructions
These instructions are used to perform serial data transfer (IN/OUT) from the D_7 bit of accumulator. Ex:- SID, SOD etc.

5.) Branching instructions → These instructions are used to direct the flow of operation of a microprocessor operation depending upon the conditions like for eg. status of flag. These are also used to introduce sub-routines in a main program.

Ex:- JMP, CALL etc.

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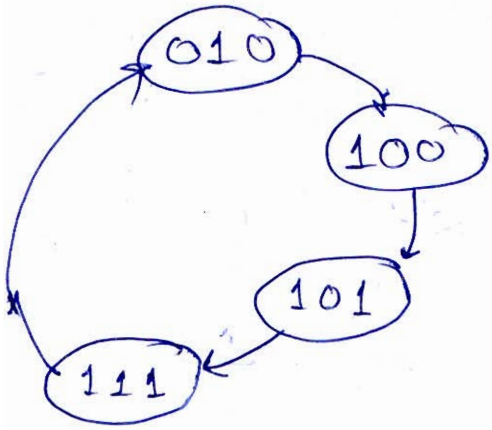
Q.1 (e) A bar code scanner scans the boxes being shipped from the loading dock and record all the codes in computer memory; the end of the data is indicated by the byte 00. The code 10100011 (A3H) is assigned to 19″ television sets. Write a program to count the number of 19″ television sets that were shipped from the following data set: Data: FA, 67, A3, B8, A3, A3, FA, 00. Write comments in the program.

[12 marks]

Q.2 (a) Design a 3-bit counter that goes through the states 2, 4, 5, 7, 2, 4, ... using *T*-flip flops. Assume the unused states as don't cares. Check whether the designed counter is self starting or not and there by give the complete sequence diagram for the designed counter.

[20 marks]

Sol^n:-



	Present state			Next state					
	Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	T_2	T_1	T_0
2	0	1	0	1	0	0	1	1	0
4	1	0	0	1	0	1	0	0	1
5	1	0	1	1	1	1	0	1	0
7	1	1	1	0	1	0	1	0	1

T_2

	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1 \bar{Q}_0$
$\bar{Q}_2$	0: X	1: X	3: X	2: 1
Q_2	4: 0	5: 0	6: X	7: 1

$$\boxed{T_2 = Q_1}$$

T_1

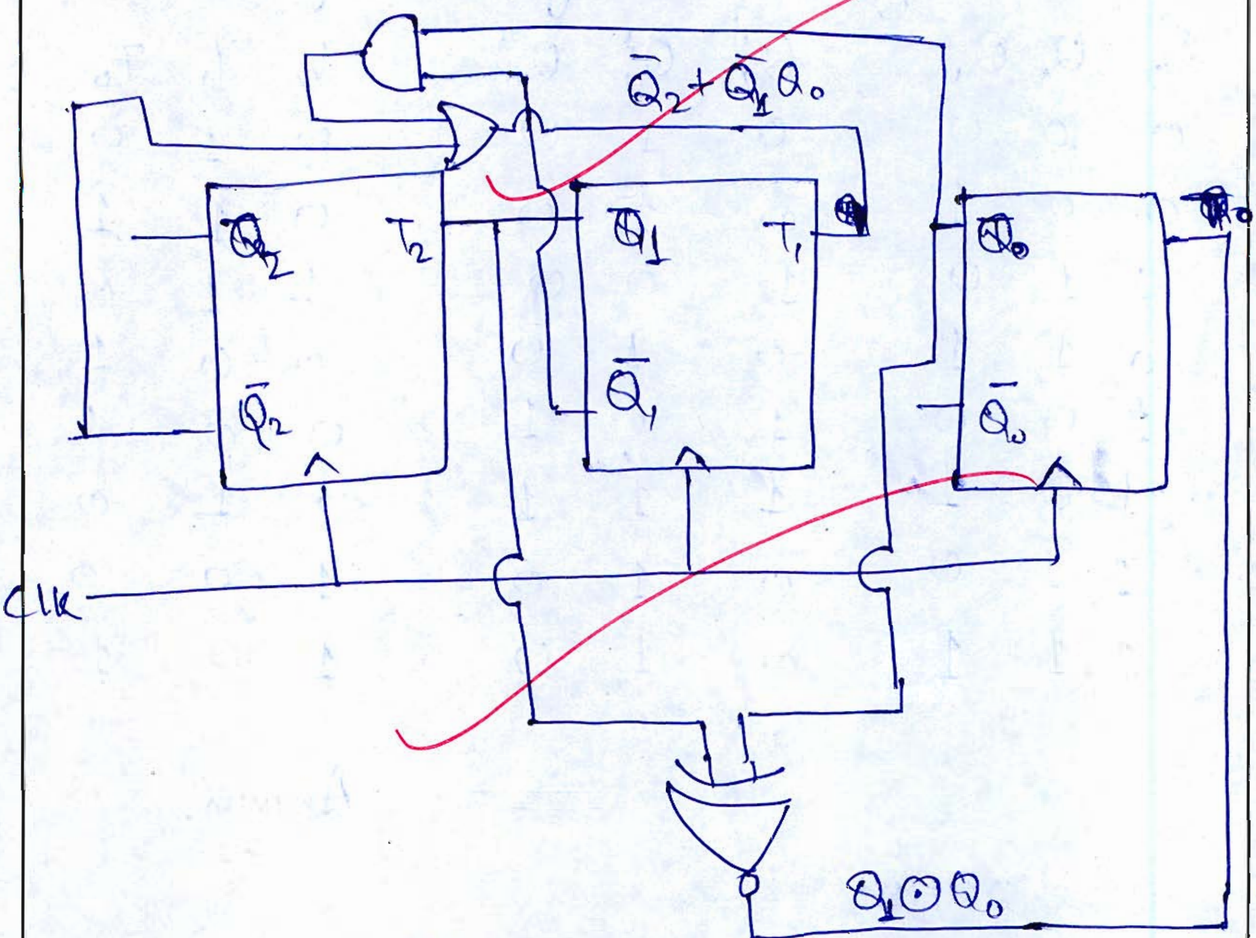
	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1 \bar{Q}_0$
$\bar{Q}_2$	0: X	1: X	3: X	2: 1
Q_2	4: 0	5: 1	7: 0	6: X

$$\boxed{T_1 = \bar{Q}_2 + \bar{Q}_1 Q_0}$$

T_0	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1 Q_0$	$Q_1 Q_0$	$Q_1\bar{Q}_0$
$\bar{Q}_2$	X (0)	X (1)	X (3)	0 (2)
Q_2	1 (4)	0 (5)	1 (7)	X (6)

$T_0 = \bar{Q}_1\bar{Q}_0 + Q_1 Q_0$

$$\boxed{T_0 = Q_1 \odot Q_0}$$



<u>Answer</u>

For $Q_2\, Q_1\, Q_0 \Rightarrow 0\,0\,0$

$T_2 = 0$

$T_1 = 1 + 1 \times 0 = 1$

$T_0 = 1$

So, Next State will be 011 = (3) → unused state.

So, the designed counter is not self-starting.

For making it self starting we fine a state. let 010 (2

Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	T_2	T_1	T_0
0	0	0	0	1	0	0	1	0
0	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	1	0
0	1	1	0	1	0	0	0	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	1	0	1	0
1	1	0	0	1	0	1	0	0
1	1	1	0	1	0	1	0	1

Answer.

15

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Q.2 (b) (i) Calculate the delay produced by the following subroutine program of an 8085 microprocessor, which is operating with a clock frequency of 2 MHz.

DELAY :	MVI B, 02H	→ No. of T-state = 7
LOOP2 :	MVI C, FFH	→ No. of T-state = 7
LOOP1 :	DCR C	→ 4 T state
	JNZ LOOP1	
	DCR B	
	JNZ LOOP2	
	RET	

[14 marks]

Sol^n^:

Delay: MVI B, 02H
↳ Have 7-T State.

Loop2 : MVI C, FFH
↳ Have 7-T state.

Loop 1: DCR C → Have 4-T states.

JNZ Loop 1
→ No. of T-state: correct → 13 T-states FRRR; false → 11 T-states.

DCR B
↳ 4-T-state.

JNZ loop2 → correct → 13 T-states; false → 11 T-state.

RET → 4 T-State

T-state = $\frac{1}{2\times10^6}$ = 0.5 µsec.

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Good Approach

Q.2 (b) (ii) Explain the features of the three sources of interrupts in the 8086 microprocessor.
[6 marks]

Soln. Sources of Interrupts—

Hardware interrupts:— These includes interrupts such as power failure e.t.c. They can be edge triggered or level triggered. Have higher priority than software interrupts. ~~They~~ ~~are~~ ~~a~~ There memory address have a vector address location.

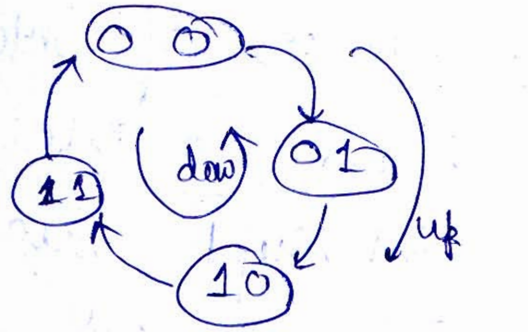
Software interrupts:— Do not have there own address and uses ~~~~ INTA for there operation. They do not posses address locations and thus are non-vectored interrupts.

5

Q.2 (c) Draw the state diagram of a modulo-4 UP/DOWN counter. Design the circuit using JK flip flops.

[20 marks]

Q_3	Q_2	Q_1	y
0	0	0	0
0	0	0	1
0	0	1	0



Sol^n:

$y = 0 \rightarrow$ UP
$y = 1 \rightarrow$ down

Q_1	Q_0	y	Q_1^+	Q_0^+	J_1	K_1	J_0	K_0
0	0	0	0	1	0	X	1	X
0	0	1	1	1	1	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	0	0	X	X	1
1	0	0	1	1	X	0	1	X
1	0	1	0	1	X	1	1	X
1	1	0	0	0	X	1	X	1
1	1	1	1	0	X	0	X	1

J_1

	$\bar{Q}_0\bar{y}$	$\bar{Q}_0 y$	$Q_0 y$	$Q_0\bar{y}$
$\bar{Q}_1$	0 (0)	1 (1)	0 (3)	1 (2)
Q_1	X (4)	X (5)	X (7)	X (6)

$$\boxed{J_1 = \bar{Q}_0 y + Q_0\bar{y} = Q_0 \oplus y}$$

K_1

	$\bar{Q}_0\bar{y}$	$\bar{Q}_0 y$	$Q_0 y$	$Q_0\bar{y}$
$\bar{Q}_1$	X	X	X	X
Q_1	0	1	0	1

$$\boxed{K_1 = Q_0 \oplus y}$$

J_0

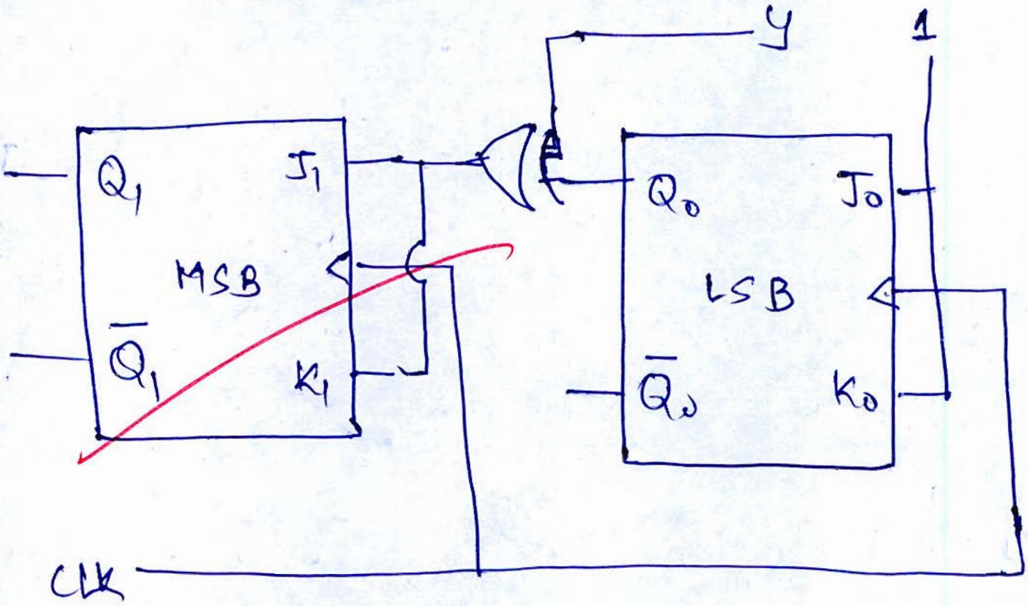
1	1	x	x
1	1	x	x

$\boxed{J_0 = 1}$

K_0

x	x	1	1
x	x	1	1

$\boxed{K_0 = 1}$



Modulo-4 UP/DOWN counter using J-K F-F

15

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Q.3 (a) (i) Design a synchronous counter using *D*-flip flop that counts in the following sequence: 6, 3, 5, 0, 2, 6, 3, 5, 0, 2, 6

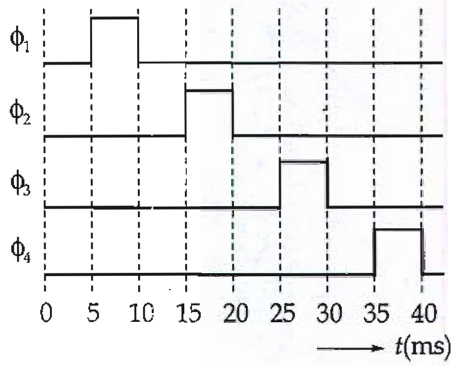
[10 marks]

Q.3 (a) (ii) Explain the working of 3-bit flash type ADC.

[10 marks]

Q.3 (b) **(i)** Design a synchronous 3-bit binary up-counter using D-flip flops.

(ii) A stepper-motor drive circuit requires four periodic signal waveforms, each with a period of 40 ms, as shown below. By using the counter circuit obtained in part (i), design a circuit to generate the necessary signal waveforms for this stepper motor.



[10 + 10 marks]

Q.3 (c) (i) Write a short note on the flag register of 8086 microprocessor.

[14 marks]

Q.3 (c) (ii) Write down the purpose of each bit in SIM (Set Interrupt Mask) Instruction. Give three different functions of SIM instruction.

[6 marks]

Q.4 (a) (i) Write the steps involved in DMA data transfer. Also, describe the functions of 8085 pins which are used in DMA data transfer.

[10 marks]

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Q.4 (a) **(ii)** How can we generate a square wave with a variable bit rate, using microprocessor? Output should be available on a chosen port, using bit 'O'.

[10 marks]

Q.4 (b) A set of five 16-bit readings of the power consumption of industrial control units is monitored by meters and stored at memory locations starting at 2050H. Each reading occupies two memory locations: the lower order byte is stored first, followed by the higher order byte. The corresponding maximum limits for each control unit are stored at memory locations starting at 2090H, also with the lower order byte stored first followed by the higher order byte. Write an 8085 assembly language program to subtract each reading from its specified maximum limit and store the difference at the same memory locations of the readings. Also provide a provision in the program to call the indicator subroutine if the reading is higher than its maximum limit and then continue checking the remaining reading.

[20 marks]

Q.4 (c) **(i)** Given that $(292)_{10} = 1204$ in some number system, find the base of that system.

(ii) In the following series, the same integer is expressed in different number systems. Determine the missing number of the series : 10000, 121, 100, ? , 24, 22, 20.

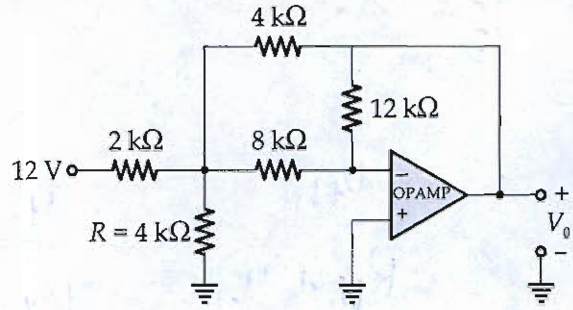
(iii) Add the binary numbers of 1101.101 and 111.011. Find its decimal equivalent.

(iv) Subtract 14 from 46 using 8-bit 2's complement arithmetic.

[5 × 4 marks]

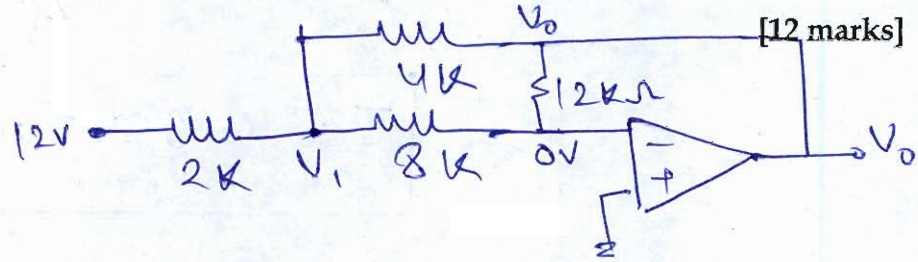
Section B : Electrical Circuits-1 + Systems and Signal Processing-1

Q.5 (a) In the operational amplifier circuit shown in figure below, calculate the current in $R = 4\ \text{k}\Omega$ resistor, using Thevenin's theorem.



[12 marks]

Sol^n:- $\underline{\underline{V_{th} \rightarrow}}$



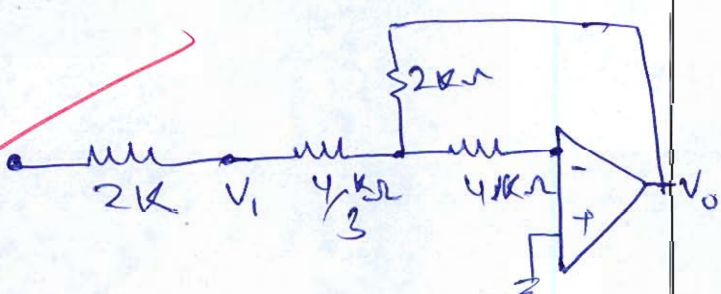
$$\frac{12-V_1}{2K} = \frac{V_1 - 0}{8K} + \frac{V_1 - V_0}{4K}$$

$$6 - \frac{V_1}{2} = \frac{V_1}{8} + \frac{V_1 - V_0}{4}$$

$$6 = \frac{V_1}{2} + \frac{V_1}{8} + \frac{V_1}{4} - \frac{V_0}{4} = \frac{7V_1}{8} - \frac{V_0}{4}$$

$$48 = 7V_1 - 2V_0 \quad -(1)$$

$$\cancel{\frac{V_1 - V_0}{4}} = \cancel{\frac{V_0}{12}}$$



$$\frac{V_1}{4/3\,K} = \frac{0 - V_0}{2K}$$

$$\frac{3V_1}{4} = \frac{-V_0}{2} \Rightarrow V_0 = \frac{-3V_1}{2}$$

So, $48 = 7V_1 - \cancel{2} \times \frac{3V_1}{\cancel{2}}$

$$48 = 7V_1 + 3V_1$$

$$10V_1 = 48 \Rightarrow V_1 = 4.8V$$

$$\boxed{V_{th} = 4.8V}$$

Op-Amp is high-impedence device $(Z \approx \infty)$

$$R_{Th} = 4/3 + 2 = \frac{10}{3}\,\Omega$$

So; $I_{4\Omega} = \dfrac{V_{th}}{R_{Th} + 4\Omega} = \dfrac{4.8}{10/3 + 4}$

$$\boxed{I = 0.6545 \text{ Amp}}$$

6

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Q.5 (b) The input $x[n]$ and the impulse response $h[n]$ of a discrete time LTI system are given by:

$$x[n] = u[n]$$

$$h[n] = \alpha^n u[n]; \quad 0 < \alpha < 1$$

Compute the output $y[n]$ by method of convolution.

[12 marks]

Soln:- $x[n] = u[n]$

$h[n] = \alpha^n u[n]$; $0 < \alpha < 1$

From the property of convolution

$y[n] = x[n] * h[n]$

$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$= \sum_{k=-\infty}^{\infty} u[k]\, \alpha^{n-k}\, u[n-k]$

$= \alpha^n \sum_{k=-\infty}^{\infty} u[k] \cdot u[n-k]$

$= \alpha^n \sum_{k=0}^{n} 1 \cdot \alpha^{-k}$ for $n > 0$

$= 0$ for $n < 0$

$= \alpha^n \left[1 + \alpha^{-1} + \alpha^{-2} + \cdots + \alpha^{-n} \right]$

$= \alpha^n \left[\frac{1\,[1 - (\alpha^{-1})^{n+1}]}{1 - \alpha^{-1}} \right]$

$$\boxed{y[n] = \alpha^n \left[\frac{1 - (\alpha^{-1})^{n+1}}{1 - \alpha^{-1}} \right]}$$

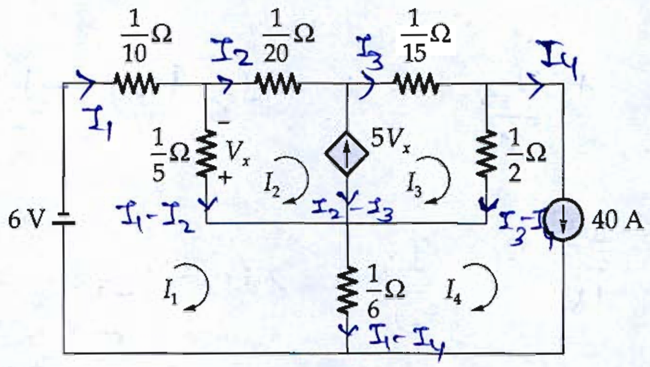
11

Good Approach

So; $\boxed{y(n) = \dfrac{(1-\alpha^{n+1})}{1-\alpha} u[n]}$

Ans

Q.5 (c) Find currents I_1, I_2, I_3 and I_4.



[12 marks]

Soln:- Applying KVL in loop (1)

$$-6 = \frac{I_1}{10} + \frac{I_1 - I_2}{5} + \frac{I_1 - I_4}{6}$$

$$-6 = \frac{I_1}{10} + \frac{I_1}{5} - \frac{I_2}{5} + \frac{I_1}{6} - \frac{I_4}{6}$$

$$-6 = \frac{7}{15}I_1 - \frac{I_2}{5} - \frac{I_4}{6} \quad \text{—(i)}$$

$$I_3 = I_2 + 5V_x$$

$$-V_x = \frac{I_1 - I_2}{5} \Rightarrow V_x = \frac{I_2 - I_1}{5}$$

$$I_3 = I_2 + \cancel{5} \times \left(\frac{I_2 - I_1}{\cancel{5}}\right) = 2I_2 - I_1 \quad \text{—(ii)}$$

$$I_3 = 2I_2 - I_1 \Rightarrow -I_1 + 2I_2 - I_3 = 0$$

$$I_4 = 40\text{A}$$

So eqn (1) $\quad -6 = \frac{7}{15}I_1 - \frac{I_2}{5} - \frac{40}{6}$

$$-6 + \frac{20}{3} = \frac{7}{15}I_1 - \frac{I_2}{5}$$

$$\frac{2}{3} = \frac{7}{15}I_1 - \frac{I_2}{5}$$

$$10 = 7I_1 - 3I_2 \quad \text{—(iii)}$$

Do not write in this margin

From KVL

$$0 = \frac{1}{20} I_2 + \frac{I_3}{15} + \frac{1}{2}(I_3 - 40) - \frac{1}{5}[I_1 - I_2]$$

$$0 = \frac{I_2}{20} + \frac{I_3}{15} + \frac{I_3}{2} - 20 - \frac{I_1}{5} + \frac{I_2}{5}$$

$$20 = -\frac{I_1}{5} + I_2\left[\frac{1}{4}\right] + I_3\left[\frac{17}{30}\right] \quad \text{—(iv)}$$

From eqn. (ii), (iii), (iv)

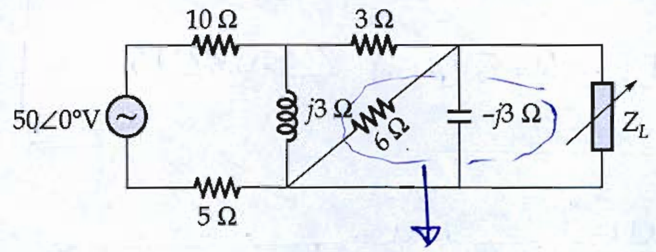
$$\boxed{\begin{aligned} I_1 &= 10\,\text{Amp} \\ I_2 &= 20\,\text{Amp} \\ I_3 &= 30\,\text{Amp} \\ I_4 &= 40\,\text{Amp (Given)} \end{aligned}}$$

Ans

Good Approach

11

Q.5 (d) Find the impedance Z_L so that maximum power can be transferred to it in the network shown below. Also, find the maximum power delivered to load Z_L.



[12 marks]

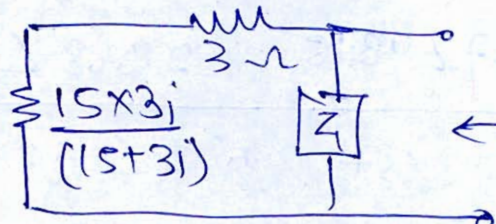
Soln:

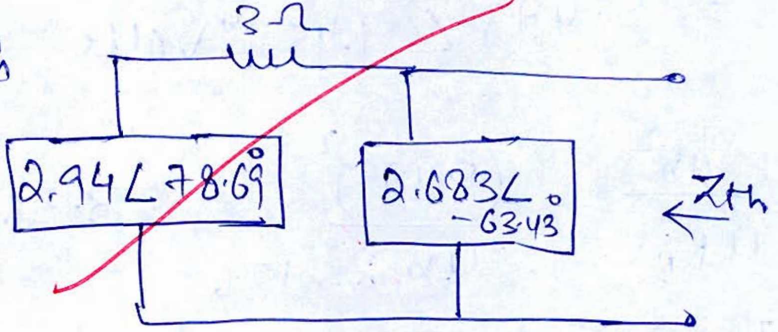
6Ω & −j3Ω are connected in parallel.

So; $Z_1 = (6 \,||\, -j3)$

$= 2.683\angle -63.435^\circ \;\Omega$

Z_{th}





$Z_{th} = (3 + 2.94\angle 78.69^\circ) \,||\, (2.683\angle -63.43^\circ)$

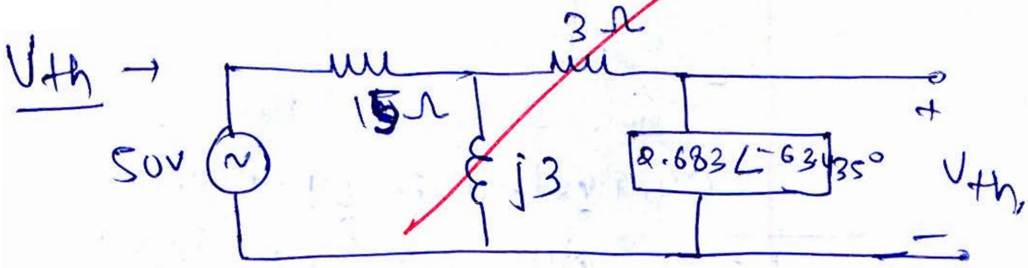
$= (4.595\angle 38.88^\circ) \,||\, (2.683\angle -63.435^\circ)$

$\boxed{Z_{th} = 2.5677\angle -30.346^\circ \,\Omega}$

for P_{max} $Z_L = Z_{th}^*$

$$\boxed{Z_L = 2.5677 \angle 30.346^\circ \, \Omega}$$ Ans

$V_{th} \rightarrow$



$$\frac{50 - V}{15} = \frac{V}{j3} + \frac{V}{3 + 2.683\angle -63.435^\circ}$$

$$\frac{10}{3} = (0.3374 \angle -43.15^\circ)\, V$$

$$V = 9.87 \angle 43.15^\circ \text{ Volts}$$

$$V_{th} = \frac{9.87 \angle 43.15^\circ \times 2.683 \angle -63.435^\circ}{3 + 2.683 \angle -63.435^\circ}$$

$$V_{th} = 5.479 \angle 9.459^\circ \text{ Volts}$$

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{(5.479)^2}{4 \times 2.2156} = 3.3869 \text{ W}$$

$$\boxed{P_{max} = 3.3869 \text{ watts}}$$

Good Approach

11

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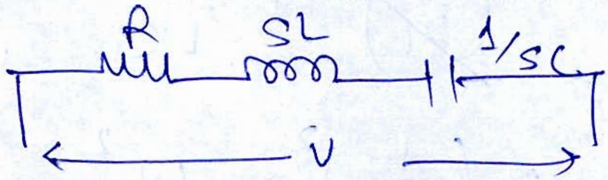
.5 (e) Show that the resonant frequency ω_0 of an *R-L-C* series circuit is geometric mean of ω_1 and ω_2, the lower and upper half-power frequencies respectively.

[12 marks]

Sol^n:-



$$V = I\left[R + SL + \frac{1}{SC}\right]$$

$$\frac{V}{I} = \frac{RSC + S^2LC + 1}{SC} = \frac{LC\left[S^2 + \frac{1}{LC} + \frac{RSC}{LC}\right]}{SC}$$

Characteristic eqn -

$$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$$

$$S_1, S_2 = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\omega_1 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\omega_2 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\omega_1 \cdot \omega_2 = \left(-\frac{R}{2L}\right)^2 - \left(\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)^2$$

$$= \left(-\frac{R}{2L}\right)^2 - \left(\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}\right)$$

=

$$\omega_1 \cdot \omega_2 = \cancel{\left(\frac{-R}{2L}\right)^2} - \cancel{\left(\frac{R}{2L}\right)^2} + \frac{1}{LC}$$

$$\omega_1 \cdot \omega_2 = \frac{1}{LC} = \left(\frac{1}{\sqrt{LC}}\right)^2 = \omega_0^2$$

So, $\omega_0^2 = \omega_1 \cdot \omega_2$

$$\Rightarrow \boxed{\omega_0 = \sqrt{\omega_1 \cdot \omega_2}}$$

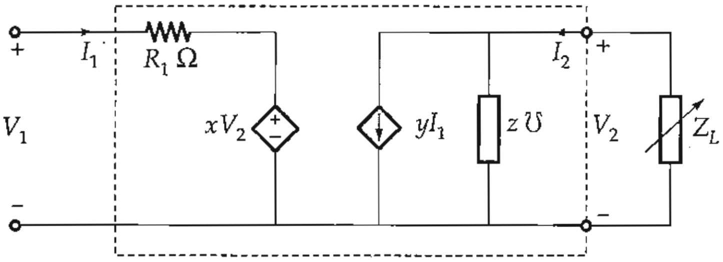
Hence, proved

So, resonant frequency ω_0 of an R-L-C series ckt. is geometric mean of upper & lower half-power frequencies.

11

Good Approach

Q.6 (a) Consider a two port network shown in figure below,



If transmission parameters matrix of the network is $\begin{bmatrix} 10^{-2} & 10^{2} \\ 0 & 10^{-1} \end{bmatrix}$.

Then, calculate:

(i) parameters of the circuit: R_1, x, y and z.

(ii) the value of load impedance (Z_L), for maximum power transfer.

(iii) maximum power transfer to load for $V_1 = 0.1$ volt.

[20 marks]

Soln:- From fig -

$$V_1 = I_1 R_1 + x V_2 \quad \text{— (1)}$$

$$I_2 = y I_1 + Z V_2 \quad \text{— (2)}$$

Transmission parameter matrix of network is given by

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$V_1 = A V_2 - B I_2$$

$$I_1 = C V_2 - D I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}$$

In eqn. (2)

$$0 = y I_1 + Z V_2 \Rightarrow y I_1 = -Z V_2 \Rightarrow I_1 = \frac{-Z V_2}{y}$$

Substituting in eqn. (1)

$$V_1 = -\frac{Z R_1}{y} V_2 + x V_2$$

So $\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \left(x - \frac{ZR_1}{y}\right)}$

$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

Eqn-(2)

$I_2 = y I_1 \Rightarrow I_1 = \frac{I_2}{y}$

Eqn-(1)

$V_1 = \frac{R_1 I_2}{y} \Rightarrow \frac{V_1}{-I_2} = -\frac{R_1}{y}$

$\boxed{B = -\frac{R_1}{y}}$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{-Z}{y}$; $D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = -\frac{1}{y}$

(i) Given $T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 10^{-2} & 10^{2} \\ 0 & 10^{-1} \end{bmatrix}$

So; $-\frac{Z}{y} = 0 \Rightarrow \boxed{Z = 0}$

$-\frac{1}{y} = 10^{-1} \Rightarrow -\frac{1}{y} = \frac{1}{10} \Rightarrow \boxed{y = -10}$

$x - \frac{ZR_1}{y} = 10^{-2} \Rightarrow \boxed{x = 10^{-2}}$

$-\frac{R_1}{y} = 10^{2} \Rightarrow -R_1 = 10^{2} \times -10$

$\boxed{R_1 = 10^{3}}$

(ii) For Z_L for maxm power transfer

$$Z_L = Z_{th}^*$$

$Z_{th} \rightarrow$ $V_1 = 0$; $I_2 = 1$

$0 = I_1 R_1 + x V_2$

$1 = y I_1 + z V_2 \Rightarrow y I_1 = 1 \Rightarrow I_1 = \frac{1}{y}$ $(\because z = 0)$

$0 = \frac{1}{y} R_1 + x V_2 \Rightarrow x V_2 = -\frac{R_1}{y} \Rightarrow V_2 = \frac{-R_1}{xy}$

$V_2 = \frac{+10^2}{10^{-2} \times 10} = 10^4$

$Z_{th} = 10^4 \Omega$

$\boxed{|Z_L| = 10^4 \Omega}$ For maxm power transfer.

(i) For $V_1 = 0.1 V$; $I_2 = 0$; $V_2 = V_{th}$

$V_{th} \rightarrow$ $0.1 = I_1 R_1 + x V_2 \Rightarrow I_1 = \frac{0.1 - x V_2}{R_1}$

$0 = y I_1 + z V_2$

$y I_1 = 0 \quad \because z = 0$

$I_1 = 0$

So; $0.1 = x V_2 \Rightarrow V_2 = V_{th} = \frac{0.1}{10^{-2}} = 0.1 \times 100 = 10 V$

$P_{max} = \frac{V_{th}^2}{4 Z_{th}} = \frac{10^2}{4 \times 10^4} = \frac{10^{-2}}{4} = 2.5 mW$

$\boxed{P_{max} = 2.5 mW}$

18 Good Approach

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Q.6 (b) (i) The transfer function relating the input $x(t)$ to the output $y(t)$ of a system is given by $G(s) = \frac{1}{(s+3)}$. A unit-step input is applied to the system at time $t = 0$. Assuming that $y(0) = 3$, find the value of $y(t)$ at time $t = 1$.

[12 marks]

Soln:

$$\frac{Y(s)}{X(s)} = \frac{1}{s+3} \Rightarrow sY(s) + 3Y(s) = X(s)$$

$$\Rightarrow \frac{dy(t)}{dt} + 3y(t) = x(t) \quad -(1)$$

for $x(t) = u(t)$

$y(0) = 3$

Applying laplace trnf. on eqn (1)

$$[sy(s) - y(0)] + 3y(s) = X(s) = \frac{1}{s}$$

$$(s+3)y(s) - 3 = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s+3}\left[\frac{1}{s} + 3\right]$$

$$Y(s) = \frac{1}{s(s+3)} + \frac{3}{s+3}$$

$$Y(s) = \frac{1}{3}\frac{3}{s(s+3)} + \frac{3}{s+3}$$

$$= \frac{1}{3}\left[\frac{1}{s} - \frac{1}{s+3}\right] + \frac{3}{s+3}$$

Applying inverse laplace trnf.

$$y(t) = \frac{1}{3}\left[1 - e^{-3t}\right] + 3e^{-3t}$$

at $t = 1$

$$y(1) = \frac{1}{3}\left[1 - e^{-3}\right] + 3e^{-3}$$

$$\boxed{y(1) = 0.466 \approx 0.5}$$

Ans

11

Good Approach

Q.6 (b) **(ii)** Consider the signal $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$, where t is in seconds. Find its fundamental time period.

(iii) For a periodic signal $v(t) = 30\sin 100t + 10\cos 300t + 6\sin\left(500t + \frac{\pi}{4}\right)$, find the fundamental frequency in rad/s.

[6 + 2 marks]

(ii) $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \ldots\right)$

$\omega_1 = \pi$ rad/sec $\quad \omega_2 = 2\pi/3$ rad/sec $\quad \omega_3 = \pi/2$ r/s

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\pi} = 2$ sec.

$T_2 = \frac{2\pi}{2\pi/3} = 3$ sec. ; $T_3 = \frac{2\pi}{\pi/2} = 4$ sec.

Fundamental time period $T_0 = \text{LCM}(T_1, T_2, T_3)$

$= \text{LCM}[2, 3, 4]$

$\boxed{T_0 = 12 \text{ sec}}$

(iii) $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$

$\omega_1 = 100$ r/s

$\omega_2 = 300$ r/sec

$\omega_3 = 500$ rad/sec

Fundamental frequency $= \text{HCF}(\omega_1, \omega_2, \omega_3)$

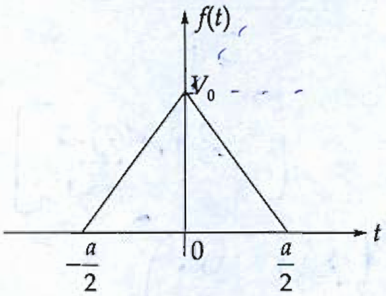
$\boxed{\omega_0 = 100 \text{ rad/sec.}}$

8

Good Approach

.6 (c) (i) The figure shows a triangular pulse which is zero for all time except $-\frac{a}{2} \le t \le \frac{a}{2}$. For this pulse:

1. determine the Fourier transform,
2. sketch the continuous amplitude spectrum.



[12 marks]

Solⁿ:

$$f(t) = \frac{2V_0}{a} r(t+a/2) - \frac{2V_0}{a} r(t) - \frac{2V_0}{a} r(t) + \frac{2V_0}{a} r(t-a/2)$$

Differentiating →

$$\frac{df(t)}{dt} = \frac{2V_0}{a} u(t+a/2) - \frac{2V_0}{a} u(t) - \frac{2V_0}{a} u(t) + \frac{2V_0}{a} u(t-a/2)$$

$$\frac{d^2f(t)}{dt^2} = \frac{2V_0}{a} \delta(t+a/2) - \frac{4V_0}{a} \delta(t) + \frac{2V_0}{a} \delta(t-a/2)$$

$$f(t) \rightleftharpoons F(\omega)$$

$$\frac{d^n f(t)}{dt^n} \rightleftharpoons (j\omega)^n F(\omega)$$

$$(j\omega)^2 \cdot F(\omega) = \frac{2V_0}{a} \cdot e^{\frac{j\omega a}{2}} - \frac{4V_0}{a} + \frac{2V_0}{a} e^{-\frac{j\omega a}{2}}$$

$$(j\omega)^2 F(\omega) = \frac{2V_0}{a} \left[e^{j\omega a/2} + e^{-\frac{j\omega a}{2}} - 2 \right]$$

$$= \frac{2V_0}{a} \left[\left(e^{\frac{j\omega a}{4}}\right)^2 + \left(e^{-\frac{j\omega a}{4}}\right)^2 - 2 e^{\frac{j\omega a}{4}} e^{-\frac{j\omega a}{4}} \right]$$

$$(j\omega)^2 F(\omega) = \frac{2V_0}{a}\left[e^{\frac{j\omega a}{4}} - e^{\frac{-j\omega a}{4}}\right]^2$$

$$(j\omega)^2 F(\omega) = \frac{2V_0}{a}\left[\frac{e^{\frac{j\omega a}{4}} - e^{\frac{-j\omega a}{4}}}{2j}\right]^2 \times (2j)^2$$

$$\omega^2 f(\omega) = \frac{2V_0}{a}\times 4\left[\sin\frac{\omega a}{4}\right]^2$$

$$F(\omega) = \frac{8V_0}{a\omega^2}\left(\sin\frac{\omega a}{4}\right)^2$$

$$= \frac{8V_0}{a\omega^2} \times \frac{\left(\sin\frac{\omega a}{4}\right)^2}{\left(\frac{\omega a}{4}\right)^2} \times \left(\frac{\omega a}{4}\right)^2$$

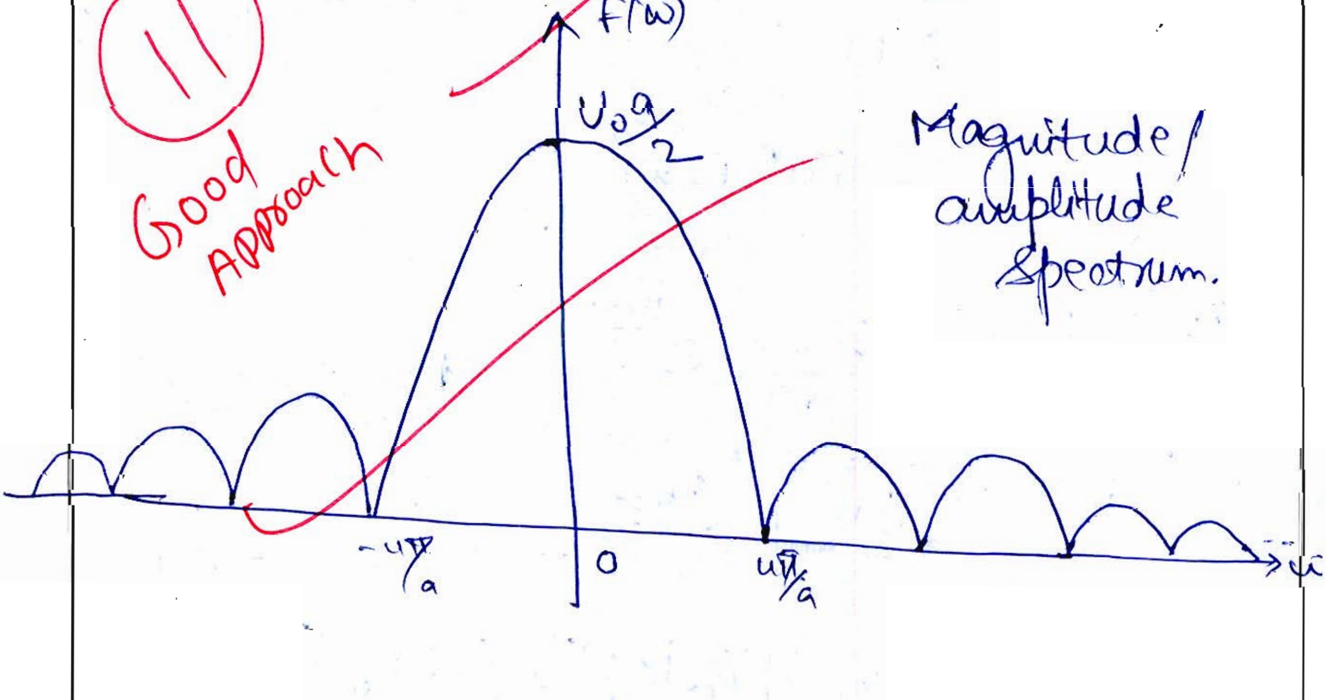
$$f(\omega) = \frac{\cancel{8}V_0\cancel{\omega^2}\times a^{\cancel{2}}}{\cancel{a}\,\cancel{\omega^2}\times\cancel{16}_{\,2}} \cdot Sa^2\left(\frac{\omega a}{4}\right)$$

$$\boxed{f(\omega) = \frac{V_0 a}{2}\, Sa^2\left(\frac{\omega a}{4}\right)}$$

Ans

11

Good Approach



Magnitude / amplitude Spectrum.

.6 (c) **(ii)** Using duality property show that the Fourier transform of $\left[\frac{1}{1+j2\pi t}\right]$ is equal to $e^{f}u(-f)$ where $u(t)$ is the unit step.

[8 marks]

Sol^n^:- We know that

$\cancel{u(t) \xrightleftharpoons{F.T} \frac{1}{j\omega}}$ $e^{-at}u(t) \xrightleftharpoons{F.T} \frac{1}{a+j\omega}$

Substituting $a=1$

$e^{-t}u(t) \xrightleftharpoons{F.T} \frac{1}{1+j\omega} = \frac{1}{1+j2\pi f}$

$t \to -t \quad f \to -f$

So; $e^{t}u(-t) \xrightleftharpoons{F.T} \frac{1}{1-j2\pi f}$

So; By applying duality

$\frac{1}{1-j2\pi t} \xrightleftharpoons{F.T} e^{-f}u(f)$

$t \to -t \quad f \to -f$

$$\boxed{\frac{1}{1+j2\pi t} \rightleftharpoons e^{f}u(-f)}$$

Hence, proved.

8

Good Approach

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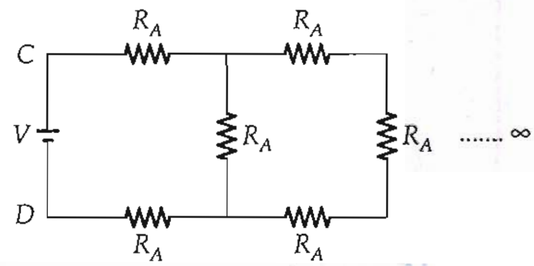
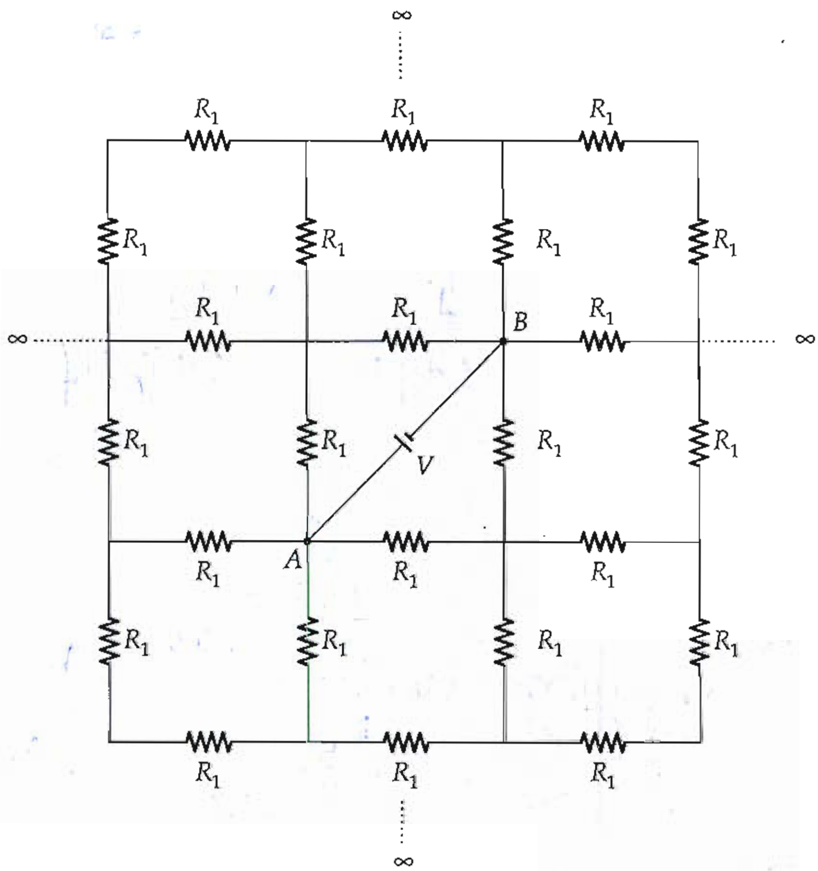
Q.7 (a) For the networks shown in figure below,



Network (i)



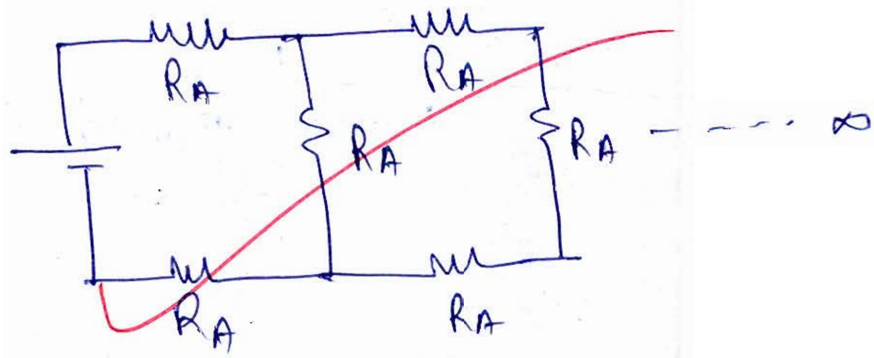
Network (ii)

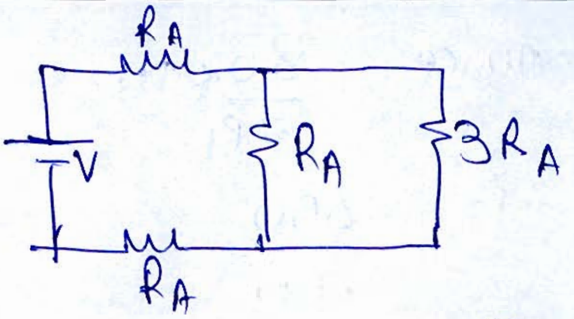
On applying 10 V voltage source across terminal CD and AB respectively in network (i) and (ii), we get same power delivered by the source. Derive the relationship between the resistance present in network (i) and (ii). And also calculate the power delivered by the source for $R_A = 10\ \Omega$.

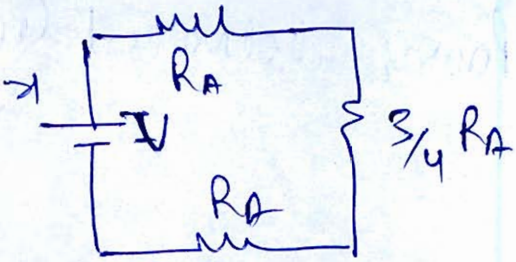
[20 marks]

Soln:-

V





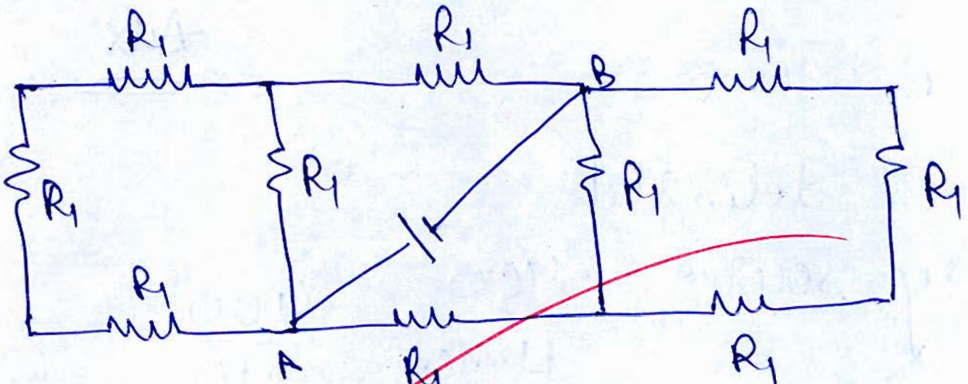


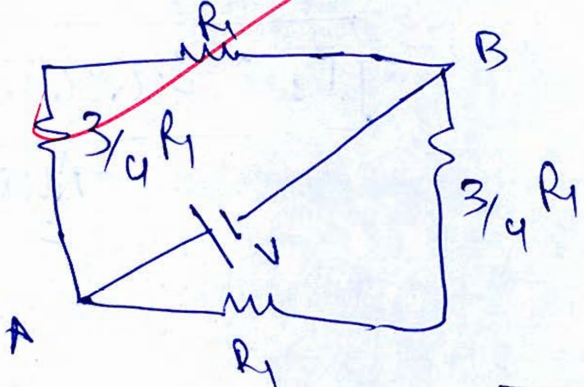
$$I = \frac{10}{\left(\frac{3}{4}+2\right)R_A} = \frac{40}{11R_A} \text{ Amp} \qquad V = 10V$$

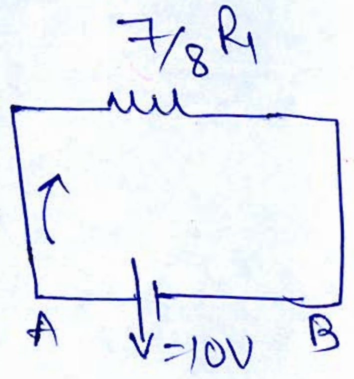
Power delivered by source $= VI = 10 \times I$

$$\boxed{P_1 = \frac{400}{11R_A}}$$

for N/w-(ii)







$$I = \frac{10}{\frac{7}{8}R_1} = \frac{80}{7R_1} \text{ Amp}$$

Power delivered by source = $\frac{80 \times 10}{7R_1}$

$P_2 = \frac{800}{7R_1}$

Given $P_1 = P_2$

$$\frac{\cancel{400}}{11 R_A} = \frac{\cancel{800}\ 2}{7 R_1}$$

$$\frac{1}{11 R_A} = \frac{2}{7 R_1}$$

$$\frac{R_1}{R_A} = \frac{22}{7} \rightarrow \boxed{R_1 = \frac{22}{7} R_A}$$

Ans

For $R_A = 10\Omega$

Power delivered by source = $\frac{400}{11 \times 10} = \frac{400}{110}$

$$\boxed{P_1 = 3.636 \text{ watt}}$$

Ans

18

Good Approach

Q.7 (b) Consider a signal $x(t)$ with Fourier transform $X(j\omega)$. Suppose we are given the following facts :

1. $x(t)$ is real and non-negative.
2. $F^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$, where A is independent of t.
3. $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi$

Determine a closed-form expression of $x(t)$.

[20 marks]

Soln:- $F^{-1}[(1+j\omega)X(j\omega)] = Ae^{-2t}u(t)$

$\Rightarrow (1+j\omega)X(j\omega) = F(Ae^{-2t}u(t))$

$= \frac{A}{2+j\omega}$

$\Rightarrow X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)}$

$X(j\omega) = A\left[\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right]$

Taking inverse fourier transform

$x(t) = A\left[e^{-t} - e^{-2t}\right]u(t)$

Given $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$

$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = \int_{-\infty}^{\infty} |x(t)|^2 dt = 1$

$\int_{0}^{\infty} A^2\left[e^{-2t} + e^{-4t} - 2e^{-3t}\right] = 1$

$$A^2 \left[\frac{e^{-2t}}{-2} + \frac{e^{-4t}}{-4} - \frac{2}{-3}e^{-3t}\right]_0^{\infty} = 1$$

$$-A^2\left[-\frac{1}{2} - \frac{1}{4} + \frac{2}{3}\right] = 1$$

$$\frac{A^2}{12} = 1 \Rightarrow A = \sqrt{12}$$

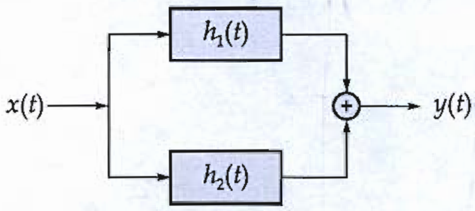
So,
$$\boxed{x(t) = \sqrt{12}\left[e^{-t} - e^{-2t}\right]u(t)}$$

Ans

18

Good
Approach

7 (c) (i) Consider the parallel combination of two LTI systems shown in the figure,



The impulse responses of the systems are

$$h_1(t) = 2\delta(t+2) - 3\delta(t+1);$$
$$h_2(t) = \delta(t-2)$$

If the input $x(t)$ is a unit step signal, then find the energy of $y(t)$.

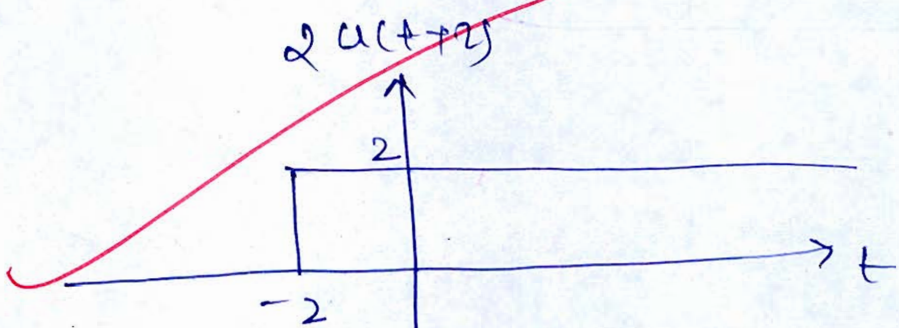
[10 marks]

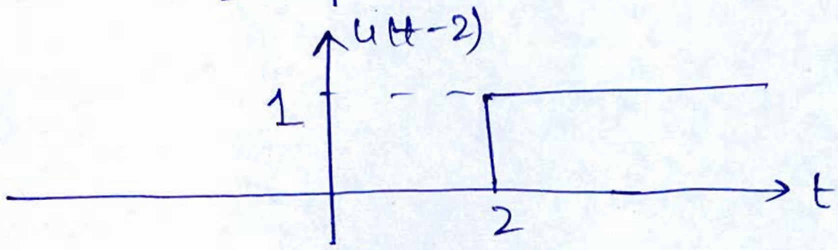
Sol^n - $y(t) = x(t) * h_1(t) + x(t) * h_2(t)$

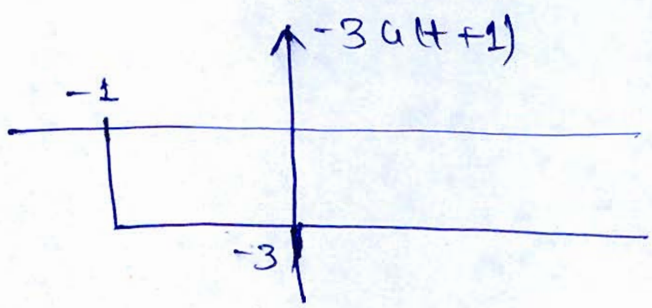
$x(t) = u(t)$ [Unit step function]

$y(t) = u(t) * [2\delta(t+2) - 3\delta(t+1)]$
$+ u(t) * [\delta(t-2)]$

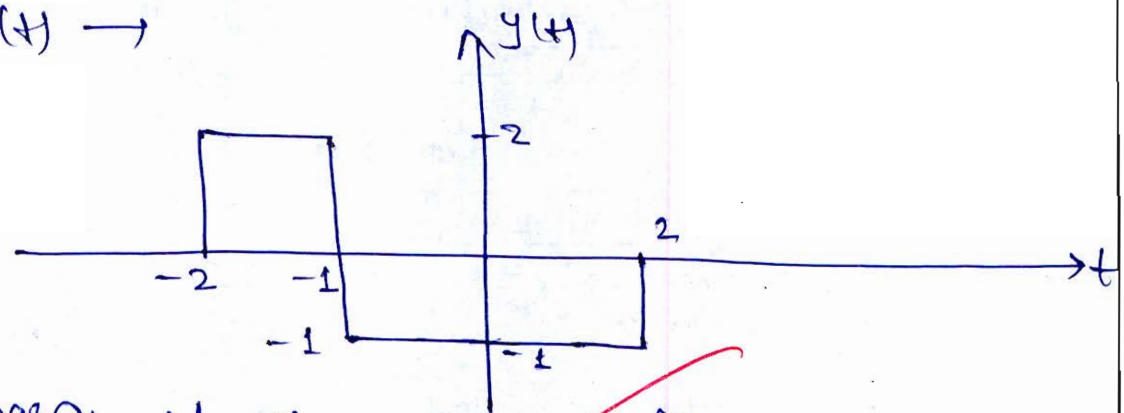
$y(t) = 2u(t+2) - 3u(t+1) + u(t-2)$







$y(t) \rightarrow$



Energy of sig. $y(t) = \int_{-\infty}^{\infty} |y(t)|^2 \, dt$

$$= \int_{-2}^{-1} 2^2 \, dt + \int_{-1}^{2} (-1)^2 \, dt$$

$$= 4 \times [-1+2] + 1\,[2-(-1)]$$

$$= 4 + 1 \times 3 = 7$$

$$\boxed{E_{y(t)} = 7 \text{ units}}$$

9

Good Approach

7 (c) **(ii)** The exponential Fourier series representation of a continuous-time periodic signal $x(t)$ is defined as $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$, where ω_o is the fundamental angular frequency of $x(t)$ and the coefficients of the series are a_k. The following information is given about $x(t)$ and a_k.

1. $x(t)$ is real and even, having a fundamental period of 6.
2. The average value of $x(t)$ is 2.
3. $a_k = \begin{cases} k, & 1 \le k \le 3 \\ 0, & k > 3 \end{cases}$

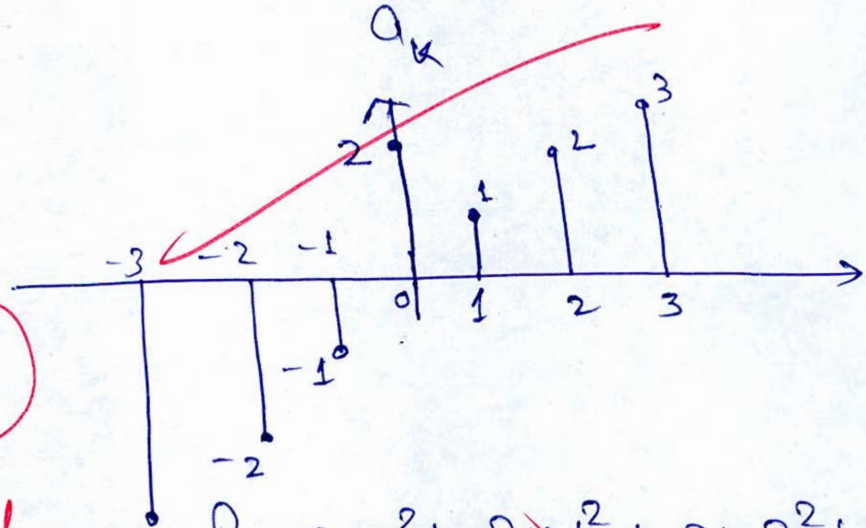
Find the average power of the signal $x(t)$.

[10 marks]

Sol^n:- $T_0 = 6$

Power of Signal $= \sum |a_k|^2$

Given $a_0 = 2$ (D.C or average value)



9

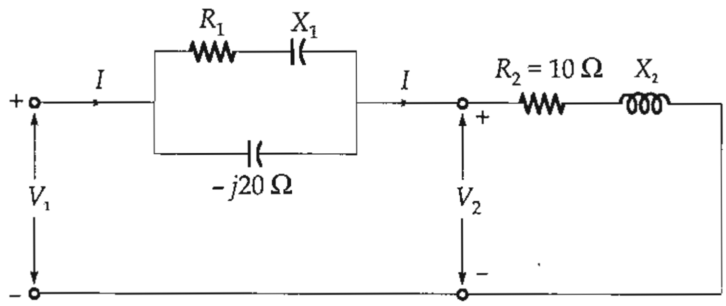
Good Approach

Power $= 2^2 + 2 \times 1^2 + 2 \times 2^2 + 2 \times 3^2$

$= 4 + 2 + 8 + 18$

$\boxed{P = 32 \text{ units}}$

8 (a) In the circuit shown in the figure below, $|V_1| = 200$ V, $V_2 = 200\angle 0°$ V and $|I| = 12$ A. The total power absorbed by the circuit is 1.8 kW. Find R_1, X_1 and X_2.



[20 marks]

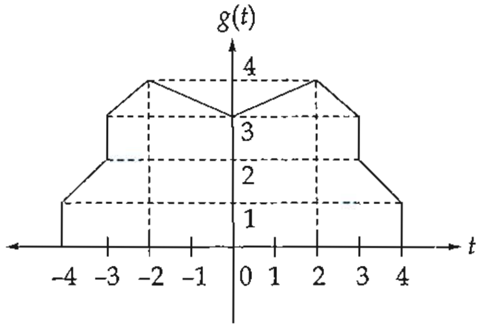
Do not write in this margin

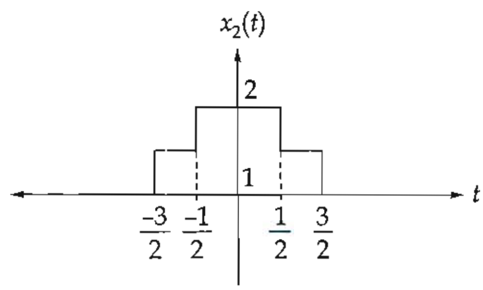
Q.8 (b) **(i)** Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by $y''(t) + y'(t) - 2y(t) = x(t)$.

1. Find the system function $H(s)$.
2. Determine the impulse response $h(t)$ for each of the following three cases : (1) The system is causal, (2) The system is stable, (3) The system is neither causal nor stable.

[13 marks]

Q.8 (b) (ii) The response of an LTI system to an input signal $x_1(t) = u(t+1) - u(t-1)$ is denoted as $g(t)$, as illustrated in the figure below. If a new input $x_2(t)$ is applied to the same system, resulting in an output $y(t)$. Determine the value of the output at $t = 0$.



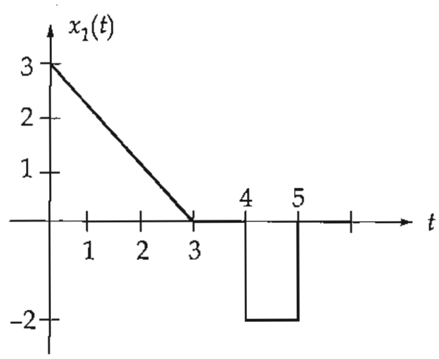


[7 marks]

3 (c) Consider a continuous-time LTI system with an impulse response $h(t)$ defined as a rectangular pulse of amplitude A and duration $T(0 < t < T)$. When the system is subjected to an input $x_1(t)$ given below, it produces an output $y_1(t)$. It is observed that $y_1(5) = 0$.

Furthermore, when the input is $x_2(t) = \sin\left(\frac{\pi t}{3}\right)u(t)$, the output $y_2(t)$ at $t = 9$ is equal to 9.

Determine the value of the product $A \times T$.



[20 marks]

OOOO

Space for Rough Work

Space for Rough Work