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Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

E & T Engineering Test No : 4

Section A : Electronic Devices & Circuits + Advanced Communication

Q.1 (a) Solution:

Given, Ideal MOS capacitor,

$$\text{Oxide thickness, } t_{ox} = 10 \text{ nm} = 1 \times 10^{-6} \text{ cm.}$$

$$\text{Oxide voltage, } V_{ox} = 0.4 \text{ V}$$

$$\in_{ox} = 40 \times 10^{-14} \text{ F/m}$$

$$\in_{\text{Si}} = 100 \times 10^{-14} \text{ F/cm}$$

(i) From the given energy-band bending diagram, bands bend downward at the semiconductor surface, therefore electron concentration increases at the surface.

This corresponds to an n-type semiconductor. Since the ideal MOS capacitor is given, semiconductor potential is equal to oxide potential,

$$\text{i.e., } \phi_s = V_{ox} = 0.4 \text{ V}$$

(ii) The work function difference ϕ_{ms} in a MOSFET is equal to the energy difference between the gate material's Fermi level ($E_{f,\,m}$) and the semiconductor substrate's Fermi level.

$$\therefore \quad \phi_{ms} = -\phi_s = -0.4 \text{ V}$$

(iii) At equilibrium, the MOS capacitance is the series combination of oxide and depletion capacitances.

$$\therefore \quad \frac{1}{C_{MOS}} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}}$$

where, oxide capacitance,

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$\therefore \quad C_{ox} = \frac{40\times10^{-14}}{1\times10^{-6}} = 4\times10^{-7}\,\text{F/cm}^2$$

given, $$C_{MOS} = \frac{1}{3}\times10^{-7} = 3.33\times10^{-8}\,\text{F/cm}^2 \quad (\text{at } V = 0)$$

$$\frac{1}{C_{dep}} = \frac{1}{C_{MOS}} - \frac{1}{C_{ox}}$$

$$\frac{1}{C_{dep}} \approx 3.0\times10^{7} - 2.5\times10^{6} \approx 2.75\times10^{7}$$

$$\therefore \quad C_{\text{dep}} \approx 3.6\times10^{-8}\,\text{F/cm}^2$$

$$C_{\text{dep}} = \frac{\epsilon_s}{W} \qquad (\text{at } V = 0)$$

$$\therefore \quad \text{W}\,(V=0) = \frac{\epsilon_s}{C_{dep}} = \frac{1\times10^{-12}}{3.6\times10^{-8}}$$

$$W(V=0) \approx 2.8\times10^{-5}\,\text{cm}$$

$$\approx 0.28\,\mu\text{m}$$

Q.1 (b) Solution:

Given,

Photogenerated current, $I_{ph} = 3.50$ A

Dark saturation current, $I_0 = 3.22\times10^{-11}$ A

Maximum power, $P_{\text{max}} = 1.75$ W

Thermal voltage, $V_T = \frac{KT}{q} = 26$ mV

For a solar cell at open circuit condition,

$$I = 0 = I_{ph} - I_0\left(e^{V_{oc}/V_T} - 1\right)$$

Thus, $$I_{ph} = I_0\left(e^{V_{oc}/V_T} - 1\right)$$

Open-circuit voltage, $$V_{oc} = V_T \ln\left(\frac{I_{ph}}{I_0} + 1\right)$$

We have, $\dfrac{I_{ph}}{I_0} = \dfrac{3.5}{3.22\times10^{-11}} = 1.087\times10^{11}$

$\therefore$ $V_{oc} = 0.026 \ln(1.087 \times 10^{11})$

$\therefore$ $V_{oc} = 0.66$ V

We have, Fill factor, (F.F) $= \dfrac{P_{max}}{V_{oc}\, I_{sc}}$

For ideal solar cell, $I_{sc} \approx I_{ph}$

Thus, F.F $= \dfrac{1.75}{0.66\times3.5}$

$\therefore$ F.F = 0.76 (or) 76%

Q.1 (c) Solution:

(i) Foliage means dense trees or vegetation of any type. Foliage loss is the loss in signal power during propagation because of presence of foliage environment in its path. Foliage loss may occur due to the sizes of trunks of trees, branches and leaves of trees including their texture and thickness; density of trees; distribution of sizes of trunks, branches, and leaves of trees in a thick forest area; height of the trees relative to the antenna heights of the cell-site and mobile unit.

(ii) Close proximity of foliage at the cell-site transmitter always heavily attenuates the received signal strength and thus degrades the received signal quality. Therefore, the cell site must be placed away from dense trees. If the heavy foliage is close to the mobile unit, there may be significant degradation in the received signal quality, and additional signal loss due to foliage must be considered in the system design.

Q.1 (d) Solution:

We have,

Frequency of transmission, f_c = 900MHz

Transmitting antenna height, h_t = 100m

Receiving antenna height, h_r = 2 m

Distance between Tx and Rx, r = 4km

Using the Hata Model, the median path loss in urban area is given by

$$L_{pH(\text{urban})}\text{(dB)} = 69.55+26.16 \log f_c\,(\text{MHz}) - 13.82 \log h_t\,(\text{m}) - \alpha_r + (44.9 - 6.55 \log h_t\,\text{m}) \log r\,(\text{km})$$

where α_r is the correction factor for effective mobile antenna height, which depends on the size of the coverage area.

α_r for a large city is given by

$$\alpha_r(\text{dB}) = 8.29\,(\log 1.54h_r)^2 - 1.1 \text{ for } f_c \le 300 \text{ MHz}$$

$$\alpha_r(\text{dB}) = 3.2\,(\log 11.75h_r)^2 - 4.97 \text{ for } f_c > 300 \text{ MHz}$$

As f_c = 900 MHz, thus

$$\alpha_r(\text{dB}) = 3.2\,(\log 11.75h_r)^2 - 4.97$$

$$\alpha_r(\text{dB}) = 3.2\,(\log 11.75 \times 2)^2 - 4.97$$

$$\alpha_r(\text{dB}) = 1.045 \approx 1\text{dB}$$

Now, propagation path loss $L_{pH\,(\text{urban})}$ (dB) for specified cellular system parameters is

$$L_{pH\,(\text{urban})}\,(\text{dB}) = 69.55 + 26.16 \log 900 - 13.82 \log 100 - 1 + (44.9 - 6.55 \log 100) \log 4$$

Hence, $L_{pH\,(\text{urban})}$ (dB) = 137.3 dB

Q.1 (e) Solution:

We have,

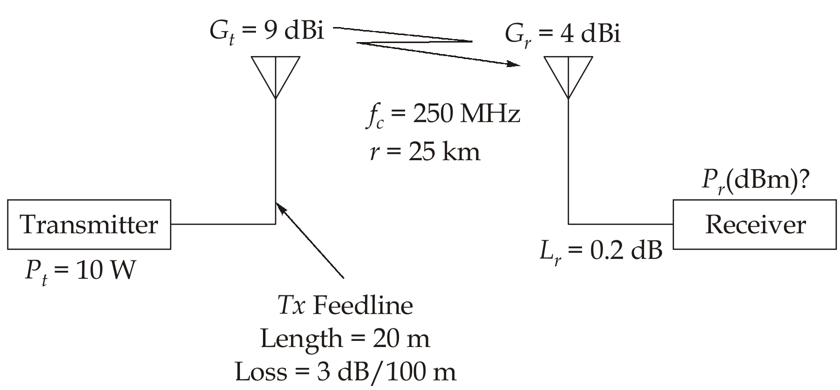
Transmitter output power, P_t = 10 W or 10,000 mW

Transmitter antenna gain, G_t = 9 dBi

Receiver antenna gain, G_r = 4 dBi

Carrier frequency, f_c = 250MHz

Distance between transmitter and receiver, r = 25 km



To convert $P_t(W)$ in P_t (dBm)

We know that, P_t (dBm) = 10 log[P_t(mW)]

Therefore, P_t(dBm) = 10 log[10000] = +40 dBm

Now, free-space path loss, L_{pf} (dB) is

$$L_{pf}\,(\text{dB}) = 32.44 + 20 \log r(\text{km}) + 20 \log f_c\,(\text{MHz})$$

$$= 32.44 + 20 \log 25 + 20 \log 250 = 108.3 \text{ dB}$$

The T_x antenna RF cable loss, L_t (dB)

$$\text{Cable length} = 20 \text{ m (given)}$$

$$\text{Cable attenuation} = 3 \text{ dB/100 m (given)}$$

Therefore, Tx antenna RF cable loss,

$$L_t = 20 \text{ m} \times 3 \text{ dB }/100 \text{ m}$$

$$Tx \text{ antenna } RF \text{ cable loss, } L_t = 0.6 \text{ dB}$$

and, Rx antenna RF cable loss, L_r = 0.2 dB (given) [Due to mismatch]

Received power, $$P_r(\text{dBm}) = P_t(\text{dBm}) - L_t(\text{dB}) + G_t(\text{dB}) - L_{pf}(\text{dB}) + G_r(\text{dB}) - L_r(\text{dB})$$

$$P_r(\text{dBm}) = 40 - 0.6 + 9 - 108.3 + 4 - 0.2 = -56.1 \text{ dBm}$$

Hence, power delivered to the receiver is –56.1 dBm = $10^{-5.61} = 2.45 \times 10^{-6}$ mW

Q.2 (a) Solution:

We have,

Service area of a cellular system, A_{sys} = 4200 km²

Coverage area of a cell, A_{cell} = 12 km²

Total number of available channels, N = 1001

(i) To calculate number of clusters, cell capacity, and system capacity

As, Cluster size, K = 7

The coverage area of a cluster,

$$A_{cluster} = K \times A_{cell}$$

Therefore, $$A_{cluster} = 7 \times 12 \text{ km}^2 = 84 \text{ km}^2$$

The number of times that the cluster has to be replicated to cover the entire service area of cellular system

$$= A_{sys}/A_{cluster}$$

Or, number of clusters, M = 4200/84

Hence,

number of clusters, M = 50 clusters

Since total number of available channels are allocated to one cluster, therefore, the number of channels per cell,

$$J = N/K$$

Or, cell capacity, J = 1001/7

Hence, cell capacity, J = 143 channels/cell

The system capacity, $C = N \times M$

Or, system capacity, $C = 1001 \times 50$

Hence, the system capacity,

$$C = 50{,}050 \text{ channels}$$

(ii) To calculate new system capacity for reduced K

As, new cluster size, $K = 4$

The coverage area of a cluster, $A_{cluster'} = K \times A_{cell}$

Therefore, $A_{cluster'} = 4 \times 12 \text{ km}^2 = 48 \text{ km}^2$

The number of times that the cluster has to be replicated to cover the entire service area of a cellular system = $A_{sys}/A_{cluster'}$

Or, number of clusters, $M = 4200/48$

Hence,

number of clusters, $M = 87$ (approx.)

The system capacity, $C = N \times M$

Or, system capacity, $C = 1001 \times 87$

Hence,

system capacity, $C = 87{,}000$ channels

Comments on the results: From (i) and (ii) above, it is seen that for decrease in cluster size from 7 to 4 results into an increase in number of clusters from 50 to 87 for a given service area. The system capacity is increased from 50,050 channels to 87,000 channels. Therefore, decreasing the cluster size does increase the system capacity. However, the average signal-to-cochannel interference also increases which has to be kept at an acceptable level in order to achieve desirable signal quality.

Q.2 (b) Solution:

Given, uniformly doped silicon npn BJT at $T = 300$ K.

Emitter doping, $N_E = 2 \times 10^{18} \text{ cm}^{-3}$

Base doping, $N_B = 2 \times 10^{16} \text{ cm}^{-3}$

Collector doping, $N_C = 2 \times 10^{15} \text{ cm}^{-3}$

Neutral base width at zero bias,

$$x_{B0} = 0.85 \text{ µm}$$

Electron diffusion coefficient in base, $D_n = 25 \text{ cm}^2/\text{s}$.

Also, given $x_{B0} << L_n$.

(i) For a short base width,

$$x_{B0} << L_n,$$

the electron diffusion current density in the base is given by

$$J_n = qD_n \frac{n_{b0}}{x_B} \exp\left(\frac{V_{BE}}{V_T}\right)$$

Effective basewidth, $x_B = x_{B0} - W_{CB}$

where, W_{CB} : Collector-base depletion width in base side (dominating)

$$\therefore \quad W_{CB} \approx \sqrt{\frac{2\in_{si}}{q} \frac{N_C}{N_B(N_B + N_C)}(V_{CB} + V_{bi})}$$

where $$V_{bi} = V_T \ln\left(\frac{N_B N_C}{n_i^2}\right)$$

$$= 0.026 \ln\left(\frac{2\times10^{16} \times 2\times10^{15}}{10^{20}}\right) = 0.695 \text{ V}$$

Thus, $$\omega_{CB} = \sqrt{\frac{2\times11.7\times8.854\times10^{-14}}{1.6\times10^{-19}} \times \frac{2\times10^{15}}{2\times10^{16}\times2.2\times10^{16}} \times (V_{CB} + 0.695)}$$

$$= 0.076\sqrt{(V_{CB} + 0.695)}\mu\text{m}$$

$$\therefore \quad J_n = qD_n \frac{n_{b0}}{x_{B0} - W_{CB}} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$= qD_n \frac{n_i^2}{N_B(x_{B0} - W_{CB})} \exp\left(\frac{V_{BE}}{V_T}\right)$$

1. For $V_{CB} = 4$ V,

$$W_{CB} = 0.076\sqrt{(4 + 0.695)}\ \mu\text{m} = 0.165\ \mu\text{m}$$

$$J_n \approx 1.6\times10^{-19} \times 25 \frac{10^{20}}{(2\times10^{16})(0.85 - 0.165)\times10^{-4}} \exp\left(\frac{0.65}{0.026}\right)$$

$$\therefore \quad J_n \approx 21.02 \text{ A/cm}^2$$

2. For $V_{CB} = 8$ V

$$W_{CB} = 0.076\sqrt{(8+0.695)}\ \mu\text{m} = 0.22\ \mu\text{m}$$

$$J_n \approx 1.6\times10^{-19}\times25\times\frac{10^{20}}{(2\times10^{16})(0.85-0.22)\times10^{-4}}\exp\left(\frac{0.65}{0.026}\right)$$

$$\approx 22.86\ \text{A/cm}^2$$

3. For $V_{CB} = 12$ V,

$$W_{CB} = 0.076\sqrt{(12+0.695)}\ \mu\text{m} = 0.27\ \mu\text{m}$$

$$J_n \approx 1.6\times10^{-19}\times25\times\frac{10^{20}}{(2\times10^{16})(0.85-0.27)\times10^{-4}}\exp\left(\frac{0.65}{0.026}\right)$$

$$J_n \approx 24.83\ \text{A/cm}^2$$

(ii) In an uniformly doped npn BJT, the collector current density varies as,

$$J_n \propto \frac{1}{x_B}$$

We can write,

$$J_n = \frac{K}{x_B}$$

Using linear approximation, the base width decreases approximately linearly with increase in V_{CB}, and this dependence is expressed using the Early voltage V_A:

$$x_B \approx x_{B0}\left(1-\frac{V_{CB}}{V_A}\right)$$

Differentiating with respect to V_{CB}:

$$\frac{dx_B}{dV_{CB}} = -\frac{x_{B0}}{V_A}$$

Hence, the change in base width is:

$$\Delta x_B \approx -\frac{x_{B0}}{V_A}\Delta V_{CB}$$

Thus, we can write,

$$\Delta J_n = \frac{dJ_n}{dx_B}\Delta x_B = \frac{K}{x_B^2}\times\frac{x_{B0}}{V_A}\Delta V_{CB}$$

Assuming $x_{B0} \approx x_B$, we get

$$\Delta J_n = J_n \times \frac{\Delta V_{CB}}{V_A}$$

$$\frac{\Delta J_n}{J_n} \approx \frac{\Delta V_{CB}}{V_A}$$

From part (i), V_{CB} : 4 V to 12 V

$$\frac{\Delta J_n}{J_n} \approx \frac{J_{n(12\text{ V})} - J_{n(4\text{ V})}}{J_{n(4\text{ V})}} = \frac{24.83 - 21.02}{21.02} = 0.181$$

$$\Delta V_{CB} \approx 12 - 4 = 8\text{ V}$$

$\therefore$ Early voltage, $V_A \approx \dfrac{8}{0.181} \approx 44.2\text{ V}$

Q.2 (c) Solution:

(i) Linear Scattering Losses: Linear scattering mechanisms cause the transfer of some or all of the optical power contained within one propagating mode to be transferred linearly (proportionally to the mode power) into a different mode. This process tends to result in attenuation of the transmitted light as the transfer may be to a leaky or radiation mode which does not continue to propagate within the fiber core, but is radiated from the fiber. It must be noted that as with all linear processes, there is no change of frequency on scattering.

Linear scattering may be categorized into two major types:

(a) Rayleigh scattering

(b) Mie scattering

Both result from the non-ideal physical properties of the manufactured fiber which are difficult and, in certain cases, impossible to eradicate at present.

(a) Rayleigh Scattering: Rayleigh scattering is the dominant intrinsic loss mechanism in the low-absorption window between the ultraviolet and infrared absorption tails. It results from inhomogeneities of a random nature occurring on a small scale compared with the wavelength of the light. These inhomogeneities manifest themselves as refractive index fluctuations and arise from density and compositional variations which are frozen into the glass lattice on cooling. The compositional variations may be reduced by improved fabrication, but the index fluctuations caused by the freezing-in of density inhomogeneities are fundamental and cannot be avoided. The subsequent scattering due to density fluctuations, which is in almost all directions produces an attenuation proportional to $1/\lambda^4$.

(b) Mie Scattering: Linear scattering may also occur at inhomogeneities which are comparable in size with the guided wavelength. These result from the nonperfect cylindrical structure of the waveguide and may be caused by fiber imperfections such as irregularities in the core-cladding interface, core-cladding refractive index differences along the fiber length, diameter fluctuations, strains and bubbles. When the scattering inhomogeneity size is greater than $\lambda/10$, the scattered intensity which has an angular dependence can be very large.

The scattering created by such inhomogeneities is mainly in the forward direction and is called Mie scattering. Depending upon the fiber material, design and manufacture, Mie scattering can cause significant losses. The inhomogeneities may be reduced by:

1. Removing imperfections due to the glass manufacturing process;
2. Carefully controlled extrusion and coating of the fiber;
3. Increasing the fiber guidance by increasing the relative refractive index difference.

By these means it is possible to reduce Mie scattering to insignificant levels.

(ii) 1. Grade of Service (GoS):

- GoS is the measure of traffic congestion in telephone network.
- GoS is index of quality of service offered by network.
- GoS is referred as call congestion or loss probability i.e. probability that a call will be blocked or delayed in a telecommunications network.
- Grade of Service (GoS) is defined as the ratio of lost traffic to offered traffic.

$$\text{GoS} = \frac{\text{Lost traffic}}{\text{Offerred traffic}} = \frac{A - A_0}{A}$$

2. Offered Traffic

- Offered traffic is the product of the average number of calls generated by the users and the average holding time per call.
- We have, $A = \lambda T$

  where A = Load offered to the network

  λ = Message (call) arrival rate

  T = Average service time per message

3. Delay Sytem

- When call or message arrivals are serviced in queue as per the availability of resources, these systems are called delay systems, waiting call systems or queuing systems.
- When incoming calls arrive during congestion, wait in a queue until an outgoing trunk becomes free. Thus they are delayed but not lost. Such systems are therefore called queuing systems or delay system.
- Message switching is an example of delay system. It operates by storing entire messages at intermediate nodes and forwarding them only when resources are available.

4. Loss System

- In loss systems, the traffic carried by the network is generally lower than the actual traffic offered to the network by the users. The overload traffic is rejected and hence is not carried by the network.
- The amount of the traffic rejected by the network is an index of the quality of the service offered by the network.

Q.3 (a) Solution:

(i) **1.** The dispersion per unit length may be acquired simply by dividing the total dispersion by the total length of the fiber.

∴ Dispersion per unit length,

$$= \frac{\tau}{L} = \frac{0.5\times10^{-6}}{10} = 50 \text{ ns-km}^{-1}$$

2. For NRZ pulse:

$$(\text{B.W})_{max} = \frac{R_b}{2} = \frac{1}{4\tau} \quad \text{.... where } R_b = \frac{1}{2\tau}$$

$$= \frac{1}{4\times0.5\times10^{-6}} = 0.5 \text{ MHz}$$

3. Bandwidth length product of fiber is given as:

$$(\text{B.W.})\times(L)_{fiber} = 0.5\times10$$

$$= 5 \text{ MHz-km}$$

(ii) We have, $L = 9$ km; $n_1 = 1.5$, $n_2 = 1.45$

∴ Relative refractive index $(\Delta) = \dfrac{n_1^2 - n_2^2}{2n_1^2} = \dfrac{(1.5)^2-(1.45)^2}{2(1.5)^2} = 0.032$

1. The RMS pulse broadening:

$$\sigma_s = \frac{Ln_1\Delta}{2\sqrt{3}c} \qquad c = \text{speed of light}$$

$$= \frac{9\times10^3\times1.5\times0.032}{2\sqrt{3}\times3\times10^8} = 4.156 \times 10^{-7} \text{ sec}$$

2. Delay difference between the fastest and slowest modes at the fiber output

$$\delta T_s = \frac{Ln_1\Delta}{c}$$

$$= \frac{9\times10^3\times1.5\times0.032}{3\times10^8} = 1.44 \times 10^{-6} \text{ sec}$$

Q.3 (b) Solution:

(i) Given,

Donor concentration, $N_D = 10^{15} \text{ cm}^{-3}$

Generation rate, $g_{op} = 10^{21} \text{ EHP/cm}^3\text{-s}$

Carrier lifetimes, $\tau_n = \tau_p = 1 \ \mu\text{sec} = 10^{-6}$ sec

Excess carrier concentration,

$$\Delta n = \Delta p = g_{op} \cdot \tau$$

$$\Delta n = 10^{21} \times 10^{-6} = 10^{15} \text{ cm}^{-3}$$

For n-type semiconductor,

Equilibrium electron concentration,

$$n_0 \approx N_D = 10^{15}$$

Minority holes, $p_0 = \dfrac{n_i^2}{n_0}$

∴ Equilibrium hole concentration,

$$p_0 = \frac{(1.5\times10^{10})^2}{10^{15}}$$

$$p_0 = 2.25 \times 10^5 \text{ cm}^{-3}$$

Under illumination, electron concentration

$$n = n_0 + \Delta n$$

$$n = 10^{15} + 10^{15} = 2 \times 10^{15} \text{ cm}^{-3}$$

and hole concentration, $p = p_0 + \Delta p$

∴ $p \simeq 10^{15} \text{ cm}^{-3}$

Separation of Quasi-Fermi levels,

$$F_n - F_p = KT\ln\left(\frac{np}{n_i^2}\right)$$

We have,

$$np = (2 \times 10^{15})(10^{15})$$

$$np = 2 \times 10^{30}$$

$$n_i^2 = (1.5 \times 10^{10})^2 = 2.25 \times 10^{20}$$

$\therefore$

$$F_n - F_p = KT\ln\left[\frac{2\times10^{30}}{2.25\times10^{20}}\right] = 0.026\ln\left[\frac{2\times10^{10}}{2.25}\right]$$

$$F_n - F_p \simeq 0.596 \text{ eV}$$

(ii) When a semiconductor is exposed to continuous photonic excitation for a long time ($t < 0$), it reaches steady-state condition with a constant excess carrier concentration.

At $t = 0$, the excitation is removed. The excess carriers then recombine with time, causing their concentration to decay.

From the continuity equation, the excess carriers due to photonic excitation in the semiconductor is,

$$\frac{d(\Delta n)}{dt} = G - R$$

where, G = generation rate

R = Recombination rate

Δn = excess electron concentration

After the light is removed at $t = 0$;

$$G = 0$$

$$\frac{d(\Delta n)}{dt} = -R \qquad \text{...(i)}$$

Under lowlevel injection, recombination rate is proportional to excess carriers:

$\therefore$

$$R = \frac{\Delta n}{\tau}$$

where, τ is carrier lifetime.

Substituting in equation (i),

$$\frac{d(\Delta n)}{dt} = -\frac{\Delta n}{\tau}$$

Solution of the above differential equation gives,

$$\frac{d(\Delta n)}{\Delta n} = \frac{-dt}{\tau}$$

$$\int \frac{d(\Delta n)}{\Delta n} = -\int \frac{dt}{\tau}$$

$$\ln(\Delta n) = -\frac{t}{\tau} + c \quad \text{...(ii)}$$

At $t = 0$;

$$\Delta n = \Delta n_0$$

$\therefore$
$$c = \ln(\Delta n_0)$$

From equation (ii),

$$\ln(\Delta n) = -\frac{t}{\tau} + \ln(\Delta n_0)$$

$$\ln(\Delta n) - \ln[\Delta n_0] = \frac{-t}{\tau}$$

$$\ln\left[\frac{\Delta n}{\Delta n_0}\right] = \frac{-t}{\tau}$$

$$\frac{\Delta n}{\Delta n_0} = e^{-t/\tau}$$

$\therefore$
$$\Delta n(t) = \Delta n_0 e^{-t/\tau}$$

Similarly,
$$\Delta p(t) = \Delta p_0 e^{-t/\tau}$$

Q.3 (c) Solution:

(i) Thevenin equivalent of the one-port network is given as,

V_{TH} = Thevenin voltage, R_{TH} = Thevenin resistance

When the LED is connected directly,

$$I_D = \frac{V_{TH} - V_D}{R_{TH}}$$

LED characteristics are given as,

$$I_D = 0, \quad V_D < 1.5 \text{ V}$$

$$I_D = 0.03V_D - 0.045, \quad V_D \geq 1.5 \text{ V}$$

Operating point (load-line method)

Since LED will conduct only for $V_D \geq 1.5$ V.

Equating network current and diode current,

$$\frac{V_{TH} - V_D}{R_{TH}} = 0.03\,V_D - 0.045$$

$$V_{TH} - V_D = R_{TH}(0.03V_D - 0.045)$$

$$V_{TH} = V_D(1 + 0.03R_{TH}) - 0.045R_{TH}$$

Hence, $$V_D = \frac{V_{TH} + 0.045R_{TH}}{1 + 0.03R_{TH}}$$

Then $$I_D = 0.03V_D - 0.045$$

$\therefore$ $$I_D = 0.03\left(\frac{V_{TH} + 0.045R_{TH}}{1 + 0.03R_{TH}}\right) - 0.045$$

(ii) Given,

Emitter efficiency, $\gamma_e = 0.995$

Base doping, $N_A = 1.2 \times 10^{16}\ \text{cm}^{-3}$

Electron diffusion length, $L_e \approx W_b = 0.7\ \mu\text{m}$

Minority carrier diffusion coefficient,

$D_{pe} = 12\ \text{cm}^2/\text{s}$ (Hole diffusion coefficient in emitter)

$D_{nb} = 30\ \text{cm}^2/\text{s}$ (Electron diffusion coefficient in base)

Minority electron concentration in base,

$$n_{b0} = 1.88 \times 10^4\ \text{cm}^{-3}$$

We know that,

Emitter efficiency, $\gamma_e = \dfrac{J_{nE}}{J_{nE} + J_{pE}}$ for npn transistor

$$\gamma_e = \frac{1}{1 + \dfrac{J_{pE}}{J_{nE}}}$$

where, $$\frac{J_{pE}}{J_{nE}} = \frac{D_{pe}W_b p_{e0}}{D_{nb}L_e n_{b0}}$$

where minority hole concentration in emitter,

$$p_{e0} = \frac{n_i^2}{N_D}$$

Let the ratio $\frac{J_{pE}}{J_{nE}} = R$

$$\therefore \quad \gamma_e = \frac{1}{1+R}$$

$$R = \frac{1-\gamma_e}{\gamma_e}$$

$$R = \frac{1-0.995}{0.995}$$

$$\therefore \quad R = 0.00503$$

$$\Rightarrow \quad R = \frac{D_{pe} W_b p_{e0}}{D_{nb} L_e n_{b0}}$$

given, $W_b \approx L_e$

$$\therefore \quad R = \frac{D_{pe} p_{e0}}{D_{nb} n_{b0}}$$

$$R = \frac{12}{30} \times \frac{p_{e0}}{1.88 \times 10^4}$$

$$0.00503 = \frac{p_{e0}}{2.5 \times 1.88 \times 10^4}$$

$$\therefore \quad p_{e0} = 236.41 \times 10^6 \text{ cm}^{-3}$$

but $$p_{e0} = \frac{n_i^2}{N_D}$$

$$N_D = \frac{n_i^2}{P_{e0}} = \frac{(1.5 \times 10^{10})^2}{236.41}$$

$$\therefore \quad N_D = 9.52 \times 10^{17} \text{ cm}^{-3}$$

Q.4 (a) Solution:

(i) Assuming ΔL is the width of the depletion region at pinched-off portion. Since, the depletion region extends equally into the channel and drain side neutral regions, we can write

$$L' = L - \frac{\Delta L}{2} = 0.90\,L$$

$$\Delta L = 2(L - 0.90L) = 0.20L$$

We have, $\Delta L = 0.20L = \sqrt{\dfrac{2\varepsilon_{si}}{qN_D}[V_{DS} - V_{DS(sat)}]}$

$$L = 5\sqrt{\frac{2\varepsilon_{si}}{qN_D}[V_{DS} - V_{DS(sat)}]} \quad ...(i)$$

When $V_{GS} = 0$, $V_{DS(sat)} = |V_P|$.

$$|V_P| = V_{P0} - V_{bi}$$

where $V_{P0} = \dfrac{qa^2N_D}{2\varepsilon_{si}} = \dfrac{1.6\times10^{-19}\times(0.40\times10^{-4})^2\times3\times10^{16}}{2\times1.06\times10^{-12}} = 3.62 \text{ V}$

and $V_{bi} = V_t \ln\left(\dfrac{N_AN_D}{n_i^2}\right) = 0.026\ln\left(\dfrac{10^{19}\times3\times10^{16}}{2.25\times10^{20}}\right) = 0.9 \text{ V}$

So, $V_{DS(sat)} = 3.62 - 0.9 = 2.72 \text{ V}$

The applied drain to source voltage is $V_{DS} = 5$ V. Thus,

$$L = 5\sqrt{\frac{2\times1.06\times10^{-12}}{1.6\times10^{-19}\times3\times10^{16}}(5-2.72)} \text{ cm}$$

$$= 1.6\ \mu\text{m}$$

(ii) Given, n-type Si semiconductor,

donor concentration, $N_D = 10^{16}$ cm^{-3}

optical generation rate, $G = 10^{21}$ cm^{-3} s^{-1}

Trap (recombination center) density,

$$N_t = 10^{15} \text{ cm}^{-3}$$

Cross sections, $\sigma_n = \sigma_p = 10^{-16}$ cm^2

Thermal velocity, $V_t = 10^7$ cm/sec

Intrinsic concentration, $n_i = 10^{10}$ cm^{-3}

Electron and hole lifetimes,

$$\tau_n = \frac{1}{\sigma_n V_t N_t};\ \tau_p = \frac{1}{\sigma_p V_t N_t}$$

Since, $\sigma_n = \sigma_p$

$$\tau_n = \tau_p = \frac{1}{10^{-16}\times10^7\times10^{15}}$$

$\therefore$ $\tau_n = \tau_p = 10^{-6}$ sec

Equilibrium electron concentration (without illumination),

$$n_0 \approx N_D = 10^{16}\ \text{cm}^{-3}$$

Equilibrium hole concentration (without illumination),

$$p_0 = \frac{n_i^2}{n_0} = \frac{(10^{10})^2}{10^{16}} = 10^4\,\text{cm}^{-3}$$

Under steady illumination, generation rate,

$$G = \frac{\Delta n}{\tau_{eff}}$$

For midgap traps ($E_t = E_i$) and low-level injection in n-type material ($n >> p$),

$$\tau_{\text{eff}} \approx \tau_p$$

Excess carrier concentration,

$$\Delta n = \Delta p = G\tau_p$$

$$\Delta n = 10^{21} \times 10^{-6} = 10^{15}\ \text{cm}^{-3} = \Delta p$$

Total electron concentration,

$$n = n_0 + \Delta_n = 10^{16} + 10^{15} = 1.1 \times 10^{16}\ \text{cm}^{-3}$$

Total hole concentration, $p = p_0 + \Delta p = 10^4 + 10^{15}$

$\therefore \qquad p \simeq 10^{15}\ \text{cm}^{-3}$

Q.4 (b) Solution:

(i) Since the MOSFET is in saturation region,

$$I_D = k_n(V_{GS} - V_T)^2$$

For $V_{GS} = 1$ V, $\quad k_n(1 - V_T)^2 = 50\ \mu\text{A}$

For $V_{GS} = 3$ V, $\quad k_n(3 - V_T)^2 = 200\ \mu\text{A}$

So, $$\frac{(1-V_T)^2}{(3-V_T)^2} = \frac{1}{4}$$

$$4(V_T^2 - 2V_T + 1) = (V_T^2 - 6V_T + 9)$$

$$3V_T^2 - 2V_T - 5 = 0$$

After solving, we get,

$$V_T = -1\ \text{V},\ \frac{5}{3}\ \text{V}$$

For transistor to conduct, $V_{GS} > V_T$. If we consider $V_T = 1.67$ V, $V_{GS} < V_T$ for $V_{GS} = 1$ V and the MOSFET would be in cutoff. Thus, $V_T = -1$ V.

(ii) The threshold voltage of an ideal MOSFET can be given by,

$$V_T = 2\phi_F + \frac{2\sqrt{\varepsilon_{si}\, qN_A\phi_F}}{C_{ox}}$$

where $$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-13}}{500\times10^{-8}}\ \text{F/cm}^2 = 6.9\times10^{-8}\ \text{F/cm}^2$$

and $$\phi_F = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) = 0.026\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right)\text{V} = 0.35\ \text{V}$$

So, $$V_T = 2(0.35) + \frac{2\sqrt{1.06\times10^{-12}\times1.6\times10^{-19}\times10^{16}\times0.35}}{6.9\times10^{-8}}\ \text{V}$$

$$= 1.4\ \text{V}$$

Q.4 (c) Solution:

Given: EIRP = 36 dBW

Downlink frequency, f = 4 GHz

Slant distance = 39,500 km

To find: $\left[\frac{C}{N_0}\right]$ at the receiver output.

Steps to be followed:

Step I : To calculate path loss.

Step II: To calculate the gain of receiving antenna.

Step III: To find the system noise temperature.

Step IV: To find $\left[\frac{G_r}{T_s}\right]$ ratio at the receiver.

Step V : To find the carrier to noise ratio.

Step I: To calculate path loss:

$$\text{Path loss} = 20\log_{10}\left[\frac{4\pi R}{\lambda}\right]$$

where, R = 39,500 km = 39.5 × 10^6 m

$$\lambda = \frac{c}{f} = \frac{3\times10^8\ \text{m/s}}{4\times10^9\ \text{Hz}}$$

$$\lambda = 0.075\ \text{m}$$

$$\therefore \qquad \text{Path loss} = 20\log_{10}\left[\frac{4\pi\times 39.5\times 10^6}{0.075}\right]$$

$$\text{Path loss} = 196.41 \text{ dB}$$

Step II: To calculate the gain of the receiving antenna:

The diameter of the antenna is given as,

$$D = 10 \text{ ft}$$

$$1 \text{ ft} = 0.305 \text{ m}$$

$$D = 10 \text{ ft} = 10 \times 0.305$$

$$D = 3.05 \text{ m}$$

Area of the antenna is

$$A_r = \frac{\pi D^2}{4}$$

Given, antenna efficiency $\eta_A = 0.65$. Hence, the effective area will be

$$A_e = \eta_A \cdot \frac{\pi D^2}{4}$$

$$A_e = 0.65 \cdot \frac{\pi(3.05)^2}{4}$$

$$A_e = 4.75 \text{ m}^2$$

The downlink operates at a frequency of 4 GHz. We have,

$$G_r = \frac{4\pi A_e}{\lambda^2}$$

$$\therefore \qquad G_r = \frac{4\pi\times 4.75}{(0.075)^2} = 10.612\times 10^3$$

$$[G_r] = 10 \log_{10}(10.612 \times 10^3)$$

$$[G_r] = 40.257 \text{ dB}$$

Step III: To find the system noise temperature:

$$T_s = T_{\text{ant}} + T_{\text{feed}} + \frac{T_{\text{LNA}}}{G_{\text{feed}}} + \frac{T_{\text{receiver}}}{G_{\text{feed}}\times G_{LNA}}$$

Given,

$$T_{\text{ant}} = 32°\text{K}$$

$$G_{\text{feed}} = 0.98$$

$$G_{\text{LNA}} = 50 \text{ dB} = 10^5$$

$$T_{\text{LNA}} = 140° \text{ K}$$

$$T_{\text{receiver}} = 261° \text{ K}$$

$\therefore$
$$T_s = 32 + 0 + \frac{140}{0.98} + \frac{261}{0.98 \times 10^5}$$

$$T_s = 32 + 142.85 + 2.66 \times 10^{-3} = 174.86$$

$$[T_s] = 10 \log_{10}(174.86)$$

$$[T_s] = 22.42 \text{ dBK}$$

Step IV: To find $\left[\frac{G}{T_s}\right]$ ratio at the receive:

$$[G_r] = 40.257 \text{ dB}$$

$\therefore$
$$\left[\frac{G_r}{T_s}\right] = [G_r] - [T_s]$$

$$\left[\frac{G_r}{T_s}\right] = 40.257 - 22.42$$

$$\left[\frac{G_r}{T_s}\right] = 17.83 \text{ dB/K}$$

Step V : To find $\frac{C}{N}$ ratio:

$$B = 30 \text{ MHz}$$

$$[B] = 10 \log_{10}(30 \times 10^6)$$

$$[B] = 74.77 \text{ dBHz}$$

$$k = 1.39 \times 10^{-23} \text{ J/K}$$

$$[k] = 10 \log_{10}(1.39 \times 10^{-23})$$

$$[k] = -228.57 \text{ dB}$$

Thus,
$$\left[\frac{C}{N}\right] = [\text{EIRP}] - [\text{Path loss}] + \left[\frac{G_r}{T_s}\right] - [k] - [B]$$

$$\left[\frac{C}{N}\right] = 36 - 196.41 + 17.83 - (-228.57) - 74.77$$

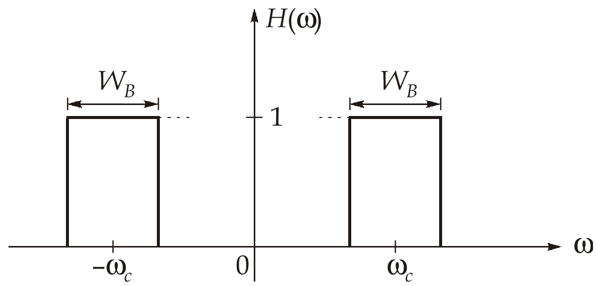
$\therefore$
$$\left[\frac{C}{N}\right] = 11.22 \text{ dB}$$

The carrier to noise ratio at demodulator input will be 11.22 dB.

Section B : Analog & Digital Communication Systems-1
Digital Circuit-2 + Microprocessors and Microcontroller-2

Q.5 (a) Solution:

(i) Frequency response of an ideal band-pass filter is depicted below,



From the figure, we have

$$\max |H(\omega)|^2 = 1$$

then
$$B_{eq} = \frac{1}{2\pi} \cdot \frac{\int_0^\infty |H(\omega)|^2 \, d\omega}{|H(\omega)|^2_{max}}$$

$$B_{eq} = \frac{1}{2\pi} \cdot \frac{W_B}{1} = \frac{W_B}{2\pi} \text{Hz}$$

Hence, Equivalent Noise Bandwidth (B_{eq}) of the ideal band-pass filter is given as $\frac{W_B}{2\pi}$.

For the low-pass RC filter, frequency response is expressed by,

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

then
$$\max |H(\omega)|^2 = |H(0)|^2 = 1$$

Hence,
$$B_{eq} = \frac{1}{2\pi} \int_0^\infty |H(\omega)|^2 \, d\omega$$

$$B_{eq} = \frac{1}{2\pi} \int_0^\infty \frac{1}{1 + (\omega RC)^2} d\omega = \frac{1}{2\pi} \cdot \frac{1}{RC} \left[\tan^{-1} \omega RC \Big|_0^\infty \right]$$

$$B_{eq} = \frac{1}{4RC} \text{Hz}$$

Hence, equivalent noise bandwidth (B_{eq}) of the low-pass RC filter is given as $\frac{1}{4RC}$ Hz.

(ii) For bipolar baseband signalling, the input signal is expressed as,

$$S_i(t)\begin{cases} S_1(t) = +A; & 0 \le t \le T \\ S_2(t) = -A; & 0 \le t \le T \end{cases}$$

Then, at the filter input, energy of the difference signal is given by,

$$E_d = \int_0^T [S_1(t) - S_2(t)]^2\, dt$$

$$E_d = \int_0^T (2A)^2 . dt$$

$$E_d = 4A^2T \qquad \text{...(1)}$$

For a baseband signalling scheme,

$$\text{Probability of error, } P_e = Q\left[\sqrt{\frac{E_d}{2\eta}}\right]$$

where $\eta/2$ is the PSD of noise at the filter input. Thus, from equation (1), we get

$$P_e = Q\left[\sqrt{\frac{4A^2T}{2\eta}}\right] = Q\left[\sqrt{\frac{2A^2T}{\eta}}\right]$$

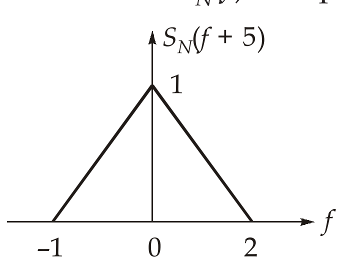
where $Q(\cdot)$ is complementary error function.

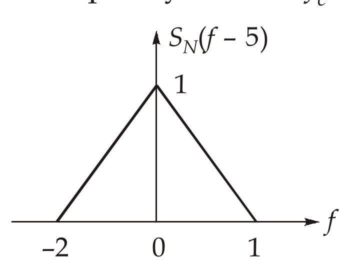
Q.5 (b) Solution:

1. For narrowband noise, the PSD of the in-phase (I) and quadrature (Q) components is given by the formula:

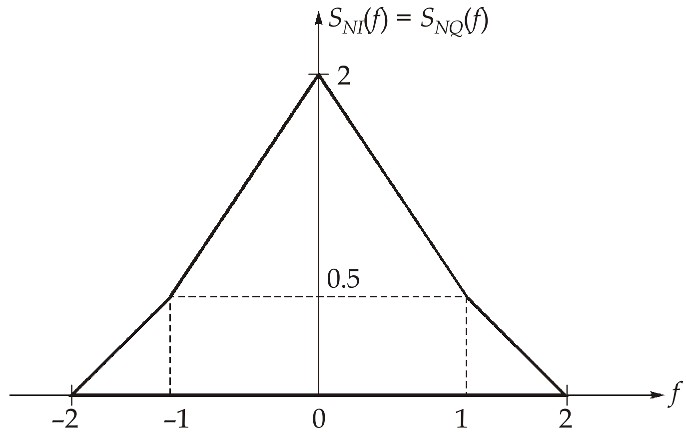
$$S_{NI}(f) = S_{NQ}(f) = \begin{cases} S_N(f + f_c) + S_N(f - f_c)\,; & -B \le f \le B \\ 0 & ;\ \text{otherwise} \end{cases}$$

where it is assumed that $S_N(f)$ occupies the frequency interval $f_c - B \le f \le f_c + B$.





So, $S_{NI}(f)$ and $S_{NQ}(f)$ can be drawn as:



2. The total power (P_n) is the area under the $S_N(f)$ curve. Mathematically:

$$P_n = \int_{-\infty}^{\infty} S_N(f)df$$

Since the PSD is symmetric, we calculate the area of one triangle and multiply by 2.

$$P_n = 2*\left[\frac{1}{2}\times 3\times 1\right]$$

$$P_n = 3 \text{ watts}$$

Hence, total power is 3 Watt.

3. The relationship between the PSD of the noise $S_N(f)$ and the cross-spectral density $S_{NI,\,NQ}(f)$ is given by:

$$S_{NI,NQ}(f) = \begin{cases} j[S_N(f+f_c) - S_N(f-f_c)] & ; -B \le f \le B \\ 0 & ; \text{ otherwise} \end{cases}$$

where $f_c = 5$ Hz

To find if the components are uncorrelated, we check if $S_{NI,NQ}(f) = 0$, which only happens if $S_N(f + f_c) = S_N(f - f_c)$. This requires the PSD to be locally symmetric about f_c.

The given PSD $S_N(f)$ is asymmetric about the carrier frequency f_c. Because of this asymmetry

$$S_N(f+5) \neq S_N(f-5)$$

Thus, $$S_{NI,\,NQ}(f) \neq 0$$

Hence, the in-phase and quadrature components are correlated (not uncorrelated)

Q.5 (c) Solution:

Given, output range of DAC: 0 to 10 V

For a n-bit DAC, output voltage,

$$V_0 = \frac{V_{ref}}{2^n}[2^{n-1}B_{n-1} + 2^{n-2}B_{n-1} + \ldots + 2B_1 + B_0]$$

$$V_0 = V_{ref}[2^{-1}B_{n-1} + 2^{-2}B_{n-2} + \ldots + 2^{-(n-1)}B_1 + 2^{-n}B_0]$$

where, B : Binary number

(i) Given input binary number, 10.

i.e., $B_1 = 1$, $B_0 = 0$

$$V_0 = 10\text{ V}\left[1\times\frac{1}{2}+0\times\frac{1}{4}\right] = 10\left[\frac{1}{2}\right] = 5\text{ V}$$

(ii) Given input binary number, 0110

i.e., $B_3 = 0$, $B_2 = 1$, $B_1 = 1$, $B_0 = 0$

$$V_0 = 10\left[0\times\frac{1}{2}+1\times\frac{1}{2^2}+1\times\frac{1}{2^3}+0\times\frac{1}{2^4}\right] = 10\left[\frac{1}{4}+\frac{1}{8}\right]$$

$$\therefore \quad V_0 = 3.75\text{ V}$$

(iii) Given input binary code, 10111100

i.e., $B_5 = 1$, $B_6 = 0$, $B_5 = B_4 = B_3 = B_2 = 1$, $B_7 = B_8 = 0$

$$\text{output voltage, } V_0 = 10\text{ V}\left[1\times\frac{1}{2}+0\times\frac{1}{2^2}+1\times\frac{1}{2^3}+1\times\frac{1}{2^4}+1\times\frac{1}{2^5}\right.$$

$$\left.+1\times\frac{1}{2^6}+0\times\frac{1}{2^7}+0\times\frac{1}{2^8}\right]$$

$$= 10\text{ V}\left[\frac{1}{2}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}\right]$$

$$V_0 = 7.34\text{ V}$$

Q.5 (d) Solution:

Drawing the excitation table of J-K flip flop, we get,

Q_n	J	K	Q_{n+1}
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Thus the state table can be drawn as,

Q_n	J	K	Q_{n+1}	D
0	0	0	0	0
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

Thus, the *K*-map to obtain boolean expression for '*D*' can be drawn as

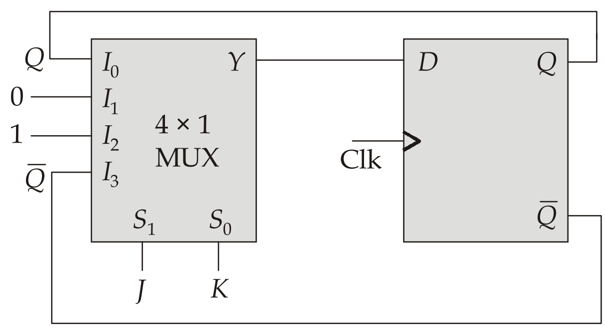
Q_n \ JK	00	01	11	10
0	0	0	1	1
1	1	0	0	1

$\therefore$ $$D = J\bar{Q}_n + \bar{K}Q_n$$

Now, we need to implement the above equation using a 4 × 1 MUX.

Let the 4 × 1 MUX have select lines equal to J and K.

	I_0	I_1	I_2	I_3
JK	00	01	10	11
$\bar{Q}$	0	0	(1)	(1)
Q	(1)	0	(1)	0
	Q	0	1	$\bar{Q}$



Q.5 (e) Solution:

Given, $F_1 = \Sigma m(2, 4, 5, 10, 12, 13, 14)$

By using 4-variable K-map,

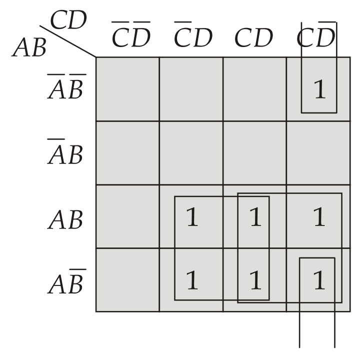
$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				1
$\bar{A}B$	1	1		
AB	1	1		1
$A\bar{B}$				1

The minimized expression for F_1 is

$$F_1 = \bar{B}C\bar{D} + B\bar{C} + AC\bar{D}$$

Consider $F_2 = \Sigma m(2, 9, 10, 11, 13, 14, 15)$

By using 4-variable K-map,



$$F_2 = AD + AC + \overline{B}C\overline{D}$$

For PLA implementation of F_1 and F_2, the required product terms are $B\overline{C}, AC\overline{D}, \overline{B}C\overline{D}, AD, AC$.

PLA programming table:

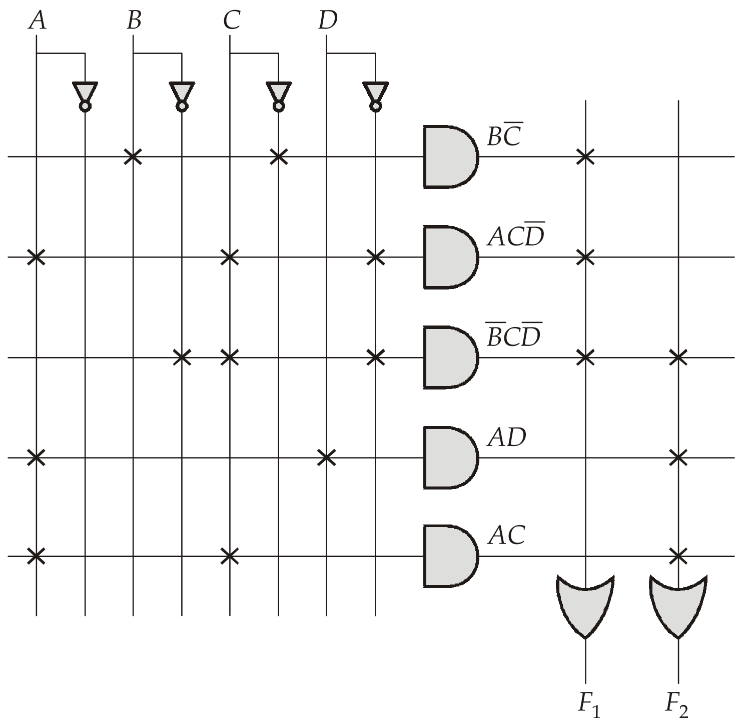
Product term	**A**	**B**	**C**	**D**	F_1	F_2
$P1 = B\overline{C}$	–	1	0	–	1	0
$P2 = AC\overline{D}$	1	–	1	0	1	0
$P3 = \overline{B}C\overline{D}$	–	0	1	0	1	1
$P4 = AD$	1	–	–	1	0	1
$P5 = AC$	1	–	1	–	0	1

∴ PLA equations are

$$F1 = P1 + P2 + P3 = B\overline{C} + AC\overline{D} + \overline{B}C\overline{D}$$

$$F2 = P3 + P4 + P5 = \overline{B}C\overline{D} + AD + AC$$

Implementation of PLA,



Q.6 (a) Solution:

(i) The impulse response of the filter matched to $s(t)$ is

$$h(t) = s(T - t) = s(3 - t) = s(t)$$

Thus, $h(t)$ has the same plot as $s(t)$.

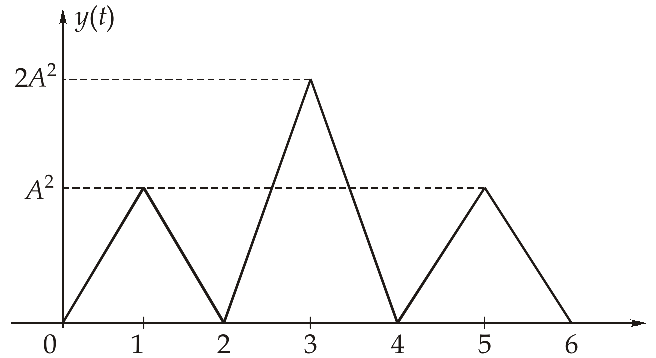
(ii) The output of the matched filter is

$$y(t) = h(t) * s(t) = s(t) * s(t)$$

$$= \int_{-\infty}^{\infty} s(\tau)s(t-\tau)d\tau$$

$$y(t) = \begin{cases} 0 & ; \quad t<0 \\ A^2 t & ; \ 0 \le t < 1 \\ A^2(2-t) & ; \ 1 \le t < 2 \\ 2A^2(t-2) & ; \ 2 \le t < 3 \\ 2A^2(4-t) & ; \ 3 \le t < 4 \\ A^2(t-4) & ; \ 4 \le t < 5 \\ A^2(6-t) & ; \ 5 \le t < 6 \\ 0 & ; \quad t \ge 6 \end{cases}$$

A sketch of $y(t)$ is depicted as shown below:



(iii) At the output of the matched filter and for $t = T = 3$, the noise is

$$n_T = \int_{-\infty}^{\infty} n(\tau)h(t-t)d\tau$$

$$= \int_{-\infty}^{\infty} n(\tau)s(T-(T-\tau))d\tau$$

$$= \int_{0}^{T} n(\tau)s(\tau)d\tau$$

The variance of the noise is

$$\sigma_{nT}^2 = E\left[\int_0^T\int_0^T n(\tau)n(V)s(\tau)s(V)d\tau\, dV\right]$$

$$= \int_0^T\int_0^T s(\tau)s(V)E[n(\tau)n(V)]d\tau\, dV$$

$$= \frac{N_0}{2}\int_0^T\int_0^T s(\tau)s(V)\delta(\tau-V)d\tau\, dV$$

$$= \frac{N_0}{2}\int_0^T s^2(\tau)\cdot d\tau = \frac{N_0}{3}\times 2A^2$$

$$\sigma_{nT}^2 = N_0 A^2$$

(iv) For antipodal equiprobable signals and using matched filter, the probability of error is

$$P(e) = Q\left(\sqrt{\frac{2E}{N_0}}\right) = Q\left[\sqrt{\left(\frac{S}{N}\right)_{0\max}}\right]$$

where, $\left(\frac{S}{N}\right)_{0\,\max}$ is the maximum output SNR from the matched filter. Since

$$\left(\frac{S}{N}\right)_{0\max} = \frac{y^2(t)\big|_{\max}}{E\left[n_T^2\right]} = \frac{4A^4}{N_0 A^2}$$

$$\left(\frac{S}{N}\right)_{0\max} = \frac{4A^2}{N_0}$$

$$P(e) = Q\left[\sqrt{\frac{4A^2}{N_0}}\right]$$

Q.6 (b) Solution:

(i) **1.** Given $\Delta f = 3(\text{BW})_{\text{AM}} = 3 \times 2f_m = 6f_m$. Thus, for FM signal,

$$m_f = \beta = \frac{\Delta f}{f_m} = \frac{6f_m}{f_m} = 6$$

2. Let A_1 be the peak amplitude of the carrier in the AM system and A_2 be peak amplitude of the carrier in the FM system.

Total power in AM system = $\frac{A_1^2}{2} + \frac{1}{4}\mu^2 A_1^2$

where μ is the modulation index.

Total power in FM system = $\frac{A_2^2}{2}$

Magnitude of the first sideband in FM system is $A_2|J_1(\beta)| = A_2|J_1(6)|$

Magnitude of the first sideband in AM system is $\frac{\mu}{2}A_1$

Given: (i) $$\frac{A_2^2}{2} = \frac{A_1^2}{2} + \frac{\mu^2 A_1^2}{4}$$

(ii) $$A_2|J_1(6)| = \frac{\mu}{2}A_1$$

$$J_1(6) = -0.28 \quad \text{(Given)}$$

(i) and (ii) can be solved simultaneously to yield

$$\mu = \frac{2|J_1(6)|}{\sqrt{1-2J_1^2(6)}} = 0.61$$

(ii) Let τ is any positive time interval, then

$$R_{XX}(t, t+\tau) = E[X(t)\, X(t+\tau)]$$

$= A^2 P\,[X(t)$ and $X(t+\tau)$ have same signs$] + (-A^2)\, P[X(t)$ and $X(t+\tau)$ have different signs$]$

$$= A^2 P\,[z \text{ is even in } (t, t+\tau)] - A^2 P\,[z \text{ is odd in } (t, t+\tau)]$$

$$= A^2 \sum_{k,\text{even}} e^{-\alpha\tau}\frac{(\alpha\tau)^k}{k!} - A^2 \sum_{k\,\text{odd}} e^{-\alpha\tau}\frac{(\alpha\tau)^k}{k!}$$

$$= A^2 e^{-\alpha\tau}\sum_{k=0}^{\infty}\frac{(\alpha\tau)^k}{k!}(-1)^k$$

$$R_{XX}(t, t+\tau) = A^2 e^{-\alpha\tau}\sum_{k=0}^{\infty}\frac{(-\alpha\tau)^k}{k!} = A^2 e^{-\alpha\tau}e^{-\alpha\tau} = A^2 e^{-2\alpha\tau}$$

The complete solution that includes $\tau < 0$ also is

$$R_{XX}(\tau) = A^2 e^{-2\alpha|\tau|}$$

Taking Fourier transform on both sides, we get the power spectral density of $X(t)$ as

$$S_{XX}(\omega) = A^2 \cdot \frac{4\alpha}{\omega^2 + (2\alpha)^2}$$

Q.6 (c) Solution:

Given, 6-bit DAC

$$\text{Full scale output, } V_{FS} = 1.260 \text{ V}$$

$$\text{Accuracy} = \pm 0.1\% \text{ of Full scale}$$

$$\text{Offset error} = \pm 1 \text{ mV}$$

$$\text{Step size, } \Delta = \frac{V_{FS}}{2^n - 1}$$

$$= \frac{1.260}{2^6 - 1} = \frac{1.260}{63}$$

$$\therefore \quad \Delta = 0.02 \text{ V} = 20 \text{ mV}$$

$$\text{Accuracy} = \pm 0.1\% \text{ of full scale}$$

$$= \pm\frac{0.1}{100}[1.260]$$

$$= 0.00126 \text{ V}$$

$$= 1.26 \text{ mV}$$

$$\text{Total error} = \pm(\text{Offset error} + \text{Accuracy})$$
$$= \pm(1 + 1.26) \text{ mV}$$
$$= \pm 2.26 \text{ mV}$$

∴ Maximum error possible $= \pm 2.26$ mV

We have,

$$\text{ideal DAC output, } V_{\text{ideal}} = \frac{\text{Decimal equivalent of digital code}}{2^n - 1} \times V_{FS}$$
$$= \frac{\text{Decimal equivalent of digital code}}{2^6 - 1} \times 1.26 \text{ V}$$
$$V_{\text{ideal}} = \frac{\text{Decimal equivalent of digital code}}{63} \times 1.26 \text{ V}$$

For the measurement to be within the specification,

$$|V_{\text{measured}} - V_{\text{ideal}}| \leq \text{Maximum allowable error}$$

i.e.,
$$|V_{\text{measured}} - V_{\text{ideal}}| \leq 2.26 \text{ mV}$$

(i) For the given digital code 000010,

$$V_{\text{measured}} = 41.5 \text{ mV}$$
$$V_{\text{ideal}} = \frac{(000010)_2}{63} \times 1.26 = \frac{2}{63} \times 1.26 = 40 \text{ mV}$$

Since $|41.5 \text{ mV} - 40 \text{ mV}| = 1.5 \text{ mV} < 2.26 \text{ mV}$

∴ The measurement corresponding to digital code 000010 is within the given DAC's specification.

(ii) For the given digital code input 000111,

$$V_{\text{measured}} = 140.2 \text{ mV}$$
$$V_{\text{ideal}} = \frac{(000111)_2}{63} \times 1.26$$
$$= \frac{7}{63} \times 1.26$$
$$= 140 \text{ mV}$$

Since $|140.2 \text{ mV} - 140 \text{ mV}| = 0.2 \text{ mV} < 2.26 \text{ mV}$

∴ The measurement corresponding to digital code 000111 is within the given DAC's specification.

(iii) For the given digital code input 001100,

$$V_{\text{measured}} = 242.5 \text{ mV}$$

$$V_{\text{ideal}} = \frac{(001100)_2}{63} \times 1.26$$

$$= \frac{12}{63} \times 1.26$$

$$= 240 \text{ mV}$$

Since $|242.5 \text{ mV} - 240 \text{ mV}| = 2.5 \text{ mV} > 2.26 \text{ mV}$

Thus, the measurement corresponding to the digital code 001100 is not within the DAC's specification.

(iv) For the given digital code input 111111,

$$V_{\text{measured}} = 1.258 \text{ mV}$$

$$V_{\text{ideal}} = \frac{(111111)_2}{63} \times 1.26$$

$$= \frac{63}{63} \times 1.26$$

$$= 1.26 \text{ V}$$

$$\therefore \quad |V_{\text{measured}} - V_{\text{ideal}}| = |1.258 \text{ V} - 1.260 \text{ V}|$$

$$= 0.002 \text{ V}$$

Since $2 \text{ mV} < 2.26 \text{ mV}$

Hence, the measurement corresponding to the digital code 111111 is within the DAC's specification.

Q.7 (a) Solution:

(i) 1. Let L denotes inductive component and C denotes capacitive component of each varactor diode due to bias voltage V_b alone, thus

$$C_0 = 100 \text{ V}^{-1/2} \text{ pF}$$

The capacitance of varactor diodes is in series, thus frequency of oscillations is

$$f_0 = \frac{1}{2\pi\sqrt{L\left(C + \frac{C_0}{2}\right)}}$$

So,
$$10^6 = \frac{1}{2\pi\sqrt{200\times10^{-6}\left(100\times10^{-12} + 50V_b^{-1/2}\times10^{-12}\right)}}$$

So,
$$V_b = 3.52 \text{ volts}$$

2. Frequency multiplication ratio is 64, so the modulation frequency is also multiplied by 64. Thus, the modulation index of FM signal at the output of frequency multiplier is

$$\beta = \frac{5}{64} = 0.078$$

This indicates that it is NBFM. Which means that amplitude A_m is small compared to V_b.

The instantaneous frequency can be written as

$$f_i(t) = \frac{1}{2\pi\sqrt{L\left(C+\frac{C_0}{2}\right)}}$$

$$f_i(t) = \frac{1}{2\pi}\left[200\times10^{-6}\{(100\times10^{-12})+50\times10^{-12}(3.52+A_m\sin 2\pi f_m t\})^{-1/2}\right]^{-1/2}$$

$$= \frac{10^7}{2\sqrt{2}\pi}\left(1+0.266\left(1+\frac{A_m}{3.52}\sin 2\pi f_m t\right)^{-1/2}\right)^{-1/2}$$

$$= \frac{10^7}{2\sqrt{2}\pi}\left(1+0.266\left(1-\frac{A_m}{7.04}\sin 2\pi f_m t\right)\right)^{-1/2}$$

$$= \frac{10^7}{2\sqrt{2}\pi}(1.266-0.038A_m\sin 2\pi f_m t)^{-1/2}$$

$$= 10^6[1-0.03A_m\sin 2\pi f_m t]^{-1/2}$$

$$= 10^6[1 + 0.015\, A_m \sin 2\,\pi f_m t]$$

$$= 10^6 + 0.015 \times 10^6\, A_m \sin 2\pi f_m t$$

We have

$$\beta = 0.078$$

So, $\Delta f = \beta \cdot f_m$

$$= 0.078 \times 10^4 \text{ Hz} = 780 \text{ Hz}$$

Thus, we have

$$\Delta f = 0.015 \times 10^6 A_m \quad \Rightarrow \quad 780 = 0.015 \times 10^6 A_m$$

So, $A_m = 52 \times 10^{-3}$ Volts

(ii) **1.** The standard PM signal is given by

$$x_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$

$$= 10\cos[2\pi(10^6)t + 10m(t)] \qquad [\text{Given, } k_p = 10 \text{ rad/V}]$$

Given, $$x_{PM}(t) = 10\cos[2\pi(10^6)t + 0.1\sin(10^3)\pi t]$$

On comparing, we get

$$m(t) = 0.01\sin(10^3)\pi t$$

2. The standard FM signal is given by

$$x_{FM}(t) = A\cos\left[\omega_c t + k_f\int_{-\infty}^{t} m(\lambda)d\lambda\right] \qquad [\text{Given, } k_p = 10\pi \text{ rad/V}]$$

Further, $$X_{FM}(t) = 10\cos[2\pi(10^6)t + 0.1\sin(10^3)\pi t]$$

Assuming $$m(t) = a_m\cos(10^3)\pi t$$

we get, $$10\pi\int_{-\infty}^{t} m(\lambda)d\lambda = 10\pi a_m\int_{-\infty}^{t}\cos(10^3)\pi\lambda\, d\lambda$$

$$= \frac{a_m}{100}\sin(10^3)\pi t = 0.1\sin(10^3)\pi t$$

thus, $$a_m = 10 \text{ and } m(t) = 10\cos(10^3)\pi t$$

Q.7 (b) Solution:

(i) Given port addresses:

PORT A = 40 H

PORT B = 41 H

PORT C = 42 H

CONTROL REGISTER = 43 H

Fire detector is connected to port A (PA_0), hence it acts as input port.

Fire alarm is connected to port C (PC_5), hence it acts as output port.

Considering Mode 0 operation,

8255 control word format (Mode 0)

D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0
1	M2	M1	PA	PC0	MB	PB	PC_L
= 1 (I/O mode)	00 (Mode 0)		= 1 (input port)	= 0 (Port C upper as output port)	= 0 (output port)	Assume = 0 (output port)	= 0 (Port C lower as output port)

∴ Control word = 10010000_2 = 90 H

(ii) Assembly language program:

$PA_0 = 1$ means fire detected, hence turn ON alarm (set $PC_5 = 1$)

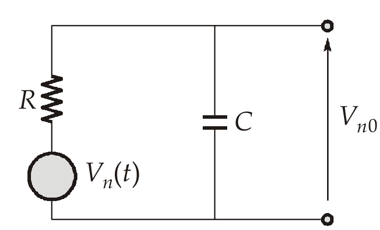
$PA_0 = 0$ means No fire, hence turn OFF alarm. (clear PC_5)

```
        ORG 2000 H
        MVI A, 90 H     ;  initialize 8255
        OUT 43 H        ;  Send control word
START:
        IN 40 H         ;  Read port A
        ANI 01 H        ;  Mask PA0
        JZ NO_FIRE      ;  If Z = 0  ⇒  no fire
                        ;  else FIRE DETECTED
        MVI A, 20 H     ;  00100 000 → set PC5 = 1
        OUT 42 H        ;  Send to port C
        JMP START
NO_FIRE:
        OUT A, 00 H     ;  Clear port C
        OUT 42 H
        JMP START
        END
```

Q.7 (c) Solution:

(i)

$$\frac{V_{n0}}{V_n} = H(j\omega) = \frac{1}{1 + j\omega RC}$$



The mean square value of the output noise voltage appearing across the terminals of parallel RC combination is given by

$$\overline{V_{n0}^{\,2}} = P_0 = \frac{KT}{C}$$

$$= \frac{1.38 \times 10^{-23} \times 300}{100 \times 10^{-12}} = 4.14 \times 10^{-11}$$

So, $V_{n0} = \sqrt{4.14 \times 10^{-11}} = 0.6434 \times 10^{-5}$ V = 6.434 μV

The two-sided PSD of thermal noise given by $S_n(\omega) = 2KTR$ volt^2/Hz . The output noise power spectral density is given by $S_n(\omega)\,|H(\omega)|^2$.

Now, let B_N be the noise bandwidth.

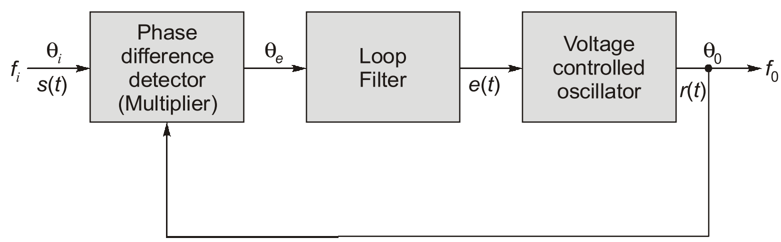
$\Rightarrow$ $2 \times B_N$ (Peak value of output power spectral density) $= P_0$

$$\Rightarrow 2B_N(2KTR)\left|(H(j\omega))_{\max}\right|^2 = P_0$$

$$\Rightarrow \quad 2B_N(2\,KTR)\,(1) = \frac{KT}{C}$$

$$\Rightarrow \quad B_N = \frac{1}{4\,RC} = \frac{1}{4\times 100\times 10^3\times 100\times 10^{-12}} = 25\text{ kHz}$$

(ii) The given PLL circuit can be drawn as below:



1. The noiseless PSK signal is given by,

$$s(t) = A_0\cos(2\pi f_c t + k_p m(t)]$$
$$= A_0\cos(2\pi f_c t)\cos[k_p m(t)] - A_c\sin(2\pi f_c t)\sin[k_p m(t)]$$

Since, $m(t) = \pm 1$, it follows that,

$$\cos[k_p m(t)] = \cos(\pm k_p) = \cos(k_p)$$
$$\sin[k_p m(t)] = \sin(\pm k_p) = \pm\sin(k_p) = m(t)\sin(k_p)$$

Therefore, $s(t) = A_c\cos(k_p)\cos(2\pi f_c t) - A_c m(t)\sin(k_p)\sin(2\pi f_c t)$

The VCO output is

$$r(t) = A_v\sin[2\pi f_c t + \theta(t)]$$

The multiplier output is therefore,

$$r(t)\,s(t) = \frac{1}{2}A_c A_v\cos(k_p)\{\sin[(\theta(t)] + \sin(4\pi f_c t + \theta(t)]\}$$
$$= -\frac{1}{2}A_c A_v\, m(t)\sin(k_p)\{\cos[(\theta(t)] - \cos(4\pi f_c t + \theta(t)]\}$$

The loop filter removes the components with frequency $2f_c$, producing the output,

$$e(t) = \frac{1}{2}A_c A_v\cos(k_p)\sin[\theta(t)] - \frac{1}{2}A_c A_v\, m(t)\sin(k_p)\cos[\theta(t)]$$

Note that if $k_p = \frac{\pi}{2}$, (i.e., the carrier is fully deviated), there would be no carrier.

2. Since the error signal tends to drive the loop into phase locked state (i.e., $\theta(t)$ approaches zero), the loop filter output reduces to

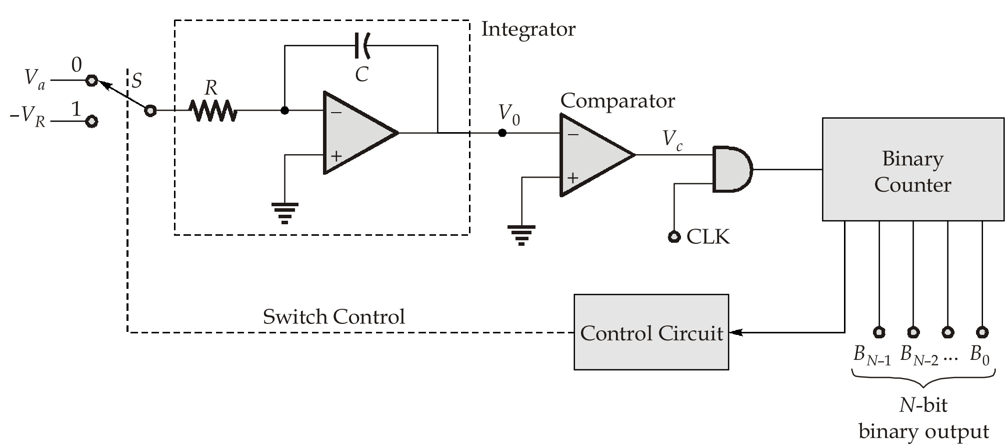
$$e(t) = -\frac{1}{2}A_c A_v \sin(k_p)\, m(t)$$

which is proportional to the desired data signal $m(t)$. Hence, the phase-locked loop may be used to recover $m(t)$.

Q.8 (a) Solution:

(i) Dual-Slope Integrating Type ADC

- The dual-slope ADC has one of the slowest conversion time (typically 10 to 100 ms) but has the advantage of relatively low cost because it does not require precision components such as DAC or VCO, and also they are insusceptible to noise and parameter variations due to the change in temperature.
- The block diagram of a dual-slope integrating type ADC has four major building blocks:
  (i) An integrater
  (ii) A voltage comparator
  (iii) A binary counter
  (iv) A switching/control circuit

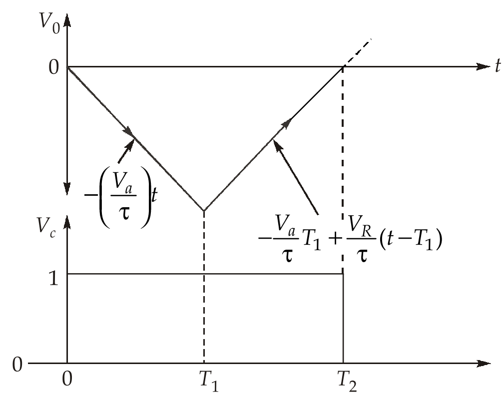


- The basic operation of this ADC involves the linear charging and discharging of capacitor 'C', using constant currents.
- The conversion process starts at $t = 0$, when switch 'S' is in position '0', connecting the analog input V_a to the input of the integrator. The output of the integrator is,

$$V_0 = -\frac{1}{\tau}\int_0^t V_a dt = -\left(\frac{V_a}{\tau}\right)t$$

- This results in HIGH V_c thus, enabling the AND gate and the clock pulses reach the CLK input terminal of the counter which was initially clear. The counter counts from 00 .. 00 to 111 ... 11 when $2^N - 1$ clock pulses are applied. At the next clock pulse 2^N, the counter is cleared and Q becomes 1. This controls the state of S_1 which now moves to position 1 at T_1, thereby connecting $-V_R$ to the input of the integrator. The output of the integrator now starts to move in the positive direction. The counter continues to count until $V_0 < 0$. As soon as V_0 goes positive at T_2, V_C goes LOW disabling the AND gate. The counter will stop counting in the absence of the CLK pulses.

Waveforms of Dual-slope ADC



Since, $T_1 = 2^N \times T_C$

where, T_C = time period of the CLK pulse.

When, S_1 is at '1' then, $V_0 = -\frac{V_a}{\tau}T_1 + \frac{V_R}{\tau}(t - T_1)$

At $t = T_2$, $V_0 = 0$

Thus, $$(T_2 - T_1)\frac{V_R}{\tau} = \frac{V_a}{\tau} \times T_1$$

$$T_2 - T_1 = \frac{V_a}{V_R} \cdot T_1 = \frac{V_a}{V_R} \cdot 2^N T_C \quad \text{...(i)}$$

Let the count recorded in the counter be 'n' at T_2.

Then, $$T_2 - T_1 = n \times T_C \quad \text{...(ii)}$$

From (i) and (ii) we get,

$$n \times T_C = \frac{V_a}{V_R} 2^N T_C$$

$$\Rightarrow \quad n = \frac{V_a}{V_R} \cdot 2^N$$

Conversion Time (t_c)

Maximum total conversion time for an N-bit ADC is given by,

$$t_{c(\max)} = T_1 + T_2 = (2^N + n_{\max})T_C$$

Since $|V_r| > |V_a|$, $n_{\max} = 2^N$. Thus,

$$t_{c(\max)} = (2^{n+1}) \text{ CLK cycles}$$

It is most accurate but slowest ADC. It is often used in "digital voltmeter" because of its good conversion accuracy and low cost.

(ii) For a dual slope converter, we have

$$\text{Number of bits '}n\text{'} = 4$$

$$\text{Clock rate} = 3.2 \text{ kHz}$$

For a dual slope integrating ADC, the total conversion time for n-bit is given as

$$t = 2^n T_c + \frac{V_a}{V_R} \cdot 2^n T_c$$

The maximum analog voltage that can be applied is

$$V_{0(\max)} = V_R$$

$$\therefore \quad t = 2^{n+1} T_c$$

Thus,

$$f \le \frac{1}{t} = \frac{1}{2^{n+1}} \cdot \frac{1}{T_c} = \frac{3.2 \times 10^3}{2^5} = 100 \text{ Hz}$$

$\therefore$ Maximum sampling rate,

$$f_{\max} = 100 \text{ Hz}$$

Q.8 (b) Solution:

Given 8085 microprocessor is interfaced with 8255 (PPI) to drive 7-Segment Display from the given figure.

$PA_0 - PA_6$ (i.e., PORT A) of 8255 are connected to 7-segment display LEDs (a-g).

So, PORT A must be programmed as OUTPUT. Therefore, 8255 should operate in I/O Mode and may be configured for Mode 0 with all ports acting as output.

$\therefore$ Control Word would be as below:

D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	
1	0	0	0	0	0	0	0	$\rightarrow$ 80 H

7-Segment Code Table:

Digit	a	b	c	d	e	f	g	Hex code
0	1	1	1	1	1	1	0	7E H
1	0	1	1	0	0	0	0	30 H
2	1	1	0	1	1	0	1	6D H
3	1	1	1	1	0	0	1	79 H
4	0	1	1	0	0	1	1	33 H
5	1	0	1	1	0	1	1	5B H
6	1	0	1	1	1	1	1	5F H
7	1	1	1	0	0	0	0	70 H
8	1	1	1	1	1	1	1	7F H
9	1	1	1	1	0	1	1	7B H

Let us assume the last six digits of mobile number to be displayed on 7-segment display is 123456. Corresponding hexadecimal values will be 30 H, 6D H, 79 H, 33 H, 5B H, 5F H

Assembly language program:

```
ORG     2000 H
        MVI     A, 80 H    ; Control Word-Mode 0
        OUT     03 H       ; Send to control register
START:
        MVI     A, 30 H
        OUT     00 H       ; Display First Digit
        CALL    DELAY
        MVI     A, 6D H
        OUT     00 H       ; Display Second Digit
        CALL    DELAY
        MVI     A, 79 H
        OUT     00 H       ; Display Third Digit
        CALL    DELAY
        MVI     A, 33 H    ; Display Fourth Digit
        OUT     00 H
        CALL    DELAY
        MVI     A, 5B H
        OUT     00 H       ; Display Fifth Digit
        CALL    DELAY
        MVI     A, 5F H
```

```
        OUT    00 H     ; Display Sixth Digit
        CALL   DELAY
        JMP    START
DELAY:
        MVI    B, FF H
L1:     MVI    C, FF H
L2:     DCR    C
        JNZ    L2
        DCR    B
        JNZ    L1
        RET
        END
```

Q.8 (c) Solution:

(i) Given,

$$f_C = 555 \text{ KHz}$$
$$f_{LO} = 1010 \text{ KHz}$$
$$f_{IF} = \text{Intermediate frequency}$$
$$= f_{LO} - f_C = 1010 - 555 = 455 \text{ kHz}$$

Now,

$$f_i = \text{Image frequency}$$
$$= f_{LO} + f_{IF} = 1010 + 455 = 1465 \text{ kHz}$$
$$Q = 40$$

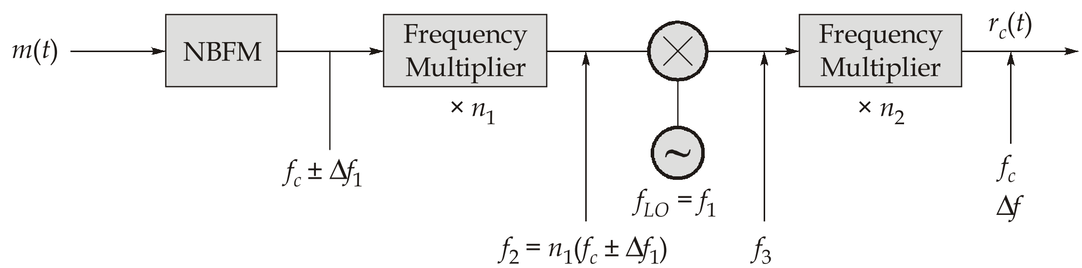
Rejection ratio, $$\alpha_{IRR} = \sqrt{1+Q^2\left(\frac{f_i}{f_C}-\frac{f_C}{f_i}\right)^2}$$

$$= \sqrt{1+1600\left(\frac{1465}{555}-\frac{555}{1465}\right)^2} = 90.4375$$

Express it in dB IRR (dB) $= 20\log_{10}(\alpha_{IRR})$

$$= 20\log_{10} 90.43$$
$$= 39.12 \text{ dB}$$

(ii) Armstrong modulator generate a narrow band FM wave and then use the frequency multipliers and mixer to obtain the required values of frequency deviation, carrier and modulation index.The block diagram for the Armstrong method for the generation of FM signal is as below:



Here, $$f_2 = n_1(f_c \pm \Delta f_1)$$

$$\Delta f_1 = \beta f_m = 0.2 \times 50 = 10 \text{ Hz}$$

(considering f_m = 50 Hz as it requires maximum multiplication factor)

$$\frac{\Delta f}{\Delta f_1} = \frac{80\times10^3}{10} = 8000 = n_1 n_2$$

$$f_2 = n_1 f_1 = n_1\, 2 \times 10^5 \text{ Hz} \quad ...(i)$$

Assuming down conversion, we have,

$$f_2 - f_{LO} = \frac{f_c}{n_2}$$

Thus, $$f_{LO} = n_1 f_1 - \frac{f_c}{n_2}$$

$$= \frac{8000\times2\times10^5 - 108\times10^6}{n_2} = \frac{1492}{n_2}\times10^6 \text{ Hz}$$

Let, $$n_2 = 200, \text{ we obtain}$$

$$n_1 = \frac{8000}{200} = 40$$

and $$f_{LO} = \frac{1492}{200} = 7.46 \text{ MHz}$$

❍❍❍❍