MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2026 Mains Test Series

Mechanical Engineering Test No : 4

Section A : Theory of Machines **[All Topics]**

Section B : Fluid Mechanics & Turbo Machinery-1 **[Part Syllabus]**

Strength of Materials & Mechanics-2 **[Part Syllabus]**

Section A : Theory of Machines

1. (a) Solution:

Given : $m = 10$ mm; $\alpha = 18°$; $z_1 = 20$; $z_2 = 40$

$$r_1 = \frac{mz_1}{2} = \frac{10 \times 20}{2} = 100 \text{ mm}$$

$$r_2 = \frac{mz_2}{2} = \frac{10 \times 40}{2} = 200 \text{ mm}$$

Let, the pinion be the driver

Maximum possible length of approach = $r_1 \sin\alpha$

$$= 100 \sin 18° = 30.901 \text{ mm}$$

$$\text{Actual length of approach} = \sqrt{R_{a_2}^2 - (200\cos 18°)} - 200\sin 18°$$

$$= \sqrt{R_{a_2}^2 - (190.211)^2} - 61.803$$

Also,

Actual length of approach = 0.6 × maximum possible length of approach

$$\sqrt{R_{a_2}^2 - (190.211)^2} - 61.803 = 0.6 \times 30.901$$

$$R_{a_2} = 206.483 \text{ mm}$$

Addendum for gear, $A_2 = R_{a_2} - r_2$

$$A_2 = 206.483 - 200$$

$$A_2 = 6.483 \text{ mm}$$

Maximum possible length of recess $= r_2 \sin\alpha$

$$= 200 \sin 18°$$

$$= 61.803 \text{ mm}$$

Actual length of recess $= \sqrt{r_{a_1}^2 - (r_1 \cos\alpha)^2} - r_1 \sin\alpha$

$$= \sqrt{r_{a_1}^2 - (100 \cos 18°)^2} - 100 \sin 18°$$

$$= \sqrt{r_{a_1}^2 - (95.105)^2} - 30.901$$

Also, Actual length of recess = 0.6 × Maximum possible length of recess

$$\sqrt{r_a^2 - (95.105)^2} - 30.901 = 0.6 \times 61.803$$

$$r_{a_1} = 116.904 \text{ mm}$$

Addendum for pinion, $A_1 = r_{a_1} - r_1$

$$= 116.904 - 100 = 16.904 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\alpha} = \frac{0.6 \times (r_1 + r_2) \sin\alpha}{\cos\alpha}$$

$$= 0.6 \times (100 + 200) \times \tan 18°$$

$$= 58.49 \text{ mm}$$

1. (b) Solution:

Given data : s_{ut} = 225 MPa, d = 40 mm; t = 30 mm; l = 90 mm; v = 15 m/s; k_e = 0.03

Punching force, $F = \pi d t s_{ut}$

$$= \pi \times 40 \times 30 \times 225$$

$$= 848230.016 \text{ N}$$

$$\text{Punching time per hole} = \frac{60}{6} = 10 \text{ sec}$$

Energy required in punching one hole,

$$E_1 = \frac{1}{2} \times F \times t$$

$$E_1 = \frac{1}{2} \times 848230.016 \times 0.03 = 12723.45 \text{ Nm}$$

$$\text{Power required} = \frac{E_1}{\text{Punching time}} = \frac{12723.45}{10} = 1272.34 \text{ Watt}$$

The punch travels a total distance of 2 × 90 = 180 mm in 10 sec

Time required to punch a hole in 30 mm thick plate = $\frac{10 \times 30}{180} = 1.666$ sec

Energy required to be supplied by motor in 10 sec = 12723.45 Nm

Energy supplied by the motor in 1.666 sec = $\frac{12723.45}{10} \times 1.666 = 2119.726$ Nm

Energy supplied by flywheel, E_f = 12723.45 – 2119.726 = 10603.724 Nm

Now, coefficient of fluctuation of energy

$$K_e = \frac{E_f \text{(Energy fluctuation)}}{E\text{(Mean energy)}}$$

$$E = \frac{E_f}{k_e} = \frac{10603.724}{0.03}$$

$$E = 353457.46 \text{ Nm}$$

If M is the mass of flywheel, then,

$$\frac{1}{2} M V_{mean}^2 = E$$

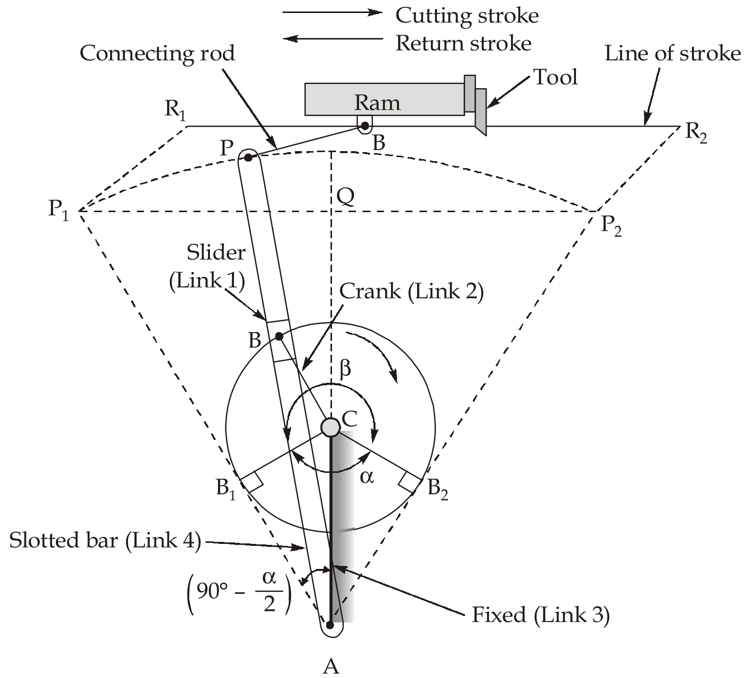
$$\frac{1}{2} M \times 15^2 = 353451.46$$

$$M = 3141.84 \text{ kg}$$ **Ans.**

1. (c) Solution:

Given : Quick return ratio, QRR = 2

Length of stroke, L = 280 mm



Crank and slotted lever quick return motion mechanism

$$\frac{[V_{s,\max}]_{return}}{[V_{s,\max}]_{forward}} = \frac{c+r}{c-r} = 2$$

where, c = centre distance between fixed centre; r = crank radius

$$\Rightarrow \quad c + r = 2c - 2r$$

$$\Rightarrow \quad c = 3r \quad \text{...(i)}$$

$$\Rightarrow \quad \cos\left(\frac{\alpha}{2}\right) = \frac{r}{c} = \frac{r}{3r} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{\alpha}{2} = \cos^{-1}\left(\frac{1}{3}\right) = 70.528°$$

$$\Rightarrow \quad \alpha = 141.057°$$

$$\therefore \quad QRR = \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{360° - \alpha}{\alpha}$$

$$\Rightarrow \quad = \frac{360° - 141.057°}{141.057°} = 1.552$$

$$\Rightarrow \quad \alpha = 120° \text{ and } \beta = 360° - 120° = 240°$$

$$\text{Length of slotted lever, } L_s = \frac{\left(\frac{L}{2}\right)}{\sin\left(90^\circ - \frac{\alpha}{2}\right)} = \frac{\left(\frac{280}{2}\right)}{\sin(90^\circ - 70.525^\circ)}$$

$$= 419.984 \text{ mm}$$ **Ans.**

$$\text{Maximum cutting speed, } (V_s)_{max} = \omega \cdot r \cdot \frac{L_s}{c+r} = \left(\frac{2\pi \times 45}{60}\right) \times \frac{419.984 \times r}{(3r+r)} \times \frac{1}{1000}$$

$$= 0.495 \text{ m/s}$$ **Ans.**

1. (d) Solution:

Given data : m = 15 kg; d = 0.025 m; l = 1.2 m; N = 2100 rpm;
e = 0.12 mm; $E = 220 \times 10^9$ N/m^2

$$\text{Moment of inertia, } I = \frac{\pi d^4}{64} = \frac{\pi \times (0.025)^4}{64} = 1.9174 \times 10^{-8} \text{ m}^4$$

$$\text{Deflection, } \delta = \frac{mgl^3}{48EI} = \frac{15 \times 9.81 \times 1.2^4}{48 \times 220 \times 10^9 \times 1.9174 \times 10^{-8}}$$

$$\delta = 1.5069 \times 10^{-3} \text{ m}$$

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{1.5069 \times 10^{-3}}} = 80.682 \text{ rad/s}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2100}{60} = 219.911 \text{ rad/s}$$

$$\text{Amplitude, } y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

$$y = \frac{0.12}{\left(\frac{80.682}{219.911}\right)^2 - 1} = -0.1386 \text{ mm}$$

The –ve sign indicates that the displacement is out of phase with the centrifugal force.

$$\text{Dynamic force on the bearing} = s \times y = m\omega_n^2 y$$

$$= 15 \times (80.682)^2 \times 0.1386 \times 10^{-3} = 13.539 \text{ N}$$

$$\text{Total load on each bearing} = \frac{mg}{2} + \frac{sy}{2} = \frac{15 \times 9.81}{2} + \frac{13.539}{2}$$

$$= 80.344 \text{ N}$$ **Ans.**

1. (e) Solution:

Given data : N_B = 1100 rpm (CCW from right)

$$Z_B = 25, Z_F = 36, Z_C = 90, Z_D = 70, Z_E = 35$$

Operation	Arm A	Gear B	Compound Gears D, E	Gear C	Gear F (output shaft)
Arm A fixed +1 revolution to gear B, CCW	0	1	$\pm \frac{Z_B}{Z_D}$	$-\frac{Z_B}{Z_C}$	$-\frac{Z_B}{Z_D}\frac{Z_E}{Z_F}$
Multiply by x	0	x	$\pm x\frac{Z_B}{Z_D}$	$-x \times \frac{Z_B}{Z_C}$	$-x \times \frac{Z_B}{Z_D}\frac{Z_E}{Z_F}$
Add y	y	$x+y$	$\pm x\frac{Z_B}{Z_D}+y$	$-x\frac{Z_B}{Z_C}+y$	$-x \times \frac{Z_B}{Z_D}\frac{Z_E}{Z_F}+y$

(i) Gear C is fixed

$$-x\left(\frac{Z_B}{Z_C}\right)+y = 0$$

$$-x\left(\frac{25}{90}\right)+y = 0$$

$$y = 0.2777x \quad \text{...(i)}$$

Also, $x + y = 1100$ ...(ii)

From (i) and (ii), $x = 860.92$ rpm, $y = 239.07$ rpm

$$N_F = -x\left(\frac{Z_B}{Z_D}\right)\left(\frac{Z_E}{Z_F}\right)+y$$

$$N_F = -x\left(\frac{25}{70}\right)\left(\frac{35}{36}\right)+0.2777x$$

$$N_F = -0.0695 \times 860.92$$

$$N_F = -59.83 \text{ rpm}$$

or $N_F = 59.83$ rpm (Clockwise)

(ii) When gear C is rotating at 10 rpm clockwise

$$-x\left(\frac{Z_B}{Z_C}\right)+y = -10$$

$$-x\left(\frac{25}{90}\right)+y = -10 \quad \text{...(iii)}$$

From equation (ii) and (iii),

$$x = 868.69 \text{ rpm and } y = 231.304 \text{ rpm}$$

$$N_F = -x\left(\frac{Z_B}{Z_D}\right)\left(\frac{Z_E}{Z_F}\right) + y$$

$$N_F = -868.69 \times \left(\frac{25}{70}\right)\left(\frac{35}{36}\right) + 231.304$$

$$N_F = -70.32 \text{ rpm or } 70.32 \text{ rpm (Clockwise)}$$

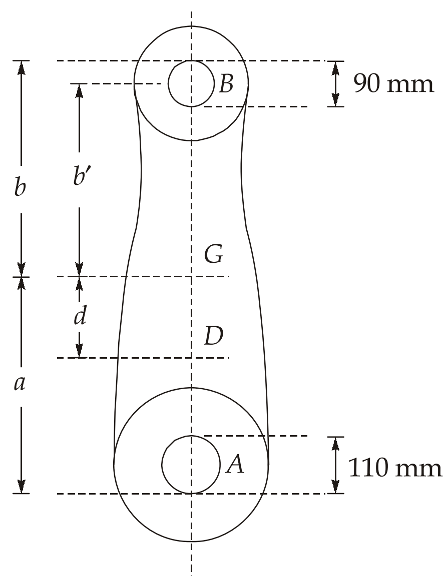
2. (a) Solution:

Let, L_a = Length of equivalent simple pendulum when suspended from the top of big-end bearing.

L_b = Length of equivalent simple pendulum when suspended from the top of small-end bearing

a = Distance of centre of mass G from top of big end bearing.

b = Distance of centre of mass G from top of small end bearing.



$$t_a = 2\pi\sqrt{\frac{L_a}{g}}$$

$$1.8 = 2\pi\sqrt{\frac{L_a}{9.81}}$$

$$L_a = 0.8051 \text{ m}$$

$$t_b = 2\pi\sqrt{\frac{L_b}{g}}$$

$$1.95 = 2\pi\sqrt{\frac{L_b}{9.81}}$$

$$L_b = 0.9448 \text{ m}$$

Also, $a + \frac{k^2}{a} = L_a$ and $b + \frac{k^2}{b} = L_b$

$$a + \frac{k^2}{a} = 0.8051 \qquad b + \frac{k^2}{b} = 0.9448$$

$$k^2 = 0.8051a - a^2 \quad \text{...(i)} \qquad k^2 = 0.9448b - b^2 \quad \text{...(ii)}$$

From equation (i) and (ii),

$$0.8051a - a^2 = 0.9448b - b^2 \quad \text{...(iii)}$$

$$0.8051 \times (1.1 - b) - (1.1 - b)^2 = 0.9448b - b^2$$

$$\left\{\left[a + b = 1000 + \frac{110}{2} + \frac{90}{2}\right] \quad a + b = 1100 \text{ mm or } 1.1 \text{ m} \quad \therefore\ a = (1.1 - b)\right\}$$

$$b = 0.7207 \text{ m}$$

$$a = 1.1 - 0.7207$$

$$a = 0.37929 \text{ m}$$

Ans.

From equation (i)

$$k^2 = 0.8051 \times 0.37929 - (0.37929)^2$$

$$k = 0.40187 \text{ m}$$

(ii) Moment of inertia, $I = mk^2$

$$I = 60 \times (0.40187)^2$$

$$I = 9.6903 \text{ kgm}^2$$

Ans.

The distance of centre of mass of connecting rod from the centre of small end bearing,

$$b' = 720.7 - \frac{90}{2} = 675.7 \text{ mm} = 0.6757 \text{ m}$$

Let, the second mass be placed at D

Take $GD = d$ and m_d = mass at D

then, $$d = \frac{k^2}{b'} = \frac{0.16149}{0.6757} = 0.2390 \text{ m}$$

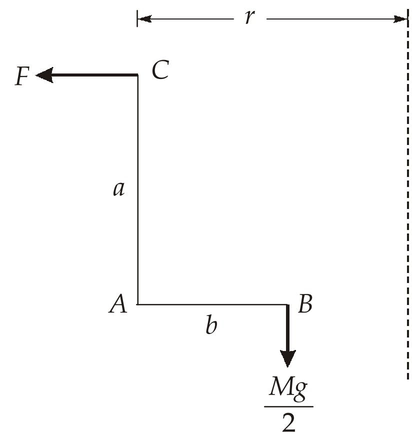
$$m_d = \frac{m \times b'}{b' + d} = \frac{60 \times 0.6757}{0.6757 + 0.239} = 44.322 \text{ kg}$$

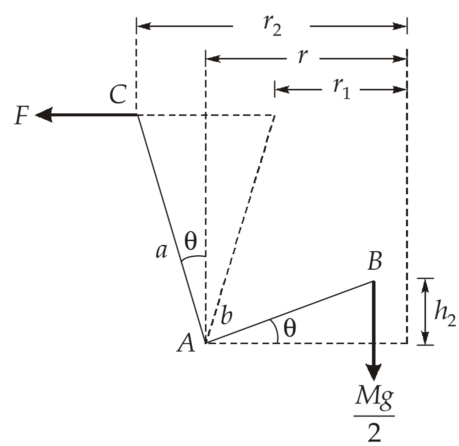
$$m_b' = 60 - 44.322$$

Mass at the small end bearing centre, $m_b' = 15.677$ kg

2. (b) Solution:

Given data : $a = 150$ mm; $b = 130$ mm; $r = 170$ mm; $N = 280$ rpm; $m = 6$ kg; $h_2 = 14$ mm





$$\omega = \frac{2\pi \times 280}{60} = 29.32 \text{ rad/s}$$

$$N_2 = 1.05 \times 280 = 294 \text{ rad/s}$$

$$\omega_2 = 1.05 \times 29.32 = 30.78 \text{ rad/s}$$

$$\frac{r_2 - r}{a} = \frac{h_2}{b}$$

$$\Rightarrow \quad r_2 = \frac{ah_2}{b} + r$$

$$\Rightarrow \quad r_2 = \frac{150}{130} \times 14 + 170$$

$$r_2 = 186.154 \text{ mm}$$

$$F = mr\omega^2 = 6 \times 0.17 \times (29.32)^2 = 876.85 \text{ N}$$

$$F_2 = mr_2\omega_2^2 = 6 \times 0.1861 \times (30.78)^2 = 1058.18 \text{ N}$$

(i)

$$s = 2 \times \frac{a^2}{b^2}\left[\frac{F_2 - F}{r_2 - r}\right]$$

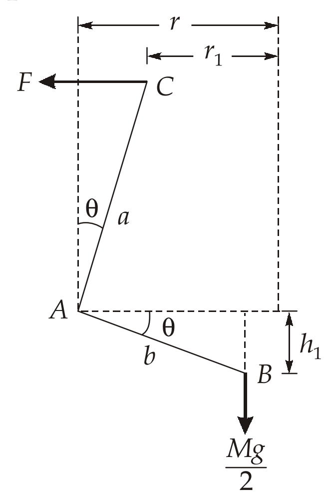
$$= 2 \times \left[\frac{150}{130}\right]^2 \times \left[\frac{1058.18 - 876.85}{0.1861 - 0.170}\right] \text{ N/m}$$

$$= 29.989 \text{ N/mm}$$

(ii)

As sleeve moves by 28 mm means it moves 14 mm from its mean position

$\therefore$ $h_1 = 14$ mm



Also, $\dfrac{r - r_1}{a} = \dfrac{h_1}{b}$

$\Rightarrow$ $r_1 = r - \dfrac{ah_1}{b}$

$\Rightarrow$ $r_1 = 170 - \dfrac{150}{130} \times 14$

$\Rightarrow$ $r_1 = 153.846$ mm

Again, $s = 2 \times \left(\dfrac{a}{b}\right)^2 \left[\dfrac{F_2 - F_1}{r_2 - r_1}\right]$

$$29.989 \times 10^3 = 2 \times \left(\frac{150}{130}\right)^2 \times \left[\frac{1058.18 - F_1}{0.1861 - 0.1538}\right]$$

$$\Rightarrow \qquad F_1 = 694.40 \text{ N}$$

$$F_1 = m \times r_1 \times (\omega_1)^2$$

$$694.40 = 6 \times 0.1538 \times \left(\frac{2 \times \pi \times N_1}{60}\right)^2$$

$$N_1 = 261.95 \text{ rpm}$$

(iii)

$$\text{Sensitiveness} = \frac{2(N_2 - N_1)}{N_2 + N_1} = \frac{2 \times (294 - 261.95)}{294 + 261.95}$$

$$= 0.11528 \text{ or } 11.53\%$$

(iv) For isochronous governor,

$$F_1 = mr_1\omega^2 = 6 \times 0.1538 \times (29.32)^2 = 793.29 \text{ N}$$

$$F_2 = mr_2\omega^2 = 6 \times 0.1861 \times (29.32)^2 = 959.89 \text{ N}$$

$$s = 2 \times \left(\frac{0.15}{0.13}\right)^2 \times \left[\frac{959.89 - 793.29}{0.1861 - 0.1538}\right] \text{ N/m}$$

$$s = 13.734 \text{ N/mm}$$

2. (c) Solution:

Given : $M = 100$ kg; $m = 2.5$ kg; $\varepsilon = \frac{1}{20}$, $N = 900$ rpm; $r = \frac{110}{2} = 55$ mm;

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.24 \text{ rad/s}$$

In the absence of damping, $\xi = 0$

$$\varepsilon = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\frac{1}{20} = \frac{1}{\left(\frac{94.24}{\omega_n}\right)^2 - 1}$$

$$\omega_n = 20.564 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{s}{M}}$$

$$20.564 = \sqrt{\frac{s}{100}}$$

$\Rightarrow$ Combined stiffness, $s = 42291.32$ N/m = 42.291 N/mm

(i) $$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = \ln\left[\frac{x_1}{x_2}\right] = \ln\left[\frac{1}{1-0.25}\right]$$

On solving, $\xi = 0.0457$

Now,
$$\varepsilon = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\varepsilon = \frac{\sqrt{1+\left(2\times 0.0457\times\frac{94.24}{20.564}\right)^2}}{\sqrt{\left[1-\left(\frac{94.24}{20.564}\right)^2\right]^2+\left(2\times 0.0457\times\frac{94.24}{20.564}\right)^2}}$$

$$\varepsilon = 0.05419$$

The maximum unbalanced force on the machine due to reciprocating parts,

$$F = mr\omega^2$$

$$F = 2.5 \times 0.055 \times 94.24^2 = 1221.16 \text{ N}$$

$$\varepsilon = \frac{F_t}{F} \text{ or } 0.05419 = \frac{F_t}{1221.16}$$

$$F_t = 66.174 \text{ N}$$

(ii) At resonance, $\frac{\omega}{\omega_n} = 1$

$$\varepsilon = \frac{\sqrt{1+(2\xi)^2}}{2\xi} = \frac{\sqrt{1+(2\times 0.0457)^2}}{2\times 0.0457} = 10.99$$

Maximum unbalanced force on the machine due to reciprocating parts at resonance, i.e. when $\omega = \omega_n$

$$F = 2.5 \times 0.055 \times (20.564)^2$$

$$F = 58.145 \text{ N}$$

$$F_t = \varepsilon F$$

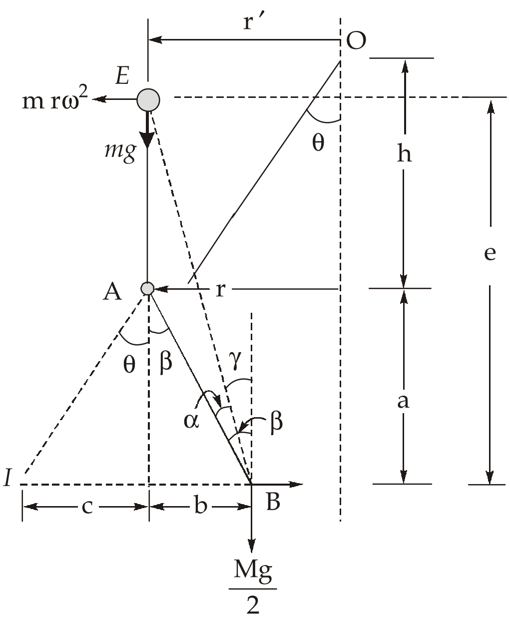
$$F_t = 10.99 \times 58.145 = 639.02 \text{ N}$$

(iii) $$\text{Amplitude} = \frac{\text{Force transmitted at resonance}}{\text{Stiffness}}$$

$$= \frac{639.02}{42.291} = 15.11 \text{ mm}$$ **Ans.**

3. (a) Solution:

Given : $m = 8$ kg; $M = 90$ kg



When AE is vertical, $r' = r = 185$ mm

$$mr\omega^2 = \frac{a}{e}\tan\theta\left[mg + \frac{Mg}{2}(1+k)\right] \quad \text{...(i)}$$

Friction is neglected, $a = \sqrt{(350)^2 - (185-45)^2}$

$$a = 320.78 \text{ mm}$$

$$e = a + AE$$

$$= 320.78 + 105 = 425.78 \text{ mm}$$

$$\sin\theta = \frac{r}{OA} = \frac{185}{350} = 31.908°$$

$$\tan\theta = \tan(31.908) = 0.6226$$

$$\sin\beta = \frac{185 - 45}{350} = 0.4$$

$$\beta = 23.578°$$

$$\tan\beta = \tan(23.578) = 0.43643$$

$$k = \frac{\tan\beta}{\tan\theta} = \frac{0.43643}{0.6226} = 0.7009$$

On putting values in equation (i),

$$8 \times 0.185 \times \omega^2 = \frac{320.78}{425.78} \times 0.6226 \times \left[8 \times 9.81 + \frac{90 \times 9.81}{2} \times (1 + 0.7009)\right]$$

$$\omega = 16.212 \text{ rad/s}$$

$$N = 154.82 \text{ rpm}$$

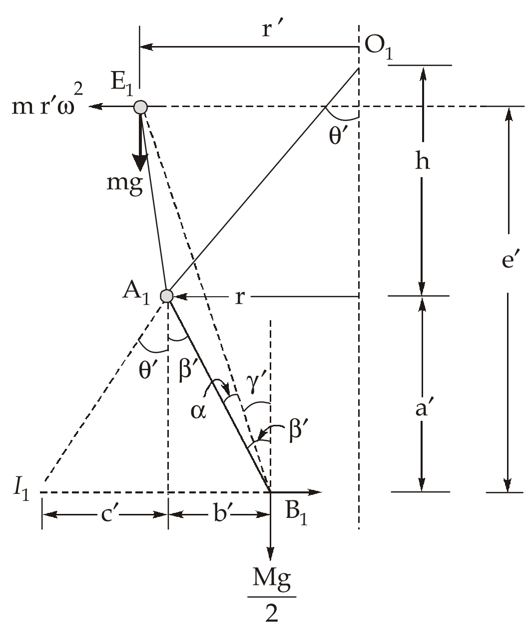
$$BE = \sqrt{e^2 + b^2} = \sqrt{425.78^2 + (185 - 45)^2}$$

$$BE = 448.205 \text{ mm}$$

$$\cos\gamma = \frac{e}{BE} = \frac{425.78}{448.205}$$

$\Rightarrow$ $$r = 18.20°$$

$$\alpha = \beta - \gamma = 23.578 - 18.20 = 5.378°$$



$$\sin\beta' = \frac{b'}{A_1B_1} = \frac{245-45}{350}$$

$$\beta' = 34.849°$$

$$\gamma' = \beta' - \alpha = 34.849° - 5.378° = 29.47°$$

$$e' = B_1E_1 \cos\gamma'$$

$$= BE\cos\gamma' = 448.205 \times \cos 29.47°$$

$$e' = 390.206 \text{ mm}$$

$$r' = B_1E_1 \sin\gamma' + 45$$

$$r' = 448.205 \times \sin 29.47° + 45$$

$$= 265.502 \text{ mm}$$

$$b' = 200 \text{ mm}$$

$$a' = A_1B_1 \cos\beta'$$

$$a' = 350 \cos 34.849$$

$$a' = 287.23 \text{ mm}$$

$$\sin\theta' = \frac{245}{350} = 0.07$$

$$\theta' = 44.427°$$

$$c' = a'\tan\theta'$$

$$c' = 287.23 \tan(44.427°)$$

$$c' = 281.541 \text{ mm}$$

Taking moments about I_1

$$mr'\omega^2 e' = mg(c' + r - r') + \frac{Mg}{2}(c' + b')$$

$$8 \times 0.2655 \times \omega^2 \times 0.3902 = 8 \times 9.81 \times (0.28154 + 0.245 - 0.2655) + \frac{90 \times 9.81}{2} \times (0.28154 + 0.200)$$

$$\omega = 16.769 \text{ rad/s}$$

$$N = \frac{60\omega}{2\pi} = \frac{60 \times 16.769}{2 \times \pi} = 160.135 \text{ rpm}$$

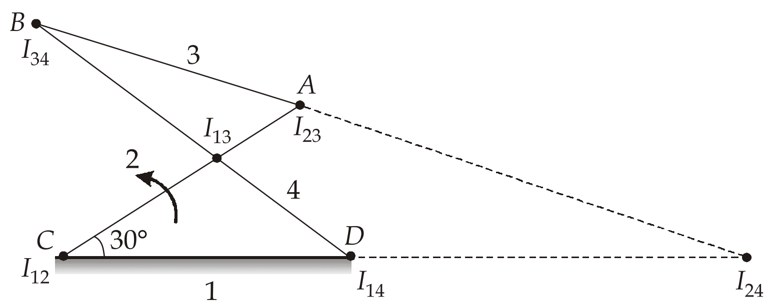
$$N = 160.135 \text{ rpm}$$ **Ans.**

3. (b) Solution:

Given : CD = 65 mm; CA = 60 mm; DB = 80 mm; AB = 55 mm; N_{AC} = 100 rpm

$$\text{Number of } I\text{-centre} = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 2 \times 3 = 6$$

All the instantaneous centres of given mechanism are shown in figure.



***I*-centre**

Angular velocity of link AC, $\omega_2 = \frac{2\pi N}{60} = \frac{2\pi \times 100}{600} = 10.472$ rad/s

We know that, $\omega_2 \times I_{12}I_{23} = \omega_3 \times I_{13}I_{23}$

$$\omega_3 = \omega_2 \times \frac{I_{12}I_{23}}{I_{13}I_{23}}$$

$$\omega_3 = \frac{10.472 \times 60}{22} = 28.56 \text{ rad/s}$$

$\therefore$ $\omega_{AB} = \omega_3 = 28.56$ rad/s

Also, $\omega_2 \times I_{12}I_{24} = \omega_4 \times I_{14}I_{24}$

$$\omega_4 = \omega_2 \times \frac{I_{12}I_{24}}{I_{14}I_{24}}$$

$$\omega_4 = \frac{10.472 \times 209}{144} = 15.199 \text{ rad/s}$$

$\therefore$ $\omega_{BD} = \omega_4 = 15.199$ rad/s

3. (c) Solution:

Given data : m = 17 kg; l = 1.5 m; e = 0.0004 m; $E = 220 \times 10^9$ N/m^2; d = 0.015 m; σ_{per} = 70 MPa

As shaft is held in the long bearings, it may be assumed to be fixed at the ends.

$$\Delta_{st} = \frac{mgl^3}{192EI} = \frac{17 \times 9.81 \times (1.5)^3}{192 \times 220 \times 10^9 \times \frac{\pi}{64} \times (0.015)^4}$$

$$\Delta_{st} = 5.362 \times 10^{-3} \text{ m}$$

$$\omega_n = \sqrt{\frac{g}{\Delta_{st}}} = \sqrt{\frac{9.81}{5.362 \times 10^{-3}}}$$

$$\omega_n = 42.772 \text{ rad/s}$$

$$N = \frac{60 \times \omega_n}{2\pi} = \frac{60 \times 42.772}{2\pi}$$

Critical speed, N = 408.44 rpm **Ans.**

Also,
$$\sqrt{\frac{s}{m}} = \omega_n$$

$$s = m\omega_n^2 = 17 \times (42.772)^2$$

$$= 31100.54 \text{ N/m}$$

When the shaft rotates, additional dynamic load on the shaft can be obtained from the relation.

$$\frac{M}{I} = \frac{\sigma_{per}}{y}$$

$$\Rightarrow \quad \frac{\frac{W_1 l}{8}}{\frac{\pi}{64} d^4} = \frac{\sigma_{per}}{\frac{d}{2}}$$

$$\frac{\frac{W_1 \times 1.5}{8}}{\frac{\pi}{64} \times (0.015)^4} = \frac{70 \times 10^6}{0.015} \times 2$$

$$W_1 = 123.7 \text{ N}$$

Additional deflection due to W_1, $\delta_1 = \dfrac{W_1}{s}$

$$\delta_1 = \frac{123.7}{31100.54}\text{m} = 3.977 \times 10^{-3}\text{m}$$

For whirling shaft,
$$\delta_1 = y = \frac{\pm e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} = \frac{\pm e}{\left(\frac{N_n}{N}\right)^2 - 1}$$

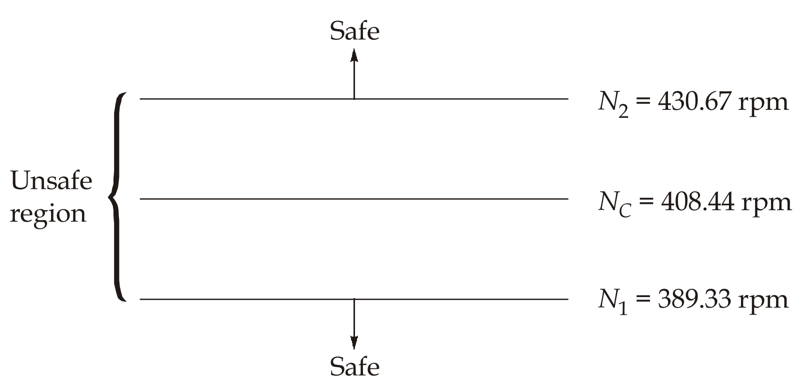
$$3.977 \times 10^{-3} = \frac{\pm 0.0004}{\left(\frac{408.44}{N}\right)^2 - 1}$$

+ve,
$$\left(\frac{408.44}{N}\right)^2 - 1 = \frac{0.0004}{3.977 \times 10^{-3}}$$

$$N_1 = 389.33 \text{ rpm}$$

–ve,
$$\left(\frac{408.44}{N}\right)^2 - 1 = \frac{-0.0004}{3.977 \times 10^{-3}}$$

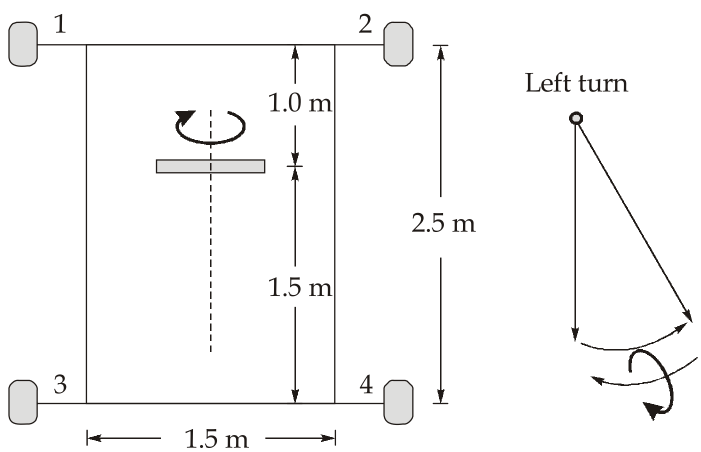
$$N_2 = 430.67 \text{ rpm}$$



4. (a) Solution:

Given data : M = 2000 kg; m = 150 kg; w = 1.5 m; h = 0.6 m; b = 2.5 m; I_w = 0.6 kgm^2
k = 0.16 m; G = 5

$$r = \frac{0.7}{2} = 0.35 \text{ m}\,;\ R = 110 \text{ m and } v = \frac{75 \times 1000}{3600} = 20.833 \text{ m/s}$$



Reaction due to weight

$$\text{Total weight} = 2000 \times 9.81 = 19620 \text{ N}$$

$$Rw_{1,2} = 19620 \times \frac{1.5}{2.5} \times \frac{1}{2} = 5886 \text{ N (upwards)}$$

$$Rw_{3,4} = 19620 \times \frac{1}{2.5} \times \frac{1}{2} = 3924 \text{ N (upwards)}$$

Reaction due to gyroscopic couples

$$\text{Due to wheels, } c_w = 4I_w \frac{v^2}{rR} = \frac{4 \times 0.6 \times 20.833^2}{0.35 \times 110} = 27.055 \text{ Nm}$$

For outer wheels, $$R'_{G,2,4} = \frac{C_w}{2w} = \frac{27.055}{2 \times 1.5} = 9.018 \text{ N} \quad \text{(Upwards)}$$

For inner wheels, $$R'_{G,1,3} = 9.018 \text{ N} \quad \text{(Downwards)}$$

Now, $$I_e = mk^2$$
$$= 150 \times (0.16)^2 = 3.84 \text{ kgm}^2$$

$$C_e = I_e G \omega_w \omega_p = I_e G \frac{v^2}{rR}$$

$$C_e = \frac{3.84 \times 5 \times 20.833^2}{0.35 \times 110}$$

$$C_e = 216.44 \text{ Nm}$$

For front wheels, $R''_{G,1,2} = \dfrac{C_e}{2b} = \dfrac{216.44}{2 \times 2.5} = 43.288\,\text{N}$ [Upwards]

For rear wheels, $R''_{G,3,4} = 43.288$ N [Downwards]

(iii) Reaction due to centrifugal couple,

$$C_c = \frac{Mv^2}{R}h = \frac{2000 \times 20.833^2}{110} \times 0.6$$

$$C_c = 4734.696 \text{ Nm}$$

For outer wheels, $R_{c2,4} = \dfrac{C_c}{2w} = \dfrac{4734.696}{2 \times 1.5} = 1578.23$ N [Upwards]

For rear wheels, $R_{c1,3} = 1578.23$ N [Downwards]

Therefore, reaction on wheels,

$$R = R_w + R'_G + R''_G + R_c$$

$$R_1 = 5886 - 9.018 + 43.288 - 1578.23 = 4342.03 \text{ N}$$

$$R_2 = 5886 + 9.018 + 43.288 + 1578.23 = 7516.53 \text{ N}$$

$$R_3 = 3924 - 9.018 - 43.288 - 1578.23 = 2293.46 \text{ N}$$

$$R_4 = 3924 + 9.018 - 43.288 + 1578.23 = 5467.96 \text{ N}$$ **Ans.**

4. (b) Solution:

Given data : $T = 1200 + 350 \sin 2\theta - 600 \cos 2\theta$; $N = 280$ rpm; $m = 400$ kg; $k = 0.350$ m

$$\omega = \frac{2\pi \times N}{60} = \frac{2\pi \times 280}{60} = 29.32 \text{ rad/s}$$

(i)

$$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta = \frac{1}{\pi}\int_0^{\pi} (1200 + 350 \sin 2\theta - 600 \cos 2\theta) d\theta$$

$$T_{mean} = 1200 \text{ Nm}$$

$$\text{Power} = T_{mean} \times \omega$$

$$P = 1200 \times 29.32 \times 10^{-3} \text{ kW}$$

$$P = 35.185 \text{ kW}$$

(ii) At any instant, $\Delta T = T - T_{mean}$

$$= [1200 + 350 \sin 2\theta - 600 \cos 2\theta] - 1200$$

$$= 350 \sin 2\theta - 600 \cos 2\theta$$

ΔT is zero, when $350 \sin 2\theta - 600 \cos 2\theta = 0$

$$\tan 2\theta = \frac{600}{350} = 1.71428$$

$$2\theta = 59.743°, 239.743°$$
$$\theta = 29.871°, 119.871°$$
$$E_{max} = \int_{\theta_1}^{\theta_2} \Delta T dQ$$
$$E_{max} = \int_{29.871°}^{119.871°} (350 \sin 2\theta - 600 \cos 2\theta) d\theta$$
$$E_{max} = \left[-\frac{350}{2}\cos 2\theta - \frac{600}{2}\sin 2\theta\right]_{29.871°}^{119.871°}$$
$$E_{max} = 694.41 \text{ Nm}$$

Coefficient of fluctuation of speed,

$$C_s = \frac{E_{max}}{I\omega^2} = \frac{694.41}{400 \times 0.35^2 \times 29.32^2}$$
$$C_s = 0.01648$$
$$C_s = 1.648\%$$

(iii) Acceleration or deceleration is produced by excess or deficit torque than the mean value at any instant

$$(\Delta T)_{at\ \theta = 70°} = 350 \sin 140° - 600 \cos 140°$$
$$= 684.602 \text{ Nm}$$
$$(\Delta T) = I\alpha$$
$$684.602 = 400 \times 0.35^2 \times \alpha$$
$$\alpha = 13.971 \text{ rad/s}^2$$

(iv) For $(\Delta T)_{max}$ and $(\Delta T)_{min}$

$$\frac{d}{d\theta}(\Delta T) = 0$$
$$\frac{d}{d\theta}(350 \sin 2\theta - 600 \cos 2\theta) = 0$$
$$350 \times 2 \times \cos 2\theta + 600 \times 2 \times \sin 2\theta = 0$$
$$\tan 2\theta = -\frac{7}{12}$$
$$2\theta = 149.743°, 329.743°$$

or $\theta = 74.871°, 164.87°$

$$(\Delta T)_{\theta = 74.871°} = 350 \times \sin 149.743 - 600 \cos 149.743$$

$$\Delta T = 694.622 \text{ Nm} \quad \text{[Maximum]}$$

$$(\Delta T)_{\theta = 164.87°} = 350 \times \sin 329.743 - 600 \cos 329.743$$

$$\Delta T = -694.622 \text{ Nm} \quad \text{[Minimum]}$$

Since, magnitude of maximum and minimum torque are same, therefore maximum acceleration and retardation is equal.

$$\Delta T = I\alpha$$

$$\frac{694.622}{400 \times 0.35^2} = \alpha$$

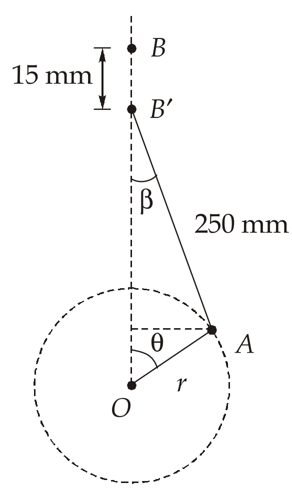
Maximum acceleration or retardation,

$$\alpha = 14.175 \text{ rad/s}^2 \quad \textbf{Ans.}$$

4. (c) Solution:

Given data : $N = 1900$ rpm; $r = 50$ mm; $l = 250$ mm ; $n = \frac{l}{r} = \frac{250}{50} = 5$

$d = 88$ mm; $m_{rec} = 1.5$ kg; $x = 15$ mm; $P = 750$ kPa



$$\omega = \frac{2\pi \times 1900}{60} = 198.96 \text{ rad/s}$$

$$OB = l + r = BB' + l\cos\beta + r\cos\theta$$

$$l + r = x + l\cos\beta + r\cos\theta \quad \text{...(i)}$$

Also,

$$l\sin\beta = r\sin\theta$$

$$\sin\beta = \frac{\sin\theta}{l/r} = \frac{\sin\theta}{n}$$

$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{\sin^2\theta}{n^2}} \quad \text{...(ii)}$$

Substitute in equation (i)

$$l + r = x + l\sqrt{1-\left(\frac{\sin\theta}{n}\right)^2} + r\cos\theta$$

Putting values,

$$250 + 50 = 15 + 250 \times \frac{\sqrt{5^2 - \sin^2\theta}}{5} + 50\cos\theta$$

$$\frac{285}{50} = \frac{5\sqrt{5^2 - \sin^2\theta}}{5} + \cos\theta$$

$$(5.7 - \cos\theta)^2 = 5^2 - \sin^2\theta$$

$$(5.7)^2 + \cos^2\theta - 2 \times 5.7\cos\theta = 5^2 - \sin^2\theta$$

$$\cos\theta = 0.7447$$

$$\theta = 41.863°$$

From equation (ii),

$$\cos\beta = \sqrt{1-\frac{\sin^2 41.863°}{5^2}}$$

$$\beta = 7.670°$$

Force due to gas pressure, $f_p = \frac{\pi}{4}d_p^2 \times p = \frac{\pi}{4} \times 88^2 \times 0.75 = 4561.59 \text{ N}$

Accelerating force, $f_b = mr\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$$= 1.5 \times 0.05 \times 198.96^2\left[\cos 41.86° + \frac{\cos(2\times 41.86°)}{5}\right]$$

$$= 2276.107 \text{ N}$$

Downward force due to gravity $= mg = 1.5 \times 9.81 = 14.715$ N

(i) Force on the piston, $f = f_p + mg - f_b$

$$f = 4561.59 + 14.715 - 2276.107$$

$$= 2300.198 \text{ N}$$

(ii) Thrust in the connecting rod,

$$f_c = \frac{f}{\cos\beta} = \frac{2300.198}{\cos 7.670}$$

$$f_c = 2320.963 \text{ N}$$

(iii) Thrust on the sides of cylinder walls,

$$f_n = f \tan \beta$$

$$f_n = 2300.198 \times \tan 7.670°$$

$$f_n = 309.77 \text{ N}$$

(iv) The above values are zero at the speed when the force on the piston f is zero.

$$f = f_p - mr\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right] + mg$$

$$0 = 4561.59 - 1.5 \times 0.05 \times \omega^2 \times \left[\cos 41.86° + \frac{\cos 83.72°}{5}\right] + 14.715$$

On solving, $\omega = 282.115$ rad/s or N = 2693.99 rpm **Ans.**

Section B : Fluid Mechanics & Turbo Machinery-1 + Strength of Materials & Mechanics-2

5. (a) Solution:

$$P = f(\rho) = 3000 P_0\left[\left(\frac{\rho}{\rho_o}\right)^7 - 1\right] \quad \text{...(i)}$$

Take $P_0 = 100 \text{ kN/m}^2$

We know that

$$\text{Bulk modulus of elasticity, } K = \frac{1}{\text{Compressibility } (\beta)}$$

$$K = \frac{1}{\beta} = \frac{\sigma_v}{\epsilon_v} = \frac{dP}{-\frac{dV}{V}} \quad \text{...(ii)}$$

$\because$ $m = \rho V$ = Constant on compression

$\Rightarrow$ $\rho dV + V d\rho = 0$

$\Rightarrow$ $-\frac{dV}{V} = \frac{d\rho}{\rho}$

$\therefore$ $K = \frac{\rho dP}{d\rho}$

From (i)

$$P = (3000)P_0\left[\left(\frac{\rho}{\rho_0}\right)^7 - 1\right]$$

$$\frac{dP}{d\rho} = (3000)P_0\left[\frac{7\rho^6}{\rho_0^7}\right] \quad \text{...(iii)}$$

From (ii) and (iii),

$$K = (3000)P_0\frac{7\rho^7}{\rho_0^7}$$

$$K = \left\{21000\left(\frac{\rho}{\rho_0}\right)^7\right\}P_0$$

From equation (i)

$$\left(\frac{\rho}{\rho_0}\right)^7 = \frac{P}{3000P_0} + 1$$

$$\therefore \quad K = 7(P + 3000P_0) = f(P)$$

$$K_{1.1\text{atm}} = 7 \times [1.1 \times 101.325 + 3000 \times 100]$$

$$= 2.10077 \text{ GPa}$$ **Ans.**

$$\beta_{1.1\text{atm}} = \frac{1}{K_{1.1\text{atm}}} = 4.76\times10^{-7}\,\text{m}^2/\text{kN}$$ **Ans.**

$$K_{100\text{atm}} = 7 \times [100 \times 101.325 + 3000 \times 100]$$

$$= 2.1709 \text{ GPa}$$ **Ans.**

$$\beta_{100\text{atm}} = \frac{1}{K_{100atm}} = 4.606\times10^{-7}\,\text{m}^2/\text{kN}$$ **Ans.**

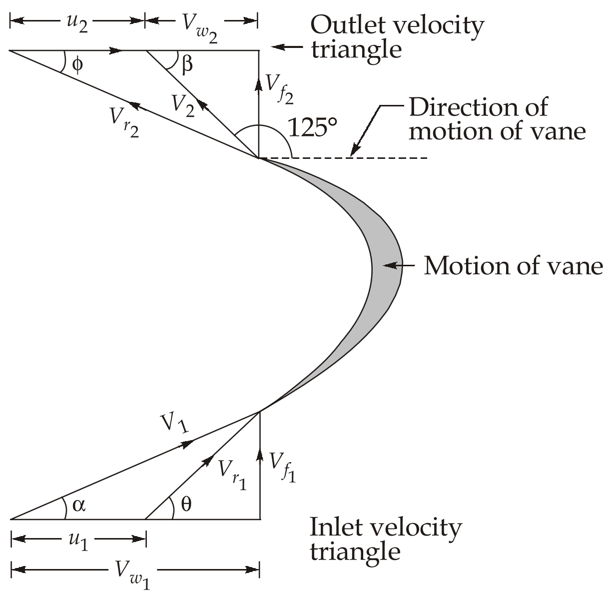
5. (b) Solution:

Given : $V_1 = 30$ m/s, $u_1 = u_2 = 14$, $\alpha = 25°$

Angle made by the jet at the outlet with the direction of motion of vanes = 125°

$$\therefore \quad \text{Angle, } \beta = 180° - 125°$$

$$\beta = 55°$$



1. Angle of vanes tips, $V_{w_1} = V_1 \cos\alpha = 30\cos 25° = 27.19°$

$$V_{f_1} = V_1 \sin\alpha = 30 \sin 25°$$

$$V_{f_1} = 12.68 \text{ m/s}$$

and,
$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{12.68}{27.19 - 14}$$

$$\theta = 43.87°$$

By sine rule,
$$\frac{V_{r_1}}{\sin 90} = \frac{V_{f_1}}{\sin\theta}$$

$$V_{f_1} = \frac{12.68}{\sin(43.87°)}$$

Now,
$$V_{r_1} = V_{r_2} = 18.296 \text{ m/s}$$

From outlet velocity triangle, by sine rule

$$\frac{V_{r_2}}{\sin 125} = \frac{u_2}{\sin(55-\phi)}$$

$$\frac{18.296}{\sin 125} = \frac{14}{\sin(55-\phi)}$$

$$55 - \phi = 38.814$$

$$\phi = 16.186°$$

2. Work done per unit weight of water entering = $\frac{1}{g}\left(V_{w_1} + V_{w_2}\right)u_1$

$$V_{w1} = 27.19 \text{ m/s and } u_1 = 14$$

The value of V_{w2} is obtained from outlet velocity triangle

$$V_{w2} = V_{r2}\cos\phi - u_2$$

$$= 18.296\cos(16.186) - 14$$

$$V_{w2} = 3.57 \text{ m/s}$$

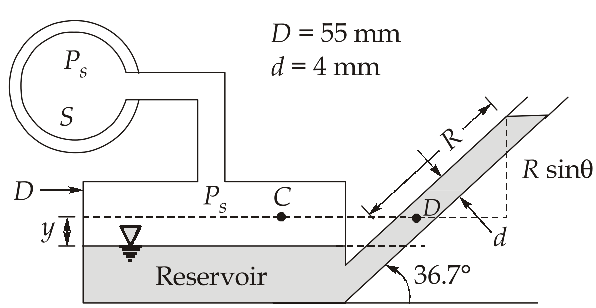
$$\therefore \quad \frac{\text{Work done}}{\text{Unit weight}} = \frac{1}{9.81}[27.19 + 3.57] \times 14 = 43.90 \text{ Nm/N}$$

3. $$\text{Efficiency} = \frac{\text{Work done per kg}}{\text{energy supplied per kg}}$$

$$= \frac{43.90}{\frac{V_1^2}{2g}} = \frac{43.90 \times 2 \times 9.81}{30 \times 30}$$

$$\eta = 95.70\%$$

5. (c) Solution:



Let, with the application of pressure P_s the level of gauge fluid in the reservoir drops by 'y'.

$$P_c = P_D \qquad \text{(same level)}$$

$$P_s = \rho g\,(R\sin\theta + y) \quad ..(i)$$

Also; from continuity equation,

$$Ay = aR$$

$$y = \left(\frac{a}{A}\right)R \quad ...(ii)$$

where A and a are cross-sectional area of reservoir and tube respectively.

from (i) and (ii),

$$P_s = \rho g R\sin\theta\left[1+\frac{a}{A}\times\frac{1}{\sin\theta}\right] \quad ...(iii)$$

Let, the pressure P_s be measured as P_s' from the gauge reading R only (neglecting the reservoir deflection 'y')

$$P_s' = \rho g R\sin\theta \quad ...(iv)$$

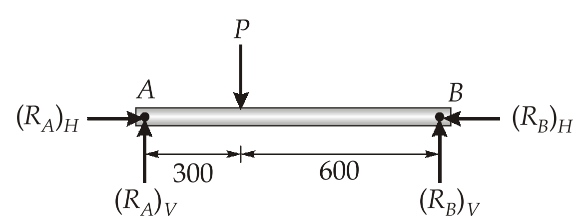
$$\%\text{Error} = \left(\frac{P_s - P_s'}{P_s}\right)\times 100 = \frac{100}{\left[1+\frac{A}{a}\sin\theta\right]}$$

$$= \frac{100}{1+\left(\frac{55}{4}\right)^2\sin 36.7}\% \simeq 0.8773\% \quad \textbf{Ans.}$$

5. (d)

Given : $m_d = 40$ kg; $\mu_s = 0.3$; $AB = 900$ mm

From FBD of link AB



(All dimensions are in mm)

$$\Sigma F_H = 0$$

$$\therefore \quad (R_A)_H = (R_B)_H$$

$$\Sigma F_V = 0$$

$$\therefore \quad (R_A)_V + (R_B)_V = P$$

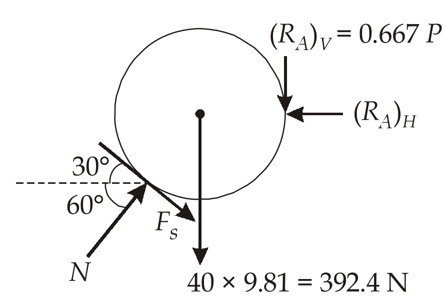
Taking moment about point B,

We get,

$$P\times 600 - (R_A)_V\times 900 = 0$$

$$(R_A)_V = 0.667\,P$$

From F.B.D. of disc



$$\Sigma F_V = 0$$

$N\sin 60° - F_s \sin 30° - 0.667P - 392.4 = 0$ ...(i)

Taking moment about centre point O,

We get,

$$F_s \times 200 - 0.667P \times 200 = 0$$

$$F_s = 0.667P$$

If the disc is on the verge of moving, slipping would have at point C. Hence,

$$F_s = \mu_s \times N = 0.3 \times N$$

$$\therefore \quad 0.3 \times N = 0.667P$$

$$N = 2.223P$$

On putting the value of F_s and N in equation (i)

$2.223P \times \sin 60° - 0.667 \sin 30° - 0.667P - 392.4 = 0$

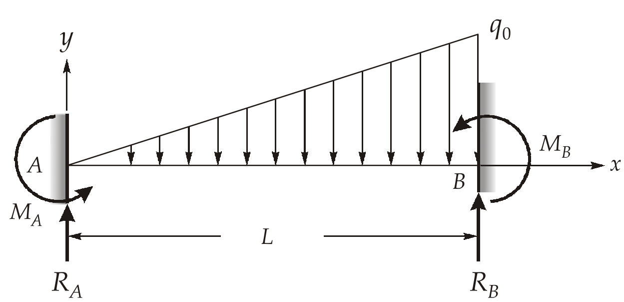
$$1.925P - 0.3335P - 0.667P = 392.4$$

$$0.9245P = 392.4$$

$$P = 424.45 \text{ N}$$

Hence, the maximum vertical force P is 424.45 N.

5. (e) Solution:



$$\text{Triangular load, } q = \frac{q_o x}{L}$$

Using,
$$EI\frac{d^4y}{dx^4} = \frac{-q_0 x}{L} \quad \text{...(i)}$$

Integrating,
$$EI\frac{d^3y}{dx^3} = \frac{-q_o x^2}{2L} + c_1 \quad \text{...(ii)}$$

$$EI\frac{d^2y}{dx^2} = \frac{-q_o x^3}{6L} + c_1 x + c_2 \quad \text{...(iii)}$$

$$EI\frac{dy}{dx} = \frac{-q_o x^4}{24L} + \frac{c_1 x^2}{2} + c_2 x + c_3 \quad \text{...(iv)}$$

$$EIy = \frac{-q_o x^5}{120L} + \frac{c_1 x^3}{6} + \frac{c_2 x^2}{2} + c_3 x + c_4 \quad \text{...(v)}$$

Boundary conditions,

BC_1 at $x = 0$ $\quad \frac{dy}{dx} = 0$

BC_2 at $x = 0$ $\quad y = 0$

BC_3 at $x = L$ $\quad \frac{dy}{dx} = 0$

BC_4 at $x = L$ $\quad y = 0$

Using , BC_1 and equation (iv)

$$c_3 = 0$$

Using , BC_2 and equation (v)

$$c_4 = 0$$

Using , BC_3 and equation (iv)

$$0 = \frac{-q_o L^4}{24L} + \frac{c_1 L^2}{2} + c_2 L$$

$$c_1 L + 2c_2 = \frac{q_0 L^2}{12} \quad \text{...(vi)}$$

Using BC_4 and equation (v)

$$0 = \frac{-q_o L^5}{120L} + \frac{c_1 L^3}{6} + \frac{c_2 L^2}{2}$$

$$c_1 L + 3c_2 = \frac{q_0 L^2}{20} \quad \text{...(vii)}$$

On solving equation (vi) and (vii)

$$c_1 = \frac{3}{20} q_0 L$$

$$c_2 = -\frac{1}{30} q_0 L^2$$

Shear force, using equation (ii)

$$V(x) = \left[\frac{-q_0 x^2}{2L} + \frac{3}{20}q_0 L\right]$$

Reactions, $R_A = V(0) = \frac{3}{20}q_0 L$ **Ans.**

$$R_B = V(L) = \left[\frac{-q_0 L}{2} + \frac{3q_0 L}{20}\right]$$

$$R_B = -\frac{7}{20}q_0 L$$ **Ans.**

Moment, using equation (iii)

$$M(x) = \left[\frac{-q_0 x^3}{6L} + \frac{3}{20}q_0 Lx + \left(-\frac{1}{30}q_0 L^2\right)\right]$$

$$M_A = M(0) = \frac{-q_0 L^2}{30}$$ **Ans.**

$$M_B = M(L) = \left[\frac{-q_0 L^3}{6L} + \frac{3}{20}q_0 L^2 - \frac{1}{30}q_0 L^2\right]$$

$$M_B = \frac{-q_0 L^2}{20}$$ **Ans.**

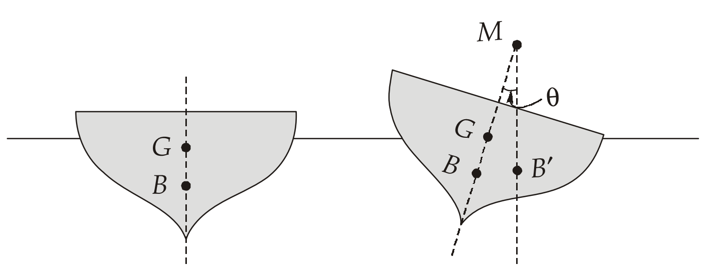
Deflection curve equation, using equation (v)

$$EIy = \frac{-q_0 x^5}{120L} + \frac{3}{20}q_0 L\frac{x^3}{6} - \frac{1}{30}q_0 L^2\frac{x^2}{2}$$

$$y = \frac{1}{120L\,EI}\left(-q_0 x^5 + 3q_0 L^2 x^3 - 2q_0 L^3 x^2\right)$$ Ans.

6. **(a) (i) Solution:**

Metacentre : It is the point of intersection of line of action of buoyant-force before and after a small angular displacement of floating body.



Metacentric height (GM)

$$GM = BM - BG$$

Distance of metacentre above centre of gravity.

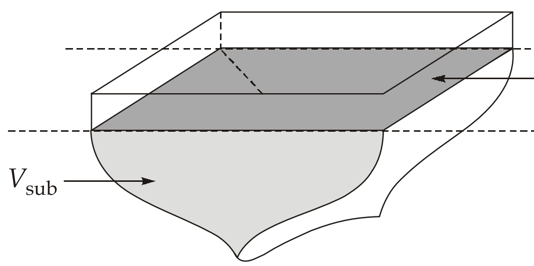
Physical significance

- Larger GM : Stable rotational equilibrium.
- M below G represents unstability in rotation.

Metacentric radius (BM)

It is distance between metacentre and centre of buoyancy.

$$BM = \frac{I_{min}}{V_{sub}}$$



Physical significance:

- Larger cross-section at interface of fluid means greater stability.
- Depends totally on geometry of submerged part.

Stability of floating bodies:

1. **Rotational stability of floating bodies :** Metacentre must lie above centre of gravity.

i.e.

$GM > 0$: Stable rotational equilibrium

$GM = 0$: Neutral equilibrium

$GM < 0$: Unstable rotational equilibrium

2. **Translational stability of floating bodies :**

$$\text{Weight} = \text{Buoyant force}$$

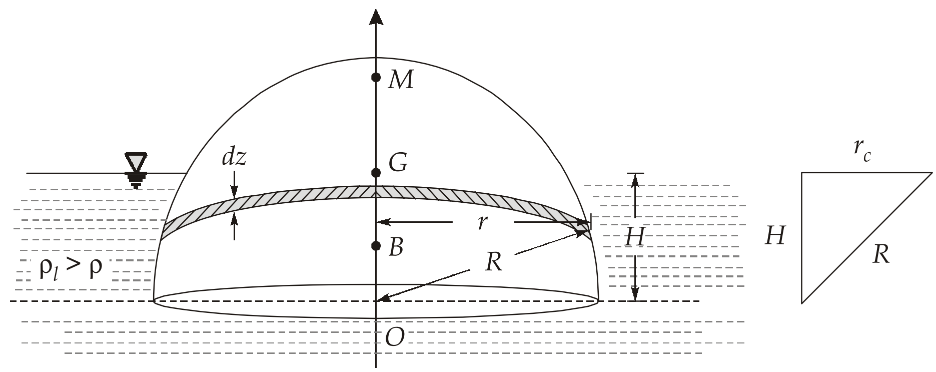
$\frac{dF_B}{dz} > 0$: Stable translational equilibrium.

$\frac{dF_B}{dz} = 0$; Neutral translational equilibrium.

$\frac{dF_B}{dz} < 0$: Unstable translational equilibrium.

6. (a) (ii) Solution:

The hemisphere in its floating condition is shown below.



For equilibrium under floating condition

$\Sigma F_y = 0$

$$\Rightarrow \quad \rho\left(\frac{2}{3}\pi R^3\right) = V_{sub} \cdot \rho_l$$

$$\Rightarrow \quad V_{sub} = \frac{\rho}{\rho_l}\left(\frac{2}{3}\pi R^3\right) \quad \text{...(i)}$$

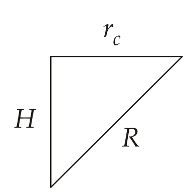
The centre of gravity will lie on axis of symmetry of hemisphere. The distance G along this line from the base of the hemisphere can be found by taking moments of elemental circular strips about the base.

$$OG = \frac{\int_0^R \pi\left(R^2 - z^2\right) z dz}{\frac{2}{3}\pi R^3} = \left(\frac{3}{8}R\right) \quad \text{...(ii)}$$

Similarly centre of buoyancy i.e. centre of immersed volume can be found out.

$$OB = \frac{\int_0^H \pi\left(R^2 - z^2\right) z dz}{\frac{2}{3}\pi R^3 \frac{\rho}{\rho_l}} = \left(\frac{3}{8}R\right)\left(\frac{\rho_l}{\rho}\right)\left(\frac{H}{R}\right)^2\left(2 - \left(\frac{H}{R}\right)^2\right) \quad \text{...(iii)}$$

Let r_c = radius of hemisphere at waterline.



$$R^2 = H^2 + r_c^2$$

$$H^2 = R^2 - r_c^2 \quad \text{...(iv)}$$

$$\therefore \quad OB = \frac{3}{8}\left(\frac{\rho_l}{\rho}\right)R\left(1-\frac{r_c^4}{R^4}\right)$$

$$BM = \frac{I}{V} = \frac{\pi r_c^4}{4\left\{\left(\frac{2}{3}\pi R^3\right)\frac{\rho}{\rho_l}\right\}} = \frac{3}{8}R\left(\frac{\rho_l}{\rho}\right)\left(\frac{r_c^4}{R^4}\right) \quad \text{...(v)}$$

$$GM = BM - BG = BM - (OG - OB)$$

$$= \frac{3}{8}\left(\frac{\rho_l}{\rho}\right)R\left(\frac{r_c}{R}\right)^4 - \frac{3}{8}R + \left[\frac{3}{8}\left(\frac{\rho_l}{\rho}\right)R\left(1-\left(\frac{r_c}{R}\right)^4\right)\right]$$

$$GM = \frac{3}{8}R\left(\frac{\rho_l}{\rho}-1\right) \quad \textbf{Ans.}$$

Since, $\rho_l > \rho$ $\quad GM > 0$ and hence, the equilibrium is stable

For $\quad \frac{\rho_l}{\rho} = 0.7,\ R = 1.5$ m

$$GM = \frac{3}{8}(1.5)\left(\frac{1}{0.7}-1\right)$$

$$GM = 0.24107 \text{ m} \quad \textbf{Ans.}$$

+ve value implies body is rotationally stable.

6. (b) Solution:

Given : $h_B = 3h_A$

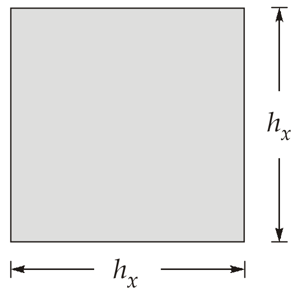
(i)

At any section x

$$h(x) = h_A + \frac{h_B - h_A}{L}x$$

$$h(x) = h_A + \frac{3h_A - h_A}{L}x$$

$$h(x) = h_A\left[1+\frac{2x}{L}\right] \quad \text{...(i)}$$



$$\text{Section modulus, } z(x) = \frac{h^3(x)}{6}$$

$$\sigma(x) = \frac{M(x)}{z(x)} = \frac{6\times P\times x}{\left[h_A\left[1+\frac{2x}{L}\right]\right]^3}$$

$$\sigma(x) = \frac{6PxL^3}{h_A^3\times(L+2x)^3} \quad \text{...(ii)}$$

For maximum stress, $\dfrac{d\sigma(x)}{dx} = 0$

$$\frac{d}{dx}\left[\frac{6PxL^3}{h_A^3\times(L+2x)^3}\right] = 0$$

$$6P\frac{L^3}{h_A^3(L+2x)^2} - 36Px\frac{L^3}{h_A^3(L+2x)^4} = 0$$

$$\frac{-L+4x}{h_A^3(L+2x)^4} = 0$$

So, $x = \dfrac{L}{4}$

$$\sigma_{max} = \sigma\left(\frac{L}{4}\right)$$

$$\sigma_{max} = \frac{6P\cdot\frac{L}{4}\cdot L^3}{h_A^3\left(L+\frac{2L}{4}\right)^3} = \frac{4PL}{9h_A^3}$$

$$\Rightarrow \quad \sigma_{max} = \frac{4PL}{9h_A^3}$$

$$\sigma_B = \sigma(L)$$

From equation (ii),

$$\sigma_B = \frac{2PL}{9h_A^3}$$

$$\frac{\sigma_{max}}{\sigma_B} = \frac{\frac{4PL}{9h_A^3}}{\frac{2PL}{9h_A^3}}$$

$$\Rightarrow \quad \frac{\sigma_{max}}{\sigma_B} = 2$$ **Ans.**

(ii)

$\Sigma F_V = 0;$ $$R_A = P$$

$$M(x) = \left[R_A x - \frac{P}{L}x\left(\frac{x}{2}\right)\right]$$

$$M(x) = Px - \frac{1}{2}x^2\frac{P}{L}$$

$$\sigma(x) = \frac{M(x)}{Z(x)} = \frac{\left[Px - \frac{1}{2}x^2\frac{P}{L}\right]\times 6}{\left[h_A\left(1+\frac{2x}{L}\right)\right]^3}$$

$$\sigma(x) = \frac{-3xP(-2L+x)L^2}{h_A^3(L+2x)^3}$$

For maximum stress, $$\frac{d}{dx}\sigma(x) = 0$$

$$\frac{d}{dx}\left[-3xP(-2L+x)\frac{L^2}{h_A^3(L+2x)^3}\right] = 0$$

$$-3P(-2L+x)\frac{L^2}{h_A^3(L+2x)^3} - 3xP\frac{L^2}{h_A^3(L+2x)^3} + 18xP(-2L+x)\frac{L^2}{h_A^3(L+2x)^4} = 0$$

On simplifying

$$L^2 - 5xL + x^2 = 0$$

$$x_{max} = \left[\frac{5-\sqrt{5^2-4}}{2}\right]L = 0.20871L$$

$$\sigma_{max} = \sigma(0.28071L)$$

$$\sigma_{max} = -3(0.28071L)P(-2L+0.280271L)\frac{L^2}{h_A^3(L+2\times 0.280271L)^3}$$

On solving,
$$\sigma_{max} = 0.394\frac{PL}{h_A^3}$$

Now,
$$\sigma_B = \sigma(L) = -3(L)(P)(-2L+L)\frac{L^2}{h_A^3(L+2L)^3}$$

$$\sigma_B = \frac{PL}{9h_A^3}$$

$$\frac{\sigma_{max}}{\sigma_B} = \frac{0.394\frac{PL}{h_A^3}}{\frac{PL}{9h_A^3}}$$

$$\frac{\sigma_{max}}{\sigma_B} = 3.546$$ **Ans.**

6. (c) Solution:

Given data : N_s = 0.075 rev.; Q = 0.05 m^3/s; $d_s = d_d$ = 150 mm; f = 0.005; η_{hyd} = 75%

Velocity in the pipes (v) $= \frac{Q}{A} = \frac{0.05}{\frac{\pi}{4}\times(0.15)^2}$

$$v = 2.8294 \text{ m/s}$$

Total losses in the pipe, $h_1 = \frac{4fl}{2gd}v^2 + \frac{3v^2}{2g} = \left[\frac{4\times 0.005\times 40}{0.15}+3\right]\times\left[\frac{2.8294^2}{2\times 9.81}\right]$

$$h_1 = 3.4 \text{ m}$$

Therefore, total head required to be developed.

$$H = 35 + 3.4 = 38.4 \text{ m}$$

The speed of pump is determined using N_s

$$N_s = \frac{N(Q)^{1/2}}{(gH)^{3/4}}$$

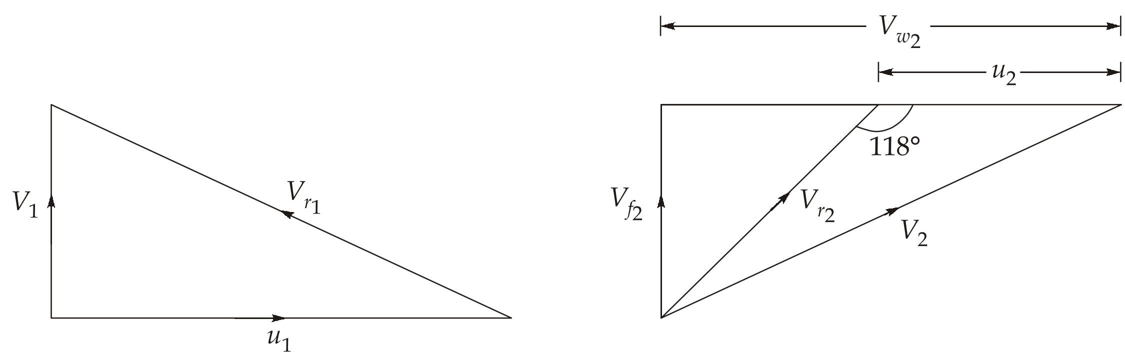
$$0.075 = \frac{N(0.05)^{1/2}}{(9.81 \times 38.4)^{3/4}}$$

$$N = 28.6798 \text{ rev./s}$$ **Ans.**

Let the impeller diameter be 'D'

Flow area perpendicular to impeller outlet periphery

$$\pi D \times \left(\frac{D}{10}\right) \times (1 - 0.06) = 0.295 D^2$$



Inlet velocity triangle **Outlet velocity triangle**

$$V_{f_2} = \frac{Q}{0.295D^2} = \frac{0.05}{0.295D^2} = \frac{10}{59D^2} \text{ m/s}$$

$$u_2 = \pi N D = 3.14 \times 28.6798 \times D = 90.1D \text{ m/s}$$

$$\text{Hydraulic efficiency } (\eta_{hyd}) = \frac{gH}{V_{w_2} u_2}$$

$$0.75 = \frac{9.81 \times 38.4}{(90.1)D \times V_{w_2}}$$

$$V_{w_2} = \frac{5.5746}{D} \text{ m/s}$$

From outlet velocity triangle,

$$\tan(180° - 118°) = \frac{V_{f_2}}{V_{w_2} - u_2} = \frac{\frac{10}{59D^2}}{\frac{5.5746}{D} - 90.1D}$$

$$1.8807 = \frac{10}{59D^2\left[\frac{5.5746 - 90.1D^2}{D}\right]}$$

$$\Rightarrow \quad 5.3171 = 59D(5.5746 - 90.1D^2)$$

$$\Rightarrow 5315.9D^3 - 328.901D + 5.3171 = 0$$

$$\Rightarrow \quad D = -0.256, 0.24022, 0.016$$

$$D = 0.24022 \text{ m or } 240.22 \text{ mm}$$ **Ans.**

7. (a) Solution:

Given data : $N = 375$ rpm; $H = 60$ m; $D = 1.5$ m; $N_s = 0.14$ rev.; $V_{f_1} = 9$ m/s; $V_{w_2} = 0$

$V_2 = 7$ m/s; $V_3 = 2$ m/s; $\eta_{hydraulic} = 95\%$

(i) Runner speed at inlet

$$u_1 = \frac{\pi DN}{60} = \frac{\pi \times 375 \times 1.5}{60} = 29.4524 \text{ m/s}$$

$$\because \quad V_{w_2} = 0 \text{ m/s}$$

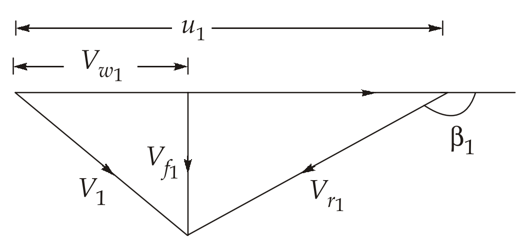
Power transferred to the runner per unit mass flow rate of water

$$P = V_{w_1}u_1 - \overset{0}{\cancel{V_{w_2}}}u_2 = V_{w_1}u_1$$

$$\text{Hydraulic efficiency, } h_{hyd} = \frac{V_{w_1}u_1}{gH}$$

$$0.95 = \frac{V_{w_1} \times 29.4524}{9.81 \times 60}$$

$$V_{w_1} = 18.9855 \text{ m/s}$$



Inlet velocity diagram

$$\tan(180° - \beta_1) = \frac{V_{f_1}}{u_1 - V_{w_1}} = \frac{9}{29.4524 - 18.9855}$$

$$= 139.30929°$$ **Ans.**

(ii) Let the loss of head in the guide-vanes be 'h_g'.

Applying Bernoulli's equation between inlet to guide-vane and exit from guide-vanes (i.e. inlet to runner)

$$\frac{P_0}{\rho g}+\frac{V_0^2}{2g}+z_0 = \frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_g$$

$$V_1^2 = (18.9855)^2 + (9)^2 = 441.4492 \text{ m}^2/\text{s}^2$$

$$\frac{P_0}{\rho g}+\frac{V_0^2}{2g}+z_0 = 60 \text{ m (Total head to turbine)}$$

$$60 = \left(35+\frac{441.4492}{2\times 9.81}+2\right)+h_g$$

$$h_g = 0.5 \text{ m}$$

Head loss in the runner 'h_r'

Applying Bernoulli's equation between points at the runner entry and runner exit.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_r+W$$

where, W = Work head delivered by the fluid to the runner

$$W = \frac{V_{w_1}u_1}{g} = \frac{18.9855\times 29.4524}{9.81} \simeq 56.99 \text{ m}$$

Therefore;

$$h_r = \left[35+\frac{441.4492}{2\times 9.81}+2\right]-\left[-2.2+\frac{7^2}{2\times 9.81}+1.7\right]-56.99$$

$$= 57.5025 - 57 = 0.5125 \text{ m}$$ **Ans.**

Loss of head in draft tube (h_d)

Applying Bernoulli's equation between the points at entry and exit of draft tube gives.

$$\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 = \frac{P_3}{\rho g}+\frac{V_3^2}{2g}+z_3+h_d$$

$$-2.2+\frac{7^2}{2\times 9.81}+1.7 = 0+\frac{2^2}{2\times 9.81}+0+h_d$$

$$h_d = 1.79 \text{ m}$$ **Ans.**

7. (b) (i) Solution:

$$v_\theta = -\frac{c\sin\theta}{r^2}$$

1. Expression for radial velocity, v_r:

The continuity equation for a 2-D, steady incompressible flow is

$$\frac{1}{r}\frac{\partial}{\partial r}(r.v_r)+\frac{\partial(v_\theta)}{r\partial\theta} = 0$$

or, $$\frac{\partial}{\partial r}(rv_r)+\frac{\partial}{\partial\theta}(v_\theta) = 0 \quad ...(i)$$

For the given velocity component:

$$\frac{\partial v_\theta}{\partial\theta} = \frac{\partial}{\partial\theta}\left(-\frac{c\sin\theta}{r^2}\right) = -\frac{c}{r^2}\cos\theta \quad ...(ii)$$

for (i) and (ii), we have:

$$\frac{\partial}{\partial r}(rv_r) = \frac{c}{r^2}\cos\theta$$

$$rv_r = \int_0^r \frac{c}{r^2}\cos\theta dr \quad \text{(Integrating both side w.r.t. r)}$$

$$rv_r = -\frac{c\cos\theta}{r}$$

$$\text{Radial component, } v_r = -\frac{c\cos\theta}{r^2} \quad \text{Answer}$$

2. Resultant velocity:

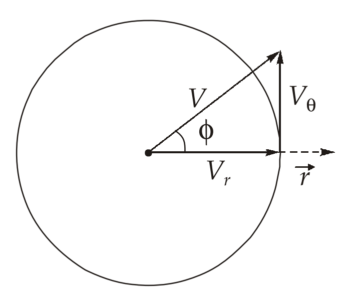
$$\text{Resultant velocity} = \sqrt{v_r^2+v_\theta^2}$$

$$= \left[\left(-\frac{c\cos\theta}{r^2}\right)^2+\left(-\frac{c\sin\theta}{r^2}\right)^2\right]^{1/2}$$

$$= \frac{c}{r^2}(\cos^2\theta+\sin^2\theta)^{1/2} = \frac{c}{r^2} \quad \text{Answer}$$

Direction of resultant velocity:

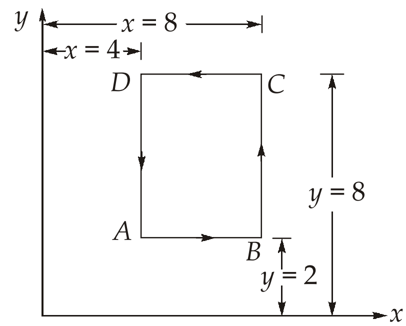
$$\tan\phi = \left(\frac{-c\dfrac{\sin\theta}{r^2}}{-c\dfrac{\cos\theta}{r^2}}\right)$$



$$\tan\phi = \tan\theta$$

$$\phi = \theta$$

7. (b) (ii) Solution:



$$u = 16y - 8x$$

$$v = 8y - 7x$$

$$\Gamma_{ABC} = \int_{AB} (udx + vdy) + \int_{BC} (udx + vdy) + \int_{CD} (udx + vdy) + \int_{DA} (udx + vdy)$$

$$= \int_4^8 (16y - 8x)dx + \int_2^2 (8y - 7x)dy + \int_8^8 (16y - 8x)dx + \int_2^8 (8y - 7x)dy + \int_8^4 (16y - 8x)dx +$$

$$\int_8^8 (8y - 7x)dy + \int_4^4 (16y - 8x)dx + \int_8^2 (8y - 7x)dy$$

$$\Gamma_{ABCD} = \int_4^8 (16y - 8x)dx + \int_2^8 (8y - 7x)dy + \int_8^4 (16y - 8x)dx + \int_8^2 (8y - 7x)dy$$

We can observe that for AB, y is invariant and for BC x is invariant and so on.

$$\Gamma_{ABCD} = \left[16yx - 4x^2\right]_4^8 + \left[4y^2 - 7xy\right]_2^8 + \left[16yx - 4x^2\right]_8^4 + \left[4y^2 - 7xy\right]_8^2$$

(i) (ii) (iii) (iv)

In integral (i): $y = 2$

In integral (ii): $x = 8$

In integral (iii): $y = 8$

In integral (iv): $x = 4$

Substituting these values, we have:

$$\Gamma_{ABCD} = [16 \times 2 \times 8 - 4 \times 8 \times 8 - 16 \times 2 \times 4 + 4^2 \times 4] + [4 \times 8^2 - 7 \times 8^2 - 4 \times 2^2 + 7 \times 2 \times 8] + [16 \times 8 \times 4 - 4 \times 4^2 - 16 \times 8 \times 8 + 4 \times 8^2] + [4 \times 2^2 - 7 \times 4 \times 2 - 4 \times 8^2 + 7 \times 4 \times 8]$$

$$\Gamma_{ABCD} = [256 - 256 - 128 + 64] + [256 - 448 - 16 + 112] + [512 - 64 - 1024 + 256] + [16 - 56 - 256 + 224]$$

$$= -64 - 96 - 320 - 72 = -552$$ Answer

Alternate solution,

Total circulation around a closed loop

$$\Gamma = \Gamma_{AB} + \Gamma_{BC} + \Gamma_{CD} + \Gamma_{DA}$$

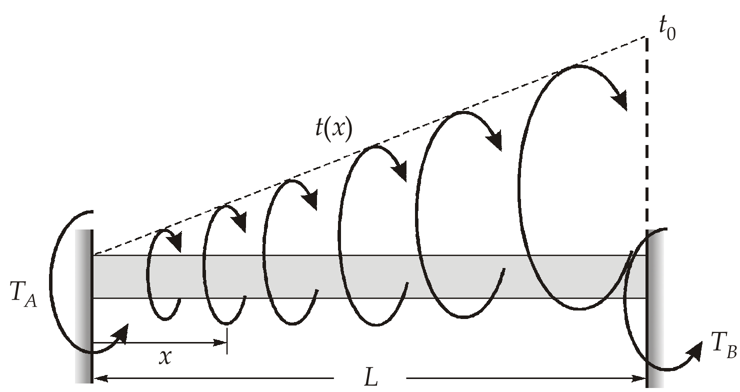
$$\left(u - \frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x + \left(v + \frac{dv}{\partial x}\frac{\delta x}{2}\right)\delta y - \left(u + \frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x - \left(v - \frac{\partial v}{\partial x}\frac{\delta x}{2}\right)\delta y$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\delta x \delta y = \left(\frac{\partial(8y - 7x)}{\partial x} - \frac{\partial(16y - 8x)}{\partial y}\right)$$

$$= (-7 - 16) \times 24 = -552$$

7. (c) (i) Solution:

1. Let T_A and T_B be the reaction torque at A and B.



Torque at a distance x, From A,

$$t_x = -T_A + \frac{1}{2} \times x \times \left(\frac{t_o}{L}x\right)$$

$$\phi = \int_0^L \frac{t_x\,dx}{GJ} = 0$$

$$\int_0^L \frac{\left(-T_A + \frac{t_o x^2}{2L}\right)}{GJ} dx = 0$$

$$-T_A \times L + \frac{t_o L^2}{6} = 0$$

$$T_A = \frac{t_0 L}{6}$$

Also,
$$T_A + T_B = \frac{1}{2} t_o L$$

$$\frac{t_oL}{6} + T_B = \frac{t_oL}{2}$$

$$T_B = \frac{t_oL}{3}$$

2.

$$\phi(x) = \int_0^x \left(-\frac{t_oL}{6} + \frac{t_ox^2}{2L}\right)\frac{dx}{GJ}$$

$$\phi(x) = \frac{1}{GJ}\left(\frac{-t_oLx}{6} + \frac{t_ox^3}{6L}\right)$$

For finding $\phi_{max} = \dfrac{d\phi(x)}{dx} = 0$

$$\Rightarrow \quad \frac{d}{dx}\left[\frac{-t_oLx}{6} + \frac{t_ox^3}{6L}\right] = 0$$

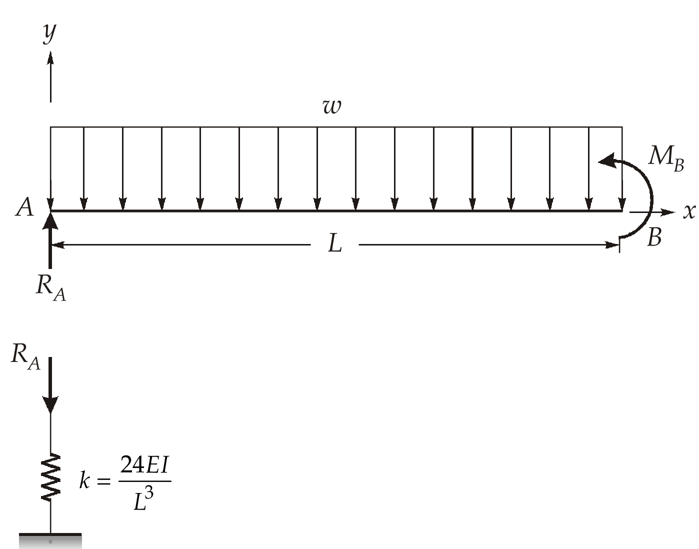
$$\Rightarrow \quad -\frac{t_oL}{6} + \frac{t_ox^2}{2L} = 0$$

$$x = \frac{L}{\sqrt{3}}$$

$$\phi_{max} = \phi\left(\frac{L}{\sqrt{3}}\right) = \frac{1}{GJ}\left[\frac{-t_oL}{6}\left(\frac{L}{\sqrt{3}}\right) + \frac{t_o}{6L}\left(\frac{L}{\sqrt{3}}\right)^3\right]$$

$$\phi_{max} = \frac{-\sqrt{3}}{27}\frac{t_oL^2}{GJ} = -0.064\frac{t_oL^2}{GJ}$$

7. (c) (ii) Solution:



For reactions, using equilibrium equation

$\Sigma F_y = 0$

$\Rightarrow \quad R_A = wL$

$\Sigma M_A = 0$

$\Rightarrow \quad M_B = \dfrac{wL^2}{2}$

Differential equation for deflection is given as

$$EI\frac{d^2y}{dx^2} = M(x) = R_A x - \frac{wx^2}{2} = wLx - \frac{wx^2}{2} \quad \text{...(i)}$$

Integrating both side

$$EI\frac{dy}{dx} = \frac{wLx^2}{2} - \frac{wx^3}{6} + c_1 \quad \text{...(ii)}$$

Again by integrating both side, we get

$$EIy = \frac{wLx^3}{6} - \frac{wx^4}{24} + c_1 x + c_2 \quad \text{...(iii)}$$

Boundary conditions, at $x = L$, $\dfrac{dy}{dx} = 0$

$\Rightarrow \quad c_1 = -\dfrac{1}{3}wL^3$

at $x = 0$, $y = \dfrac{R_A}{k} = \dfrac{wLL^3}{24EI} = \dfrac{wL^4}{24EI}$

From equation (iii)

$\therefore \quad EI \cdot \dfrac{wL^4}{24EI} = c_2$

$\Rightarrow \quad c_2 = \dfrac{wL^4}{24}$

Thus, for deflection at B $(x = L)$

$$EIy_B = \frac{wL}{6}(L)^3 - \frac{w(L)^4}{24} - \frac{1}{3}wL^3(L) + \frac{wL^4}{24}$$

or $\quad y_B = -\dfrac{1}{6}\dfrac{wL^4}{EI}$

8. (a) **Solution:**

Applying Bernoulli's equation at points 1 and 'D', we get

$$\frac{P_{atm}}{\rho g} + \frac{V_1^2}{2g} + 5.5 = \frac{P_D}{\rho g} + \frac{V_D^2}{2g} + 0$$

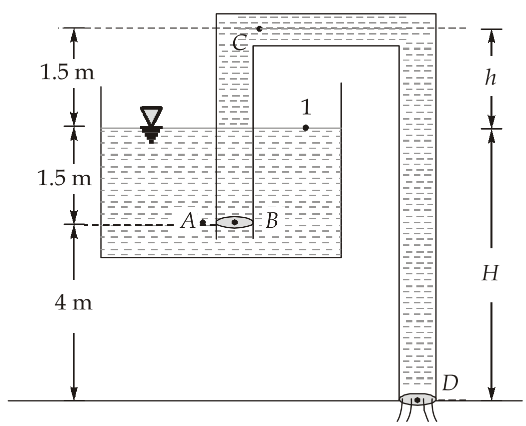
The horizontal plane through D is taken as datum

$$P_D = P_{\text{atm}} \text{ and } V_1 << V_D \quad (\because \quad A_{\text{tank}} >> A_{\text{siphon}})$$

$$\frac{P_{atm}}{\rho g} + 0 + 5.5 = \frac{P_{atm}}{\rho g} + \frac{V_D^2}{2g}$$

Velocity of oil through siphon,

$$V_D = \sqrt{2 \times g \times 5.5} = 10.39 \text{ m/s}$$



Pressure at point 'A'

$$P_A = P_{\text{atm}} + \rho \times g(1.5 \text{ m})$$

$$P_A = 101 + \frac{0.8 \times (10)^3 \times 9.81 \times 1.5}{1000}$$

$$P_A = 101 + 11.772$$

$$P_A = 112.772 \text{ kPa}$$ **Ans.**

Applying Bernoulli's equation between A and B.

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$\Rightarrow \quad \frac{P_A}{\rho g} + 0 + 4 = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + 4$$

$$\Rightarrow \quad P_B = P_A - \rho\frac{V_B^2}{2}$$

$$V_B = V_D = 10.39 \text{ m/s (from continuity equation)}$$

and $\quad P_A = 112.77$ kPa

$$\Rightarrow \quad P_B = 112.772 - 0.8\times10^3 \times \frac{(10.39)^2}{2\times10^3} = 69.592 \text{ kPa}$$

$$P_B = 69.592 \text{ kPa}$$

Applying Bernoulli's equation between 1 and C.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C$$

$$\Rightarrow \quad \frac{P_{atm}}{\rho g} + 0 + 5.5 = \frac{P_C}{\rho g} + \frac{(10.39)^2}{2g} + 7$$

$$\Rightarrow \quad \frac{P_C}{\rho g} = \frac{P_{atm}}{\rho g} - 7$$

$$\Rightarrow \quad P_c = 101 - \frac{7\times0.8\times10^3\times9.81}{10^3} = 46.06 \text{ kPa}$$

$$P_c = 46.06 \text{ kPa}$$ **Ans.**

For maximum height of summit (point 'C') above liquid level the pressure at 'C' will be the vapour-pressure of the liquid at working temperature.

Let 'h' be the maximum height of point 'C'.

Using Bernoulli's equation between 1 and C

$$\frac{P_{atm}}{\rho g} + 0 + 5.5 = \frac{30\times10^3}{\rho g} + \frac{10.39^2}{2g} + 5.5 + h$$

$$h = \frac{101\times10^3}{0.8\times10^3\times9.81} - \frac{30\times10^3}{0.8\times10^3\times9.81} - 5.502$$

$$h = 3.5469 \text{ m}$$ **Ans.**

Let H be the maximum depth which renders the pressure at C to be the vapour pressure. Then from Bernoulli's equation between 1 and D.

$$V_D = \sqrt{2gH}$$

Applying Bernoulli's equation between 1 and 'C'

$$\frac{P_{atm}}{\rho g} + 0 + H = \frac{30 \times 10^3}{0.8 \times 10^3 \times 9.81} + \frac{V_D^2}{2g} + H + 1.5$$

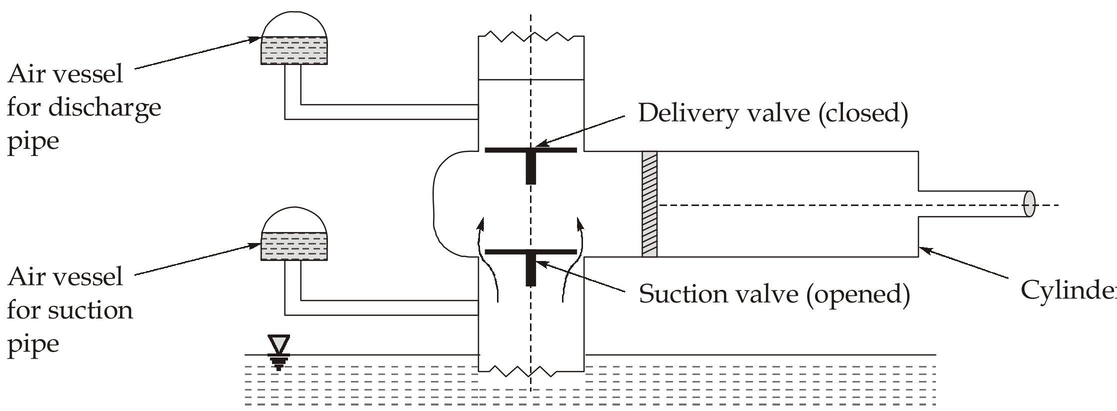
$$H = \frac{P_{atm}}{\rho g} - \frac{30 \times 10^3}{0.8 \times 10^3 \times 9.81} - 1.5$$

$$H = 7.5469 \text{ m}$$ **Ans.**

8. **(b) (i) Solution:**

Air vessels are containers containing compressed air which can contract or expand to absorb most of the pressure fluctuations.

Air vessels are used in reciprocating pump to reduce pulsation due to pressure fluctuation.



Reciprocating pump connected with air-vessels

The pulsation of pressure due to inertia or acceleration head in suction and delivery pipe and the non-uniformity of discharge during the delivery-stroke may-largely be eliminated by connecting a large and closed chamber to both the suction and delivery pipes at points close to cylinder as shown in the above diagram.

Working principle:

An air vessel in a reciprocating pump acts like a flywheel of an engine.

Whenever the pressure rises, water in excess of the mean discharge is forced into the air vessel, thereby compressing the air within the vessel. When the water pressure in the pipe falls the compressed air again eject the excess water out, thus an air vessel acts like an intermediate reservoir. On the suction side, the water first accumulates here and is then transferred to the cylinder of the pump. On the delivery side, the water first goes to the vessel and then flows with a uniform velocity in the delivery pipe. The column of water which is now fluctuating, is only between the pump cylinder and the air vessels which is very small due to the vessels being fitted near to the pump cylinder as possible.

Advantages of air-vessel attached to a reciprocating pump is as follows.

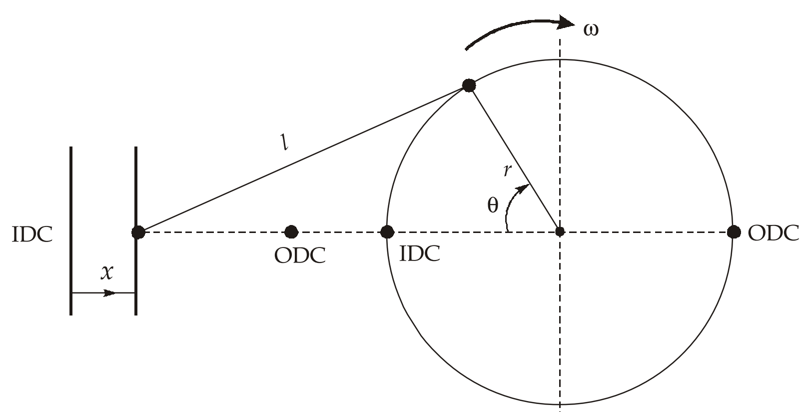
(a) Suction side

1. Reduces possibility of cavitation.
2. Pump can be run at a higher speed.
3. The length of the suction pipe below the air vessel can be increased.

(b) Delivery side

1. A large amount of power consumed in supplying the accelerating head can be saved.
2. Maintains almost a constant rate of discharge.

8. (b) (ii) Solution:



Piston displacement diagram of a reciprocating pump

Liquid mass in suction and delivery side gets accelerated and decelerated due to typical accelerating motion of piston, this causes a non-uniform additional head known as acceleration-head.

$$x = r - r\cos\theta$$

$$x = r(1 - \cos\omega t)$$

$$V = \frac{dx}{dt} = r\omega\,(\sin\omega t)$$

$$V_l = \frac{A}{a} r\omega \sin\omega t$$

where (V_l): is the velocity of liquid written using continuity equation. And 'A' and 'a' are cross-sectional areas of cylinder and pipeline.

$$a_l = \frac{dV_l}{dt} = \frac{A}{a} r\omega^2 \cos\omega t$$

Therefore; force required to accelerate liquid mass by this acceleration is given by

$$F = ma_l = (\rho l a)\left(\frac{A}{a}\right) r\omega^2 \cos\omega t$$

$$F = \rho l A r\omega^2 \cos\theta$$

$$l = \text{Length of pipeline; } \theta = \omega t$$

Pressure head caused by this force 'F'.

$$\frac{F}{a\rho g} = \frac{l}{g}\left(\frac{A}{a}\right) r\omega^2 \cos\theta$$

$$h_a = \frac{l}{g}\left(\frac{A}{a} r\omega^2 \cos\theta\right)$$

This is know as acceleration head.

For suction and delivery line it can be written as

$$(h_a)_s = \frac{l_s}{g}\left(\frac{A}{a_s} r\omega^2 \cos\theta\right) \quad \textbf{Ans.}$$

$$(h_a)_d = \frac{l_d}{g}\left(\frac{A}{a_d} r\omega^2 \cos\theta\right) \quad \textbf{Ans.}$$

8. (c) Solution:

Given area = Rectangle (OACF) – Triangle (EFG) – Quarter circle (DBC)

$$\text{Given area} = (A_{\text{total}} - A_{\text{triangle}} - A_{\text{quarter circle}})$$

$$= (A_1 - A_2 - A_3)$$

$$(x_{\text{cm}}, y_{\text{cm}}) = \left[\frac{A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3}{A_1 - A_2 - A_3}, \frac{A_1\bar{y}_1 - A_2\bar{y}_2 - A_3\bar{y}_3}{A_1 - A_2 - A_3}\right]$$

$$A_1 = 100 \text{ cm} \times 60 \text{ cm} = 6000 \text{ cm}^2$$

$$\bar{x}_1 = \frac{100}{2} = 50 \text{ cm}; \quad \bar{y}_1 = \frac{60}{2} = 30 \text{ cm}$$

$$A_2 = \frac{1}{2}(30 \times 30) = 450 \text{ cm}^2$$

$$\bar{x}_2 = \frac{30}{3}\text{ cm} = 10\text{ cm};\ \bar{y}_2 = 30 + \frac{2\times 30}{3} = 50\text{ cm}$$

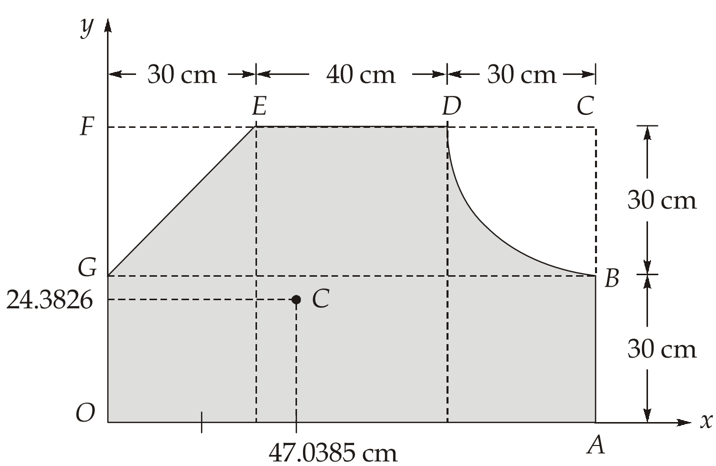
$$A_3 = \frac{\pi(30^2)}{4} = 225\pi\,\text{cm}^2$$

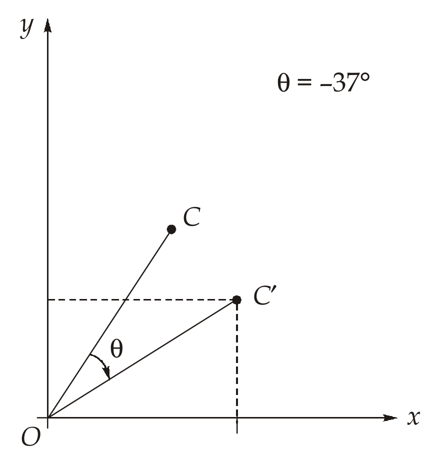
$$\bar{x}_3 = 100 - \frac{4(30)}{3\pi} = \left(100 - \frac{40}{\pi}\right)\text{ cm};$$

$$\bar{y}_3 = 60 - \frac{4(30)}{3\pi} = \left(60 - \frac{40}{\pi}\right)\text{ cm}$$

$$(x_{cm}, y_{cm}) = \left[\frac{(6000)(50)-(450)(10)-(225\pi)\left(100-\frac{40}{\pi}\right)}{6000-450-225\pi}, \frac{(6000)(30)-(450)(50)-(225\pi)\left(60-\frac{40}{\pi}\right)}{6000-450-225\pi}\right]$$

$$(x_{cm}, y_{cm}) = (48.277\text{ cm}, 25.621\text{ cm})$$ **Ans.**





$$C' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(-37°) & -\sin(-37°) \\ \sin(37°) & \cos(37°) \end{bmatrix}\begin{bmatrix} 48.277 \\ 25.621 \end{bmatrix} = \begin{bmatrix} 53.974 \\ -8.591 \end{bmatrix}$$

New centroid location $C' = (53.974\text{ cm}, -8.591\text{ cm})$ **Ans.**

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