MADE EASY

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Detailed Solutions

ESE-2026 Mains Test Series

Civil Engineering Test No : 4

Section A : Design of Concrete and Masonry Structures (All Topics)

1. (a) Solution:

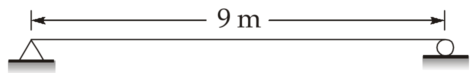
(i) Ordinary mild steel has a low yield stress and low elastic limit, so it cannot be stressed to the high levels required in prestressing. Prestressing needs steel to be stressed up to about 70–80% of its ultimate strength, which would cause plastic deformation in mild steel. Also, mild steel exhibits higher relaxation losses, leading to rapid loss of prestress. Hence, high-tensile steel is used instead.

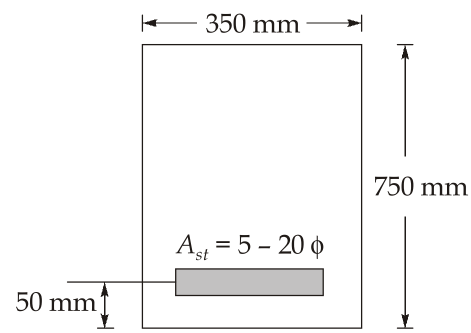
(ii) In pre-tensioned beams, all tendons are tensioned first and then released simultaneously, causing the concrete to shorten elastically at once. This elastic shortening affects all the tendons equally, resulting in higher prestress loss. In post-tensioned beams, tendons are tensioned one after another, so elastic shortening caused by earlier tendons does not affect the tendons stressed later. Therefore, the overall elastic shortening loss is smaller.

A parabolic tendon profile produces an upward vertical component of prestressing force, which acts like a uniformly distributed upward load (camber). This upward force counteracts the downward deflection caused by dead load. As a result, the net deflection is reduced, and part (or whole) of the dead load deflection is compensated.

1. (b) Solution:

Given section properties





Cross-section

$$\text{Width } (b) = 350 \text{ mm}$$

$$\text{Effective depth } (d) = 700 - 50 = 650 \text{ mm}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1570.8 \text{ mm}^2$$

Depth of Neutral Axis (x_u)

Equating total compressive force to total tensile force:

$$\Rightarrow \quad 0.36\, f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

$$\Rightarrow \quad x_u = \frac{0.87 \times 415 \times 1570.8}{0.36 \times 20 \times 350} = 225.05 \text{ mm}$$

Limiting depth of NA, $x_{u,\text{lim}} = 0.48\text{ d}$ (For Fe415)

$$= 0.48 \times 650$$

$$= 312 \text{ mm}$$

Since $x_u < x_{u,\text{lim}}$, the section is Under-Reinforced.

Ultimate Moment of Resistance (M_u)

$$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$$

$$\Rightarrow \quad M_u = 0.87 \times 415 \times 1570.8 \times (650 - 0.42 \times 225.05)$$

$$\Rightarrow \quad M_u = 315.03 \times 10^6 \text{ N-mm} = 315.03 \text{ kNm}$$

OR

$$M_u = 0.36\, f_{ck}\, b x_u\, (d - 0.42\, x_u)$$

$$\Rightarrow \quad M_u = 0.36 \times 20 \times 350 \times 225.05\,(650 - 0.42 \times 225.05)$$

$$\Rightarrow \quad M_u = 315.03 \text{ kNm}$$

Load Calculation:

For a simply supported beam with UDL:

$$M_u = \frac{w_u l_{eff}^2}{8}$$

$$\Rightarrow \quad 315.03 = \frac{w_u \times 9^2}{8}$$

$$\Rightarrow \quad w_u = 31.11 \text{ kN/m}$$

Total Working Load (w), $w = \frac{w_u}{1.5} = \frac{31.11}{1.5} = 20.74$ kN/m

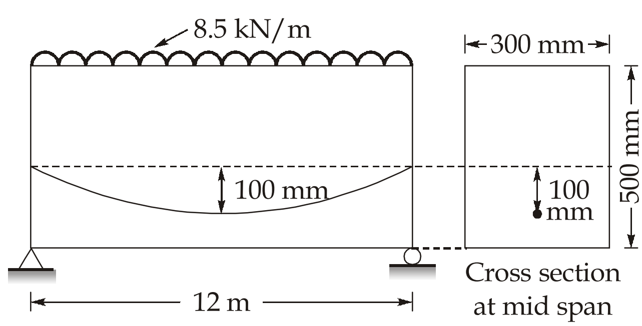
Self-Weight of Beam (w_d), $w_d = 0.35 \times 0.70 \times 25 = 6.125$ kN/m

Safe Imposed (Live) Working Load:

$$w_{live} = 20.74 - 6.125 = 14.615 \text{ kN/m}$$

1. (c) Solution:

Given:



Beam size = 300 × 500 mm

Span L = 12 m

Superimposed load = 8.5 kN/m

Unit weight of concrete = 24 kN/m^3

Prestressing force P = 650 kN

Eccentricity at mid-span e = 100 mm

Loss of prestress = 15%

Section Properties

$$A = 300 \times 500 = 150000 \text{ mm}^2$$

$$Z = \frac{bh^2}{6} = \frac{300 \times 500^2}{6} = 12.5 \times 10^6 \text{ mm}^3$$

Loads and Moments

$$\text{Self-weight, } w_d = 0.3 \times 0.5 \times 24 = 3.6 \text{ kN/m}$$

$$\text{Dead load moment, } M_d = \frac{w_d L^2}{8} = \frac{3.6 \times 12^2}{8} = 64.8 \text{ kNm}$$

$$\text{Live load moment, } M_l = \frac{8.5 \times 12^2}{8} = 153.0 \text{ kNm}$$

Stress Components

Direct stress:

$$\frac{P}{A} = \frac{650000}{150000} = 4.333 \text{ N/mm}^2$$

Bending stress due to prestress:

$$\frac{Pe}{Z} = \frac{650000 \times 100}{12.5 \times 10^6} = 5.20 \text{ N/mm}^2$$

Stress due to dead load:

$$\frac{M_d}{Z} = \frac{64.8 \times 10^6}{12.5 \times 10^6} = 5.184 \text{ N/mm}^2$$

Stress due to live load:

$$\frac{M_l}{Z} = \frac{153 \times 10^6}{12.5 \times 10^6} = 12.24 \text{ N/mm}^2$$

Stresses at Transfer Stage (Prestress + Self-weight)

Top fiber,

$$f_{top} = 4.333 - 5.20 + 5.184 = 4.32 \text{ N/mm}^2 \text{ (Compressive)}$$

Bottom fiber,

$$f_{bottom} = 4.333 + 5.20 - 5.184 = 4.35 \text{ N/mm}^2 \text{ (Compressive)}$$

Stresses at Service Stage (15% Loss) (Effective prestress + self weight + live load)

$$P_f = (1 - 0.15) \times 650 \text{ kN} = 552.5 \text{ kN}$$

Effective direct stress:

$$\frac{P_t}{A} = 4.333 \times 0.85 = 3.68 \text{ N/mm}^2$$

Effective prestress bending:

$$\frac{P_t e}{Z} = 5.20 \times 0.85 = 4.42 \text{ N/mm}^2$$

Top fiber,

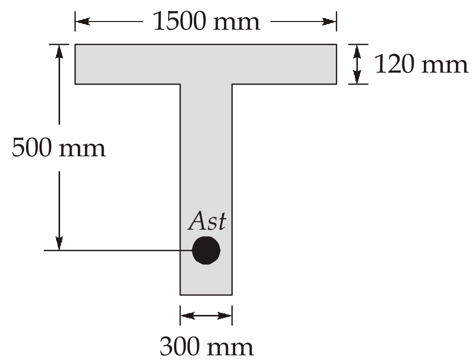
$$f_{top} = 3.68 - 4.42 + 5.18 + 12.24 = 16.68 \text{ N/mm}^2 \text{ (Compressive)}$$

Bottom fiber,

$$f_{bottom} = 3.68 + 4.42 - 5.18 - 12.24 = -9.32 \text{ N/mm}^2 \text{ (Tensile)}$$

1. (d) Solution:

Given:



$$f_{ck} = 15 \text{ N/mm}^2$$
$$f_y = 415 \text{ N/mm}^2$$
$$b_f = 1500 \text{ mm}, D_f = 120 \text{ mm}$$
$$b_w = 300 \text{ mm}, d = 500 \text{ mm}$$

Limiting Depth of Neutral Axis

For Fe-415 steel, $x_{u,\lim} = 0.48\, d$

$\Rightarrow$ $x_{u,\lim} = 0.48 \times 500 = 240 \text{ mm} > (D_f = 120 \text{ mm})$

Since, $x_{u,\lim} > D_f$

So, (limiting) Neutral axis lies in the web portion.

Stress Block Check and Effective Flange Depth

$$\frac{3}{7}x_{u,\lim} = \frac{3}{7}\times 240 = 102.86 \text{ mm} < (D_f = 120 \text{ mm})$$

$\because$ $\frac{3}{7}x_{u,\lim} < D_f$

The stress block is not uniform stressed over the flange depth.

Effective equivalent flange thickness:

$$y_f = 0.15\, x_{u,\lim} + 0.65\, D_f$$

$\Rightarrow$ $y_f = 0.15(240) + 0.65(120) = 114 \text{ mm}$

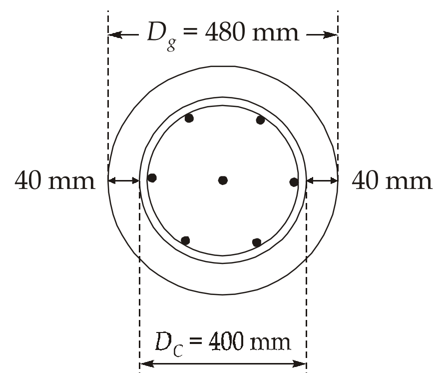
Limiting Moment of Resistance

$$M_{u,\,\lim} = 0.36\,f_{ck} b_w\, x_{u,\lim}(d - 0.42\,x_{u,\lim}) + 0.45\,f_{ck}\,(b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$

$$\Rightarrow \quad M_{u,\,\lim} = 0.36 \times 15 \times 300 \times 240 \times (500 - 0.42 \times 240) + 0.45 \times 15 \times (1500 - 300) \times 114 \times \left(500 - \frac{114}{2}\right)$$

$$\Rightarrow \quad M_{u,\,\lim} = 155.21 + 409.066 = 564.27 \text{ kN-m}$$

1. (e) Solution:



Given:

$$D_g = 480 \text{ mm}$$
$$f_{ck} = 25 \text{ N/mm}^2$$
$$f_y = 415 \text{ N/mm}^2$$

Area of main reinforcement, $A_{sc} = 6 \times \frac{\pi}{4} \times 20^2 = 1884.96 \text{ mm}^2$

Helix: 8 mm Dia, $p = 75$ mm

Clear cover = 40 mm

Check for Helical Reinforcement

Gros area, $A_g = \frac{\pi}{4} \times 480^2 = 180955.7 \text{ mm}^2$

Dia. of core, $D_c = 480 - 2(40) = 400$ mm

Area of core, $A_c = \frac{\pi}{4} \times 400^2 = 125663.7 \text{ mm}^2$

Volume of core per pitch, $V_c = 125663.7 \times 75 = 9424777.5 \text{ mm mm}^3$

$d_h = 400 - 8 = 392$ mm

Volume of helix reinforement per pitch,

$$V_h = (\pi \times 392)\left(\frac{\pi}{4} \times 8^2\right) = 61902.158 \text{ mm}^3$$

$$\frac{V_h}{V_c} = \frac{61902.158}{9424777.5} = 0.006568$$

$$0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y} = 0.36\left(\frac{480^2}{400^2} - 1\right) \times \frac{25}{415} = 0.00954$$

Since $\frac{V_h}{V_c} < 0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y}$, 5% increase not permitted.

As per clause 39.4 of IS 456:2000

Ultimate Load Capacity

$$A_{conc} = A_g - A_{sc} = 180955.7 - 1884.96 = 179070.7 \text{ mm}^2$$

Load carrying capacity of column is given by,

$$P_u = 0.4 f_{ck} A_{conc} + 0.67 f_y A_{sc}$$

$$\Rightarrow \quad P_u = (0.4 \times 25 \times 179070.7) + (0.67 \times 415 \times 1884.96)$$

$$\Rightarrow \quad P_u = 2314.82 \text{ kN}$$

2. (a) (i) Solution:

In cantilever beam, the top fibers are in tension and the bottom fibres are in compression.

Width of bottom, $b = 500$ mm

Effective length, $l_{eff} = 2.5$ m

Effective Depth, $d = 500 - 40 = 460$ mm

Area of Steel, $A_{st} = 6 \times \frac{\pi}{4} \times 16^2 = 1206.37 \text{ mm}^2$

Depth of Neutral Axis (x_u)

Equating compression (C) and tension (T):

$$\Rightarrow \quad C = T$$

$$\Rightarrow \quad T = 0.36 f_{ck}\, b\, x_u = 0.87 f_y A_{st}$$

$$\Rightarrow \quad 0.36 \times 20 \times 500 \times x_u = 0.87 \times 415 \times 1206.37$$

$$\Rightarrow \quad 3600\, x_u = 435{,}500$$

$$x_u = 120.98 \text{ mm} \qquad \text{(From bottom)}$$

Limiting depth of neutral axis

For Fe415 steel, $x_{u,\lim} = 0.48\, d = 0.48 \times 460 = 220.8$ mm

$\because \quad x_u < x_{u,\lim}$

So section is under-reinforced.

Ultimate Moment of Resistance (M_u)

$$M_u = 0.87\, f_y\, A_{st}\, (\text{d} - 0.42\, x_y)$$

$$\Rightarrow \quad M_u = 0.87 \times 415 \times 1206.37 \times (460 - 0.42 \times 120.98)$$

$$\Rightarrow \quad M_u = 178.23 \text{ kNm}$$

Maximum UDL Intensity (w_u)

For a cantilever beam with UDL, the maximum bending moment occurs at the fixed support:

$$M_{\max} = \frac{w_u L^2}{2}$$

Equating $M_{\max} = M_u$:

$$\Rightarrow \quad 178.23 = \frac{w_u \times 2.5^2}{2}$$

$$\Rightarrow \quad w_u = 57.03 \text{ kN/m}$$

2. (a) (ii) Solution:

1. Under-reinforced sections are preferred over over-reinforced sections.

   In an under-reinforced section, tensile steel yields before the concrete in compression reaches its ultimate strain. Yielding of steel provides large deflection and visible cracking before failure, giving adequate warning and ensuring ductile behavior. In an over-reinforced section, concrete crushes before steel yields. Since concrete is brittle in compression, failure occurs suddenly without warning. Hence, under reinforced sections are preferred to ensure ductility, safety, and redistribution of moments as assumed in limit state design.

2. Minimum reinforcement is specified even when bending moment is very small.

   Minimum reinforcement is provided to:

   - Control cracking due to shrinkage and temperature effects.
   - Prevent sudden brittle failure immediately after cracking.
   - Ensure adequate ductility and redistribution of stresses.
   - Account for unexpected moments due to secondary effects or construction tolerances.

   Thus, even if calculated bending moment is small, minimum reinforcement ensures crack control and structural integrity.

3. Maximum reinforcement is limited in beams as per codal provisions.

   Maximum reinforcement is restricted to:

   - Prevent congestion of reinforcement, which affects proper compaction of concrete.
   - Avoid transformation of the section into an over-reinforced (brittle) section.
   - Ensure sufficient space for concrete placement and effective bond.

   Limiting maximum reinforcement ensures ductile failure and proper constructability.

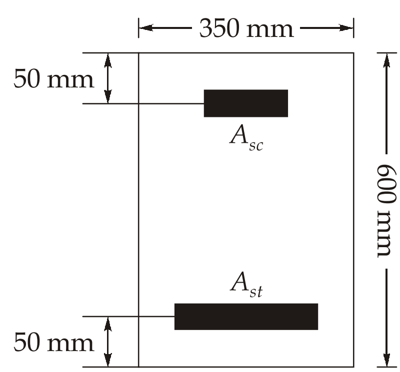
4. Shear reinforcement is required even though concrete has some shear strength.

   Although concrete can resist shear through aggregate interlock, dowel action, and uncracked compression zone, its shear strength is limited and brittle in nature. Shear reinforcement is provided to:

   - Prevent sudden diagonal tension failure.
   - Carry shear after cracking of concrete.
   - Improve ductility and energy absorption capacity.
   - Hold cracked sections together and enhance overall safety.

2. (b) Solution:

Given: $M_u = 414$ kN-m



Width (b) = 300 mm

Effective depth (d) = 600 – 50 = 550 mm

Compression cover (d') = 50 mm

Concrete strength (f_{ck}) = 20 N/mm^2

Steel strength (f_y) = 415 N/mm^2

Limiting Capacity of the Section ($M_{u,\,lim}$)

For the limiting neutral axis depth is:

$$x_{u,\lim} = 0.48\, d = 0.48 \times 550 = 264 \text{ mm}$$

$$M_{u,\lim} = 0.138 f_{ck} b d^2$$

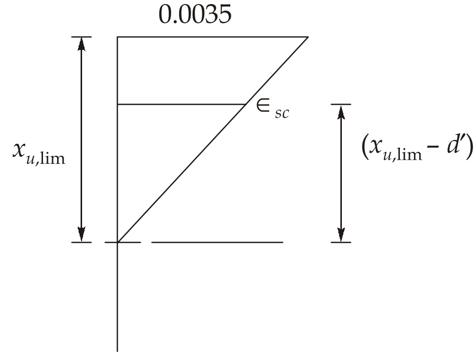
$\Rightarrow$ $$M_{u,\lim} = 0.138 \times 20 \times 300 \times 550^2 \times 10^{-6} = 250.47 \text{ kNm}$$

Since:

$$M_u = 414.0 \text{ kNm} > M_{u,\lim} = 250.47 \text{ kNm}$$

A doubly reinforced section is required.

Stress in Compression Steel (f_{sc})



Strain diagram at limit state

Strain at compression steel level

$$\in_{sc} = 0.0035\left(\frac{x_{u,\lim} - d'}{x_{u,\lim}}\right)$$

$$\in_{sc} = 0.0035\left(\frac{264 - 50}{264}\right) = 0.002837$$

Using the given design stress strain table for Fe 415

$$f_{yd} = 0.87 f_y = 0.87 \times 415 = 361.05 \text{ N/mm}^2$$

- At strain 0.00276: $0.975 f_{yd} = 352.02 \text{ N/mm}^2$
- At strain 0.00380: $1.0 f_{yd} = 361.05 \text{ N/mm}^2$

Interpolating:

$$f_{sc} = 352.02 + \frac{(0.002837 - 0.00276)}{(0.00380 - 0.00276)} \times (361.05 - 352.02)$$

$$f_{sc} = 352.69 \text{ N/mm}^2$$

$\because$ $$M_u = M_{u1} + M_{u2} = M_{u,\lim} + M_{u2}$$

Now,

For area of Compression Steel (A_{sc})

Additional moment to be resisted:

$$M_{u2} = M_u - M_{u,\lim} = 414.0 - 250.47 = 163.53 \text{ kNm}$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')}$$

$$\Rightarrow \quad A_{sc} = \frac{163.53 \times 10^6}{(352.69 - 0.45 \times 20)(550 - 50)}$$

$$\Rightarrow \quad A_{sc} = 951.61 \text{ mm}^2$$

Area of Tension Steel (A_{st})

Total tension steel:

$$A_{st} = A_{st1} + A_{st2}$$

(a) Tension Steel for $M_{u,\lim}$

$$A_{st1} = \frac{M_{u,\lim}}{0.87 f_y (d - 0.42 x_{u,\lim})}$$

$$\Rightarrow \quad A_{st1} = \frac{250.47 \times 10^6}{361.05 \times (550 - 0.42 \times 264)}$$

$$\Rightarrow \quad A_{st1} = 1576.02 \text{ mm}^2$$

(b) Tension Steel for Additional Moment (A_{st2})

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y}$$

$$A_{st2} = \frac{951.93 \times (352.52 - 8.94)}{361.05}$$

$$A_{st2} = 905.86 \text{ mm}^2$$

Total Tension Steel

$$A_{st} = 1576.02 + 905.86 = 2481.88 \text{ mm}^2$$

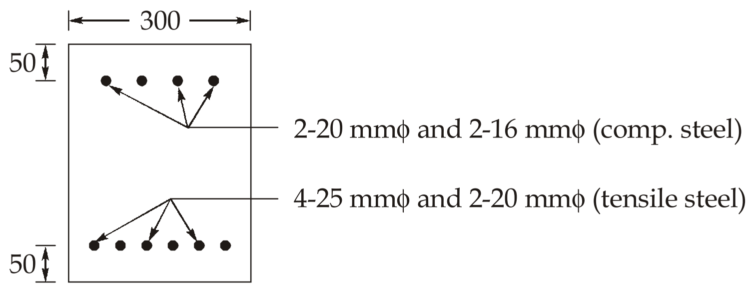
Final Reinforcement Design

- **Compression Steel:**

  $A_{sc} = 951.93 \text{ mm}^2$ (Provide 2-20 mmϕ and 2-16 mmϕ)

- **Tension Steel:**

  $A_{st} = 2481.88 \text{ mm}^2$ (Provide 4-25 mmϕ and 2-20 mmϕ)



2. (c) Solution:

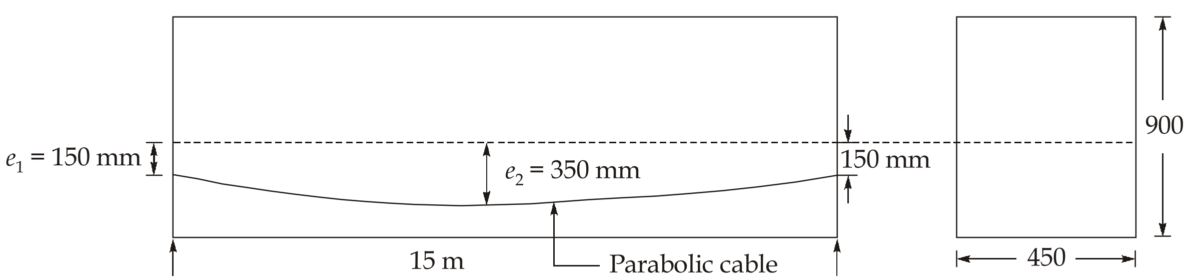
Given: Section Properties

Area, $A = 450 \times 900 = 4.05 \times 10^5 \text{ mm}^2$

Moment of Inertia, $I = \dfrac{450 \times 900^3}{12} = 2.73375 \times 10^{10} \text{ mm}^4$

Net Cable Dip, $h = e_2 - e_1 = 350 - 150 = 200 \text{ mm}$

Effective Prestress $P_e = 2200 \times (1 - 0.15) = 1870 \text{ kN}$



Load Intensities

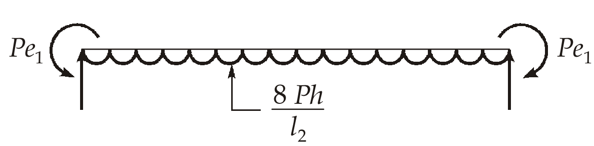
Self-weight (w_{DL}):

$$w_{DL} = (0.45 \times 0.90 \times 1) \times 25 = 10.125 \text{ kN/m}$$

Live Load (w_{LL}):

$$w_{LL} = 15 \text{ kN/m}$$

Using load balancing concept, the parabolic profile may be replaced with following Loading conditions



Deflection due to prestressing force:

$$\Delta_p = \text{due to } P_{e1} + \text{due to } UDL \text{ of } \frac{8Ph}{l^2}$$

$$\Rightarrow \quad \Delta_p = \frac{(Pe_1)l^2}{8E_cI} + \frac{5}{384} \times \frac{\left(\frac{8Ph}{l^2}\right)l^4}{E_cI}$$

$$\Rightarrow \quad \Delta_p = \frac{Pe_1l^2}{8E_cI} + \frac{5}{48} \times \left(\frac{Phl^2}{E_cI}\right)$$

Deflections:

Upward Deflection due to Prestress (Δ_p)

For a parabolic cable with end eccentricities, the upward deflection is given by

$$\Delta_p = \frac{Pe_1L^2}{8E_cI} + \frac{5Phl^2}{48E_cI}$$

$$\Delta_{p1} \text{ (due to } e_1) = \frac{2200 \times 150 \times 15000^2}{8 \times 36 \times 2.73375 \times 10^{10}} = 9.431 \text{ mm } (\uparrow)$$

$$\Delta_{p2} \text{ (due to } h) = \frac{5 \times 2200 \times 200 \times 15000^2}{48 \times 36 \times 2.73375 \times 10^{10}} = 10.478 \text{ mm } (\uparrow)$$

$$\Delta_{p\text{ (initial)}} = 19.909 \text{ mm}(\uparrow)$$

Downward Deflection due to Dead Load (Δ_{DL})

$$\Delta_{DL} = \frac{5w_{DL}l^4}{384E_cI}$$

$$\Rightarrow \quad \Delta_{DL} = \frac{5 \times 10.125 \times 10^{-3} \times 15000^4}{384 \times 36 \times 2.73375 \times 10^{10}} = 6.782 \text{ mm } (\downarrow)$$

Downward Deflection due to Live Load (Δ_{DL})

$$\Delta_{LL} = \frac{5w_{LL}L^4}{384E_cI}$$

$$\Delta_{LL} = \frac{5 \times 15 \times 10^{-3} \times 15000^4}{384 \times 36 \times 2.73375 \times 10^{10}} = 10.047 \text{ mm } (\downarrow)$$

I. Initial Deflection (Δ_{initial})

$$\Delta_{\text{initial}} = \Delta_{p\text{(initial)}} - \Delta_{DL}$$

$$\Delta_{\text{initial}} = 19.909 - 6.782 = 13.127 \text{ mm } (\uparrow)$$

II. Final Deflection (Δ_{final})

Effective prestress deflection:

$$\Delta_{p(eff)} = 19.909 \times 0.85 = 16.922 \text{ mm } (\uparrow)$$

$$\Rightarrow \quad (\Delta_{final}) = \Delta_{p(eff)} - \Delta_{DL} - \Delta_{LL}$$

$$\Rightarrow \quad (\Delta_{final}) = 16.922 - 6.782 - 10.047$$

$$(\Delta_{final}) = 0.093 \text{ mm } (\uparrow)$$

3. (a) Solution:

Given:

$$\text{Column size} = 400 \times 600 \text{ mm}; P_u = 1600 \text{ kN}$$

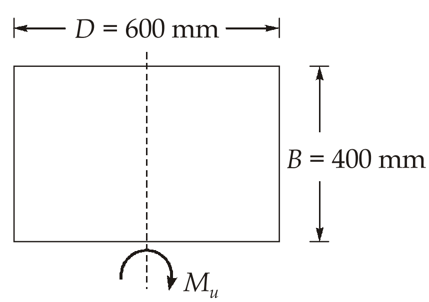
$$M_u = 250 \text{ kNm (about major axis)}$$

$$\text{Unsupported length} = 3000 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 500 \text{ N/mm}^2$$

$$\text{Effective cover } d' = 60 \text{ mm}$$



For bending about major axis,

$$D = 600 \text{ mm}$$

$$\frac{d'}{D} = \frac{60}{600} = 0.10$$

For Fe 500 steel with reinforcement on four sides and $d'/D = 0.10$, use SP-16 Chart 48 (given)

$$\frac{P_u}{f_{ck}bD} = \frac{1600 \times 10^3}{25 \times 400 \times 600} = 0.267$$

$$\frac{M_u}{f_{ck}bD^2} = \frac{250 \times 10^6}{25 \times 400 \times 600^2} = 0.069$$

From Chart 48, for

$$\frac{P_u}{f_{ck}bD} = 0.267, \frac{M_u}{f_{ck}bD^2} = 0.069$$

$$\frac{p}{f_{ck}} = 0.2$$

$$p = 0.2 \times 25 = 5\% \not> 4\%$$

Area of steel: $A_{st} = \frac{4}{100} \times 400 \times 600 = 9600 \text{ mm}^2$

Minimum steel: $A_{st\,min} = 0.8\% \times 400 \times 600 = 1920 \text{ mm}^2 < 9600 \text{ mm}^2$ (OK)

Minimum eccentricity: $e_{min} = \frac{3000}{500} + \frac{600}{30} = 6 + 20 = 26 \text{ mm} (> 20 \text{ mm})$

Actual eccentricity:

$$e = \frac{M_u}{P_u} = \frac{250 \times 10^3}{1600} = 156.25 \text{ mm}$$

$$e > e_{min}$$

Longitudinal Reinforcement

Provide 12 mm diameter bars of 32 mm diameter:

$$A_{prov} = 12 \times \frac{\pi}{4} \times 32^2 = 9650.9 \text{ mm}^2 > 9600 \text{ mm}^2$$

Steel distributed equally on four sides.

Design of lateral ties:

$$\text{Diameter of lateral ties} \not< \max^m \begin{cases} \frac{\phi_{main}}{4} = 8 \text{ mm} \\ 6 \text{ mm} \end{cases}$$

$$\phi_{\text{lateral ties}} = 8 \text{ mm}$$

$$\because \quad \frac{P_t}{f_{ck}} = \frac{4}{25} = 0.16$$

$$\frac{d'}{d} = 0.1, \frac{P_u}{f_{ck}bD} = 0.267$$

From charge $\frac{M_u}{f_{ck}bD^2} = 0.06$

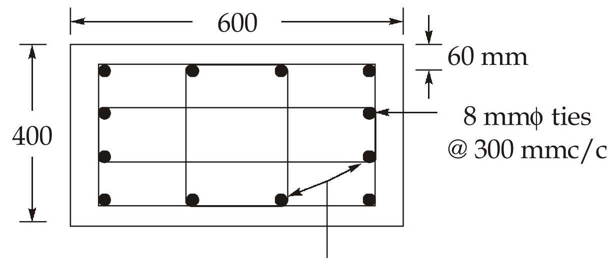
$$\Rightarrow \quad M_u = 216 \text{ kNm (Moment restrict)}$$

Use 8 mm lateral ties

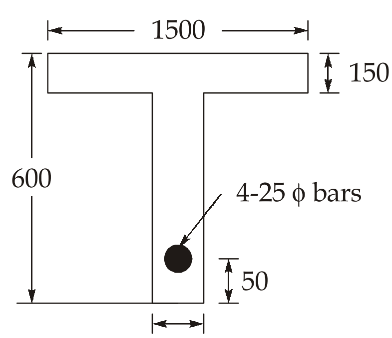
$$\text{Spacing} = \text{minimum}\begin{cases}\text{least lateral dimension} = 400 \text{ mm} \\ 16 \times \phi = 512 \text{ mm main} \\ 300 \text{ mm}\end{cases}$$

$$\text{Spacing} = 300 \text{ mm}$$

∴ Use 8 mm diameter lateral ties at 300 mm c/c



3. (b) Solution:



Given:

$$f_{ck} = 25 \text{ MPa}, f_y = 500 \text{ MPa}, E_c = 5000\sqrt{25} = 25{,}000 \text{ MPa.}$$

$$E_s = 200{,}000 \text{ MPa.}$$

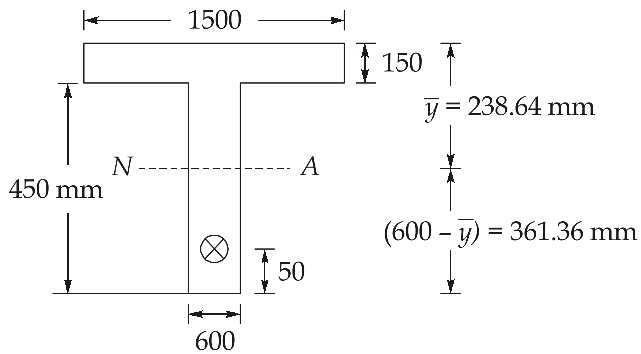
Modular Ratio (m); $$m = \frac{E_s}{E_c} = \frac{200000}{25000} = 8$$

Area of tension steel (A_{st}), $$A_{st} = 4 \times \frac{\pi \times 25^2}{4} = 1963.5 \text{ mm}^2$$

Span $$L = 6 \text{ m} = 6000 \text{ mm}$$

Sustained Load (w) = 40 kN/m. (including of self weight)

Gross Properties (I_{gr} and M_r)



Centroid from top fiber ($\bar{y}$):

$$A_1 \text{ (Flange)} = 1500 \times 150 = 225000 \text{ mm}^2$$

$$A_2 \text{ (Web)} = 600 \times 450 = 270000 \text{ mm}^2$$

$$\bar{y} = \frac{(225000 \times 75) + (270000 \times 375)}{225000 + 270000} = 238.64 \text{ mm}$$

Gross Moment of Inertia (I_{gr}) about NA:

$$\text{Flange, } I_1 = \frac{1500 \times 150^3}{12} + 225000(238.64 - 75)^2 = 6.45 \times 10^9 \text{ mm}^4$$

$$\text{web, } I_2 = \frac{600 \times 450^3}{12} + 270000(375 - 238.64)^2 = 9.58 \times 10^9 \text{ mm}^4$$

$$I_{gr} = 6.45 \times 10^9 + 9.58 \times 10^9 = 1.603 \times 10^{10} \text{ mm}^4$$

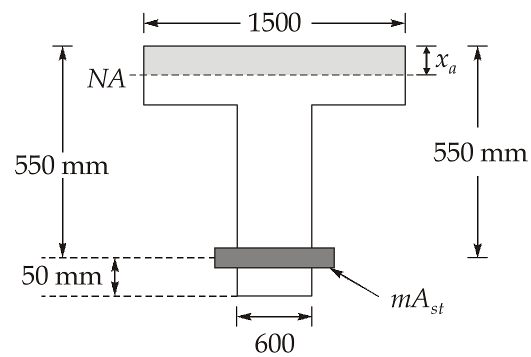
Cracking Moment (M_r):

$$f_{cr} = 0.7\sqrt{25} = 3.5 \text{ MPa}$$

$$y_{max} = 600 - 238.64 = 361.36 \text{ mm}$$

$$M_r = \frac{f_{cr}}{y_{max}}(I_{gr}) = \frac{3.5 \times 1.603 \times 10^{10}}{361.36} = 155.260 \text{ kN-m}$$

Cracked Properties (I_{cr})



Neutral Axis (x_a) (assuming $x \le D_f$):

$$\Rightarrow \quad \frac{b_f x_a^2}{2} = mA_{st}(d - x_a)$$

$$\Rightarrow \quad \frac{1500x_a^2}{2} = 8 \times 1963.5\ (550 - x_a)$$

$$\Rightarrow \quad x_a^2 + 20.94x_a - 11519.2 = 0$$

$$\Rightarrow \quad x_a = 97.37 \text{ mm}$$

(Since x_a < 150 mm, the assumption is correct.)

Cracked Moment of Inertia (I_{cr}):

$$I_{cr} = \frac{Bx_a{}^3}{3} + m\left[\frac{n\pi\phi^2}{64} + A_{st}\left(d - x_a\right)^2\right]$$

$$I_{cr} = \frac{1500 \times 97.37^3}{3} + 8\left[\frac{4 \times \pi \times 25^2}{64} + 1963.5\left(550 - 97.37\right)^2\right]$$

$$= 3680.35 \times 10^6 \text{ mm}$$

Effective Moment of Inertia (l_{eff})

Service Moment (M), $\quad M = \dfrac{40 \times 6^2}{8} = 180$ kN-m

Lever arm (z), $\quad z = d - \dfrac{x_a}{3} = 550 - \dfrac{97.37}{3} = 517.54$ mm

$$l_{eff} = \frac{l_{cr}}{1.2 - \left(\frac{M_r}{M}\right)\frac{z}{d}\left(1 - \frac{x_a}{d}\right)\frac{b_w}{b_f}}$$

$$1.2-\left(\frac{M_r}{M}\right)\frac{z}{d}\left(1-\frac{x}{d}\right)\frac{b_w}{b_f} = 1.2-\left[\frac{155.260}{180}\times\frac{517.54}{550}\times\left(1-\frac{97.37}{550}\right)\times\frac{600}{1500}\right]=0.9328$$

$$\therefore \qquad l_{eff} = \frac{3680.35\times10^6}{0.9328} = 3945.48 \times 10^6 \text{ mm}^4$$

(Since $l_{cr} < l_{eff} < l_{gr}$, the value is acceptable.)

Final Deflection Components

(i) Immediate Short-term Deflection (δ_i)

$$\delta_1 = \frac{5wL^4}{384E_c l_{eff}} = \frac{5\times40\times6000^4}{384\times25000\times3945.48\times10^6} = 6.84 \text{ mm}$$

(ii) Additional Deflection due to Creep (δ_{cp})

Using creep coefficient θ =1.6:

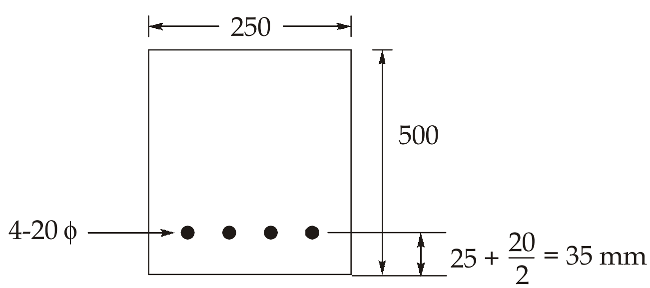
$$\delta_{cp} = \delta_i \times \theta = 6.84 \times 1.6 = 10.950 \text{ mm}$$

$$\text{Total Deflection} = 6.84 + 10.950 = 17.79 \text{ mm}$$

Deflection limit = span/250 = 6000/250 = 24 mm > (Total Deflection = 17.794 mm)

Safe

3. (c) (i) Solution:



Given:

$$b = 250 \text{ mm}$$

$$D = 500 \text{ mm}$$

$$\text{Clear cover} = 25 \text{ mm}$$

$$\text{Effective cover} = 25 + 20/2 = 35 \text{ mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.64 \text{ mm}^2$$

$$V_u = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100.53 \text{ mm}^2$$

Concrete grade: M20 (f_{ck} = 20 MPa)

Steel grade: Fe 415 (f_y = 415 MPa)

Effective Depth

$$d = D - \text{clear cover} - \frac{\phi}{2}$$

$$d = 500 - 25 - \frac{20}{2} = 465 \text{ mm}$$

Nominal Shear Stress $$\tau_v = \frac{V_u}{bd} = \frac{150\times10^3}{250\times465}$$

$$= 1.29 \text{ N/mm}^2$$

For M20 concrete,

$$\tau_{c,\max} = 0.63\sqrt{f_{ck}} = 0.63\times\sqrt{20} = 2.8 \text{ N/mm}^2$$

Since $\tau_v < \tau_{c,\max}$, the section is safe in shear.

Design Shear Strength of Concrete

Percentage of tensile steel:

$$p_t = \frac{100A_{st}}{bd} = \frac{100\times1256.64}{250\times465} = 1.08\%$$

From the given table:

For $A_{st}\% = 1.00\%$, $\tau_c = 0.62 \text{ N/mm}^2$

For $A_{st}\% = 1.25\%$, $\tau_c = 0.67 \text{ N/mm}^2$

Interpolating for 1.08%:

$$\tau_c = 0.62 + \frac{0.67-0.62}{1.25-1.00}(1.08 - 1.00) = 0.636 \text{ N/mm}^2$$

Shear to be Carried by Stirrups

Since $\tau_v > \tau_c$,

$$V_{us} = V_u - (\tau_c bd)$$

$\Rightarrow$ $$V_{us} = 150000 - (0.636 \times 250 \times 465)$$

$\Rightarrow$ $$V_{us} = 150000 - 73935 = 76065 \text{ N}$$

Spacing of Stirrups, $$s_v \le \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$s_v \le \frac{0.87\times415\times100.53\times465}{76065} = 221.8 \text{ mm}$$

Check for Minimum and Maximum Spacing

Minimum shear reinforcement spacing:

$$\frac{A_{sv}}{bSv} \geq \frac{0.4}{0.87\, f_y}$$

$$\Rightarrow \qquad S_v \leq \frac{0.87 \times 415 \times 100.53}{0.4 \times 250}$$

$$\Rightarrow \qquad S_v \leq 362.96 \text{ mm}$$

Maximum spacing limits:

$$\leq \begin{cases} 0.75d = 0.75 \times 465 = 348.75 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

Final Answer

Provide two-legged 8 mm diameter stirrups @ 220 mm c/c

3. (c) (ii) Solution:

Material and Sectional Properties

Modular Ratio (m), $\quad m = \dfrac{280}{3\sigma_{cbc}} = \dfrac{280}{3 \times 7} = 13.33$

Area of Steel (A_{st}), $\quad A_{st} = 4 \times \dfrac{\pi}{4} \times 25^2 = 1963.5 \text{ mm}^2$

Modulus of Rupture (f_{cr}), $\quad f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\sqrt{20} = 3.13 \text{ N/mm}^2$

Determination of Cracking Moment (M_{cr})

Before calculating stresses, we must verify if the concrete in the tension zone has cracked.

Gross Moment of Inertia (l_g):

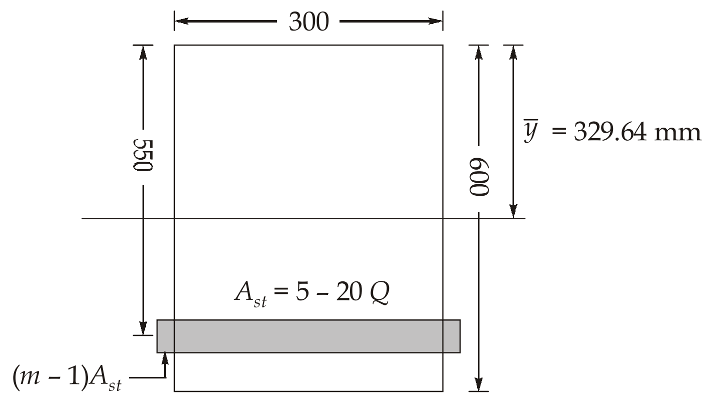
$$l_g = \frac{bD^3}{12} = \frac{300 \times 600^3}{12} = 5.4 \times 10^9 \text{ mm}^4$$

Cracking Moment (M_{cr}):

$$M_{cr} = \frac{f_{cr} \cdot I_g}{y_t} = \frac{3.13 \times 5.4 \times 10^9}{300} \times 10^{-6} = 56.34 \text{ kNm}$$

Since the applied moment (M = 45 kNm) is less than M_{cr} = 56.34 kNm, the section is uncracked.

Because the section is uncracked, the entire concrete area is effective.



[Transformed section]

Transformed Area (A_e):

$$A_e = (b \times D) + (m - 1)A_{st} = (300 \times 600) + (12.33 \times 1963.5) = 204{,}210 \text{ mm}^2$$

Neutral Axis ($\bar{y}$ from top):

$$\bar{y} = \frac{(b \times D \times D/2) + (m-1)A_{st} \times d}{A_e}$$

$$\Rightarrow \quad \bar{y} = \frac{300 \times 600 \times 300}{204{,}210} = 329.64 \text{ mm}$$

Transformed Moment of Inertia (I_{tr}):

$$I_{tr} = \left[\frac{bD^3}{12} + bD(\bar{y} - 300)^2\right] + (m-1)A_{st}(d - \bar{y})^2$$

$$\Rightarrow \quad I_{tr} = \frac{300 \times 600^3}{12} + 300 \times 600 \times (329.64 - 300)^2 + 12.33 \times 1963.5 \times (550 - 329.64)^2$$

$$\Rightarrow \quad I_{tr} = 6733.735 \times 10^6 \text{ mm}^4$$

Concrete stress (at extreme top fiber)

$$\sigma_c = \frac{M \cdot \bar{y}}{I_{tr}} = \frac{45 \times 10^6 \times 329.64}{6733.735 \times 10^6} = 2.20 \text{ N/mm}^2 \text{ (comp)}$$

Steel stress

$$\sigma_{st} = m \cdot \frac{M \cdot (d - \bar{y})}{I_{tr}} = 13.33 \times \frac{45 \times 10^6 \times (550 - 329.64)}{6733.735 \times 10^6} = 19.63 \text{ N/mm}^2 \text{ (tension)}$$

4. (a) Solution:

Lateral Earth Pressure Calculation

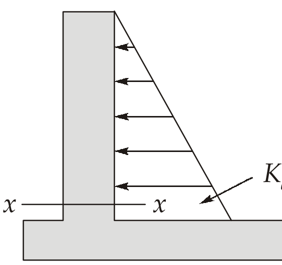
Coefficient of Active Earth Pressure (K_a)

Using Rankine's theory:

$$K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

Active Earth Pressure at Base (p_a)

$$p_a = K_a \gamma h = \frac{1}{3} \times 18 \times 5 = 30 \text{ kN/m}^2$$



Factored Bending Moment (M_u)

The stem is treated as a cantilever beam fixed at the base.

Total Lateral Force (P_a)

$$P_a = \frac{1}{2} \times p_a \times h = \frac{1}{2} \times 30 \times 5 = 75 \text{ kN/m}$$

Maximum Bending Moment at Base (M)

$$M = P_a\left(\frac{h}{3}\right) = 75 \times \frac{5}{3} = 125 \text{ kNm/m}$$

Factored Moment,

$$M_u = 1.5 \times 125 = 187.5 \text{ kNm/m}$$

$$\text{Limiting moment, } M_{u,\,lim} = 0.138 f_{ck}\, bd^2 = 0.138 \times 25 \times 1000 \times 450^2$$

$$= 698.625 \text{ kN-m/m} > M_u$$

So under reinforcement section.

Vertical (Main) Reinforcement (16 mm ϕ)

The required area of steel A_{st} is calculated using:

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]bd$$

$$\Rightarrow \qquad A_{st} = \frac{0.5\times 25}{415}\left[1-\sqrt{1-\frac{4.6\times 187.5\times 10^6}{25\times 1000\times 450^2}}\right]\times 1000\times 450$$

$$A_{st} = 1208.5 \text{ mm}^2 \approx 1210 \text{ mm}^2$$

Spacing of 16 mm Bars

Area of one 16 mm bar: $A = 201 \text{ mm}^2$

$$s = \frac{1000\times 201}{1210} = 166.11 \text{ mm}$$

Provide:

16 mm ϕ bars @ 150 mm c/c (vertical) on the rear (earth-retaining) face.

Horizontal (Secondary) Reinforcement (12 mm ϕ)

Secondary reinforcement is provided to resist temperature and shrinkage stresses.

Overall Thickness of Stem

$$D = d + \text{clear cover} + \text{radius of main bar}$$

$$D = 450 + 50 + 8 = 508 \text{ mm}$$

Required Distribution Steel (0.12% of Gross Area)

$$A_{st,\ dist} = 0.0012 \times 1000 \times 508 = 609.6 \text{ mm}^2/\text{m}$$

Spacing of 12 mm Bars

Area of one 12 mm bar:

$$A = 113.1 \text{ mm}^2$$

$$s = \frac{1000\times 113.1}{609.6} = 185.5 \text{ mm}$$

Provide: on both front and rear faces. 12 mm ϕ bars @ 180 mm c/c (horizontal)

Check for shear: (For 1 m length of wall)

$$V_u = 1.5 \times 75 = 112.5 \text{ kN}$$

$$\tau_v = \frac{112.5\times 10^3}{1000\times 450} = 0.25 \text{ N/mm}^2$$

$$p_t = \frac{\left(\frac{\pi}{4}\times 16^2\right)\times\frac{1000}{150}}{1000\times 450}\times 100 = 0.298\%$$

Since $D > 300$ mm $k_s = 1$

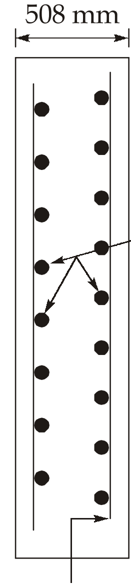
and from table given

$$\tau_c = 0.36 + \frac{(0.49 - 0.36)}{(0.50 - 0.25)} \times (0.298 - 0.25)$$

$$\tau_c = 0.385 \text{ N/mm}^2$$

Since $\tau_c > \tau_v$

Safe in shear, no shear reinforcement is required



4. (b) Solution:

Given:

Column load at service = 1500 kN

Soil bearing capacity = 120 kN/m² = q_g

Grade of steel and concrete are Fe 415 and M25 respectively.

Let the side of square footing = B

$$B \times B = \frac{1500 \times 1.1}{120} = 13.75 \text{ m}^2$$

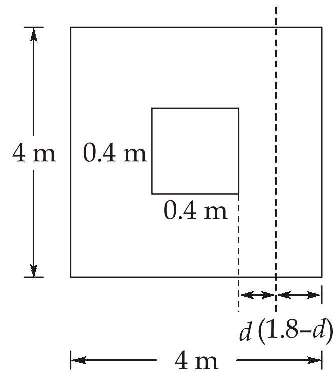
$$B = \sqrt{13.75} = 3.71 \text{ m}$$

Take B = 4 m

Factored load

$$w_u = \frac{1500 \times 1.5}{4^2} = 140.625 \text{ kN/m}^2$$

(To decide about size of footing, we work with service load condition and for design of footing we work with factored load)



One way shear

$$\tau_v = \frac{V_u}{bd} = \frac{w_u \times 4(1.8-d)}{4d}$$

$$= 140.625\left(\frac{1.8-d}{d}\right) \text{kN/m}^2$$

$$\tau_v < K\tau_c \text{ for safety, assuming } K = 1$$

$$\tau_c = 0.35 \times 10^3 \text{ kN/m}^2$$

$$\frac{140.625(1.8-d)}{d} \leq 350$$

$$d \geq 0.516 \text{ m}$$

or $$d \geq 516 \text{ mm}$$

Two way shear (Punching stress)

$$\tau_v = \frac{\left[4\times 4-(0.4+d)^2\right]\times w_u}{4(0.4+d)\times d}$$

$$= \frac{\left[16-(0.4+d)^2\right]\times 140.625}{4(0.4+d)\times d}$$

$$\tau_v \leq K_s\tau'_c$$

$$K_s = 0.5+\beta_c\frac{4}{4} = 1.5 \leq 1 so K_s = 1$$

$$\tau'_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25 \text{ N/mm}^2$$

$$= 1250 \text{ kN/m}^2$$

$$\frac{\left[16-(0.4+d)^2\right]\times 140.625}{4(0.4+d)\times d} \le 1250$$

$$d \ge 484.11 \text{ mm}$$

Adopt a effective depth of 550 mm

Bending moment criteria

Critical section is at column face

$$M_u = w_u \times 1.8 \times \frac{1.8}{2}$$

$$= 140.625 \times \frac{1.8^2}{2} = 227.81 \text{ kN-m}$$

$$M_{u\,\lim} = 0.138 f_{ck}\, bd^2$$

$$= 0.138 \times 25 \times 1000 \times 550^2$$

$$= 1044 \text{ kN-m} > M_u$$

Reinforcement calculation

$$\because \quad M_u = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]bd$$

$$\Rightarrow \quad M_u = \frac{0.5\times 25}{415}\left[1-\sqrt{1-\frac{4.6\times 227.81\times 10^6}{25\times 1000\times 550^2}}\right]1000\times 550$$

$$\Rightarrow \quad M_u = 1190.56 \text{ mm}^2$$

$$A_{st} = 1190.56 \text{ mm}^2 \text{ for 1 meter width (use 16 mm bar)}$$

$$\text{Spacing} = \frac{1000\times\frac{\pi}{4}\times 16^2}{1190.56} = 168.88 \text{ mm}$$

$\therefore$ Provide 16 mm ϕ bar @ 160 mm spacing

Total depth of footing, $D = 550 + 50 + \frac{16}{2} + 16 = 624 \text{ mm} \approx 625 \text{ mm}$

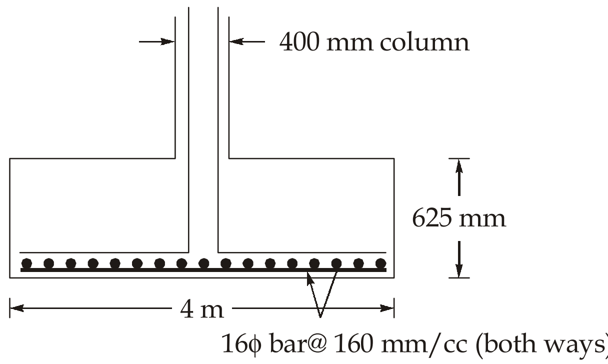
(Taking clear cover = 50 mm and 16 ϕ bar)

Reinforcement detail of the footing

Footing size = 4 m

Depth = 625 mm

Provide 16 ϕ bar @ 160 mm c/c (both ways)

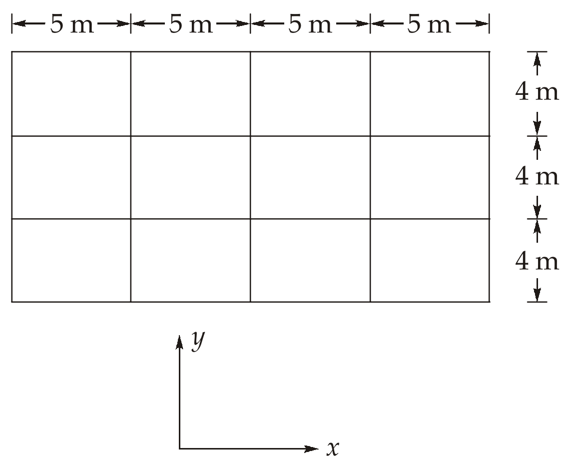


Reinforcement detail of the footing

4. (c) Solution:

Seismic Analysis of the Building (Equivalent Static Method)

Given Data



Plan of building

Number of storeys = 4

Building plan dimensions = 20 m × 12 m

4 bays @ 5 m in x-direction

3 bays @ 4 m in y-direction

Storey heights = 3.6 + 3.3 + 3.3 + 3.4 = 13.6 m

Total height (h) = 13.6 m

Dimension along direction of earthquake (d) = 12 m

Loading Data

Dead load on floors = 12 kN/m^2

Dead load on roof = 10 kN/m^2

Live load on floors = 3 kN/m^2

25% considered for seismic weight

Live load on roof = 1 kN/m^2

0% considered for seismic weight

Seismic Parameters (IS 1893)

Zone factor (Z) = 0.24

Importance factor (I) = 1.2

Response reduction factor (R) = 5

Soil type = Type I (Rock)

Seismic Weight Calculation

Area of each floor:

$$A = 20 \times 12 = 240 \text{ m}^2$$

Floors 1, 2 and 3

- Dead load = 12 × 240 = 2880 kN
- Live load contribution = 0.25 × 3 × 240 = 180 kN

$$W_1 = W_2 = W_3 = 2880 + 180 = 3060 \text{ kN}$$

Roof (Floor 4)

- Dead load = 10 × 240 = 2400 kN
- Live load contribution = 0

$$W_4 = 2400 \text{ kN}$$

Total Seismic Weight

$$W = (3 \times 3060) + 2400 = 11580 \text{ kN}$$

Fundamental Natural Period (T_a)

For RC moment-resisting frame with masonry infill:

$$T_a = \frac{0.09h}{\sqrt{d}}$$

$$T_a = \frac{0.09 \times 13.6}{\sqrt{12}} = \frac{1.224}{3.464} = 0.353 \text{ } s$$

Spectral Acceleration Coefficient (S_a/g)

For Type I (Rock) soil from given chart for time period of 0.353 s

$$\frac{S_a}{g} = 2.5$$

Design Horizontal Seismic Coefficient (A_h)

$$A_h = \frac{Z \times I \times (S_a / g)}{2R}$$

$$A_h = \frac{0.24 \times 1.2 \times 2.5}{2 \times 5} = \frac{0.72}{10} = 0.072$$

Design Base Shear (V_B)

$$V_B = A_h \times W$$

$$V_B = 0.072 \times 11580 = 833.76 \text{ kN}$$

Distribution of Design Lateral Force

Lateral force at storey i :

$$Q_i = V_B \times \frac{W_i h_i^2}{\Sigma W_i h_i^2}$$

Storey-wise Calculations

Storey	W_i (kN)	h_i(m)	$W_i h_i^2$
4 (Roof)	2400	13.6	443,904
3	3060	10.2	318,362.4
2	3060	6.9	145,686.6
1	3060	3.6	39,657.6
Total			**947,610.6**

Lateral Forces at Each Storey

- Storey 4 (Roof):

$$Q_4 = 833.76 \times \frac{443904}{947610.6} = 390.57 \text{ kN}$$

- Storey 3:

$$Q_3 = 833.76 \times \frac{318362.4}{947610.6} = 280.11 \text{ kN}$$

- Storey 2:

$$Q_2 = 833.76 \times \frac{145686.6}{947610.6} = 128.18 \text{ kN}$$

- Storey 1:

$$Q_1 = 833.76 \times \frac{39657.46}{947610.6} = 34.89 \text{ kN}$$

5. (a) Solution:

Given beam size = 60 × 60 mm

k = 200 N/mm

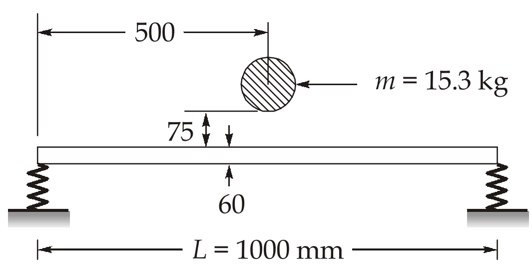
The deflection of the system due to a statically applied force of 15.3 kg = 15.3 × 9.81 = 150 N is computed first. In the first case, this deflection is that of the beam only; In the second case, the static deflection of the beam is augmented by the deflection of the springs subjected to a 75-N force each. The impact factors are then computed. Static deflections and stresses are multiplied by the impact factors to obtain the answers.

1.
$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{150\times1000^3}{48\times200\times10^3\times60^4/12} = 0.0145 \text{ mm}$$

$$\text{Impact factor} = 1+\sqrt{1+\frac{2h}{\Delta_{st}}} = 1+\sqrt{1+\frac{2\times75}{0.0145}} = 102.71$$

2.
$$\Delta_{st} = \Delta_{\text{beam}} + \Delta_{spr} = 0.0145 + \frac{75}{200} = 0.3895 \text{ mm}$$

$$\text{Impact factor} = 1+\sqrt{1+\frac{2\times75}{0.3895}} = 20.65$$



For either case, the maximum bending stress in the beam due to a static application of P is

$$(\sigma_{max})_{st} = \frac{M}{S} = \frac{PL}{4S} = \frac{150\times1000}{4\times60^3/6} = 1.042 \text{ MPa}$$

Multiplying the static deflections and stress by the respective impact factors gives the required results.

	Static		Dynamic	
	With Springs	No Springs	With Springs	No Springs
Δ_{max}, mm	0.3895	0.0145	8.043	1.489
σ_{max}, MPa	1.042	1.042	21.52	107.02

It is apparent from this table that large deflections and stresses are caused by a dynamically applied load. The stress for the condition with no springs is particularly large; however, owing to the flexibility of the beam, it is not excessive.

5. (b) Solution:

Location of centroid of section from left most fibre,

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$\Rightarrow \quad \bar{x} = \frac{(100\times8)\times4+(142\times10)79}{100\times8+142\times10}$$

$$\Rightarrow \quad \bar{x} = \frac{800\times4+1420\times79}{2220} = 51.97 \text{ mm}$$

Moment of inertia about neutral axis,

$$I_{NA} = \left[\frac{100\times8^3}{12}+(51.97-4)^2\times800\right]+\left[\frac{142^3\times10}{12}+1420\times(79-51.97)^2\right]$$

$$\Rightarrow \quad I_{NA} = [0.4267 + 184.089 + 238.607 + 103.748] \times 10^4$$

$$\Rightarrow \quad I_{NA} = 5.269 \times 10^6 \text{ mm}^4$$

Direct axial tensile stress, $\sigma_a = \dfrac{10\times10^3}{2220} = 4.505$ MPa (Tensile)

Bending stress at A, $\sigma_{ba} = \dfrac{(10\times10^3)\times(100+51.97)\times51.97}{5.269\times10^6} \simeq 15$ MPa (Tensile)

Bending stress at B, $\sigma_{bb} = \dfrac{(10\times10^3)\times(100+51.97)\times(150-51.97)}{5.269\times10^6}$

$= 28.274$ MPa (Compressive)

$\therefore$ Stress at A $= \sigma_a + \sigma_{ba} = 19.505$ MPa (Tensile)

Stress at B $= \sigma_a - \sigma_{bb} = -23.77$ MPa (Compressive)

5. (c) Solution:

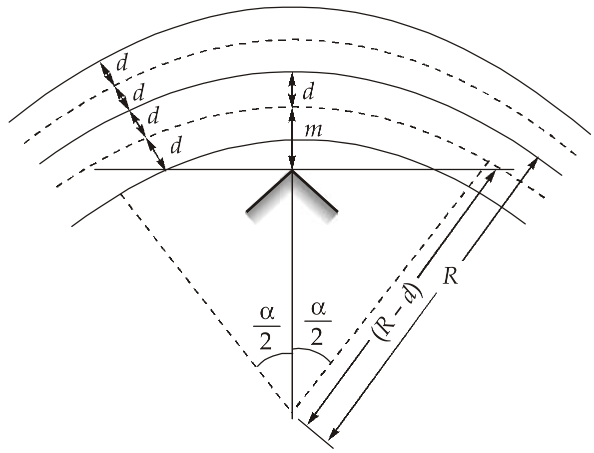
Given:

Radius curve, R = 160 m

Number of lanes, n = 2, $f = 0.40$

Width of lane = 3.5 m

$$d = \frac{3.5}{2} = 1.75\,\text{m}$$



Set back distance from centerline of inner lane

$$m = 10 - d = 10 - \frac{3.5}{2} = 8.25\text{ m}$$

Set back distance from center line of inner lane

$$m = (R-d)-(R-d)\cos\left(\frac{\alpha}{2}\right) \qquad \because \; L > S$$

$$\Rightarrow \quad 8.25 = (160-1.75)-(160-1.75)\cos\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad 158.25\cos\left(\frac{\alpha}{2}\right) = 150$$

$$\Rightarrow \quad \left(\frac{\alpha}{2}\right) = \cos^{-1}(0.947867)$$

$$\Rightarrow \quad \left(\frac{\alpha}{2}\right) = 18.582°$$

$$\Rightarrow \quad \left(\frac{\alpha}{2}\right) = 18.582° \times \frac{\pi}{180}\text{ radian}$$

$$\Rightarrow \quad \left(\frac{\alpha}{2}\right) = 0.32432\text{ radian}$$

$$\Rightarrow \quad \frac{S}{2(R-d)} = 0.32432\text{ radian}$$

$$\Rightarrow \quad S = 102.647\text{ m}$$

Now, $SSD = V \times 2.5 + \frac{V^2}{2 \times 9.81 \times 0.40}$

$\Rightarrow$ $0.1274V^2 + 2.5V - 102.647 = 0$

$\therefore$ $V = 20.22 \text{ m/sec} = 72.792 \text{ km/hr}$

5. (d) Solution:

Given Data

Initial ADT, $P = 2{,}500$ CVPD (combined both directions)

Construction period, $x = 2$ years

Design life, $n = 15$ years

Growth rates:

- During construction and first 10 years:

$$r_1 = 7.5\% = 0.075$$

- Final 5 years:

$$r_2 = 9.0\% = 0.09$$

Lane Distribution Factor, $D = 0.75$ (for a 4-lane divided highway)

Vehicle Damage Factor, F:

$$VDF_{avg} = (0.40 \times 3.5) + (0.60 \times 5.5)$$

$$VDF_{avg} = 1.4 + 3.3 = 4.7$$

Traffic at Completion of Construction (A)

Before the highway opens, traffic grows for 2 years:

$$A = P(1 + r_1)^x$$

$$A = 2500(1 + 0.075)^2$$

$$A = 2889.063 \text{ CVPD}$$

Cumulative Design Traffic (N)

Since the growth rate changes, the calculation is split into two phases.

Phase 1: Years 1 to 10 years

$$n_1 = 10,\ r_1 = 7.5\%$$

$$\Rightarrow \quad N_1 = \frac{365 \times A \times D \times F \times \left[(1 + r_1)^{n_1} - 1\right]}{r_1}$$

$$\Rightarrow \quad N_1 = \frac{365 \times 2889.063 \times 0.75 \times 4.7 \times \left[(1 + 0.075)^{10} - 1\right]}{0.075}$$

$$\Rightarrow \quad N_1 = 52586714.47 = 52.586 \text{ msa}$$

Phase 2: Years 11 to 15 years

Traffic at the start of Year 11:

$$A_{11} = A(1 + r_1)^{10}$$

$$\Rightarrow \quad A_{11} = 2889.063(1.075)^{10}$$

$$\Rightarrow \quad A_{11} = 5954.451 \text{ CVPD}$$

Now, cumulative traffic for the remaining 5 years:

$$N_2 = \frac{365 \times A_{11} \times D \times F \times \left[(1+r_2)^{n_2} - 1\right]}{r_2}$$

$$\Rightarrow \quad N_2 = \frac{365 \times 5954.451 \times 0.75 \times 4.7 \times \left[(1+0.09)^5 - 1\right]}{0.09}$$

$$\Rightarrow \quad N_2 = 45849731.38 = 45.849 \text{ msa}$$

Total Design Traffic

$$N_{total} = N_1 + N_2 = 52.586 + 45.849$$

$$\Rightarrow \quad N_{total} = 98.435 \text{ msa}$$

5. (e) Solution:

Given:

Area to be surveyed:

$$L = 30 \text{ km}, W = 15 \text{ km}$$

Scale of photograph (S):

1: 15,000

Size of photograph ($l \times w$):

23 cm × 23 cm (0.23 m × 0.23 m)

Longitudinal overlap (p_l):

$$60\% = 0.60$$

Side overlap (p_w):

$$30\% = 0.30$$

Ground Coverage of a Single Photograph

First, calculate the actual ground dimensions represented by one 23 cm × 23 cm photograph:

- Ground length (L):

$$0.23 \text{ m} \times 15{,}000 = 3{,}450 \text{ m} = 3.45 \text{ km}$$

- Ground width (W):

$$0.23 \text{ m} \times 15{,}000 = 3{,}450 \text{ m} = 3.45 \text{ km}$$

Next, calculate the effective ground coverage by subtracting the overlaps:

- Effective length (L_e):

$$L(1 - p_l) = 3.45 \times (1 - 0.60) = 1.380 \text{ km}$$

- Effective width (W_e):

$$W(1 - p_w) = 3.45 \times (1 - 0.30) = 2.415 \text{ km}$$

Number of Photographs Required

The total number of photographs (N) is the product of the number of photographs per flight line (n_1) and the number of flight lines (n_2).

Number of photographs per flight line (n_1)

Assuming the flight lines are along the 30 km side:

$$n_1 = \frac{L_{area}}{L_e} + 1 = \frac{30}{1.380} + 1 \approx 21.739 + 1 = 22.739$$

(We round up to ensure full coverage)

$$= 23 \text{ photographs per line}$$

Number of flight lines (n_2)

$$n_2 = \frac{W_{area}}{W_e} + 1 = \frac{15}{2.415} + 1 \approx 6.211 + 1 = 7.211$$

(We round up to ensure full coverage)

$$= 8 \text{ flight lines}$$

Total Number of Photographs (N)

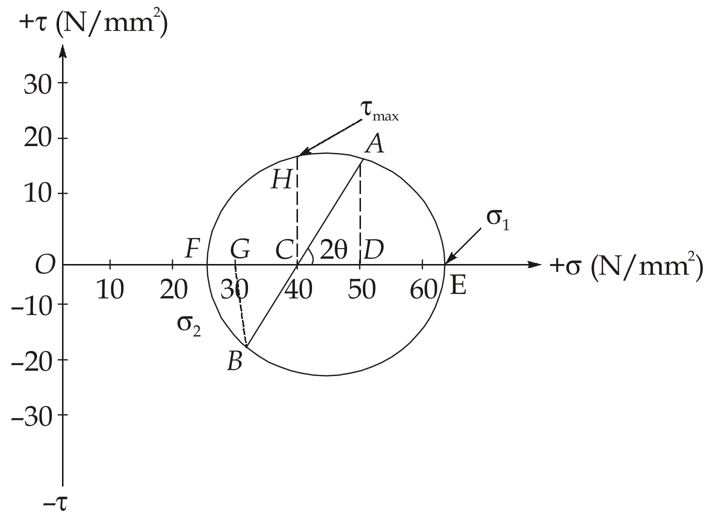
$$N = n_1 \times n_2$$

$$N = 23 \times 8 = 184$$

$$\text{Total photographs required} = 184$$

6. (a) (i) Solution:

The shearing stress acting in conjunction with σ_x are counter clockwise, hence τ_{xy} is said to be positive on the vertical axis. Similarly, the shearing stresses acting in conjunction with σ_y are clockwise, hence τ_{xy} is said to be negative on the horizontal planes. On σ-τ diagram, construct a circle with the line joining the points (σ_x, τ_{xy}) or (50, 20) and point $(\sigma_y, -\tau_{xy})(30, -20)$ as diameter as shown by A and B respectively.



The principal stresses and their directions can be obtained from a scaled drawing, but we shall calculate σ_1, σ_2 and others values.

$$DA = 20\text{MPa}$$

$$OD = \sigma_x = 50\text{MPa}$$

$$OG = \sigma_y = 30\text{MPa}$$

$$OC = \frac{OD + OG}{2} = \frac{50+30}{2} = 40\text{MPa}$$

$$CD = OD - OC = 50 - 40 = 10\text{MPa}$$

$$AC^2 = CD^2 + DA^2$$

$$\Rightarrow \quad AC^2 = 10^2 + 20^2$$

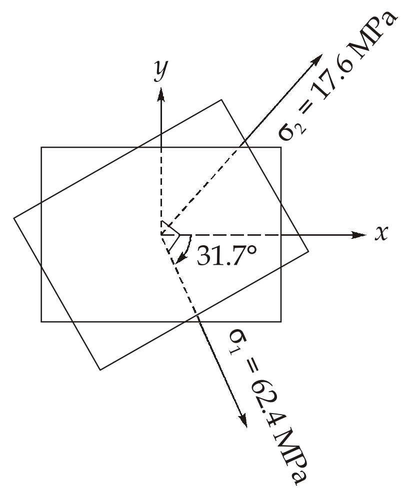
$$\Rightarrow \quad AC = 22.36 \text{ MPa}$$

Major principal stress, $\sigma_1 = OE = OC + AC = 40 + 22.36 = 62.36 \text{ MPa} \simeq 62.4 \text{ MPa}$

Minor principal stress, $\sigma_2 = OF = OC - AC = 40 - 22.36 = 17.64 \text{ MPa} \simeq 17.6 \text{ MPa}$

$$2\theta = \tan^{-1}\left(\frac{AD}{CD}\right) = \tan^{-1}\left(\frac{20}{10}\right) = 63.43°$$

$$\therefore \quad \theta = 31.7°$$

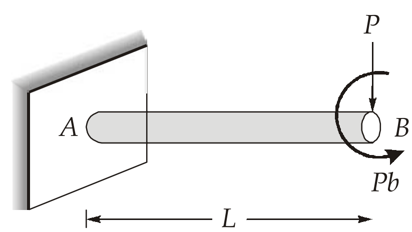


Maximum shear stress $=$ $\tau_{max} = CH = AC = 22.36$ MPa

(which occurs on planes at 45° to those of the principal stresses i.e., 76.7°)

6. (a) (ii) Solution:

Apply equal and opposite vertical load P at end B and now consider effect of BC on steel rod AB.



Torsional moment on AB, $T = P \times b$

Vertical load at $B = P$

1. Bending of AB since BC is rigid

Hence, Deflection at C = deflection of point B due to point load P at B

$$\Rightarrow \quad \delta_c = \frac{P(L_{AB})^3}{3EI} = \frac{PL^3}{3EI}$$

$$\Rightarrow \quad \delta_c = \frac{PL^3}{3E\frac{\pi d^4}{64}} = \frac{64PL^3}{3\pi Ed^4}$$

$$\Rightarrow \quad \delta_c = \left(\frac{64}{3\pi d^4}\right)\frac{PL^3}{E}$$

2. Torsion of *AB*,

Torque, $T = Pb$

Due to torque, *T*, end *B* rotates by an angle θ_B in counter clockwise direction. Hence, rigid bar *BC* will deflect downward

Rotation of rod *AB* at *B*, $$\theta_B = \frac{TL}{GI_P} = \frac{PbL}{G\times\frac{\pi d^4}{32}} = \left(\frac{32}{\pi d^4 G}\right)PbL$$

If rod *AB* rotates by an angle θ, then end *C* will deflect by,

$$= \theta_B \times b$$

$$= \left(\frac{32}{\pi d^4}\right)\frac{PbL}{G}\times b = \left(\frac{32}{\pi d^4}\right)\frac{Pb^2L}{G} \quad \text{(down ward)}$$

3. Both bending and torsion,

$$\delta_c = \left(\frac{64}{3\pi d^4}\right)\frac{PL^3}{E} + \left(\frac{32}{\pi d^4}\right)\frac{Pb^2L}{G}$$

6. (b) Solution:

(i) Given: $R = 250$ m

Deflection angle, $\phi = 180° - 150 = 30°$

Chainage at intersection point = 1250 m

Peg interval = 20 m

Least count of vernier = 20″

$$\text{Tangent length} = R\tan\left(\frac{\phi}{2}\right) = 250\tan 15° = 66.99 \simeq 67.0 \text{ m}$$

$$\text{Curve length, } l = \frac{\pi R\phi}{180} = \frac{\pi\times 250\times 30°}{180°} = 130.90 \text{ m}$$

$$\text{Apex distance} = R\left(\sec\frac{\phi}{2}-1\right) = 250(\sec 15° - 1) = 8.82 \text{ m}$$

$$\text{Versed sine of curve} = R\left(1-\cos\frac{\phi}{2}\right) = 250(1-\cos 15°) = 8.52 \text{ m}$$

Chainage at first tangent point,

$$T_1 = 1250 - 67 = 1183.0 \text{ m}$$

Chainage at second tangent point,

$$T_2 = 1183 + 130.90 = 1313.9 \text{ m}$$

Length of initial sub-chord = 1200 – 1183 = 17 m

Number of full chords (20 m) = 5

Chainage covered by 5 full chords

= 1200 + 5 × 20 = 1300 m

Length of final sub-chord = 1313.9 – 1300 = 13.9 m

Deflection angle for initial sub-chord

$$\delta_1 = \frac{180}{\pi}\left[\frac{17}{2\times 250}\right] = 1°56'53''$$

Deflection angle for full chord,

$$\delta = \frac{180}{\pi}\left[\frac{20}{2\times 250}\right] = 2°17'31''$$

Deflection angle for final sub-chord,

$$\delta_n = \frac{180}{\pi}\left[\frac{13.9}{2\times 250}\right] = 1°35'34''$$

Arithmetical check:

Total deflection angle, $\Delta_n = \delta_1 + 5\times\delta + \delta_n$

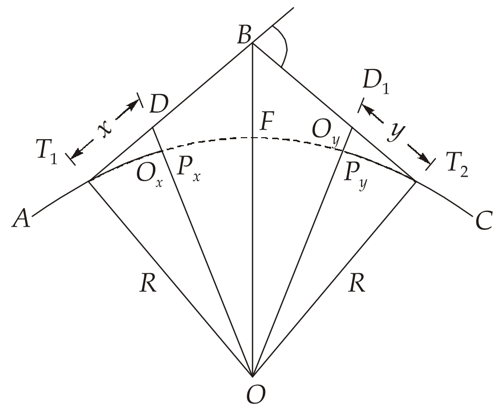
$$\Delta_n = \frac{\phi}{2} = \frac{30°}{2} = 15°$$

$$\Delta_n = 1°56'53'' + 5 \times (2°17'31'') + 1°35'34''$$

$$= 15°00'02'' \simeq 15° = \frac{\phi}{2}$$

(ii) (a) Setting out curve by radial offset method:

AB and BC are two tangents intersecting at B, and that the tangent points are T_1 and T_2.



Let us take a point D on the rear tangent AB such that

$$T_1D = x$$

Let O_x be the radial offset at D.

The point D is joined with the centre O. So, OD is the radial line. Now, from ΔT_1OD,

$$OT_1^2 + T_1D^2 = OD^2$$

where
$$OT_1 = R$$
$$OD = R + O_x$$
$$T_1D = x$$

$$\therefore \quad R^2 + x^2 = (R + O_x)^2$$

$$\Rightarrow \quad R + O_x = \sqrt{R^2 + x^2}$$

$$\Rightarrow \quad O_x = \sqrt{R^2 + x^2} - R$$

(b) The calculated distance O_x is cut off from the radial line OD to get the first point of the curve P_x.

(c) By increasing the value of x by a regular amount, a number of offsets are obtained. These are set off along the respective radial lines.

From the tangent point T_1, one half of the curve can be set out. In this case, the left half of the curve can be set out from T_1 up to the apex point F.

(d) The other half of the curve can be set out from the second tangent point T_2. Let a point D_1 be taken at a distance y from T_2. The offset O_y is then calculated as $O_y = \sqrt{R^2 + y^2} - R$.

The calculated distance O_y is set off along the radial line OD_1 to get the point P_y on the curve. Thus by increasing the value of y, the required offsets are calculated and set off along their respective radial lines to get the point on the curve for the right half.

6. (c) (i) Solution:

Greenshields Model

The Greenshields model is one of the earliest and most fundamental traffic flow models. It establishes a linear relationship between traffic speed and traffic density. The model assumes that speed decreases linearly as density increases.

According to Greenshields:

$$u = u_f\left(1 - \frac{k}{k_j}\right)$$

Where:

u = speed at density k

u_f = free flow speed

k = traffic density

k_j = jam density

When:

- $k = 0$, speed $u = u_f$ (free flow condition)
- $k = k_j$, speed $u = 0$ (jam condition)

The corresponding flow equation is obtained by multiplying speed and density:

$$q = uk = u_f k\left(1 - \frac{k}{k_j}\right) \quad \text{...(i)}$$

This results in a parabolic relationship between flow and density.

For maximum flow, $$\frac{dq}{dk} = 0$$

$$\Rightarrow \quad u_f\left(1 - \frac{2k}{k_j}\right) = 0$$

$$\Rightarrow \qquad k = \frac{k_j}{2}$$

Maximum flow (capacity) occurs when:

$$k = \frac{k_j}{2}$$

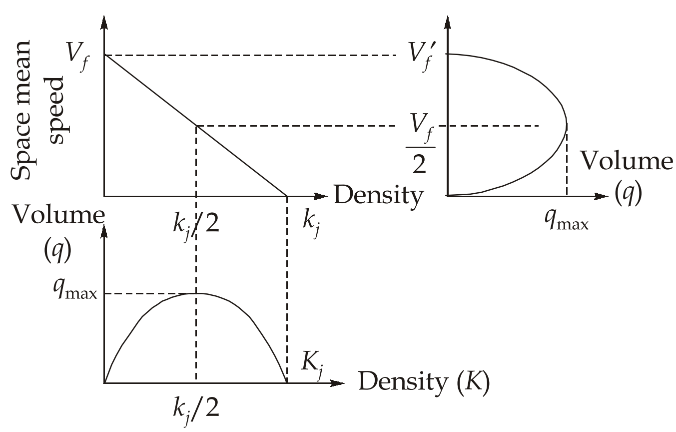
At this condition:

Traffic speed, $$u = \frac{u_f}{2}$$

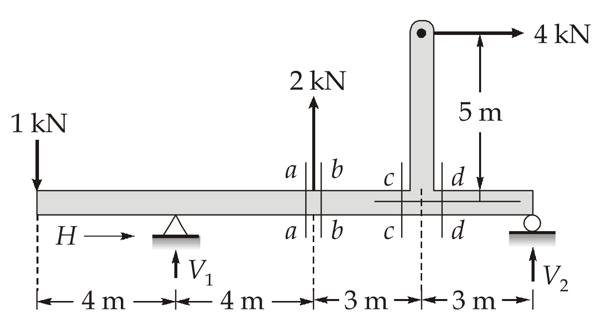
From equation (i), $$q_{max} = \frac{u_f k_j}{4}$$

Thus, according to the Greenshields model:

- Maximum flow occurs at half the jam density.
- Speed at maximum flow is half of the free flow speed.



6. (c) (ii) Solution:



Using force equilibrium equation

$\Sigma f_y = 0 \quad \Rightarrow \quad -1 + V_1 + 2 + V_2 = 0$

$$V_1 + V_2 = -1 \quad \text{...(i)}$$

$\Sigma f_x = 0 \quad \Rightarrow \quad H_1 + 4 = 0$

$$H_1 + 4 = 0$$

$$H_1 = 4 \text{ kN} (\leftarrow)$$

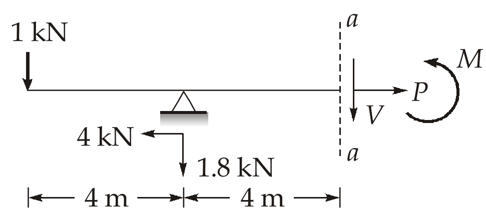
$\Sigma M = 0$ about pin support

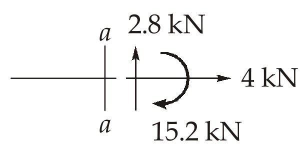
$\Rightarrow \quad -1 \times 4 - 2 \times 4 + 4 \times 5 - V_2 \times 10 = 0 \quad \Rightarrow \quad V_2 = 0.8 \text{ kN} (\uparrow)$

$$V_1 = -1 - 0.8 = -1.8 \text{ kN} (\downarrow)$$

At section *a – a*

Free body diagram





$$P = 4 \text{ kN (Tensile)}$$

$$V = -1 - 1.8 = -2.8 \text{ kN}$$

$$M = -1 \times 8 - 1.8 \times 4$$

$$= -15.2 \text{ kN-m (hogging)}$$

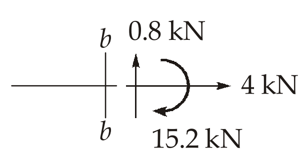
At section *b - b*

Due to concentrated level of 2 kN (upwards), only shear force well get change.

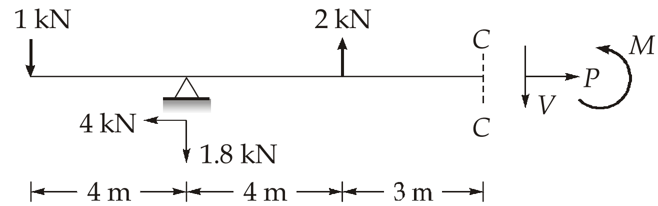
$$P = 4 \text{ kN (Tension)}$$

$$V = -2.8 + 2 = -0.8 \text{ kN}$$

$$M = -15.2 \text{ kN-m (hogging)}$$



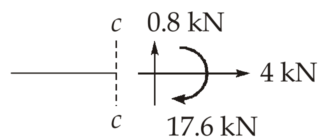
At section *c - c*



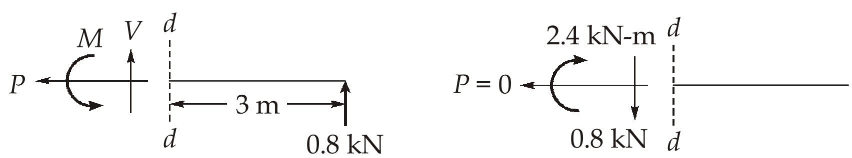
$P = 4$ kN (T)

$V = -0.8$ kN

$M = -1 \times 11 - 1.8 \times 7 + 2 \times 3 = -17.6$ kN-m



At section *d* - *d*



$P = 0$

$V = -0.8$ kN

$M = 0.8 \times 3 = 2.4$ kN-m

7. (a) (i) Solution:

Station	BS	IS	FS	HI	RL	Remarks
1	1.315			251.315	250.000	*BM*
2	2.035		1.150	252.200	250.165	*CP*
3	1.980		3.450	250.730	248.750	*CP*
4	2.625		2.255	251.100	248.475	*CP*
5		1.655			249.445	$P-1$
6		2.155			248.945	$P-2$
7		2.655			248.445	$P-3$
8		3.155			247.945	$P-4$
9			3.655		247.445	$P-5$

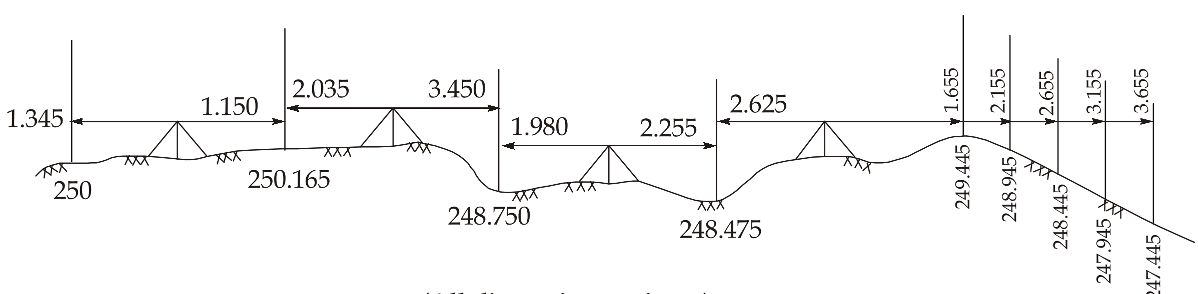
The first peg is fixed with the RL of its top at 249.445 m and the height of instrument is 251.100 m. The RL of the subsequent pegs at 20 m interval will depend upon the decreasing gradient, which is 1 in 40. Difference in level, between two consecutive readings $= \dfrac{\text{distance}}{\text{gradient}} = \dfrac{20}{40} = 0.5$ m. The ground is decreasing by 0.5 m between the consecutive pegs.

Reduced level of pegs:

$$\text{Peg 1} = 249.445 \text{ m}$$
$$\text{Peg 2} = 249.445 - 0.5 = 248.945 \text{ m}$$
$$\text{Peg 3} = 248.945 - 0.5 = 248.445 \text{ m}$$
$$\text{Peg 4} = 248.445 - 0.5 = 247.945 \text{ m}$$
$$\text{Peg 5} = 247.945 - 0.5 = 247.445 \text{ m}$$

Subsequent staff reading at pegs will be

$$\text{Peg 1} = 251.100 - 249.445 = 1.655 \text{ m}$$
$$\text{Peg 2} = 251.100 - 248.945 = 2.155 \text{ m}$$
$$\text{Peg 3} = 251.100 - 248.445 = 2.655 \text{ m}$$
$$\text{Peg 4} = 251.100 - 247.945 = 3.155 \text{ m}$$
$$\text{Peg 5} = 251.100 - 247.445 = 3.655 \text{ m}$$



(All dimension are in m)

Check: $\Sigma BS - \Sigma FS = 7.955 - 10.51 = -2.555$ m

Last RL – First RL $= 247.445 - 250.000 = -2.555$ m (OK)

7. (a) (ii) Solution:

Two-point problem: In this problem, two well-defined points whose positions have already been plotted on the plan are selected. Then, by perfectly bisecting these points, a new station is established at the required position.

Procedure:

(a) Suppose P and Q are two well-defined points whose positions are plotted on map as p and q. It is required to locate a new station at A by perfectly bisecting P and Q.

(b) An auxiliary station B is selected at a suitable position. The table is set up at B, and levelled and oriented by eye estimation. It is then clamped.

(c) With the alidade touching p and q, the points P and Q are bisected and rays are drawn. Suppose these rays intersect at b.

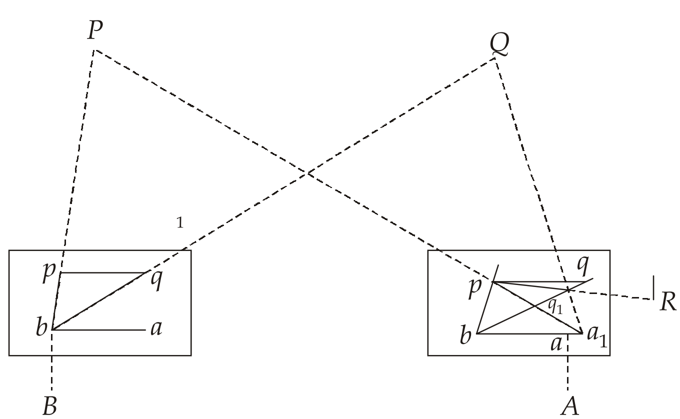
(d) With the alidade centered at b, the ranging road at A is bisected and a ray is drawn. Then, by eye estimation, a point a_1 is marked on this ray.

(e) The table is shifted and centered at A, with a_1 just over A. It is levelled and oriented by backsighting. With the alidade touching p, the point P is bisected and a ray is drawn. Suppose this ray intersects the line ba, at the point a_1, as was assumed previously.

(f) With the alidade centered at a_1, the point Q is bisected and a ray is drawn. Suppose this ray intersects the ray bq at a point q_1. The triangle pqq_1 is known as the triangle of error, and is to be eliminated.

(g) The alidade is placed along the line pq_1 and a ranging rod R is fixed at some distance from the table. Then, the alidade is placed along the line pq and the table is turned to bisect R. At this position, the table is said to be perfectly oriented.

(h) Finally, with the alidade centered at p and q, the points P and Q are bisected and rays are drawn. Suppose these rays intersect at a point a. This would represent the exact position of the required station A. Then the station A is marked on the ground.



Two-point problem

7. (b) (i) Solution:

Given:

Mass of Compacted Specimen (W_m) = 1150 g

Volume of Compacted Specimen (V_m) = 490 cc

Constituent	Weight (W_i, g)	Specific Gravity (G_i)	Percentage by Weight (P_i)
Coarse Aggregate	1250	2.66	$\frac{1250}{2520} \times 100 = 49.603\%$
Fine Aggregate	950	2.58	$\frac{950}{2520} \times 100 = 37.698\%$
Mineral Filler	200	2.72	$\frac{200}{2520} \times 100 = 7.937\%$
Bitumen	120	1.03	$\frac{120}{2520} \times 100 = 4.762\%$
Total Mix	2520	–	100%

Volumetric Calculations

1. Theoretical Maximum Specific Gravity (G_t)

   This is the specific gravity of the mix assuming zero air voids.

$$G_t = \frac{(w_1 + w_2 + w_3 + w_4)}{\left(\frac{w_1}{G_1} + \frac{w_2}{G_2} + \frac{w_3}{G_3} + \frac{w_4}{G_4}\right)} \times 100$$

$$\Rightarrow \quad G_t = \frac{100}{\frac{P_1}{G_1} + \frac{P_2}{G_2} + \frac{P_3}{G_3} + \frac{P_4}{G_4}}$$

$$\Rightarrow \quad G_t = \frac{100}{\frac{49.603}{2.66} + \frac{37.698}{2.58} + \frac{7.937}{2.72} + \frac{4.762}{1.03}} = 2.451$$

2. Bulk Specific Gravity of Compacted Specimen (G_m)

$$G_m = \frac{\text{Weight of specimen}}{\text{Volume of specimen} \times \gamma_w} = \frac{1150}{490 \times 1}$$

$$\Rightarrow \quad G_m = 2.347$$

3. Percent Air Voids (V_v)

$$V_v = \frac{G_t - G_m}{G_t} \times 100 = \frac{2.451 - 2.347}{2.451} \times 100$$

$$\Rightarrow \quad V_v = 4.243\%$$

4. Percent Volume of Bitumen (V_b)

$$V_b = \frac{G_m \times P_b}{G_b} = \frac{2.347 \times 4.762}{1.03}$$

$$\Rightarrow \quad V_b = 10.851\%$$

5. Voids in Mineral Aggregate (*VMA*)

   VMA represents the space available for bitumen and air between the aggregate particles.

$$VMA = V_v + V_b$$

$$\Rightarrow \quad VMA = 4.243 + 10.851$$

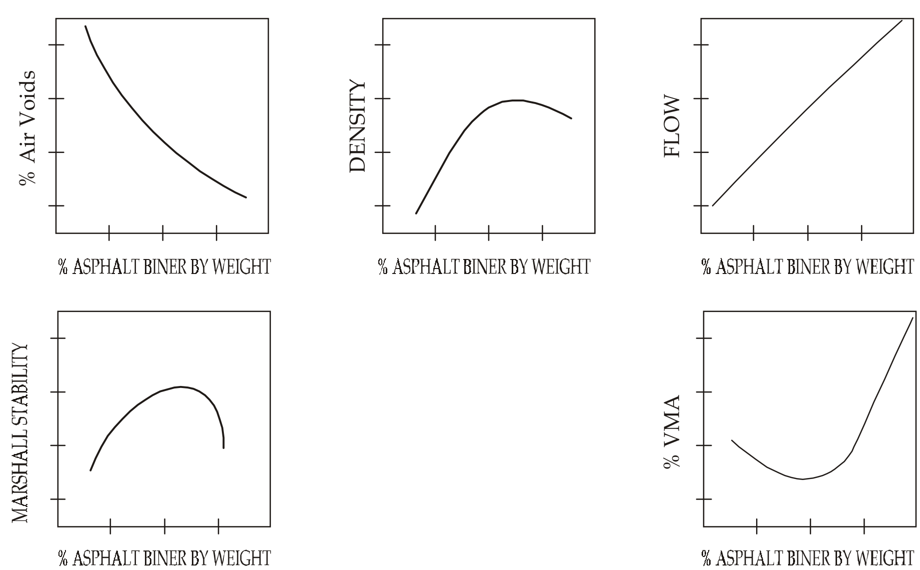
$$\Rightarrow \quad VMA = 15.094\%$$

6. Voids Filled with Bitumen (*VFB*)

$$VFB = \frac{V_b}{VMA} \times 100 = \frac{10.851}{15.094} \times 100$$

$$\Rightarrow \quad VFB = 71.889\%$$

7. (b) (ii) Solution:



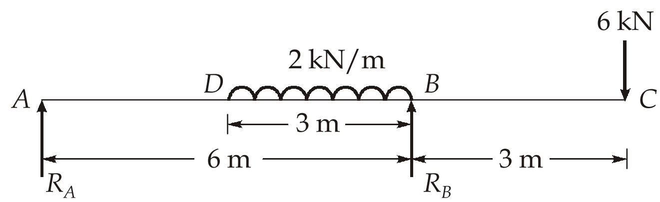
7. (c) Solution:

Given: $E = 2 \times 10^5$ N/mm^2

$I = 5 \times 10^8$ mm^4

$EI = 2 \times 10^5 \times 5 \times 10^8 = 10 \times 10^{13}$ N-mm^2

$= 10 \times 10^4$ kN-m^2

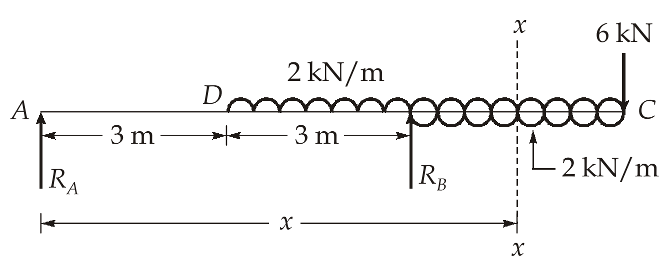


$\Sigma M_A = 0 \quad \Rightarrow \quad R_B \times 6 = (2 \times 3) \times 4.5 + 6 \times 9$

$R_B = 13.5$ kN ($\uparrow$)

$\therefore \quad R_A = 3 \times 2 + 6 - 13.5 = -1.5$ kN($\downarrow$)

Negative sign shows that R_A will be acting downwards. In order to obtain general expression for the bending moment at a distance x from the left end A, which will apply for all values of x, it is necessary to extend the uniformly distributed load upto point C, compensating with an equal upward load of 2 kN/m over the span BC as shown in Figure. Now Macaulay's method can be applied.



Writing bending moment for endspan (section $x - x$)

$$M(x) = R_A\langle x\rangle - \frac{2\langle x-3\rangle^2}{2} + R_B\langle x-6\rangle + \frac{2\langle x-6\rangle^2}{2}$$

$$\Rightarrow \quad \frac{EId^2y}{dx^2} = -1.5x - \langle x-3\rangle^2 + 13.5\langle x-6\rangle + \langle x-6\rangle^2$$

Integrating both sides,

$$EI\frac{dy}{dx} = -1.5\frac{x^2}{2} - \frac{\langle x-3\rangle^3}{3} + 13.5 \times \frac{\langle x-6\rangle^2}{2} + \frac{\langle x-6\rangle^3}{3} + c_1 \quad \text{...(i)}$$

Again integrating both sides,

$$EI(y) = -1.5\frac{x^3}{6} - \frac{\langle x-3\rangle^4}{12} + 13.5\times\frac{\langle x-6\rangle^3}{6} + \frac{\langle x-6\rangle^4}{12} + c_1x + c_2 \text{...(ii)}$$

Using boundary conditions:

At $x = 0$, $y = 0$ for equation (ii) $\Rightarrow$ $c_2 = 0$

At $x = 6$ m, $y = 0$ for equation (ii)

$$\Rightarrow \quad 0 = -1.5\times\frac{6^3}{6} - \frac{(6-3)^4}{12} + 6\times c_1$$

$$c_1 = 10.125$$

Now, slope at Point 'C'

$$EI\,\theta_c = \left(\frac{dy}{dx}\right) \text{ at } x = 9 \text{ m}$$

Considering only positive terms of x from equation (i)

$$\Rightarrow \quad EI\,\theta_c = -1.5\times\frac{9^2}{2} - \frac{(9-3)^3}{3} + 13.5\times\frac{(9-6)^2}{2} + \frac{(9-6)^3}{3} + 10.125$$

$$\Rightarrow \quad \theta_c = \frac{-52.875}{EI} = \frac{-52.875}{10\times10^4} = -5.2875\times10^{-4} \text{ rad } (\circlearrowright)$$

Deflection at c ($x = 9$ m) from equation (ii)

$$EI\delta_c = -1.5\times\frac{9^3}{6} - \frac{(9-3)^4}{12} + 13.5\times\frac{(9-6)^3}{6} + \frac{(9-6)^4}{12} + 10.125\times 9$$

$$\Rightarrow \quad \delta_c = \frac{-131.625}{EI}$$

$$= \frac{-131.625}{10\times10^4}\times1000 = -1.316 \text{ mm } (\downarrow)$$

Slope at Point D, put $x = 3$ in equation (i)

$$EI\theta_D = -1.5\times\frac{3^2}{2} + 10.125$$

$$\Rightarrow \quad \theta_D = \frac{3.375}{EI} = \frac{3.375}{10\times10^4}$$

$$\Rightarrow \quad \theta_D = 3.375\times10^{-5} \text{ Rad } (\circlearrowleft)$$

Deflection at Point D, put $x = 3$ in equation (ii)

$$EI\delta_D = -1.5 \times \frac{3^3}{6} + 10.125 \times 3$$

$$\Rightarrow \quad \delta_D = \frac{23.625}{EI}$$

$$\Rightarrow \quad \delta_D = \frac{23.625}{10 \times 10^4} \times 1000 = 0.236 \text{ mm } (\uparrow)$$

8. (a) (i) Solution:

Given: $f = 150$ mm, $H = 2500$ m, $B = 900$ m

1. Elevations of points A and B

First, we calculate the parallax (p) for each point using the formula:

$$p = x_L - x_R$$

- Parallax for A (p_A):

$$p_A = 45.20 - (-38.40) = 83.600 \text{ mm}$$

- Parallax for B (p_B):

$$p_B = 22.50 - (-52.30) = 74.800 \text{ mm}$$

Now, we use the elevation formula:

$$h = H - \frac{B \times f}{p}$$

- Elevation of A (h_A):

$$h_A = 2500 - \frac{900 \times 150}{83.600} = 2500 - 1614.833 = 885.167 \text{ m}$$

- Elevation of B (h_B):

$$h_B = 2500 - \frac{900 \times 150}{74.800} = 2500 - 1804.813 = 695.187 \text{ m}$$

2. Horizontal ground distance between A and B

We need the ground coordinates (X, Y) for both points. The formulas are:

$$X = \frac{B}{p} \times x_L, Y = \frac{B}{p} \times y_L$$

- For Point A:

$$X_A = \frac{900}{83.600} \times 45.20 = 486.603 \text{ m}$$

$$Y_A = \frac{900}{83.600} \times 52.60 = 566.268 \text{ m}$$

- For Point B:

$$X_B = \frac{900}{74.800} \times 22.50 = 270.722 \text{ m}$$

$$Y_B = \frac{900}{74.800} \times (-40.10) = -482.487 \text{ m}$$

- Horizontal Distance (L):

$$L = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$$

$$L = \sqrt{(270.722 - 486.603)^2 + (-482.487 - 566.268)^2}$$

$$L = 1070.744 \text{ m}$$

3. Gradient (percentage slope) of line AB

   The gradient is the ratio of the vertical difference (rise) to the horizontal distance (run).

   - Vertical Difference (Δh):

$$\Delta h = 885.167 - 695.187 = 189.980 \text{ m}$$

   - Horizontal Distance (L): 1070.744 m

$$\text{Gradient } \% = \left(\frac{\Delta h}{L}\right) \times 100$$

$$\text{Gradient } \% = \left(\frac{189.980}{1070.744}\right) \times 100 = 0.1774 \times 100 = 17.743\%$$

8. (a) (ii) Solution:

	Map	Photograph
Definition	A symbolic representation of the Earth's surface showing features like rivers, roads, elevations, etc.	A visual image captured by a camera showing the actual appearance of a place at a particular moment.
Nature	Abstract and symbolic	Realistic and pictorial
Scale	Represented with a scale (e.g., 1:50,000)	No scale, shows actual view
Details	Shows selected features, may generalize or omit minor details	Shows everything visible in the scene, including minor details
Purpose	To provide information, navigation, planning	To observe and record real-time appearance
Perspective	Can be from top, oblique, or thematic view	Usually from a single point of view (aerial or ground)
Interpretation	Requires reading skills (symbols, legends)	Can be interpreted directly by sight

8. (b) Solution:

Weaving width, $W = \frac{e_1 + e_2}{2} + 3.5$

Given, $e_1 = e_2 = 10$ m

$\therefore$ $W = \frac{10+10}{2} + 3.5 = 13.5$ m

Length of weaving section,

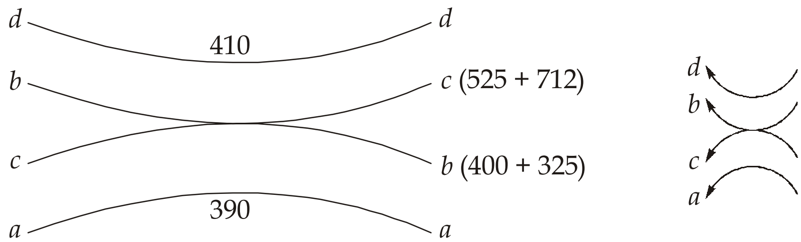
$$L = 4W$$
$$= 4 \times 13.5 = 54 \text{ m}$$

Calculation of proportion of weaving traffic to the non-weaving traffic in all the four approaches.

Let the proportion of traffic in West-North direction be denoted by p_{WN}, in North-East direction as p_{NE} in East-South direction as p_{ES} and finally in South-West direction as p_{SW} and proportion can be calculated as,

$$p = \frac{\text{Weaving and crossing traffic}}{\text{Total traffic}} = \frac{b+c}{a+b+c+d}$$

Traffic weaving in East-South direction



$$a = V_{ES},\ b = V_{EN} + V_{CW},\ c = V_{NS} + V_{WS},\ d = V_{NW}$$

$$p_{ES} = \frac{(400+325)+(525+712)}{(410)+(390)+(525+712)+(400+325)} = 0.71$$

Similarly for leg WN

$$a = V_{WN},\ b = V_{WS} + V_{WE},\ c = V_{SN} + V_{EN},\ d = V_{SE}$$

$$p_{WN} = \frac{(600+712)+(325+280)}{(570)+(200)+(600+712)+(325+280)} = 0.713$$

For leg NE, $a = V_{NE},\ b = V_{NW} + V_{NS},\ c = V_{SE} + V_{WE},\ d = V_{WS}$

$$p_{ES} = \frac{600+200+525+410}{(470)+(712)+(600+200)+(525+410)} = 0.594$$

For leg SW, $a = V_{SW},\ b = V_{SE} + V_{SN},\ c = V_{WE} + V_{SW},\ d = V_{EN}$

$$p_{SW} = \frac{280+200+400+410}{(150)+(325)+(280+200)+(410+400)} = 0.730$$

Here we observe that proportion of weaving traffic to non-weaving traffic is highest in South-West direction i.e., $p_{max} = p_{SW} = 0.730$ and lowest in North-East direction i.e., $p_{min} = 0.594$.

Capacity of rotary is given by Q_{CP},

$$Q_{CP} = \frac{280W\left(1+\frac{e}{W}\right)\left(1+\frac{p}{3}\right)}{1+\frac{W}{L}}$$

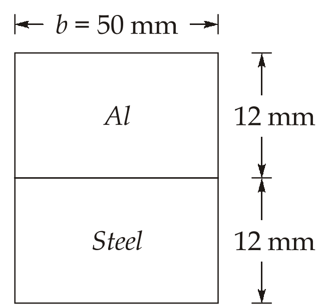
It is clear from the equation that highest proportion of weaving traffic to non-weaving traffic will give the minimum capacity.

$$p_{max} = p_{SW} = 0.730$$

$$Q_{CP} = Q_{SW} = \frac{280 \times 13.5\left(1+\frac{10}{13.5}\right)\left(1-\frac{0.730}{3}\right)}{1+\frac{13.5}{54}}$$

$$= 3983.09 \text{ veh/hr} \simeq 3983 \text{ veh/hr}$$

8. (c) Solution:



Given Parameters Dimensions:

Width, $b = 50$ mm

Depth of each plate, $d = 12$ mm

Total depth = 24 mm

Span, $L = 1.2$ m = 1200 mm

Material Properties

Steel: $E_s = 2 \times 10^5$ N/mm^2 ; $\sigma_{s,\text{ allow}} = 120$ N/mm^2

Aluminium: $E_a = 7 \times 10^4$ N/mm^2; $\sigma_{a,\text{ allow}} = 80$ N/mm^2

Moment of Inertia for One Plate

$$I = \frac{b \cdot d^3}{12} = \frac{50 \times 12^3}{12}$$

$$= 7200 \text{ mm}^4$$

(i) Plates Bend Independently (Not Connected)

When not connected, both plates share the same radius of curvature, and the total moment M is distributed based on their flexural rigidities (EI).

Stiffness Relationship

$$\frac{M_s}{E_s I} = \frac{M_a}{E_a I}$$

$$M_a = M_s\left(\frac{E_a}{E_s}\right) = M_s\left(\frac{7\times10^4}{2\times10^5}\right) = 0.35\ M_s$$

Check Limit states

If Steel Reaches 120 N/mm^2

$$M_s = \frac{\sigma_{s,allow}\cdot I}{y} = \frac{120\times7200}{6} = 144{,}000 \text{ N-mm}$$

$$M_a = 0.35 \times 144{,}000 = 50{,}400 \text{ N-mm}$$

$$\sigma_a = \frac{50400\times6}{7200} = 42 \text{ N/mm}^2 \text{ (Safe, < 80)}$$

If Aluminium Reaches 80 N/mm^2

$$M_a = 96{,}000 \text{ N-mm}$$

$$M_s = \frac{96000}{0.35} = 274{,}285 \text{ N-mm}$$

$$\sigma_s = \frac{274{,}285\times6}{7200} \approx 228 \text{ N/mm}^2 \text{ (Unsafe, > 120)}$$

Maximum Central Load (P)

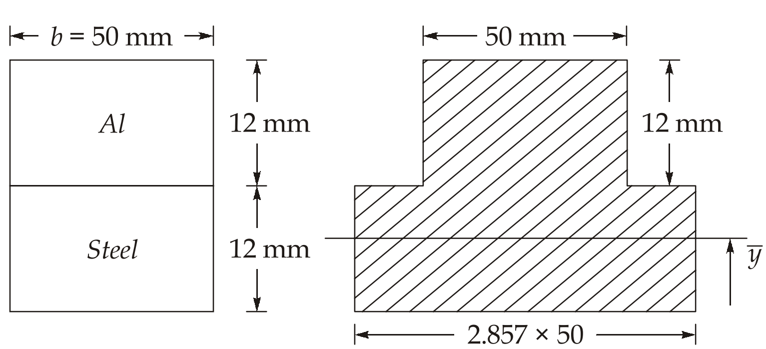
$$M_{total} = M_s + M_a = 144000 + 50400 = 194400 \text{ N-mm}$$

$$P = \frac{4\cdot M_{total}}{L} = \frac{4\times194400}{1200} = 648 \text{ N}$$

(ii) Plates Firmly Secured (Composite Action)

Modular Ratio

$$m = \frac{E_s}{E_a} = \frac{2\times10^5}{7\times10^4} = 2.857$$



Transformed section

Neutral Axis ($\overline{y}$ from Bottom)

$$\overline{y} = \frac{(m \cdot b \cdot d) \cdot 6 + (b \cdot d) \cdot 18}{(m \cdot b \cdot d) + (b \cdot d)}$$

$$\Rightarrow \quad \overline{y} = \frac{(2.857 \times 50 \times 12) \times 6 + (50 \times 12) \times 18}{(2.857 \times 50 \times 12) + (50 \times 12)}$$

$$\Rightarrow \quad \overline{y} = 9.11 \text{ mm}$$

Composite Moment of Inertia (I_{comp})

Using parallel axis theorem for the transformed aluminium section:

$$I_{comp} = \left[\frac{(2.857 \times 50) \times 12^3}{12} + (1714.2)(9.11 - 6)^2\right] + \left[\frac{50 \times 12^3}{12} + (600)(18 - 9.11)^2\right]$$

$I_{camp} = 91766.4 \text{ mm}^4$

Check Limit States

Steel Limit (120 N/mm^2 at Bottom)

$$M_S = \frac{\sigma_{s,allow} \cdot I_{comp}}{m \cdot \overline{y}} = \frac{120 \times 91,766.4}{2.857 \times 9.11} = 423094 \text{ N-mm}$$

Aluminium Limit (80 N/mm^2 at Top)

$$M_a = \frac{\sigma_{s,allow} \cdot I_{comp}}{(24 - \overline{y})} = \frac{80 \times 91,766.4}{14.89} = 49.3036 \text{ N-mm}$$

Maximum Central Load (P)

The steel governs the design.

$$P = \frac{4 \times 423094}{1200} = 1410 \text{ N}$$

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