MADE EASY

Leading Institute for ESE, GATE & PSUs

ESE 2026 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-1 : Network Theory + Signals and Systems [All Topics]

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☑	Bhopal ☐	Jaipur ☐	
Pune ☐	Hyderabad ☐		

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	35
Q.2	
Q.3	~~30~~ 42
Q.4	
Section-B	
Q.5	21
Q.6	
Q.7	34
Q.8	31+2=33
Total Marks Obtained	16~~5~~

Signature of Evaluator .... Cross Checked by ....

27/3/26

• Good keep it up.

• Practice more — to improve accu—.

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Do not write in this margin

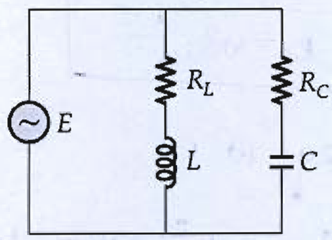
Section A : Network Theory

Q.1(a) Briefly explain sources used in network (independent and dependent sources) and their types.

[12 marks]

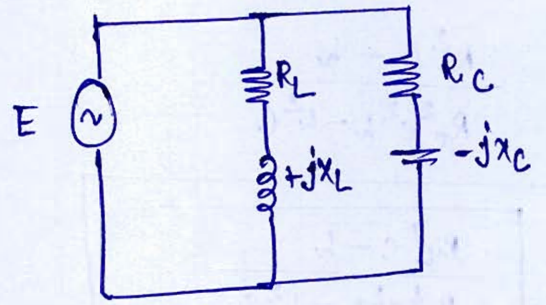
Q.1 (b) Draw the phasor diagram for the circuit shown and prove that the condition for resonance is

$$\frac{L}{C} = \frac{R_L^2 + \omega^2 L^2}{1 + R_C^2 \omega^2 C^2}$$



Also, calculate the value of ω (rad/sec) at resonance.

[12 marks]



where $jX_L = jL\omega$

$-jX_C = \frac{-j}{C\omega}$

Tetata Admitance $Y_{eq} = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$

$$Y_{eq} = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + (X_C)^2}$$

$$Y_{eq} = \left(\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}\right) + j\left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right)$$

At Resomama mamce Admittance will be real.

$$\text{Img}[Y_{eq}] = 0$$

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$\frac{X_C}{X_L} = \frac{R_C^2 + X_C^2}{R_L^2 + X_L^2}$$

$$\frac{X_L}{X_C} = \frac{R_L^2 + X_L^2}{R_C^2 + X_C^2}$$

$$\frac{L\omega}{\frac{1}{\omega C}} = \frac{R_L^2 + L^2\omega^2}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

Do not write in this margin

$$\frac{L}{\omega^2} = \frac{\omega^2 C\,(R_L^2+\omega^2L^2)}{1+R_C^2\omega^2C^2}$$

$$\boxed{\frac{L}{C} = \frac{R_L^2+\omega^2L^2}{1+R_C^2\omega^2C^2}}$$

For Resonance freq. ω:

$$L + R_C^2\omega^2C^2L = R_L^2C + \omega^2L^2C$$

$$\omega^2(R_C^2C^2L - L^2C) = R_L^2C - L$$

$$\omega^2 = \frac{R_L^2C - L}{R_C^2C^2L - L^2C}$$

Resonance Frequency: $\boxed{\omega = \sqrt{\frac{R_L^2C - L}{R_C^2C^2L - L^2C}}}$

$$\omega = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

Try to write in standard form

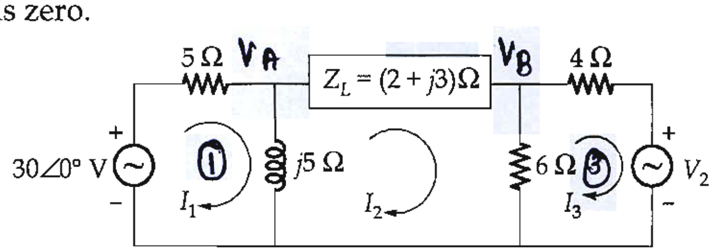
(12)

Good

Q.1 (c) An iron-cored coil takes 4A at a power factor of 0.5 when connected to a 200-V, 50 Hz supply. When the iron core is removed and the voltage is reduced to 40 V, the current rises to 5 A at a pf of 0.8. Find the iron loss in the core and inductance in each case.

[12 marks]

Q.1 (d) In the network shown below, find the value of V_2 so that the current through $Z_L = (2 + j3)\ \Omega$ impedance is zero.



[12 marks]

Let current through Z_L is zero.

$$\boxed{I_2 = 0} \quad \text{—— (1)}$$

Apply KVL in loop (1)

$$30 = (5 + j5) I_1$$

$$I_1 = \frac{30}{5 + j5}$$

Node voltage $V_A = I_1 \times j5$

$$V_A = \frac{30 \times j5}{5 + j5}$$

$$V_A = V_B \quad \{ \because I_2 = 0 \} \text{ —— (2)}$$

$$V_B = \frac{V_2 \times 6}{4 + 6} \quad [\text{By KVL voltage division}]$$

$$V_B = \frac{6V_2}{10} = V_A \quad [\text{From Eq } ②]$$

$$\frac{6V_2}{10} = \frac{30 \times j5}{5+j5}$$

$$\vec{V_2} = \frac{300 \times j5}{6(5+j5)}$$

$$V_2 = 25 + j25$$

$$\boxed{V_2 = 35.3553 \angle 45^\circ \text{ Volt}}$$

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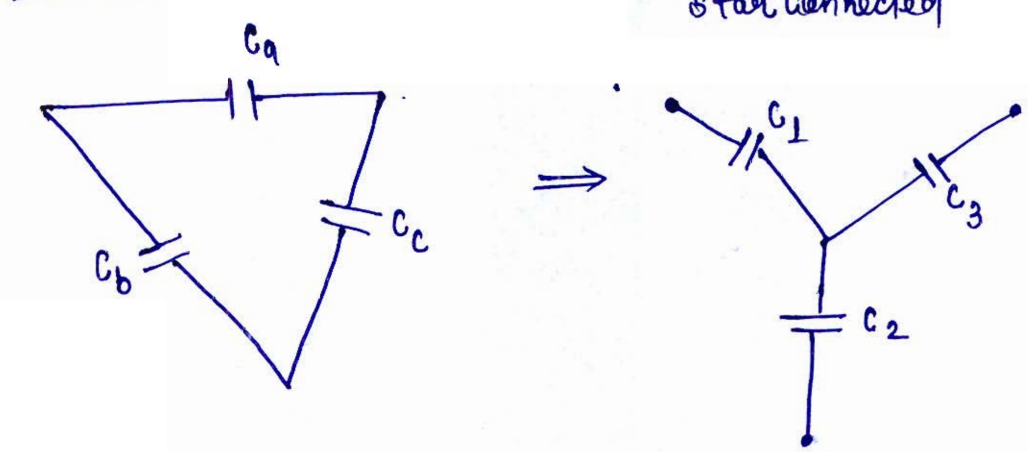
Good

Q.1 (e) Represent a Delta connected capacitive circuit as an equivalent star connection and also express the star connected capacitive elements in terms of the delta connected capacitive elements.

[12 marks]

Delta

Star Connected

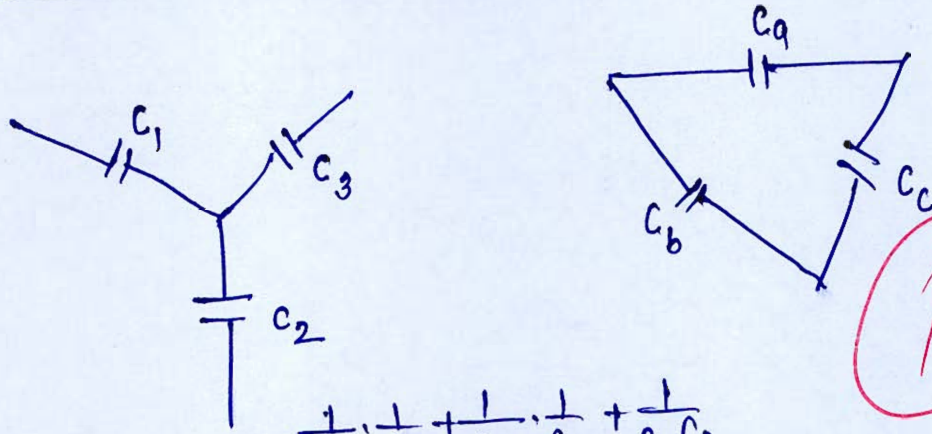


$$\frac{1}{C_1} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}} \Rightarrow C_1 = \frac{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}{\frac{1}{C_a} \cdot \frac{1}{C_b}}$$

$$C_2 = \frac{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}{\frac{1}{C_b} \cdot \frac{1}{C_c}}$$

$$C_3 = \frac{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C}}{\frac{1}{C_a} \cdot \frac{1}{C_c}}$$

STAR to Delta



11

$$\frac{1}{C_a} = \frac{\frac{1}{C_1}\cdot\frac{1}{C_2} + \frac{1}{C_1}\cdot\frac{1}{C_3} + \frac{1}{C_2 C_3}}{\frac{1}{C_2}}$$

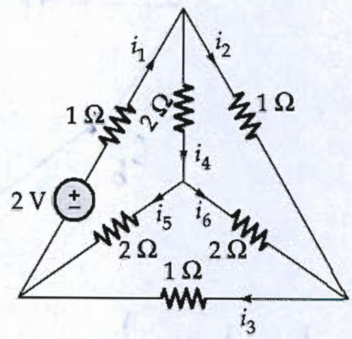
$$\boxed{C_a = \frac{\frac{1}{C_2}}{\frac{1}{C_1}\cdot\frac{1}{C_2} + \frac{1}{C_1}\cdot\frac{1}{C_3} + \frac{1}{C_2}\cdot\frac{1}{C_3}}}$$

$$C_b = \frac{\frac{1}{C_3}}{\frac{1}{C_1}\cdot\frac{1}{C_2} + \frac{1}{C_1}\cdot\frac{1}{C_3} + \frac{1}{C_2}\cdot\frac{1}{C_3}}$$

$$C_c = \frac{\frac{1}{C_1}}{\frac{1}{C_1}\cdot\frac{1}{C_2} + \frac{1}{C_1}\cdot\frac{1}{C_3} + \frac{1}{C_2}\cdot\frac{1}{C_3}}$$

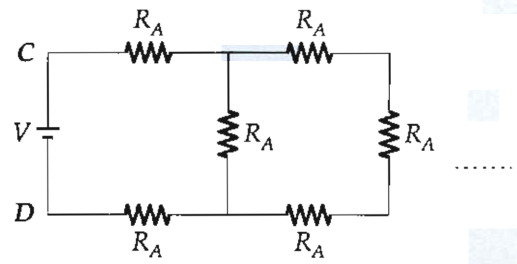
Try to write in simplified form

Q.2 (a) For the network shown in figure below, write down the tieset matrix and obtain the network equilibrium equations in matrix form using KVL. Calculate the loop currents and branch currents.

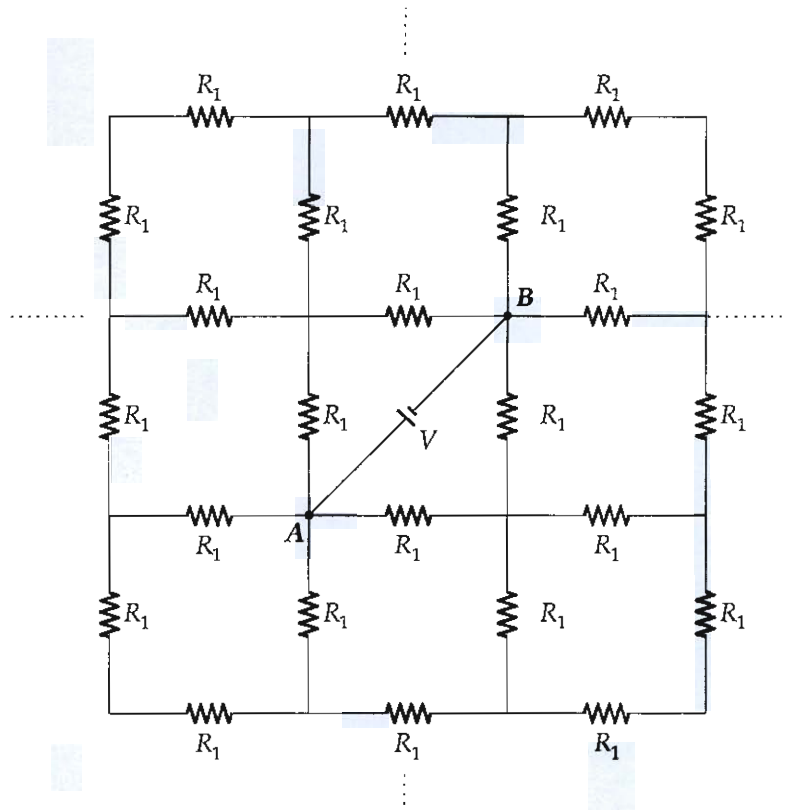


[20 marks]

Q.2 (b) For the networks shown in figure below,



Network (i)

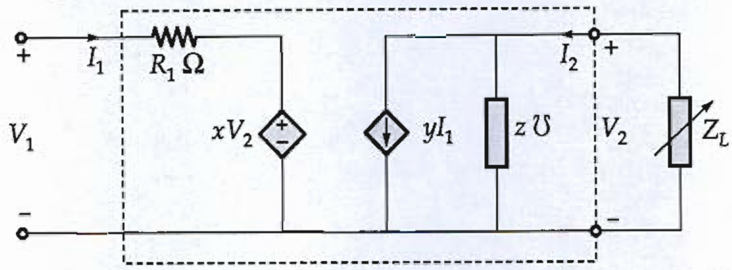


Network (ii)

On applying 10 V voltage source across terminal CD and AB respectively in network (i) and (ii), we get same power delivered by the source. Derive the relationship between the resistance present in network (i) and (ii). And also calculate the power delivered by the source for $R_A = 10\ \Omega$.

[20 marks]

Q.2 (c) Consider a two port network shown in figure below,



If transmission parameters matrix of the network is $\begin{bmatrix} 10^{-2} & 10^{2} \\ 0 & 10^{-1} \end{bmatrix}$.

Then, calculate:

(i) parameters of the circuit: R_1, x, y and z.

(ii) the value of load impedance (Z_L), for maximum power transfer.

(iii) maximum power transfer to load for $V_1 = 0.1$ volt.

[20 marks]

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Q.3 (a) (i) State 'Voltage to current source transformation' theorem. It is required to replace network N in figure (a) by a suitable equivalent network. Which of the networks of figure (b) could be valid equivalent network (s)?

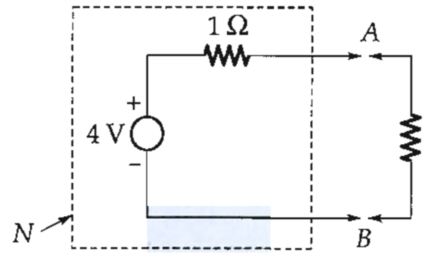
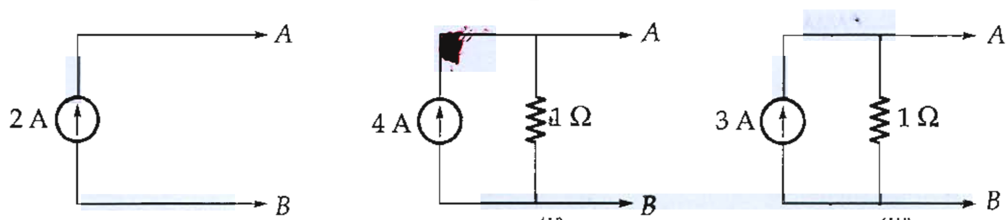


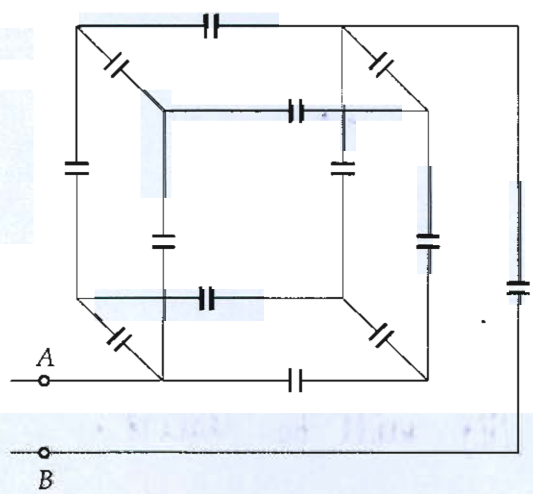
Fig. (a)



(i) (ii) (iii)

Fig. (b)

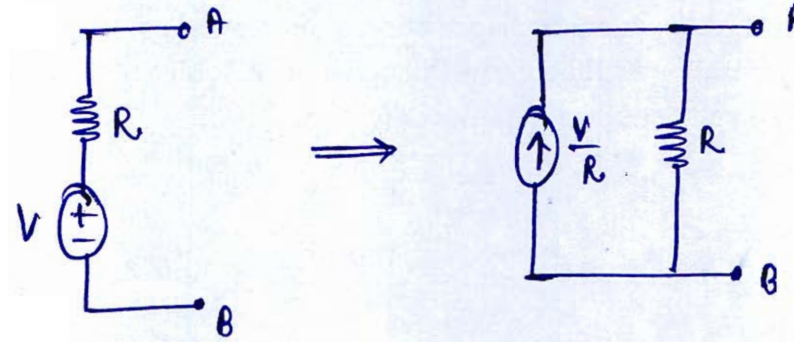
(ii) The network of capacitors in the figure below is composed of a 2F capacitor on each edge of a cube along with a capacitor of 2F connected to the vertices of the cube as shown. Find the $C_{equivalent}$ between the terminals A-B and also calculate the energy stored by the capacitive circuit if 5 V is applied across the terminal A-B.

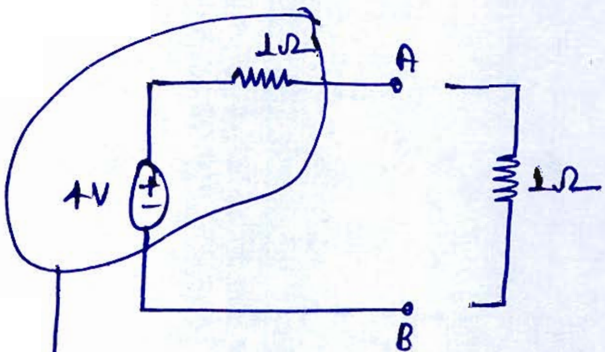


[10 + 10 marks]

(i) Voltage to current source transformation :

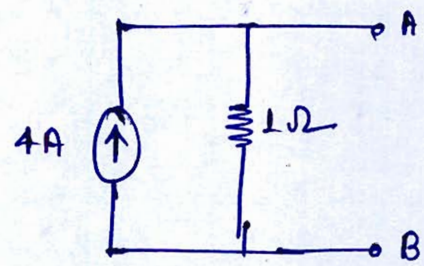
If any voltage source having voltage 'V' and series resistance R then we can replace it to it equivalent current source having current '$\frac{V}{R}$' and in parallel resistance 'R'.





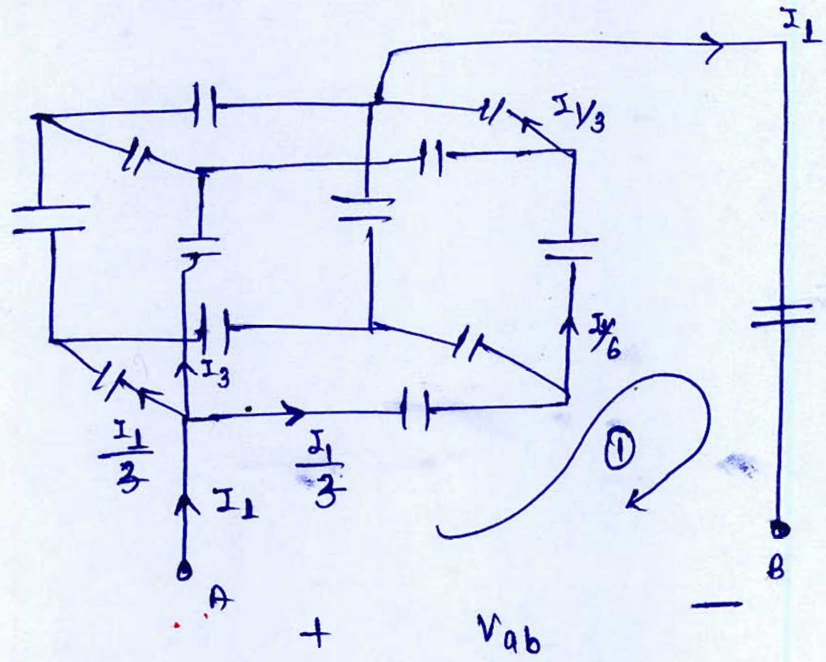
By Applying source transformation as stated above.

$$I = \frac{4}{1} = 4A$$



Hence Fig (b) (ii) will be valid.

(ii)



∵ We know that $v_C = \frac{1}{C}\int i\,dt$

Let each capacitor is C = 2F (as given)

$$V_{ab} = \frac{1}{2}\int \frac{I_1}{3}dt + \frac{1}{2}\int \frac{I_1}{6}dt + \frac{1}{2}\int \frac{I_1}{3}dt + \frac{1}{2}\int I_1 dt$$

$$\frac{1}{C_{eq}}\int I_1 dt = \int I_1 dt\left[\frac{1}{6} + \frac{1}{12} + \frac{1}{6} + \frac{1}{2}\right]$$

$$C_{eq} = \frac{1}{\frac{2+1+2+6}{12}}$$

$$\boxed{C_{eq} = \frac{12}{11}\,F}$$

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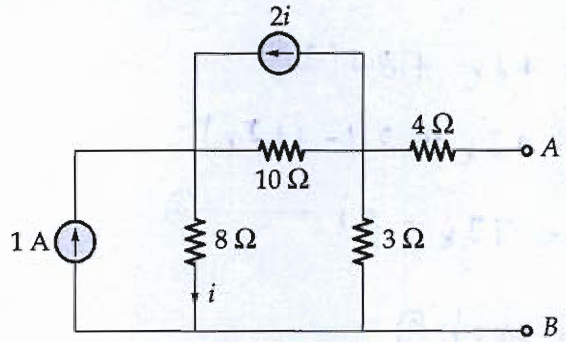
E=11

Do not write in this margin

Q.3 (b) For the circuit shown in figure below:



Calculate:

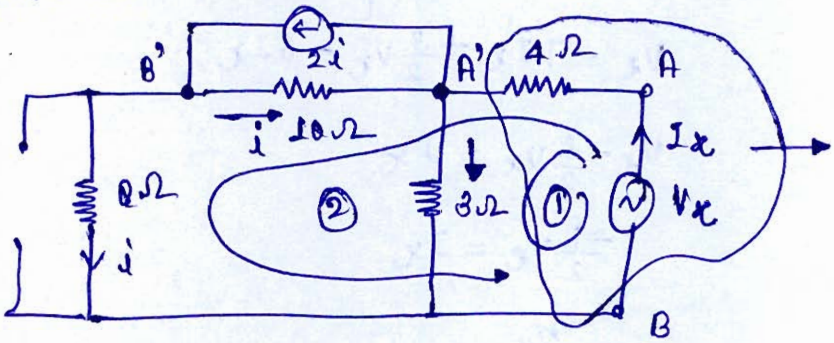
(i) Norton equivalent circuit across *A-B*.

(ii) Load impedance across *AB* for Maximum Power Transfer.

(iii) Maximum Power Transfer to load, obtained in part (ii) and also comment on the result.

[20 marks]

(i) Case 1 : Rth (deactivate All the independent sources and find the equivalent Resistance across 'AB'.



$R_{th} = R_N = \frac{V_x}{I_x}$ —— ①

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Apply KCL at Node B'

$2i = I(10\Omega) + i$

$I(10\Omega) = i$

Apply KCL at Node A'

$2i + I(3\Omega) = i + I_x$

$I(3\Omega) = -i + I_x$ —— ②

Apply KVL in loop ①

$V_x = 4I_x + 3\,I(3\Omega)$

$V_x = 4I_x + 3\,(-i + I_x)$

$V_x = 7I_x - 3i$ ——— ③

Apply KVL in loop ②

$V_x = 4I_x - 10i + 2i$

$\frac{V_x - 4I_x}{-2} = i$

Put this value in eq ③

$V_x = 7I_x - 3\left(\frac{V_x - 4I_x}{-2}\right)$

$V_x = 7I_x + \frac{3}{2}V_x - 6I_x$

$V_x - \frac{3}{2}V_x = I_x$

$-\frac{1}{2}V_x = I_x$

$\frac{V_x}{I_x} = -2$

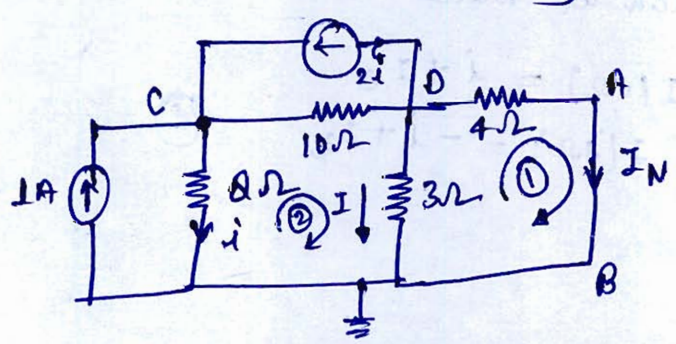
$\boxed{R_{th} = -2\Omega}$

∵ Resistance can't be −ve,

$R_{th} = R_N = 2\Omega$

See the soln

Case 2 : $I_N = I_{sc}$ (short circuit current across AB terminal)



Apply KCL at Node C :

$I_{CD} + i = 2i + 1$

$I_{CD} = i + 1$

Apply KCL at Node D

$I + 2i + I_N = i + 1$

$I = -i - I_N + 1$

Apply KVL in loop ①

$-3I + 4I_N = 0$

$-3(-i - I_N + 1) + 4I_N = 0$

$+3i + 3I_N - 3 + 4I_N = 0$

$7I_N = 3 - 3i$ —— ②

Apply KVL in Node ②

$-2i + 10(i+1) + 3(-i - I_N + 1) = 0$

$-2i + 10i + 10 - 3i - 3I_N + 3 = 0$

Put i From Eq ②

$2i - 3i + 13 - 3I_N = 0$

$3I_N = 13 - i$

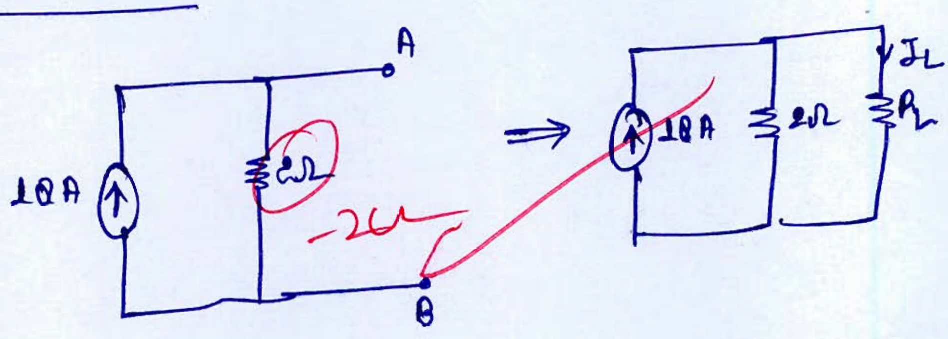
$3I_N = 13 - \left(\frac{3 - 7I_N}{3}\right)$

$9I_N = 39 - 3 + 7I_N$

$2I_N = 36$

$I_N = 18$ Amp

Equivalent Network :



Load Impedance R_L ⟶

$$P = I_L^2 R_L \qquad I_L = \frac{18 \times 2}{2+R_L} \text{ (By KCL)}$$

$$P = \left(\frac{36}{2+R_L}\right)^2 R_L$$

For max power: $\frac{dP}{dR_L} = 0$

By solving $\boxed{R_L = 2\,\Omega}$

(iii) Max Power $= P_{max} = I_L^2 R_L$

$$= \left(\frac{18 \times 2}{2+2}\right) \times 2$$

$$= \left(\frac{36}{4}\right)^2 \times 2$$

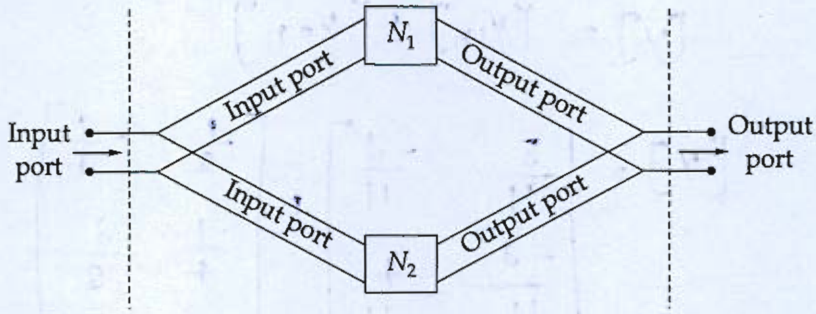
$$= 81 \times 2$$

$$= 162 \text{ watt}$$

we can say that max power transfer will only take place when $\boxed{R_L = 2\Omega = R_{th}}$

Comment ??

Q.3 (c) Consider the two-port network 'N' given below:



N_1 and N_2 are two 2-port networks connected in parallel on both input port side as well as output port side, to form a composite 2-port network N as indicated.
N_1 and N_2 are defined by the z-parameters as below:

$$[Z_{n_1}] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}\Omega, [Z_{n_2}] = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}\Omega$$

Obtain the transmission parameters for the composite 2-port network N. Also, express the transmission parameters in terms of short circuit parameters.

[20 marks]

We know:

$$[Y] = [Z]^{-1}$$

$$[Y] = \frac{1}{Z_{11}Z_{22} - Z_{21}\cdot Z_{12}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

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Network 1:

$$[Y_{n1}] = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}^{-1}$$

$$[Y_{n1}] = \frac{1}{20-9} \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$$

$$[Y_{n1}] = \frac{1}{11} \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix}$$

$$[Y_{n1}] = \begin{bmatrix} \frac{5}{11} & -\frac{3}{11} \\ \frac{-3}{11} & \frac{4}{11} \end{bmatrix}$$

Network 2:

$$[Y_{n2}] = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}^{-1}$$

$$[Y_{n2}] = \frac{1}{3\times4 - 2\times2} \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$$

$$[Y_{n2}] = \frac{1}{8} \begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix}$$

$$[Y_{n2}] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

For parallel Network we know equivalent

$$[y] = [y_{1n}] + [y_{2n}]$$

$$[y] = \begin{bmatrix} \frac{5}{11} & \frac{-3}{11} \\ \frac{-3}{11} & \frac{4}{11} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{-1}{4} & \frac{3}{8} \end{bmatrix}$$

Equivalent short circuit parameter

$$[y] = \begin{bmatrix} \frac{21}{2} & \frac{-33}{44} \\ \frac{-33}{44} & \frac{65}{88} \end{bmatrix}$$

we already define: $[z] = [y]^{-1}$

~~So Equivalent z~~

y - parameter equtian ——

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{——} ①$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{——} ②$$

~~From above ② Equation~~

Transmission parameter Equation ;

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

} we have to model the Eq ① & ② in this form.

From Eq ②

$$V_1 = -\frac{y_{22}}{y_{21}} V_2 + \frac{1}{y_{21}} I_2 \quad \text{——} ③$$

$$V_1 = AV_2 - BI_2 \text{ (By comparison)}$$

$$A = \frac{-y_{22}}{y_{21}} \qquad B = -\frac{1}{y_{21}}$$

Put Eq ③ in Eq ① the value of V_1 :

$$I_1 = y_{11}\left[\left(\frac{-y_{22}}{y_{21}}\right)V_2 + \frac{1}{y_{21}} I_2\right] + y_{12} V_2$$

$$I_1 = \left(y_{12} - \frac{y_{11} y_{22}}{y_{21}}\right)V_2 + \frac{y_{11}}{y_{21}} I_2$$

By Comparing $I_1 = CV_2 - DI_2$

$$C = y_{12} - \frac{y_{11}\, y_{22}}{y_{21}} \qquad D = \frac{-y_{11}}{y_{21}}$$

Now From above Result using Transmission line parameter.

$$A = \frac{\frac{-65}{80}2}{\frac{-33}{44}} \qquad B = -\frac{1}{\frac{-33}{44}}$$

$$A = \frac{65}{66} \qquad B = \frac{44}{33}$$

$$C = \frac{-33}{44} - \frac{\left(\frac{21}{2}\right)\left(\frac{65}{80}\right)2}{\frac{-33}{44}} \qquad D = -\frac{\frac{21}{2}}{\frac{-33}{44}}\ 22$$

$$C = -\frac{33}{44} + \frac{21 \times 65}{33 \times 4} \qquad D = \frac{+21 \times 22\ 2}{+33\ 3}$$

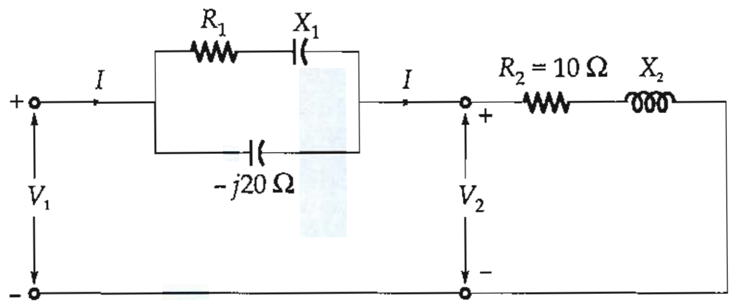
$$C = \frac{211}{22} \qquad D = \frac{42}{3}$$

Avoid Calc.

Q.4 (a) A series RC circuit has $R = 9\ k\Omega$, $C = 20\ \mu F$ and two voltage source in series given by $V_1 = 20u(-t)$ and $V_2 = 20u(t - t')$ V. Determine the complete expression for the voltage across the capacitor and plot it as a function of time, assuming t' as a positive quantity.

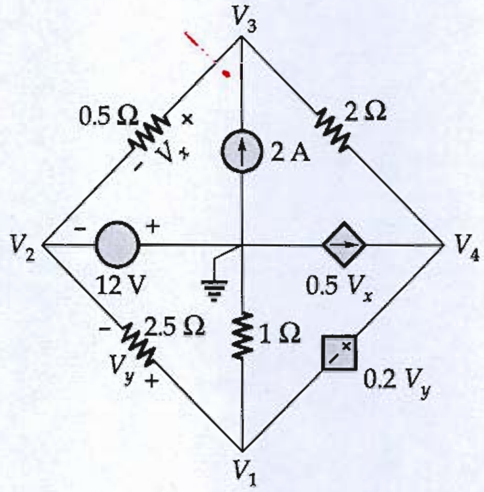
[20 marks]

Q.4 (b) In the circuit shown in the figure below, $|V_1| = 200$ V, $V_2 = 200\angle 0°$ V and $|I| = 12$ A. The total power absorbed by the circuit is 1.8 kW. Find R_1, X_1 and X_2.



[20 marks]

Q.4 (c) For the circuit shown in figure, obtain the value of voltage across 0.5 Ω and 2.5 Ω resistors using nodal analysis.



[20 marks]

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Section B : Signals and Systems

Q.5 (a) (i) State and prove commutative property and distributive property of convolution in discrete time domain.

(ii) Sketch the spectrum of modulated signal $y(t) = x(t) \cdot m(t)$, if

1. $X(f) = \begin{cases} 1-|f| \; ; & |f| \le 1 \\ 0 \; ; & \text{otherwise} \end{cases}$ and $m(t) = \cos 15\pi t$.

2. $X(f) = \text{rect}\,(0.25 f)$ and $m(t) = \cos 2\pi t$.

3. $X(f) = \text{rect}\,(f)$ and $m(t) = \cos \pi t$.

[6 + 6 marks]

Q.5 (b) The complex exponential Fourier series representation of a signal $f(t)$ over the interval $(0, T)$ is

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t} \quad \text{——— (1)}$$

(i) What is the numerical value of T?

(ii) If one of the components of $f(t)$ is $A\cos 3\pi t$, determine the value of A.

(iii) Determine the power contained by the signal $f(t)$ upto the first four harmonics as a percentage of total power of signal.

Note: $\sum_{n=-\infty}^{\infty} \left| \frac{3}{4+(n\pi)^2} \right| \approx 0.669$

[4 + 4 + 4 marks]

We know Exponential F.S.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{j\omega n t}$$

By comparing with Eq (1)

$$C_n = \frac{3}{4+(n\pi)^2} \quad \text{and} \quad \omega = \pi$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\pi}$$

$$\boxed{T = 2 \text{ sec}}$$

(ii) $A \cos 3\pi t$

$A \cos 3\omega t$

$$A\left[\frac{e^{-j3\pi t} + e^{j3\pi t}}{2}\right]$$

$$\frac{A}{2}e^{-j3\pi t} + \frac{A}{2}e^{j3\pi t}$$

So $\frac{A}{2} = C_3$ (By comparison with Eq (1))

$$A = 2 \times \frac{3}{4+(3\times\pi)^2}$$

$$A = \frac{6}{4+9\pi^2}$$

$$\boxed{A = 0.06463}$$

Power by Parsevalls theorem:

$$P = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$\frac{\text{Power up to 4 Hz}}{\text{Total power}} = \frac{\sum_{-4}^{4} |C_n|^2}{\sum_{n=-\infty}^{\infty} \left|\frac{3}{4+(n\pi)^2}\right|^2}$$

$$= \frac{C_0 + 2\sum_{1}^{4} \left(\frac{3}{4+(n\pi)^2}\right)^2}{(0.669)^2} \times 100$$

10

$$= \frac{\frac{3}{4} + 2\left[\left(\frac{3}{4+\pi^2}\right)^2 + \left(\frac{3}{4+4\pi^2}\right)^2 + \left(\frac{3}{4+9\pi^2}\right)^2 + \left(\frac{3}{4+16\pi^2}\right)^2\right]}{(0.669)^2}$$

$$= \frac{\left(\frac{3}{4}\right)^2 + 2\times 9\left[\left(\frac{1}{4+\pi^2}\right)^2 + \left(\frac{1}{4+4\pi^2}\right)^2 + \left(\frac{1}{4+9\pi^2}\right)^2 + \left(\frac{3}{4+16\pi^2}\right)^2\right]}{(0.669)^2}$$

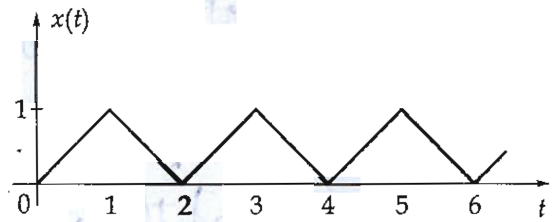
$$= \frac{\left(\frac{3}{4}\right)^2 + 2\times 9\times 0.01011359}{(0.669)^2}$$

= ~~0.932044~~ / ~~0.447561~~

In the % form $= \frac{0.610590}{(0.669)^2}$

= ??

Q.5 (c) (i) Find the Laplace transform of the triangular wave shown in figure.

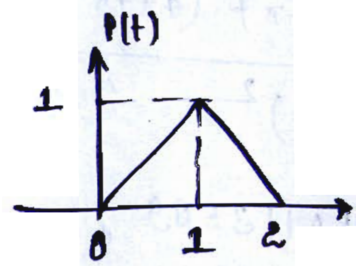


(ii) Determine whether the signal below is periodic or not and if periodic, determine the fundamental period of the signal.

$$x(n) = \text{Re}\left[e^{jn\pi/12}\right] + \text{Im}\left[e^{jn\pi/18}\right]$$

[8 + 4 marks]

(i)



$p(t) = r(t) - 2r(t-1) + r(t-2)$

Apply Laplace transform

$P(s) = \frac{1}{s^2} - \frac{2s^{-1}}{s^2} + \frac{s^{-2}}{s^2}$

we can write x(t) as

$x(t) = \sum_{n=0}^{\infty} p(t-nT)$ where $T = 2$

$x(t) = p(t) + p(t-T) + p(t-2T)$ —

Apply laplace

$X(s) = P(s) + P(s) \cdot e^{-Ts} + P(s)\, e^{-2Ts} +$ —

$$X(s) = P(s) \cdot \left[\frac{1}{1 - e^{-Ts}}\right]$$

$$X(s) = \frac{P(s)}{1 - e^{-2s}}$$

$$\boxed{X(s) = \frac{1 - 2s^{-1} + s^{-2}}{s^2(1 - e^{-2s})}}$$

(ii)

$$x(n) = Re[e^{jn\pi/12}] + Im[e^{jn\pi/18}]$$

$$x(n) = Re\left[\cos\frac{n\pi}{12} + j\sin\frac{n\pi}{12}\right] + Img\left[\cos\frac{n\pi}{18} + j\sin\frac{n\pi}{18}\right]$$

$$x(n) = \cos\frac{n\pi}{12} + \sin\frac{n\pi}{18}$$

$$N_1 = \frac{2\pi}{\pi/12} \qquad N_2 = \frac{2\pi}{\pi/18}$$

$$N_1 = 24 \qquad N_2 = 36$$

11

Fundamental period $N = LCM(N_1, N_2)$
$= LCM(24, 36)$
$= 12 \times 2 \times 3$
$= 72$

Good

$\frac{N_1}{N_2}$ = ration so given signal is periodic with period of 72.

Q.5 (d) Compute the circular convolution of the following sequences using DFT and IDFT:

$$x_1(n) = \{1, 2, 1, 2\} \text{ and } x_2(n) = \{4, 3, 2, 1\}$$

[12 marks]

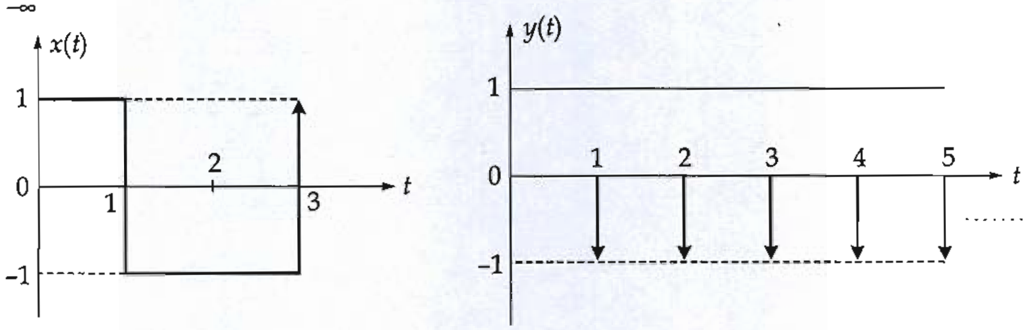
Q.5 (e) Derive the impulse response $h_d(n)$ of a highpass filter to meet the following specifications:

Cutoff frequency = 250 Hz

Sampling frequency f_s = 1 kHz and

Filter length = 7 **[12 marks]**

Q.6 (a) (i) For the signals $x(t)$ and $y(t)$ given below, determine and sketch $\int_{-\infty}^{t} x(\tau)d\tau$ and $\int_{-\infty}^{t} y(\tau)d\tau$:



(ii) For the second order FIR lattice filter with reflection coefficients $K_1 = \frac{1}{2}, K_2 = \frac{1}{4}$, draw the FIR lattice structure and find the transfer function and impulse response of the FIR system.

[8 + 12 marks]

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.6 (b) (i) Determine the time domain signals corresponding to the bilateral Laplace transforms given below. Specify the properties used:

1. $X(s) = \frac{1}{s^2} \cdot \frac{d}{ds}\left(\frac{e^{-3s}}{s}\right)$, ROC : Re(s) > 0

2. $X(s) = s\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}\right)$, ROC : Re(s) > 0

(ii) The impulse response of a relaxed linear time invariant system is $h(n) = \alpha^n u(n)$ with $|\alpha| < 1$. Determine the value of the step response as $n \to \infty$.

[5 + 5 + 10 marks]

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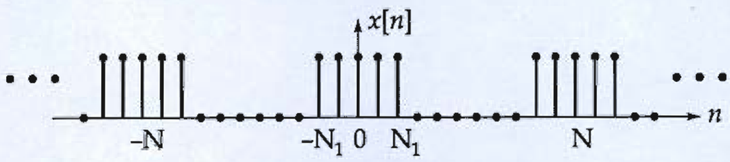
.6 (c) (i) Let $z = re^{j\omega}$ and $s = \sigma + j\Omega$. Use bilinear transformation to show that if $r < 1$, then $\sigma < 0$, and if $r > 1$, then $\sigma > 0$, and when $r = 1$, then $\sigma = 0$.

(ii) Consider a signal $x[n]$ such that

$$x[n] = \begin{cases} 1 & \text{for } -N_1 < n < N_1 \\ 0 & \text{for rest of time period} \end{cases}$$



Discrete-time periodic square wave

If time period of $x[n]$ is N where ($N > 2N_1 + 1$), then determine Fourier series coefficient a_k for signal $x[n]$.

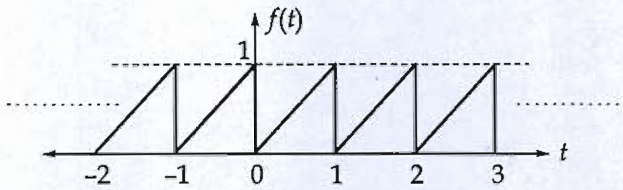
[10 + 10 marks]

Q.7 (a) **(i)** Determine the signal $x[n]$ and rational z-transform $X(z)$ for the following cases:

1. $x[n]$ is right sided, $X(z)$ has a single pole, $x[0] = 4$, $x[2] = \frac{1}{4}$.

2. $X[z]$ has poles at $z = \frac{1}{4}$ and $z = -1$, ROC includes $|z| = \frac{1}{2}$, $x[1] = 1$, $x[-1] = 1$.

(ii) **1.** Obtain exponential Fourier series representation of the periodic signal shown below.



2. Draw the magnitude and phase plot of the Fourier series coefficient of the above signal.

[10 + 10 marks]

① Let $X(z) = \frac{K}{z+P}$

Apply inverse z-transform

$x[n] = K \cdot (-P)^n u(n)$

Given $x[0] = 4$

$K \cdot (-P)^0 u(0) = 4$

$\boxed{K = 4}$

$x[2] = \frac{1}{4}$

$K(-P)^2 u(2) = \frac{1}{4}$

$4(-P)^2 \times 1 = \frac{1}{4}$

$(-P)^2 = \frac{1}{16}$

$-P = \pm \frac{1}{4}$

⊕ $P = -\frac{1}{4}$ ⊖ $P = \frac{1}{4}$

So $p = -\frac{1}{4}$ or $\frac{1}{4}$

$\therefore \quad x(n) = 2\left(\frac{1}{4}\right)^n u(n)$ or $x(n) = 2\left(\frac{-1}{4}\right)^n u(n)$

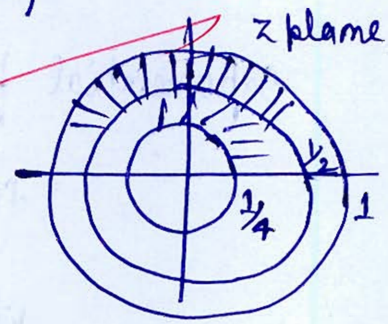
& $X(z) = \frac{2}{z+\frac{1}{4}}$ or $X(z) = \frac{2}{z-\frac{1}{4}}$

(2) $X(z)$ has pole $z = \frac{1}{4}$ & $z = -1$

then $X(z) = \frac{K}{\left(z-\frac{1}{4}\right)(z+1)}$

$\because$ ROC Include $|z| = \frac{1}{2}$

$\therefore \quad |z| > \frac{1}{4}$ and $|z| < 1$



After partial fraction

$$X(z) = \frac{A}{z-\frac{1}{4}} + \frac{B}{z+1}$$

$$X(z) = \frac{\frac{4K}{5}}{z-\frac{1}{4}} - \frac{\frac{4K}{5}}{z-(-1)}$$

Rightsided — left sided.

Apply inverse z transform —

$$x[n] = \frac{4K}{5}\left[\left(\frac{1}{4}\right)^n u(n) + (-1)^n u[-n-1]\right]$$

Given $x[1] = 1$

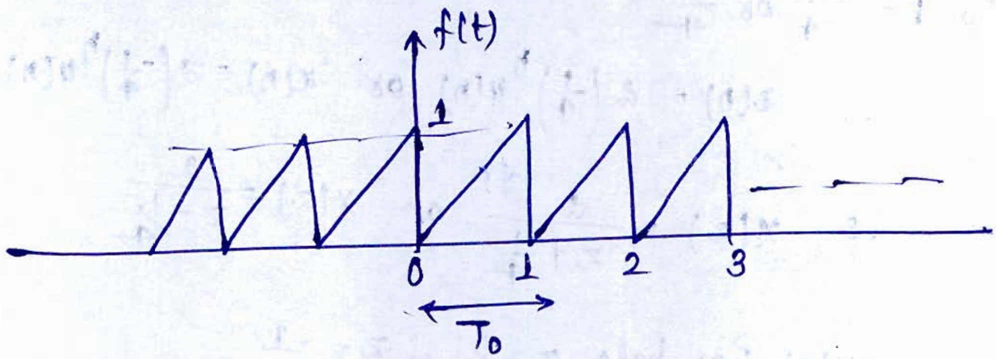
$$1 = \frac{4K}{5}\left[\left(\frac{1}{4}\right) u(1) + (-1)^1 u(-1-1)\right]$$

$$1 = \frac{4K}{5} \times \frac{1}{4} \implies K = 5$$

then $$\boxed{x(n) = 4\left[\left(\frac{1}{4}\right)^n u(n) + (-1)^n u(-n-1)\right]}$$

10

(ii)



Timeperiod, $T_0 = 1$ $\omega_0 = \frac{2\pi}{T_0}$

$\boxed{\omega_0 = 2\pi \text{ rad/sec}}$

Exponential fourier series Representation :

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega n t} \quad \text{——— (1)}$$

where $C_n = \frac{1}{T}\int_T f(t)\, e^{-j\omega n t}\, dt$

$$C_n = \frac{1}{1}\int_0^1 \underset{I}{t}\, \underset{II}{e^{-j\omega n t}}\, dt$$

$$C_n = t \cdot \frac{e^{-j\omega n t}}{-j\omega n} - \int 1 \cdot \frac{e^{-j\omega n t}}{-j\omega n}\, dt$$

$$C_n = \left[\frac{t\, e^{-j\omega n t}}{-j\omega n} + \frac{1}{j\omega n} \cdot \frac{e^{-j\omega n t}}{-j\omega n}\right]_0^1$$

$$C_n = \left(\frac{e^{-j\omega n}}{-j\omega n} + \frac{e^{-j\omega n}}{\omega^2 n^2}\right) - \frac{1}{\omega^2 n^2}$$

$\omega = 2\pi$

$$C_n = \frac{e^{-j2\pi n}}{-j2\pi n} + \frac{e^{-j2\pi n}}{4\pi^2 n^2} - \frac{1}{4\pi^2 n^2}$$

$$e^{-j2\pi n} = \cos 2\pi n - j\sin 2\pi n$$

($\cos 2\pi n \to 1$, $\sin 2\pi n \to 0$)

$= 1$

$$C_n = \frac{1}{-j2\pi n} + \frac{1}{4\pi^2 n^2} - \frac{1}{4\pi^2 n^2}$$

$$\boxed{C_n = \frac{j}{2\pi n}}$$

½

put this value in eq (1)

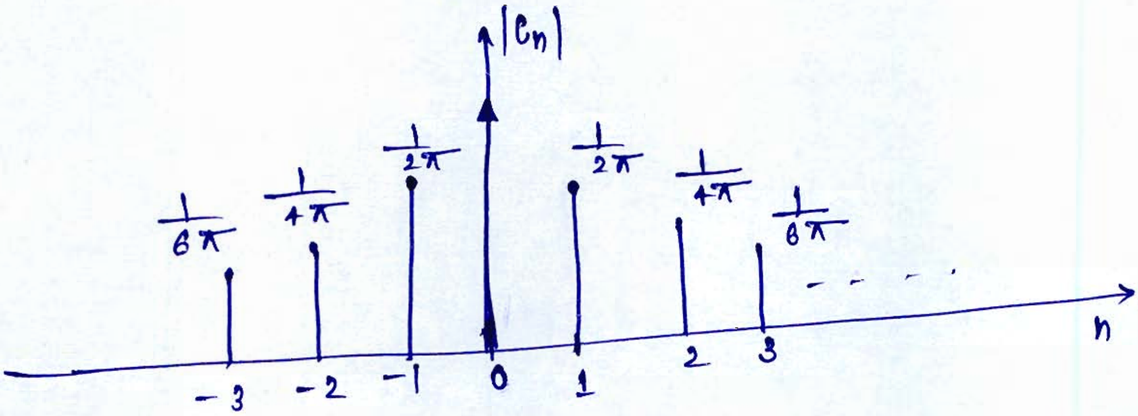
8

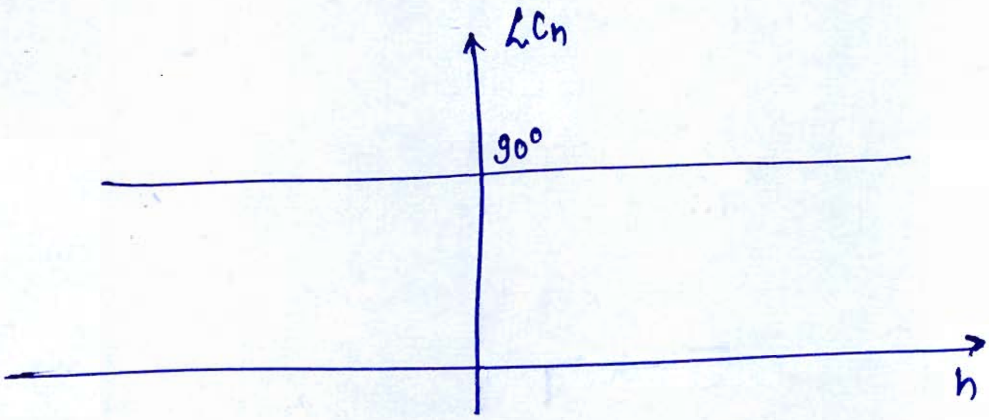
$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{j}{2\pi n} e^{j\omega_0 n t}}$$

(2)

$$C_n = \frac{j}{2\pi n}$$

$$C_n = \left|\frac{1}{2\pi n}\right| \angle 90^\circ$$





Q.7 (b) (i) Suppose we are given the following facts about an LTI system S with impulse response $h[n]$ and frequency response $H(e^{j\omega})$:

1. $\left(\frac{1}{4}\right)^n u[n] \xrightarrow{S} g[n]$, where $g[n] = 0$ for $n \geq 2$ and $n < 0$.
2. $\text{Re}\{H(e^{j\pi/2})\} = 1$.
3. $H(e^{j\omega}) = H(e^{j(\omega - \pi)})$.

Determine $h[n]$.

(ii) 1. Mention any five properties of ROC of Laplace transform.

2. Find inverse LT of $H(s) = \ln\left(\frac{1}{3s+2}\right)$.

[12 + 8 marks]

From Eq. ① $g(n) = 0$ for $n \geq 2$ & $n < 0$

then $g(1)$ & $g(0)$

ii (ii) ① Properties of ROC of laplace transform —

① ROC is always related with real part of pole.

② ROC of absolutetly integrable function contain jω axis in it.

③ For right sided signal ROC will be Right side to Right most pole in s-plane.

④ For left sided signal ROC will be left side to the left most pole.

⑤ For finite duration signal ROC will be complete s- plane except at $s = -\infty$ & $s = +\infty$.

(2) $$H(s) = \ln\left(\frac{1}{3s+2}\right)$$

Differentiate both side

$$\frac{dH(s)}{ds} = \frac{1}{\frac{1}{3s+2}} \cdot \frac{-1}{(3s+2)^2}$$

5

$$\frac{dH(s)}{ds} = \frac{-1}{3s+2}$$

$$-\frac{dH(s)}{ds} = \frac{1}{3s+2} \quad \text{--------- (1)}$$

As we know from property of frequency domain differentiation.

If $x(t) \xrightleftharpoons{L.S.} X(s)$

$t \cdot x(t) \rightleftharpoons -\frac{dX(s)}{ds}$

By this property From (1) take inverse laplace.

$$t\,h(t) = \frac{1}{3} \cdot e^{-2/3 t} u(t)$$

$$\boxed{h(t) = \frac{1}{3t} \cdot e^{-2/3 t} u(t)}$$

(c) (i) Consider the 6-length sequence defined for $0 \le n < 6$ as $x(n) = \{\underset{\uparrow}{1}, -2, 3, 0, -1, 1\}$ with a 8-point DFT $X(k)$. Evaluate the following functions of $X(k)$ without computing DFT:

1. $X(0)$
2. $X(3)$
3. $\sum_{k=0}^{5} X(k)$
4. $\sum_{k=0}^{5} |X(k)|^2$

(ii) Suppose the following facts are given about the signal $x(t)$ with Laplace transform $X(s)$:

1. $x(t)$ is real and even.
2. $X(s)$ has four poles and no zeros in the finite s-plane.
3. $X(s)$ has a pole at $s = \frac{1}{2} e^{j\frac{\pi}{4}}$.
4. $\int_{-\infty}^{\infty} x(t)dt = 4$

Determine $X(s)$.

[10 + 10 marks]

We know : $X(k) = \frac{1}{N} \sum_{n=0}^{N} x(n) e^{-j\omega nk}$ —— ①

1. Put k = 0 in eq ①

$$X(0) = \frac{1}{8} \sum_{n=0}^{7} x(n)$$

$$= \frac{1}{8} [+1 - 2 + 3 + 0 - 1 + 1]$$

$$= \frac{2}{8} = \frac{1}{4}$$

4. From Parsevals power theorem :

$$\sum_{k=0}^{5} |X(k)|^2 = \frac{1}{N} \sum_{n=0}^{N} |x(n)|^2$$

$$= \frac{1}{8} [1 + 4 + 9 + 0 + 1 + 1]$$

$$= \frac{16}{8}$$

$$= 2 \text{ unit}$$

(2) $X(3) = \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-3j\omega n}$

where $\omega = \frac{2\pi}{N} = \frac{\pi}{4}$

$$= \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-\frac{3jn\pi}{4}}$$

$$= \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-\frac{j3\pi n}{4}}$$

(3) we know $x(n) = \sum_{k=0}^{5} X(k)\, e^{j\omega nk}$

Put $n = 0$

$$\sum_{k=0}^{5} X(k) = x(0)$$

$$\boxed{\sum_{k=0}^{5} X(k) = 1}$$

(ii) ∵ $x(t)$ is real. So poles will be Conjugate pair

$P_1 = \frac{1}{2} e^{j\pi/4}$ then $P_2 = P_1^* = \frac{1}{2} e^{-j\pi/4}$

∵ $x(t)$ is even so poles will be reciprocal to each other.

$P_3 = \frac{1}{P_1}$, $P_4 = \frac{1}{P_2}$

$P_3 = 2e^{-j\pi/4}$ $P_4 = 2e^{j\pi/4}$

$$\int_{-\infty}^{\infty} x(t)\, dt = 4$$

Im: $X(s) = \int_{-\infty}^{\infty} e^{-st} x(t)\, dt$

Put $s = 0$

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt = 4 \quad \text{———— (1)}$$

Let

$$X(s) = \frac{K}{(s-P_1)(s-P_2)(s-P_3)(s-P_4)}$$

$$X(s) = \frac{K}{\left(s-\frac{1}{2}e^{j\pi/4}\right)\left(s-\frac{1}{2}e^{-j\pi/4}\right)\left(s-2e^{j\pi/4}\right)\left(s-2e^{-j\pi/4}\right)}$$

$$X(s) = \frac{K}{\left[s^2-\frac{1}{2}\left(e^{j\pi/4}+e^{-j\pi/4}\right)s+\frac{1}{4}\right]\cdot\left[s^2-2\left(e^{j\pi/4}+e^{-j\pi/4}\right)s+4\right]}$$

$$X(s) = \frac{K}{\left(s^2-\cos\pi/4\, s+\frac{1}{4}\right)\left(s^2-4\cos\pi/4\, s+4\right)}$$

$$X(s) = \frac{K}{\left(s^2-\frac{1}{\sqrt{2}}s+\frac{1}{4}\right)\left(s^2-\frac{4}{\sqrt{2}}+4\right)}$$

10

From Eq (1)

$$4 = \frac{K}{\frac{1}{4}\times 4} \Rightarrow K = 4$$

then

$$\boxed{X(s) = \frac{4}{\left(s^2-\frac{1}{\sqrt{2}}s+\frac{1}{4}\right)\left(s^2-\frac{4}{\sqrt{2}}+4\right)}}$$

Do not write in this margin

Q.8 (a) Using DIT FFT and inverse DIT FFT, determine the output of the system if input $x(n)$ and impulse response $h(n)$ are given as $x(n) = \{2, 2, 4\}$ and $h(n) = \{1, 1\}$.

[20 marks]

Do not write in this margin

(b) A discrete time system is described by the difference equation:

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + x(n-1)$$

with $y(-1) = 0$ and $y(-2) = -1$.

Find:

(i) The zero input response of the system.

(ii) The zero state response of the system due to step input $u(n)$.

[10 + 10 marks]

(i) Zero Input Response $x(n) = 0$

$y(-1) = 0 \quad y(-2) = -1$

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 0$$

Apply z-transform

$$Y(z) - \frac{3}{4}\left[z^{-1}Y(z) + z^{-1+1}y(-1)\right] + \frac{1}{8}\left[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right] = 0$$

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) - \frac{1}{8} = 0$$

$$Y(z) = \frac{\frac{1}{8}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$= \frac{\frac{1}{8}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)}$$

$$Y(z) = \frac{A}{1 - \frac{1}{4}z^{-1}} + \frac{B}{1 - \frac{1}{2}z^{-1}}$$

$$Y(z) = \frac{-\frac{1}{8}}{1 - \frac{1}{4}z^{-1}} + \frac{\frac{1}{4}}{1 - \frac{1}{2}z^{-1}}$$

Inverse z-transform

$$y_{ZIR}(n) = -\frac{1}{8}\left(\frac{1}{4}\right)^n u(n) + \frac{1}{4}\left(\frac{1}{2}\right)^n u(n)$$

(18) (20) Good

(ii) Zero state Response Initial cond = 0

$$x(n) = u(n)$$

z-transform: $X(z) = \frac{1}{1-z^{-1}}$

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + x(n-1)$$

Apply z-transform.

$$Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X(z) + z^{-1}X(z)$$

$$Y(z) = \frac{X(z)\left[1+z^{-1}\right]}{\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right]}$$

$$Y(z) = \frac{(1+z^{-1})}{(1-z^{-1})\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$Y(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}} + \frac{C}{1-z^{-1}}$$

$$A = \frac{1+2}{(1-2)\left(1-\frac{1}{4}\times 2\right)} = \frac{3}{(-1)\left(\frac{1}{2}\right)} = -6$$

$$B = \frac{1+4}{(1-4)\left(1-\frac{1}{2}\times 4\right)} = \frac{5}{(-3)(-1)} = \frac{5}{3}$$

$$C = \frac{2}{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)} = \frac{2}{\frac{1}{2}\times\frac{3}{4}} = \frac{16}{3}$$

$$Y(z) = \frac{-6}{1-\frac{1}{2}z^{-1}} + \frac{5/3}{1-\frac{1}{4}z^{-1}} + \frac{\frac{16}{3}}{1-z^{-1}}$$

Apply Inverse z-transform.

$$y_{ZSR}(n) = -6\left(\frac{1}{2}\right)^n u(n) + \frac{5}{3}\left(\frac{1}{4}\right)^n u(n) + \frac{16}{3}u(n)$$

Do not write in this margin

(c) Obtain the direct form-I and direct form-II realizations of the LTI system governed by the equation

$$y(n) = -\frac{13}{12}y(n-1) - \frac{9}{24}y(n-2) - \frac{1}{24}y(n-3) + x(n) + 4x(n-1) + 3x(n-2)$$

[20 marks]

Apply z-transform both side.

$$Y(z) = \left[-\frac{13}{12}z^{-1} - \frac{9}{24}z^{-2} - \frac{1}{24}z^{-3}\right]Y(z) + \left(1 + 4z^{-1} + 3z^{-2}\right)X(z)$$

$$Y(z)\left[1 - \left\{-\frac{13}{12}z^{-1} - \frac{9}{24}z^{-2} - \frac{1}{24}z^{-3}\right\}\right] = \left(1 + 4z^{-1} + 3z^{-2}\right)X(z)$$

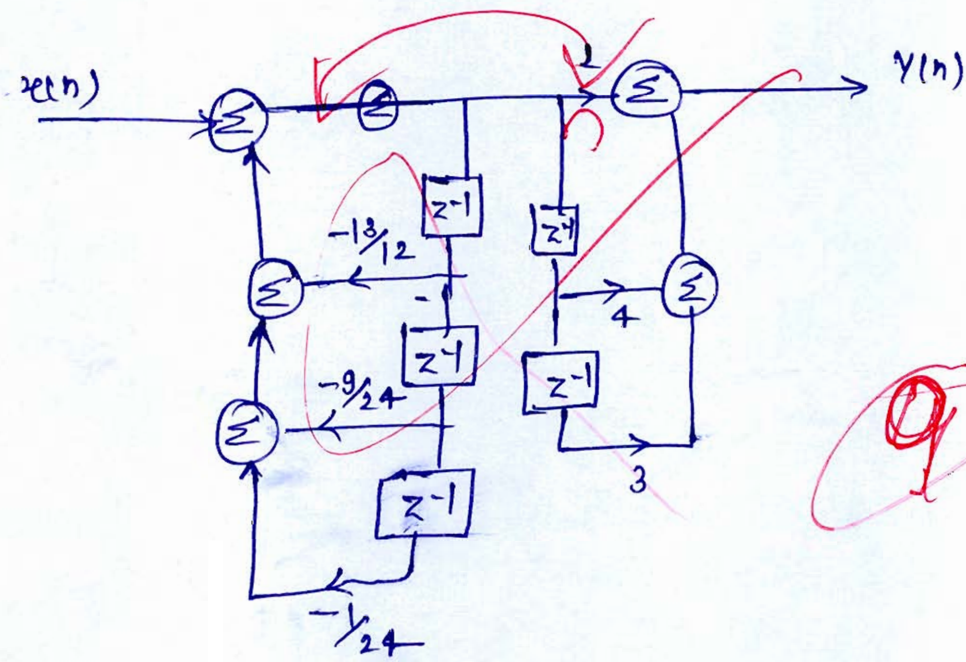
$$\frac{Y(z)}{X(z)} = \frac{1 + 4z^{-1} + 3z^{-2}}{1 - \left\{-\frac{13}{12}z^{-1} - \frac{9}{24}z^{-2} - \frac{1}{24}z^{-3}\right\}}$$

4

Direct form ①: Total delay block = order of Numerator + order of Denominator

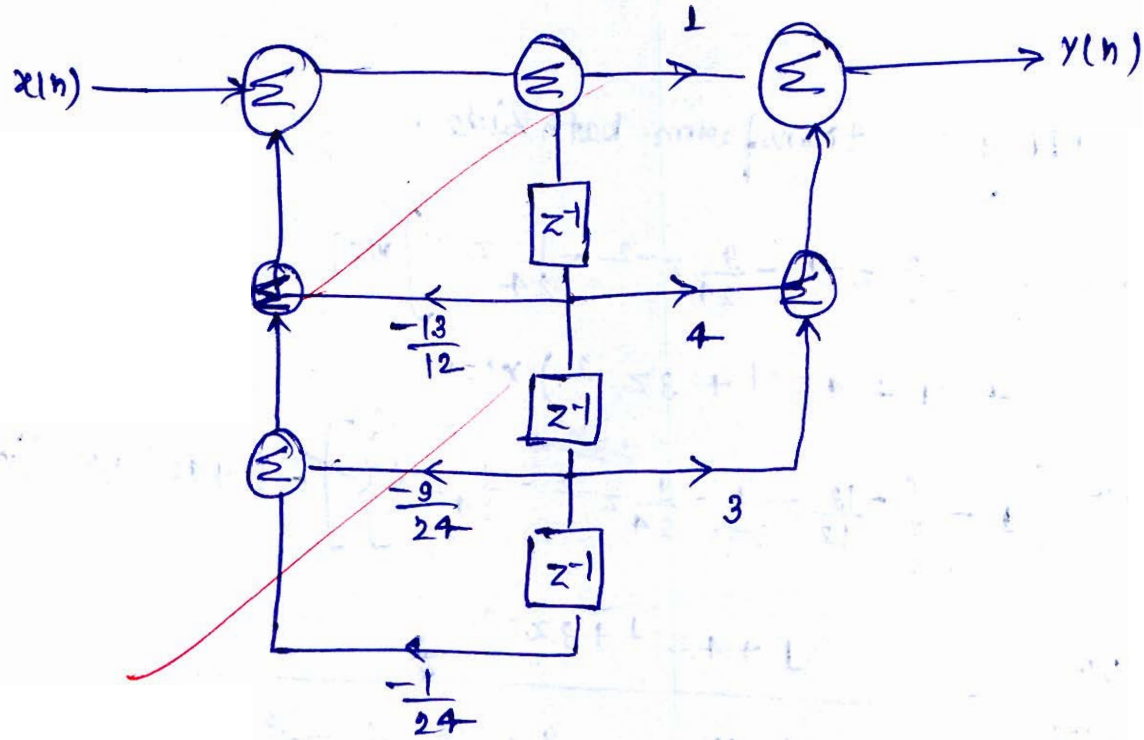
= 3+2

= 5



9

direct form II :



OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work

$$I_C = C\frac{dv_C}{dt}$$

$$v_C = \frac{1}{C}\int I\,dt$$

$$\frac{k}{\frac{1}{4}+}$$

$$\frac{4k}{5}$$

$$\frac{k}{-1-\frac{1}{4}} = -\frac{4k}{5}$$

$$\frac{\frac{1}{8}}{1-\frac{1}{2}\times 4^2}$$

$$\frac{\frac{1}{8}}{1-\frac{1}{4}\times 2} = \frac{\frac{1}{8}}{\frac{1}{2}}$$

Space for Rough Work